

Title: Recently discovered, stronger forms of quantum nonlocality

Date: Feb 06, 2018 03:30 PM

URL: <http://pirsa.org/18020077>

Abstract: <p>In this talk I will discuss recently-identified classes of quantum correlations that go beyond nonlocal classical hidden-variable models equipped with communication. First, in the bipartite scenario, I will focus on so-called instrumental causal networks, which are a primal tool in causal inference. There, I will show that it is possible to “fake” classical causal influences with quantum common causes, in a formal sense quantified by the average causal effect (ACE). Furthermore, I will show that it is possible to violate instrumental inequalities with quantum resources, both in the device-independent and in the 1-sided device-independent settings. Second, in the multipartite setting, I will present a causal hierarchy of multipartite nonlocality. I will make special emphasis on quantum correlations in the upper classes of the hierarchy that define stronger forms of genuinely multipartite quantum non-locality than those previously known. The seminar will touch upon concepts like Bell nonlocality and quantum steering as well as Bayesian nets and causal inference.</p>

Recently discovered, stronger forms of quantum non-locality

Leandro Aolita

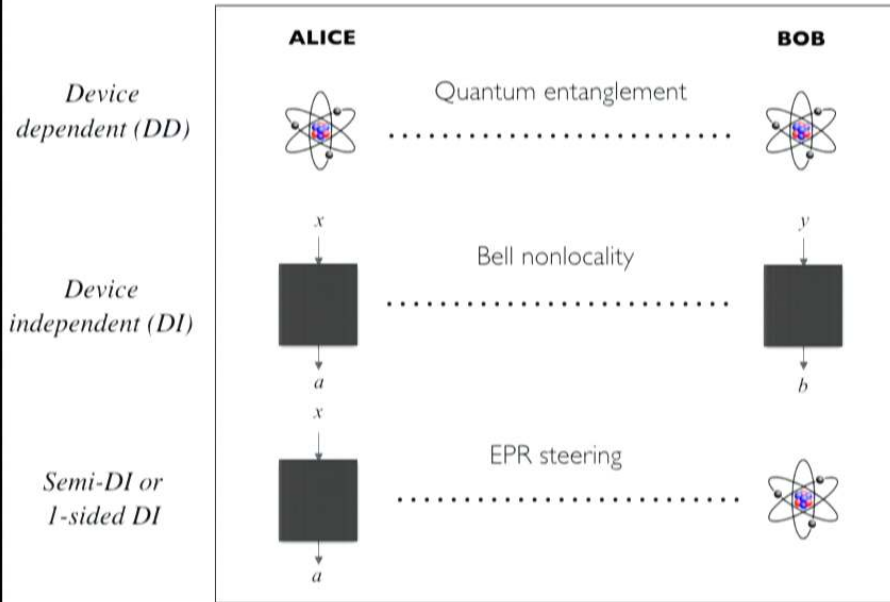
IF-UFRJ (Rio), ICTP/SAIFR (São Paulo), and IIP (Natal)

Perimeter Institute - February 2018





Bipartite quantum nonlocality in its three variants



Mathematical description

(quantum state)

$$\varrho_{AB} \neq \sum_{\lambda} p(\lambda) \varrho_A^{(\lambda)} \otimes \varrho_B^{(\lambda)}$$

R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, "Quantum entanglement", *Rev. Mod. Phys.* **81**, 865 (2009).

(black-box behaviour)

$$p(a, b|x, y) \neq \sum_{\lambda} p(\lambda) p(a|x, \lambda) p(b|y, \lambda)$$

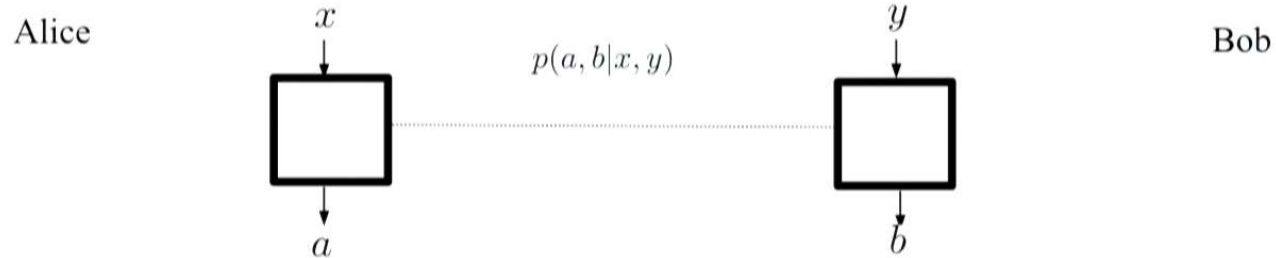
N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, and S. Wehner, "Bell nonlocality", *Rev. Mod. Phys.* **86**, 419 (2014).

$\sigma_{a|x} = p(a|x) \varrho_{a,x}$ (assemblage)

$$\neq \sum_{\lambda} p(\lambda) p(a|x, \lambda) \varrho_B^{(\lambda)}$$

M. D. Reid et al, "The EPR paradox: from concepts to applications", *Rev. Mod. Phys.* **81**, 1727 (2009); D. Cavalcanti and P. Skrzypczyk, "Quantum steering: a review with focus on semidefinite programming", *Rep. Prog. Phys.* **80**, 024001 (2017).

