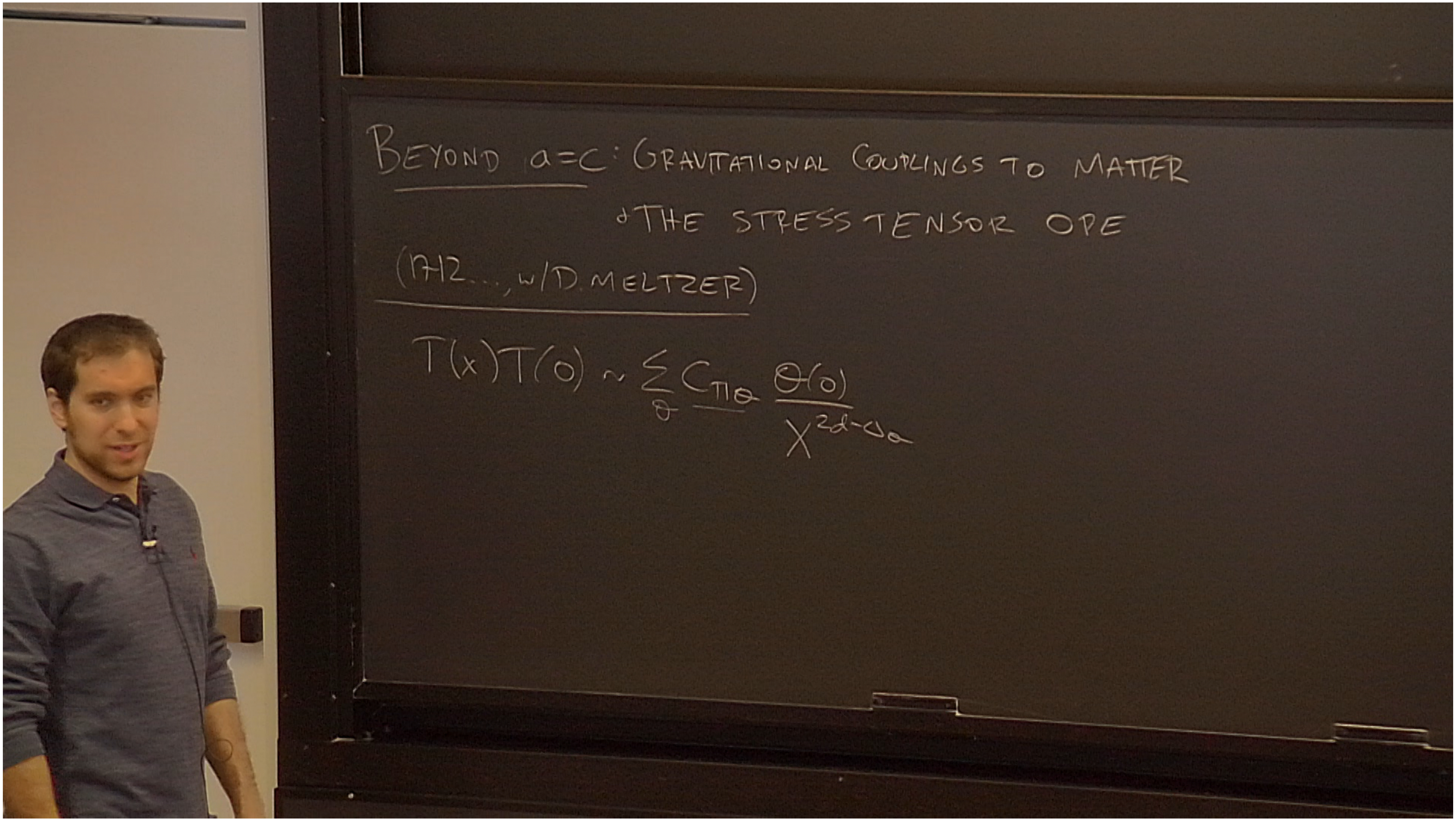
Title: Beyond a=c: Gravitational Couplings to Matter and the Stress Tensor OPE

Date: Feb 06, 2018 02:30 PM

URL: http://pirsa.org/18020078

Abstract: <p>We derive constraints on the operator product expansion of two stress tensors in conformal field theories (CFTs). In large N CFTs with a large gap to single-trace higher spin operators, we show that the coupling of two stress tensors to other single-trace operators ("TTO") is suppressed by powers of the higher spin gap, dual to the mass scale of higher spin particles in AdS. The absence of light higher spin particles is thus a necessary condition for the existence of a consistent truncation to general relativity in AdS. We are led to propose that, in general, the inverse gap scale is the CFT "dual" of an AdS derivative in a classical action. These results are derived by imposing unitarity on mixed systems of spinning four-point functions in the Regge limit. Using the same method, but without imposing a large gap, we derive new inequalities on TTO-type couplings valid in any CFT.</p>

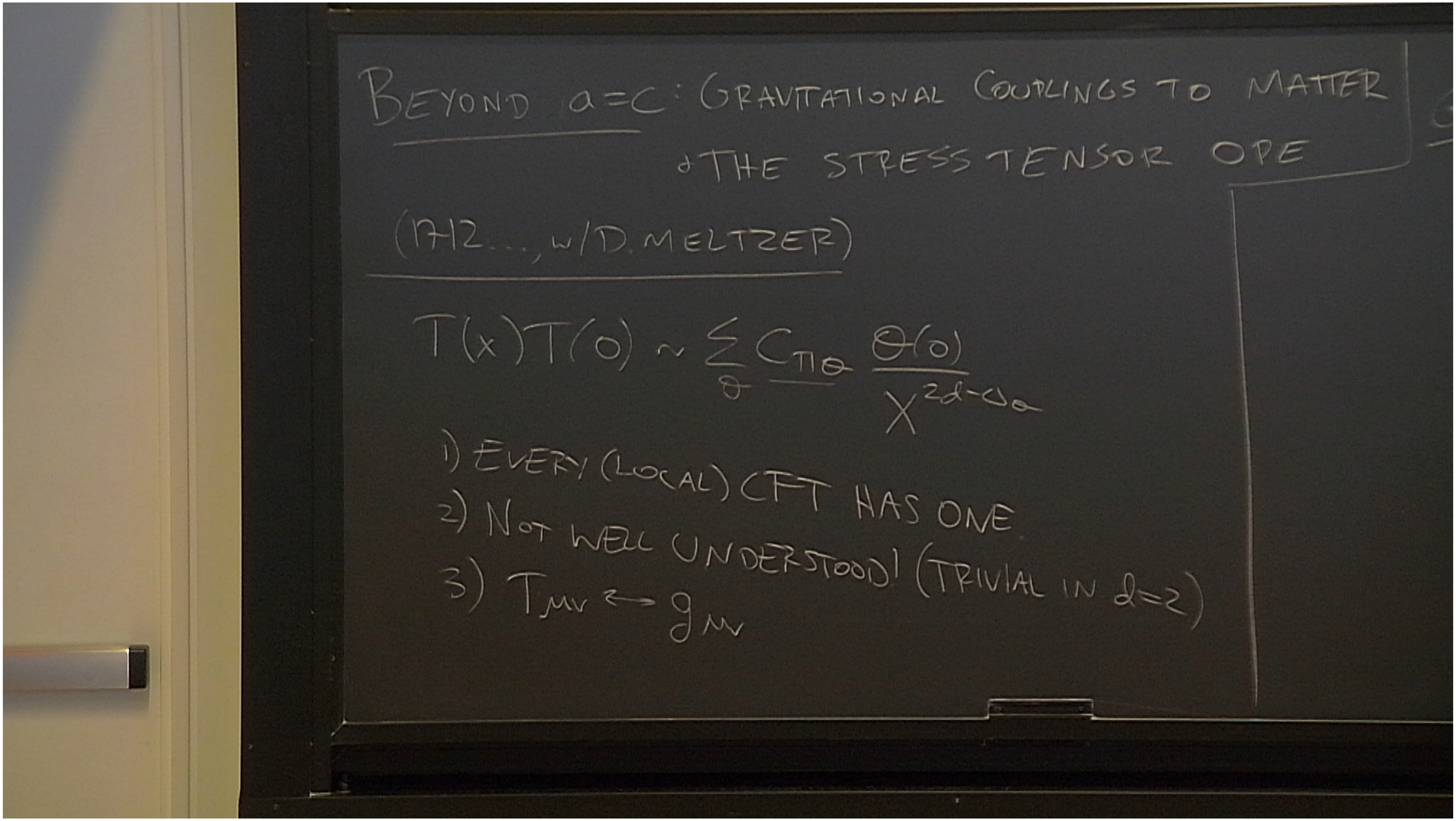


BEYOND a=c: GRAVITATIONAL COUPLINGS TO MATTER

& THE STRESS TENSOR OPE

(1712...., w/ D. MELTZER)

$$T(x)T(0) \sim \sum_{\mathcal{O}} \underline{C_{TT\mathcal{O}}} \frac{\mathcal{O}(0)}{x^{2d-\Delta_{\mathcal{O}}}}$$

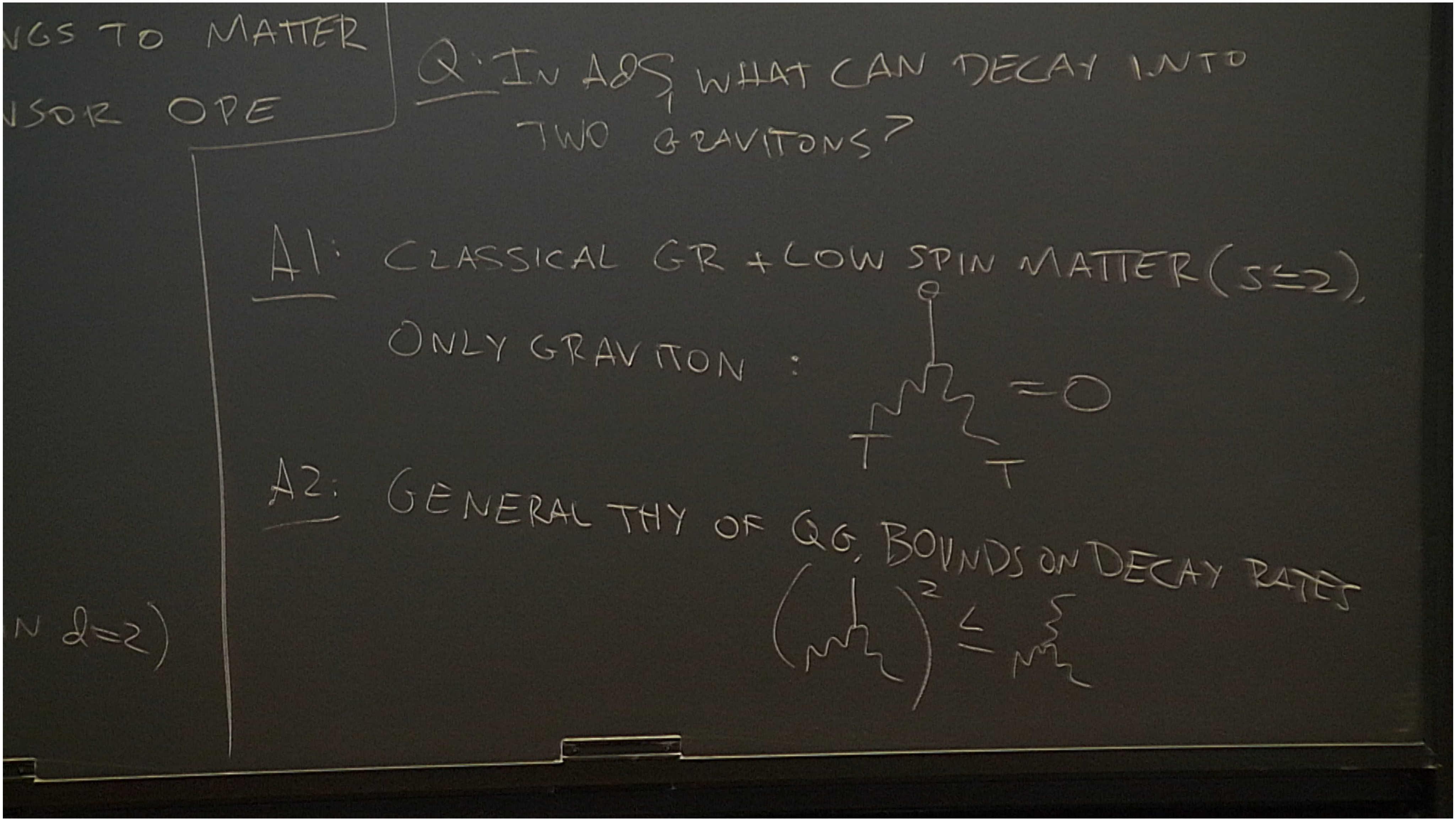


BEYOND $a=c$: GRAVITATIONAL COUPLINGS TO MATTER & THE STRESS TENSOR OPE

(1712..., w/ D. MELTZER)

