Title: PSI 17/18 - Foundations of Quantum Mechanics - Lecture 14

Date: Feb 15, 2018 10:15 AM

URL: http://pirsa.org/18020073

Abstract:

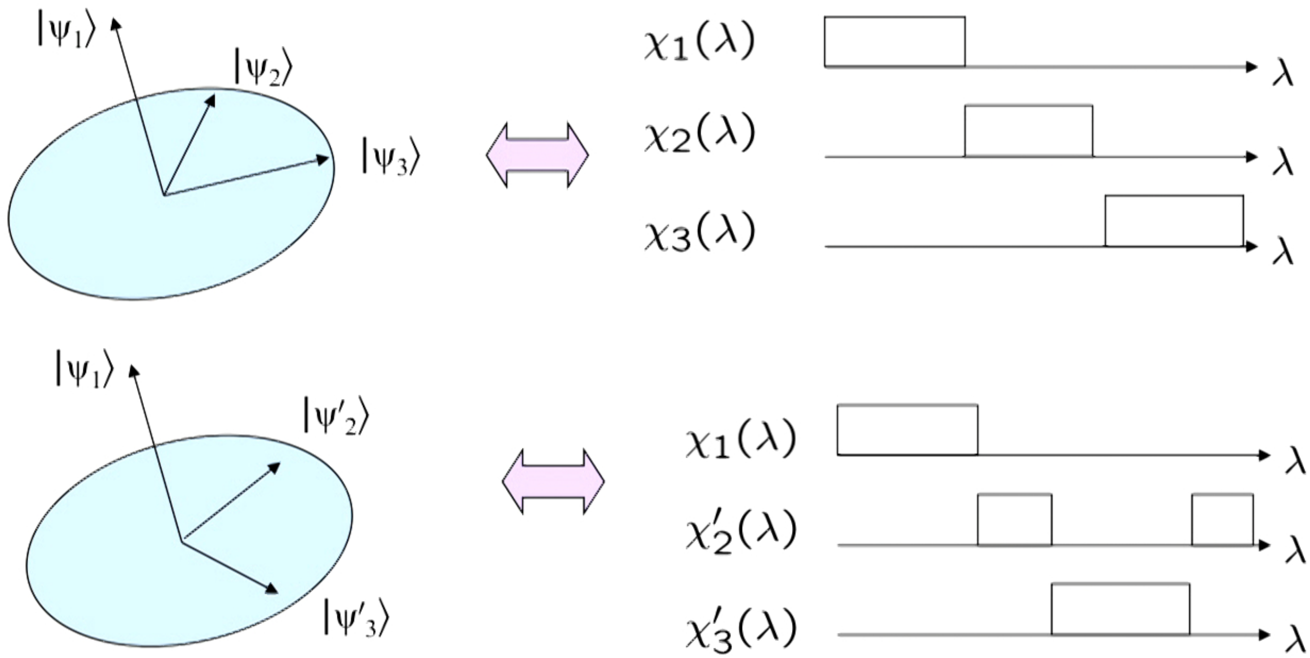
Generalized Contextuality

What we want in a notion of nonclassicality

	Subject to direct experimental test	Constitutes a resource	Applicable to a broad range of physical scenarios
Failure to admit a locally causal model	✓	✓	✗
Failure to admit a noncontextual model	✓	✓	✓

Traditional notion of noncontextuality

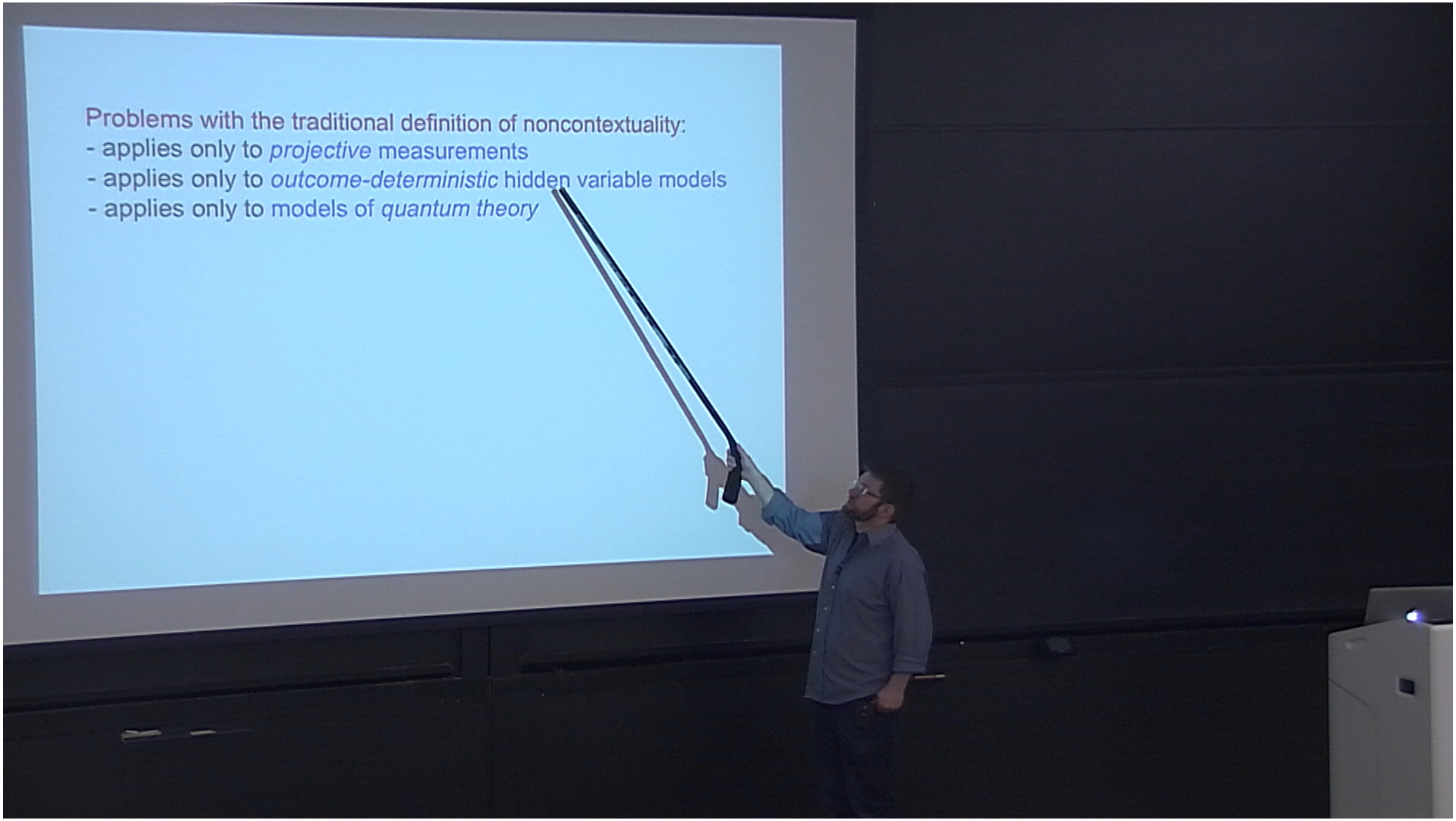
A given vector may appear in many different measurements



The traditional notion of noncontextuality:
Every vector is associated with the same $\chi(\lambda)$ regardless of how it is measured (i.e. the context)

Problems with the traditional definition of noncontextuality:

- applies only to *projective* measurements
- applies only to *outcome-deterministic* hidden variable models
- applies only to models of *quantum theory*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Problems with the traditional definition of noncontextuality:

- applies only to *projective* measurements
- applies only to *outcome-deterministic* hidden variable models
- applies only to models of *quantum theory*

An operational notion of noncontextuality would determine

- whether any given operational theory admits of a noncontextual model
- whether any given experimental data can be explained by a noncontextual model

Operational theory

$$p(X|M,P)$$

Operational theory

$$p(X|M,P)$$

Ontological model of an operational theory

$\lambda \in \Lambda$ Ontic state space

causally mediates
between P and M

Operational theory

$$p(X|M,P)$$

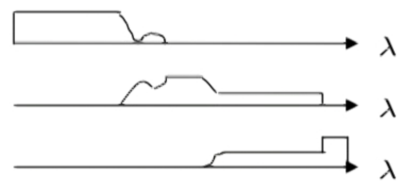
Ontological model of an operational theory

$\lambda \in \Lambda$ Ontic state space

causally mediates
between P and M

$\mathrm{P} \leftrightarrow \mu(\lambda|\mathrm{P})$

$\mathrm{M} \leftrightarrow \xi(X|\mathrm{M},\lambda)$

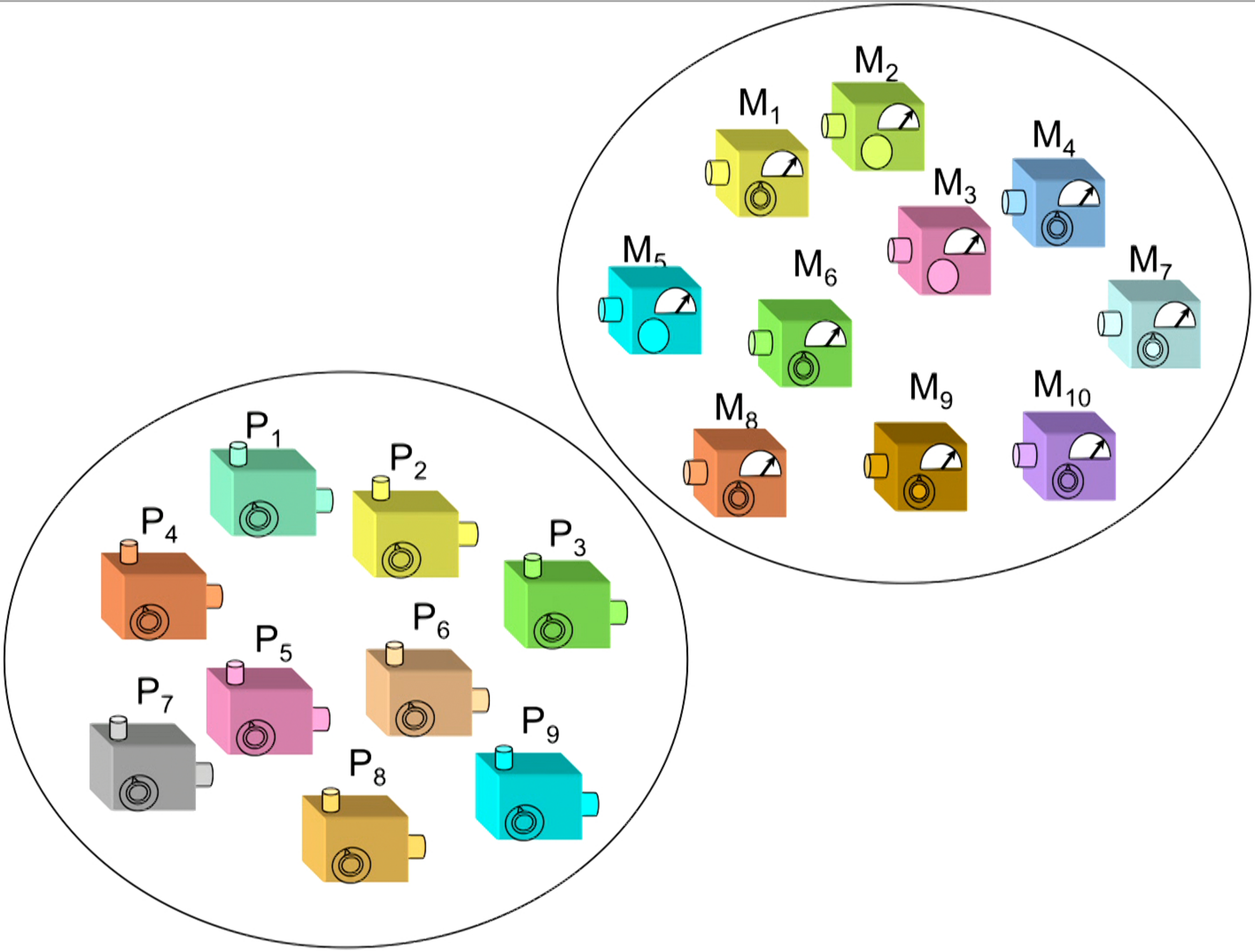


$$p(X|M,P) = \sum_\lambda \xi(X|M,\lambda)\,\mu(\lambda|P)$$

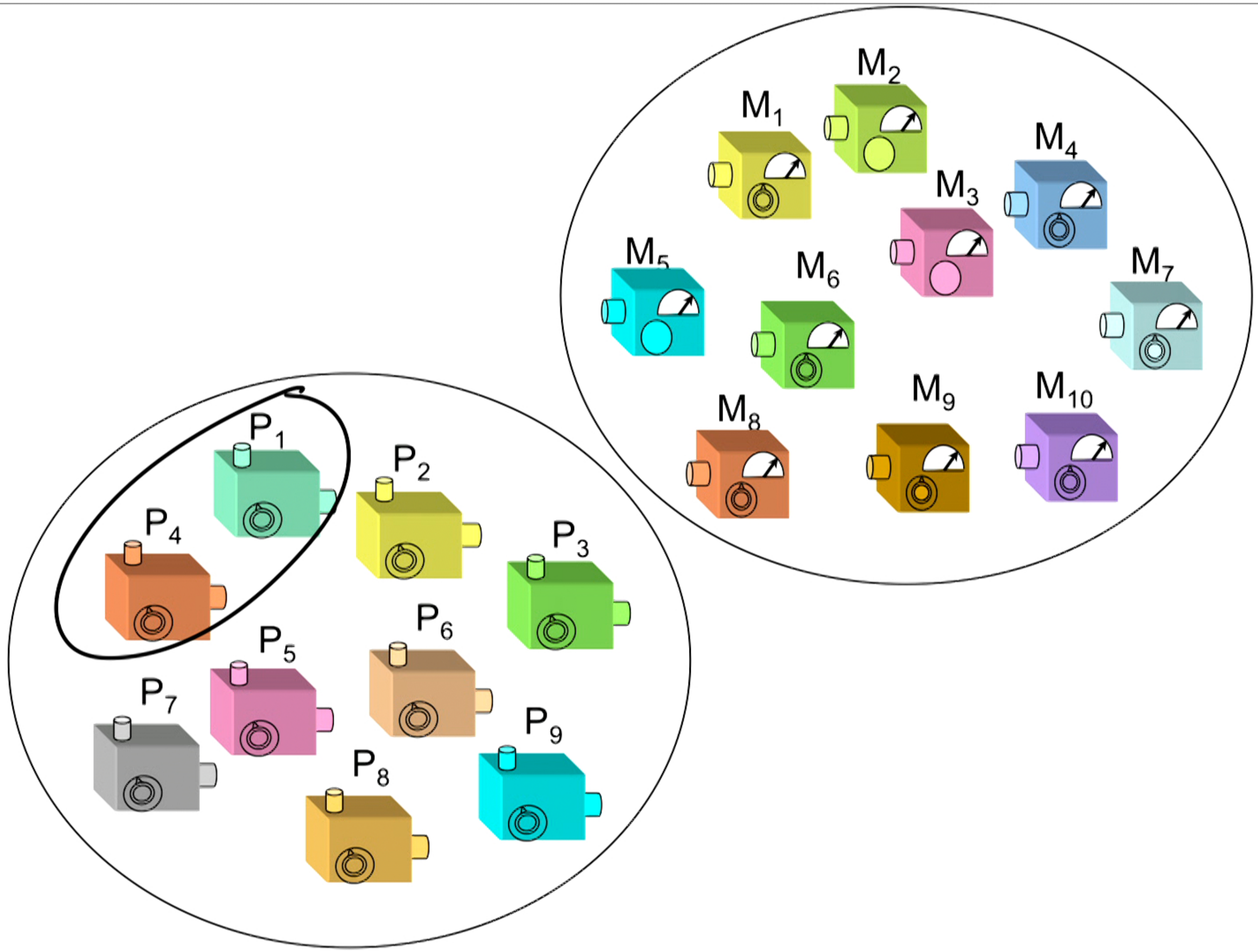
An ontological model of an operational theory is noncontextual if

Operational equivalence of two experimental procedures → Equivalent representations in the ontological model

RWS, Phys. Rev. A 71, 052108 (2005)



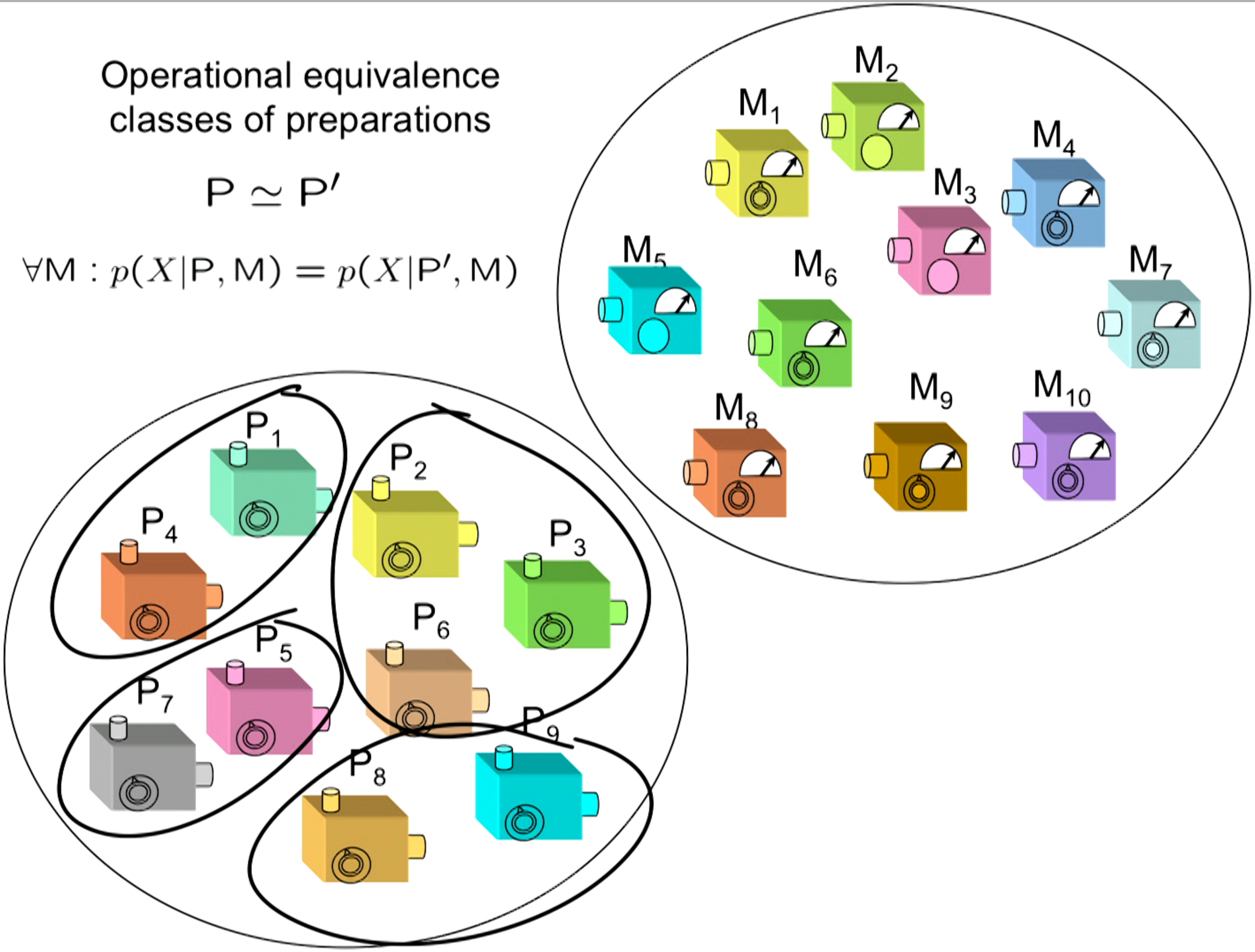
M_1
M_2
M_3
M_4
M_5
M_6
M_7
M_8
M_9
M_{10}
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9

