Title: Two-body problem in modified gravities and EOB theory

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Abstract: <p>In general relativity, the effective-one-body (EOB) approach, which consists in reducing the two-body dynamics to the motion of a test particle in an effective static, spherically symmetric metric, has proven to be a very powerful framework to describe analytically the coalescence of compact binary systems.</p>

<p>In this seminar, we address its extension to modified gravities, considering first the example of massless scalar-tensor theories (ST). We reduce the ST two-body dynamics, which is known at second post-Keplerian order, to a simple parametrized deformation of the general relativistic EOB Hamiltonian, and estimate the ST corrections to the strong-field regime; in particular, the ISCO location and orbital frequency.</p>

<p>We then discuss the class of Einstein-Maxwell-dilaton (EMD) theories, which provide simple examples of "hairy" black holes. We compute the EMD post-Keplerian two-body Lagrangian, and show that it can, as well, be incorporated within the EOB framework. Finally, we highlight that, depending on their scalar environment, EMD black holes can transition to a regime where they strongly couple to the scalar and vector fields, inducing large deviations from the general relativistic two-body dynamics.</p>

Two-body problem in modified gravities and effective-one-body theory

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Based on :

[Phys. Rev. D**95** 124054] FLJ, N.Deruelle
[JCAP **1801** 026] FLJ

Motivations

- GW150914 : first observation of a BBH coalescence by LIGO-Virgo
- GW170817 : first BNS with EM counterparts (multimessenger astronomy)

$\rightarrow$ **new era in gravitational wave astronomy.**

Opportunity to bring **new tests of modified gravities**, in the strong-field regime near merger, a topic which is for the moment still in infancy.

Motivations

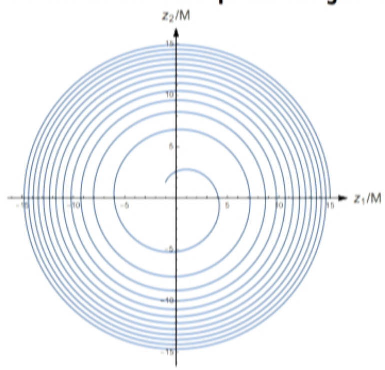
In general relativity, "effective-one-body" (EOB) :

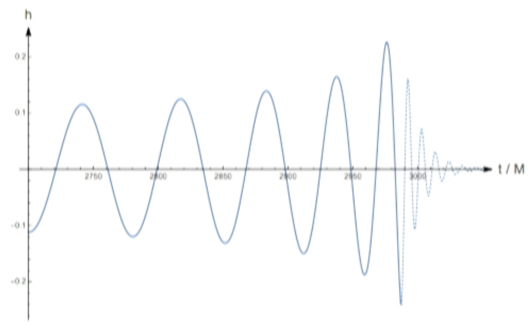
• Map the two-body PN dynamics to the motion of a **test particle** in an **effective SSS metric** [Buonanno-Damour 98]

$$H(Q,P)\ ,\quad \epsilon=\left(\frac{v}{c}\right)^2 \qquad \longrightarrow \qquad H_e(q,p)\ ,\quad ds_e^2=g^e_{\mu\nu}dx^\mu dx^\nu$$

$$\boxed{H_e = f_{\rm EOB}(H)}$$

• Defines a resummation of the PN dynamics, hence describes **analytically** the coalescence of 2 compact objects in **general relativity**, from inspiral **to merger**.





• Instrumental to build libraries of waveform templates for LIGO-Virgo

Motivations

[Phys. Rev. D 95 124054 (2017)] FLJ - N.Deruelle

- Can we extend the EOB approach to modified gravities ?
- Consider the simplest and most studied example of **massless scalar-tensor theories (ST)**.

- First buiding block : map the conservative part of the two-body dynamics onto the geodesic of an effective metric.

- ST-extension of [Buonanno-Damour 98]

Scalar-Tensor theories

We adopt the conventions of Damour and Esposito-Farèse [DEF 92, 95]

ST action in the Einstein-frame ($G_* \equiv c \equiv 1$)

$$S_{EF} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(R - 2g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right) + S_m \left[\Psi, \mathcal{A}^2(\varphi) g_{\mu\nu} \right]$$

- Einstein metric $g_{\mu\nu}$ free dynamics : Einstein-Hilbert term ; ordinary kinematical term for φ ;
- **BUT** matter Ψ is minimally coupled to the Jordan metric $\tilde{g}_{\mu\nu}$:

$$\boxed{\tilde{g}_{\mu\nu} \equiv \mathcal{A}^2(\varphi) g_{\mu\nu}}$$

where $\mathcal{A}(\varphi)$ **defines** the ST theory (GR : $\mathcal{A}(\varphi) = cst$).

- Encompass the Einstein Equivalence Principle

Scalar-Tensor theories

what about S_m ?

N-body problem in Scalar-Tensor theories

Phenomenological approach : Skeletonize extended bodies as point particles

- Negligible self-gravity

$$\boxed{S_m = -\sum_A \int d\lambda \sqrt{-\tilde{g}_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}\, \tilde{m}_A}$$

i.e. particles follow **geodesics** of $\tilde{g}_{\mu\nu}$ (weak equivalence principle).

- When self-gravity is not negligible (neutron stars, black holes),

$$\boxed{S_m = -\sum_A \int d\lambda \sqrt{-\tilde{g}_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}\, \tilde{m}_A(\varphi)}$$

$\tilde{m}_A(\varphi)$ is a function of the local value of φ to encompass the effect of the background scalar field on the equilibrium of a body. [Eardley 75, DEF 92]
$\tilde{m}_A(\varphi)$ depends on the theory $\mathcal{A}(\varphi)$ and on the EOS of body A.

$\longrightarrow$ strong equivalence principle violation

The two-body Lagrangian

Our starting point : what is known today

Two-body Scalar-Tensor Lagrangian [DEF 93][Mirshekari, Will 13]

- Harmonic coordinates $\partial_\mu(\sqrt{-g}g^{\mu\nu}) = 0$
- conservative 2PK dynamics : $\mathcal{O}((\frac{v}{c})^4) \sim \mathcal{O}((\frac{G_* m}{r})^2)$ corrections to Kepler (to be compared with GR)
- Weak field expansion

$$\boxed{\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + \delta g_{\mu\nu} \\ \varphi &= \varphi_0 + \delta\varphi \end{aligned}}$$

- the fundamental functions $m_A(\varphi)$ and $m_B(\varphi)$ are expanded around φ_0 :

$$\boxed{\begin{aligned} \ln m_A(\varphi) &\equiv \ln m_A^0 + \alpha_A^0(\varphi - \varphi_0) + \frac{1}{2}\beta_A^0(\varphi-\varphi_0)^2 + \frac{1}{6}{\beta'_A}^0(\varphi-\varphi_0)^3 + \cdots \\ \ln m_B(\varphi) &\equiv \ln m_B^0 + \alpha_B^0(\varphi - \varphi_0) + \frac{1}{2}\beta_B^0(\varphi-\varphi_0)^2 + \frac{1}{6}{\beta'_B}^0(\varphi-\varphi_0)^3 + \cdots \end{aligned}}$$

i.e. the 2PK Lagrangian depends on 8 fundamental **parameters**.

The two-body Lagrangian

Two-body 2PK Lagrangian

$$L = -m_A^0 - m_B^0 + L_{\mathrm{K}} + L_{1\mathrm{PK}} + L_{2\mathrm{PK}} + \cdots$$

$$\vec{N} \equiv \frac{\vec{Z}_A - \vec{Z}_B}{R}, \quad \vec{V}_A \equiv \frac{d\vec{Z}_A}{dt}, \quad R \equiv |\, \vec{Z}_A - \vec{Z}_B \,|, \quad \vec{A}_A \equiv \frac{d\vec{V}_A}{dt}$$

- Keplerian order :

$$\boxed{L_{\mathrm{K}} = \frac{1}{2} m_A^0 V_A^2 + \frac{1}{2} m_B^0 V_B^2 + \frac{G_{AB} m_A^0 m_B^0}{R}} \qquad \text{where} \qquad G_{AB} \equiv 1 + \alpha_A^0 \alpha_B^0$$

