Title: Black Holes, Quantum Information, and Unification

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Abstract: <p>The study of black holes has revealed a deep connection between quantum information and spacetime geometry. Its origin must lie in a quantum theory of gravity, so it offers a valuable hint in our search for a unified theory. Precise formulations of this relation recently led to new insights in Quantum Field Theory, some of which have been rigorously proven. An important example is our discovery of the first universal lower bound on the local energy density. The energy near a point can be negative, but it is bounded below by a quantity related to the information flowing past the point.</p>

Black Holes, Quantum Information, and Unification

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Quantum Information and Quantum Gravity

The study of Quantum Information is revolutionizing how we pursue Quantum Gravity.

As a byproduct, we are discovering new results in nongravitational physics (QFT), which can be (laboriously) proven.

All of these developments originate with the study of black holes.

Black Hole Event Horizon

The event horizon of a black hole is the surface of no return.

Example: Schwarzschild black hole, sphere with $R = 2GM$.

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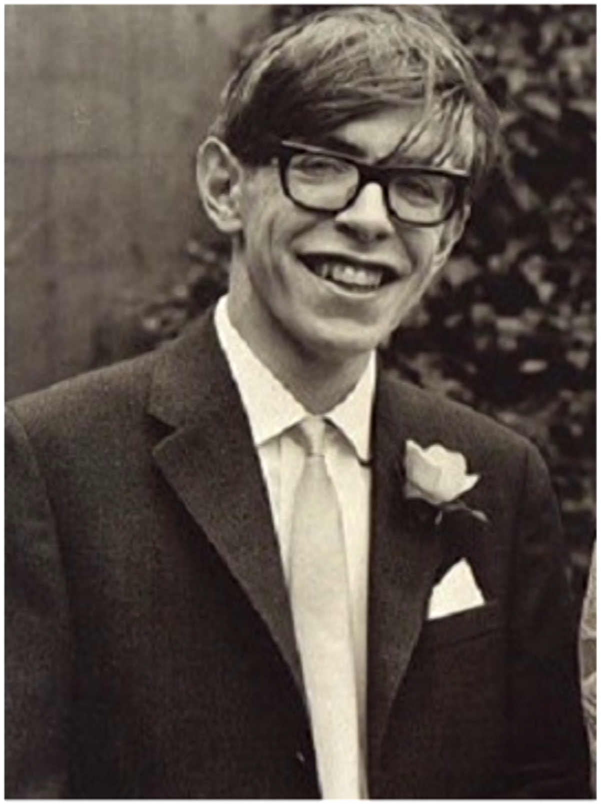
Suppose that matter falls into a black hole, or two black holes merge.

What happens to the horizon area?

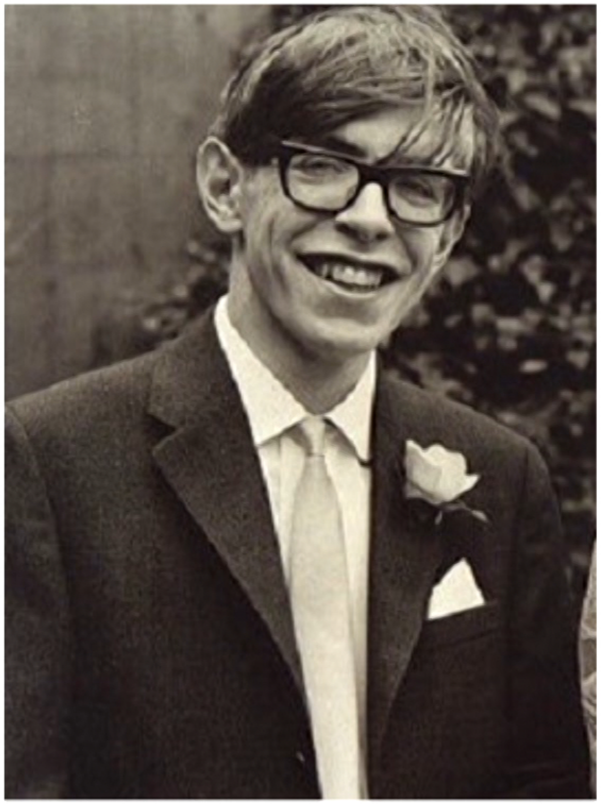
Area Theorem for Event Horizons



Hawking 1971: In GR, the total area of all event horizons cannot decrease:

$$dA \geq 0\ .$$

Area Theorem for Event Horizons



Hawking 1971: In GR, the total area of all event horizons cannot decrease:

$$dA \geq 0 \ .$$

✓Proven, assuming the Null Energy Condition (NEC):

$$T_{kk} \equiv T_{\mu\nu} k^{\mu} k^{\nu} \geq 0 \ .$$

True for classical matter.

Black Holes and the Second Law

What happens to the entropy of a matter system that is lost into a black hole?

Cannot be accessed, not even by jumping after it.

2nd Law violated/transcended?

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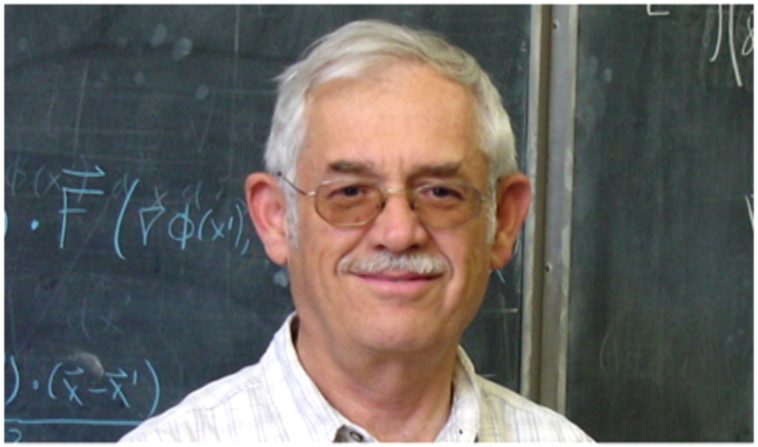
Bekenstein 1972: Black holes must themselves have entropy!



Inspired by area theorem:

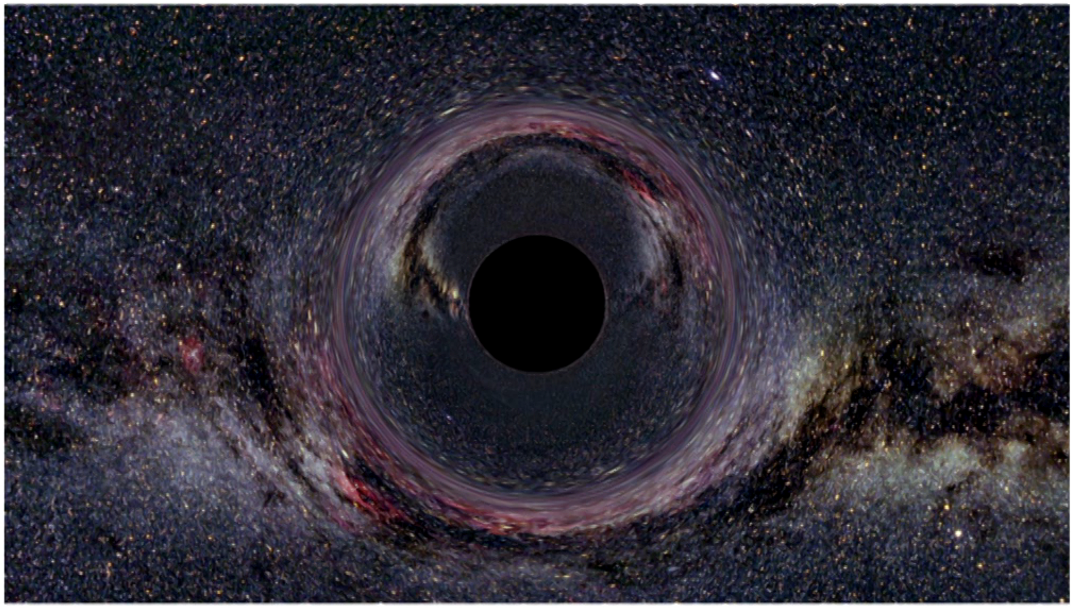
$$S_{BH} \equiv \frac{A}{4G\hbar}$$

up to $O(1)$ factor.

Generalized Entropy

To an outside observer, the total entropy consists of the entropy of all the matter outside the black hole, plus the entropy of the black hole.

$$S_{\text{gen}} \equiv \frac{A}{4G\hbar} + S_{\text{out}} .$$



Generalized Entropy
=
Geometry
+
Information

Generalized Second Law for Event Horizons

In the presence of black holes, the ordinary 2nd law becomes the Generalized Second Law: Bekenstein 1972, 1973, 1974

$$dS_{\text{gen}} \geq 0 \ ,$$

where

$$S_{\text{gen}} \equiv \frac{A}{4G\hbar} + S_{\text{out}} + \ldots \ ; \quad S_{\text{out}} = -\text{Tr}\rho_{\text{out}} \log \rho_{\text{out}} \ .$$

✓Proof (semiclassical limit)
Wall 2011

Hawking Radiation

- Had to be there since $T^{-1} = dS/dE$.
- Found by explicit calculation. Hawking (1974)
- Black holes are thermodynamic objects.

The area decreases as they evaporate!

This is possible because the Null Energy Condition is violated. (Also, e.g., in Casimir energy.)

Amazingly, the Generalized Second Law still holds.

Hawking radiation increases S_{out} enough to compensate for area loss.

The Facts So Far

The GSL is simultaneously a statement about geometry and about quantum info!

It becomes Hawking's area theorem in the classical limit.

It becomes the ordinary second law in the case where there are no black holes.

But neither law survives on its own, if black holes are present and treated at the quantum level.