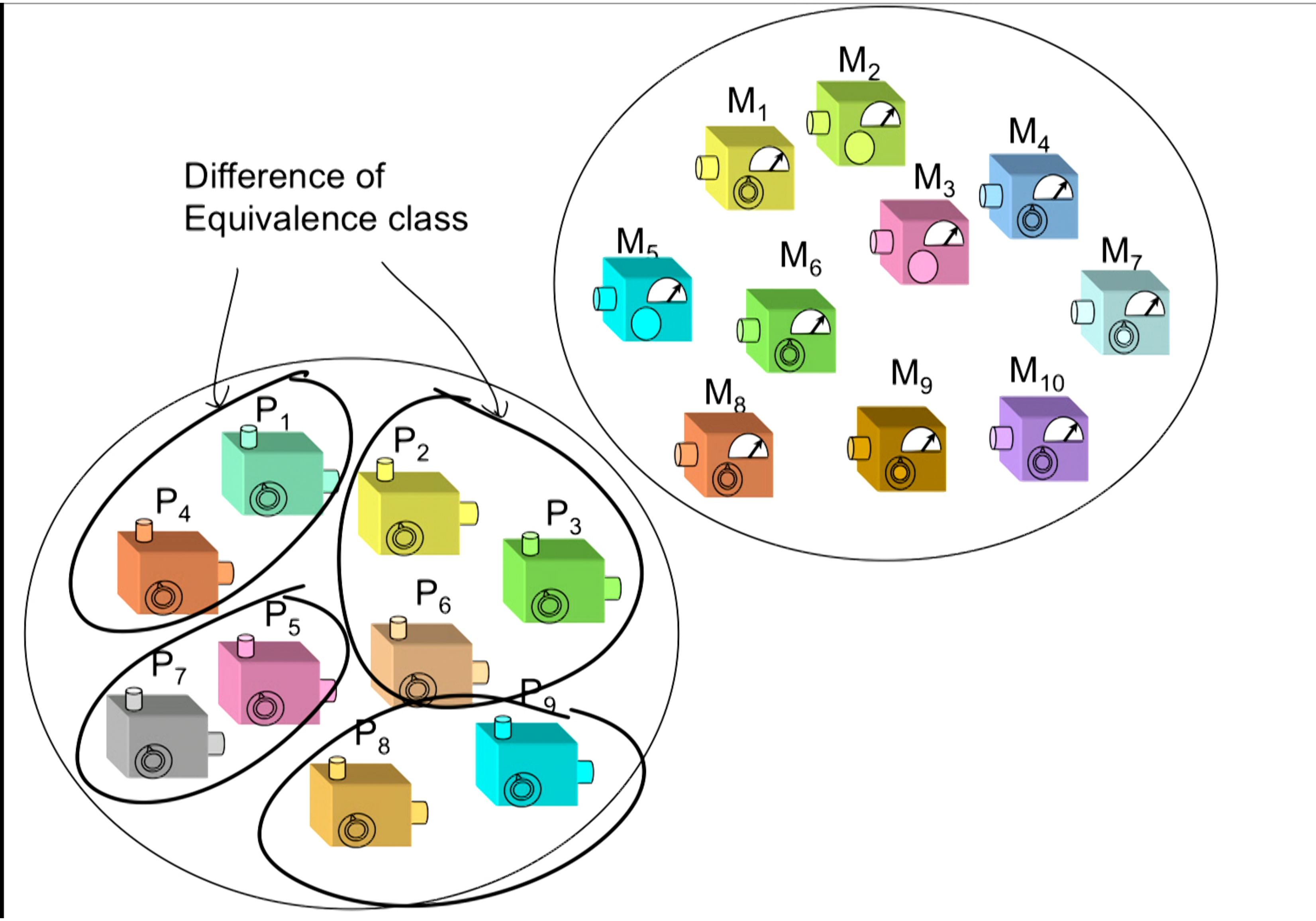

M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P1
P2
P3
P4
P5
P6
P7
P8
P9

Operational equivalence classes of preparations

$$\mathsf{P} \simeq \mathsf{P}'$$

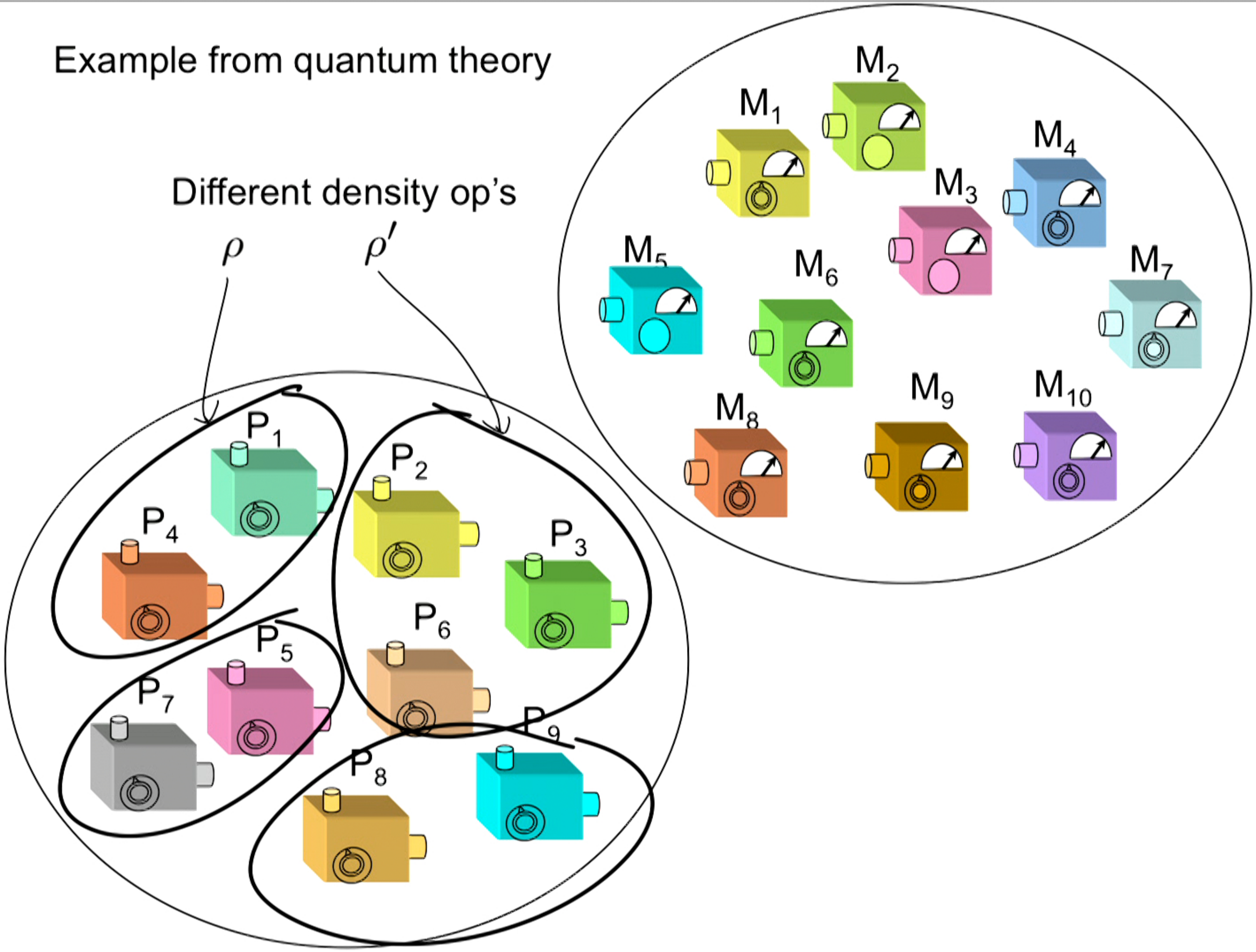
$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$





Difference of
Equivalence class
P1
P2
P3
P4
P5
P6
P7
P8
P9
M1
M2
M3
M4
M5
M6
M7
M8
M9
M10

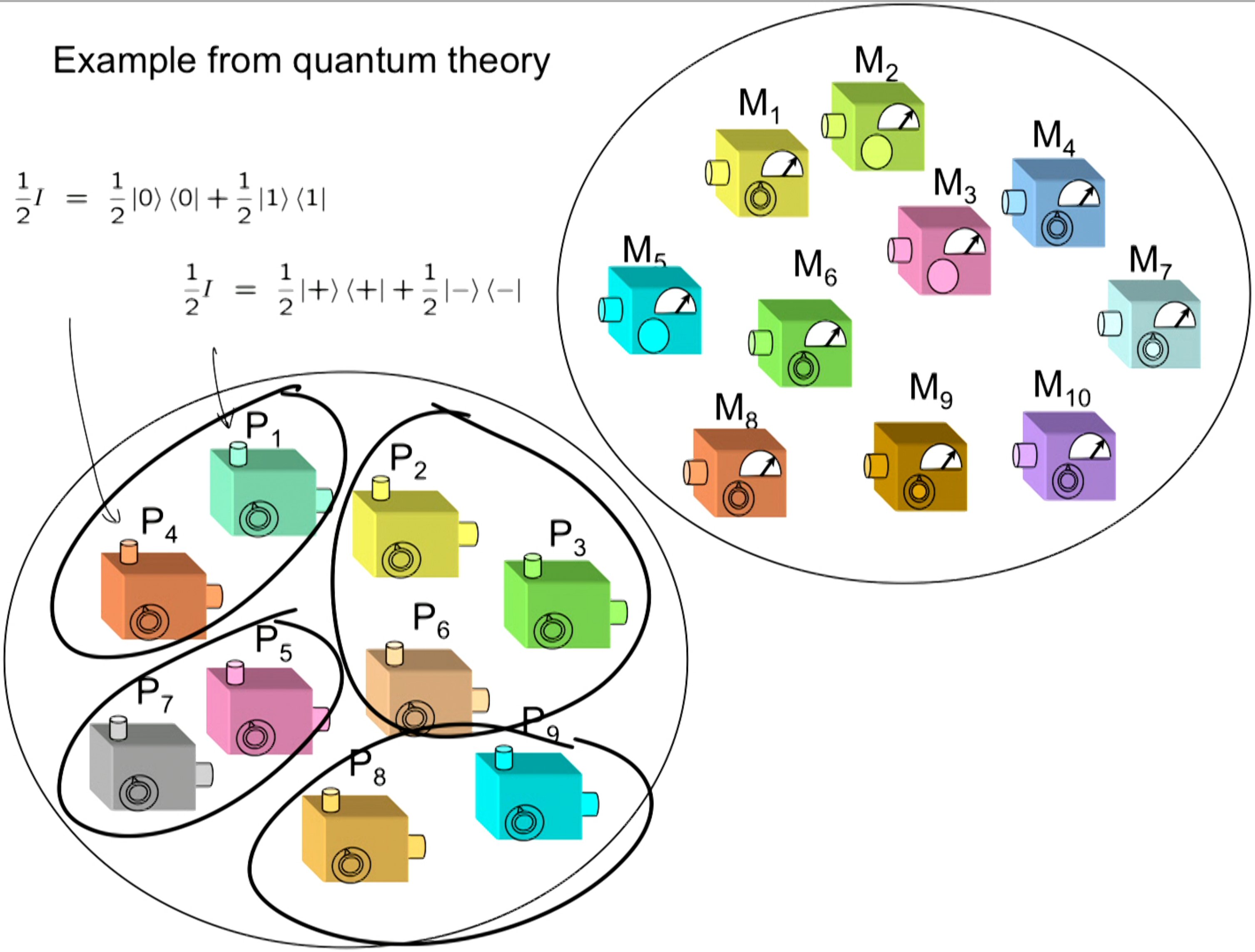
Example from quantum theory



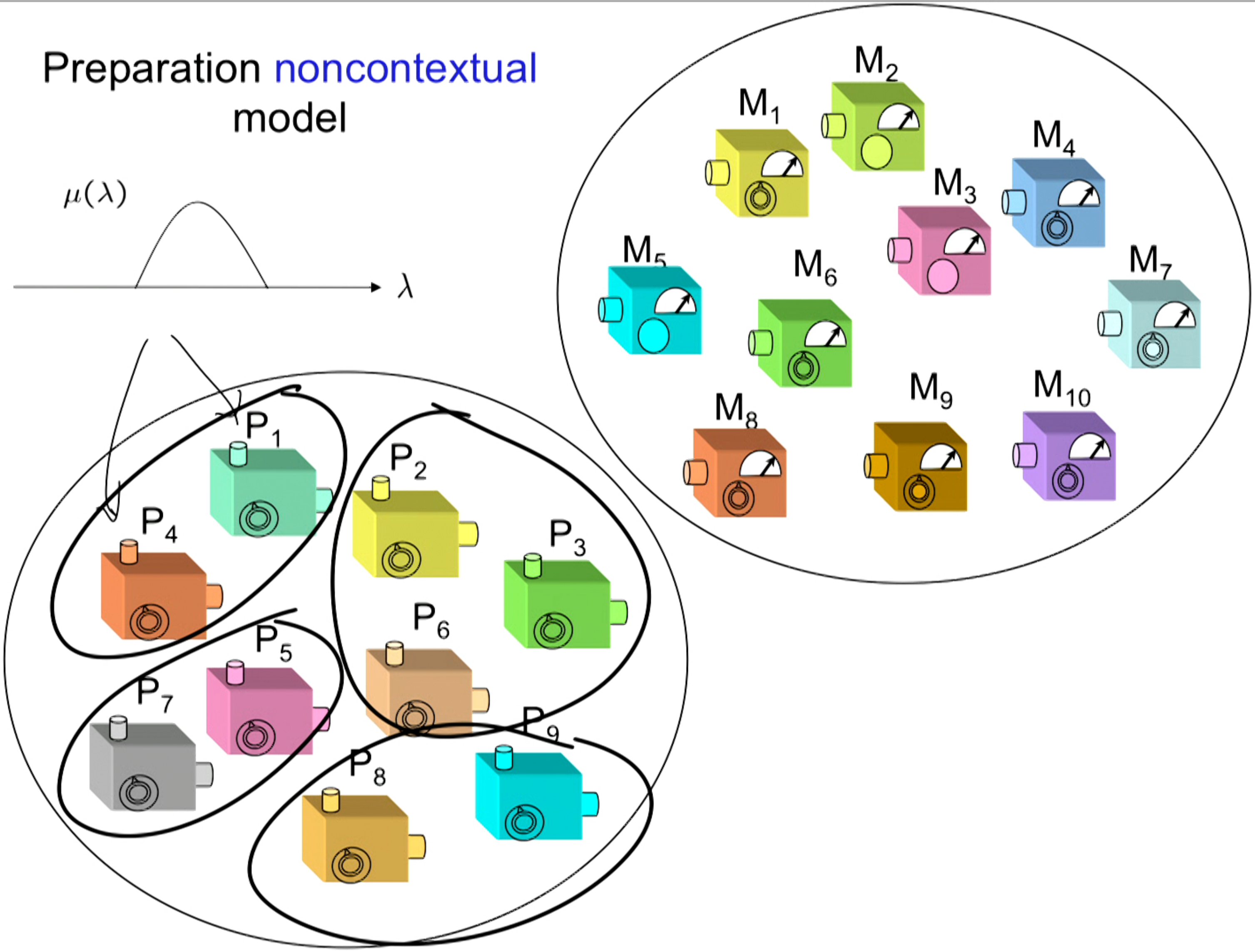
Example from quantum theory

$$\frac{1}{2}I = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

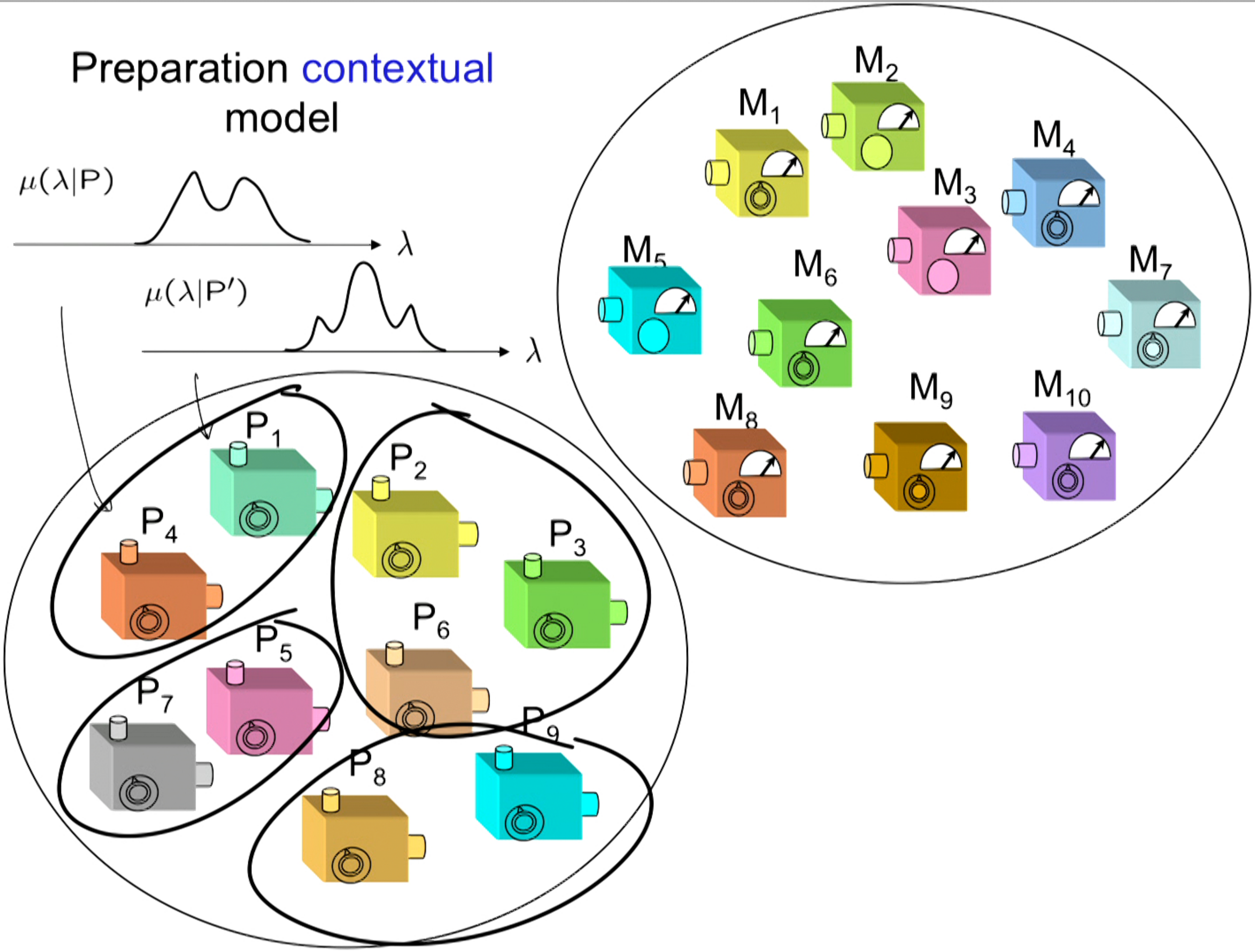
$$\frac{1}{2}I = \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$

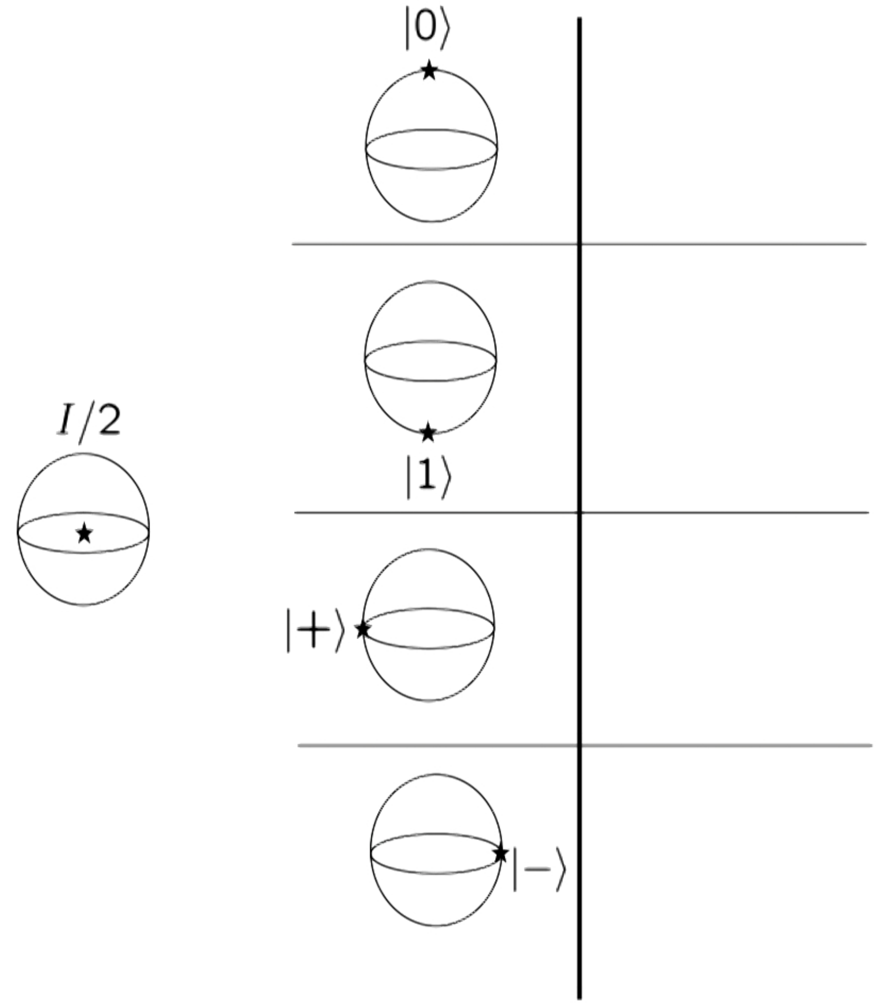


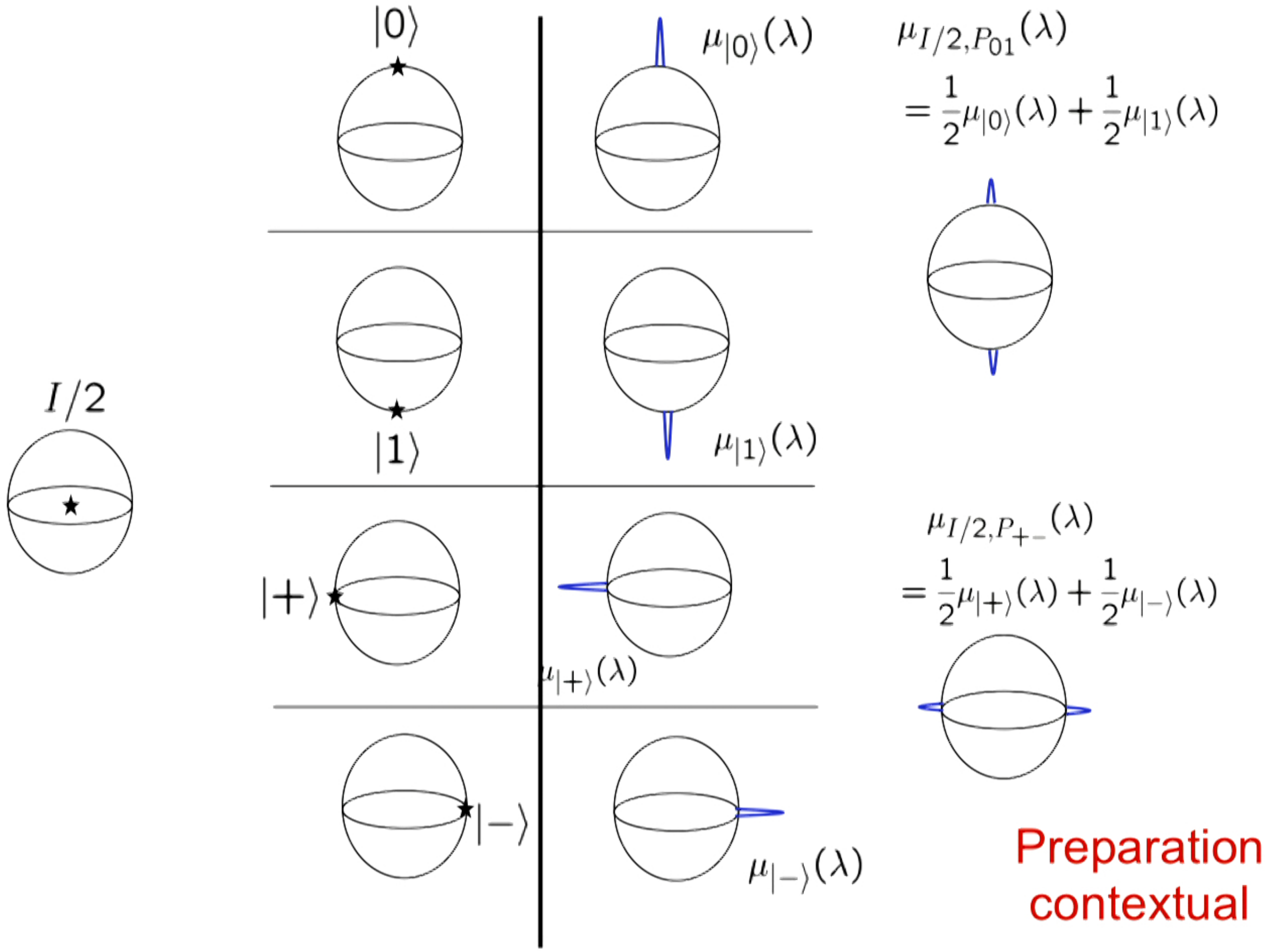
Preparation noncontextual model



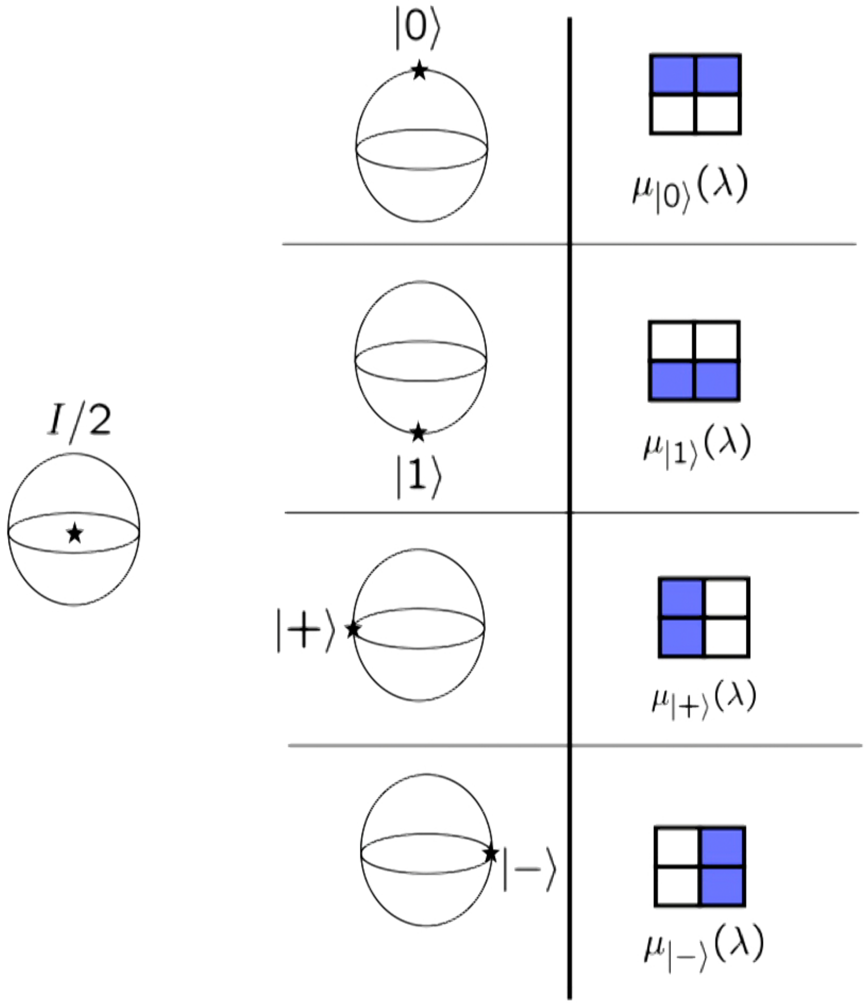
Preparation contextual model



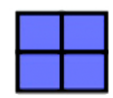

$|0\rangle$
$I/2$
$|1\rangle$
$|+\rangle$
$|-\rangle$



$|0\rangle$
$\mu_{|0\rangle}(\lambda)$
$\mu_{I/2,P_{01}}(\lambda)$
$= \frac{1}{2}\mu_{|0\rangle}(\lambda) + \frac{1}{2}\mu_{|1\rangle}(\lambda)$
$I/2$
$|1\rangle$
$\mu_{|1\rangle}(\lambda)$
$\mu_{I/2,P_{+-}}(\lambda)$
$= \frac{1}{2}\mu_{|+\rangle}(\lambda) + \frac{1}{2}\mu_{|-\rangle}(\lambda)$
$|+\rangle$
$\mu_{|+\rangle}(\lambda)$
$|-\rangle$
$\mu_{|-\rangle}(\lambda)$
Preparation contextual