Bell correlations (fully DI scenario)



Local correlations:

$$p(a, b|x, y) = \sum_{\lambda} p(\lambda) p(a|x, \lambda) p(b|y, \lambda)$$

Quantum correlations:

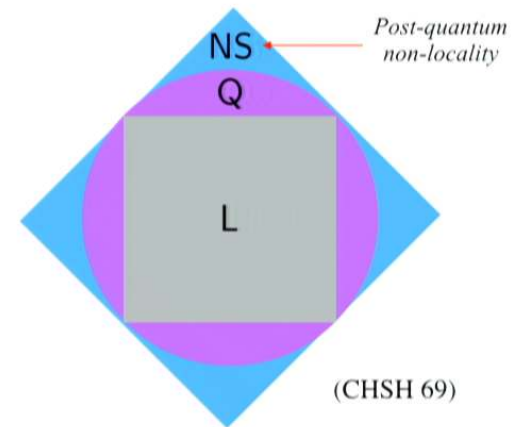
$$p(a, b|x, y) = \text{Tr}[M_x^{(a)} \otimes M_y^{(b)} \rho_{AB}] \quad (\text{Born's rule})$$

No-signalling correlations:

$$\begin{cases} p(a|x, y) := \sum_b p(a, b|x, y) = p(a|x) \\ p(b|x, y) := \sum_a p(a, b|x, y) = p(b|y) \end{cases}$$

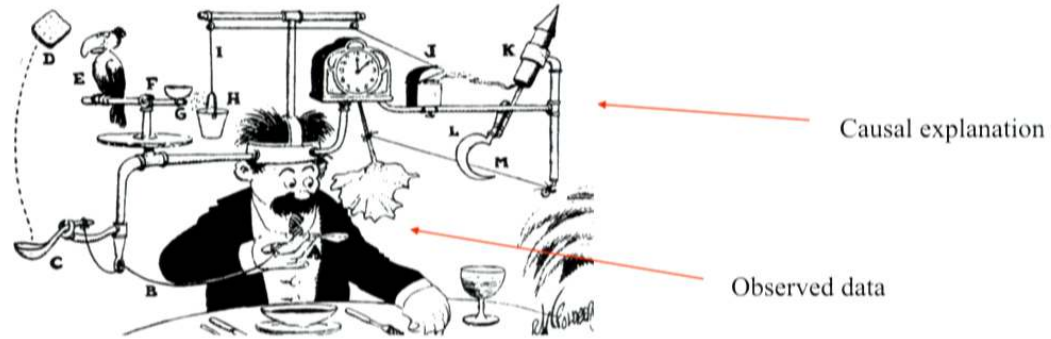
$$L \subset Q \subseteq NS$$

\downarrow
 (Bell's theorem 64)



Bell tests as causal-inference problems

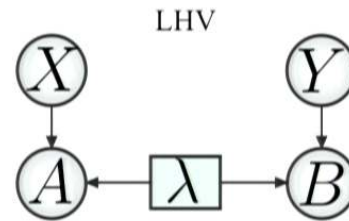
Causal inference:



A typical Bell experiment:



Can I explain experimental observed correlations with an LHV causal model?



$$p(a, b|x, y) = \sum_{\lambda} p(\lambda) p(a|x, \lambda) p(b|y, \lambda)$$

How much can one strengthen LHV models while still having a gap with quantum correlations?

What is the simplest causal structure that admits a gap between classical and quantum causal models?

*Superior forms of quantum nonlocality
(stronger than non-local hidden variable models)!!!*

Outline of the talk:

- DI bipartite case with outcome communication.

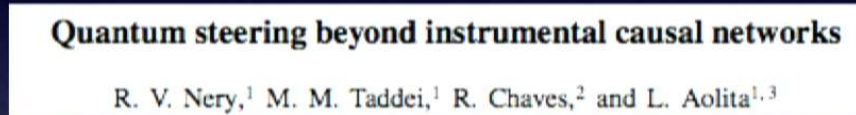


nature physics ARTICLES
<https://doi.org/10.1038/s41567-017-0008-5>

Quantum violation of an instrumental test

Rafael Chaves^{1*}, Gonzalo Carvacho², Iris Agresti², Valerio Di Giulio², Leandro Aolita³, Sandro Giacomini² and Fabio Sciarrino^{2*}

- 1-sided DI bipartite case with outcome communication.



Quantum steering beyond instrumental causal networks

R. V. Nery,¹ M. M. Taddei,¹ R. Chaves,² and L. Aolita^{1,3}

- DI multipartite case with no communication.



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the open journal for quantum science

Causal hierarchy of multipartite Bell nonlocality

Rafael Chaves^{1,2}, Daniel Cavalcanti³, and Leandro Aolita⁴

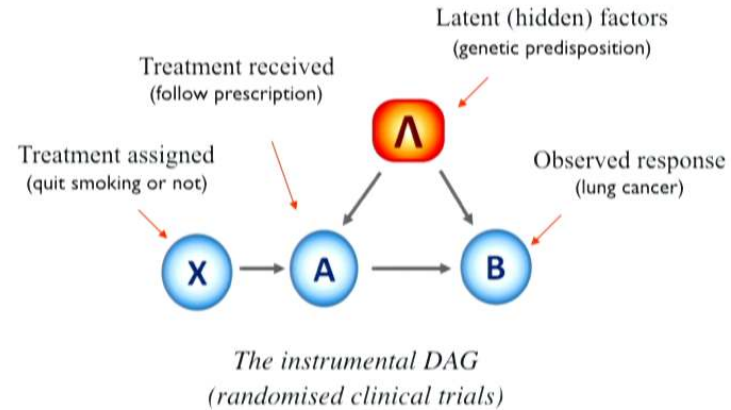
¹International Institute of Physics, Federal University of Rio Grande do Norte, 59078-970, P. O. Box 1613, Natal, Brazil
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³ICFO-Institut de Ciències Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain
⁴Instituto de Física, Universidade Federal do Rio de Janeiro, P. O. Box 68528, Rio de Janeiro, RJ 21941-972, Brazil

Device-independent bipartite case with outcome communication

R. Chaves, G. Carvacho, I. Agresti, V. Di Giulio, L. Aolita, S. Giacomini, and F. Sciarrino, Nature Physics (2017).

Classical causal models: Bayesian networks

- Causal structures represented by *directed acyclic graphs (DAGs)*:



- Causal models represented by distributions *Markov* with respect to the DAG:

$$p(a, b, x, \lambda) = p(\lambda) p(x) p(a|x, \lambda) p(b|a, \lambda)$$

Parents of b

P. Spirtes, N. Glymour, and R. Scheines (01); J. Pearl (09).

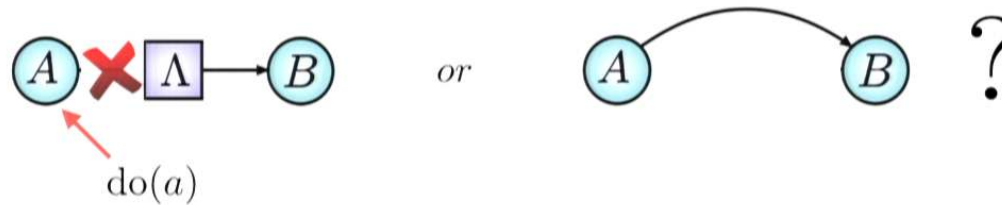
What is the simplest causal structure displaying a gap between quantum and classical correlations?