- post-Keplerian (1PK) :

$$\boxed{\begin{aligned} L_{1\mathrm{PK}} &= \frac{1}{8} m_A^0 V_A^4 + \frac{1}{8} m_B^0 V_B^4 \\ &+ \frac{G_{AB} m_A^0 m_B^0}{R} \left(\frac{3}{2}(V_A^2 + V_B^2) - \frac{7}{2} \vec{V}_A \cdot \vec{V}_B - \frac{1}{2} (\vec{N} \cdot \vec{V}_A)(\vec{N} \cdot \vec{V}_B) + \bar{\gamma}_{AB} (\vec{V}_A - \vec{V}_B)^2 \right) \\ &- \frac{G_{AB}^2 m_A^0 m_B^0}{2R^2} \left(m_A^0 (1 + 2\bar{\beta}_B) + m_B^0 (1 + 2\bar{\beta}_A) \right) \end{aligned}}$$

$$\text{where} \quad \bar{\gamma}_{AB} \equiv -\frac{2\alpha_A^0 \alpha_B^0}{1+\alpha_A^0\alpha_B^0} \qquad \bar{\beta}_A \equiv \frac{1}{2} \frac{\beta_A^0 (\alpha_B^0)^2}{(1+\alpha_A^0\alpha_B^0)^2} \; (A \leftrightarrow B)$$

The two-body Lagrangian

- post-post-Keplerian (2PK) :

$$
\begin{aligned}
L_{\mathrm{2PK}} &= \frac{1}{16} m_A^0 V_A^6 \\
&+ \frac{G_{AB} m_A^0 m_B^0}{R} \Bigg[\frac{1}{8}(7+4\bar{\gamma}_{AB})\left(V_A^4 - V_A^2(\vec{N}\cdot\vec{V}_B)^2\right) - (2+\bar{\gamma}_{AB})V_A^2(\vec{V}_A\cdot\vec{V}_B) + \frac{1}{8}(\vec{V}_A\cdot\vec{V}_B)^2 \\
&\qquad + \frac{1}{16}(15+8\bar{\gamma}_{AB})V_A^2 V_B^2 + \frac{3}{16}(\vec{N}\cdot\vec{V}_A)^2(\vec{N}\cdot\vec{V}_B)^2 + \frac{1}{4}(3+2\bar{\gamma}_{AB})\vec{V}_A\cdot\vec{V}_B(\vec{N}\cdot\vec{V}_A)(\vec{N}\cdot\vec{V}_B)\Bigg] \\
&+ \frac{G_{AB}^2 m_B^0 (m_A^0)^2}{R^2} \Bigg[\frac{1}{8}\left(2+12\bar{\gamma}_{AB}+7\bar{\gamma}_{AB}^2+8\bar{\beta}_B-4\delta_A\right)V_A^2 + \frac{1}{8}\left(14+20\bar{\gamma}_{AB}+7\bar{\gamma}_{AB}^2+4\bar{\beta}_B-4\delta_A\right)V_B^2 \\
&\qquad - \frac{1}{4}\left(7+16\bar{\gamma}_{AB}+7\bar{\gamma}_{AB}^2+4\bar{\beta}_B-4\delta_A\right)\vec{V}_A\cdot\vec{V}_B - \frac{1}{4}\left(14+12\bar{\gamma}_{AB}+\bar{\gamma}_{AB}^2-8\bar{\beta}_B+4\delta_A\right)(\vec{V}_A\cdot\vec{N})(\vec{V}_B\cdot\vec{N}) \\
&\qquad + \frac{1}{8}\left(28+20\bar{\gamma}_{AB}+\bar{\gamma}_{AB}^2-8\bar{\beta}_B+4\delta_A\right)(\vec{N}\cdot\vec{V}_A)^2 + \frac{1}{8}\left(4+4\bar{\gamma}_{AB}+\bar{\gamma}_{AB}^2+4\delta_A\right)(\vec{N}\cdot\vec{V}_B)^2\Bigg] \\
&+ \frac{G_{AB}^3 (m_A^0)^3 m_B^0}{2R^3}\left[1+\frac{2}{3}\bar{\gamma}_{AB}+\frac{1}{6}\bar{\gamma}_{AB}^2+2\bar{\beta}_B+\frac{2}{3}\delta_A+\frac{1}{3}\epsilon_B\right] + \frac{G_{AB}^3 (m_A^0)^2 (m_B^0)^2}{8R^3}\left[19+8\bar{\gamma}_{AB}+8(\bar{\beta}_A+\bar{\beta}_B)+4\zeta\right] \\
&- \frac{1}{8} G_{AB} m_A^0 m_B^0 \left(2(7+4\bar{\gamma}_{AB})\vec{A}_A\cdot\vec{V}_B(\vec{N}\cdot\vec{V}_B) + \vec{N}\cdot\vec{A}_A(\vec{N}\cdot\vec{V}_B)^2 - (7+4\bar{\gamma}_{AB})\vec{N}\cdot\vec{A}_A V_B^2\right) \\
&\qquad + (A\leftrightarrow B)
\end{aligned}
$$

where $\delta_A \equiv \frac{(\alpha_A^0)^2}{(1+\alpha_A^0\alpha_B^0)^2}$ $\quad \epsilon_A \equiv \frac{(\beta'_A\alpha_B^3)^0}{(1+\alpha_A^0\alpha_B^0)^3}$ $\quad \zeta \equiv \frac{\beta_A^0\alpha_A^0\alpha_B^0\beta_B^0}{(1+\alpha_A^0\alpha_B^0)^3}$ $(A\leftrightarrow B)$

The centre-of-mass two-body 2PK Hamiltonians

In the centre-of-mass frame : $\boxed{\vec{P}_A + \vec{P}_B \equiv \vec{0}}$

17 coefficients (polar coordinates)

$$H = M + \left(\frac{P^2}{2\mu} - \mu\frac{G_{AB}M}{R}\right) + H^{1\mathrm{PK}} + H^{2\mathrm{PK}} + \cdots$$

- $\frac{H^{1\mathrm{PK}}}{\mu} = \left(h_1^{1\mathrm{PK}}\hat{P}^4 + h_2^{1\mathrm{PK}}\hat{P}^2\hat{P}_R^2 + h_3^{1\mathrm{PK}}\hat{P}_R^4\right) + \frac{1}{\hat{R}}\left(h_4^{1\mathrm{PK}}\hat{P}^2 + h_5^{1\mathrm{PK}}\hat{P}_R^2\right) + \frac{h_6^{1\mathrm{PK}}}{\hat{R}^2}$
- $\frac{H^{2\mathrm{PK}}}{\mu} = \left(h_1^{2\mathrm{PK}}\hat{P}^6 + h_2^{2\mathrm{PK}}\hat{P}^4\hat{P}_R^2 + h_3^{2\mathrm{PK}}\hat{P}^2\hat{P}_R^4 + h_4^{2\mathrm{PK}}\hat{P}_R^6\right)$
  $+ \frac{1}{\hat{R}}\left(h_5^{2\mathrm{PK}}\hat{P}^4 + h_6^{2\mathrm{PK}}\hat{P}_R^2\hat{P}^2 + h_7^{2\mathrm{PK}}\hat{P}_R^4\right) + \frac{1}{\hat{R}^2}\left(h_8^{2\mathrm{PK}}\hat{P}^2 + h_9^{2\mathrm{PK}}\hat{P}_R^2\right) + \frac{h_{10}^{2\mathrm{PK}}}{\hat{R}^3}$

The 17 $h_i^{N\mathrm{PK}}$ coefficients are computed explicitly and depend on :

- the coordinate system
- the 8 fundamental parameters built from $m_A(\varphi)$ and $m_B(\varphi)$

The effective Hamiltonian H_e

Geodesic motion in a static, spherically symmetric metric

In Schwarzschild-Droste coordinates (equatorial plane $\theta = \pi/2$) :

$$\boxed{ds_e^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\phi^2}$$

$A(r)$ and $B(r)$ are arbitrary.

Effective Hamiltonian $H_e(q,p)$:

$$H_e(q,p) = \sqrt{A\left(\mu^2 + \frac{p_r^2}{B} + \frac{p_\phi^2}{r^2}\right)} \quad \text{with} \quad p_r \equiv \frac{\partial L_e}{\partial \dot{r}} \quad , \quad p_\phi \equiv \frac{\partial L_e}{\partial \dot{\phi}}$$

Can be **expanded** :

$$\boxed{\begin{aligned} A(r) &= 1 + \frac{a_1}{r} + \frac{a_2}{r^2} + \frac{a_3}{r^3} + \cdots \\ B(r) &= 1 + \frac{b_1}{r} + \frac{b_2}{r^2} + \cdots \end{aligned}}$$

i.e. depends on **5 effective parameters** at 2PK order, to be determined.