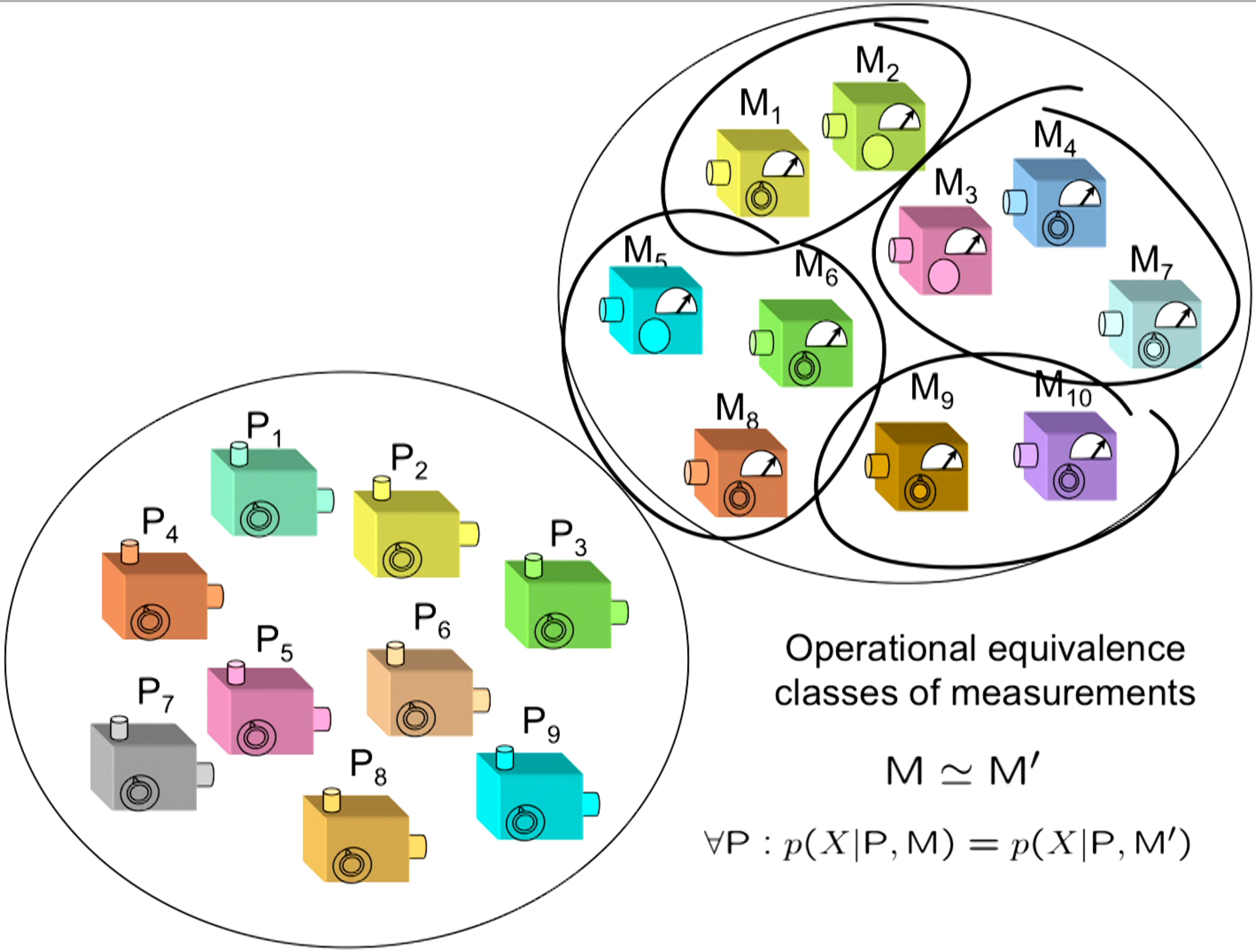


$$\mu_{I/2}(\lambda)$$
$$= \frac{1}{2}\mu_{|0\rangle}(\lambda) + \frac{1}{2}\mu_{|1\rangle}(\lambda)$$
$$= \frac{1}{2}\mu_{|+\rangle}(\lambda) + \frac{1}{2}\mu_{|-\rangle}(\lambda)$$



Preparation noncontextual

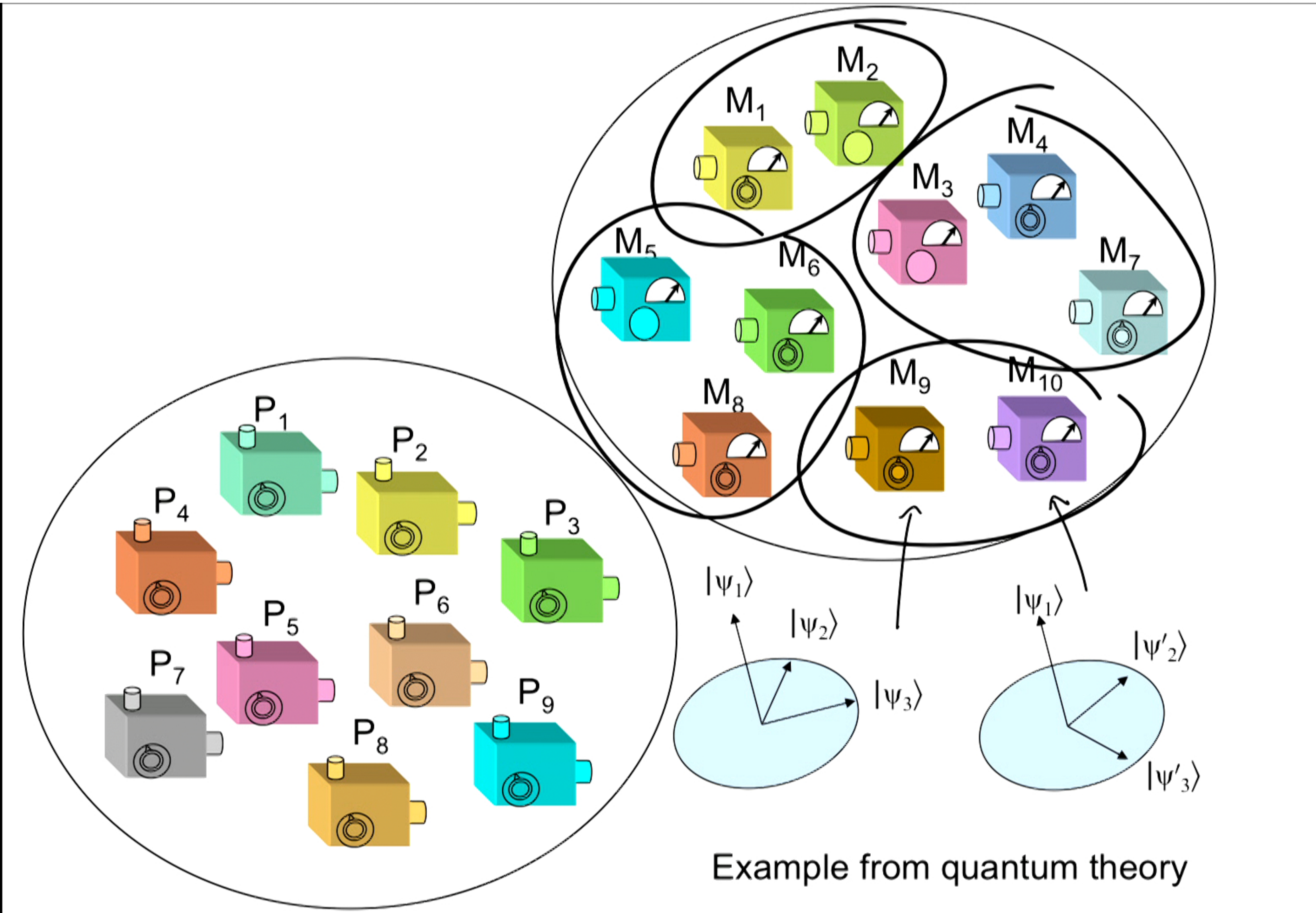
Measurement Noncontextuality



Operational equivalence classes of measurements

$$\mathsf{M} \simeq \mathsf{M}'$$

$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}')$$



P1
P2
P3
P4
P5
P6
P7
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P9
M1
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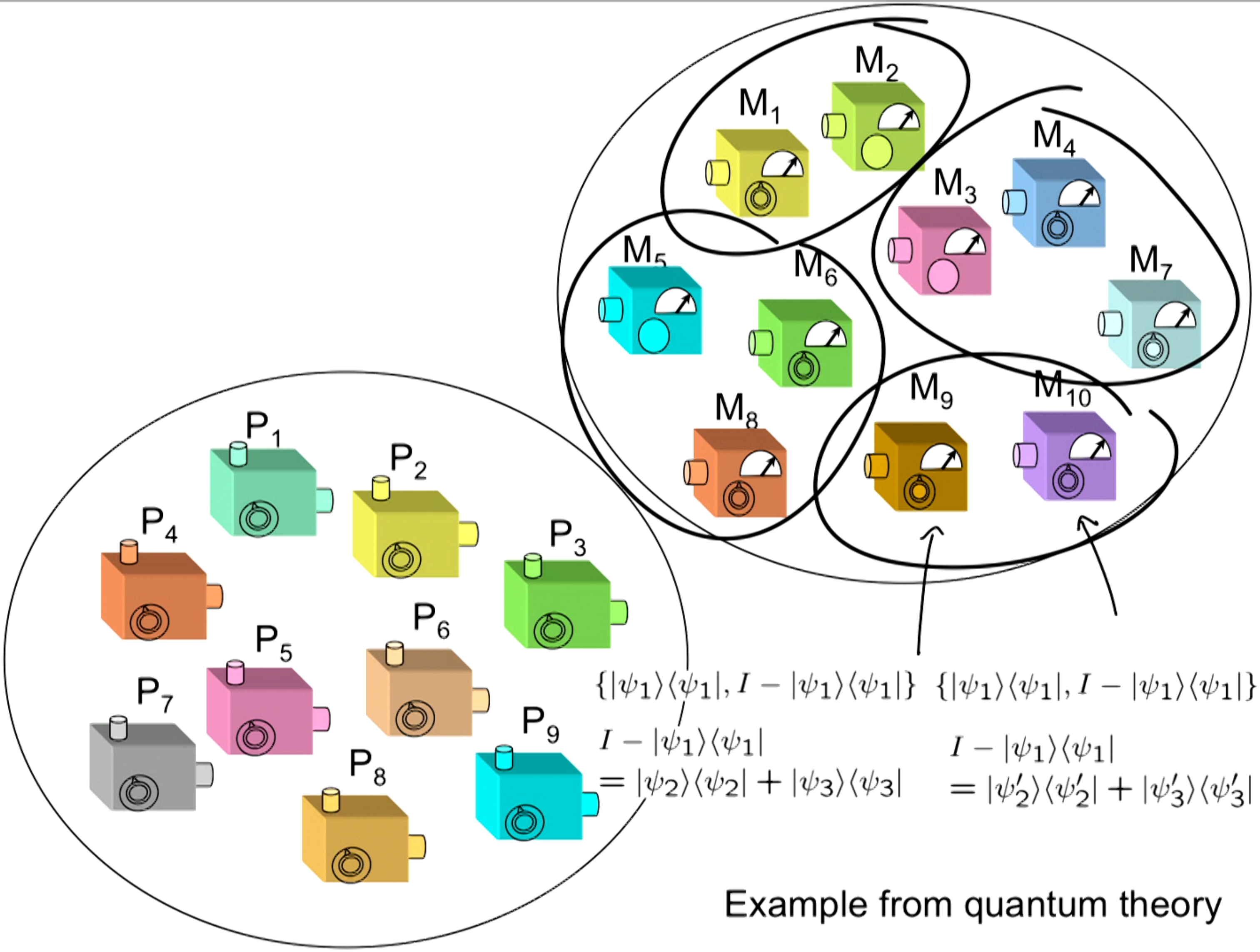
M6
M7
M8
M9
M10
|ψ1⟩
|ψ2⟩
|ψ3⟩
|ψ1⟩
|ψ′2⟩
|ψ′3⟩
Example from quantum theory

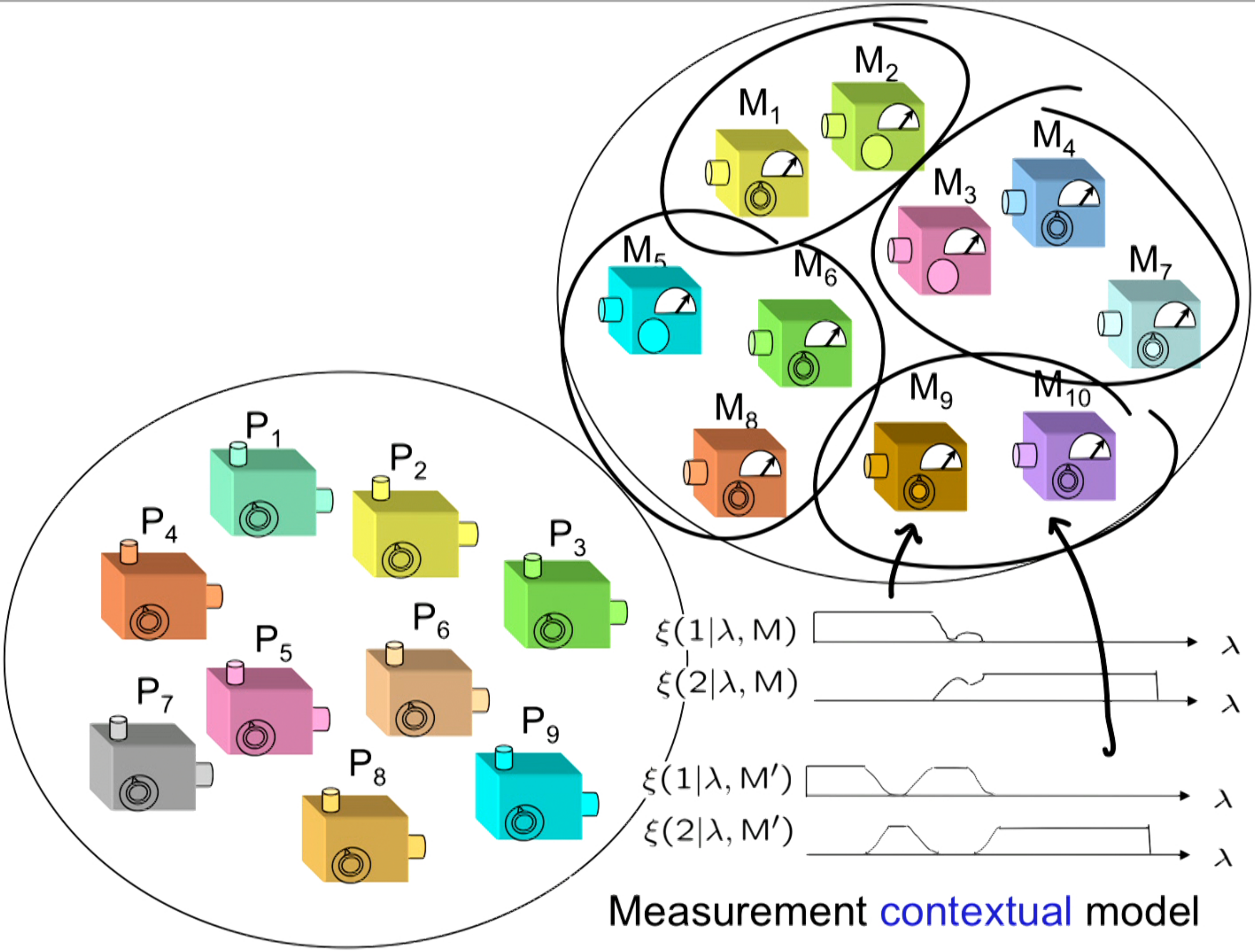


$$\{|\psi_1\rangle\langle\psi_1|, I - |\psi_1\rangle\langle\psi_1|\} \quad \{|\psi_1\rangle\langle\psi_1|, I - |\psi_1\rangle\langle\psi_1|\}$$

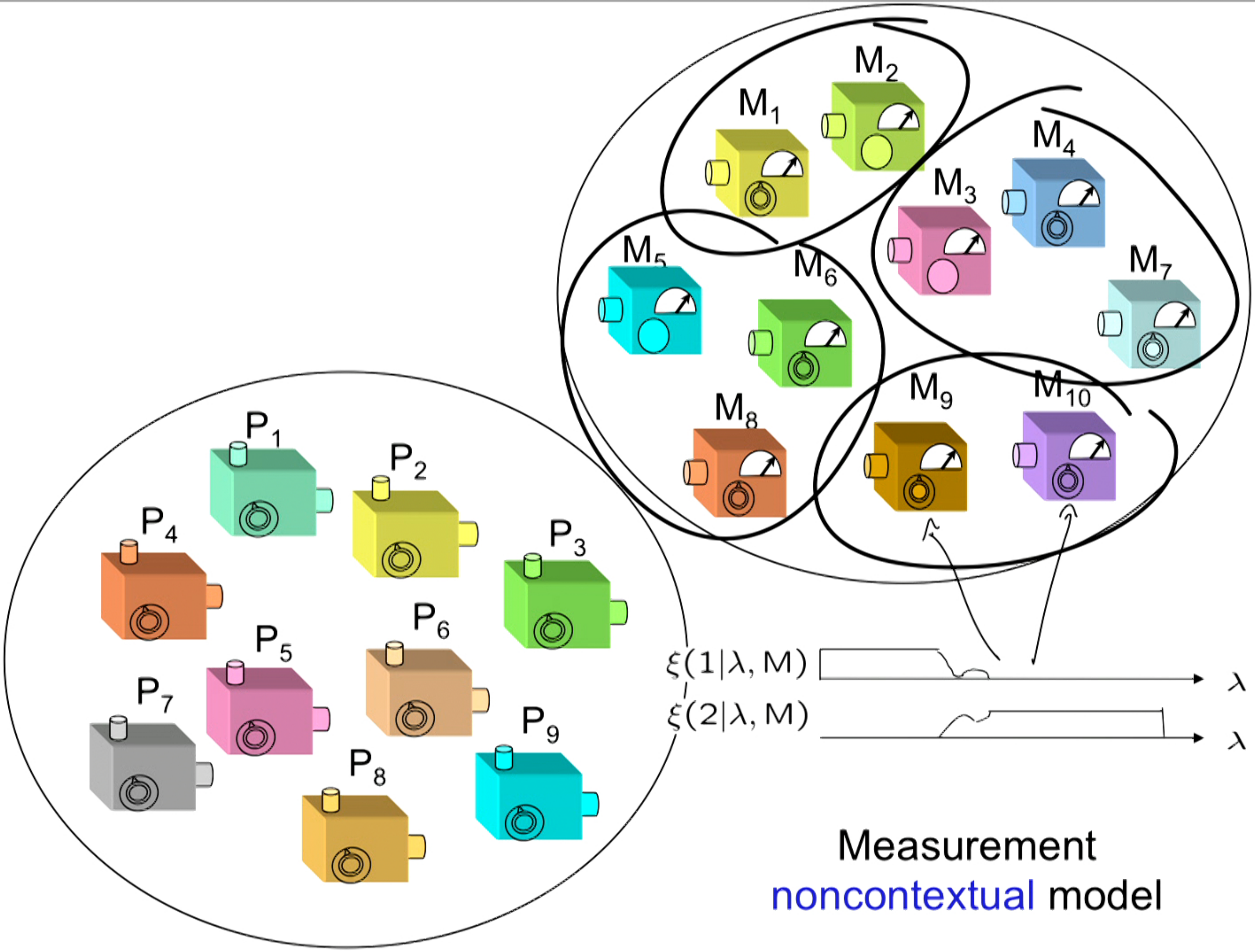
$$I - |\psi_1\rangle\langle\psi_1| = |\psi_2\rangle\langle\psi_2| + |\psi_3\rangle\langle\psi_3|$$

$$I - |\psi_1\rangle\langle\psi_1| = |\psi_2'\rangle\langle\psi_2'| + |\psi_3'\rangle\langle\psi_3'|$$