$$T(x)T(0) \sim \sum_{\mathcal{O}} \underline{C_{TT\mathcal{O}}} \frac{\mathcal{O}(0)}{x^{2d-\Delta_{\mathcal{O}}}}$$

1) EVERY (LOCAL) CFT HAS ONE
2) NOT WELL UNDERSTOOD! (TRIVIAL IN $d=2$)
3) $T_{\mu\nu} \leftrightarrow g_{\mu\nu}$



NGS TO MATTER

NSOR OPE

N $d=2$)

Q: IN AdS, WHAT CAN DECAY INTO TWO GRAVITONS?

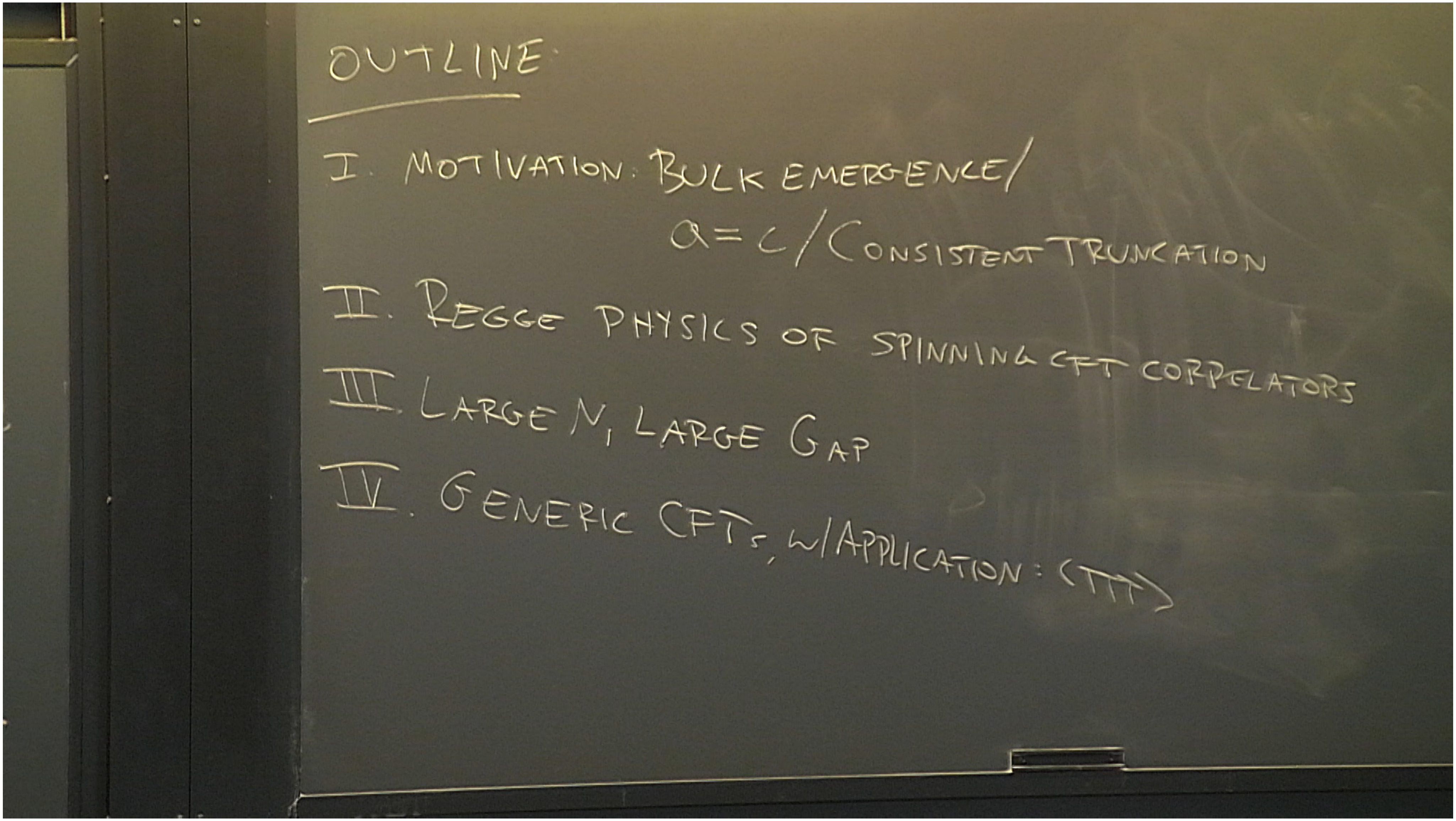
A1: CLASSICAL GR + LOW SPIN MATTER ($s \leq 2$),

ONLY GRAVITON :

$= 0$

A2: GENERAL THY OF QG, BOUNDS ON DECAY RATES

$(\ )^2 \leq \sum$



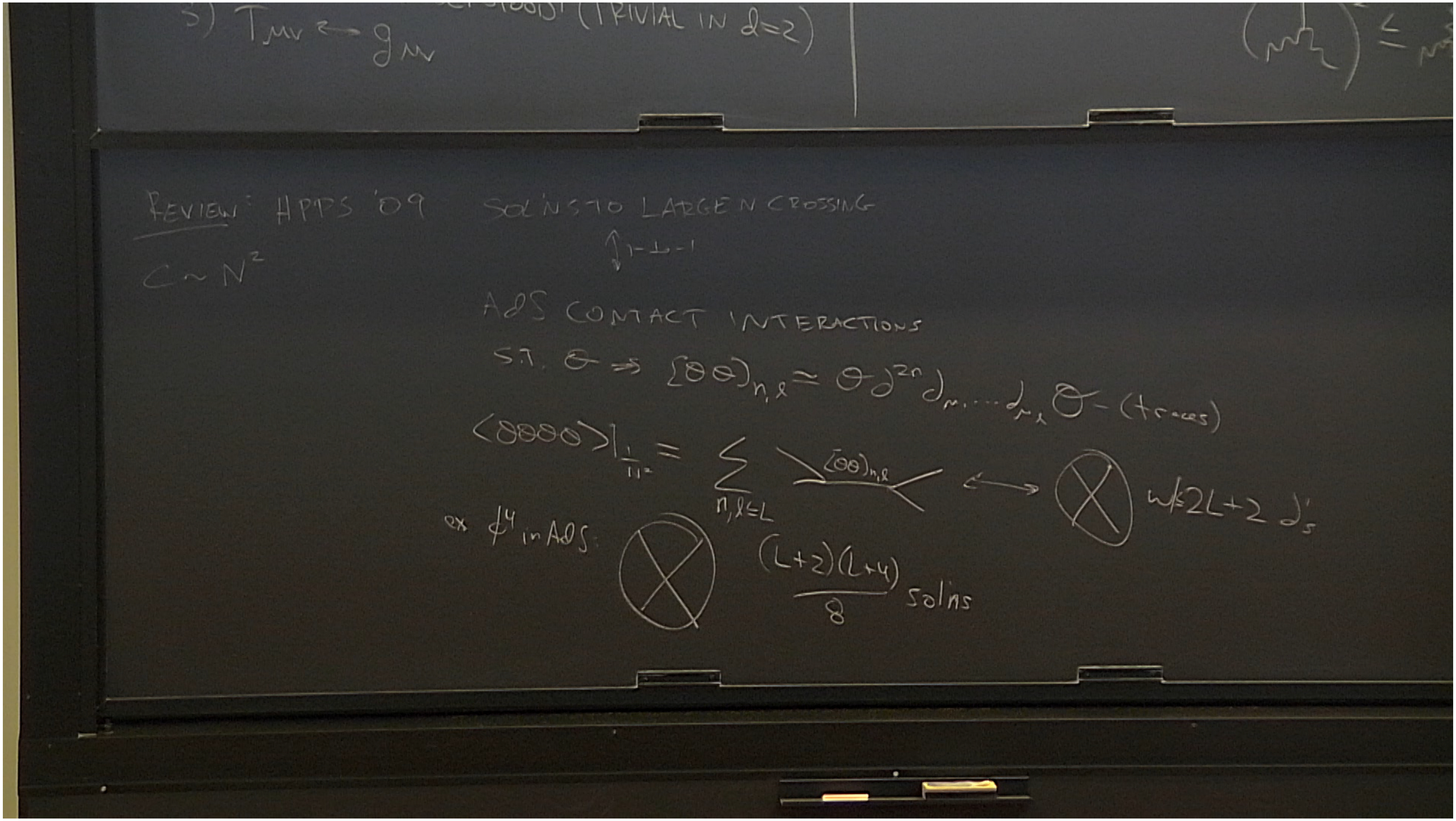
OUTLINE:

I. MOTIVATION: BULK EMERGENCE/ $a=c$/CONSISTENT TRUNCATION

II. REGGE PHYSICS OF SPINNING CFT CORRELATORS

III. LARGE N, LARGE GAP

IV. GENERIC CFTs, w/APPLICATION: $\langle TTT \rangle$



5) $T_{\mu\nu} \leftrightarrow g_{\mu\nu}$ (TRIVIAL IN $d=2$)