Alternative Fact

From the New York *Alternative Times*,
December 12, 1974:

Stephen Hawking Discovers
"2nd Law of Thermodynamics"

Claims It Follows From General Relativity

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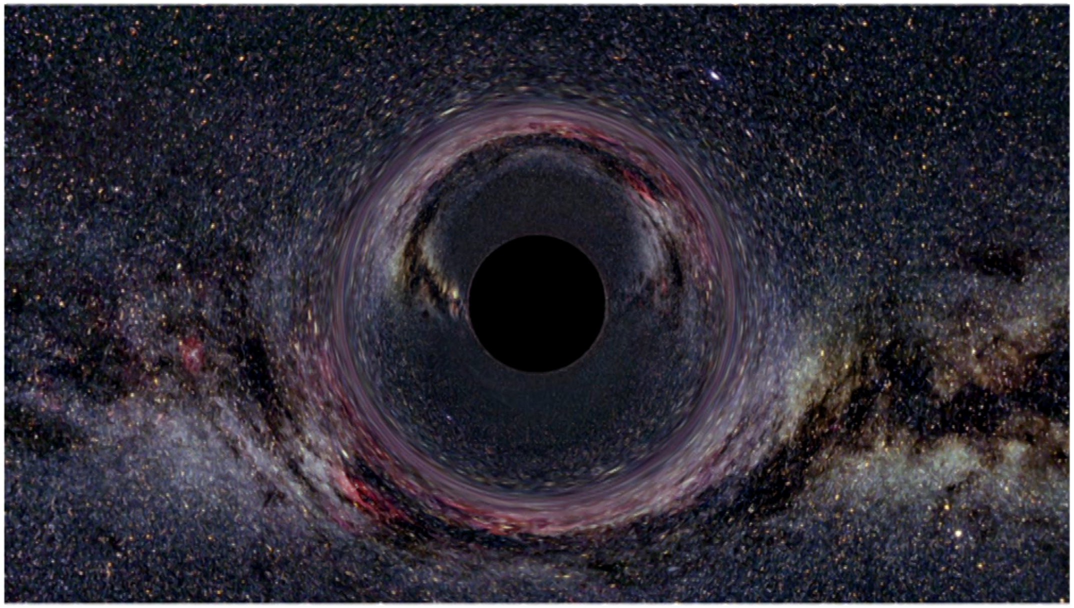

Generalized Entropy
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General Relativity as a Discovery Tool

1. Start with a classical gravity theorem involving area.
2. Add a quantum correction to make it robust against violations of the Null Energy Condition:

$$A \to A + 4G\hbar S_{\text{out}} \ .$$

3. Take a limit where gravity becomes unimportant.
4. Obtain a quantum law.

Can we actually do this, starting with other GR theorems?

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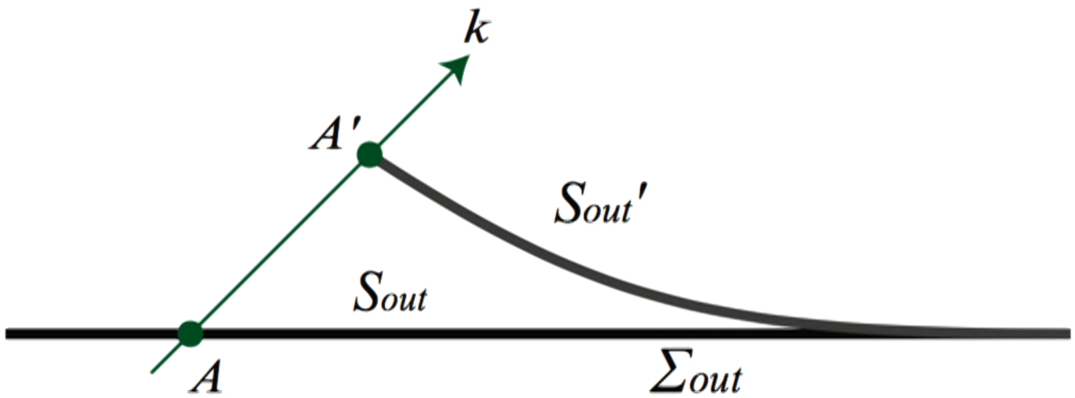
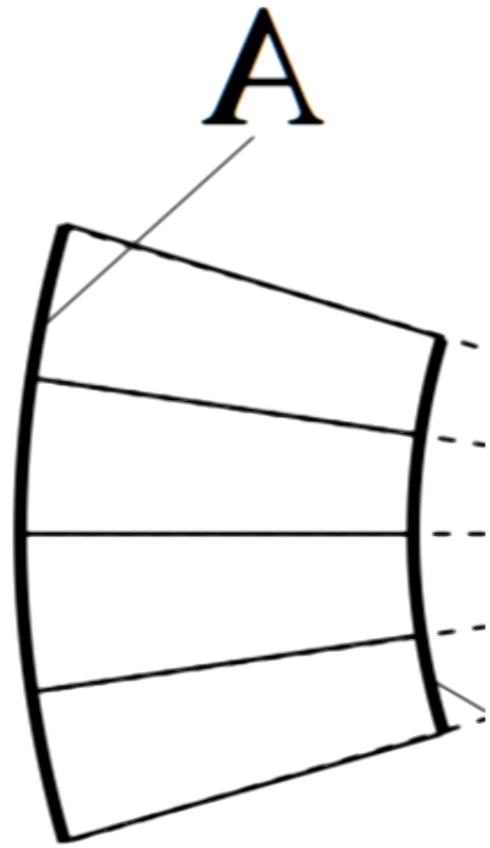
Can we actually do this, starting with other GR theorems?

Yes: Classical Focussing Theorem $\longrightarrow$ Quantum Focussing

Expansion of Light-rays

The classical expansion, θ, is the (logarithmic) derivative of an area element, when transported along orthogonal light-rays.

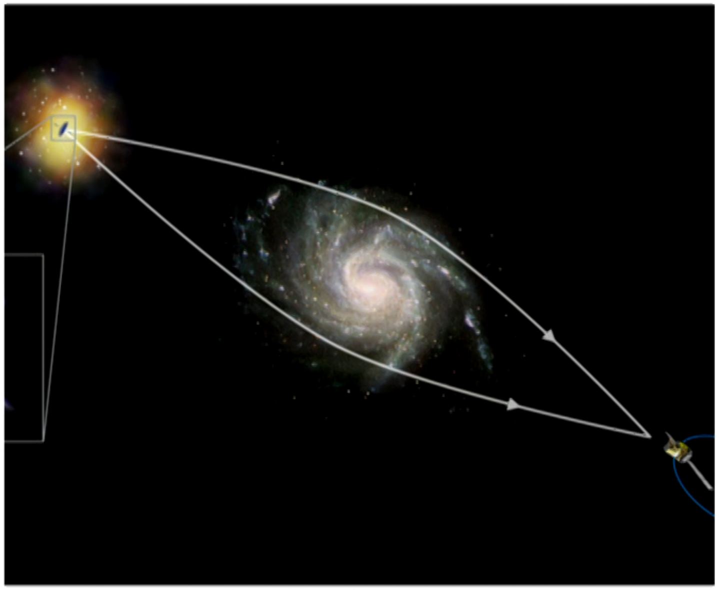




Classical Focussing Theorem

In General Relativity,
matter focusses light:

$$\theta' \leq 0 .$$



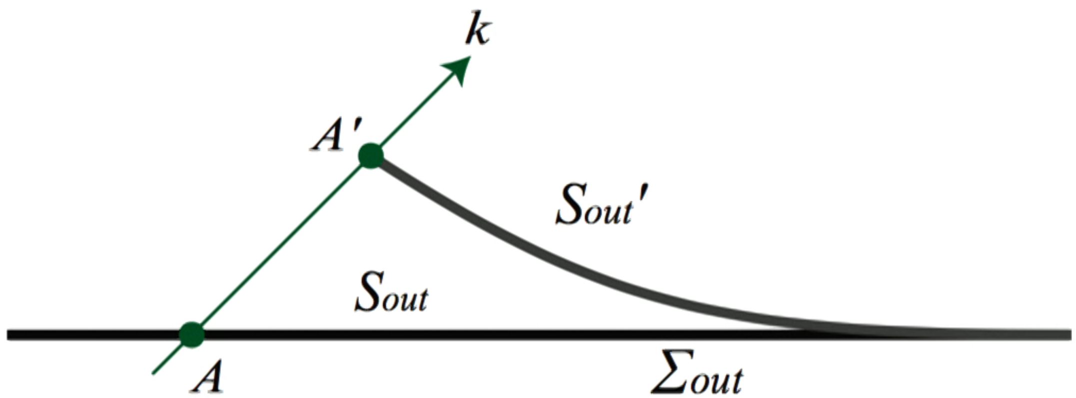
Like the Area Theorem, this assumes the Null Energy Condition, $T_{kk} \geq 0$.

Quantum effects can violate this. Example: evaporating black hole.

→ Formulate a more robust, quantum-corrected focussing theorem!