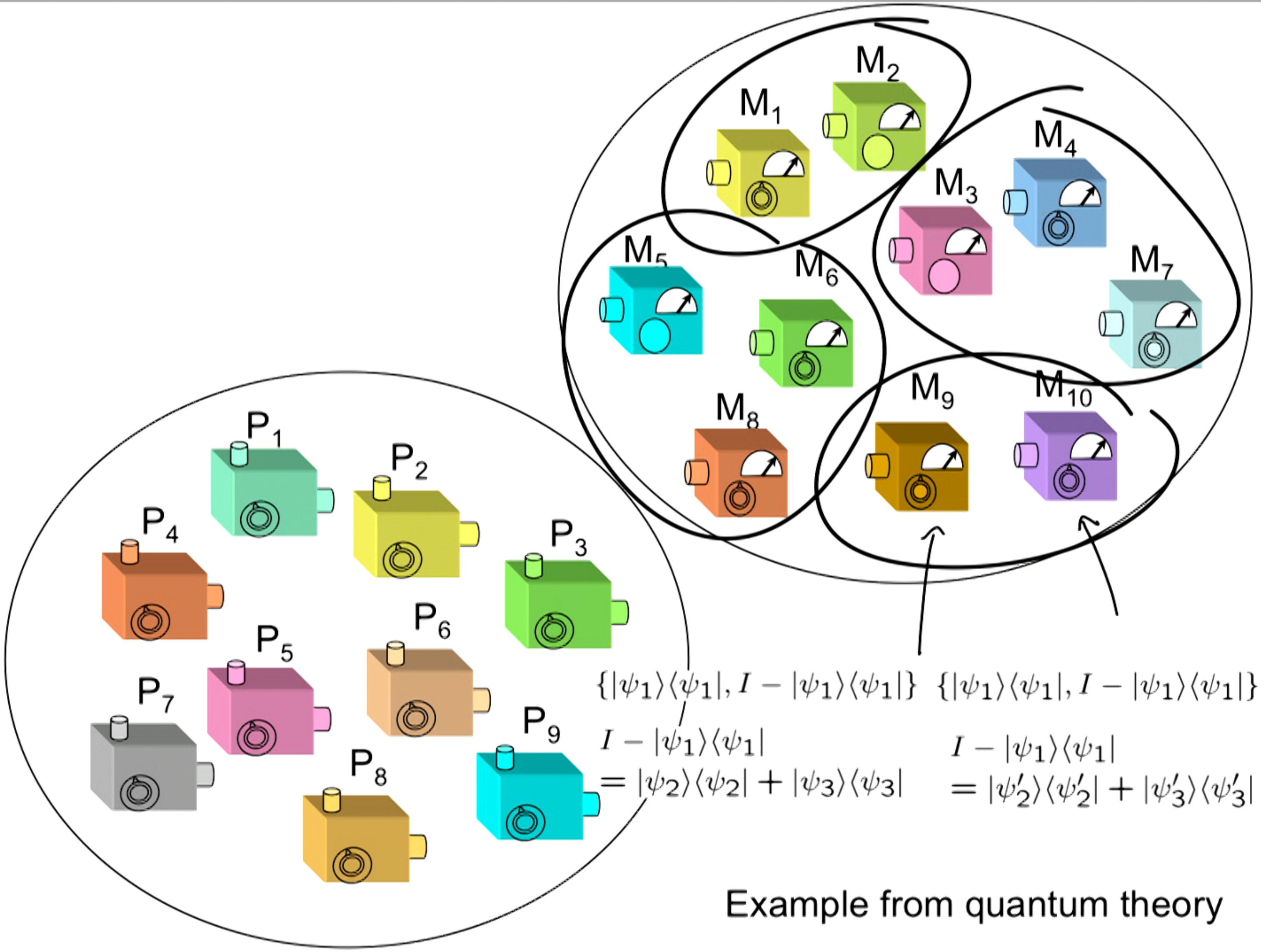
Example from quantum theory



Measurement contextual model



Measurement noncontextual model

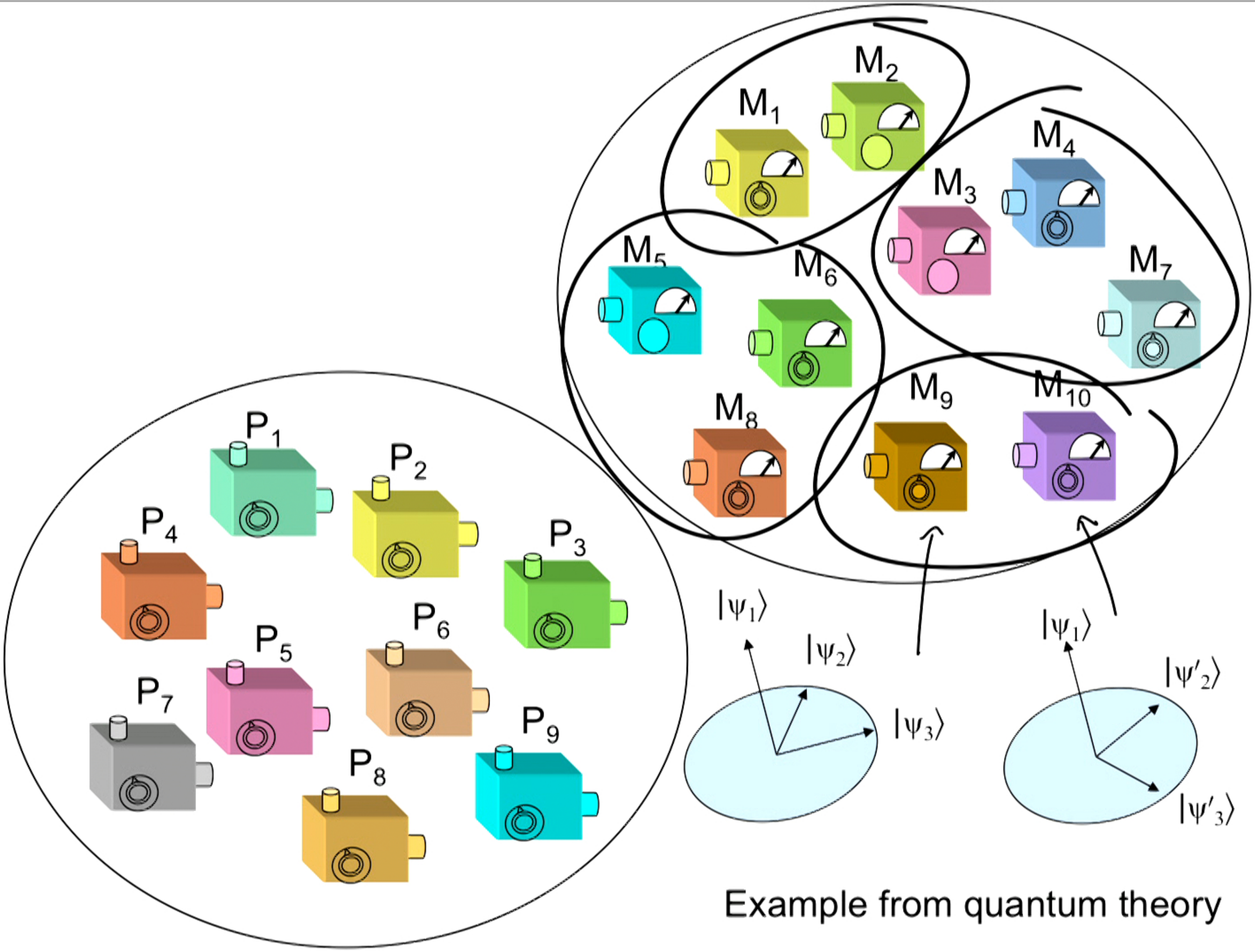


$\{|\psi_1\rangle\langle\psi_1|, I - |\psi_1\rangle\langle\psi_1|\}$ $\{|\psi_1\rangle\langle\psi_1|, I - |\psi_1\rangle\langle\psi_1|\}$

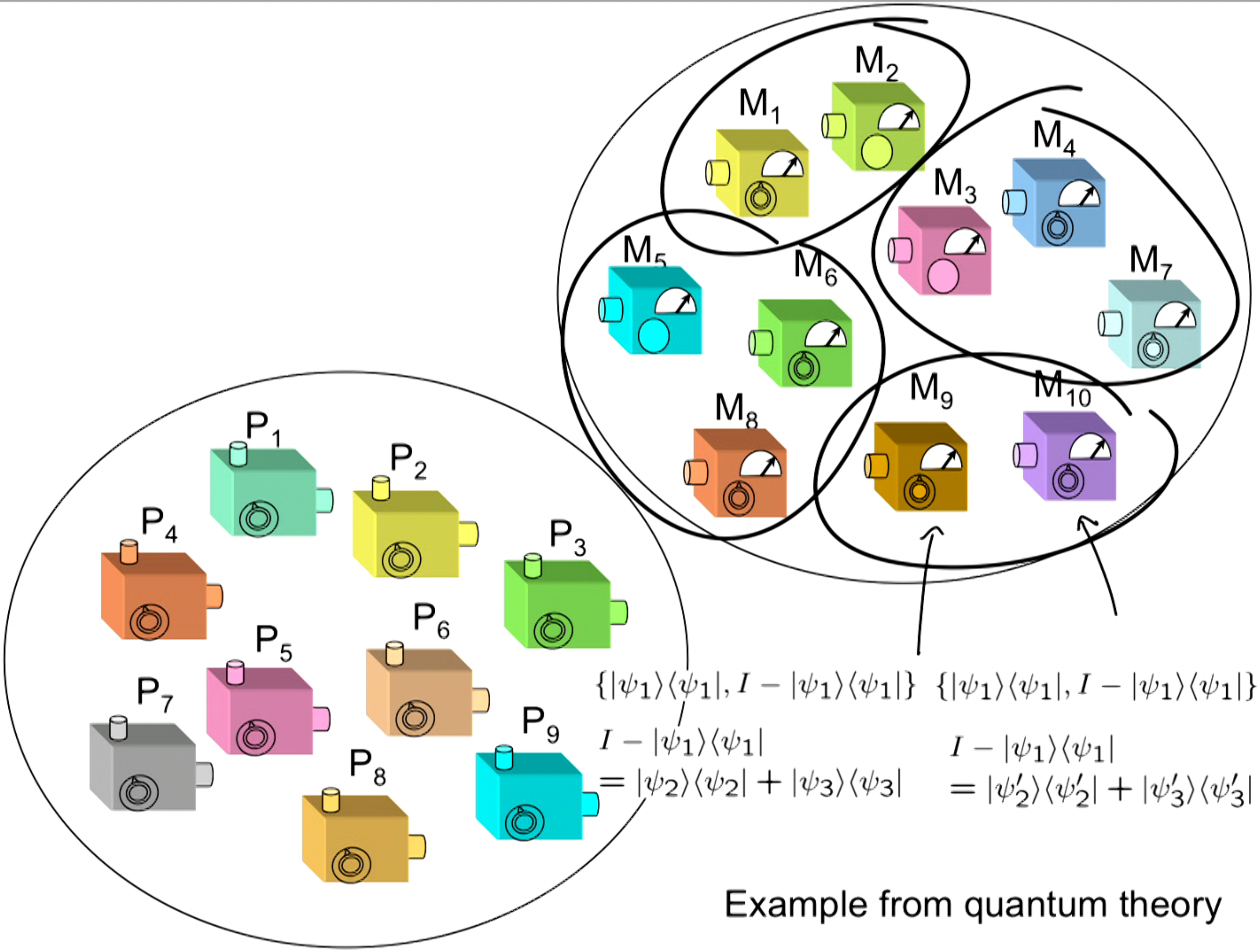
$$I - |\psi_1\rangle\langle\psi_1| = |\psi_2\rangle\langle\psi_2| + |\psi_3\rangle\langle\psi_3|$$

$$I - |\psi_1\rangle\langle\psi_1| = |\psi_2'\rangle\langle\psi_2'| + |\psi_3'\rangle\langle\psi_3'|$$

Example from quantum theory



Example from quantum theory



$\{|\psi_1\rangle\langle\psi_1|, I - |\psi_1\rangle\langle\psi_1|\}$ $\{|\psi_1\rangle\langle\psi_1|, I - |\psi_1\rangle\langle\psi_1|\}$

$I - |\psi_1\rangle\langle\psi_1|$
$= |\psi_2\rangle\langle\psi_2| + |\psi_3\rangle\langle\psi_3|$

$I - |\psi_1\rangle\langle\psi_1|$
$= |\psi_2'\rangle\langle\psi_2'| + |\psi_3'\rangle\langle\psi_3'|$

Example from quantum theory

$$\mathsf{P} \simeq \mathsf{P}'$$

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M})$$

Preparation noncontextuality $\longrightarrow$

$$\mu(\lambda|\mathsf{P}) = \mu(\lambda|\mathsf{P}')$$

$$\mathsf{M} \simeq \mathsf{M}'$$

$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}')$$

Measurement noncontextuality $\longrightarrow$

$$\xi(X|\lambda,\mathsf{M}) = \xi(X|\lambda,\mathsf{M}')$$

The best explanation of context-independence at the operational level is context-independence at the ontological level

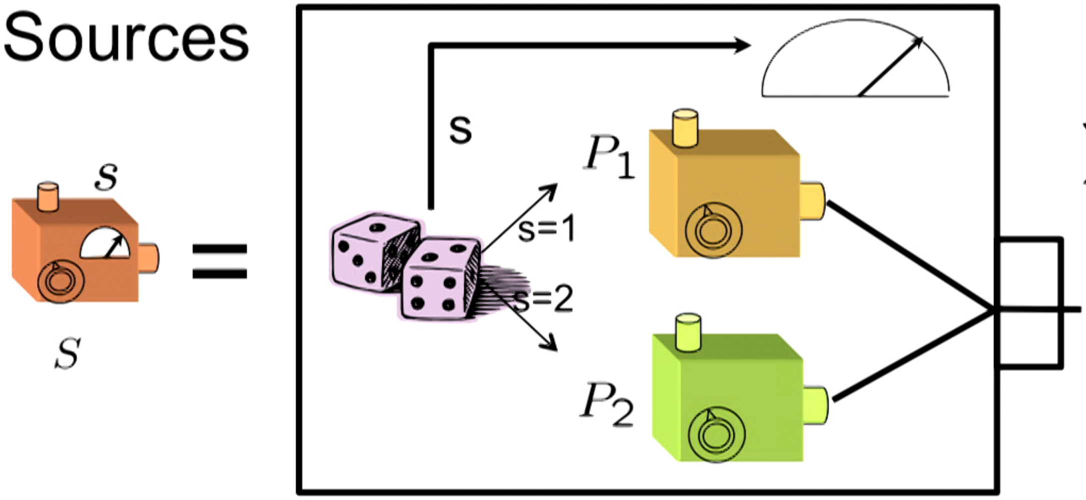
Transformation noncontextuality is defined similarly

The only natural assumption is universal noncontextuality, i.e. noncontextuality for preparations, transformations and measurements

Deriving noncontextuality inequalities

using **only** preparation noncontextuality

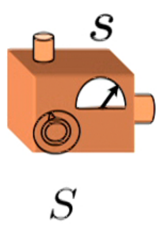
Sources



$$\mu(\lambda, s|S)$$

$$\sum_{s} \sum_{\lambda \in \Lambda} \mu(\lambda, s|S) = 1$$

Sources



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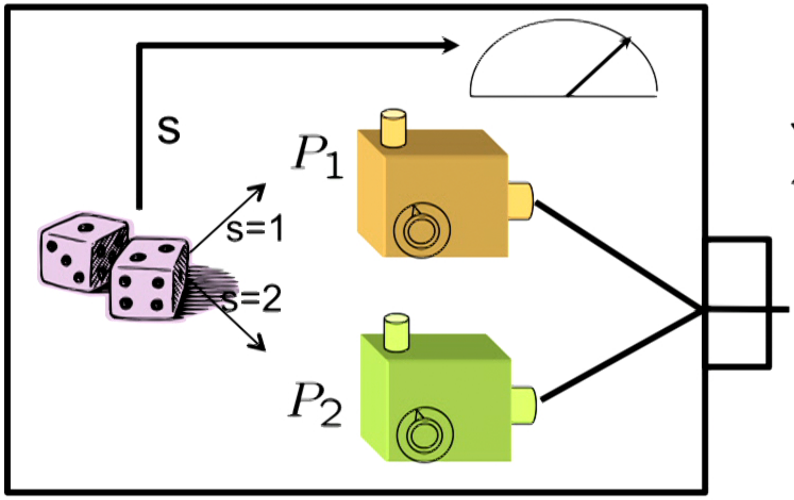
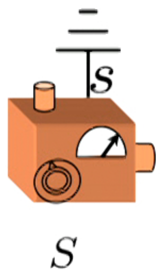


$$\mu(\lambda, s|S)$$

$$\sum_{s} \sum_{\lambda \in \Lambda} \mu(\lambda, s|S) = 1$$

P_S^{ave}



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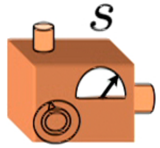

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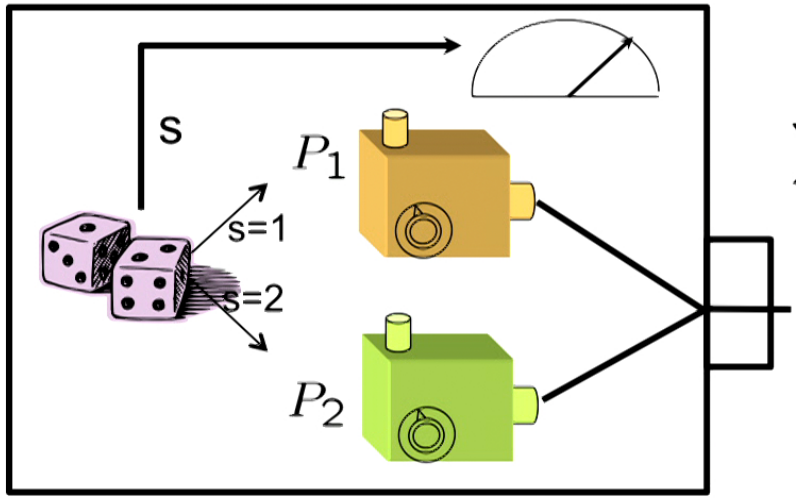
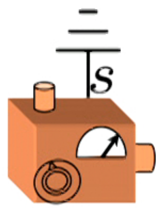


$$\mu(\lambda, s|S)$$

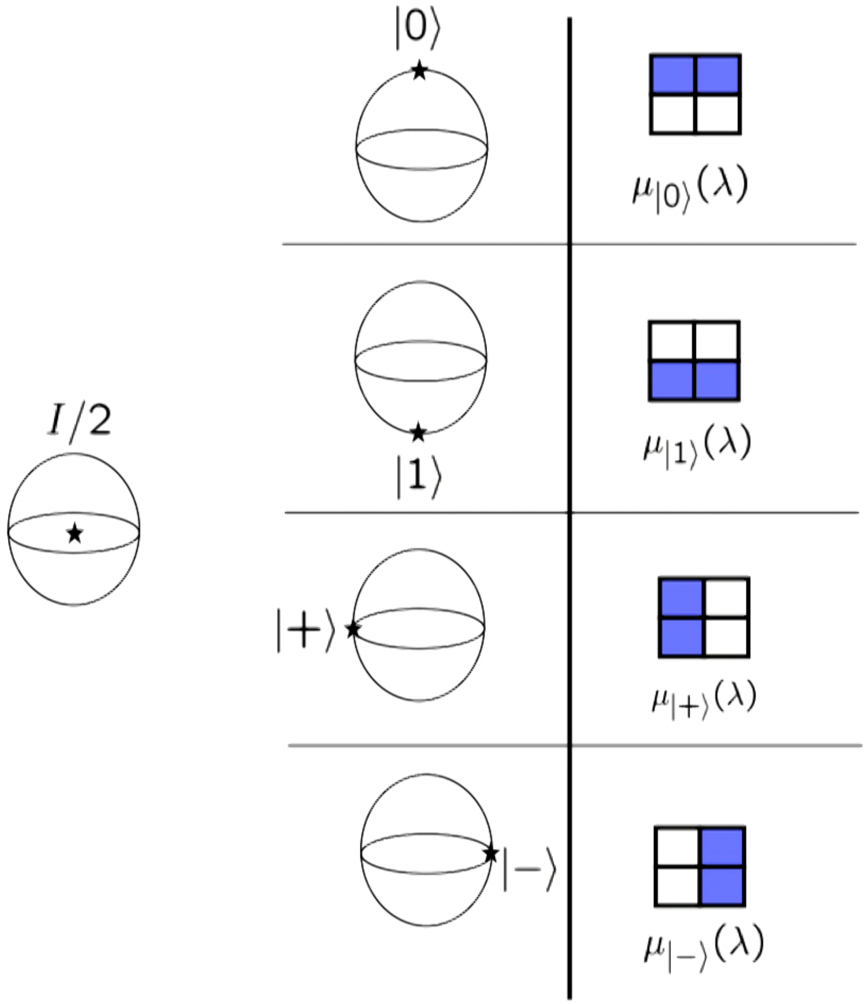
$$\sum_s \sum_{\lambda \in \Lambda} \mu(\lambda, s|S) = 1$$

P_S^{ave}

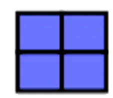


$$\mu(\lambda|P_S^{\text{ave}}) = \sum_s \mu(\lambda, s|S)$$
$$= \mu(\lambda|S)$$

$$P_S^{\text{ave}} \simeq P_{S'}^{\text{ave}} \xrightarrow{\text{Preparation noncontextuality}} \mu(\lambda|S) = \mu(\lambda|S')$$

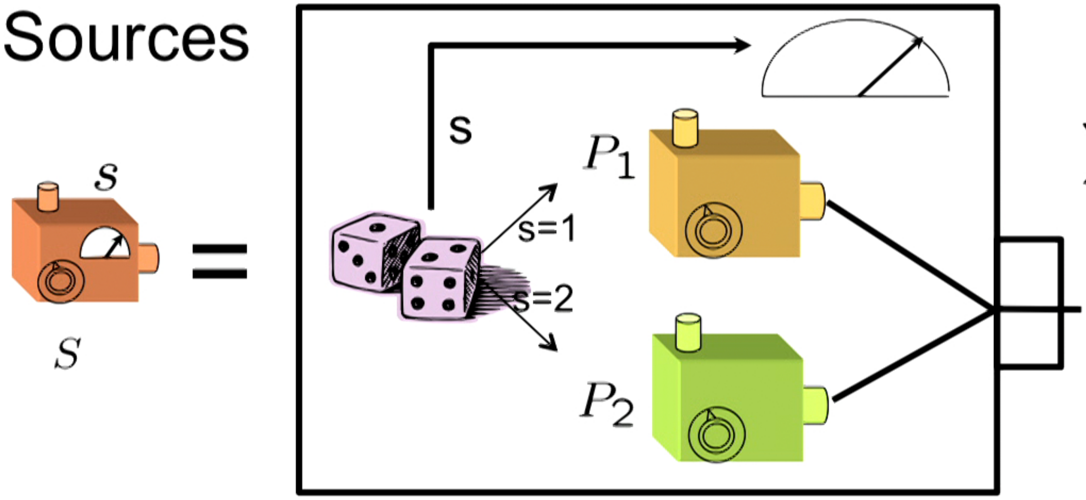


$$\mu_{I/2}(\lambda)$$
$$= \frac{1}{2}\mu_{|0\rangle}(\lambda) + \frac{1}{2}\mu_{|1\rangle}(\lambda)$$
$$= \frac{1}{2}\mu_{|+\rangle}(\lambda) + \frac{1}{2}\mu_{|-\rangle}(\lambda)$$



Preparation noncontextual

Sources



$\mu(\lambda, s|S)$

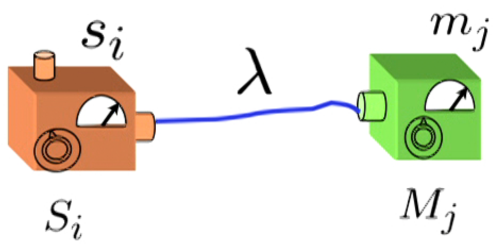
$$\sum_s \sum_{\lambda \in \Lambda} \mu(\lambda, s|S) = 1$$

By preparation noncontextuality

$$\forall i, i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}} \implies \forall i, i' : \mu(\lambda|\mathcal{S}_i) = \mu(\lambda|\mathcal{S}_{i'}) \equiv \nu(\lambda)$$