REVIEW: HPPS '09

$C \sim N^2$

SOL'NS TO LARGE N CROSSING

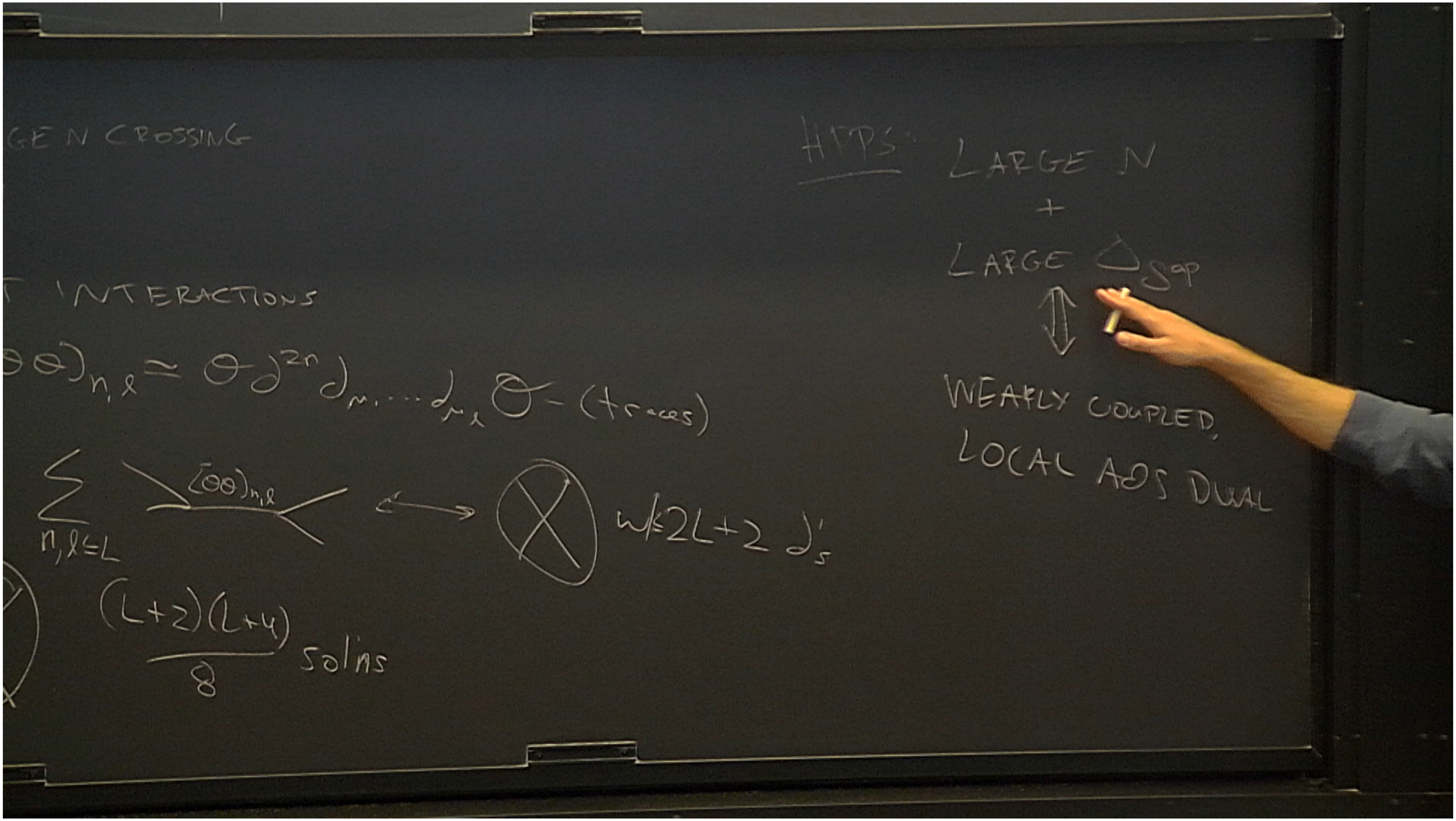
$\updownarrow$ 1-1-1

AdS CONTACT INTERACTIONS

S.T. $\mathcal{O} \Rightarrow [\mathcal{O}\mathcal{O}]_{n,\ell} \simeq \mathcal{O}\partial^{2n}\partial_{\mu_1}\cdots\partial_{\mu_\ell}\mathcal{O} - (\text{traces})$

$\langle\mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle\big|_{\frac{1}{N^2}} = \sum_{n,\ell\le L}$ $[\mathcal{O}\mathcal{O}]_{n,\ell}$ $\longleftrightarrow$ w/ $\le 2L+2$ ∂'s

ex ϕ^4 in AdS: $\frac{(L+2)(L+4)}{8}$ solns



GEN CROSSING

T INTERACTIONS

$$\mathcal{O}\mathcal{O})_{n,\ell} \simeq \mathcal{O}\,\partial^{2n}\partial_{\mu_1}\cdots\partial_{\mu_\ell}\mathcal{O} - (\text{traces})$$

$$\sum_{n,\ell\le L} \quad [\mathcal{O}\mathcal{O}]_{n,\ell} \quad \longleftrightarrow \quad \text{w/} \le 2L+2\ \partial\text{'s}$$