Generalized Entropy Off the Horizon

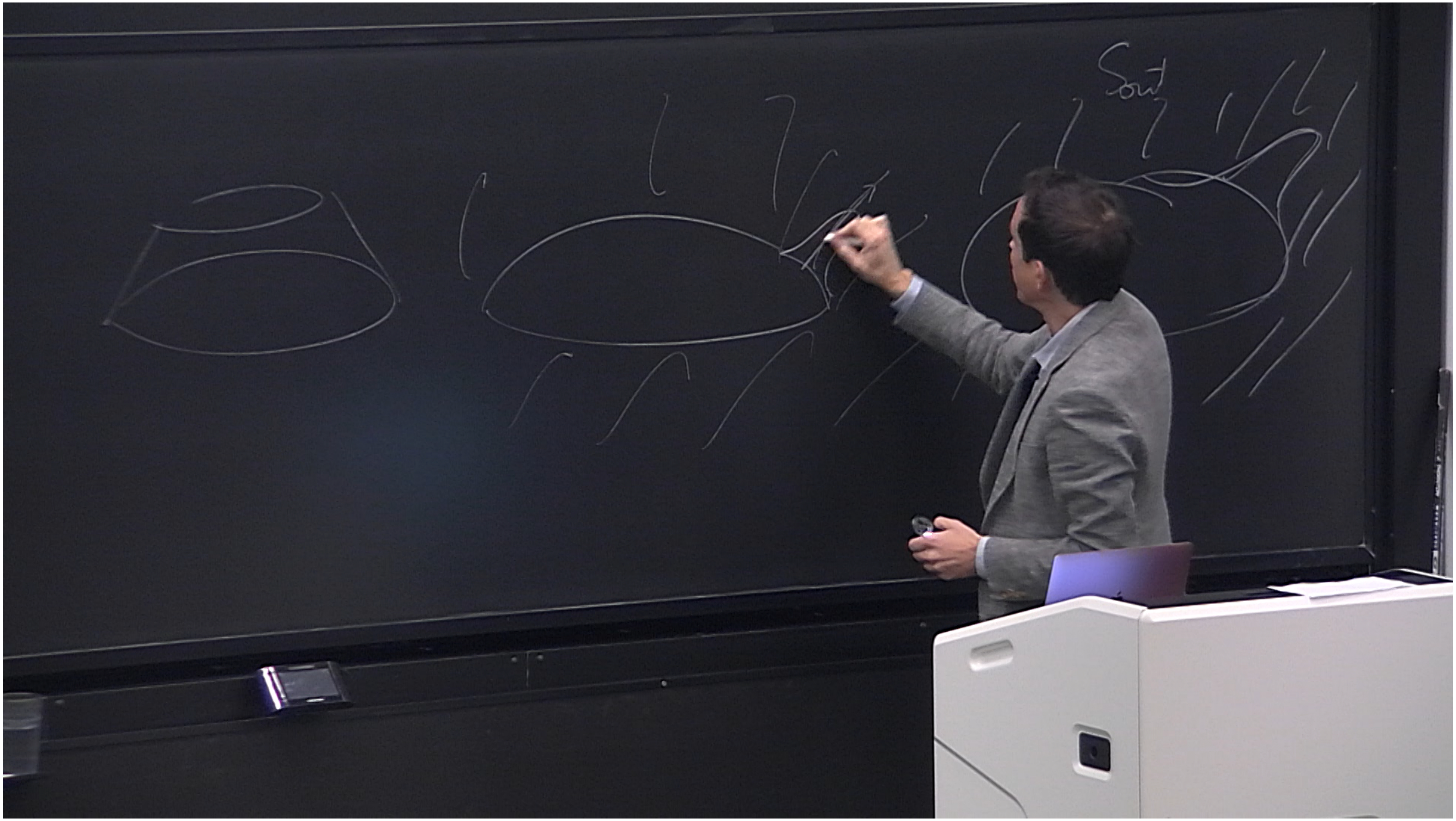
Generalized entropy can be defined not just for slices of event horizons. . .

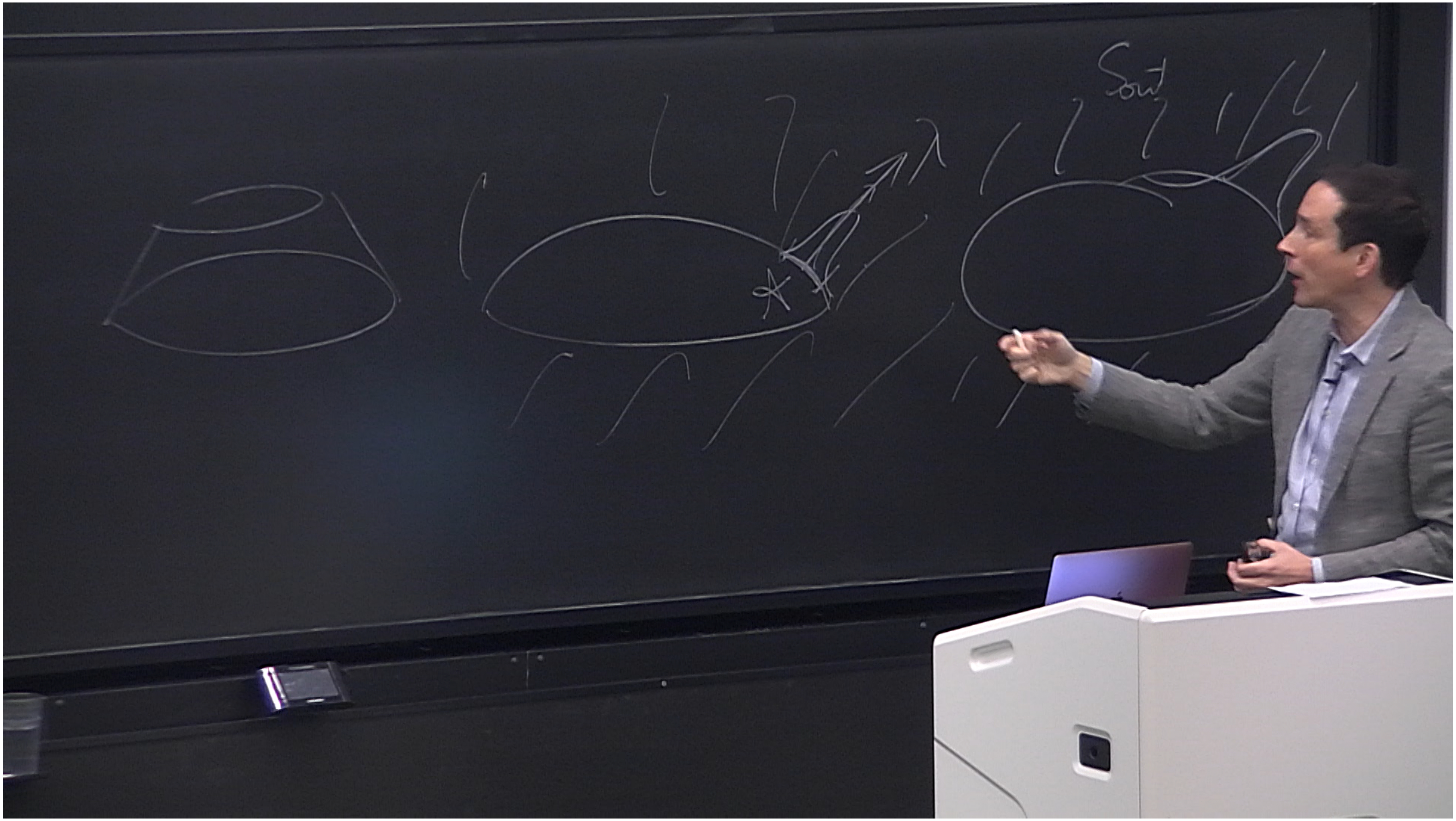


. . . but for any 2D surface σ that divides space into two sides.

This means we can consider what GR tells us about general surfaces.

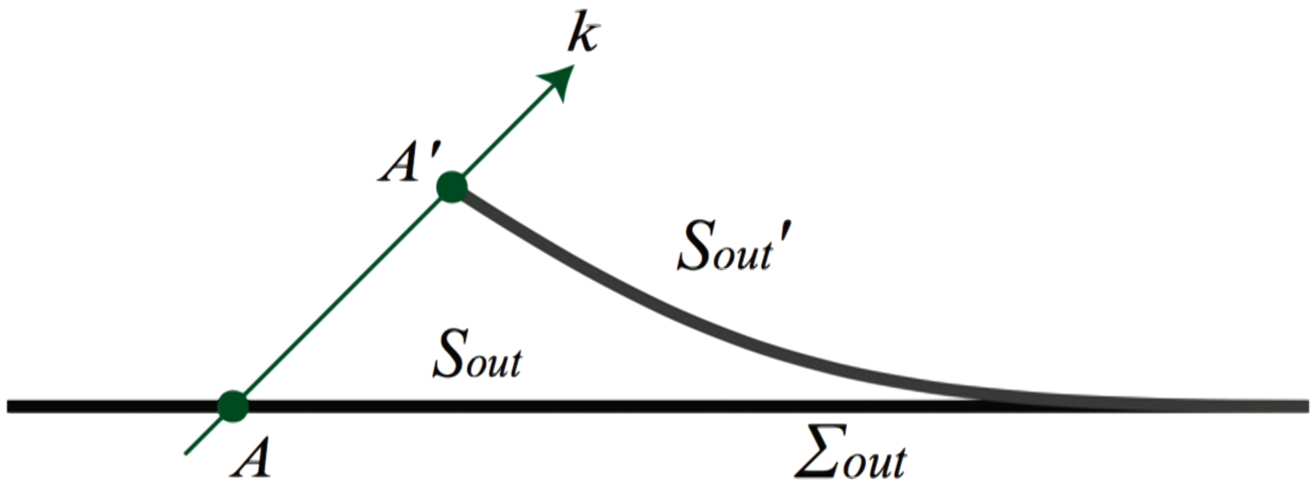
Let's see what happens when we add a quantum correction, $A \to A + 4G\hbar S_{\text{out}}$ to appropriate GR formulas.