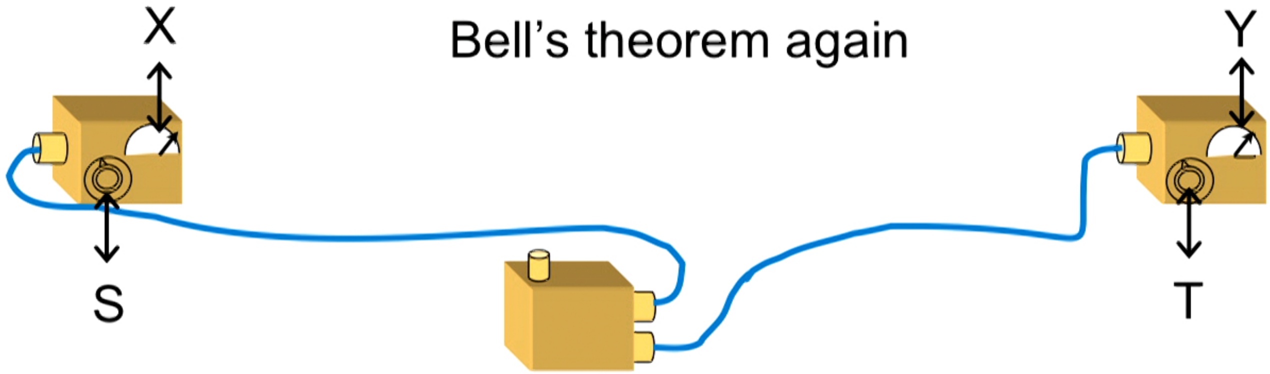
By preparation noncontextuality

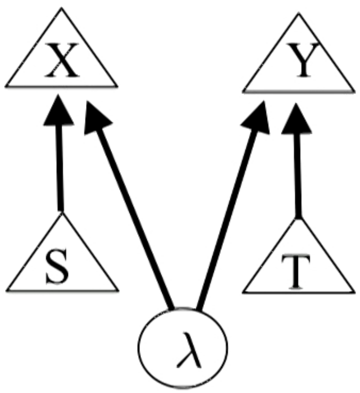
$$\forall i,i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}} \implies \forall i,i' : \mu(\lambda|\mathcal{S}_i) = \mu(\lambda|\mathcal{S}_{i'}) \equiv \nu(\lambda)$$



$$\begin{aligned}\text{pr}(m_j, s_i|M_j, S_i) &= \sum_{\lambda\in\Lambda} \xi(m_j|M_j,\lambda)\mu(s_i,\lambda|S_i) \\ &= \sum_{\lambda\in\Lambda} \xi(m_j|M_j,\lambda)\mu(s_i|\lambda,S_i)\mu(\lambda|S_i) \\ &= \sum_{\lambda\in\Lambda} \xi(m_j|M_j,\lambda)\mu(s_i|\lambda,S_i)\nu(\lambda)\end{aligned}$$

Bell's theorem again





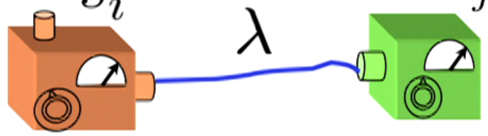
$P(\lambda)$
$P(X|\lambda, S)$
$P(Y|\lambda, T)$

$$P(X, Y|S, T) = \sum_\lambda P(X|S, \lambda)P(Y|T, \lambda)P(\lambda)$$

Satisfies the
Bell inequalities

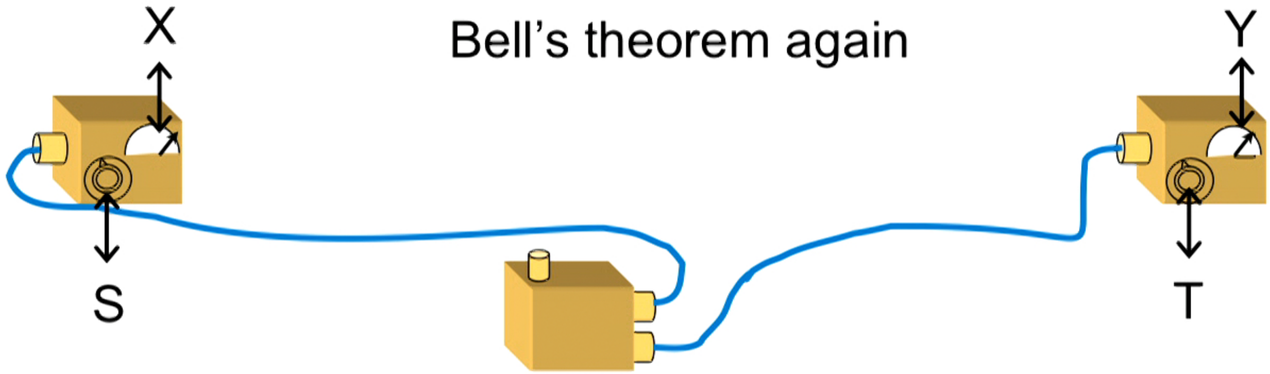
By preparation noncontextuality

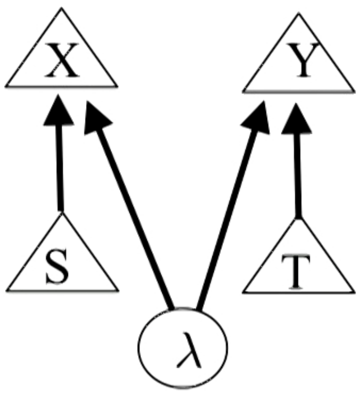
$$\forall i, i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}} \quad \Longrightarrow \quad \forall i, i' : \mu(\lambda|\mathcal{S}_i) = \mu(\lambda|\mathcal{S}_{i'}) \equiv \nu(\lambda)$$



$$\begin{aligned}
\text{pr}(m_j, s_i|M_j, S_i) &= \sum_{\lambda\in\Lambda} \xi(m_j|M_j,\lambda)\mu(s_i,\lambda|S_i) \\
&= \sum_{\lambda\in\Lambda} \xi(m_j|M_j,\lambda)\mu(s_i|\lambda,S_i)\mu(\lambda|S_i) \\
&= \sum_{\lambda\in\Lambda} \xi(m_j|M_j,\lambda)\mu(s_i|\lambda,S_i)\nu(\lambda)
\end{aligned}$$

Bell's theorem again





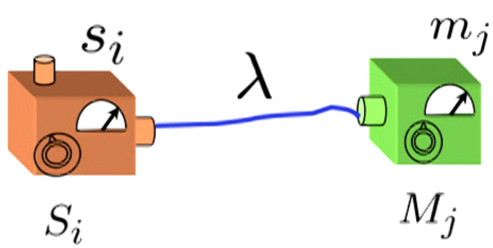
$P(\lambda)$
$P(X|\lambda, S)$
$P(Y|\lambda, T)$

$$P(X, Y|S, T) = \sum_\lambda P(X|S, \lambda)P(Y|T, \lambda)P(\lambda)$$

Satisfies the
Bell inequalities

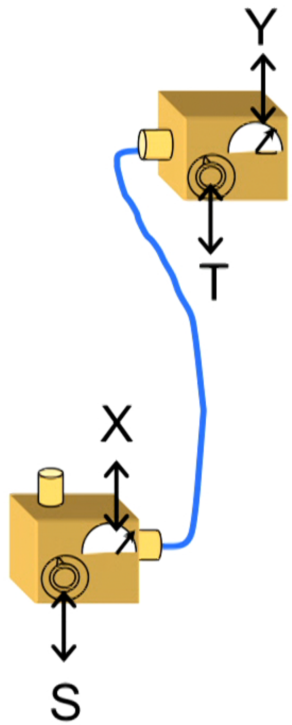
By preparation noncontextuality

$$\forall i, i' : P^{\text{ave}}_{S_i} \simeq P^{\text{ave}}_{S_{i'}} \quad \Longrightarrow \quad \forall i, i' : \mu(\lambda|\mathcal{S}_i) = \mu(\lambda|\mathcal{S}_{i'}) \equiv \nu(\lambda)$$



$$\begin{aligned}
\text{pr}(m_j, s_i|M_j, S_i) &= \sum_{\lambda\in\Lambda} \xi(m_j|M_j,\lambda)\mu(s_i,\lambda|S_i) \\
&= \sum_{\lambda\in\Lambda} \xi(m_j|M_j,\lambda)\mu(s_i|\lambda,S_i)\mu(\lambda|S_i) \\
&= \sum_{\lambda\in\Lambda} \xi(m_j|M_j,\lambda)\mu(s_i|\lambda,S_i)\nu(\lambda)
\end{aligned}$$

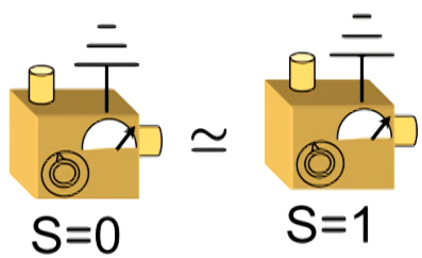
satisfies the Bell inequalities!

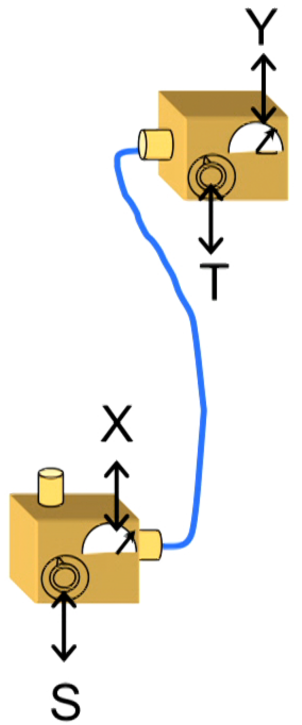


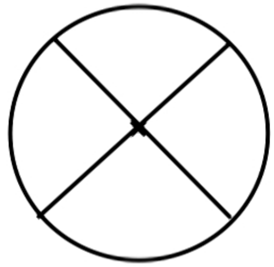
$$p(\text{success}) = \tfrac{1}{4}[\, p(\text{agree}|00) + p(\text{agree}|01) + p(\text{agree}|10) + p(\text{disagree}|11)]$$

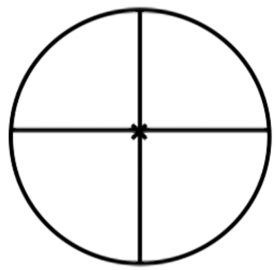
$$p(\text{success}) \leq 0.75$$

Noncontextuality inequality



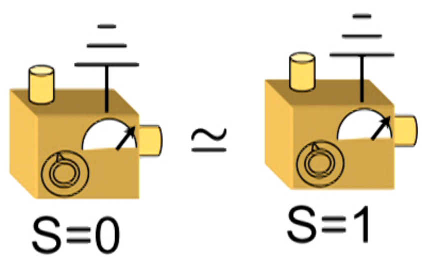


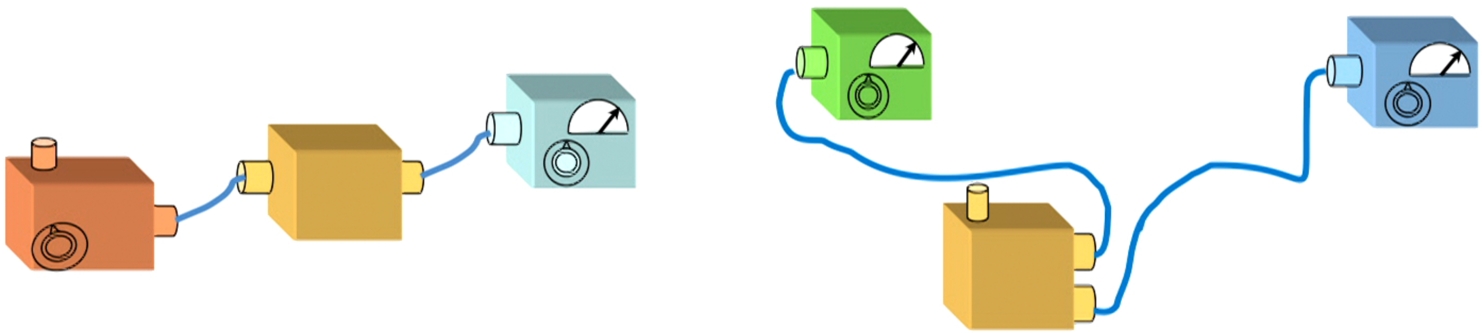




Quantum violation

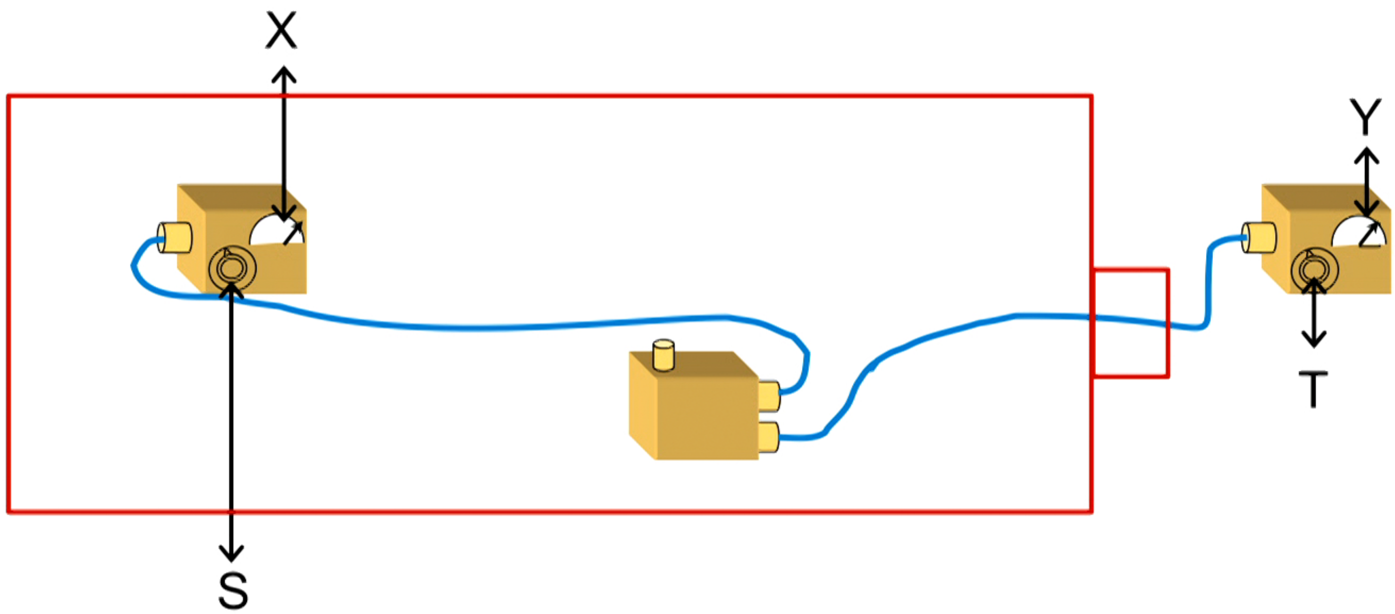
$$\mathrm{p(success)} = \frac{1}{2} + \frac{1}{2\sqrt{2}}$$
$$\simeq 0.85$$

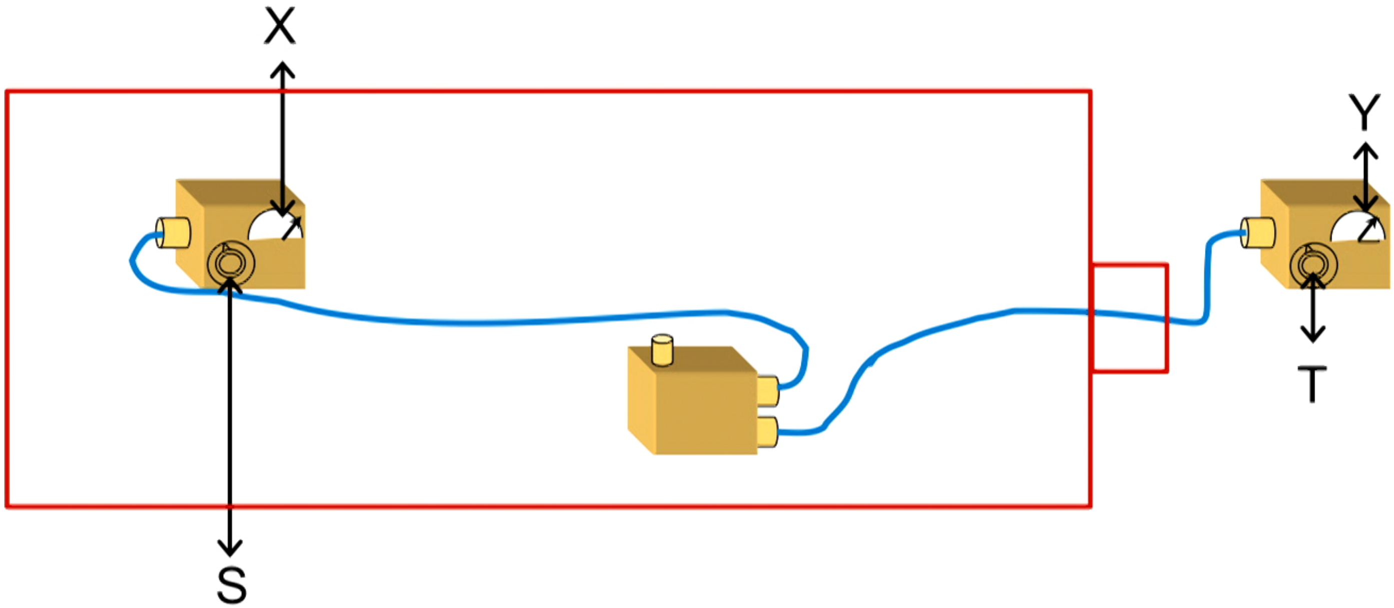




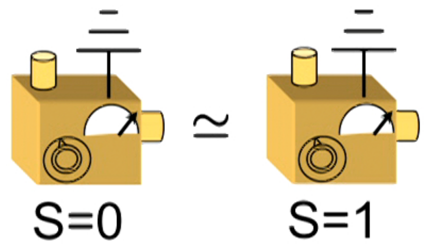
The intervening transformation plays the role of the entangled bipartite state