$$\frac{(L+2)(L+4)}{8}\ \text{sol'ns}$$

HPPS: LARGE N

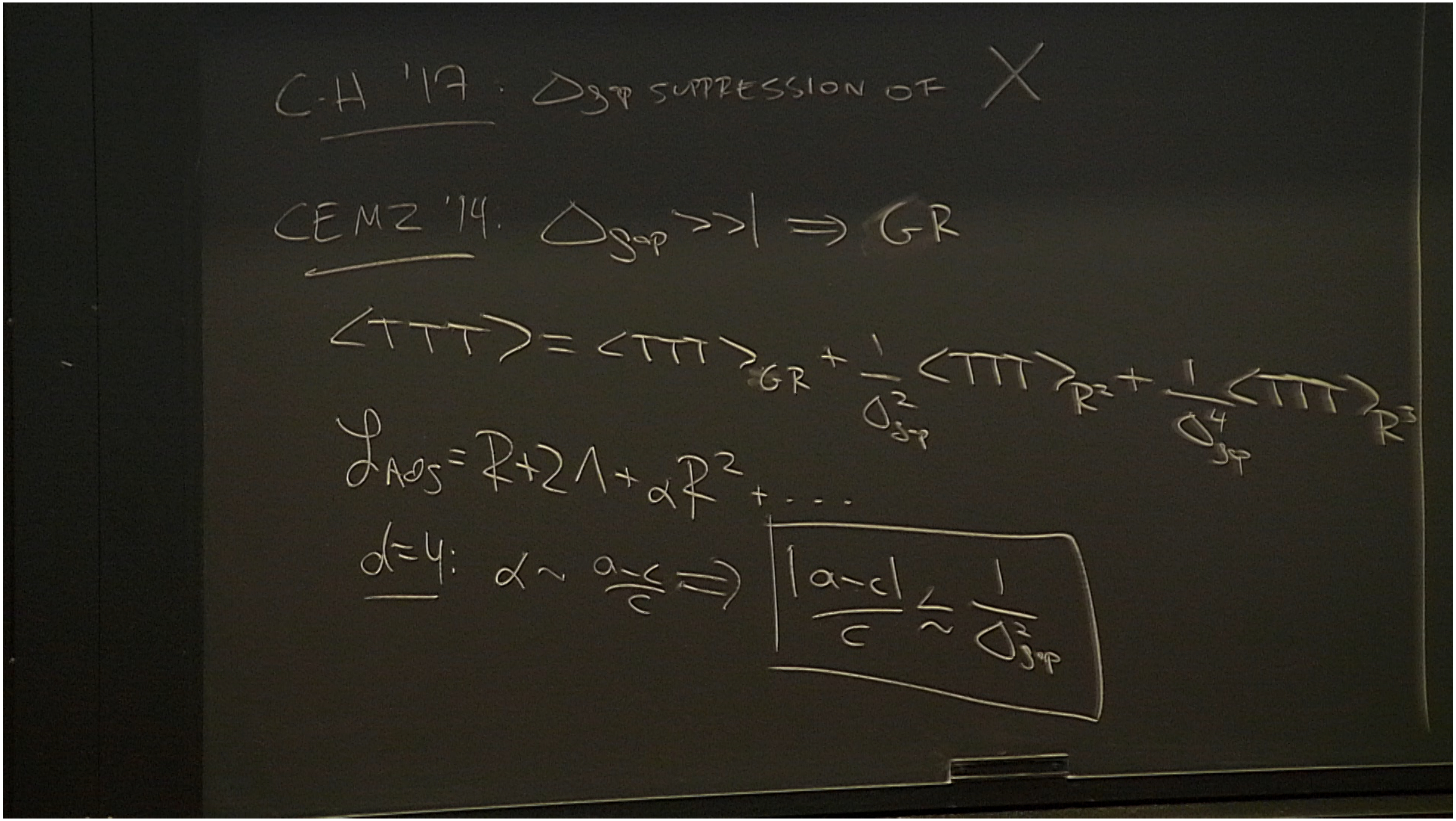
\+

LARGE Δ_{gap}

$\Updownarrow$

WEAKLY COUPLED,

LOCAL AdS DUAL



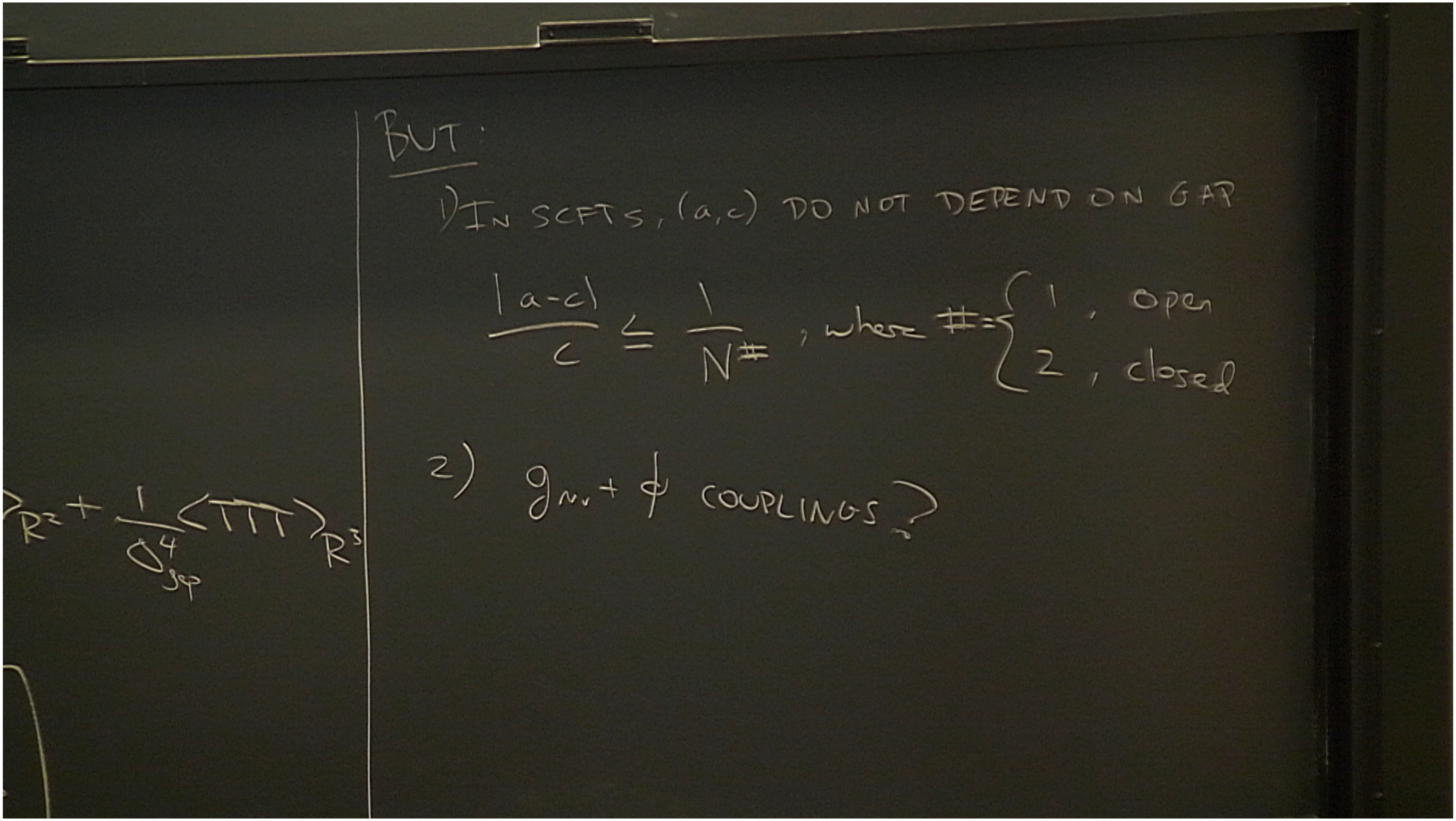
C·H '17: Δ_{gap} SUPPRESSION OF X

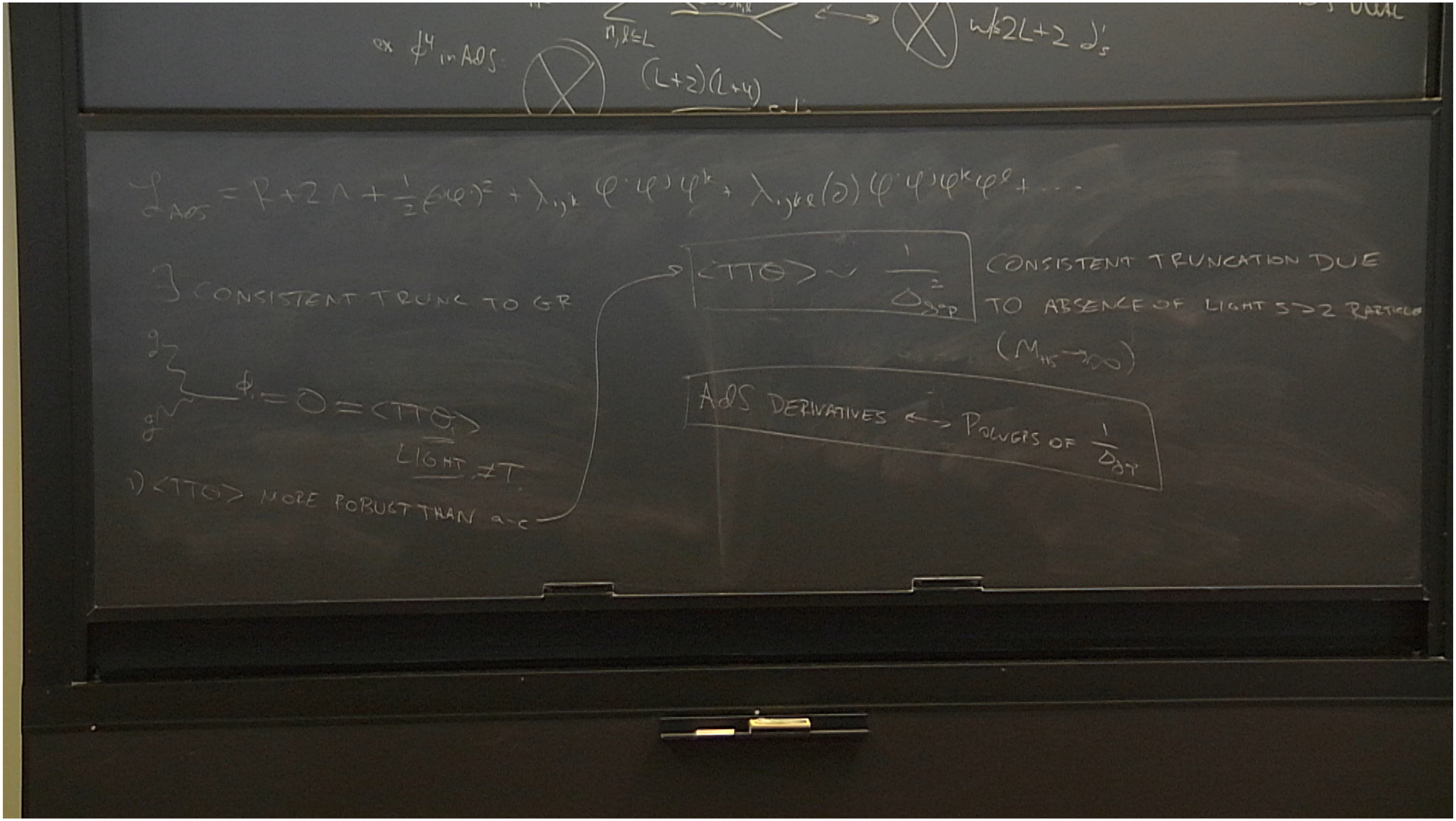
CEMZ '14: $\Delta_{gap} >> 1 \Rightarrow$ GR

$$\langle TTT \rangle = \langle TTT \rangle_{GR} + \frac{1}{\Delta_{gap}^2}\langle TTT \rangle_{R^2} + \frac{1}{\Delta_{gap}^4}\langle TTT \rangle_{R^3}$$

$$\mathcal{L}_{AdS} = R + 2\Lambda + \alpha R^2 + \dots$$

d=4: $\alpha \sim \frac{a-c}{c} \Rightarrow \boxed{\frac{|a-c|}{c} \lesssim \frac{1}{\Delta_{gap}^2}}$





ex ϕ^4 in AdS

$n, \ell \le L$

$(L+2)(L+4)$

w/ $\le 2L+2$ ∂'s

$$\mathcal{L}_{AdS} = R + 2\Lambda + \frac{1}{2}(\partial\varphi^i)^2 + \lambda_{ijk}\,\varphi^i\varphi^j\varphi^k + \lambda_{ijkl}(\partial)\,\varphi^i\varphi^j\varphi^k\varphi^l + \dots$$

$\exists$ CONSISTENT TRUNC TO GR

$\phi_i = 0 = \langle TT\underline{O_i} \rangle$

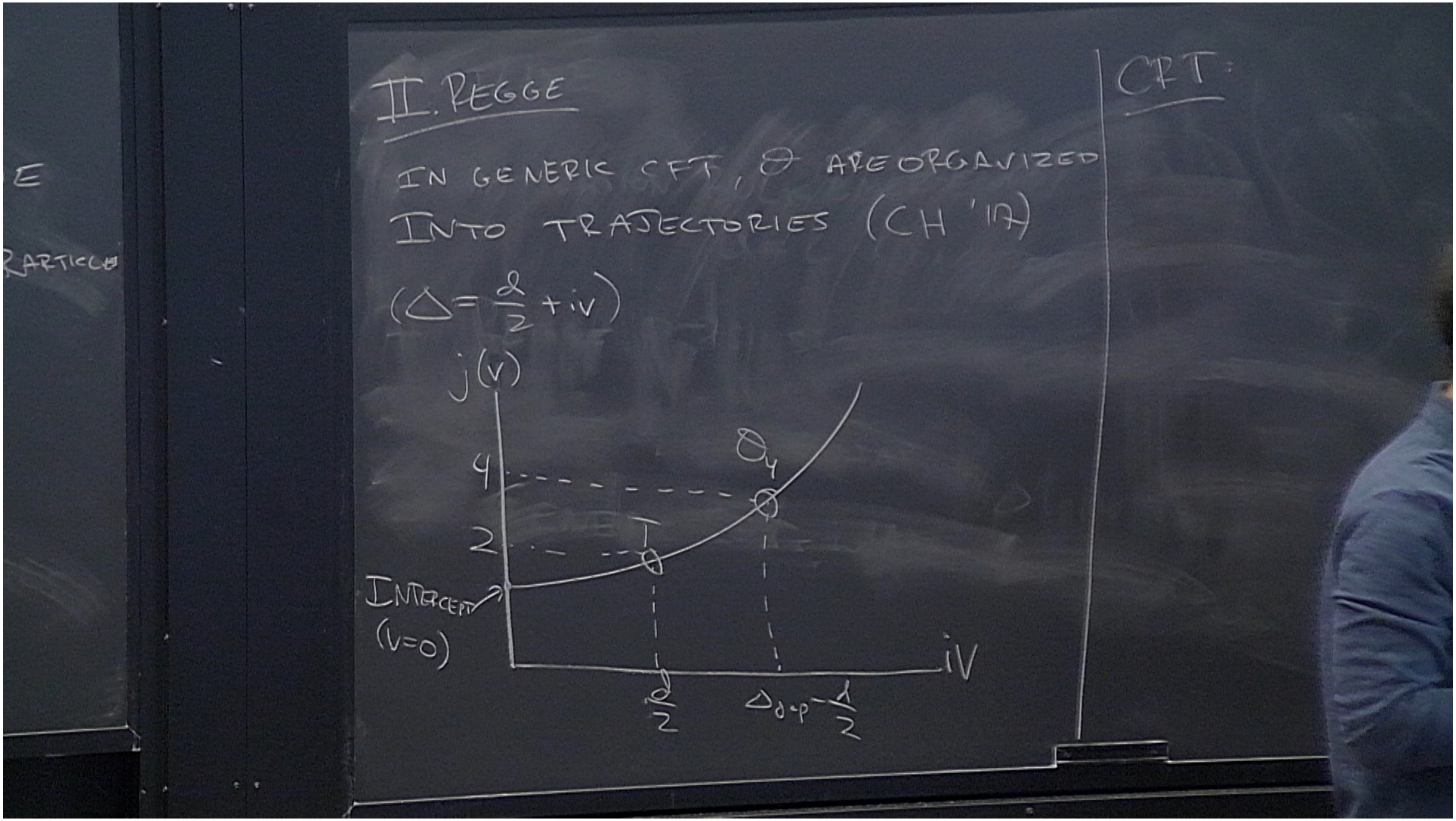
$\underline{\text{LIGHT}}, \neq T$

1) $\langle TTO \rangle$ MORE ROBUST THAN $a-c$

$$\boxed{\langle TTO \rangle \sim \frac{1}{\Delta_{gap}^2}}$$

CONSISTENT TRUNCATION DUE TO ABSENCE OF LIGHT $S>2$ PARTICLES ($M_{HS} \to \infty$)

$$\boxed{\text{AdS DERIVATIVES} \longleftrightarrow \text{POWERS OF } \frac{1}{\Delta_{gap}}}$$