Sout


S_out

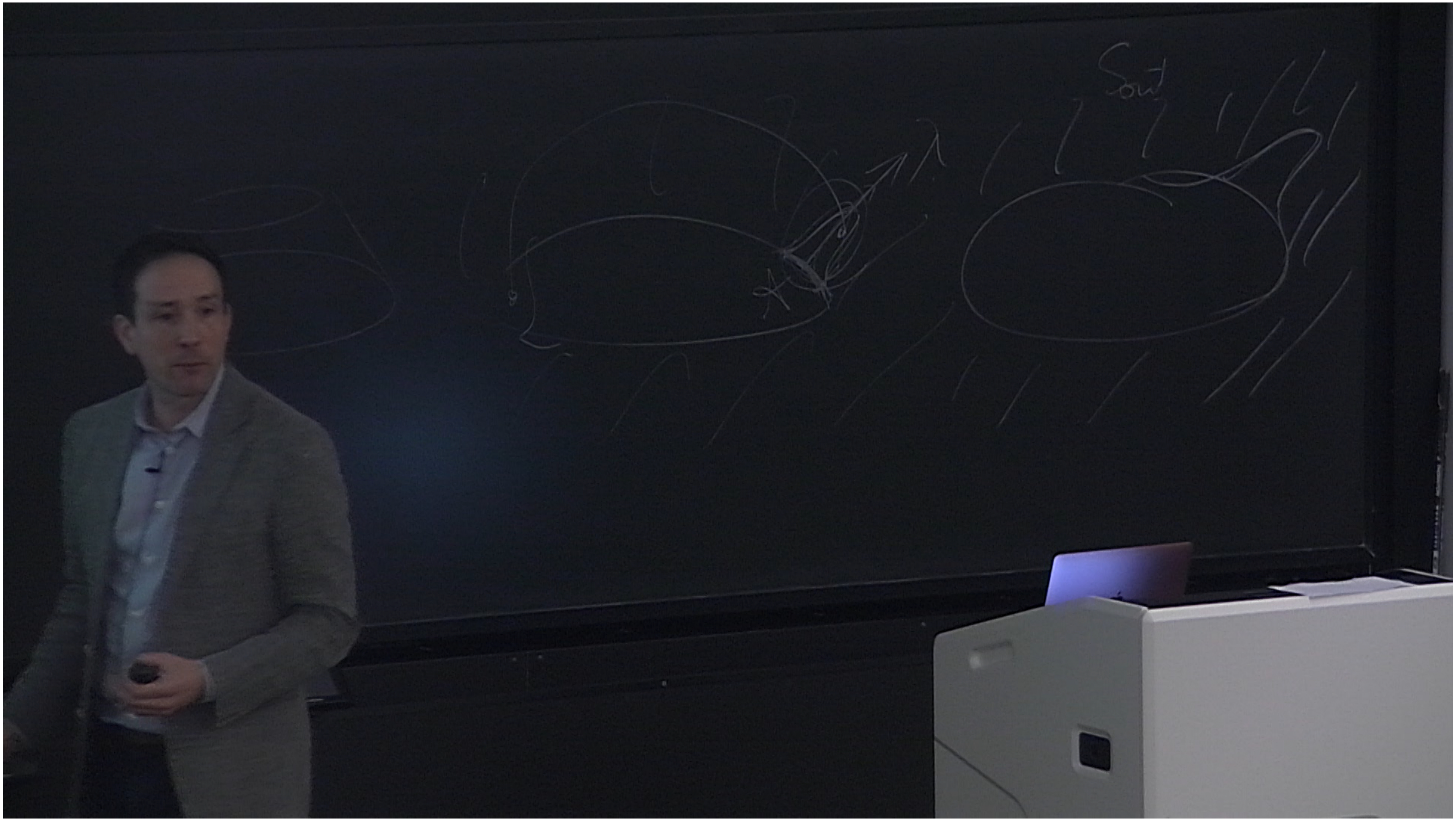
Classical Expansion → Quantum Expansion

Define a quantum expansion using $A \to S_{\text{gen}} \equiv A + 4G\hbar S_{\text{out}}$:



$\Theta[\sigma; y_1]$ is the rate (per unit area) at which the generalized entropy changes when an infinitesimal area element of σ at y_1 is deformed in one of its future orthogonal null directions.

RB, Fisher, Leichenauer & Wall, 2015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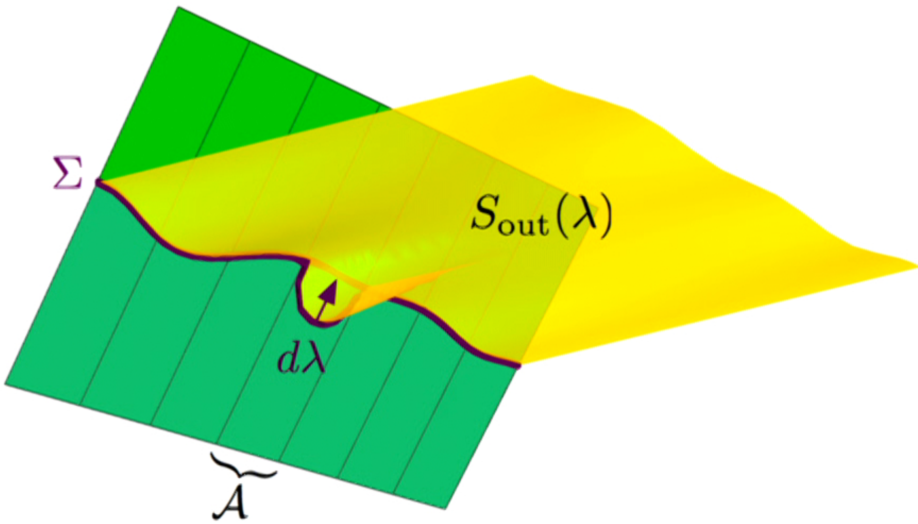

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Classical Expansion $\to$ Quantum Expansion

Define a quantum expansion using $A \to S_{\text{gen}} \equiv A + 4G\hbar S_{\text{out}}$:

$$\Theta = \theta + \frac{4G\hbar}{\mathcal{A}} S'_{\text{out}} \, .$$

$\Theta[\sigma; y_1]$ is the rate (per unit area) at which the generalized entropy changes when an infinitesimal area element of σ at y_1 is deformed in one of its future orthogonal null directions.

RB, Fisher, Leichenauer & Wall, 2015

Classical $\to$ Quantum Focussing Conjecture

The classical expansion will not increase along any light-ray,

$$\theta' \leq 0 \,,$$

if................................ the NEC holds.

Classical $\to$ Quantum Focussing Conjecture

The quantum expansion will not increase along any light-ray,

$$\Theta' \leq 0 \; ,$$

regardless of whether the NEC holds.

RB, Fisher, Leichenauer & Wall, 2015

Quantum Focussing Conjecture

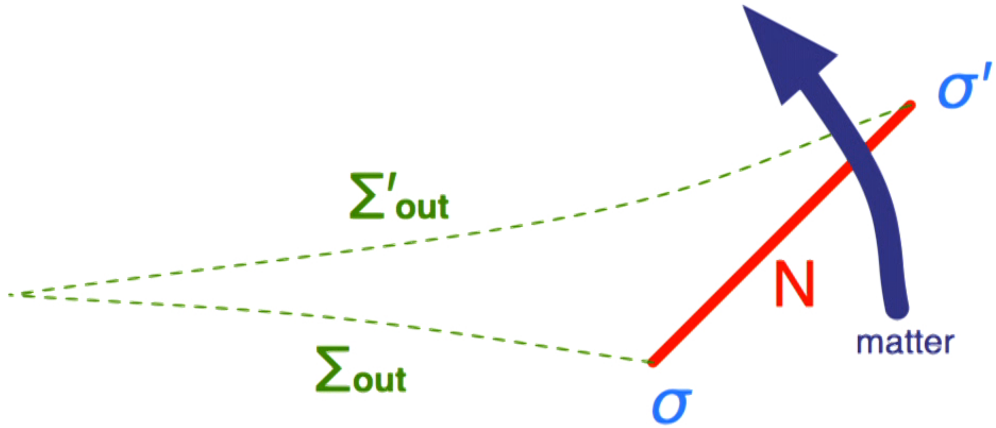
The QFC appears to be quite powerful. It implies:

1. Classical focussing theorem
2. Bekenstein's GSL (and so Hawking's area theorem) for black holes
3. Covariant Entropy Bound
4. New GSL for cosmology (and a new area theorem)
5. Quantum Null Energy Condition

I will briefly describe items 3 and 4, then 5 in more detail.

QFC Implies the Covariant Entropy Bound

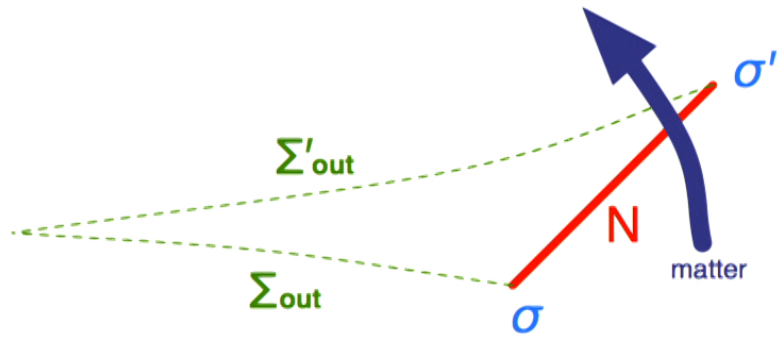
Consider the case where the generalized entropy is initially decreasing away from the surface σ.