A unitary channel corresponds to maximal entanglement

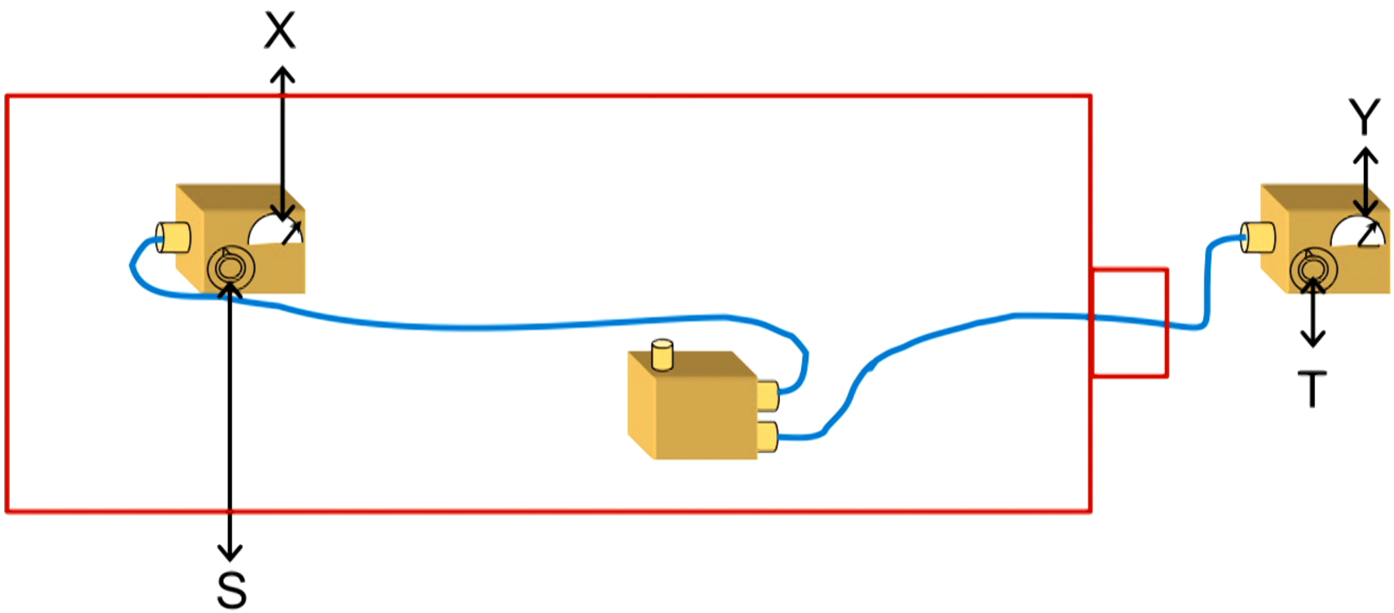

X
Y
T
S

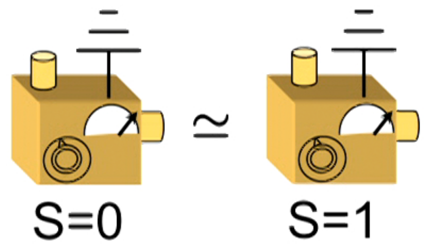


No signalling implies



Local causality implies $\mu(\lambda|S=0) = \mu(\lambda|S=1)$



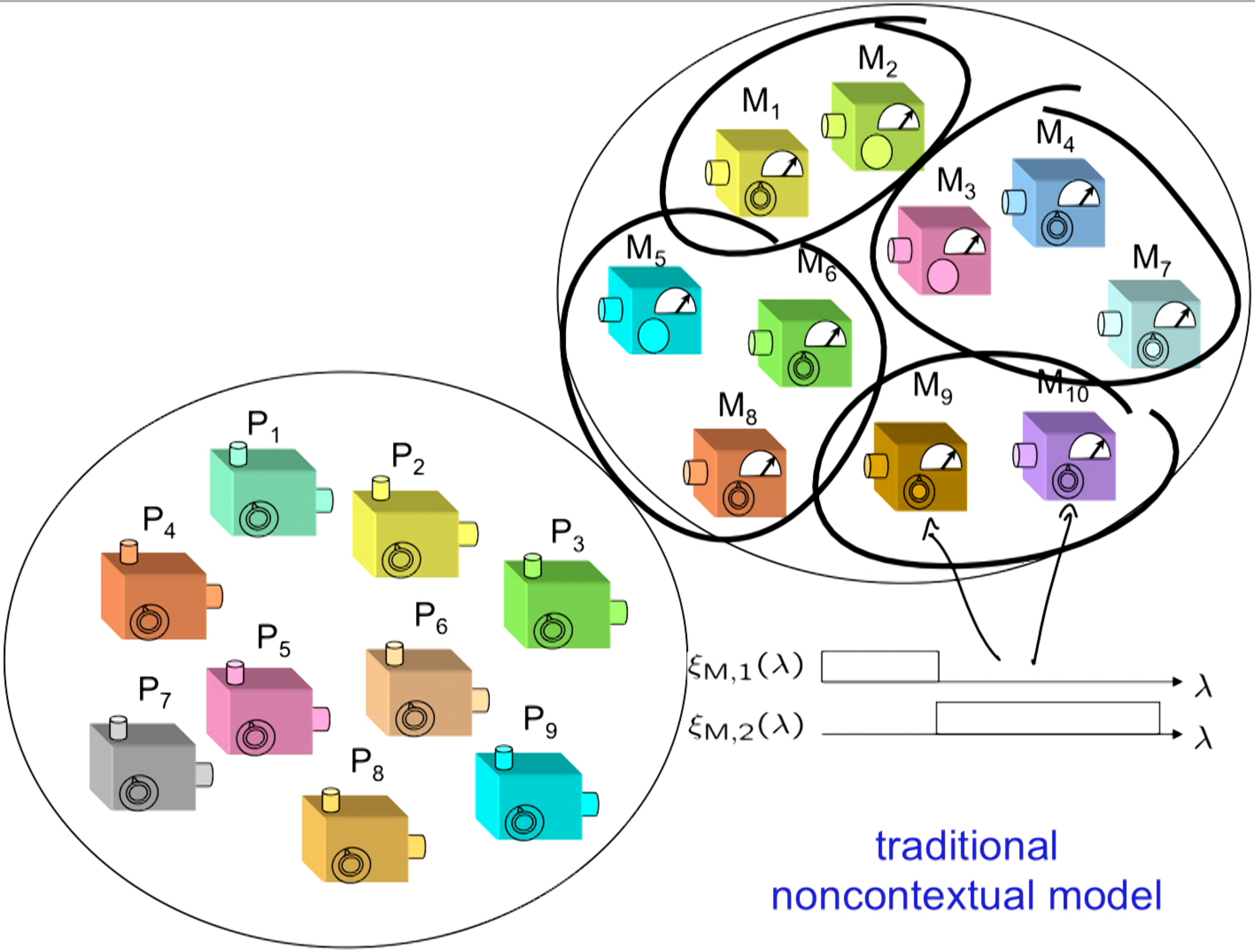
No signalling implies 

Local causality implies $\mu(\lambda|S=0) = \mu(\lambda|S=1)$

Here, local causality ← → preparation noncontextuality → contradiction

Universal noncontextuality subsumes the notion of local causality

Relation of measurement noncontextuality to traditional noncontextuality



traditional
noncontextual model

Traditional noncontextuality

+

Operational equivalences in the set of mmts

$\Rightarrow$ Contradiction

State independent no-go

measurement noncontextuality

+

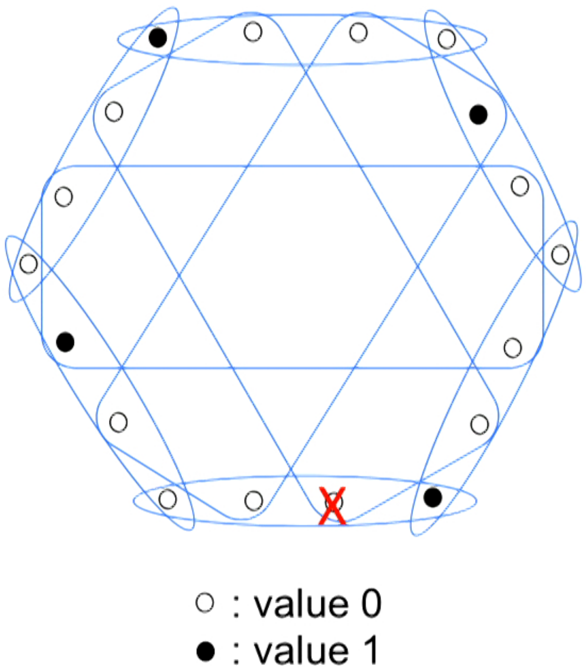
outcome determinism

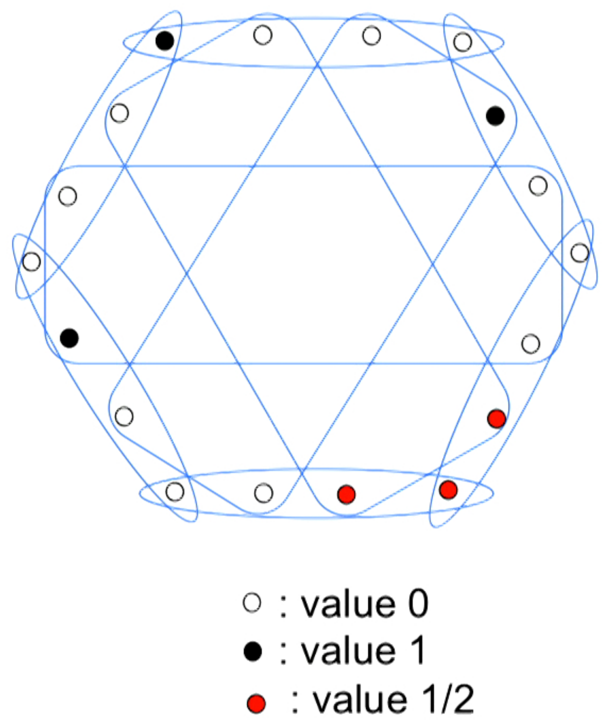
+

Operational equivalences in the sets of mmts

$\Rightarrow$ Contradiction

In face of contradiction, we could give up outcome determinism

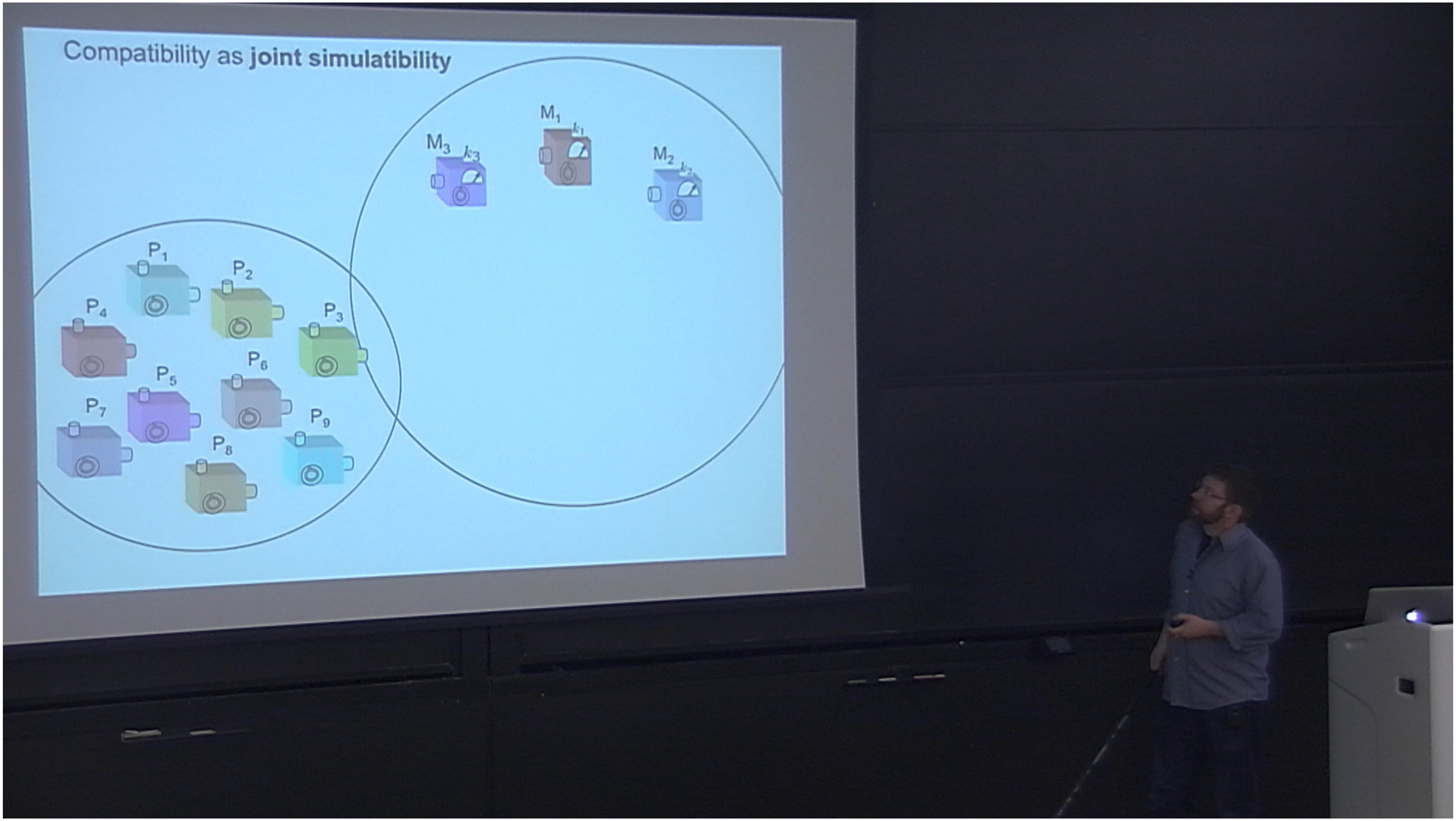

○ : value 0
● : value 1


○ : value 0
● : value 1
● : value 1/2

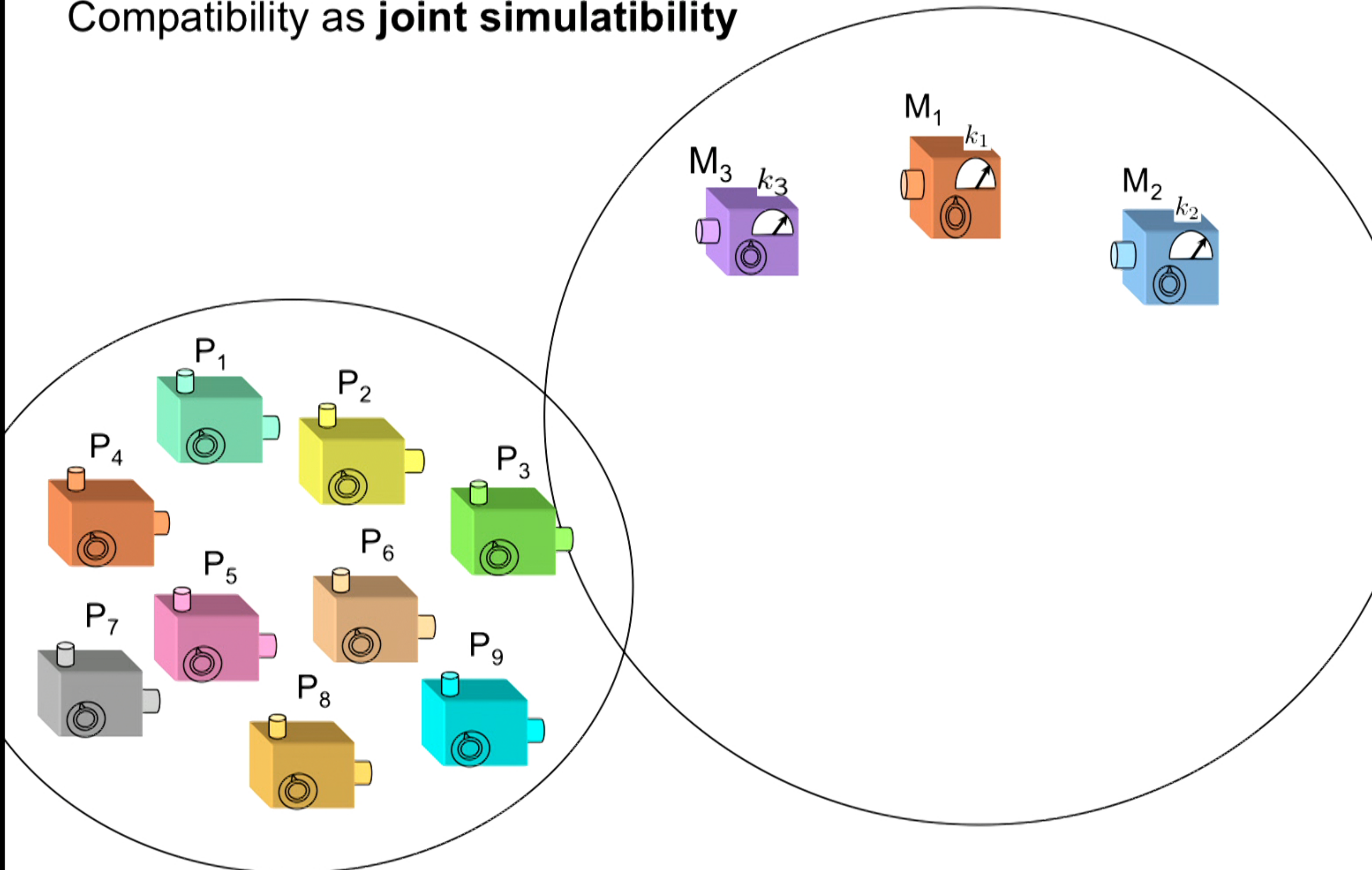
Deriving noncontextuality inequalities

using preparation noncontextuality and measurement noncontextuality

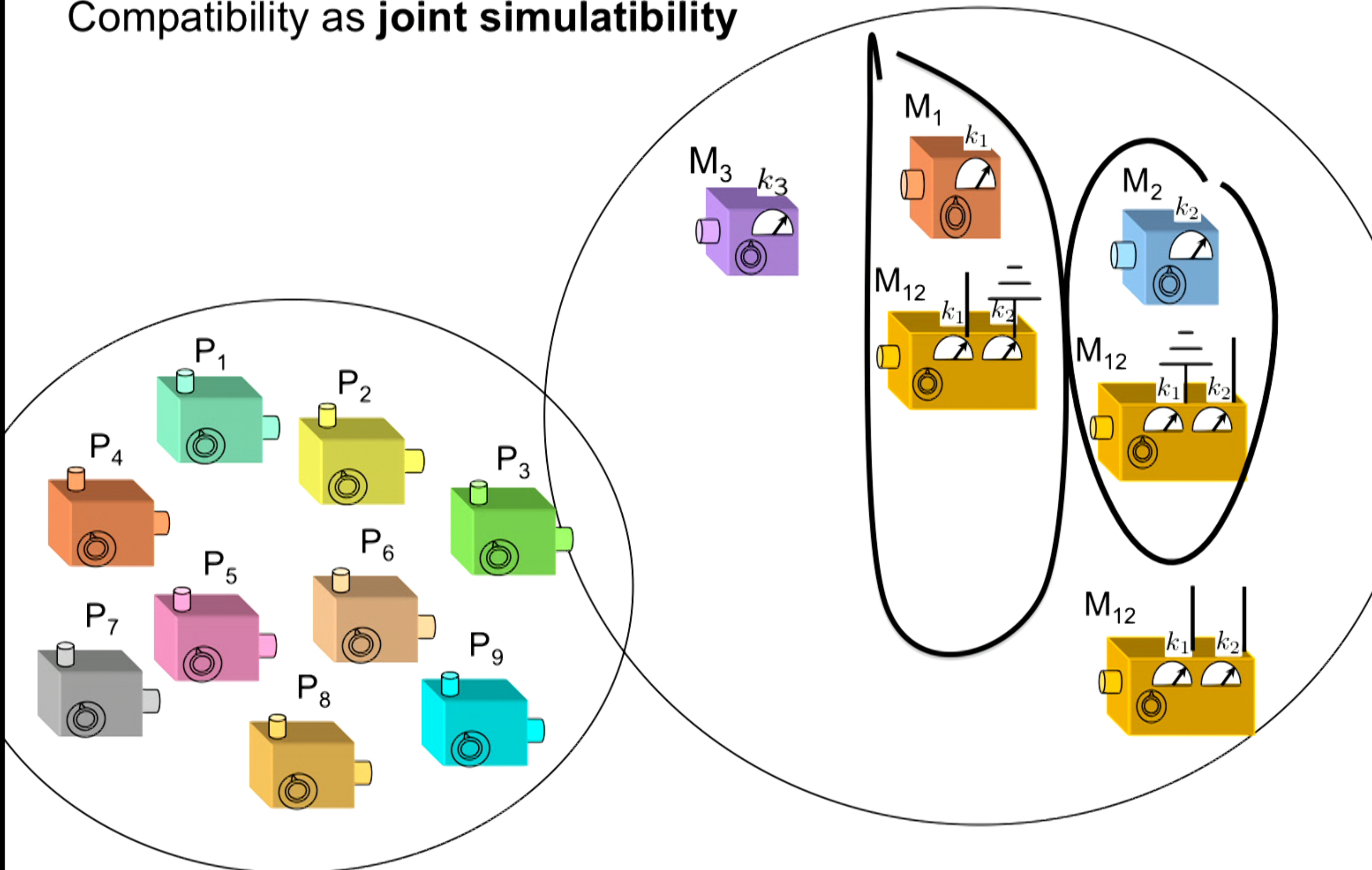
Kunjwal and RWS, PRL 115, 110403 (2015)
Krishna, RWS, Wolfe, NJP 19, 123031 (2017)


Compatibility as **joint simulatibility**
M₁
M₃
M₂
P₁
P₂
P₃
P₄
P₅
P₆
P₇
P₈
P₉

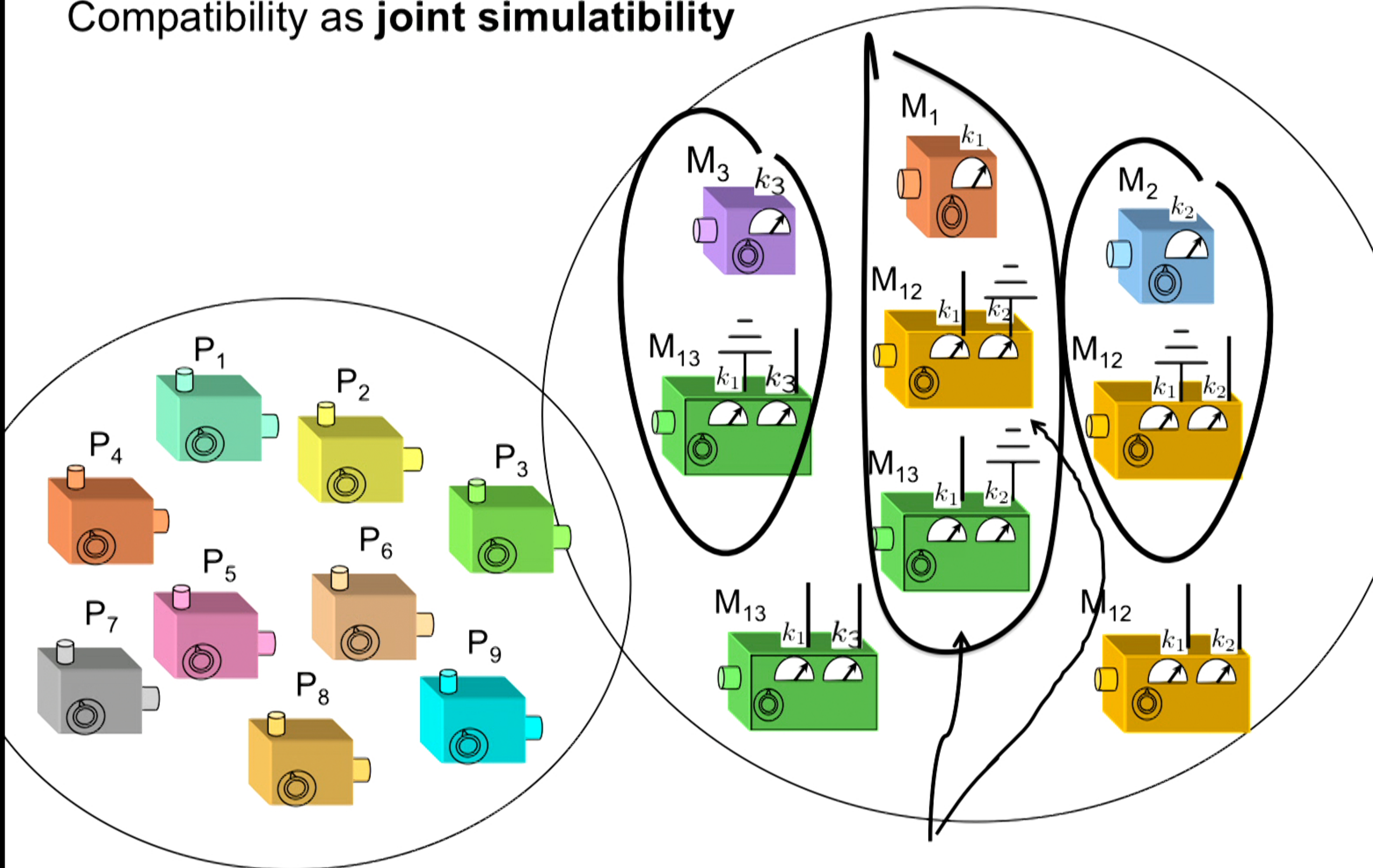
Compatibility as **joint simulatibility**



Compatibility as **joint simulatibility**



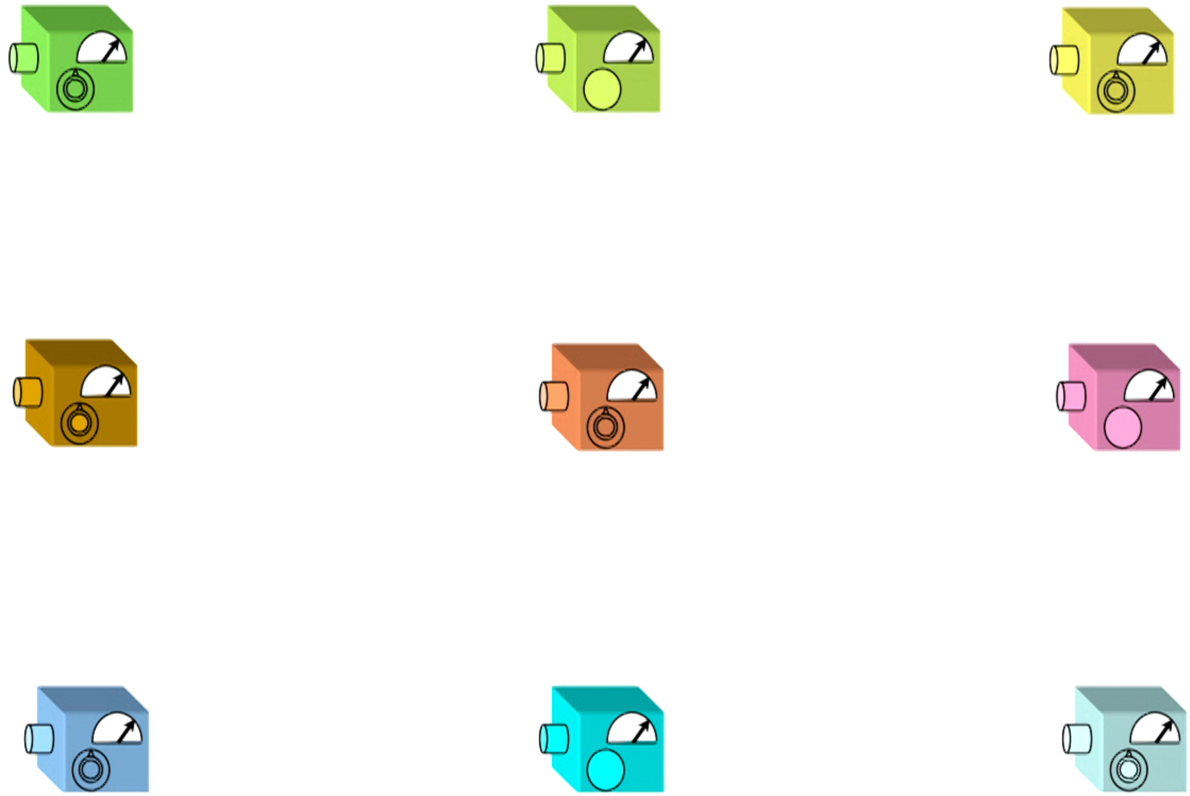
Compatibility as **joint simulatibility**

