Title: Conditional entanglement of purification

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Abstract: <p>We study the conjectured holographic duality between entanglement of purification and the entanglement wedge cross-section. We generalize both quantities and prove several information theoretic inequalities involving them. These include upper bounds on conditional mutual information and tripartite information, as well as a lower bound for tripartite information. These inequalities are proven both holographically and for general quantum states. In addition, we use the cyclic entropy inequalities to derive a new holographic inequality for the entanglement wedge cross-section, and provide numerical evidence that the corresponding inequality for the entanglement of purification may be true in general. Finally, we use intuition from bit threads to extend the conjecture to holographic duals of suboptimal purifications.</p>

Conditional Entanglement of Purification

1710.07643, 1709.07424, 1708.09393

Entanglement: the quantum or classical correlation between two subsystems.

For a pure state, the entanglement entropy quantifies the amt of ent. between a subsystem of the pure state and the rest of the pure state

$$\rho_A, \quad \rho_A = \mathrm{tr}_B |\psi\rangle\langle\psi|_{AB}, \quad S(\rho_A) = -\mathrm{tr}\, \rho_A \log \rho_A .$$

$$\rho_A, \quad \rho_A = \mathrm{tr}_B\, \rho_{AB}, \quad I(A:B) = S(A) + S(B) - S(AB),$$

Ent. entropy is known to be inv. under LOCC.

For all quantum states

$$S(A) \geq 0$$

$$S(A) + S(B) - S(AB) \geq 0$$

$$S(AB) + S(BC) - S(B) - S(ABC) \geq 0$$

Purification

Purification

$\rho_{AB} \rightarrow |\psi\rangle\langle\psi|_{ABC}$, as long as $\dim \mathcal{H}_C \geq 2^{S(AB)}$.

$S(A)$ is independent of your choice of purification!

What is the "minimal" purification of ρ_{AB}

Ent. of purification (Terhal et al. 2002)

$$E_p(A:B) = \inf_{A',B'} S(AA'), \text{ when } \rho_{AB} = \mathrm{tr}_{A'B'} |\psi\rangle\langle\psi|_{AA'BB'}, \ \mathcal{H}_{AA'} \otimes \mathcal{H}_{BB'}$$

observation

For pure ρ_{AB}, $E_p(A:B)=S(A)=S(B)$.

For product $\rho_{AB}=\rho_A\otimes\rho_B$, $E_p(A:B)=0$.

For $\rho_{AB}=\sum p_i |i\rangle\langle i|_A\otimes|i\rangle\langle i|_B$, $E_p(A:B)=-p_i\log p_i \equiv$ Shannon entropy

1. $0\le E_p(A:B)\le \min(S(A),S(B))$
2. $E_p(A:BC)\ge E_p(A:B)$
3. $E_p(A:B)\ge \frac{I(A:B)}{2}$
4. $E_p(A:BC)\ge \frac{I(A:B)}{2}+\frac{I(A:C)}{2}$
5. For pure ρ_{ABC}, $E_p(A:BC)\ge E_p(A:B)+E_p(A:C)$ "polygamous"

3. For pure ρ_{ABC}, $E_p(A:BC) \geq E_p($ …

1. standard purification $\Rightarrow S(AA') = S(A)$ or $S(B)$.

2. For pure ρ_{ABC} $\quad E_p(A:BC) = S(A) \geq E_p(A:B)$ ✓

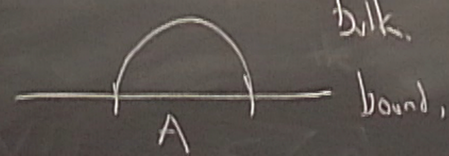
mixed " " constrained opt on the LHS, less cons. opt on the RHS

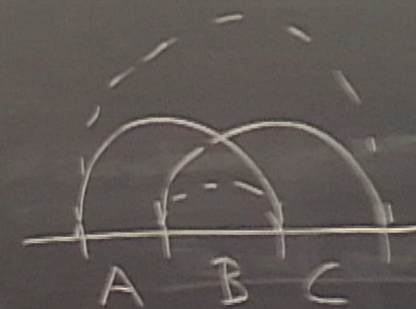
3. $S(AA'|AB') \leq S(A'|A) + S(B'|B)$.