E

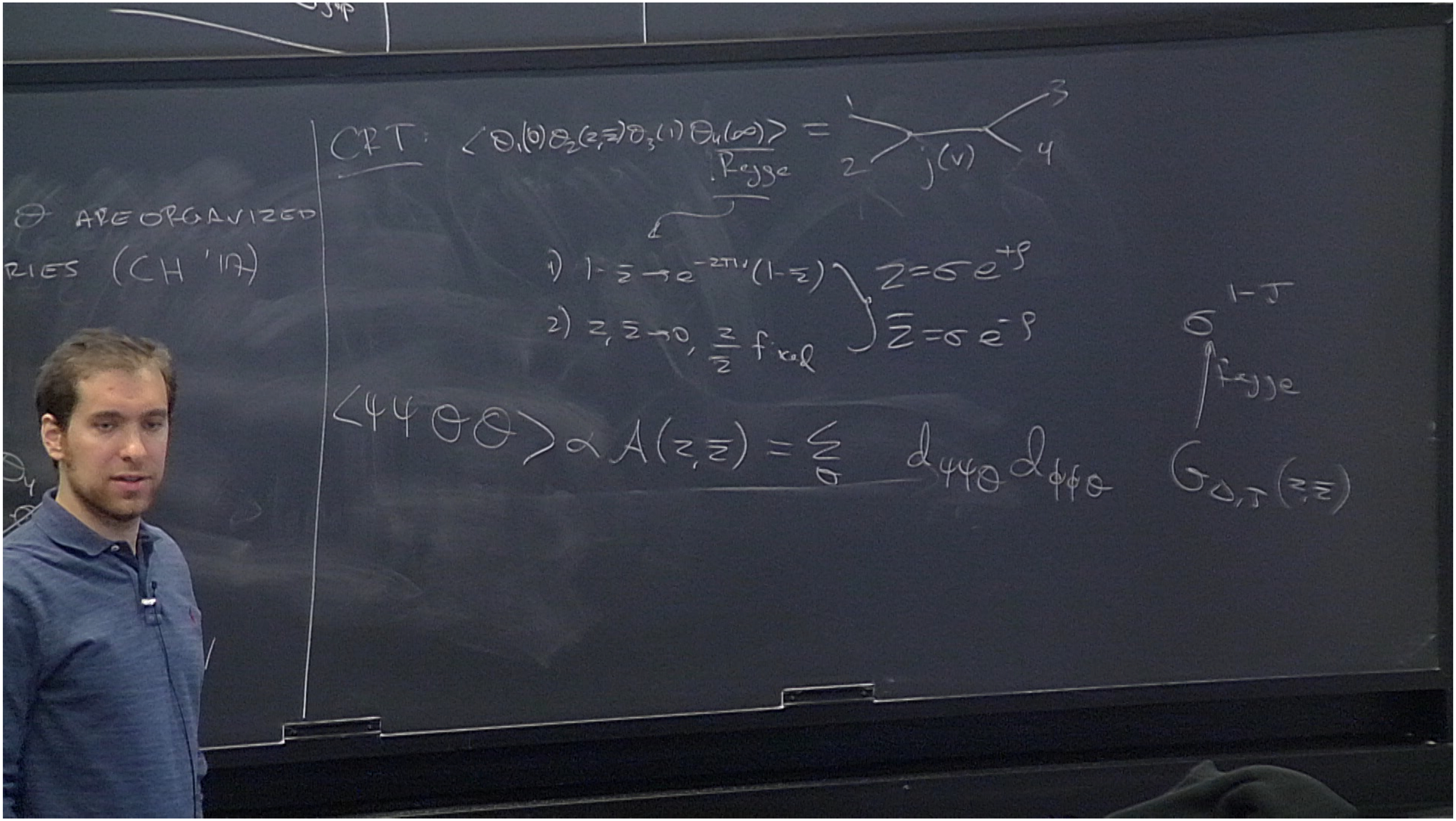
PARTICLE

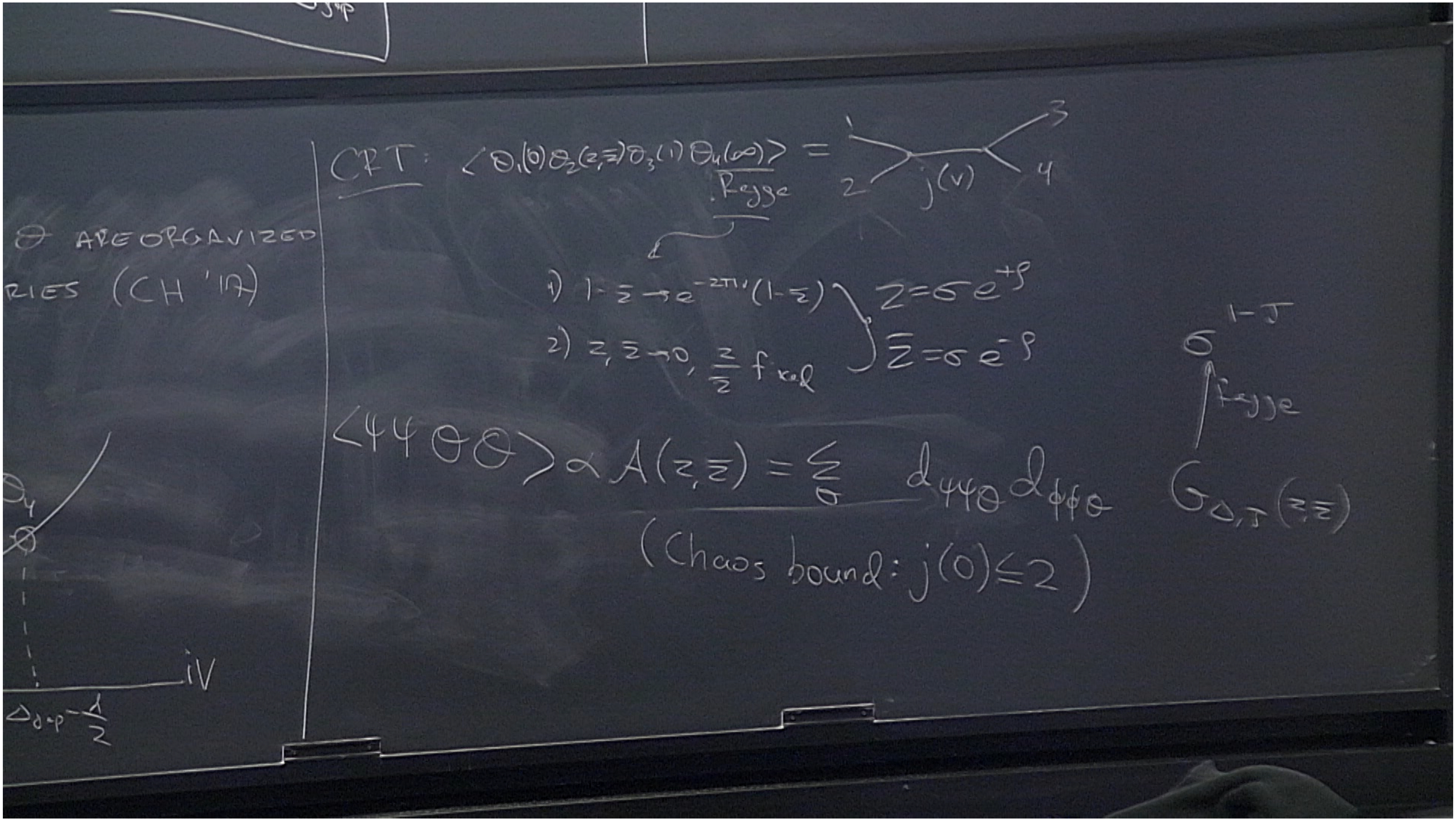
II. REGGE

IN GENERIC CFT, $\mathcal{O}$ ARE ORGANIZED INTO TRAJECTORIES (CH '17)

$(\Delta = \frac{d}{2} + i\nu)$

CRT:


O ARE ORGANIZED
RIES (CH '17)
CRT: $\langle O_1(0) O_2(z,\bar z) O_3(1) O_4(\infty)\rangle =$
1
2
3
4
$j(\nu)$
Regge
1) $1-\bar z \to e^{-2\pi i}(1-\bar z)$
2) $z, \bar z \to 0$, $\frac{z}{\bar z}$ fixed
$z = \sigma e^{+\rho}$
$\bar z = \sigma e^{-\rho}$
$\langle \psi\psi O O\rangle \propto A(z,\bar z) = \sum_{O} \lambda_{\psi\psi O}\lambda_{\phi\phi O}\, G_{\Delta,J}(z,\bar z)$
σ^{1-J}
Regge



$\mathcal{O}$ ARE ORGANIZED

RIES (CH '17)

CRT: $\langle \mathcal{O}_1(0)\mathcal{O}_2(z,\bar{z})\mathcal{O}_3(1)\mathcal{O}_4(\infty)\rangle =$

Regge

1) $1-\bar{z} \to e^{-2\pi i}(1-\bar{z})$

2) $z,\bar{z}\to 0$, $\frac{z}{\bar{z}}$ fixed

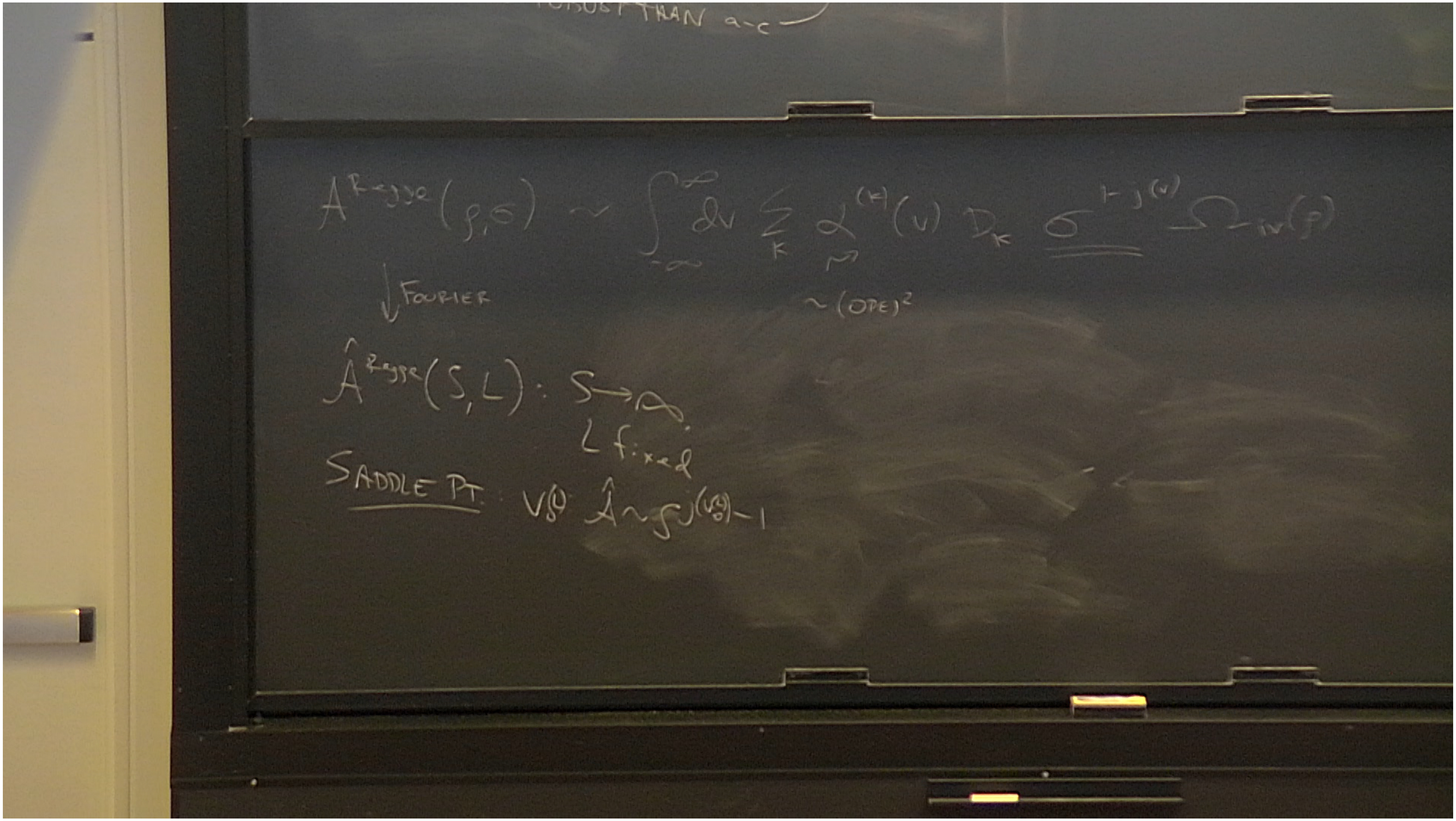
$z=\sigma e^{+\rho}$

$\bar{z}=\sigma e^{-\rho}$

$$\langle \psi\psi\mathcal{O}\mathcal{O}\rangle \propto A(z,\bar{z}) = \sum_{\mathcal{O}} d_{\psi\psi\mathcal{O}}\, d_{\phi\phi\mathcal{O}}\, G_{\Delta,J}(z,\bar{z})$$

$$G_{\Delta,J}(z,\bar{z}) \xrightarrow{\text{Regge}} \sigma^{1-J}$$

(Chaos bound: $j(0)\leq 2$)



OBUST THAN $a-c$

$$A^{\text{Regge}}(\rho,\sigma) \sim \int_{-\infty}^{\infty} d\nu \sum_{k} \underset{\nearrow}{\alpha^{(k)}(\nu)}\, D_k\, \underline{\underline{\sigma}}^{1-j(\nu)}\, \Omega_{i\nu}(\rho)$$

$\sim (\text{OPE})^2$

$\downarrow$ FOURIER

$\hat{A}^{\text{Regge}}(S,L)$: $S \to \infty$.