... the answer was always in front of our noses. 

The simplest causal inference problem



Does A causally influence B ?



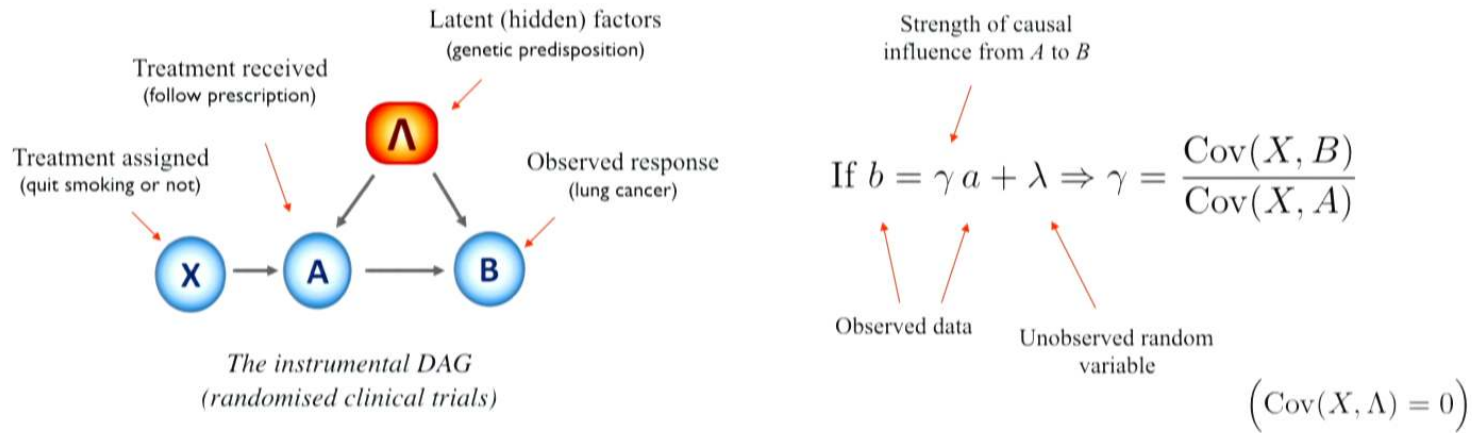
... but both causal models can explain all distributions $p(a, b)$ 😞

Interventions allow one to *infer* the underlying causal structure: $p(b|a) \neq p(b|\text{do}(a))$?

... but they assume one can **physically modify the internal dynamics** of the system, including hidden nodes (**not device-independent**) 😞

Instrumental variables

Introduced in econometrics (Wright, 1928) to estimate parameters in linear models of supply and demand.



*In full generality, the causal influence from A to B can be quantitatively assessed **even in the presence of hidden common causes and without interventions (device-independent).***

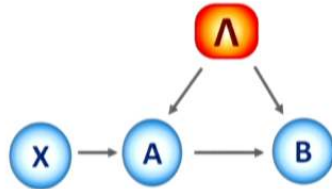


A. Balke and J. Pearl, "Bounds on treatment effects from studies with imperfect compliance", JASA (1997).

... but how does one know if a given observed statistics has an instrumental model as underlying causal explanation?

Instrumental inequalities

Instrumental causal models:



$$p(a, b|x) = \sum_{\lambda} p(\lambda) p(a|x, \lambda) p(b|a, \lambda)$$

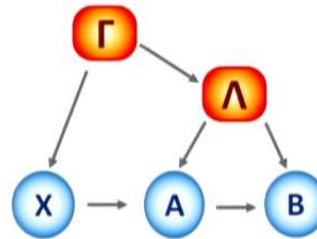
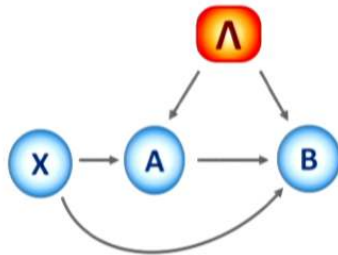
... they all satisfy:

$$\max_a \sum_b \max_x p(a, b|x) \leq 1$$

The instrumental inequalities

J. Pearl, UAI (1995).

- *Classical* instrumental-inequality *violations possible only by non-instrumental causal models:*



- Quantum mechanically, *no violation by quantum instrumental causal models.* 😞

J. Henson, R. Lal, and M. Pusey, New J. Phys. **16**, 113043 (2014) .

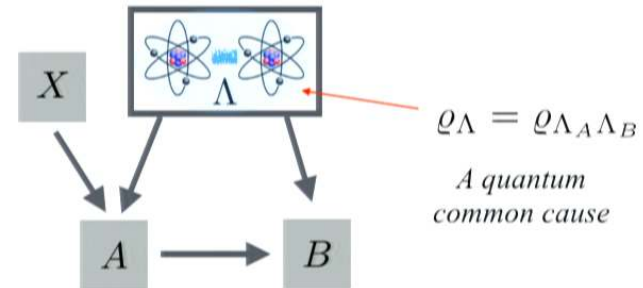
Even without violations, do quantum instrumental offer an observable discrepancy with classical ones?

Quantum instrumental causal models

Quantum instrumental causal models:

$$p_Q(a, b|x) := \text{Tr} \left[M_x^{(a)} \otimes M_a^{(b)} \varrho_\Lambda \right]$$

(measurement setting x ,
measurement outcome a)
(measurement setting a ,
measurement outcome b)



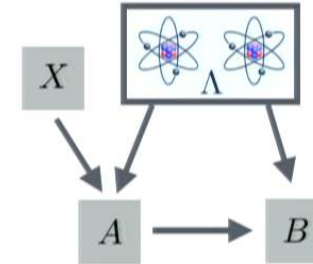
L. Hardy (2005–2007);
 F. Costa and S. Shrapnel, *New J. Phys.* **18**, 063032 (2016);
 J.-M. Allen, J. Barrett, D. C. Horsman, C. M. Lee, and R. W. Spekkens, *Phys. Rev. X* **7**, 031021 (2017).