$$\underbrace{S(AA'BB')}_{0} - S(AB) \leq \underbrace{S(AA')}_{E_p} - S(A) + \underbrace{S(BB')}_{E_p} - S(B)$$

Holography

$$S(A) = \frac{\text{Area}(A)}{4G}$$





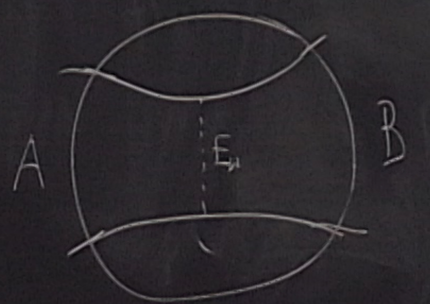
Converts hard lin. alg. problems into geometric questions.

holo. states:
$$S(AB)+S(AC)+S(BC) \geq S(A)+S(B)+S(C)+S(ABC)$$
$$S(ABC)+S(BCD)+\ldots S(EAB) \geq S(AB)+S(BC)+\ldots S(EA)+S(ABCDE)$$
$$\downarrow$$

HPPS, constraints on theories, entropy consts constrain states.

A recent conjecture by Tak. et al., Swingle, Zaletel, et al.

$\exists$ A holo. description of E_P, E_W, the entanglement wedge cross section.

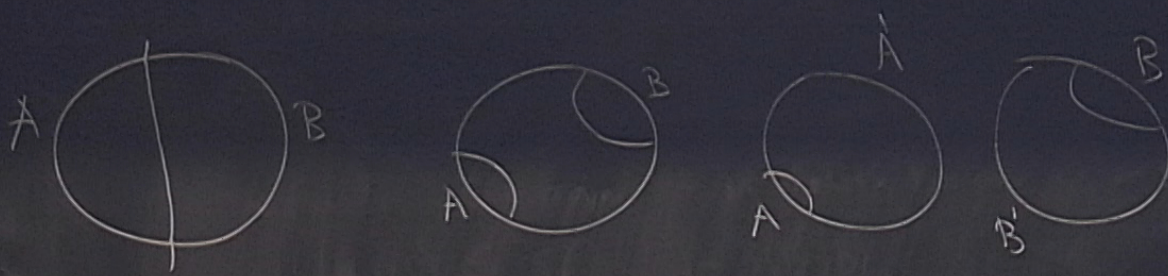


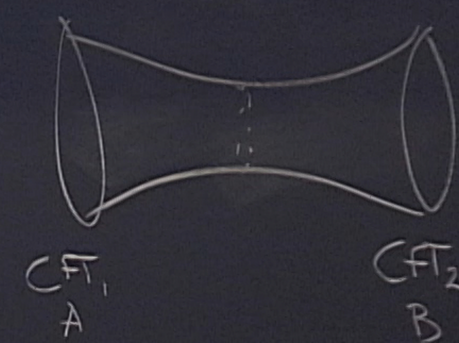
$E_W = \inf_{\text{partitions}}$ that separate A from B in the ent. wedge.

E_W obeys all the same obs. and ineqs that E_P is known to obey.