EOB mapping : [Buonanno, Damour 98]

1) Use of a canonical transformation :

$$H(Q,P) \to H(q,p)$$

Generic ansatz $G(Q,p)$ that depends on 9 parameters at 2PK order :

$$\boxed{G(Q,p) = R\,p_r \left[\left(\alpha_1 \mathcal{P}^2 + \beta_1 \hat{p}_r^2 + \frac{\gamma_1}{\hat{R}} \right) + \left(\alpha_2 \mathcal{P}^4 + \beta_2 \mathcal{P}^2 \hat{p}_r^2 + \gamma_2 \hat{p}_r^4 + \delta_2 \frac{\mathcal{P}^2}{\hat{R}} + \epsilon_2 \frac{\hat{p}_r^2}{\hat{R}} + \frac{\eta_2}{\hat{R}^2} \right) + \cdots \right]}$$

2) Relate H to H_e through the quadratic relation [Damour 2016]

$$\boxed{\frac{H_e(q,p)}{\mu} - 1 = \left(\frac{H(q,p) - M}{\mu} \right) \left[1 + \frac{\nu}{2} \left(\frac{H(q,p) - M}{\mu} \right) \right]}$$

where $\nu = \dfrac{m_A^0 m_B^0}{(m_A^0 + m_B^0)^2}$, $M = m_A^0 + m_B^0$, $\mu = \dfrac{m_A^0 m_B^0}{M}$

The EOB mapping

$$\frac{H_e(q,p)}{\mu} - 1 = \left(\frac{H(q,p)-M}{\mu}\right)\left[1+\frac{\nu}{2}\left(\frac{H(q,p)-M}{\mu}\right)\right]$$

- H_e depends on 5 parameters

$$A(r) = 1 + \frac{a_1}{r} + \frac{a_2}{r^2} + \frac{a_3}{r^3} + \cdots \ , \quad B(r) = 1 + \frac{b_1}{r} + \frac{b_2}{r^2} + \cdots$$

- H depends on 17 coefficients (h_i^{NPK}) ;
- The canonical transformation depends on 9 parameters $(\alpha_i,\ \beta_i, \ldots)$;

$$\boxed{17 = 9 + 5 + 3}$$

Hence, 3 constraints on the h_i^{NPK} coefficients of the two-body Hamiltonian.

$\rightarrow$ **The two-body problem can be mapped towards geodesic motion only for a subclass of theories**

The constraints

- At 1PK order, one constraint :

$$\boxed{2h_2^{\rm 1PK} + 3h_3^{\rm 1PK} = 0}$$

By **Lorentz invariance** of the kinematical terms $m_A^0\sqrt{1-V_A^2}$ at the Lagrangian level , $h_2^{\rm 1PK} = h_3^{\rm 1PK} = 0$ in ST theory.

- At 2PK order, two constraints : the first one

$$\boxed{h_4^{\rm 2PK} = -\frac{2}{45}\left(12h_2^{\rm 2PK} + 18h_3^{\rm 2PK} + (h_2^{\rm 1PK})^2\right)}$$

is no more restrictive.

The constraints

However, the second one

$$\boxed{\begin{aligned}&h_1^{\mathrm{2PK}} + \frac{7}{3}h_2^{\mathrm{2PK}} + h_3^{\mathrm{2PK}} + h_5^{\mathrm{2PK}} + h_6^{\mathrm{2PK}} + h_7^{\mathrm{2PK}} = \\ &- \frac{h^{\mathrm{K}}}{128}(5 + 2\nu + 5\nu^2) + \frac{1}{8}(1+\nu)\Big((3h_1^{\mathrm{1PK}} + h_2^{\mathrm{1PK}})h^{\mathrm{K}} + h_4^{\mathrm{1PK}} + h_5^{\mathrm{1PK}}\Big) + \frac{5}{2}h_1^{\mathrm{1PK}}\Big(7h_1^{\mathrm{1PK}}h^{\mathrm{K}} + 2(h_4^{\mathrm{1PK}} + h_5^{\mathrm{1PK}})\Big) \\ &+ \frac{1}{6}h_2^{\mathrm{1PK}}\Big(13h_2^{\mathrm{1PK}}h^{\mathrm{K}} + 10(h_4^{\mathrm{1PK}} + h_5^{\mathrm{1PK}})\Big) + \frac{35}{3}h_1^{\mathrm{1PK}}h_2^{\mathrm{1PK}}h^{\mathrm{K}}\ ,\end{aligned}}$$

is restrictive.

- satisfied by the scalar-tensor coefficients
- but not by electrodynamics.

The Scalar-Tensor effective metric

$$ds_e^2 = -A(r)dt + B(r)dr^2 + r^2 d\phi^2$$

Yields a **unique** solution in scalar-tensor theories (coordinate-independent)

Scalar-Tensor effective metric

$$A(r) = 1 - 2\left(\frac{G_{AB}M}{r}\right) + 2\left[\langle\bar{\beta}\rangle - \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \left[2\nu + \delta a_3^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^3 + \cdots$$

$$B(r) = 1 + 2\left[1 + \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right) + \left[2(2-3\nu) + \delta b_2^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \cdots$$

Reduces to GR when $m_A(\varphi) = cst$

General Relativity 2PN effective metric [Buonanno, Damour 98]

$$A_{\mathrm{GR}}(r) = 1 - 2\left(\frac{G_* M}{r}\right) + 2\nu\left(\frac{G_* M}{r}\right)^3 + \cdots$$

$$B_{\mathrm{GR}}(r) = 1 + 2\left(\frac{G_* M}{r}\right) + 2(2-3\nu)\left(\frac{G_* M}{r}\right)^2 + \cdots$$

The Scalar-Tensor effective metric

(i) The "bare" gravitational constant G_* is replaced by the effective one

$$\boxed{G_* \to G_{AB} \equiv 1 + \alpha_A^0 \alpha_B^0}$$

at all orders (but cannot distinguish from GR).

(ii) At 1PK level,

$$A(r) = 1 - 2\left(\frac{G_{AB}M}{r}\right) + 2\left[\langle\bar{\beta}\rangle - \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \cdots$$

$$B(r) = 1 + 2\left[1 + \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right) + \cdots$$

one recognizes the **PPN Eddington metric** written in Droste coordinates, with :

$$\boxed{\beta^{\mathrm{Edd}} = 1 + \langle\bar{\beta}\rangle \ , \quad \gamma^{\mathrm{Edd}} = 1 + \bar{\gamma}_{AB}}$$

Where

$$\langle\bar{\beta}\rangle \equiv \frac{m_A^0 \bar{\beta}_B + m_B^0 \bar{\beta}_A}{m_A^0 + m_B^0} \qquad \bar{\gamma}_{AB} \equiv -\frac{2\alpha_A^0 \alpha_B^0}{1 + \alpha_A^0 \alpha_B^0} \qquad \bar{\beta}_A \equiv \frac{1}{2}\frac{\beta_A^0 (\alpha_B^0)^2}{(1 + \alpha_A^0 \alpha_B^0)^2}$$

The Scalar-Tensor effective metric

Scalar-Tensor effective metric

$$A(r) = 1 - 2\left(\frac{G_{AB}M}{r}\right) + 2\left[\langle\bar{\beta}\rangle - \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \left[2\nu + \delta a_3^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^3 + \cdots$$

$$B(r) = 1 + 2\left[1 + \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right) + \left[2(2-3\nu) + \delta b_2^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \cdots$$

(iii) 2PK corrections

$$\delta a_3^{\mathrm{ST}} \equiv \frac{1}{12}\Big[-20\bar{\gamma}_{AB} - 35\bar{\gamma}_{AB}^2 - 24\langle\bar{\beta}\rangle(1-2\bar{\gamma}_{AB}) + 4(\langle\delta\rangle - \langle\epsilon\rangle)$$
$$+\nu\Big(-36(\bar{\beta}_A + \bar{\beta}_B) + 4\bar{\gamma}_{AB}(10+\bar{\gamma}_{AB}) + 4(\epsilon_A+\epsilon_B) + 8(\delta_A+\delta_B) - 24\zeta\Big)\Big]$$

$$\delta b_2^{\mathrm{ST}} \equiv \left[4\langle\bar{\beta}\rangle - \langle\delta\rangle + \bar{\gamma}_{AB}\left(9 + \frac{19}{4}\bar{\gamma}_{AB}\right) + \nu\left(2\langle\bar{\beta}\rangle - 4\bar{\gamma}_{AB}\right)\right]$$

$$\delta_A \equiv \frac{(\alpha_A^0)^2}{(1+\alpha_A^0\alpha_B^0)^2} \qquad \epsilon_A \equiv \frac{(\beta_A'\alpha_B^3)^0}{(1+\alpha_A^0\alpha_B^0)^3} \qquad \zeta \equiv \frac{\beta_A^0\alpha_A^0\alpha_B^0\beta_B^0}{(1+\alpha_A^0\alpha_B^0)^3}$$

EOB dynamics

- The inversion of $\frac{H_e(q,p)}{\mu} - 1 = \left(\frac{H(q,p)-M}{\mu}\right)\left[1 + \frac{\nu}{2}\left(\frac{H(q,p)-M}{\mu}\right)\right]$

defines a "resummed" EOB Hamiltonian :

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_e}{\mu} - 1\right)} \quad \text{where} \quad H_e = \sqrt{A\left(\mu^2 + \frac{p_r^2}{B} + \frac{p_\phi^2}{r^2}\right)}$$

The dynamics deduced from $H_{\rm EOB}$ and the "real" Hamiltonians H are, by construction, equivalent up to 2PK order.