R. Chaves, G. Carvacho, I. Agresti, V. Di Giulio, L. Aolita, S. Giacomini, and F. Sciarrino, *Nature Physics* (2017).

Quantum average causal effect (QACE)

The quantum average causal effect (QACE):

$$\begin{aligned} \text{QACE}_{A \rightarrow B} &= \sup_{a, a', b} |p_Q(b|\text{do}(a)) - p_Q(b|\text{do}(a'))| \\ &= \sup_{a, a', b} \left| \text{Tr} \left[\left(M_b^{\text{do}(a)} - M_b^{\text{do}(a')} \right) \varrho_{\Lambda_B} \right] \right| \\ &\equiv 0 \quad (\text{for all } v!!!) \end{aligned}$$



$$p_Q(a, b|x) = \text{Tr} [M_a^x \otimes M_b^a \varrho_\Lambda]$$

$$\begin{aligned} \Rightarrow p_Q(b|\text{do}(a)) &= \text{Tr} \left[\mathbf{1} \otimes M_b^{\text{do}(a)} \varrho_\Lambda \right] \\ &= \text{Tr}_B \left[M_b^{\text{do}(a)} \varrho_{\Lambda_B} \right] \end{aligned}$$

- The average statistics observable at B must be *more sensitive to interventions in A if the causal model is classical!* 😊
- Can be seen as a *quantum enhancement in terms of the causal strength* required to reproduce data! 😊
- Observation of this discrepancy *requires interventions* (not DI!) 😞

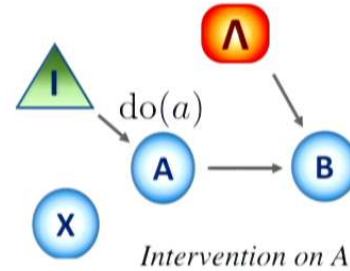
$$\begin{aligned} \varrho_\Lambda &= v |\Phi^+\rangle\langle\Phi^+| + (1 - v) \mathbf{1}/4 \\ \Rightarrow \rho_{\Lambda_B} &= \text{Tr}_A [\varrho_\Lambda] = \frac{\mathbf{1}}{2} \end{aligned}$$

Average causal effect (ACE)

The classical average causal effect (ACE):

$$ACE_{A \rightarrow B} = \sup_{a, a', b} |p(b|\text{do}(a)) - p(b|\text{do}(a'))|$$

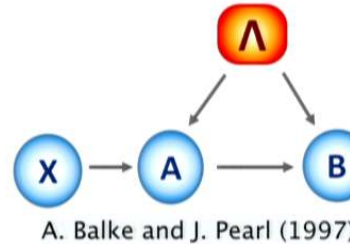
Maximum observable effect on B caused by interventions on A (on average)



DI lower bound to the ACE:

$$ACE_{A \rightarrow B} \geq 2p(a = 0, b = 0|x = 0) - 2p(a = 1, b = 1|x = 0) + p(b = 1|x = 1)$$

(for any classical instrumental causal model)



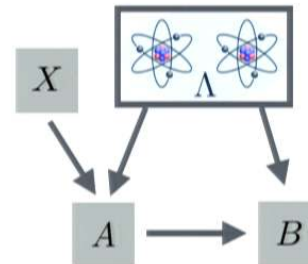
Example:

$$p_Q(a, b|x) = \text{Tr} [M_a^x \otimes M_b^a \varrho_\Lambda]$$

$$\sigma_Z \text{ or } \sigma_X \quad -\sin(\pi/8)\sigma_X + \cos(\pi/8)\sigma_Z \text{ or } \frac{\sigma_X + \sigma_Z}{\sqrt{2}}$$

(compatible with a classical instrumental model for all v)

$$ACE_{A \rightarrow B} \gtrsim 0.91v - 0.75$$



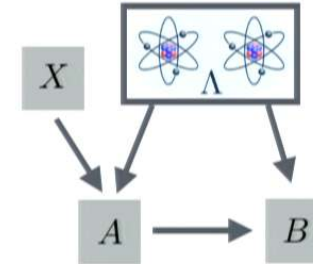
$$\varrho_\Lambda = v|\Phi^+\rangle\langle\Phi^+| + (1-v)\mathbf{1}/4$$

$$|\Phi^+\rangle := (|0_{\Lambda_A} 0_{\Lambda_B}\rangle + |1_{\Lambda_A} 1_{\Lambda_B}\rangle)/\sqrt{2}$$

Quantum average causal effect (QACE)

The quantum average causal effect (QACE):

$$\begin{aligned} \text{QACE}_{A \rightarrow B} &= \sup_{a, a', b} |p_Q(b|\text{do}(a)) - p_Q(b|\text{do}(a'))| \\ &= \sup_{a, a', b} \left| \text{Tr} \left[\left(M_b^{\text{do}(a)} - M_b^{\text{do}(a')} \right) \varrho_{\Lambda_B} \right] \right| \\ &\equiv 0 \quad (\text{for all } v!!!) \end{aligned}$$



$$p_Q(a, b|x) = \text{Tr} [M_a^x \otimes M_b^a \varrho_\Lambda]$$

$$\begin{aligned} \Rightarrow p_Q(b|\text{do}(a)) &= \text{Tr} \left[\mathbf{1} \otimes M_b^{\text{do}(a)} \varrho_\Lambda \right] \\ &= \text{Tr}_B \left[M_b^{\text{do}(a)} \varrho_{\Lambda_B} \right] \end{aligned}$$

$$\varrho_\Lambda = v |\Phi^+\rangle\langle\Phi^+| + (1 - v) \mathbf{1}/4$$

$$\Rightarrow \rho_{\Lambda_B} = \text{Tr}_A [\varrho_\Lambda] = \frac{\mathbf{1}}{2}$$

enhance
to reproduce

Can one observe a discrepancy device-independently?