L fixed

SADDLE PT : $\nu_0(L)$ $\hat{A} \sim S^{j(\nu_0)-1}$

$$\sum_{k} \underset{\nearrow}{\alpha^{(k)}}(v)\, D_k\, \underline{\underline{\sigma^{1-j(v)}}}\, \Omega_{iv}(\rho)$$

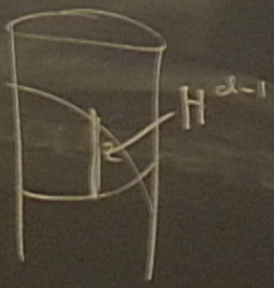
$\sim (\mathrm{OPE})^2$

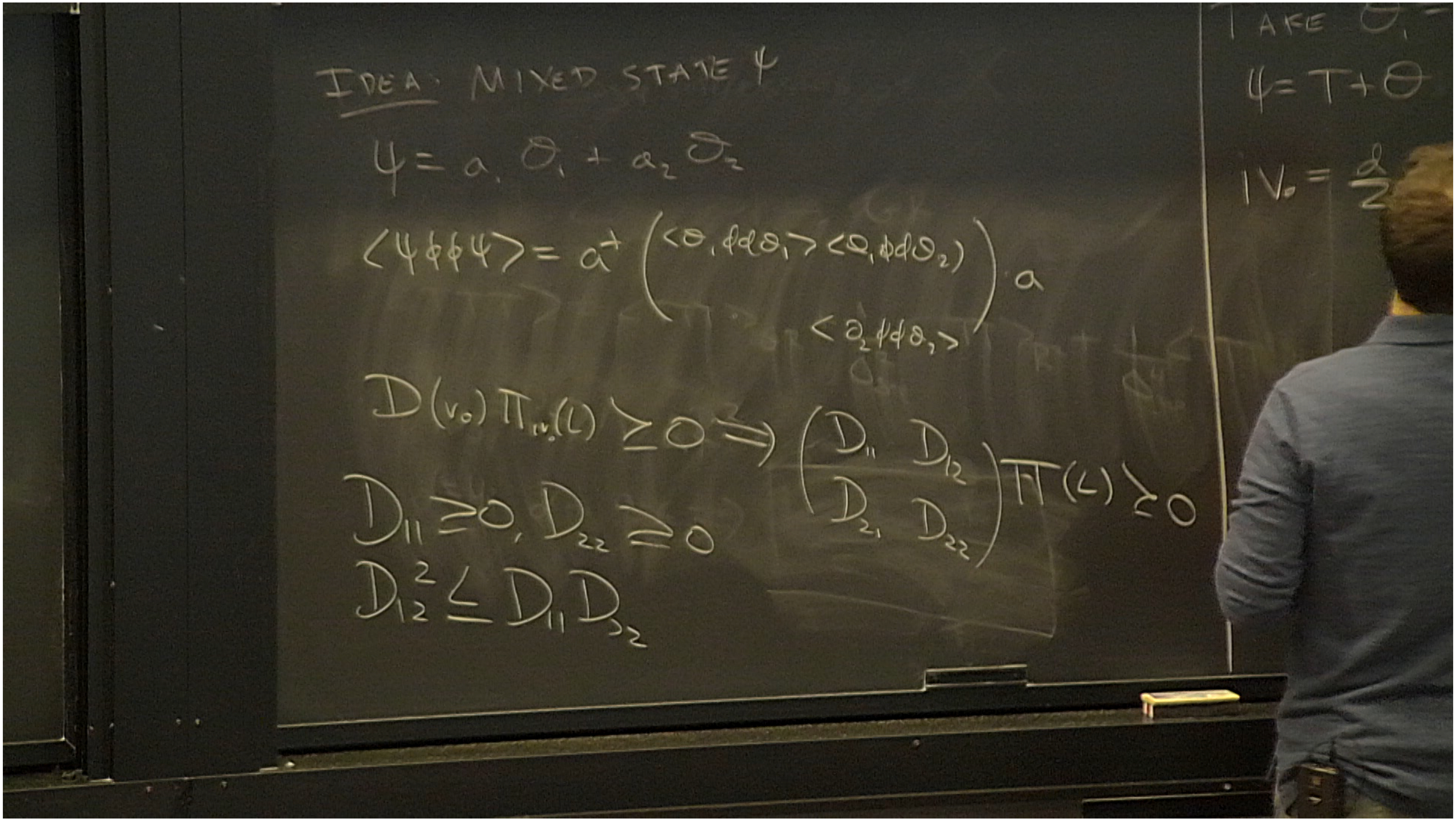
UNITARITY

$$\boxed{D(v_0)\, \Pi_{iv_0}(L) \geq 0}$$

$$D(v_0) \equiv B_{\phi\phi j(v_0)} \sum_{k} B^{(k)}_{\psi\psi j(v_0)}\, \hat{D}_k$$

$$\Pi_{iv_0}(L) = H^{d-1}{}_{\text{propagator}}$$





IDEA: MIXED STATE ψ

$$\psi = a_1 \theta_1 + a_2 \theta_2$$

$$\langle \psi \phi \phi \psi \rangle = a^{+} \begin{pmatrix} \langle \theta_1 \phi \phi \theta_1 \rangle & \langle \theta_1 \phi \phi \theta_2) \\ & \langle \theta_2 \phi \phi \theta_2 \rangle \end{pmatrix} a$$

$$D(v_0)\,\Pi_{v_0}(L) \geq 0 \Rightarrow \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} \Pi(L) \geq 0$$