4. $E_p(A{:}BC) \geq \frac{I(A{:}B)}{2} + \frac{I(A{:}C)}{2}$

5. For pure ρ_{ABC}, $E_p(A{:}BC) \geq E_p(A{:}B) + E_p(A{:}C)$ "polygamous"



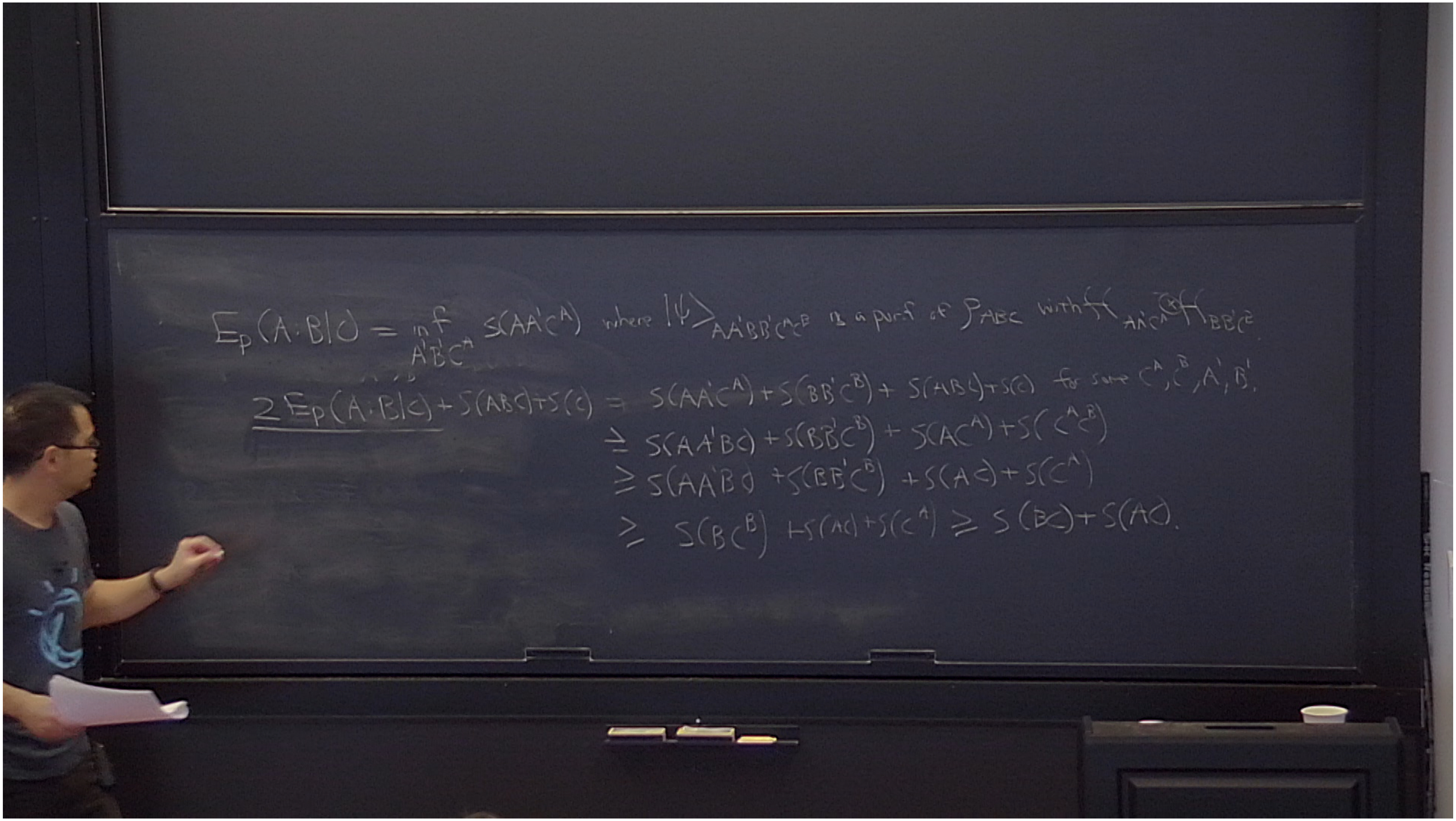


$$E_W(A{:}BC) \geq \frac{1}{2} I(A{:}BC) \geq \frac{1}{2} I(A{:}B) + \frac{1}{2} I(A{:}C) \geq 0$$