How about other instrumental inequalities (potentially violated with quantum common causes)?

Violation of a classical instrumental test with quantum instrumental causal models

If X is trichotomic, another instrumental inequality appears:

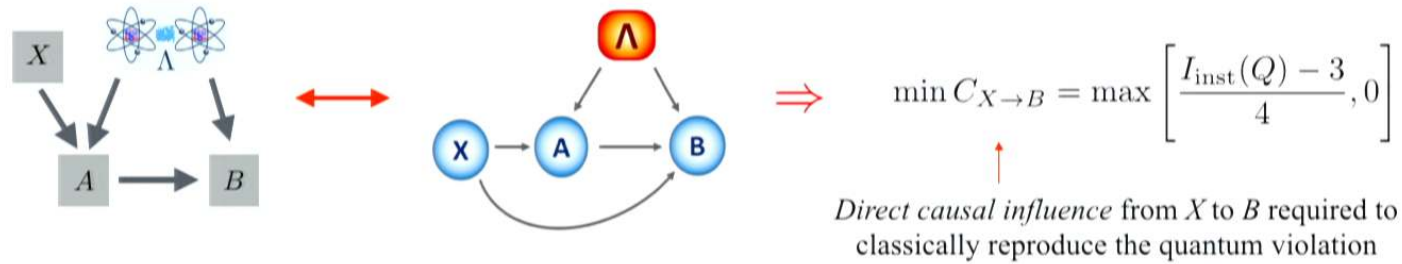
$$I_{\text{inst}} := -\langle B \rangle_{x=1} + 2\langle B \rangle_{x=2} + \langle A \rangle_{x=1} - \langle AB \rangle_{x=1} + 2\langle AB \rangle_{x=3} \leq 3$$

B. Bonet, UAI (2001).

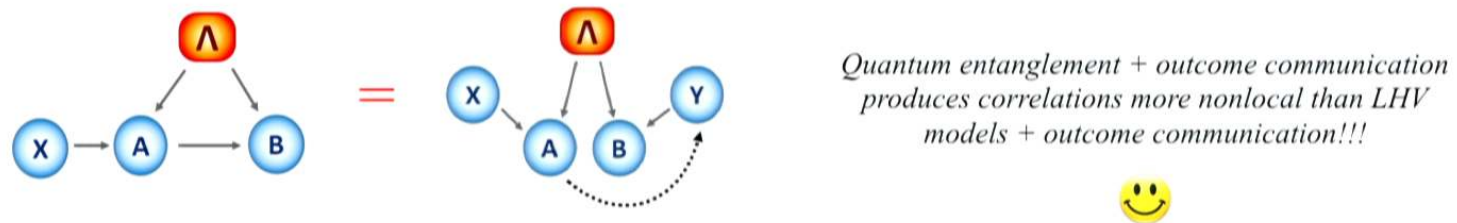
R. Chaves, G. Carvacho, I. Agresti, V. Di Giulio, L. Aolita, S. Giacomini, and F. Sciarrino, Nature Physics (2017).

Interpretations of quantum violations

• **Causal-inference viewpoint:** Quantum effects *change the interpretation* of instrumental-inequality violations:



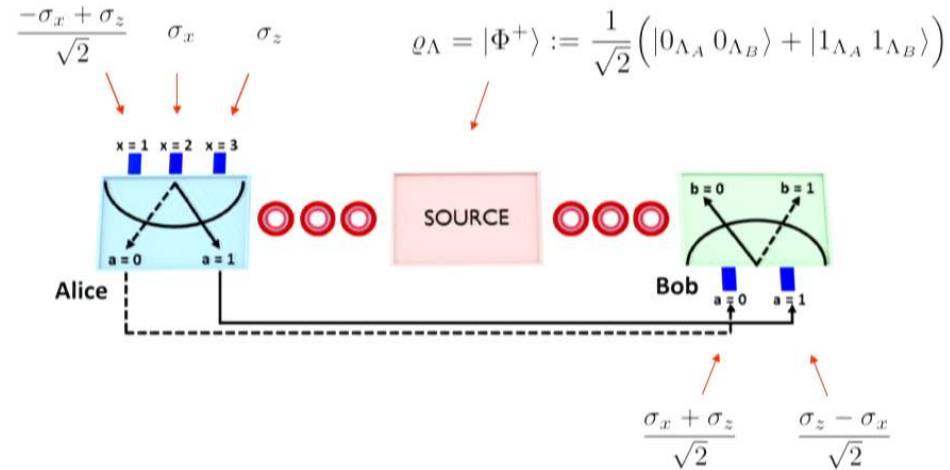
• **Bell non-locality viewpoint:** Instrumental-inequality violations by quantum instrumental causal models can be seen as a *novel, stronger form of non-classicality*.



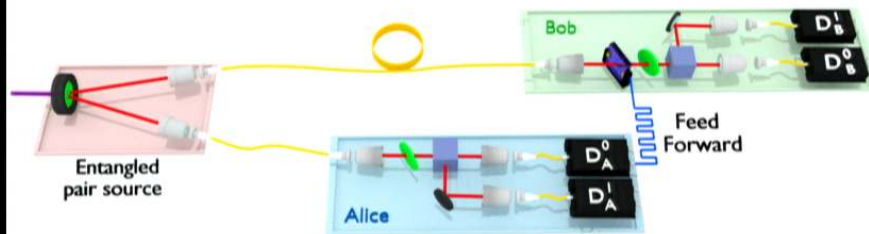
R. Chaves, G. Carvacho, I. Agresti, V. Di Giulio, L. Aolita, S. Giacomini, and F. Sciarrino, Nature Physics (2017).