- $H_{\rm EOB}$ hence defines a resummed dynamics, that may capture some features of the strong field regime.

Scalar-tensor dynamics near merger ?
(equal-mass case : $\nu = 1/4$)

ST corrections to the strong-field regime

$$H_{\mathrm{EOB}} = M\sqrt{1+2\nu\left(\frac{H_e}{\mu}-1\right)}\,, \quad \text{where} \quad H_e = \sqrt{A\left(\mu^2+\frac{p_r^2}{B}+\frac{p_\phi^2}{r^2}\right)}$$

But H_{EOB} and H_e are conservative :

$$\Rightarrow \quad \boxed{\left(\frac{\partial H_{\mathrm{EOB}}}{\partial H_e}\right) = \frac{1}{\sqrt{1+2\nu(E-1)}}} \quad \text{since} \quad H_e = \mu E \quad \text{on-shell}$$

Hence the two-body eom, deduced from H_{EOB}

$$\frac{dq}{dt} = \frac{\partial H_{\mathrm{EOB}}}{\partial H_e}\frac{\partial H_e}{\partial p}\,, \quad \frac{dp}{dt} = -\frac{\partial H_{\mathrm{EOB}}}{\partial H_e}\frac{\partial H_e}{\partial q}$$

are identical to the effective ones, deduced from H_e, to within a simple time rescaling

$$\boxed{t \to t\sqrt{1+2\nu(E-1)}}$$

ST corrections to the strong-field regime

$$ds_e^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\phi^2$$

$$\left(\frac{dr}{d\lambda}\right)^2 = \frac{1}{AB}F(u) \quad \text{with} \quad F(u) \equiv E^2 - A(u)\left(1 + j^2 u^2\right) \ , \quad u = \frac{G_{AB}M}{r}$$

- circular orbits : $F(u) = F'(u) = 0$

$$\boxed{j^2(u) = -\frac{A'}{(Au^2)'} \ , \quad E(u) = A\sqrt{\frac{2u}{(Au^2)'}}}$$

- ISCO location : $F''(u_{\text{ISCO}}) = 0$

$$\boxed{\frac{A''}{A'} = \frac{(Au^2)''}{(Au^2)'}}$$

$\rightarrow$ ST corrections to the ISCO location and orbital frequency ?
("peak chirp frequency")

$$\Omega = \frac{\partial H_{\text{EOB}}}{\partial H_e}\frac{\partial H_e}{\partial p_\phi} = \frac{j u^2 A}{G_{AB}ME\sqrt{1 + 2\nu(E-1)}} \ , \quad u = \frac{G_{AB}M}{r}$$

(depend only on $A(u) = -g_{00}^e$)

ST corrections to the strong-field regime

Last ingredient : the ST-corrected $A(u\,;\nu)$

$$\boxed{u \equiv \frac{G_{AB} M}{r} \ , \quad \nu = \frac{m_A^0 m_B^0}{(m_A^0 + m_B^0)^2}}$$

ST-corrected $A(u\,;\nu)$

$$A(u\,;\nu) = A_{\rm 2PN}^{\rm GR}(u\,;\nu) + 2\epsilon_{\rm 1PK} u^2 + (\epsilon_{\rm 2PK}^0 + \nu \epsilon_{\rm 2PK}^\nu) u^3$$

where

$$\begin{aligned}
\epsilon_{\rm 1PK} &\equiv \langle \bar\beta \rangle - \bar\gamma_{AB} \\
\epsilon_{\rm 2PK}^0 &\equiv \frac{1}{12}\Big[-20\bar\gamma_{AB} - 35\bar\gamma_{AB}^2 - 24\langle\bar\beta\rangle(1-2\bar\gamma_{AB}) + 4\big(\langle\delta\rangle - \langle\epsilon\rangle\big)\Big] \\
\epsilon_{\rm 2PK}^\nu &\equiv -3(\bar\beta_A + \bar\beta_B) + \frac{1}{3}\bar\gamma_{AB}(10 + \bar\gamma_{AB}) + \frac{1}{3}(\epsilon_A + \epsilon_B) + \frac{2}{3}(\delta_A + \delta_B) - 2\zeta
\end{aligned}$$

ST-Corrections described by 3 parameters, $(\epsilon_{\rm 1PK}, \epsilon_{\rm 2PK}^0, \epsilon_{\rm 2PK}^\nu)$

- **BUT** numerically driven by $(\alpha_A^0)^2$ (c.f. DEF, diagrammatic methods)
- When $(\alpha_A^0)^2 << 1$, $\boxed{\epsilon_{\rm 1PK} \sim \epsilon_{\rm 2PK}^0 \sim \epsilon_{\rm 2PK}^\nu}$ and ST-corrections are perturbative

Félix-Louis Julié   Two-body problem in modified gravities and effective-one-body theory

ST corrections to the strong-field regime

In this perturbative approach, **best available EOB-NR function** for GR :

$$A^{\rm GR}_{\rm 2PN}(u\,;\nu) \to \boxed{A^{\rm GR}_{\rm EOBNR}(u\,;\nu) = \mathcal{P}^1_5[A^{Taylor}_{\rm 5PN}]}$$

i.e. the $(1,5)$ Padé approximant of the truncated 5PN expansion :

$$A^{Taylor}_{\rm 5PN} = 1 - 2u + 2\nu u^3 + \nu a_4 u^4 + (a_5^c + a_5^{\ln} \ln u) u^5 + \nu (a_6^c + a_6^{\ln} \ln u) u^6$$

[Damour, Nagar, Reisswig, Pollney 2016]

- smoothly connected to Schwarzschild when $\nu \to 0$
- $a_6^c(\nu)$ is obtained by calibration with Numerical Relativity

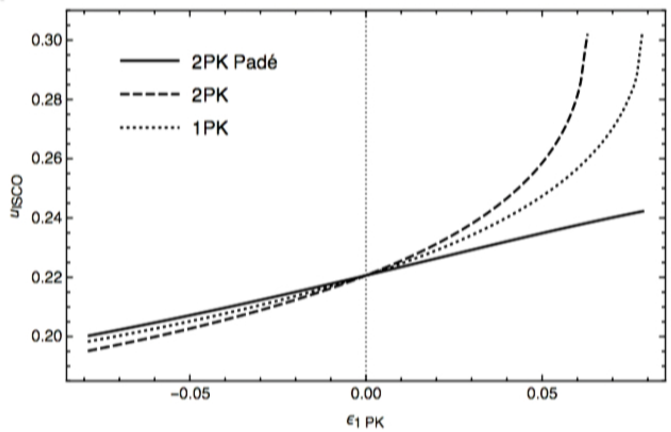
ST corrections to the strong-field regime

ISCO location and orbital frequency,
equal-mass case ($\nu = 1/4$), setting $\epsilon_{1\mathrm{PK}} \equiv \epsilon^{0}_{2\mathrm{PK}} \equiv \epsilon^{\nu}_{2\mathrm{PK}}$