Then the QFC implies that S_{gen} cannot increase anywhere along N, and hence

$$S_{gen}[\sigma'] \leq S_{gen}[\sigma] \ .$$

QFC Implies the Covariant Entropy Bound



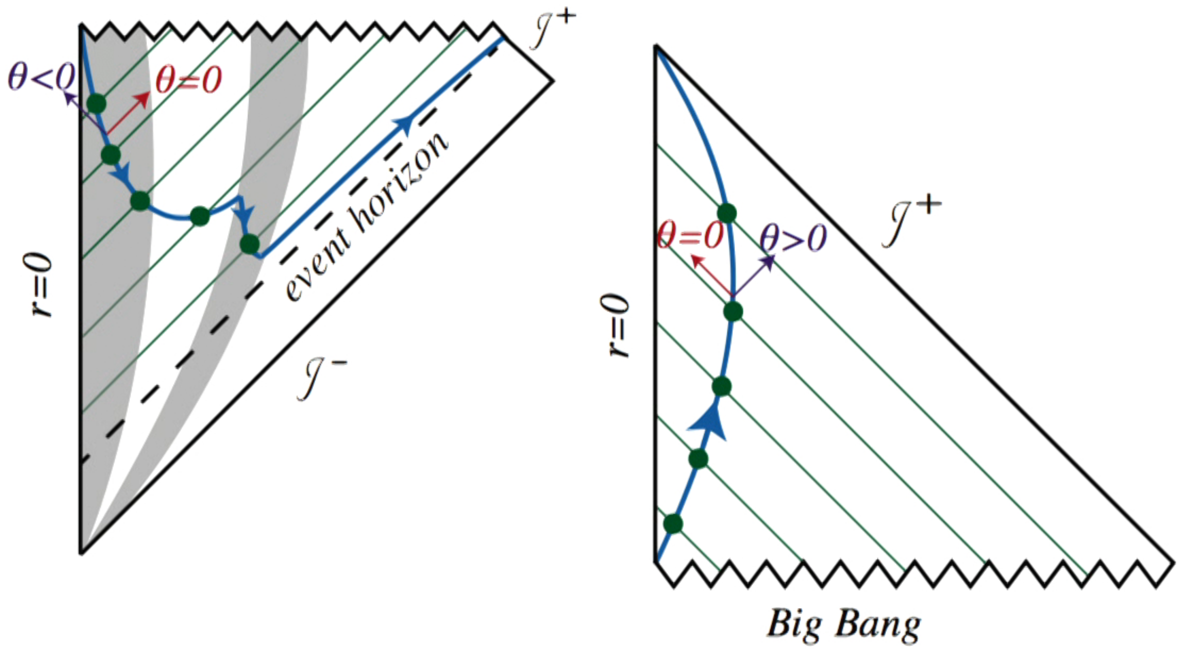
Unpack $S_{\text{gen}} = S_{\text{out}} + A/4G\hbar \rightarrow$

$$\frac{A[\sigma] - A[\sigma']}{4G\hbar} \geq S_{\text{out}}[\sigma'] - S_{\text{out}}[\sigma] \ .$$

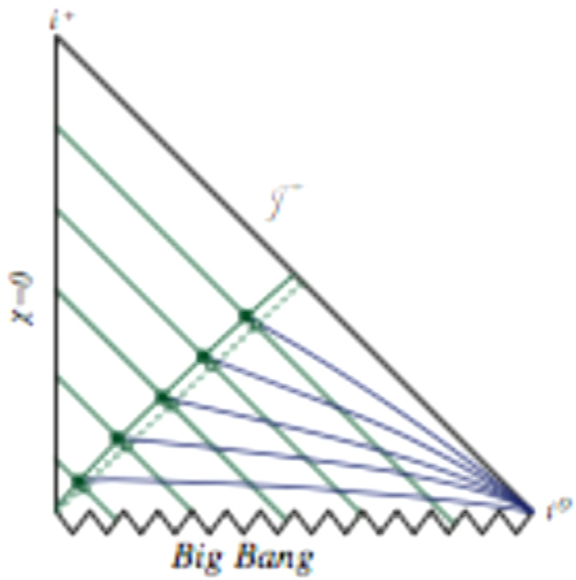
For isolated matter systems on N, and in the special case where $A[\sigma'] = 0$, we recover the Covariant Entropy Bound, $S(N) \leq A/4G\hbar$.

Area Theorem for Holographic Screens

A future holographic screen is a 2+1D hypersurface foliated by marginally trapped 2-surfaces $\sigma(r)$.



2nd Law for Cosmology



Definition: A future (past) Q-screen is a hypersurface foliated by marginally quantum (anti-)trapped surfaces.

Conjecture: A past or future Q-Screen obeys the GSL:

$$dS_{\text{gen}} \geq 0 \ .$$

The cosmological 2nd law, too, is implied by the Quantum Focussing Conjecture.

RB & Engelhardt, 2015c

GR $\rightarrow$ QG $\rightarrow$ QFT

We lifted a GR theorem to a (semi-classical) quantum gravity conjecture,

$$\Theta' \leq 0 \, .$$

Now, let's throw away* the gravity part, and learn something new about QFT! We obtain the Quantum Null Energy Condition.

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From the QFC to the QNEC

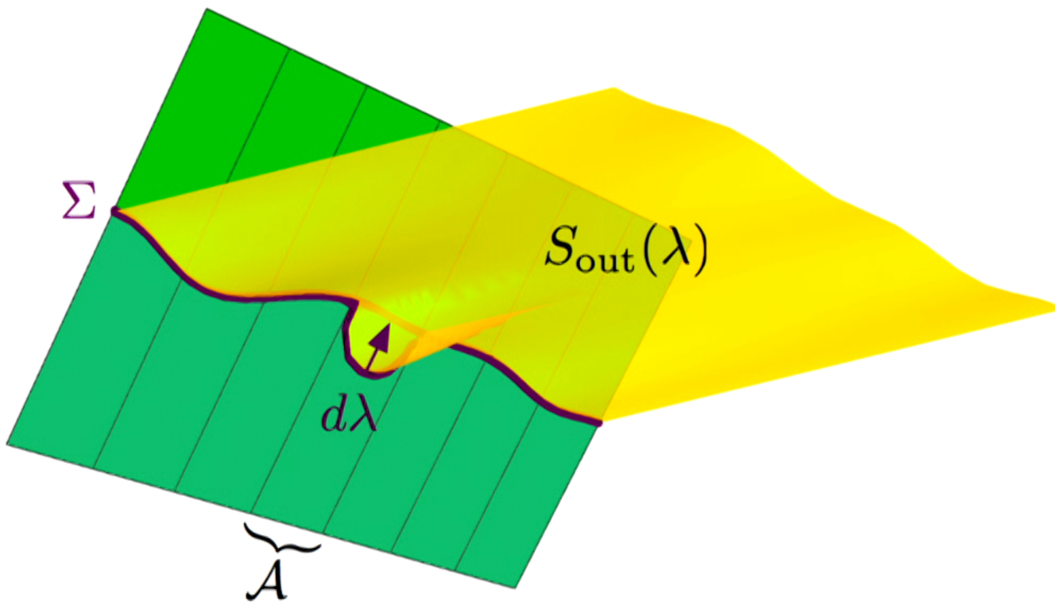
$$\Theta = \theta + \frac{4G\hbar}{\mathcal{A}} S'_{\text{out}} \cdot$$

Expanding Θ into its classical and quantum part, we notice that the first term generally dominates, because it is $O(G^0)$.

For example, if the initial surface is a sphere in Minkowski space, $\theta = 2/R$.

From the QFC to the QNEC

We can suppress such geometric contributions to θ, if we choose the initial surface to be a flat plane in Minkowski space. Then initially $\theta = 0$.



From the QFC to the QNEC

$$\Theta = \theta + \frac{4G\hbar}{\mathcal{A}} S'_{\text{out}} \ .$$

Away from the initial surface, θ will not vanish, because gravity bends the light-rays.

But now, the leading contributions to θ are $O(G)$, and so are of the same order as the "quantum correction."

From the QFC to the QNEC

$$\Theta = \theta + \frac{4G\hbar}{\mathcal{A}} S'_{\text{out}} \cdot$$

Finally, we can "tame" the effects of gravity by taking $G \to 0$.
This ensures that *only* the $O(G)$ term contributes to θ.

It also means that G cancels out as an overall factor. This is why the final result makes no reference to gravity at all. It is a QFT statement.

From the QFC to the QNEC

The QFC becomes

$$0 \geq \Theta' \;=\; \theta' + \frac{4G\hbar}{\mathcal{A}}(S''_{\text{out}} - S'_{\text{out}}\theta)$$
$$\;=\; -\frac{1}{2}\theta^2 - \varsigma^2 - 8\pi G\langle T_{kk}\rangle + \frac{4G\hbar}{\mathcal{A}}(S''_{\text{out}} - S'_{\text{out}}\theta)$$

For a null surface with vanishing classical shear and expansion, $\theta = \varsigma = 0$, this implies

$$\langle T_{kk}\rangle \geq \frac{\hbar}{2\pi} \lim_{\mathcal{A}\to 0} \frac{S''_{\text{out}}}{\mathcal{A}} \qquad (QNEC) \; .$$

Quantum Null Energy Condition

$$\langle T_{kk} \rangle \geq \frac{\hbar}{2\pi} \lim_{\mathcal{A}\to 0} \frac{S''_{\text{out}}}{\mathcal{A}} \,.$$

First lower bound on the local energy density.

RHS: nonlocal, information-theoretic quantity.

Conversely, the local energy density limits how rapidly one can increase the rate at which information is acquired.

Quantum Null Energy Condition

$$\langle T_{kk} \rangle \geq \frac{\hbar}{2\pi} \lim_{\mathcal{A}\to 0} \frac{S''_{\text{out}}}{\mathcal{A}} .$$

Since *G* dropped out, we can try to prove this statement within QFT.