Experimental demonstration (Rome)

Experimental scheme:



Implementation:



Partial Conclusions:

- The instrumental causal structure is a *cornerstone of classical causal inference*: allows to *estimate causal influences* in the presence of *hidden common causes and without interventions*.
- Introduction of quantum ACE. Classical causality concepts can lead to a significant overestimation of the ACE for quantum causal models.
- Can also be seen as a *quantum advantage* in terms of descriptive power of observed data.
- Simplest causal structure with a *quantum-classical gap* (and we found the quantum violation)!
- Quantum violation changes the *interpretation of instrumental-inequality violations*.
- Experiment with entangled-photon pairs + active feedforward: *demonstration of a novel, stronger form of non-classicality*.

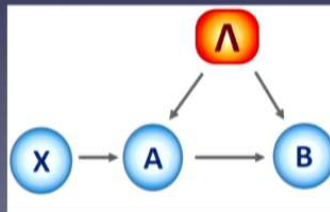
Partial outlook:

1. Multi-partite generalisations: anything interesting?

2. Quantum causal inference: DI lower bound to QACE? definition of *quantum direct causal influence*?

3. Post-quantum instrumental causal models.

4. Quantum instrumental causal models as a resource? ... for what?



Randomness generation ?

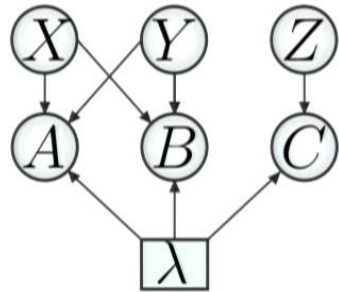
R. Gallego, L. Masanes, G. de la Torre, C. Dhara, L. Aolita, and A. Acín, "Full randomness from arbitrarily deterministic events", Nature Communications 4, 2654 (2013).

Device-independent multipartite no-signalling case

R. Chaves, D. Cavalcanti, and L. Aolita, Quantum **1**, 23 (2017).

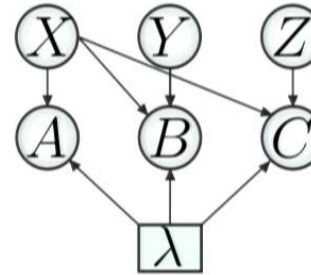
Relaxations of locality in the multipartite scenario

Bi-LHV (split into 2 halves)



G. Svetlichny (87); D. Collins et al. (02); M. Seevinck and G. Svetlichny (02); J. Lavoie, R. Kaltenbaek, and K. J. Resch (09).

Input-broadcasting



J. D. Bancal, C. Branciard, N. Gisin, and S. Pironio (09).

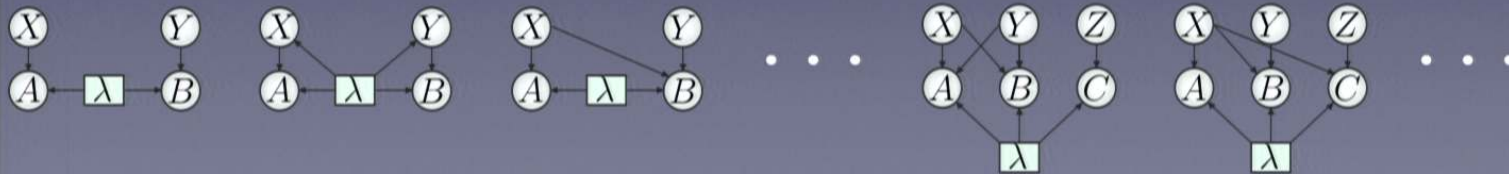
- These models give rise to *genuinely multipartite Bell non-locality*.
- They satisfy the *Svetlichny inequality*:

$$-\langle A_0 B_0 C_0 \rangle + \langle A_0 B_0 C_1 \rangle + \langle A_0 B_1 C_0 \rangle + \langle A_0 B_1 C_1 \rangle + \langle A_1 B_0 C_0 \rangle + \langle A_1 B_0 C_1 \rangle + \langle A_1 B_1 C_0 \rangle - \langle A_1 B_1 C_1 \rangle \leq 4.$$

- The Svetlichny inequality is *violated by* quantum correlations from *the GHZ state*: $|\Psi_{\text{GHZ}}\rangle := (|000\rangle + |111\rangle)/\sqrt{2}$.
- *No stronger inequality known violated by quantum correlations.* 😞

Bell DAGs are exponentially complex objects, can one find a unifying, systematic, and practical way to classify them?