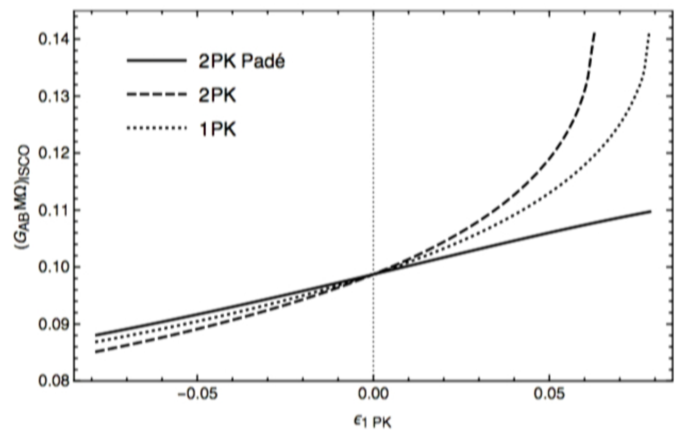
- 2PK Padeed corrections,

$$A = \mathcal{P}^1_5[A^{\mathrm{GR}}_{\mathrm{EOBNR}}(u\,;\nu) + 2\epsilon_{1\mathrm{PK}}u^2 + (\epsilon^0_{2\mathrm{PK}} + \nu\epsilon^{\nu}_{2\mathrm{PK}})u^3]$$

$u_{\mathrm{ISCO}}(\epsilon_{1\mathrm{PK}})$



$G_{AB}M\Omega_{\mathrm{ISCO}}(\epsilon_{1\mathrm{PK}})$



$$\left.\frac{d(G_{AB}M\Omega)_{ISCO}}{d\epsilon_{1\mathrm{PK}}}\right|_{\nu=1/4} \simeq 0.13$$

relative correction to GR significant ($\sim 10\%$) when $\epsilon_{1\mathrm{PK}} \sim 10^{-2} - 10^{-1}$

Recap

• Remarkably, the EOB framework can be extended beyond GR. In **scalar-tensor theories :**

$$A^{2\mathrm{PK}}(u) \equiv \mathcal{P}_5^1[A_{5\mathrm{PN}}^{Taylor} + 2\epsilon_{1\mathrm{PK}} u^2 + (\epsilon_{2\mathrm{PK}}^0 + \nu\, \epsilon_{2\mathrm{PK}}^{\nu}) u^3]$$

• But also applicable for **any theory** whose coefficients $h_i^{N\mathrm{PK}}$ satisfy the 3 mapping conditions.

Remarks

• Binary pulsar experiments have put **stringent constraints on ST theories** (no dipolar radiation)

$$\boxed{(\alpha_A^0)^2 < 4 \times 10^{-6}}$$

For **any** body A, regardless of its EOS or self-gravity.

• The ISCO ST-correction (significant for $(\alpha_A^0)^2 \gtrsim 10^{-2}$) seems unlikely to improve binary pulsar constraints.

However :

• However, stars subject to dynamical scalarization can develop non perturbative $(\alpha_A)^2$ near merger [Barausse, Palenzuela, Ponce, Lehner 2013]. EOB is well-suited to investigate this regime !

• The interferometers LIGO-Virgo or even LISA are designed to detect highly redshifted sources. Cosmological history of ST theories ?

Black holes :

• Are known in these ST theories to carry no scalar hair : $m_A(\varphi) = cst$ i.e. no deviation to GR.

• Induce scalar hair by means of a vector gauge field, as in, e.g., **Einstein-Maxwell-dilaton theories** ?

Einstein-Maxwell-dilaton theories (EMD)

EMD action in the Einstein-frame ($G_* \equiv c \equiv 1$)

$$S_{\mathrm{EMD}} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - e^{-2a\varphi}F^{\mu\nu}F_{\mu\nu}\right) + S_{\mathrm{m}}\left[\Psi, \mathcal{A}^2(\varphi)g_{\mu\nu}, A_\mu\right]$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

- $U(1)$ "graviphoton" gauge vector A_μ

$$A_\mu \to A_\mu + \partial_\mu \chi$$

- Fondamental parameter, a : non-minimal coupling between φ and A_μ
- When $a = 0$, reduces to Einstein-Maxwell minimally coupled to φ
- When $a \neq 0$, shift symmetry $\varphi \to \varphi + cst$ broken in vacuum : **"hairy" black holes**

Include Einstein-Maxwell-dilaton theories within the EOB formalism ?

- Can be mapped to geodesic motion **at 1PK** from **Lorentz symmetry**.
- First step : compute the two-body Lagrangian

Two-body Lagrangian in Einstein-Maxwell-dilaton theories

How to reduce compact bodies to point particles in EMD theories ?

Most generic ansatz for compact bodies [JCAP **1801** 026 (2018)] FLJ

$$S_{\rm m} \quad \rightarrow \quad S_{\rm m}^{\rm pp}[g_{\mu\nu}, A_\mu, \varphi, \{x_A^\mu\}] = -\sum_A \int m_A(\varphi) ds_A + \sum_A q_A \int A_\mu dx_A^\mu$$

where $ds_A = \sqrt{-g_{\mu\nu} dx_A^\mu dx_A^\nu}$.

- Covariance : $m_A(\varphi)$ is a **scalar** function of $\varphi(x_A^\mu(s))$
- Preserves $U(1)$ gauge symmetry ($A_\mu \rightarrow A_\mu + \partial_\mu \chi$) iff q_A **is a conserved charge**

$$\boxed{\partial_\mu j^\mu = 0 \ , \qquad \text{where} \qquad j^\mu(y) = \sum_A q_A \, \delta^{(3)}(\vec{y} - \vec{x}_A(t)) \frac{dx_A^\mu}{dt}}$$

- No gradients $\partial_\mu = \{\partial_t, \partial_i\}$ of the fields : no finite-size (e.g., tidal) nor out of equilibrium effects

$m_A(\varphi)$ and q_A can be computed for a given body (e.g. NS or BH)

Two-body Lagrangian in Einstein-Maxwell-dilaton theories

Two-body action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2\partial_\mu\varphi\partial_\nu\varphi - e^{-2a\varphi}F^2\right) - \sum_A \int m_A(\varphi)\, ds_A + \sum_A q_A \int A_\mu\, dx_A^\mu$$

- The two-body problem depends on a, on two functions $m_A(\varphi)$ **and** two parameters q_A.

- Field equations

$$\boxed{\begin{aligned} R_{\mu\nu} &= 2\partial_\mu\varphi\partial_\nu\varphi + 2e^{-2a\varphi}\left(F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}g_{\mu\nu}F^2\right) + 8\pi\sum_A\left(T^A_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^A\right) \\ \nabla_\nu\left(e^{-2a\varphi}F^{\mu\nu}\right) &= \frac{4\pi}{\sqrt{-g}}\sum_A q_A\,\delta^{(3)}\left(\vec{x} - \vec{x}_A(t)\right)\frac{dx_A^\mu}{dt} \\ \Box\varphi &= -\frac{a}{2}e^{-2a\varphi}F^2 + \frac{4\pi}{\sqrt{-g}}\sum_A \frac{ds_A}{dt}\frac{dm_A}{d\varphi}\delta^{(3)}\left(\vec{x} - \vec{x}_A(t)\right) \end{aligned}}$$

$$T_A^{\mu\nu} = m_A(\varphi)\frac{\delta^{(3)}(\vec{x} - \vec{x}_A(t))}{\sqrt{g g_{\sigma\tau}\frac{dx_A^\sigma}{dt}\frac{dx_A^\tau}{dt}}}\frac{dx_A^\mu}{dt}\frac{dx_A^\nu}{dt}$$

Two-body Lagrangian in Einstein-Maxwell-dilaton theories

Two-body Lagrangian at post-Keplerian (1PK) order

- Harmonic and Lorenz gauges : $\partial_\mu(\sqrt{-g}g^{\mu\nu}) = 0$, and $\nabla_\mu A^\mu = 0$
- 1PK : $\mathcal{O}\left(\left(\frac{v}{c}\right)^2\right) \sim \mathcal{O}\left(\frac{G_* m}{r}\right)$ corrections to Keplerian dynamics
- Weak field expansion (background : $\eta_{\mu\nu}$, φ_0, $A_\mu^\infty = 0$)

$$\boxed{\begin{array}{lll} g_{00} = -e^{-2U} + \mathcal{O}(v^6) & A_t = \delta A_t + \mathcal{O}(v^6) & \varphi = \varphi_0 + \delta\varphi + \mathcal{O}(v^6) \\ g_{0i} = -4g_i + \mathcal{O}(v^5) & A_i = \delta A_i + \mathcal{O}(v^5) & \\ g_{ij} = \delta_{ij} e^{2U} + \mathcal{O}(v^4) & & \end{array}}$$