Proof for Free Fields

4.1 The Replica Trick

The replica trick prescription is to use the following formula for the von Neumann entropy [50]:

$$S_{\rm out} = -\,{\rm Tr}[\rho \log \rho] = (1 - n\partial_n) \log {\rm Tr}[\rho^n]\Big|_{n=1} \,. \tag{4.1}$$

This can be written as

$$S_{\rm out} = \mathcal{D} \log \tilde{Z}_n \tag{4.2}$$

where $\tilde{Z}_n \equiv {\rm Tr}[\rho^n]$[6] and the operator $\mathcal{D}$ is defined by

$$\mathcal{D} f(n) \equiv (1 - n\partial_n) f(n)\big|_{n=1} \tag{4.3}$$

where $f(n)$ is some function of n. Since $\tilde{Z}_n$ is only defined for integer values of n, we first must analytically continue to real $n > 0$ in order to apply the $\mathcal{D}$ operator. The analytic continuation step is in general quite tricky, and will require care in our calculation. (Our analytic continuation is performed in Section 4.4.)

On general grounds discussed above, we must study the second-order term in a perturbative expansion of the entropy about the state $\rho^{(0)}$. Suppressing all λ dependence, we have

$$\tilde{Z}_n = {\rm Tr}\left[(\rho^{(0)} + \sigma)^n\right] \,. \tag{4.4}$$

Expanding $\tilde{Z}_n$ to quadratic order to isolate $S^{(2)''}$, we have

$$\tilde{Z}_n = {\rm Tr}\left[(\rho^{(0)})^n\right] + n\,{\rm Tr}\left[\sigma(\rho^{(0)})^{n-1}\right] + \frac{n}{2}\sum_{k=0}^{n-2} {\rm Tr}\left[(\rho^{(0)})^k \sigma (\rho^{(0)})^{n-k-2}\sigma\right] + \cdots \,. \tag{4.5}$$

Using the notation introduced in (3.10) we can write

$$\tilde{Z}_n = {\rm Tr}\left[(\rho^{(0)})^n\right] + n\,{\rm Tr}\left[\mathcal{O}(\rho^{(0)})^n\right] + \frac{n}{2}\sum_{k=1}^{n-1} {\rm Tr}\left[(\rho^{(0)})^{-k}\mathcal{O}(\rho^{(0)})^k \mathcal{O}(\rho^{(0)})^n\right] + \cdots \,. \tag{4.6}$$

We denote by $\mathcal{O}^{(k)}$ the operator $\mathcal{O}$ conjugated by $(\rho^{(0)})^k$:

$$\mathcal{O}^{(k)} \equiv (\rho^{(0)})^{-k}\mathcal{O}(\rho^{(0)})^k \tag{4.7}$$

$$= e^{2\pi k K}\mathcal{O} e^{-2\pi k K} \,. \tag{4.8}$$

[6] In the replica trick one often works with the partition function Z_n, in terms of which $\tilde{Z}_n = Z_n/(Z_1)^n$. Choosing Z_n over $\tilde{Z}_n$ is equivalent to choosing a different normalization for ρ, but we find it convenient to keep ${\rm Tr}\,\rho = 1$.

Since $\mathcal{O}$ is the integral of operators with angles $0 \le \theta < 2\pi$, it follows that $\mathcal{O}^{(k)}$ will be an integral over operators with angles $2\pi k < \theta < 2\pi(k+1)$.[7] Furthermore, since rotations by $2\pi k$ commute with translations by λ, we can obtain $\mathcal{O}^{(k)}$ from $\mathcal{O}$ simply by letting the range of integration that defines $\mathcal{O}$ shift from $[0, 2\pi]$ to $[2\pi k, 2\pi(k+1)]$, as long as we define $f_{ij}(r, \theta)$ to be periodic in θ with period 2π.

It will also be convenient to introduce an angle-ordered expectation value, defined as

$$\langle \ldots \rangle_n \equiv \frac{{\rm Tr}[(\rho^{(0)})^n \mathcal{T}[\ldots]]}{{\rm Tr}[(\rho^{(0)})^n]}, \tag{4.9}$$

where $\mathcal{T}[\ldots]$ is θ-ordering. Then (4.6) can be written

$$\tilde{Z}_n = {\rm Tr}\left[(\rho^{(0)})^n\right]\left(1 + n\langle\mathcal{O}\rangle_n + \frac{n}{2}\sum_{k=1}^{n-1}\langle\mathcal{O}^{(k)}\mathcal{O}\rangle_n\right) + \cdots \,. \tag{4.10}$$

Taking the logarithm of $\tilde{Z}_n$ and extracting the part quadratic in σ gives

$$\log \tilde{Z}_n \supset \frac{n}{2}\sum_{k=1}^{n-1}\langle\mathcal{O}^{(k)}\mathcal{O}\rangle_n - \frac{n^2}{2}\langle\mathcal{O}\rangle_n^2 \,, \tag{4.11}$$

where we have kept only the part quadratic in $\mathcal{O}$. The contribution of the second term to the entanglement entropy will be proportional to $\langle\mathcal{O}\rangle$, which vanishes because of the tracelessness of σ. Therefore we only need to consider the first term.

Since we are considering angle-ordered expectation values, we have the identity

$$\left\langle\left(\sum_{k=0}^{n-1}\mathcal{O}^{(k)}\right)^2\right\rangle_n = n\sum_{k=0}^{n-1}\langle\mathcal{O}^{(k)}\mathcal{O}\rangle_n \,, \tag{4.12}$$

and so from the first term in (4.11) the relevant part of $\log \tilde{Z}_n$ can be written as

$$\log \tilde{Z}_n \supset -\frac{n}{2}\langle\mathcal{O}\mathcal{O}\rangle_n + \frac{1}{2}\left\langle\left(\sum_{k=0}^{n-1}\mathcal{O}^{(k)}\right)^2\right\rangle_n \,. \tag{4.13}$$

Restoring the λ dependence and taking λ derivatives gives

$$S^{(2)''} = \frac{\partial^2}{\partial\lambda^2}\bigg|_{\lambda=0} \mathcal{D} \log \tilde{Z}_n(\lambda) \tag{4.14}$$

$$= \mathcal{D}\,\frac{-n}{2}\langle\mathcal{O}\mathcal{O}\rangle_n'' + \mathcal{D}\,\frac{1}{2}\left\langle\left(\sum_{k=0}^{n-1}\mathcal{O}^{(k)}\right)^2\right\rangle_n'' \,. \tag{4.15}$$

[7] One could worry that the phase factor in (3.5) spoils this relation, but notice that the phase has period 2π in θ and so does not appear when shifting by $2\pi k$.

Proof for Free Fields

- applies to free or superrenormalizable bosonic fields, stationary null surfaces
- null quantization $\rightarrow$ operator algebra factorizes over generators ("pencils")
- each pencil is 1+1 CFT
- in any global finite energy state, individual pencils are near the vacuum $\rightarrow$ small expansion parameter

RB, Fisher, Koeller, Leichenauer & Wall, 2015

Proof for Free Fields

- expand state, $\rho = \rho_0 + \sigma(\lambda)$
- expand entropy in powers of σ, $S = \sum S^{(i)}$
- find that $(S^{(0)} + S^{(1)})''$ would saturate the QNEC
- compute $S^{(2)''}$ using replica trick, prove < 0.