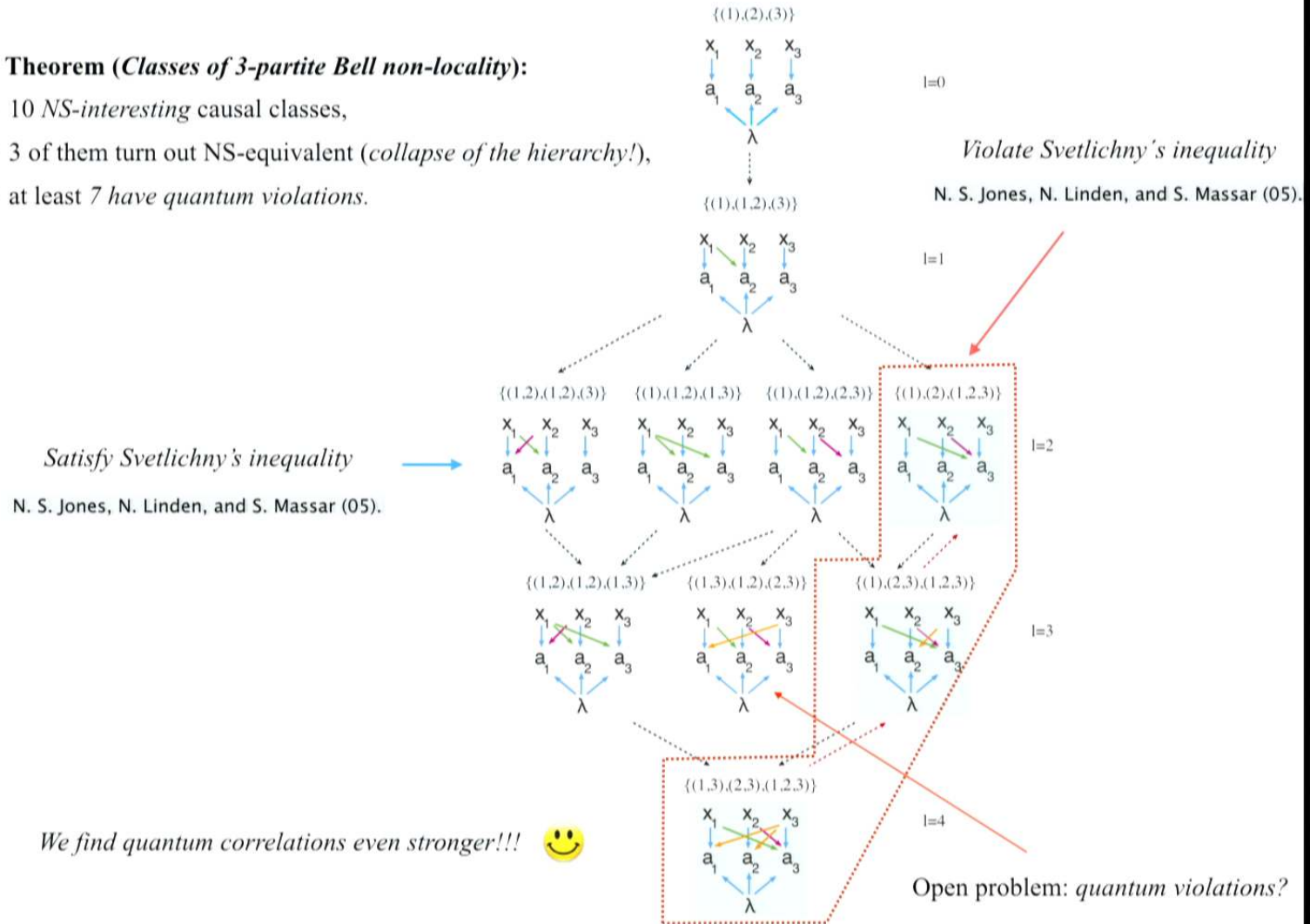
Idea: exploit the no-signalling constraint!!!



Causal hierarchy of classes of multipartite Bell non-locality

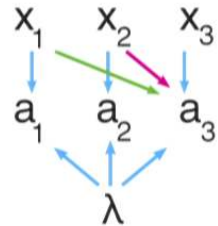
Theorem (Classes of 3-partite Bell non-locality):

- 10 NS-interesting causal classes,
- 3 of them turn out NS-equivalent (collapse of the hierarchy!),
- at least 7 have quantum violations.



Quantum genuinely multipartite “super non-locality”

The *star* Bell DAG: one party receives all inputs.

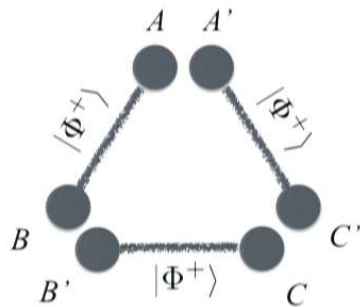


It satisfies all inequalities of the generic form:

$$I_3 := I_2(A, B) + I_2(A', C) + I_2(B', C') \leq \beta_L + 2\beta_{NS}$$

- These *inequalities are maximally violated by quantum correlations!!!*

A. Cabello (01); T. Yang et al. (05); J. Barrett, A. Kent, and S. Pironio (06); L. Aolita et al. (11); T. E. Stuart et al. (12); B. G. Christensen et al. (12).



Type of state required for the violation

*The strongest form of multipartite quantum nonlocality known
(even stronger than conventional genuinely multipartite)!!!*



Partial conclusions:

- We studied how to reproduce multipartite NS correlations with generic causal relaxations.
- The NS principle imposes non-trivial equivalences between different Bell DAGs.
- Practical classification of multipartite Bell non-locality: the causal hierarchy.



The screenshot shows the top of a Quantum journal article page. The header includes the Quantum logo and navigation links for PAPERS, VIEWS, BLOG, and INSTRUCTIONS. The article title is "Causality: relaxing before exploring", posted on 2017-08-04 by Quantum Views. The text below the title states: "This is a Perspective on 'Causal hierarchy of multipartite Bell nonlocality' by Rafael Chaves, Daniel Cavalcanti, and Leandro Aolita, published in Quantum 1, 23 (2017). Quantum Views 1, 3 (2017). https://doi.org/10.22331/qv-2017-08-04-3 By Paul Skrzypczyk, School of Physics, University of Bristol, UK." To the right of the text is a diagram showing three parties (Alice, Bob, Charlie) with their respective causal structures and a larger diagram of a causal hierarchy.

*The true power of the results comes in the **unifying and simplified view** they provide for studying relaxed causal structures. **What was previously a vast forest is now a well organised playground, ready to be explored, and played in**, as we continue to push forward our understanding of quantum non-locality.*

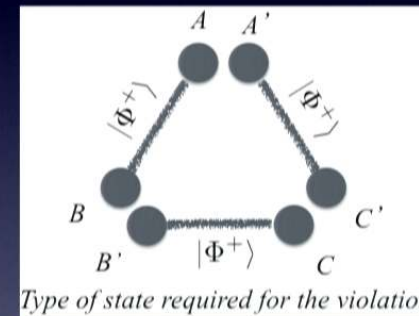


- Quantum correlations even stronger than genuinely multipartite nonlocality discovered (open problem since 05).
- Post-quantum (?) tripartite super non-locality.

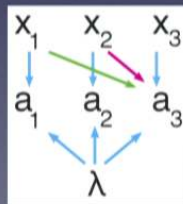
Partial outlook:



1. More compact/practical quantum realisations of multipartite super non-locality?, experiments?



2. Multipartite super non-locality as a resource?, for what?



Useful for DI secret sharing

So, for what stronger form of multipartite secrecy is super non-locality useful?

L. Aolita, R. Gallego, A. Cabello, and A. Acín, "Fully nonlocal, monogamous, and random genuinely multipartite quantum correlations", Phys. Rev. Lett. **108**, 100401 (2012).