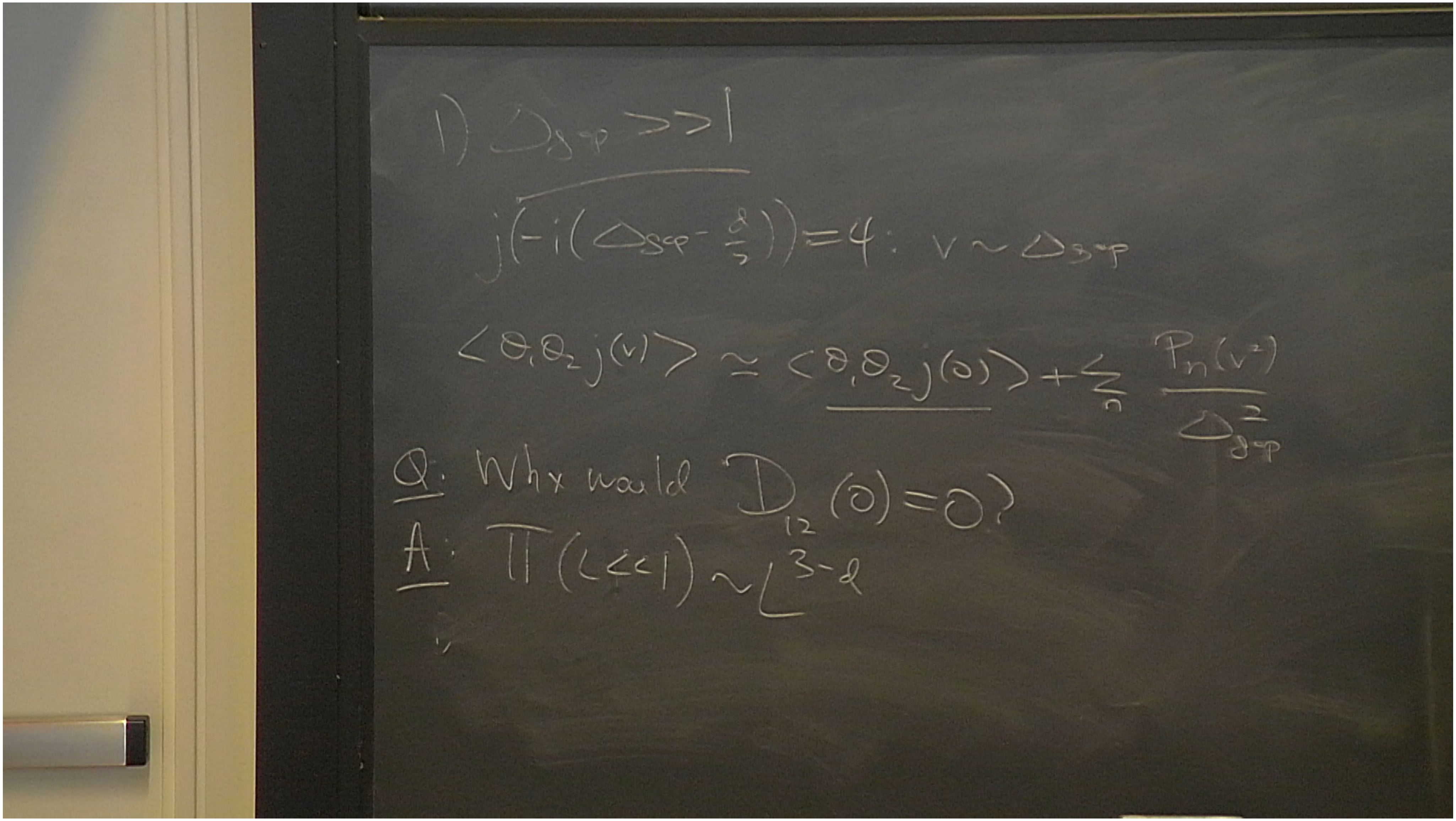
$$D_{11} \geq 0,\ D_{22} \geq 0$$

$$D_{12}^2 \leq D_{11} D_{22}$$

TAKE $\theta_1 =$

$$\psi = T + \theta$$

$$|V_0 = \frac{\alpha}{2}$$



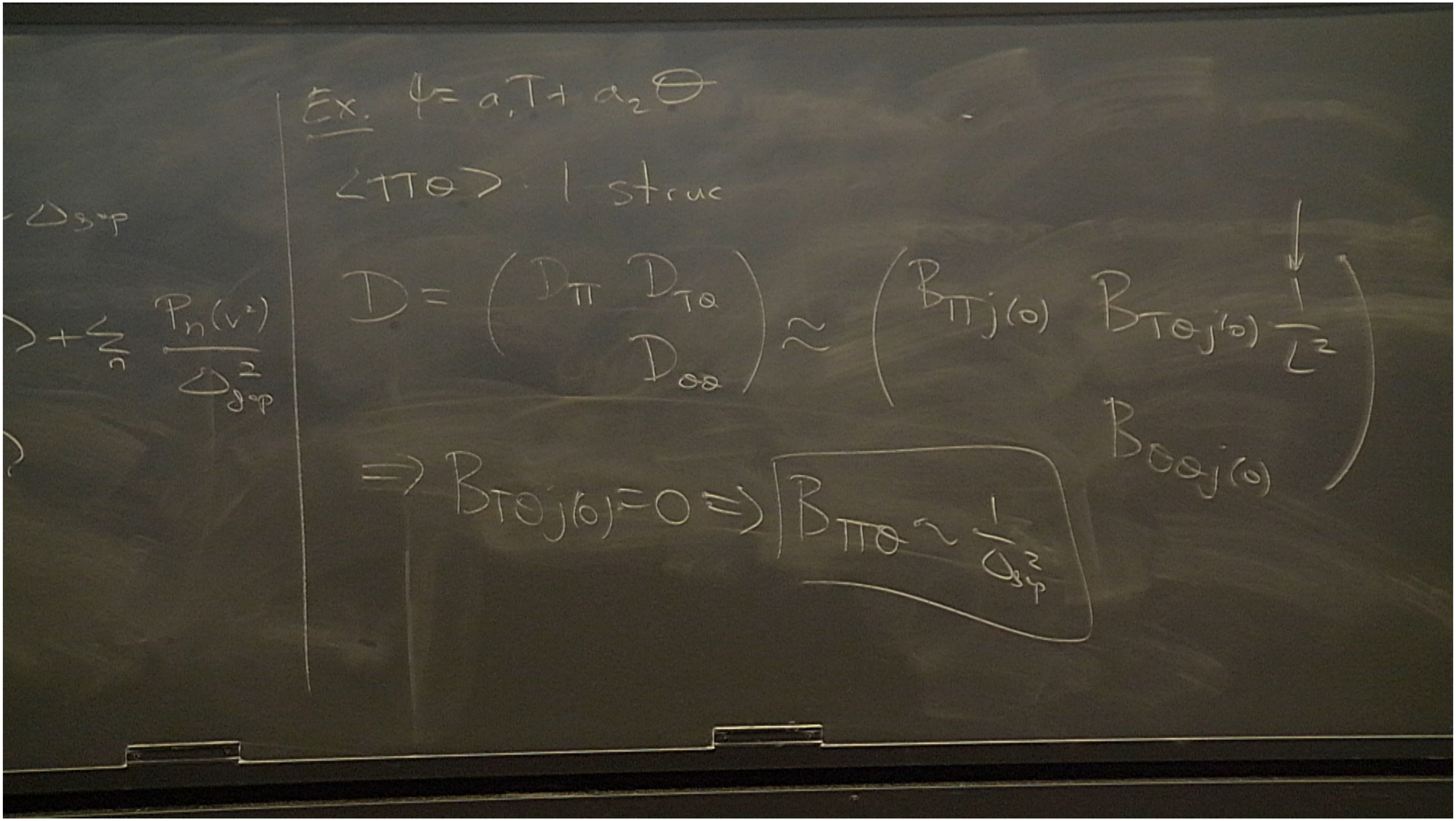
1) $\Delta_{gap} >> 1$

$$j(-i(\Delta_{gap} - \tfrac{d}{2})) = 4 : \quad \nu \sim \Delta_{gap}$$

$$\langle O_1 O_2 j(\nu) \rangle \simeq \underline{\langle O_1 O_2 j(0) \rangle} + \sum_n \frac{P_n(\nu^2)}{\Delta_{gap}^2}$$

Q: Why would $D_{12}(0) = 0$?

A: $\Pi(L << 1) \sim L^{3-d}$



$-\Delta_{gap}$

$) + \sum_n \frac{P_n(v^2)}{\Delta_{gap}^2}$

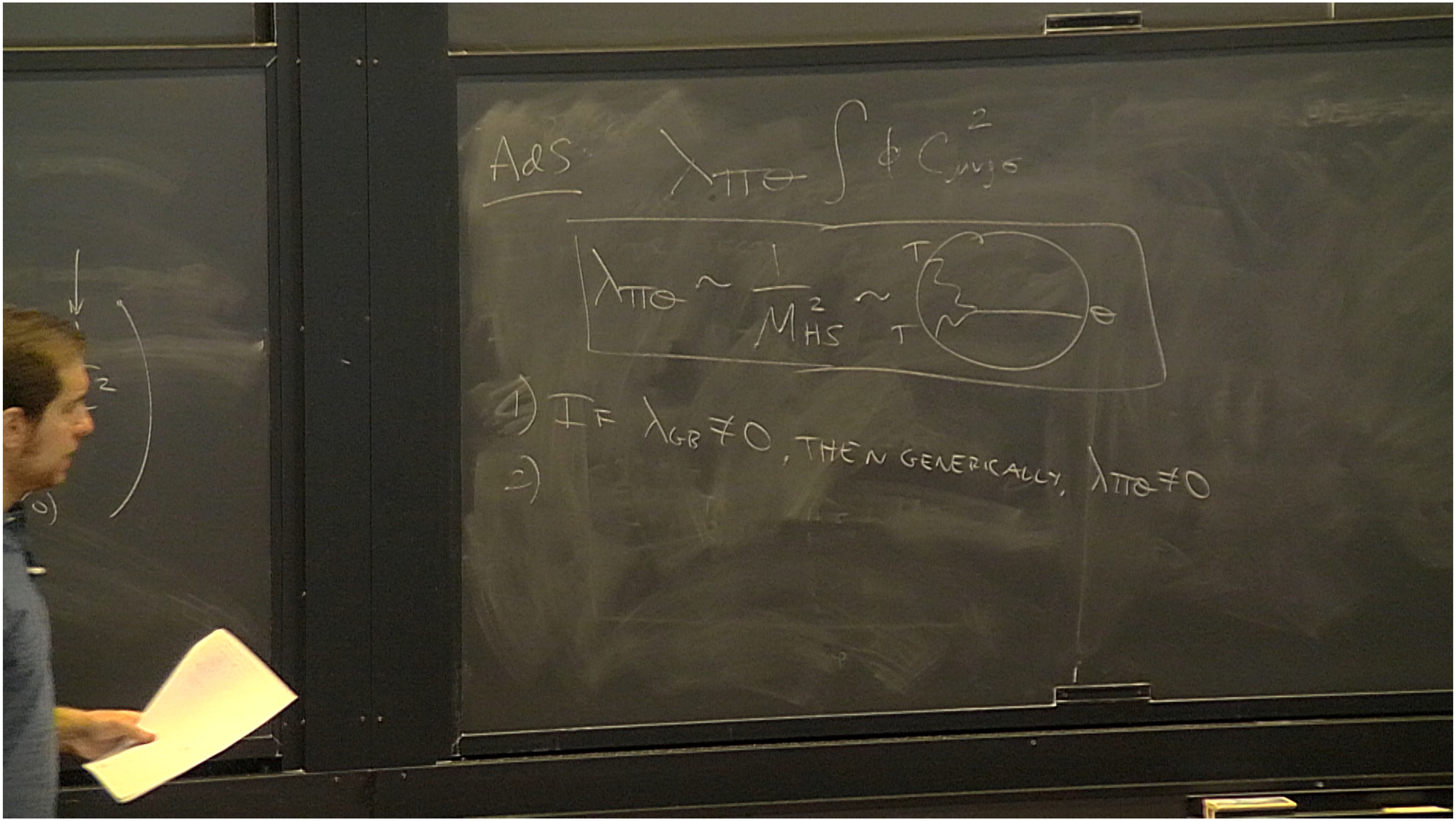
$)$

Ex. $\phi = a_1 T + a_2 \Theta$

$\langle TT\Theta \rangle$: 1 struc

$$D = \begin{pmatrix} D_{TT} & D_{T\Theta} \\ & D_{\Theta\Theta} \end{pmatrix} \approx \begin{pmatrix} B_{TT}\, j(0) & B_{T\Theta}\, j'(0) \overset{\downarrow}{\frac{1}{L^2}} \\ & B_{\Theta\Theta}\, j(0) \end{pmatrix}$$

$$\Rightarrow B_{T\Theta}\, j(0) = 0 \Rightarrow \boxed{B_{TT\Theta} \sim \frac{1}{\Delta_{gap}^2}}$$

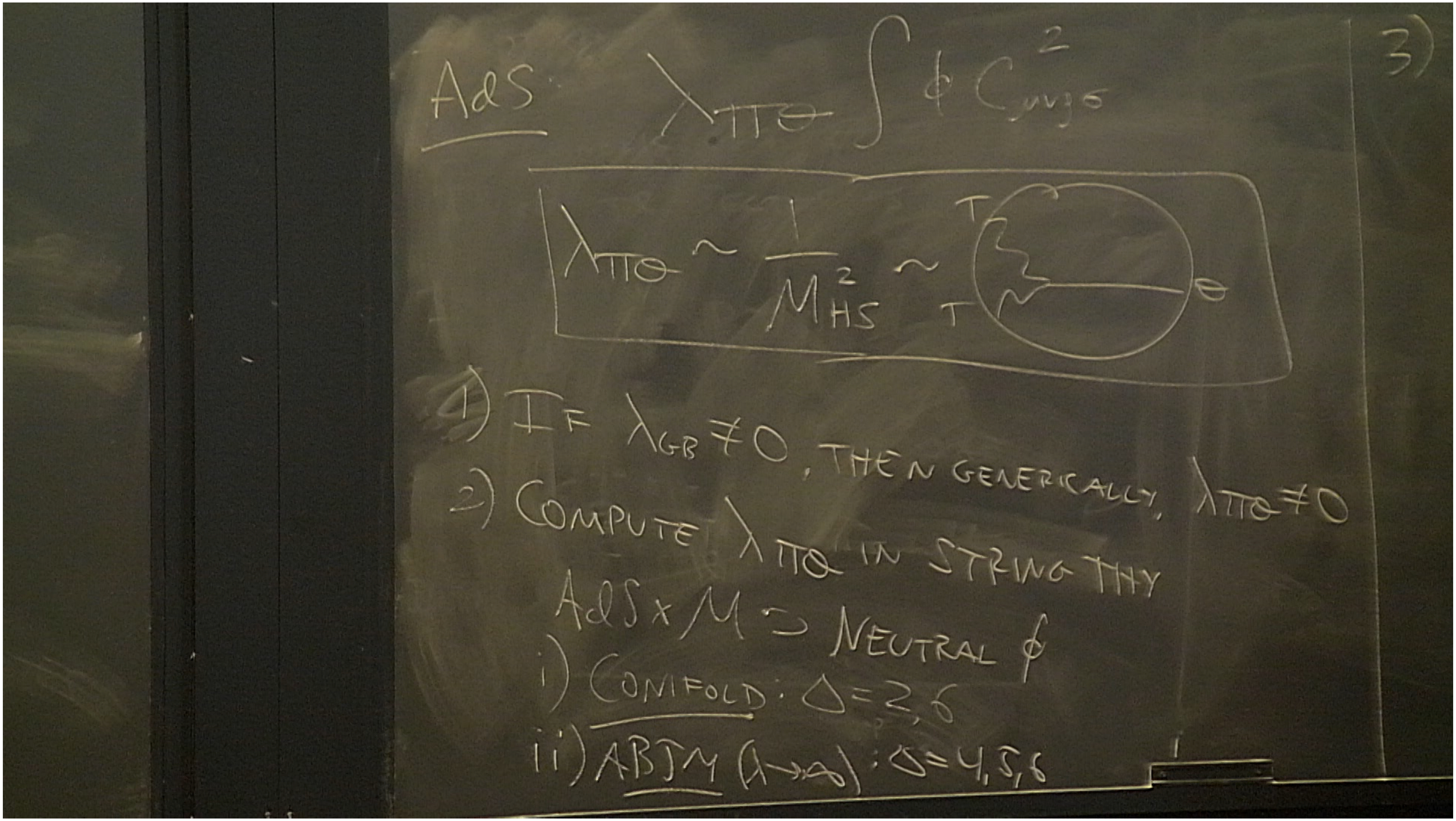


AdS: $\lambda_{TT\Theta} \int \phi \, C^2_{\mu\nu\rho\sigma}$

$$\boxed{\lambda_{TT\Theta} \sim \frac{1}{M^2_{HS}} \sim}$$

1) IF $\lambda_{GB} \neq 0$, THEN GENERICALLY, $\lambda_{TT\Theta} \neq 0$

2)