- the functions $m_A(\varphi)$ and $m_B(\varphi)$ are expanded aroud φ_0

$$\boxed{\begin{aligned} \ln m_A(\varphi) &= \ln m_A^0 + \alpha_A^0(\varphi - \varphi_0) + \frac{1}{2}\beta_A^0(\varphi - \varphi_0)^2 + \cdots \\ \ln m_B(\varphi) &= \ln m_B^0 + \alpha_B^0(\varphi - \varphi_0) + \frac{1}{2}\beta_B^0(\varphi - \varphi_0)^2 + \cdots \end{aligned}}$$

i.e. the 1PK EMD Lagrangian depends on 6+2+1=**9 fundamental parameters**

Two-body Lagrangian in Einstein-Maxwell-dilaton theories

Final result : EMD two-body Lagrangian at 1PK order

$$
\begin{aligned}
L_{\rm AB}^{\rm EMD} = &-m_A^0 - m_B^0 + \frac{1}{2}m_A^0 V_A^2 + \frac{1}{2}m_B^0 V_B^2 + \frac{G_{AB}m_A^0 m_B^0}{R} \\
&+ \frac{1}{8}m_A^0 V_A^4 + \frac{1}{8}m_B^0 V_B^4 + \frac{G_{AB}m_A^0 m_B^0}{R}\left[\frac{3}{2}(V_A^2+V_B^2) - \frac{7}{2}(V_A.V_B) - \frac{1}{2}(N.V_A)(N.V_B) + \bar{\gamma}_{AB}(\vec{V}_A - \vec{V}_B)^2\right] \\
&- \frac{G_{AB}^2 m_A^0 m_B^0}{2R^2}\left[m_A^0(1+2\bar{\beta}_B) + m_B^0(1+2\bar{\beta}_A)\right]
\end{aligned}
$$

- $L_{\rm AB}^{\rm EMD}$ has exactly the **same structure** as $L_{\rm AB}^{\rm ST}$ at 1PK !

$$
\boxed{
\begin{aligned}
G_{AB} &= 1 + \alpha_A^0\alpha_B^0 - e_A e_B \\
\bar{\gamma}_{AB} &= \frac{-4\alpha_A^0\alpha_B^0 + 3e_A e_B}{2(1+\alpha_A^0\alpha_B^0 - e_A e_B)} \\
\bar{\beta}_A &= \frac{1}{2}\frac{\beta_A^0{\alpha_B^0}^2 - 2e_A e_B(a\alpha_B^0 - \alpha_A^0\alpha_B^0) + e_B^2(1 + a\alpha_A^0 - e_A^2)}{1+\alpha_A^0\alpha_B^0 - e_A e_B}
\end{aligned}
}
$$

where $\quad e_A \equiv (q_A/m_A^0)e^{a\varphi_0} \quad , \quad e_B \equiv (q_B/m_B^0)e^{a\varphi_0}$

- reduce to ST in the limit $e_A = e_B = 0$, i.e. when $q_A = q_B = 0$.

Einstein-Maxwell-dilaton theories and EOB

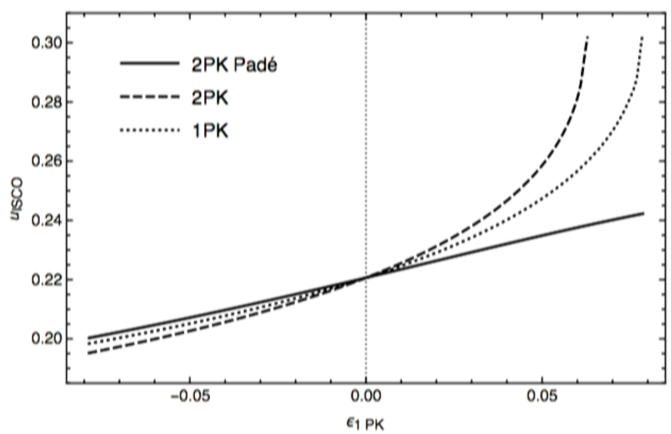
- As a consequence, we have **already included** Einstein-Maxwell-dilaton theories into the EOB framework (at 1PK order) :

Recall

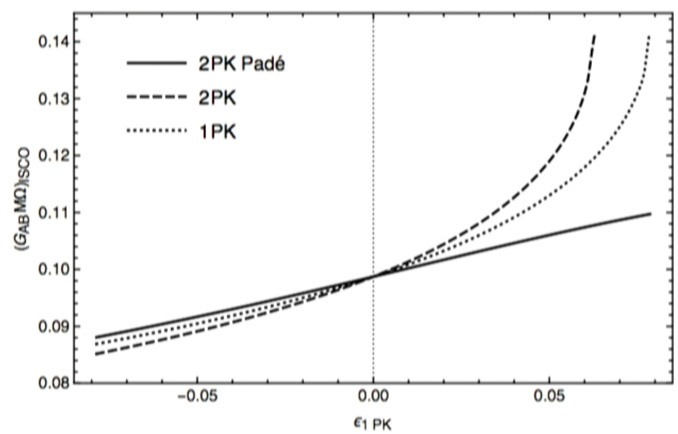
$$A(u;\nu) = \mathcal{P}_5^1[A_{\mathrm{EOBNR}}^{GR} + 2\left(\langle\bar{\beta}\rangle - \bar{\gamma}_{AB}\right)u^2]\ , \qquad u = \frac{G_{AB}M}{r}$$

where $\langle\bar{\beta}\rangle = (m_A^0\bar{\beta}_B + m_B^0\bar{\beta}_A)/(m_A^0 + m_B^0)$.

$u_{\mathrm{ISCO}}(\epsilon_{1\mathrm{PK}})$



$G_{AB}M\Omega_{\mathrm{ISCO}}(\epsilon_{1\mathrm{PK}})$



Recap

Recap :

- Start from the skeleton action, which simplifies computations

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2\partial_\mu\varphi\partial_\nu\varphi - e^{-2a\varphi}F^2\right) - \sum_A \int m_A(\varphi)\, ds_A + \sum_A q_A \int A_\mu\, dx_A^\mu$$

- At 1PK, the two-body Lagrangian depends on the two functions $m_A(\varphi)$ through

$$m_A^0 = m_A(\varphi_0)\ , \quad \alpha_A^0 = \frac{d\ln m_A}{d\varphi}(\varphi_0)\ , \quad \beta_A^0 = \frac{d^2\ln m_A}{d\varphi^2}(\varphi_0)$$

and on the two constants q_A.

- Generically describes any compact body in EMD theories

How to relate $m_A(\varphi)$ and q_A to a specific body ? Say, a "hairy" black hole ?

EMD black holes

$$S_{\rm EMD}^{vac} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - e^{-2a\varphi}F^{\mu\nu}F_{\mu\nu}\right)$$

SSS, electrically charged BH with "secondary" scalar hair [Gibbons, Maeda 88]

$$ds^2 = -\left(1-\frac{r_+}{r}\right)\left(1-\frac{r_-}{r}\right)^{\frac{1-a^2}{1+a^2}}dt^2 + \left(1-\frac{r_+}{r}\right)^{-1}\left(1-\frac{r_-}{r}\right)^{-\frac{1-a^2}{1+a^2}}dr^2 + r^2\left(1-\frac{r_-}{r}\right)^{\frac{2a^2}{1+a^2}}d\Omega^2$$

$$A_t = -\frac{Q\,e^{2a\varphi_\infty}}{r}\,,\quad A_i = 0 \quad \text{with} \quad Q^2 = \frac{r_+r_-}{1+a^2}\,e^{-2a\varphi_\infty}\,,$$

$$\varphi = \varphi_\infty + \frac{a}{1+a^2}\ln\left(1-\frac{r_-}{r}\right)\,.$$

- **3 integration constants** : r_+, r_-, φ_∞ (imposed by the faraway companion).
- No electric charge $Q = 0 \Leftrightarrow r_- = 0$: BH reduces to **Schwarzschild's**
- Decoupling limit $a = 0$: BH reduces to **Reissner-Nordström's**

To be "skeletonized"

Skeletonization of EMD black holes

SSS and electrically charged BH [Gibbons, Maeda 88]

$$ds^2 = -\left(1-\frac{r_+}{r}\right)\left(1-\frac{r_-}{r}\right)^{\frac{1-a^2}{1+a^2}} dt^2 + \left(1-\frac{r_+}{r}\right)^{-1}\left(1-\frac{r_-}{r}\right)^{-\frac{1-a^2}{1+a^2}} dr^2 + r^2\left(1-\frac{r_-}{r}\right)^{\frac{2a^2}{1+a^2}} d\Omega^2$$

$$A_t = -\frac{Q\, e^{2a\varphi_\infty}}{r}\,, \quad A_i = 0 \quad \text{with} \quad Q^2 = \frac{r_+ r_-}{1+a^2}\, e^{-2a\varphi_\infty}\,,$$

$$\varphi = \varphi_\infty + \frac{a}{1+a^2} \ln\left(1-\frac{r_-}{r}\right)\,.$$

1) Asymptotic expansion at infinity (isotropic coordinates $r = \tilde{r} + [r_+ + r_-]/2 + \cdots$)

$$\boxed{\begin{aligned} \tilde{g}_{\mu\nu} &= \eta_{\mu\nu} + \delta_{\mu\nu}\left(\frac{r_+ + \frac{1-a^2}{1+a^2} r_-}{\tilde{r}}\right) + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right)\,, \\ A_t &= -\frac{Q\, e^{2a\varphi_\infty}}{\tilde{r}} + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right)\,, \\ \varphi &= \varphi_\infty - \frac{a}{1+a^2}\frac{r_-}{\tilde{r}} + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right)\,. \end{aligned}}$$

- encode **sufficient information** to fix uniquely $m_A(\varphi)$ and q_A.