RB, Fisher, Koeller, Leichenauer & Wall, 2015

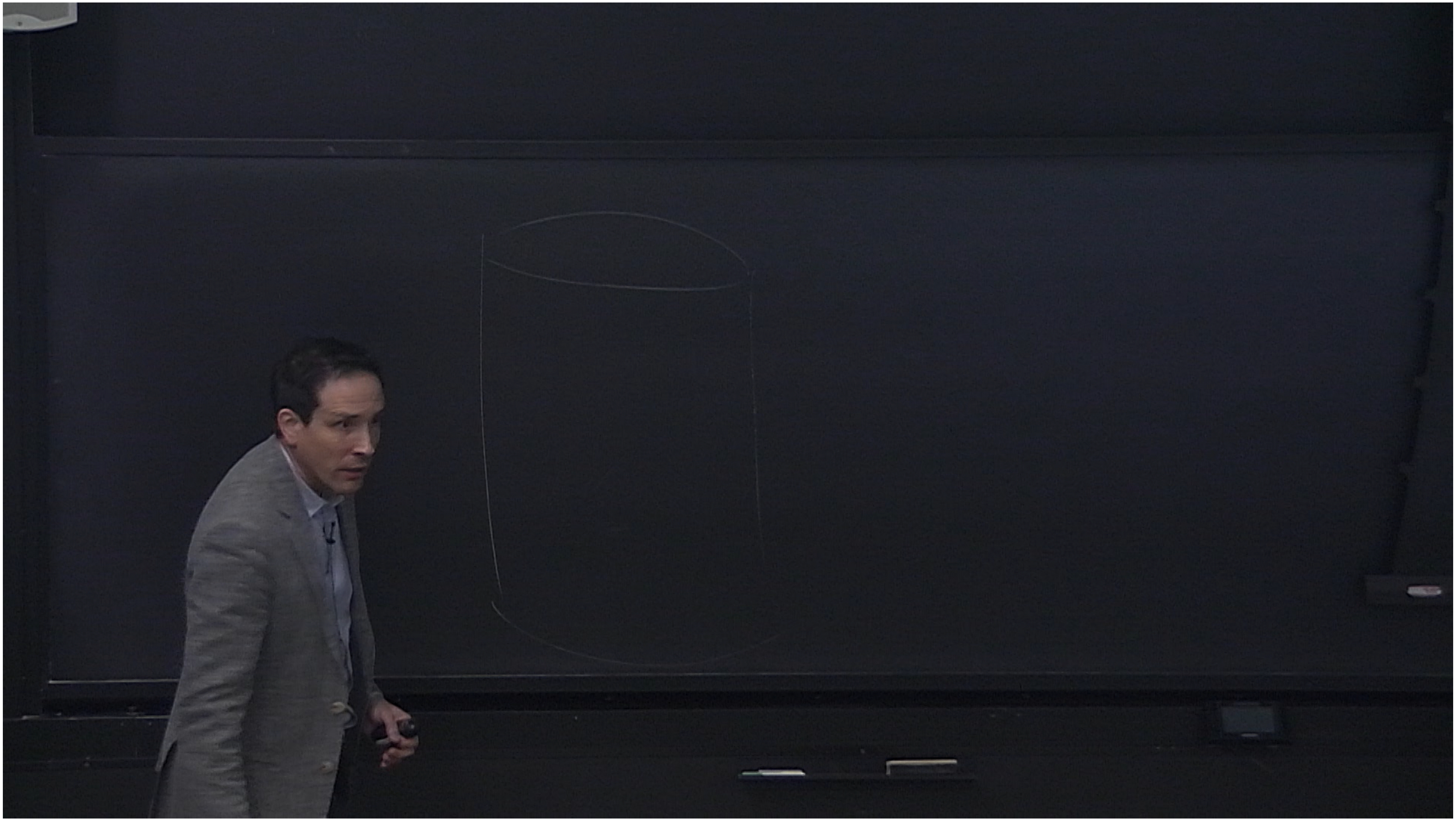
Holographic Proof

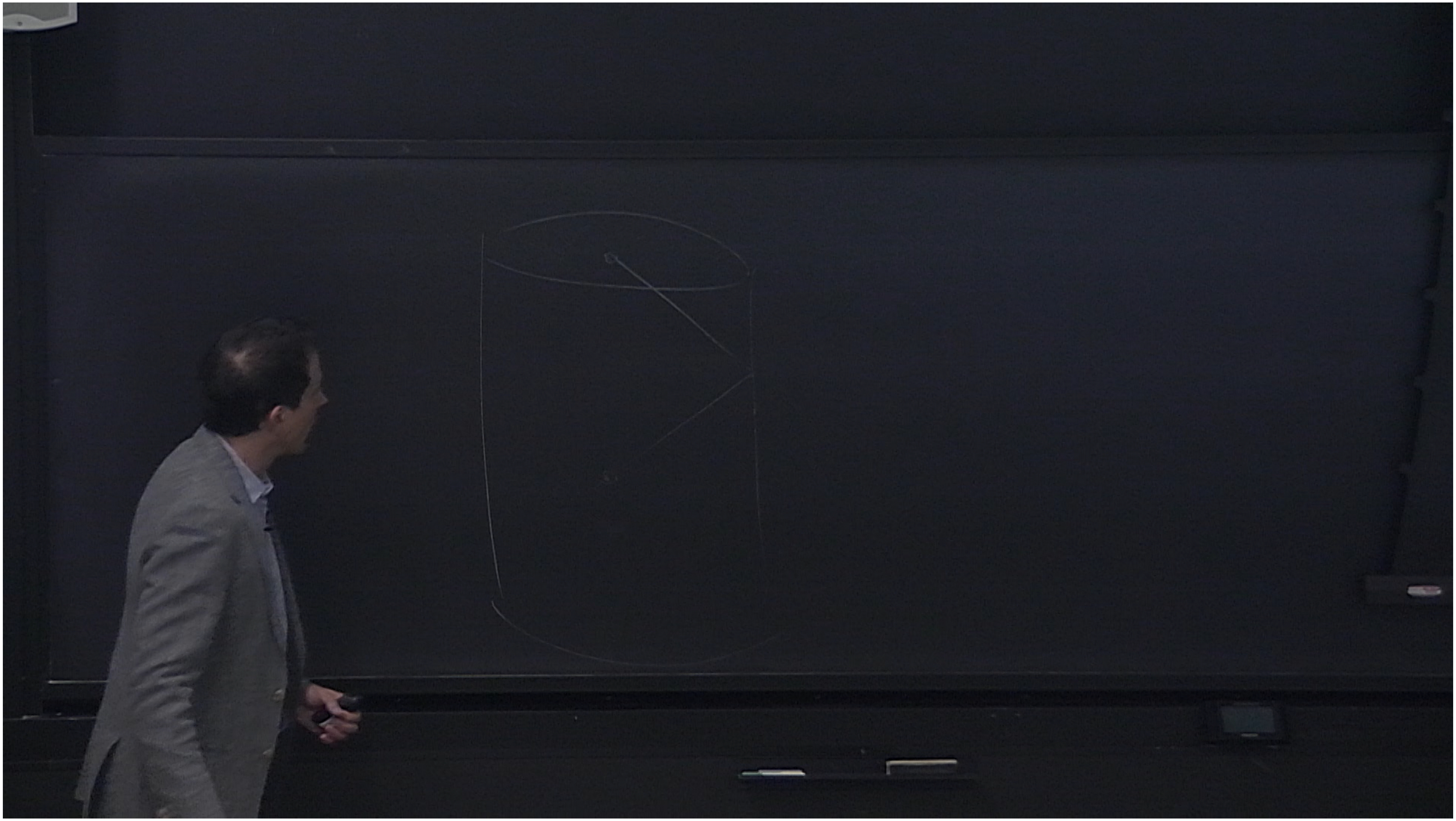
Interacting theories with a gravity dual ("AdS/CFT") satisfy the QNEC.

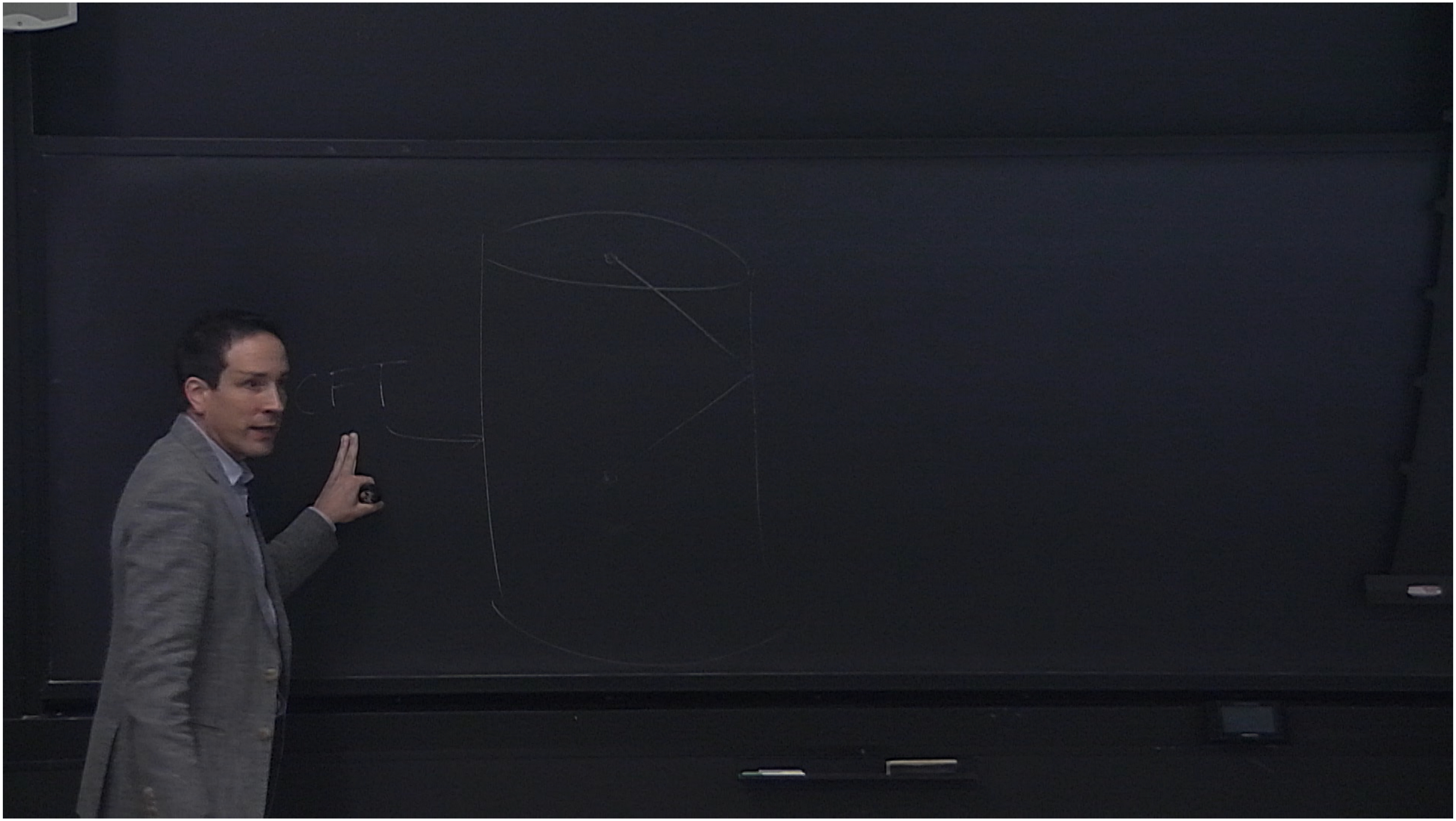
Koeller & Leichenauer, 2015

This follows

- via Ryu-Takayanagi (2006)
- from entanglement wedge nesting (Wall 2012),
- which in turn is necessary for consistent subregion duality.