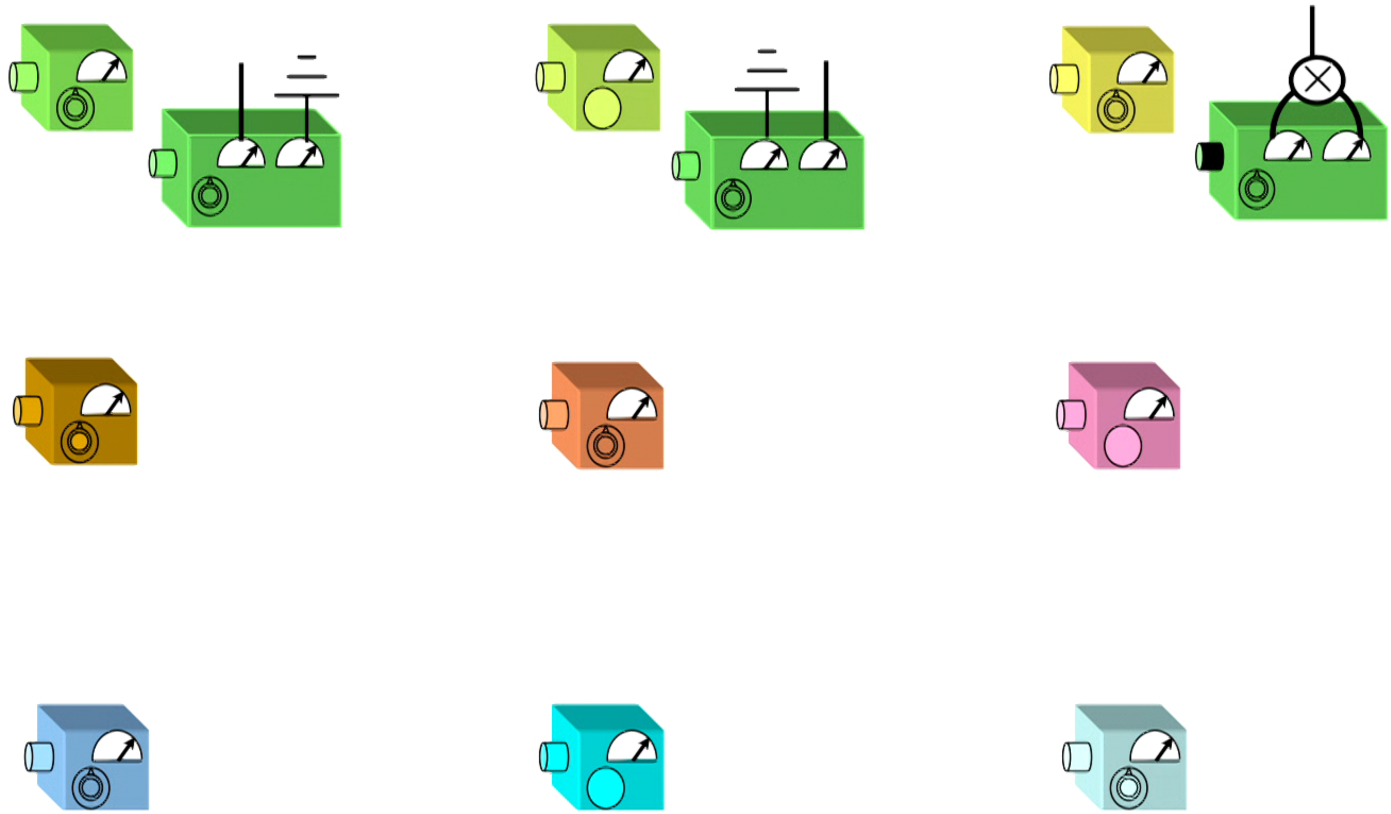


The Peres-Mermin square

$X \otimes I$	$I \otimes X$	$X \otimes X$
$I \otimes Z$	$Z \otimes I$	$Z \otimes Z$
$X \otimes Z$	$Z \otimes X$	$Y \otimes Y$

$$
\begin{aligned}
(X \otimes I)(I \otimes X)(X \otimes X) &= I \otimes I, \\
(I \otimes Z)(Z \otimes I)(Z \otimes Z) &= I \otimes I, \\
(X \otimes Z)(Z \otimes X)(Y \otimes Y) &= I \otimes I, \\
(X \otimes I)(I \otimes Z)(X \otimes Z) &= I \otimes I, \\
(I \otimes X)(Z \otimes I)(Z \otimes X) &= I \otimes I, \\
(X \otimes X)(Z \otimes Z)(Y \otimes Y) &= - I \otimes I.
\end{aligned}
$$

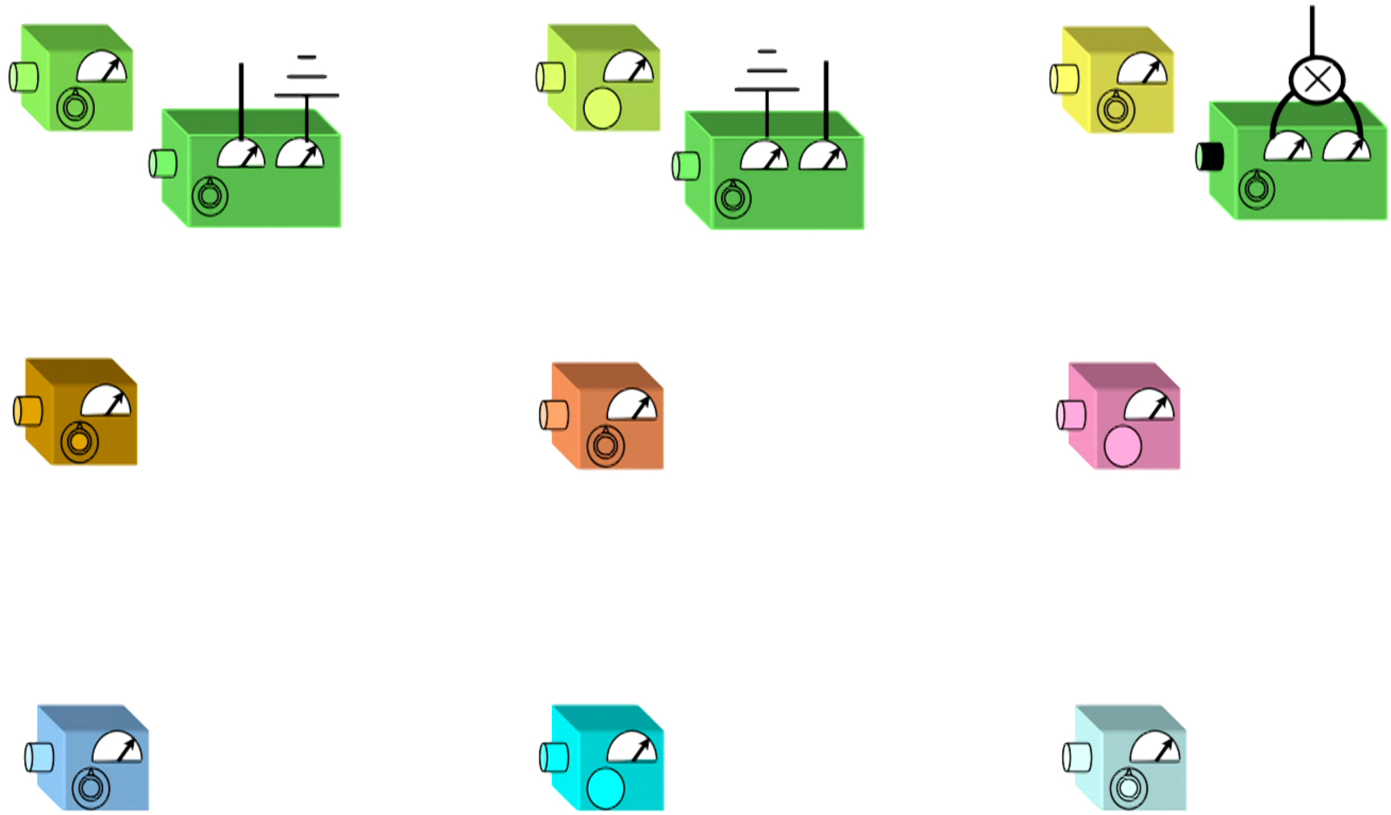


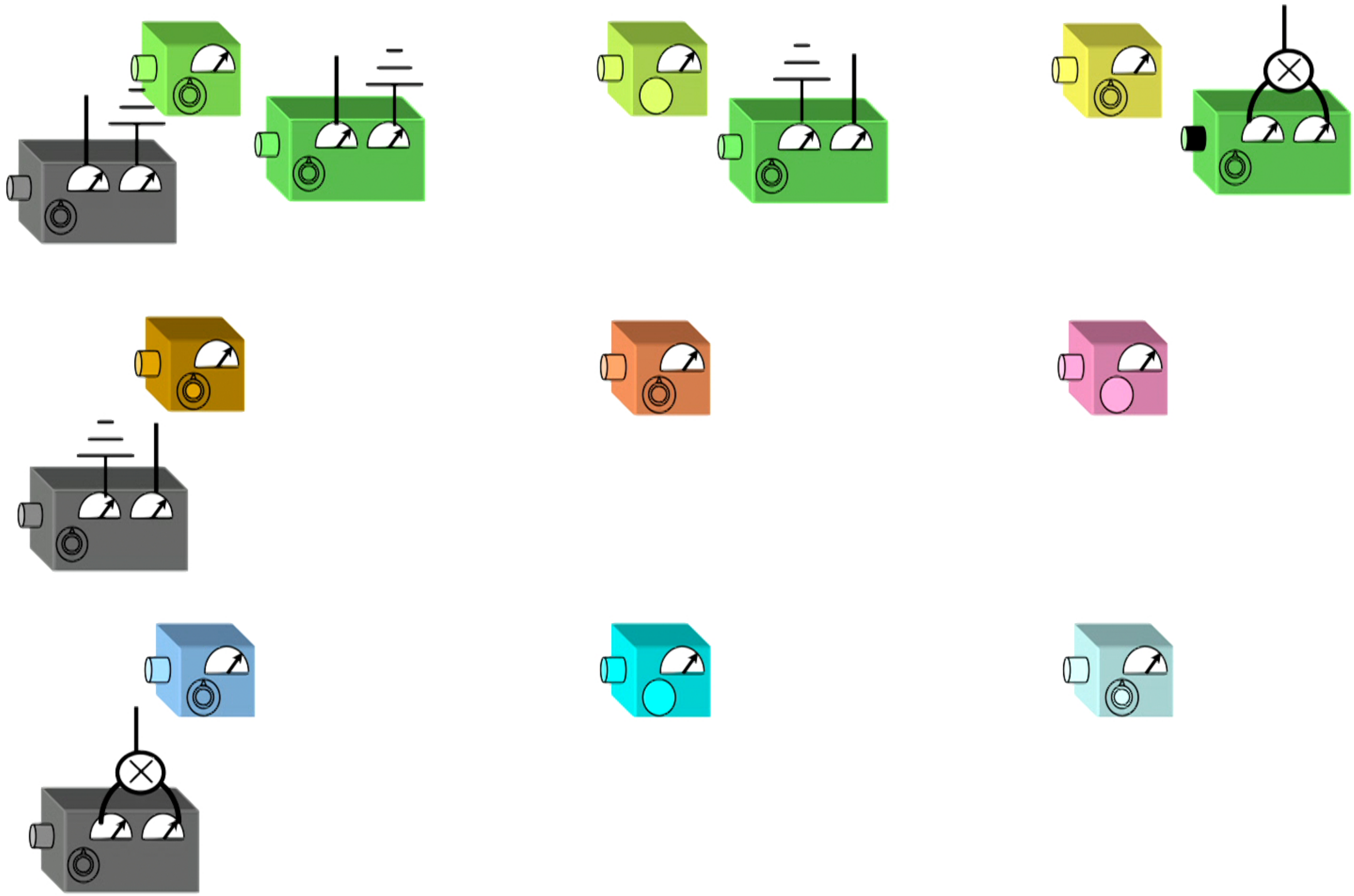


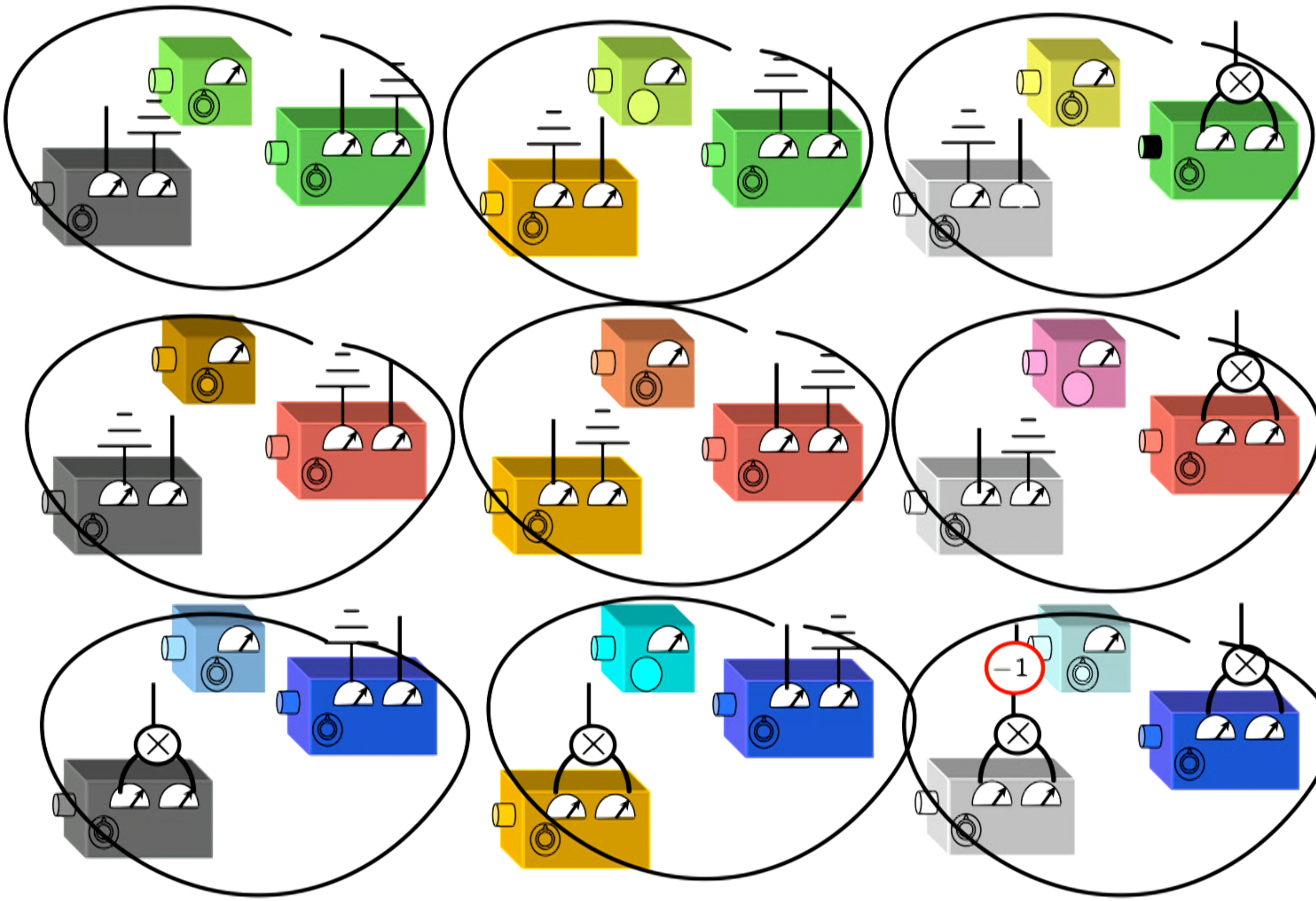
The Peres-Mermin square

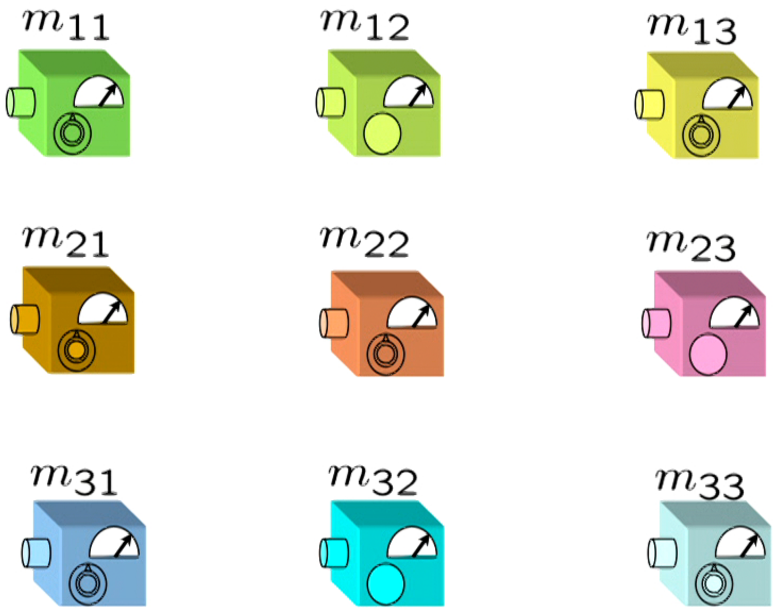
$X \otimes I$	$I \otimes X$	$X \otimes X$
$I \otimes Z$	$Z \otimes I$	$Z \otimes Z$
$X \otimes Z$	$Z \otimes X$	$Y \otimes Y$

$$
\begin{aligned}
\lfloor X \otimes I \rfloor_\lambda \lfloor I \otimes X \rfloor_\lambda \lfloor X \otimes X \rfloor_\lambda &= +1, \\
\lfloor I \otimes Z \rfloor_\lambda \lfloor Z \otimes I \rfloor_\lambda \lfloor Z \otimes Z \rfloor_\lambda &= +1, \\
\lfloor X \otimes Z \rfloor_\lambda \lfloor Z \otimes X \rfloor_\lambda \lfloor Y \otimes Y \rfloor_\lambda &= +1, \\
\lfloor X \otimes I \rfloor_\lambda \lfloor I \otimes Z \rfloor_\lambda \lfloor X \otimes Z \rfloor_\lambda &= +1, \\
\lfloor I \otimes X \rfloor_\lambda \lfloor Z \otimes I \rfloor_\lambda \lfloor Z \otimes X \rfloor_\lambda &= +1, \\
\lfloor X \otimes X \rfloor_\lambda \lfloor Z \otimes Z \rfloor_\lambda \lfloor Y \otimes Y \rfloor_\lambda &= -1.
\end{aligned}
$$






−1



Distn's have support only on values such that

$$m_{11}m_{12}m_{13} = 1$$

$$m_{21}m_{22}m_{23} = 1$$

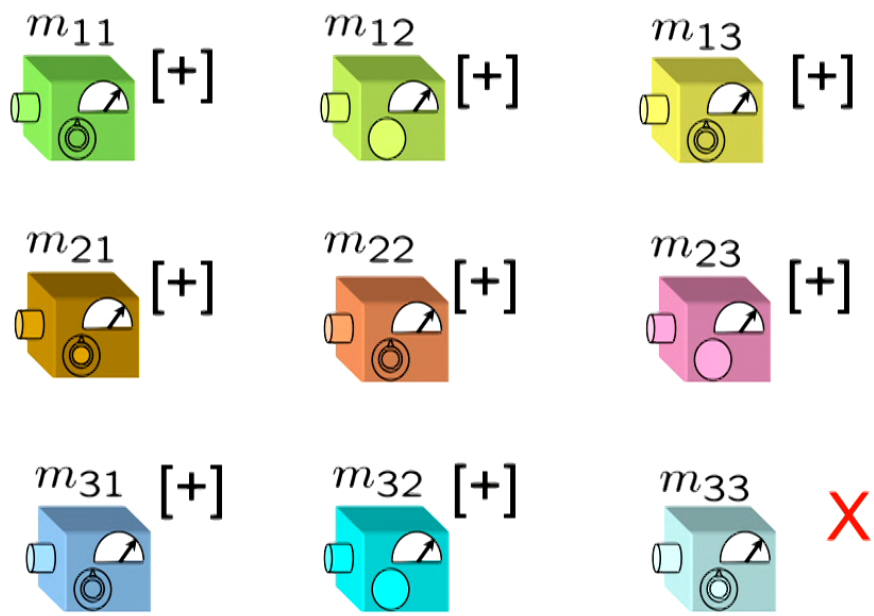
$$m_{31}m_{32}m_{33} = 1$$

$$m_{11}m_{21}m_{31} = 1$$

$$m_{12}m_{22}m_{32} = 1$$

$$m_{13}m_{23}m_{33} = -1$$

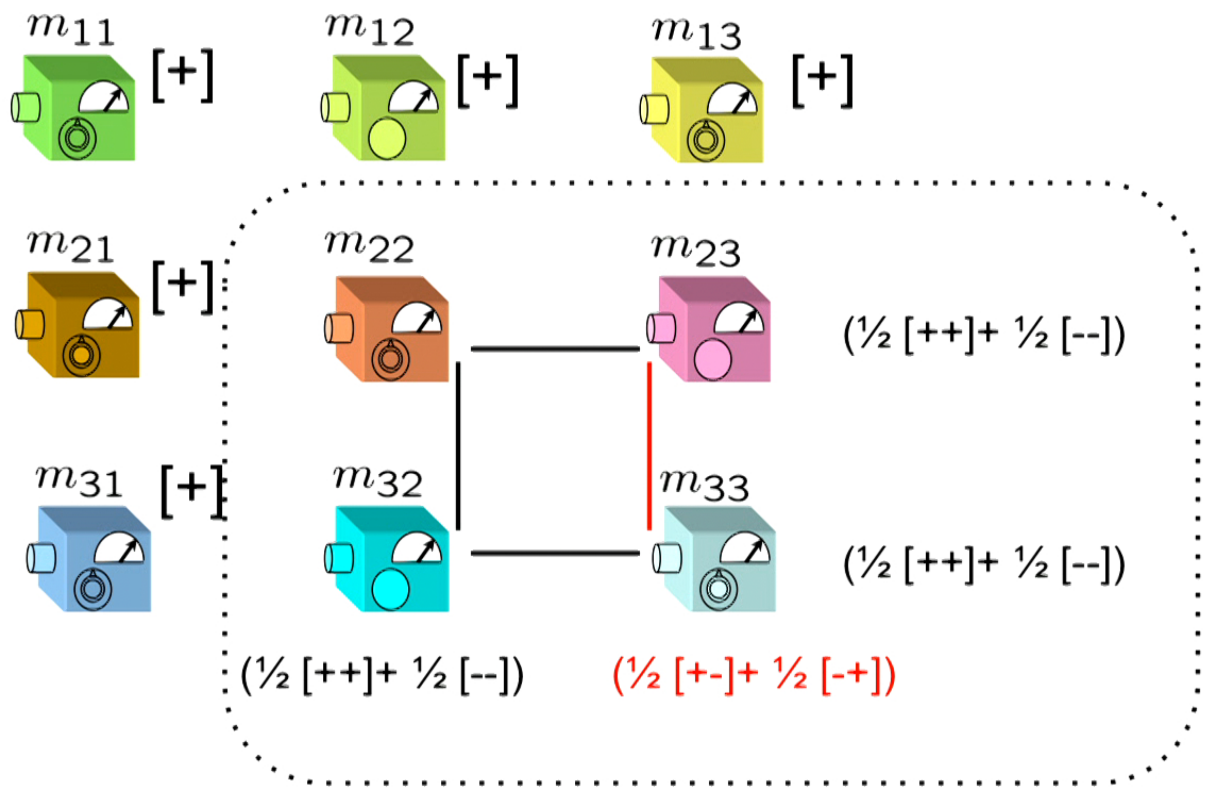
Recall: no deterministic noncontextual assignments



$$m_{11}m_{12}m_{13} = 1$$
$$m_{21}m_{22}m_{23} = 1$$
$$m_{31}m_{32}m_{33} = 1$$
$$m_{11}m_{21}m_{31} = 1$$
$$m_{12}m_{22}m_{32} = 1$$
$$m_{13}m_{23}m_{33} = -1$$