Skeletonization of EMD black holes

2) Near-worldline region of particle A

Recall : skeleton action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - 2\partial_\mu\varphi\partial_\nu\varphi - e^{-2a\varphi}F^2\right) - \int m_A(\varphi)\, ds_A + q_A\int A_\mu\, dx^\mu$$

$$\boxed{\begin{aligned} R_{\mu\nu} &= 2\partial_\mu\varphi\partial_\nu\varphi + 2e^{-2a\varphi}\left(F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}g_{\mu\nu}F^2\right) + 8\pi\left(T^A_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^A\right) \\ \nabla_\nu\left(e^{-2a\varphi}F^{\mu\nu}\right) &= \frac{4\pi}{\sqrt{-g}}q_A\,\delta^{(3)}\left(\vec{y} - \vec{x}_A(t)\right)\frac{dx^\mu_A}{dt} \\ \Box\varphi &= -\frac{a}{2}e^{-2a\varphi}F^2 + \frac{4\pi}{\sqrt{-g}}\frac{ds_A}{dt}\frac{dm_A}{d\varphi}\delta^{(3)}\left(\vec{y} - \vec{x}_A(t)\right) \end{aligned}}$$

where $$T^{\mu\nu}_A = m_A(\varphi)\frac{\delta^{(3)}(\vec{y} - \vec{x}_A(t))}{\sqrt{gg_{\sigma\tau}\frac{dx^\sigma_A}{dt}\frac{dx^\tau_A}{dt}}}\frac{dx^\mu_A}{dt}\frac{dx^\nu_A}{dt}$$

Skeletonization of EMD black holes

Asymptotic expansion of the skeleton fields

- Rest-frame of the particle $\vec{x}_A = \vec{0}$
- Harmonic coordinates $\partial_\mu(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}) = 0$
- Expansion around the BH's background :

$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \ , \quad A_t = \delta A_t \ , \quad \varphi = \varphi_\infty + \delta\varphi$$

$$\boxed{\begin{aligned} \tilde{g}_{\mu\nu} &= \eta_{\mu\nu} + \delta_{\mu\nu}\left(\frac{2m_A(\varphi_\infty)}{\tilde{r}}\right) + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right) \ , \\ A_t &= -\frac{q_A\, e^{2a\varphi_\infty}}{\tilde{r}} + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right) \ , \\ \varphi &= \varphi_\infty - \frac{1}{\tilde{r}}\frac{dm_A}{d\varphi}(\varphi_\infty) + \mathcal{O}\left(\frac{1}{\tilde{r}^2}\right) \ . \end{aligned}}$$

- Asymptotic expansion in terms of $m_A(\varphi_\infty)$, $m'_A(\varphi_\infty)$, and q_A.

Skeletonization of EMD black holes

3) Matching

- the **identification** yields (harmonic and isotropic coordinates coincide)

Matching conditions

$$m_A(\varphi_\infty) = \frac{1}{2}\left(r_+ + \frac{1-a^2}{1+a^2}\, r_-\right)$$

$$q_A = Q$$

$$\frac{dm_A}{d\varphi}(\varphi_\infty) = \frac{a\, r_-}{1+a^2}$$

- enable to **compute** q_A and $m_A(\varphi)$ in terms of the "real" source.
- **For black holes**, $Q^2 = \frac{r_+ r_-}{1+a^2}\, e^{-2a\varphi_\infty}$ yields a first order differential equation :

$$\left(\frac{dm_A}{d\varphi}\right)\left(m_A(\varphi) - \frac{1-a^2}{2a}\frac{dm_A}{d\varphi}\right) = \frac{a}{2}\, q_A^2 e^{2a\varphi}$$

Skeletonization of EMD black holes

Recall : skeleton action for compact bodies

$$S^{\rm pp}_{\rm m,A}[g_{\mu\nu}, A_\mu, \varphi, x^\mu_A] = -\int m_A(\varphi) ds_A + q_A \int A_\mu dx^\mu_A$$

where, for electrically charged EMD **black holes** :

$$\left(\frac{dm_A}{d\varphi}\right)\left(m_A(\varphi) - \frac{1-a^2}{2a}\frac{dm_A}{d\varphi}\right) = \frac{a}{2}\, q_A^2 e^{2a\varphi}$$

→ Once the theory is fixed (i.e. a), a **black hole** is entirely described by **two constant parameters** :

- its electric charge q_A
- a unique integration constant, μ_A.

The sensitivity of a hairy black hole

$$\left(\frac{dm_A}{d\varphi}\right)\left(m_A(\varphi) - \frac{1-a^2}{2a}\frac{dm_A}{d\varphi}\right) = \frac{a}{2}q_A^2 e^{2a\varphi}$$

A simple example : $a = 1$

$$\boxed{m_A(\varphi) = \sqrt{\mu_A^2 + q_A^2\frac{e^{2\varphi}}{2}}}$$

$m_A(\varphi)$ depends on **two constant parameters** :

- The **electric charge** of the BH, $q_A = Q$. When $q_A = 0$, Schwarzschild.
- The integration constant,

$$\mu_A^2 = \frac{r_+(r_+ - r_-)}{4} = \frac{A_H}{16\pi}$$

is related to the **horizon area** of the black hole !

When φ_∞ varies adiabatically, the black hole readjusts its equilibrum configuration (r_+, r_-), in keeping its **charge** and **area** fixed.

The two-body Lagrangian

Consequence: 1PK dynamics of EMD black holes, deviations from GR

$$\boxed{\begin{aligned} G_{AB} &= 1+\alpha_A^0\alpha_B^0 - e_A e_B \\ \bar{\gamma}_{AB} &= \frac{-4\alpha_A^0\alpha_B^0+3e_Ae_B}{2(1+\alpha_A^0\alpha_B^0-e_Ae_B)} \\ \bar{\beta}_A &= \frac{1}{2}\frac{\beta_A^0{\alpha_B^0}^2-2e_Ae_B(a\alpha_B^0-\alpha_A^0\alpha_B^0)+e_B^2(1+a\alpha_A^0-e_A^2)}{1+\alpha_A^0\alpha_B^0-e_Ae_B} \end{aligned}}$$

where

$$\alpha_A^0 = \frac{d\ln m_A}{d\varphi}(\varphi_0) \quad , \quad \beta_A^0 = \frac{d^2\ln m_A}{d\varphi^2}(\varphi_0) \quad , \quad e_A \equiv (q_A/m_A^0)e^{a\varphi_0}$$

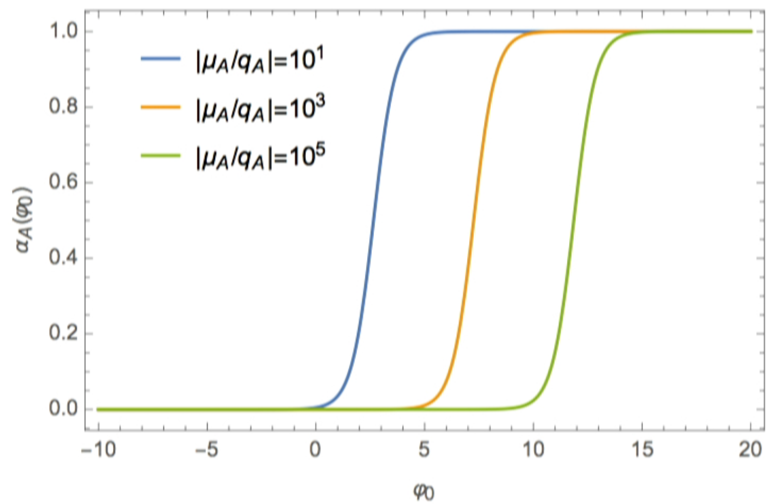
- These parameters characterize deviations from GR and can now be **computed for our EMD black holes**
- They depend on **two constant parameters** (q_A, μ_A) per black hole.