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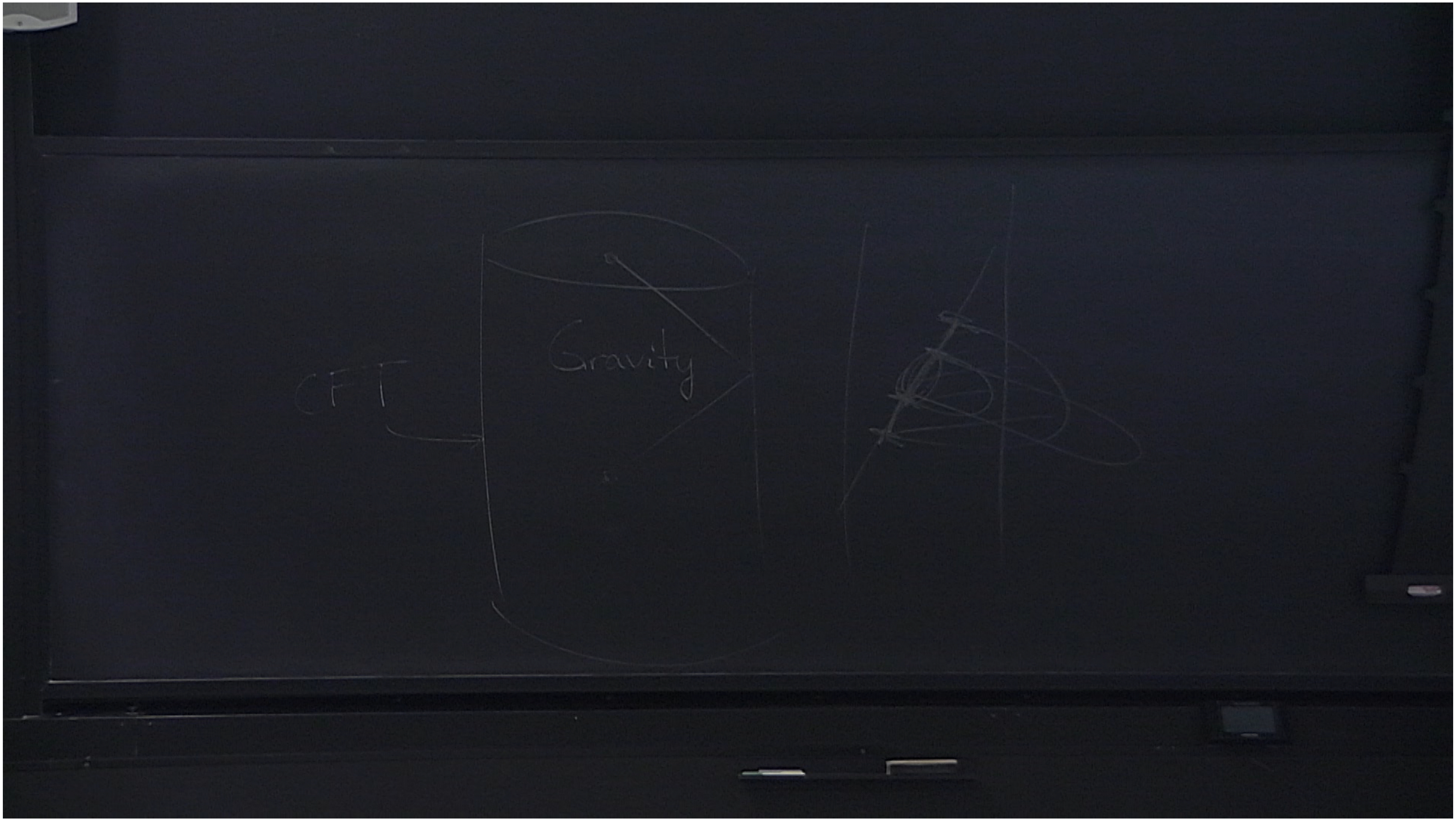
CFT
Gravity

Proof for Interacting Fields

A General Proof of the Quantum Null Energy Condition

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Abstract

We prove a conjectured lower bound on $\langle T_{--}(x)\rangle_\psi$ in any state ψ of a relativistic QFT dubbed the Quantum Null Energy Condition (QNEC). The bound is given by the second order shape deformation, in the null direction, of the geometric entanglement entropy of an entangling cut passing through x. Our proof involves a combination of the two independent methods that were used recently to prove the weaker Averaged Null Energy Condition (ANEC). In particular the properties of modular Hamiltonians under shape deformations for the state ψ play an important role, as do causality considerations. We study the two point function of a "probe" operator $\mathcal{O}$ in the state ψ and use a lightcone limit to evaluate this correlator. Instead of causality in time we consider *causality in modular time* for the modular evolved probe operators, which we constrain using Tomita-Takesaki theory as well as certain generalizations pertaining to the theory of modular inclusions. The QNEC follows from very similar considerations to the derivation of the chaos bound and the causality sum rule. We use a kind of defect Operator Product Expansion to apply the replica trick to these modular flow computations, and the displacement operator plays an important role. Our approach was inspired by the AdS/CFT proof of the QNEC which follows from properties of the Ryu-Takayanagi (RT) surface near the boundary of AdS, combined with the requirement of entanglement wedge nesting. Our methods were, as such, designed as a precise probe of the RT surface close to the boundary of a putative gravitational/stringy dual of *any* QFT with an interacting UV fixed point. We also prove a higher spin version of the QNEC.

Proof for Interacting Fields

Combines techniques used in two recent proofs of the ANEC (from quantum info/from causality):

- Modular flow
- Monotonicity of the full modular Hamiltonian

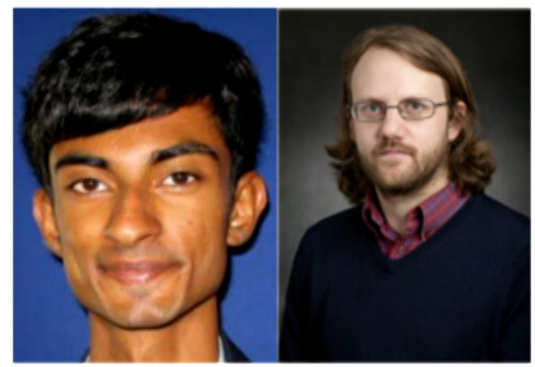
Recent and Ongoing Work

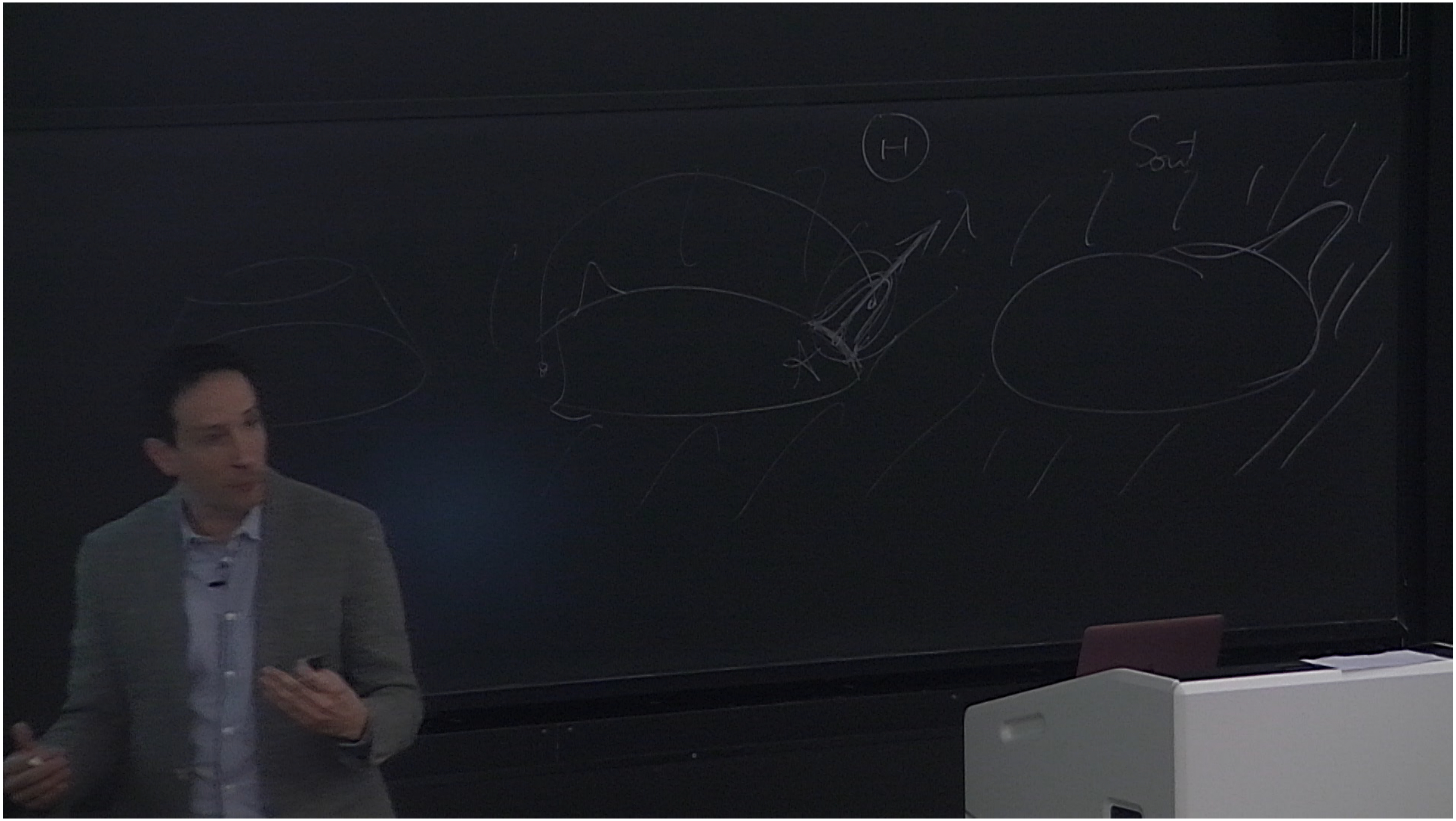
Saturation of the Diagonal QNEC!

Holographic proof & general conjecture

Leichenauer, Levine, Shahbazi-Moghaddam (2018)

Defect OPE +Faulkner, Chandrasekharan, Balakrishnan *(in progress)*

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