General conclusions:

- Quantisation of the classical notion of causality not trivial.
- Stronger forms of quantum correlations discovered.
- Quantum multipartite super non-locality and supra-instrumental correlations are just-born babies.
- Potential applications in multipartite cryptographic schemes and randomness generations may appear, any other?
- Quantum causal inference, resource-theory approaches, novel information-theoretic physical protocols, novel experiments, etc.....
- Many exciting open questions!

Collaborators at Rio:



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Permanent Faculty



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PhD student



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PhD student



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Waterloo



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Thank you for your attention!

Quantum violation of an instrumental test:

R. Chaves, G. Carvacho, I. Agresti, V. Di Giulio, L. Aolita, S. Giacomini, and F. Sciarrino, *Nature Physics* (2017).

Quantum steering beyond instrumental causal networks:

R. Neri, M. Taddei, R. Chaves, and L. Aolita, arXiv: 1712.08624 (2017).

Quantum multipartite super non-locality and the causal hierarchy:

R. Chaves, D. Cavalcanti, and L. Aolita, *Quantum* **1**, 23 (2017);

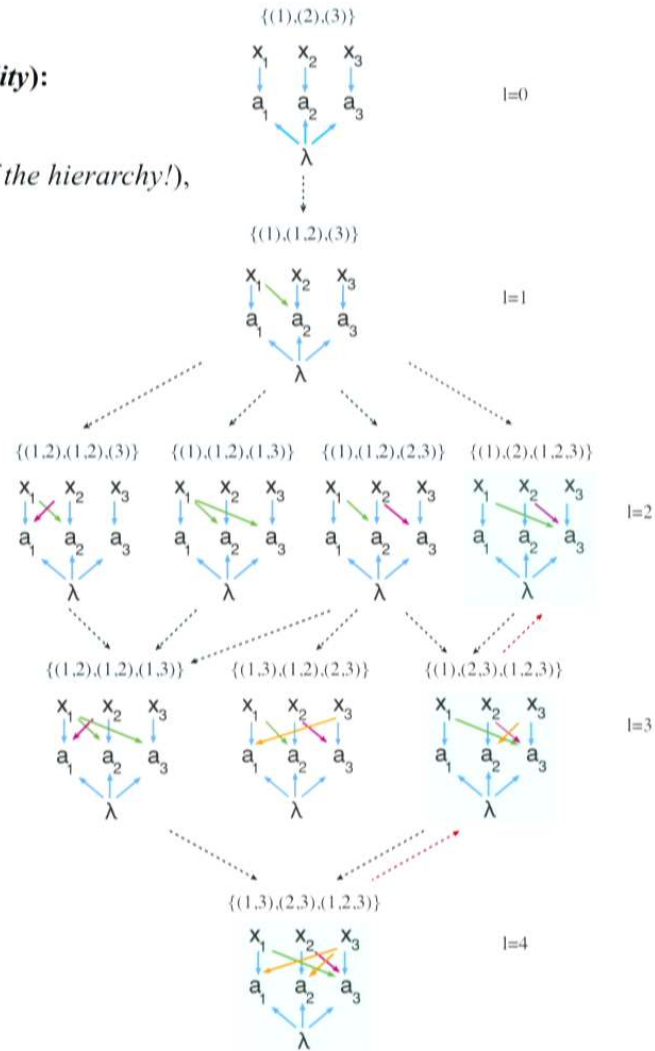
P. Skrzypczyk, *Quantum Views* **1**, 3 (2017).

Causal hierarchy of classes of multipartite Bell non-locality

Theorem (Classes of 3-partite Bell non-locality):

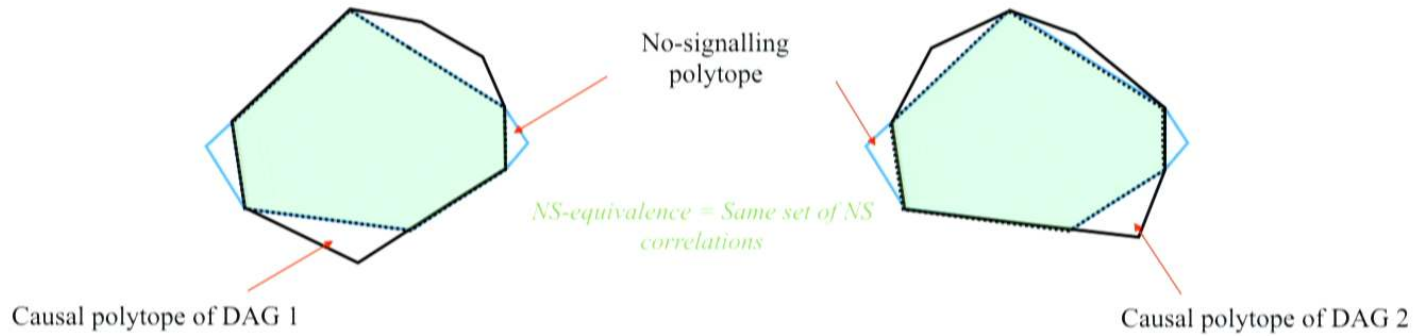
- 10 NS-interesting causal classes,
- 3 of them turn out NS-equivalent (*collapse of the hierarchy!*),
- at least 7 have quantum violations.

Satisfy Svetlichny's inequality
 N. S. Jones, N. Linden, and S. Massar (05).

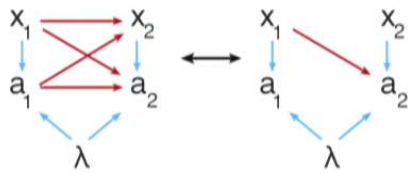


No-signalling equivalent Bell DAGs

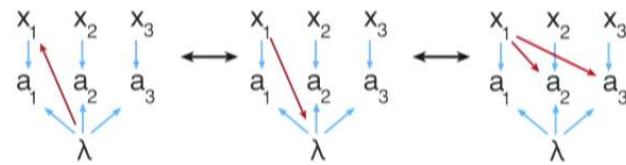
No-signalling equivalence: same set of observable no-signalling correlations.



Lemma 1: The most general locality relaxation is NS-equivalent to an input-output locality relaxation.



Lemma 2: Any measurement independence relaxation is NS-equivalent to input broadcasting.



simplifies the problem