Case $a = 1$ $\quad m_A(\varphi) = \sqrt{\mu_A^2 + q_A^2 \frac{e^{2\varphi}}{2}}$

"Fermi-Dirac" distribution !

$$\alpha_A(\varphi_0) = \frac{1}{1 + e^{2\left(\ln\left|\frac{\mu_A\sqrt{2}}{q_A}\right| - \varphi_0\right)}} , \qquad \beta_A^0 = 2\alpha_A^0(1-\alpha_A^0) , \qquad (e_A)^2 = 2\alpha_A^0$$

$$\alpha_A^0 = \alpha_A(\varphi_0)$$



- Steep transition between two *universal* regimes:

Schwarzschild-like regime ↔ **"scalarized" regime**: large deviations from GR

- Note: specific values (μ_A, q_A) only influence the location of the transition

The two-body Lagrangian

Consequence: 1PK dynamics of EMD black holes, deviations from GR

$$G_{AB} = 1 + \alpha_A^0\alpha_B^0 - e_A e_B$$

$$\bar{\gamma}_{AB} = \frac{-4\alpha_A^0\alpha_B^0 + 3e_A e_B}{2(1 + \alpha_A^0\alpha_B^0 - e_A e_B)}$$

$$\bar{\beta}_A = \frac{1}{2}\frac{\beta_A^0{\alpha_B^0}^2 - 2e_A e_B(a\alpha_B^0 - \alpha_A^0\alpha_B^0) + e_B^2(1 + a\alpha_A^0 - e_A^2)}{1 + \alpha_A^0\alpha_B^0 - e_A e_B}$$

where

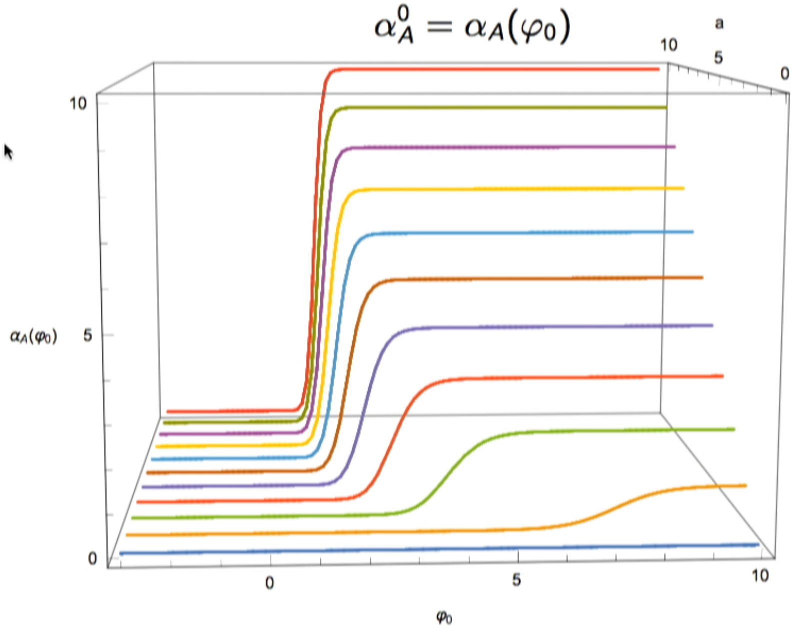
$$\alpha_A^0 = \frac{d\ln m_A}{d\varphi}(\varphi_0) \quad , \quad \beta_A^0 = \frac{d^2\ln m_A}{d\varphi^2}(\varphi_0) \quad , \quad e_A \equiv (q_A/m_A^0)e^{a\varphi_0}$$

- These parameters characterize deviations from GR and can now be **computed for our EMD black holes**
- They depend on **two constant parameters** (q_A, μ_A) per black hole.

Generic EMD theories

$$\left(\frac{dm_A}{d\varphi}\right)\left(m_A(\varphi) - \frac{1-a^2}{2a}\frac{dm_A}{d\varphi}\right) = \frac{a}{2}\, q_A^2 e^{2a\varphi}$$

$$\alpha_A^0 = \alpha_A(\varphi_0)$$



(**settings:** $a \in [\![0, 10]\!]$, $|\mu_A/q_A| = 10^3$)

$$\beta_A^0 = \alpha_A^0\,(a - \alpha_A^0)\left[\frac{(1-a^2)\alpha_A^0 - 2a}{(1-a^2)\alpha_A^0 - a}\right]$$

$$(e_A)^2 = \alpha_A^0\left[\frac{2a - (1-a^2)\alpha_A^0}{a^2}\right]$$

- Generalized "scalarization": $(\alpha_A^0, \beta_A^0, e_A) \to (a, 0, \pm\sqrt{1+a^2})$.
- The dynamics of EMD black holes depends crucially on their cosmological environment φ_0.

Quasi-extremal black holes

A striking example: two "scalarized" black holes

- Two EMD black holes
- Charges q_A and q_B of the same sign
- When φ_0 is large enough, $\alpha^0_{A/B} \to a\,, \quad \beta^0_{A/B} \to 0\,, \quad e_{A/B} \to \sqrt{1+a^2}\,.$

$$G_{AB} \to 0\,, \qquad G_{AB}\,\bar{\gamma}_{AB} \to (3-a^2)/2\,, \qquad G^2_{AB}\,\bar{\beta}_{A/B} \to 0$$

EMD-extended "Majumdar-Papapetrou" Lagrangian

$$L_{AB} \to -m^0_A\sqrt{1-V_A^2} - m^0_B\sqrt{1-V_B^2} + \left(\frac{3-a^2}{2}\right)\frac{m^0_A m^0_B}{R}(\vec{V}_A - \vec{V}_B)^2 + \mathcal{O}(V^6)$$

- Scalarized BH are in fact **quasi-extremal**: $\boxed{(e_A)^2 \equiv (q_A/m^0_A)^2 e^{2a\varphi_0} \to 1 + a^2}$
- When φ_0 increases, universal "self-tuning".

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Conclusion

Concluding remarks :

- Remarkably, the EOB approach is valid beyond the scope of general relativity. In **scalar-tensor theories :**

$$A^{\rm 2PK}(u) \equiv \mathcal{P}^1_5[A^{Taylor}_{\rm 5PN} + 2\epsilon_{\rm 1PK}\, u^2 + (\epsilon^0_{\rm 2PK} + \nu\, \epsilon^\nu_{\rm 2PK})u^3]$$

- But also applicable for **any theory** whose coefficients $h_i^{N{\rm PK}}$ satisfy the 3 mapping conditions, e.g. **Einstein-Maxwell-dilaton theories** at 1PK order.

- The ST and EMD examples suggest a generic ansatz

$$\boxed{A^{\rm PEOB}(u) \equiv \mathcal{P}^1_5[A^{Taylor}_{\rm 5PN} + 2(\epsilon^0_{\rm 1PK} + \nu\, \epsilon^\nu_{\rm 1PK})u^2 + (\epsilon^0_{\rm 2PK} + \nu\, \epsilon^\nu_{\rm 2PK})u^3]}$$

where $\epsilon^0_{\rm 1PK}$, $\epsilon^\nu_{\rm 1PK}$, $\epsilon^0_{\rm 2PK}$, and $\epsilon^\nu_{\rm 2PK}$ are theory-agnostic Parametrized EOB (PEOB) coefficients.

Conclusion

- We generalized Eardley's "**sensitivity**" $m_A(\varphi)$ to **hairy black holes**, and shed light on the "scalarization" of EMD black holes to a **quasi-extremal** regime. Note the simplicity in comparison to NS (eos, Jordan metric,...).

- Necessity to observe sources emitting from **various redshifts**. Contrarily to GR, in ST and EMD theories, the cosmological background φ_0 is determinant for the two-body dynamics.

Next step : ST and EMD corrections to the radiation reaction force

- Known in scalar-tensor theories at 1.5PK and 2.5PK [Mirshekari, Will 13]

$$\vec{\mathcal{F}} = \frac{8}{5}\frac{(G_{AB}m_A^0 m_B^0)^2}{MR^3}\left[(\vec{N}.\vec{\mathcal{V}})\vec{N}(A_{1.5}+A_{2.5}) - \vec{\mathcal{V}}(B_{1.5}+B_{2.5})\right]$$

- Unknown is Einstein-Maxwell-dilaton theories (**ongoing work**).