AdS: $\lambda_{TT\Theta} \int \phi\, C^2_{\mu\nu\rho\sigma}$

$$\boxed{\lambda_{TT\Theta} \sim \frac{1}{M^2_{HS}} \sim}$$

1) IF $\lambda_{GB} \neq 0$, THEN GENERICALLY, $\lambda_{TT\Theta} \neq 0$

2) COMPUTE $\lambda_{TT\Theta}$ IN STRING THY

AdS × M ⊃ NEUTRAL ϕ

i) CONIFOLD: $\Delta = 2, 6$

ii) ABJM $(\lambda \to \infty)$: $\Delta = 4, 5, 6$

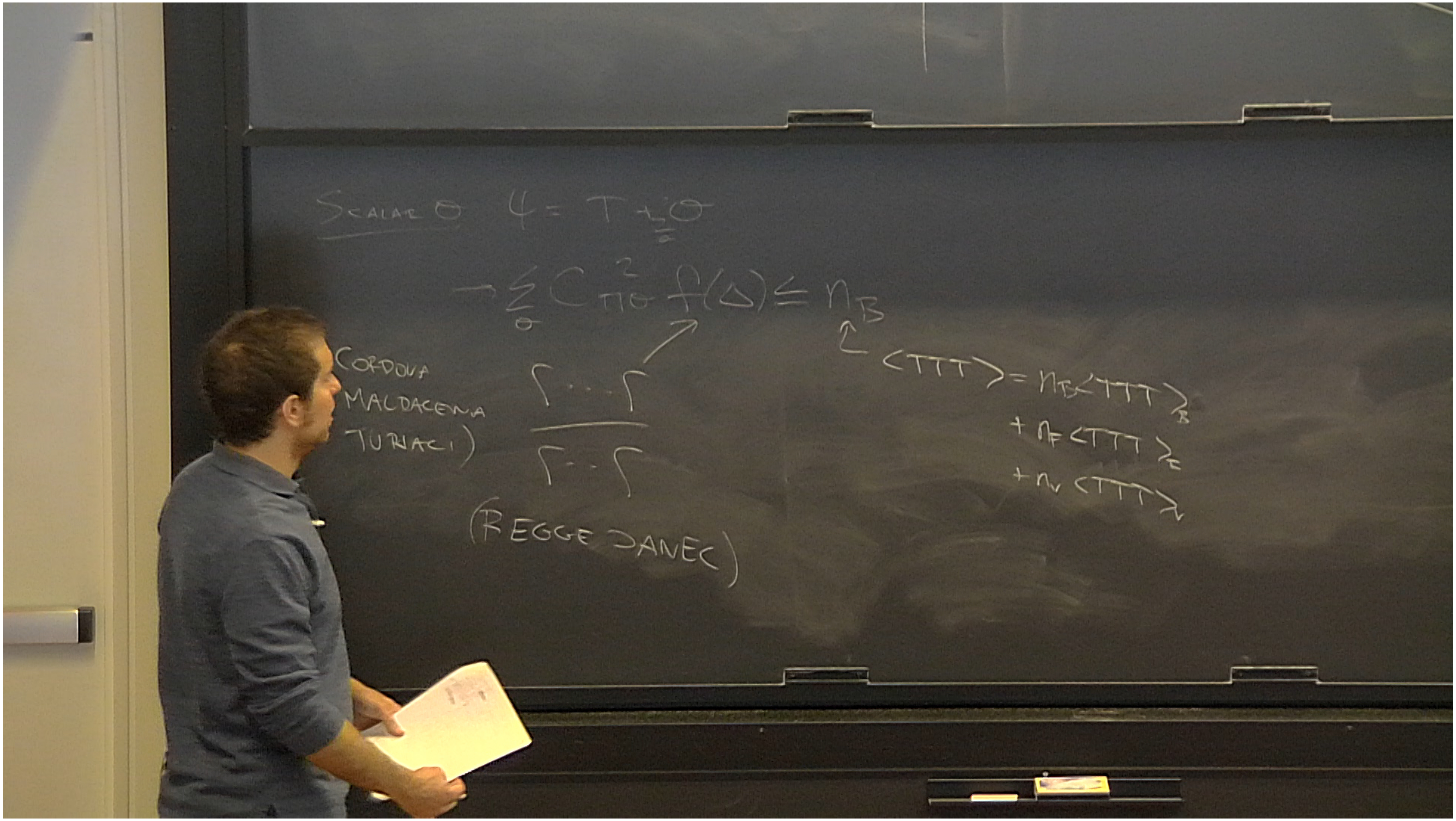
3)

$\lambda_{TT\Theta} \neq 0$

3) $\delta C_T \propto B_{TT\Theta} \sim \frac{1}{\Delta^2_{gap}}$

4)

CFT	AdS
$\langle TT\Theta \rangle$	ϕC^2
$\langle TT\Theta \rangle_{odd}$	$\phi C \tilde{C}$
$\langle TTV \rangle_{odd}$	$A \wedge R \wedge R$
$\langle TTM \rangle$	



SCALAR $\mathcal{O}$ $\quad \psi = T + \epsilon\, \mathcal{O}$

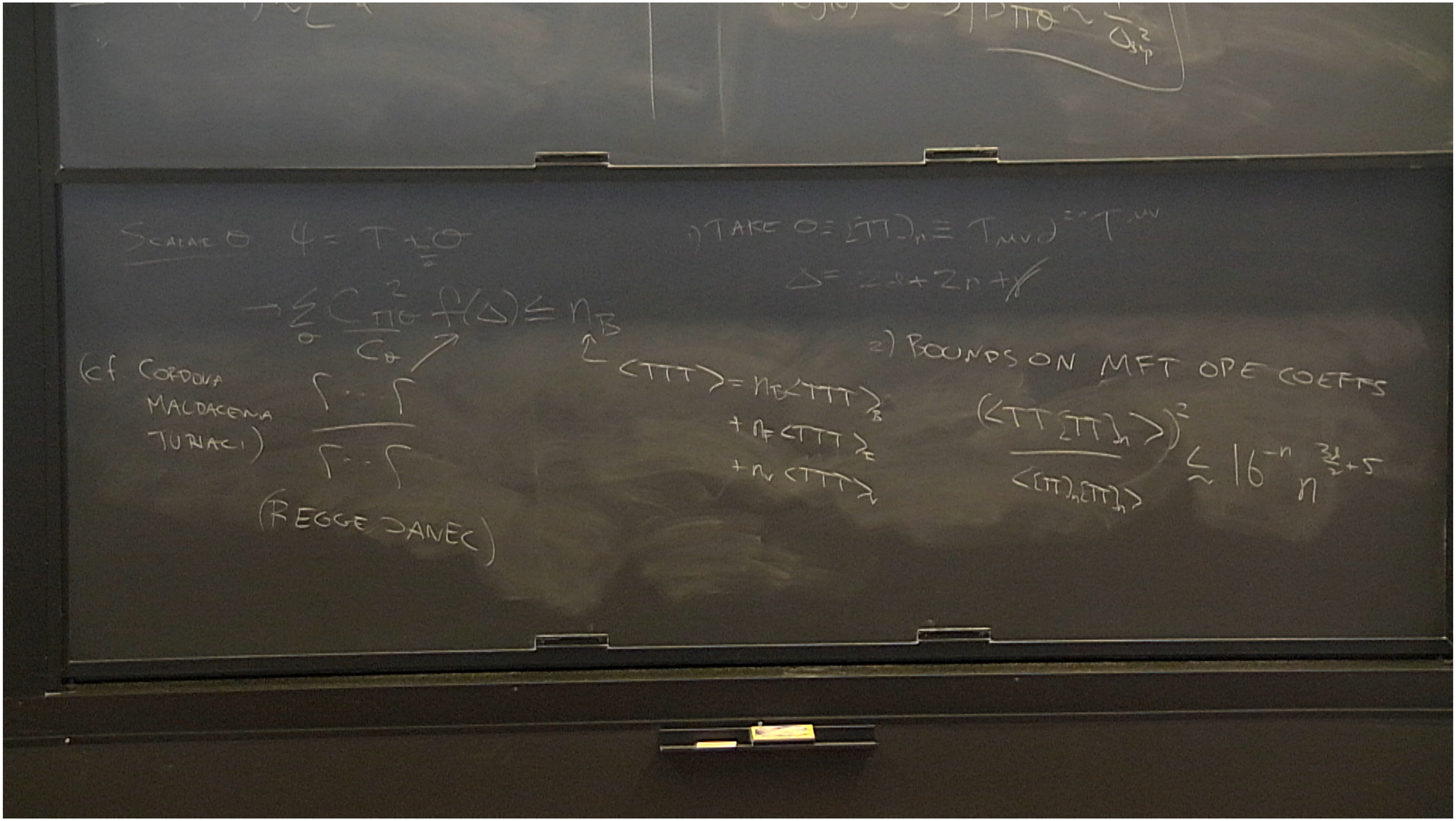
$$\rightarrow \sum_{\mathcal{O}} C_{TT\mathcal{O}}^{2} f(\Delta) \le n_B$$

(CORDOVA MALDACENA TURIACI)

$$\frac{\int \cdots \int}{\int \cdots \int}$$

(REGGE DANEC)

$$\langle TTT \rangle = n_B \langle TTT \rangle_B + n_F \langle TTT \rangle_F + n_V \langle TTT \rangle_V$$



SCALAR $\mathcal{O}$ $\quad \psi = T + \epsilon\,\mathcal{O}$

$$\rightarrow \sum_{\mathcal{O}} \frac{C_{TT\mathcal{O}}^2}{C_{\mathcal{O}}} f(\Delta) \le n_B$$

(cf. CORDOVA MALDACENA TURIACI)

$$f(\Delta) = \frac{\Gamma \cdots \Gamma}{\Gamma \cdots \Gamma}$$

(REGGE $\supset$ ANEC)

$$\langle TTT \rangle = n_B \langle TTT \rangle_B + n_F \langle TTT \rangle_F + n_V \langle TTT \rangle_V$$

1) TAKE $\mathcal{O} = [TT]_n \equiv T_{\mu\nu}\partial^{\cdots}T^{\cdot\mu\nu}$

$\Delta = 2d + 2n + \cancel{\gamma}$

2) BOUNDS ON MFT OPE COEFFS

$$\frac{\left(\langle TT[TT]_n \rangle\right)^2}{\langle [TT]_n [TT]_n \rangle} \lesssim 16^{-n}\, n^{\frac{3d}{2}+5}$$