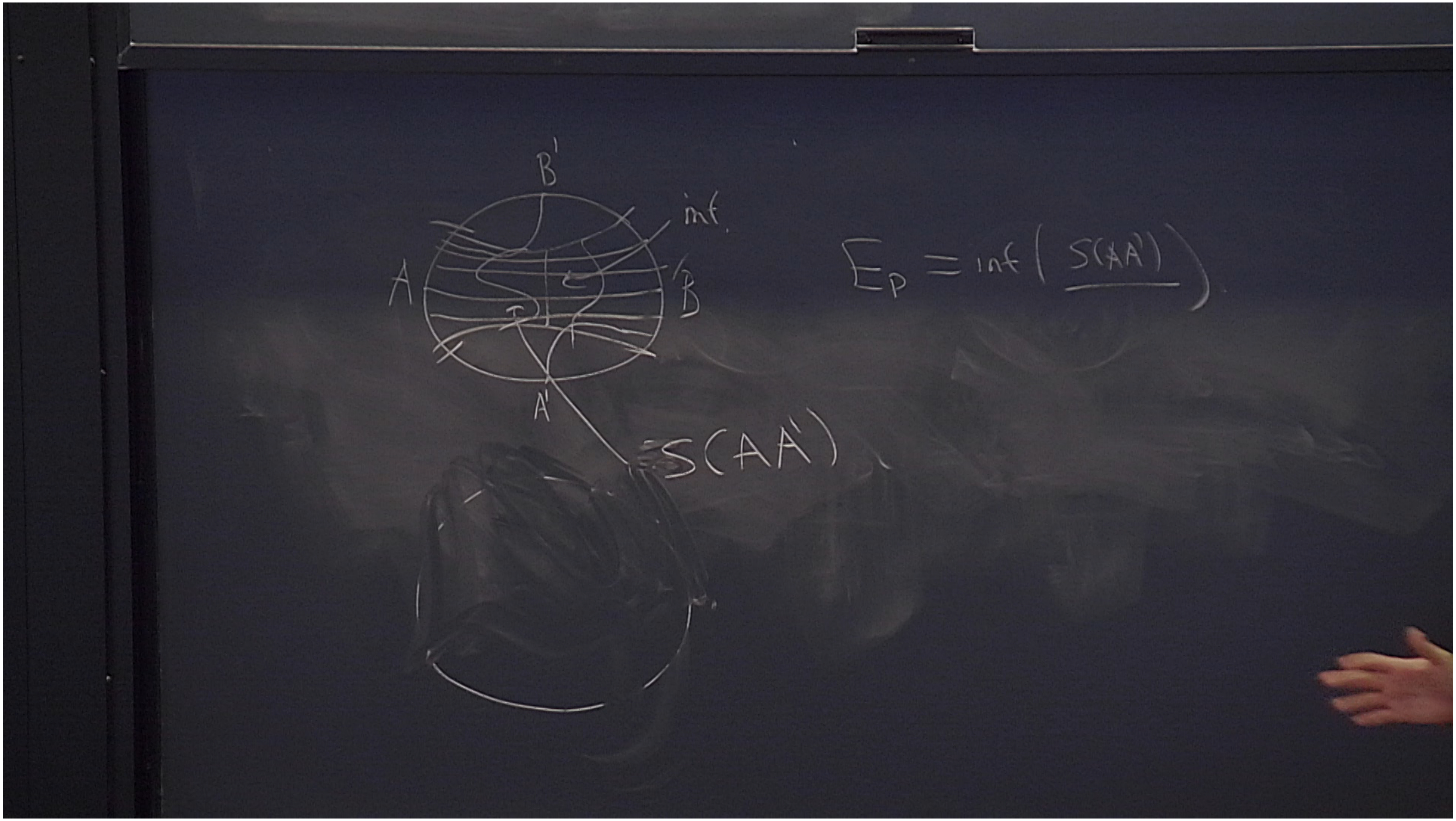
$E_W(A:B|C)$ is the min partitioning of the Ent Wedge of ABC / Ent Wedge of C. into regions adjacent to A or B.

$$f_1(E_W(A:B|C)) \geq I(A:B:C) \geq 0$$

$$f_2(E_W(A:B|C)) \geq C_K(\ \cdot\ ) \geq 0$$



$E_p(A:B|C) = \inf_{A'B'C^A} S(AA'C^A)$ where $|\psi\rangle_{AA'BB'C^AC^B}$ is a purif of ρ_{ABC} with $\mathcal{H}_{A'C^A} \otimes \mathcal{H}_{B'C^B}$.
$\underline{2E_p(A:B|C)} + S(ABC) + S(C) = S(AA'C^A) + S(BB'C^B) + S(ABC) + S(C)$ for some C^A, C^B, A', B'.
$\geq S(AA'BC) + S(BB'C^B) + S(AC^A) + S(C^AC^B)$
$\geq S(AA'BC) + S(BB'C^B) + S(AC) + S(C^A)$
$\geq S(BC^B) + S(AC) + S(C^A) \geq S(BC) + S(AC)$.


B'
inf
A
B
A'
S(AA')
E_P = inf (S(AA'))