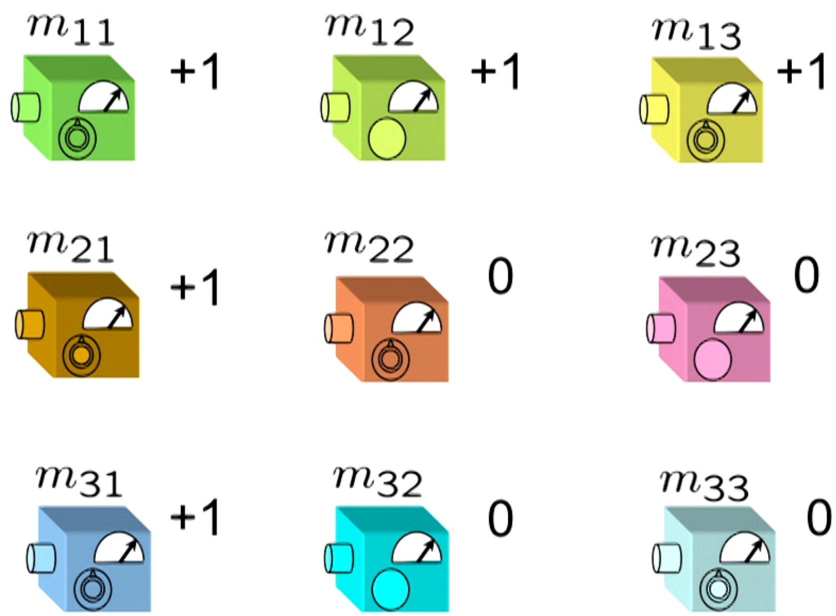
Probability dist'ns over values of each outcome variable here [+]:=$\delta_{\mathbf{m,+1}}$

An indeterministic noncontextual assignment



Assignments of probability dist'ns over values to each outcome variable; here [+]:=$\delta_{m,+1}$

An indeterministic noncontextual assignment



Expectation value for each outcome variable

Source version of the Peres-Mermin square

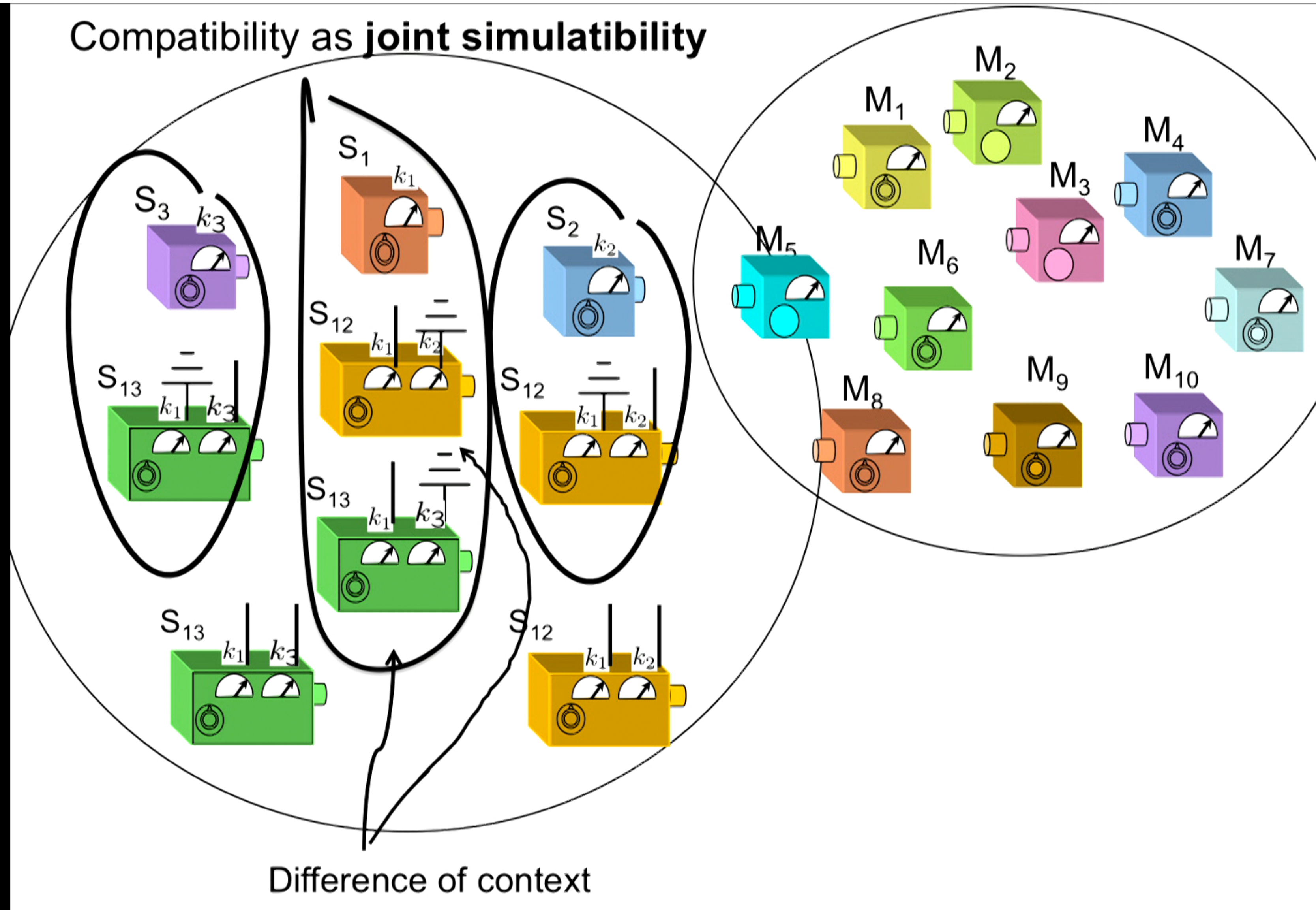
$X \otimes I$	$I \otimes X$	$X \otimes X$
$I \otimes Z$	$Z \otimes I$	$Z \otimes Z$
$X \otimes Z$	$Z \otimes X$	$Y \otimes Y$

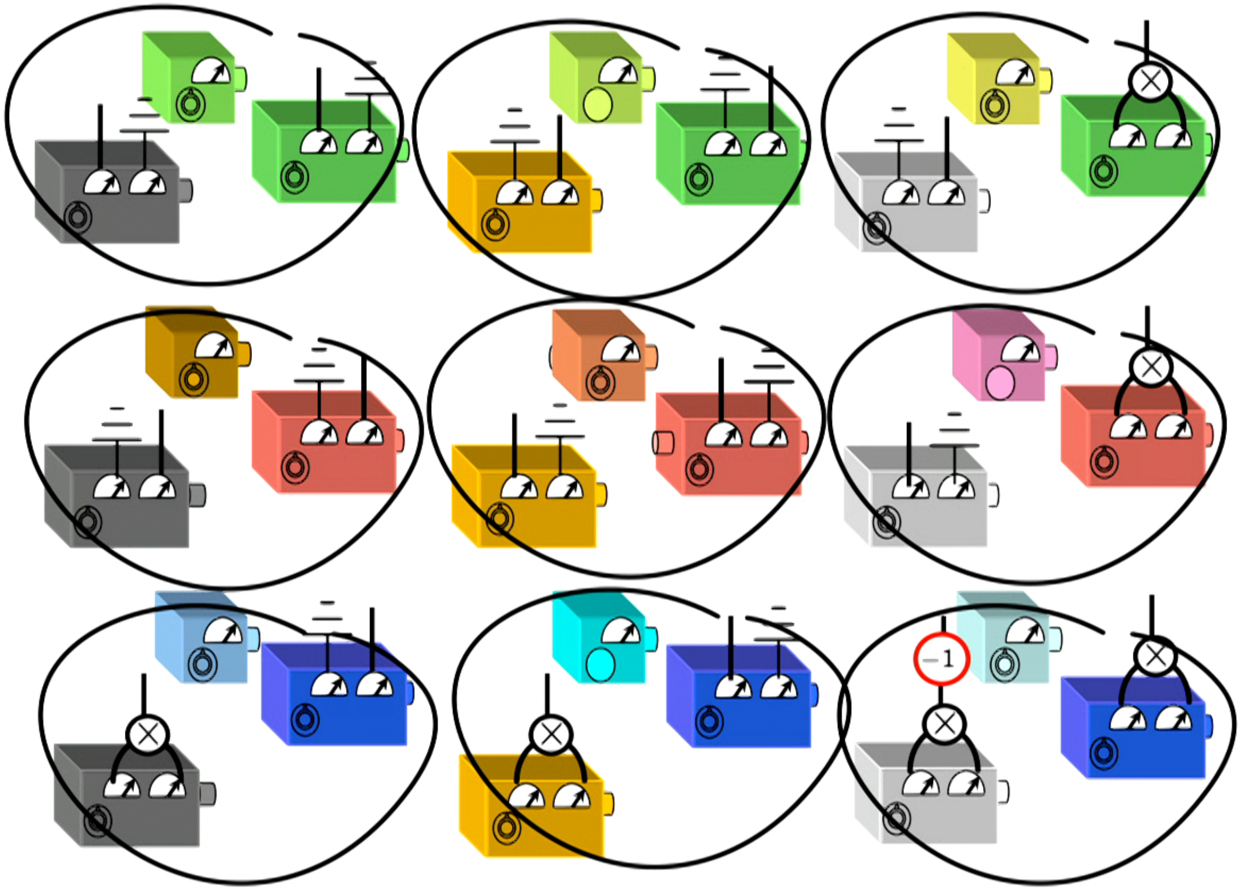
9 binary-outcome sources preparing the mixed states onto the eigenspaces of these observables

All 9 sources are compatible in triples & they define the same average preparation

$$
\begin{aligned}
(X \otimes I)(I \otimes X)(X \otimes X) &= I \otimes I, \\
(I \otimes Z)(Z \otimes I)(Z \otimes Z) &= I \otimes I, \\
(X \otimes Z)(Z \otimes X)(Y \otimes Y) &= I \otimes I, \\
(X \otimes I)(I \otimes Z)(X \otimes Z) &= I \otimes I, \\
(I \otimes X)(Z \otimes I)(Z \otimes X) &= I \otimes I, \\
(X \otimes X)(Z \otimes Z)(Y \otimes Y) &= -I \otimes I.
\end{aligned}
$$

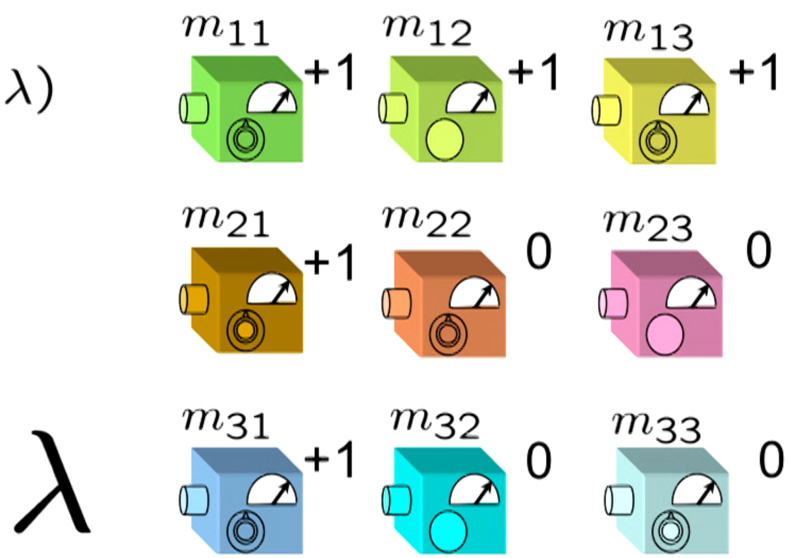
Compatibility as **joint simulatibility**

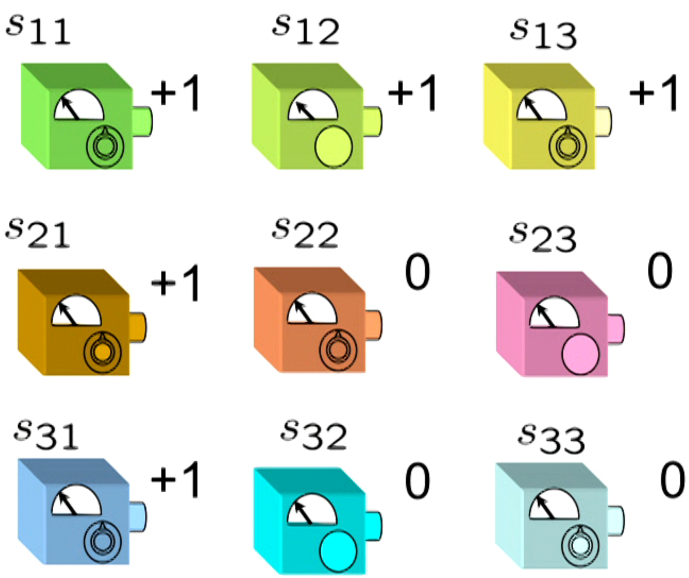




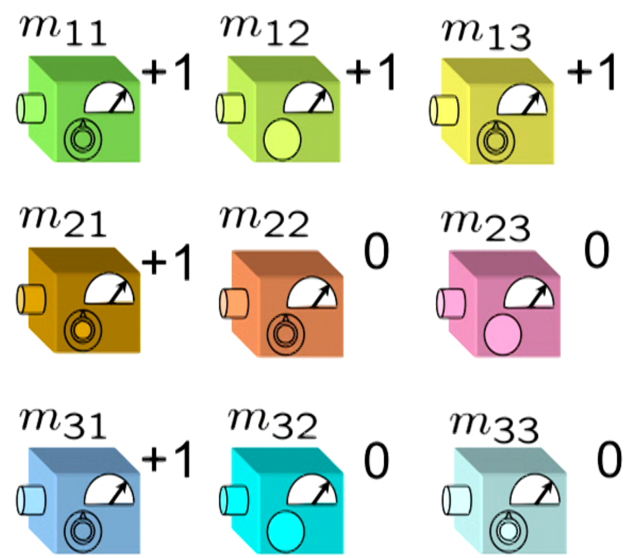
$$\forall i, i' : P_{S_i}^{\mathsf{ave}} \simeq P_{S_{i'}}^{\mathsf{ave}}$$

$$\mathrm{pr}(m_j, s_i | M_j, S_i) = \sum_{\lambda \in \Lambda} \xi(m_j | M_j, \lambda)\mu(s_i | \lambda, S_i)\nu(\lambda)$$

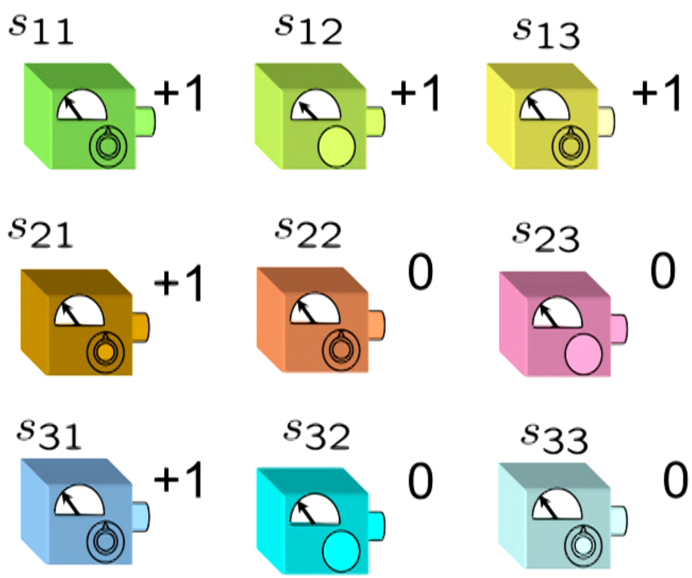




$$\mathrm{pr}(m_j, s_i | M_j, S_i)$$
$$= \sum_{\lambda \in \Lambda} \xi(m_j | M_j, \lambda) \mu(s_i | \lambda, S_i) \nu(\lambda)$$



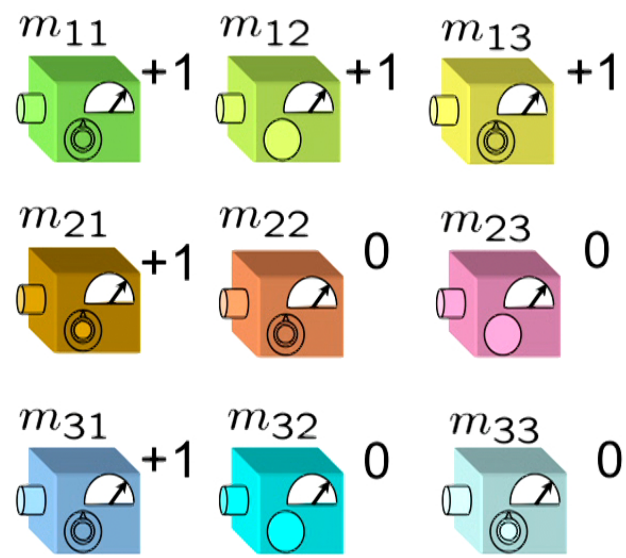
λ



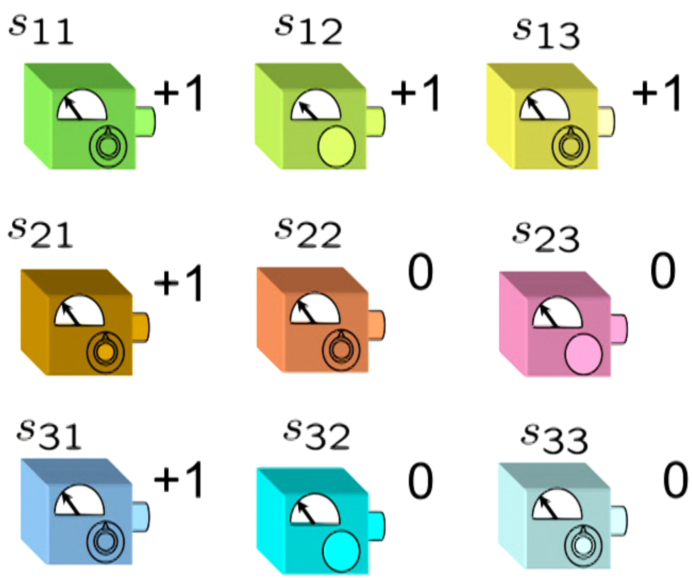
$$\mathrm{Corr} = \frac{1}{9} \sum_{i=1}^{9} \langle s_i m_i \rangle_{S_i, M_i}$$

$$\text{pr}(m_j, s_i | M_j, S_i)$$
$$= \sum_{\lambda \in \Lambda} \xi(m_j | M_j, \lambda) \mu(s_i | \lambda, S_i) \nu(\lambda)$$

$$\langle s_i m_i \rangle_{S_i, M_i} = \sum_{s_i m_i} s_i m_i \text{pr}(m_i s_i | M_i S_i)$$
$$\langle s_i \rangle_\lambda = \sum_{s_i} s_i \mu(s_i | \lambda, S_i)$$
$$\langle m_i \rangle_\lambda = \sum_{m_i} m_i \xi(m_i | \lambda, M_i)$$



λ



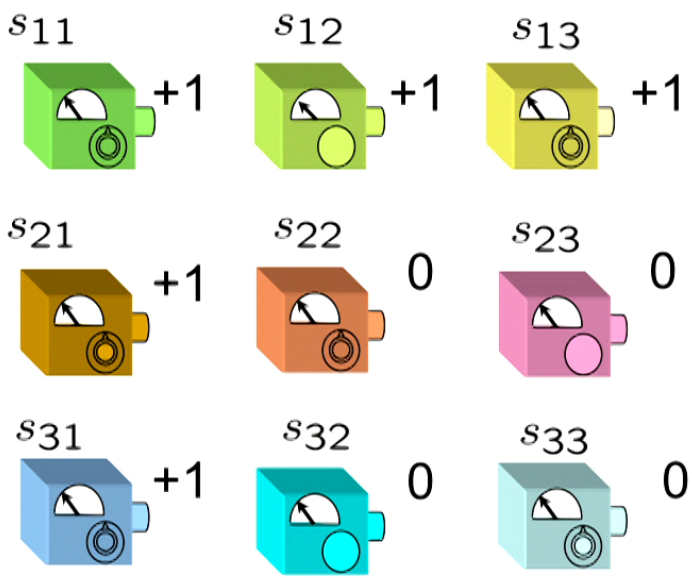
$$\text{Corr} = \tfrac{1}{9} \sum_{i=1}^{9} \langle s_i m_i \rangle_{S_i, M_i}$$
$$= \sum_\lambda \text{Corr}(\lambda) \nu(\lambda)$$
$$\text{Corr}(\lambda) = \tfrac{1}{9} \sum_{i=1}^{9} \langle s_i \rangle_\lambda \langle m_i \rangle_\lambda$$

$$\mathrm{pr}(m_j, s_i | M_j, S_i) = \sum_{\lambda \in \Lambda} \xi(m_j | M_j, \lambda) \mu(s_i | \lambda, S_i) \nu(\lambda)$$

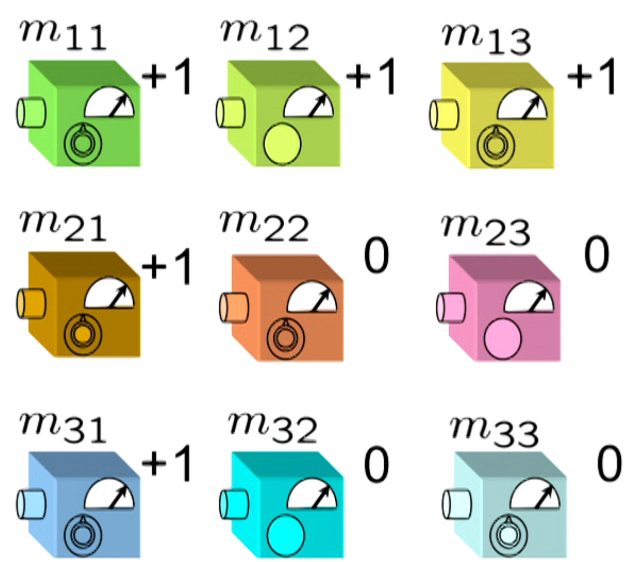
$$\langle s_i m_i \rangle_{S_i, M_i} = \sum_{s_i m_i} s_i m_i \mathrm{pr}(m_i s_i | M_i S_i)$$

$$\langle s_i \rangle_\lambda = \sum_{s_i} s_i \mu(s_i | \lambda, S_i)$$

$$\langle m_i \rangle_\lambda = \sum_{m_i} m_i \xi(m_i | \lambda, M_i)$$



λ



$$\mathrm{Corr} = \frac{1}{9} \sum_{i=1}^{9} \langle s_i m_i \rangle_{S_i, M_i}$$

$$= \sum_\lambda \mathrm{Corr}(\lambda) \nu(\lambda)$$

$$\mathrm{Corr}(\lambda) = \frac{1}{9} \sum_{i=1}^{9} \langle s_i \rangle_\lambda \langle m_i \rangle_\lambda$$

$$\mathrm{Corr} \leq \frac{5}{9}$$

Noncontextuality inequality

$X \otimes I$ $\quad I \otimes X$ $\quad X \otimes X$

$I \otimes Z$ $\quad Z \otimes I$ $\quad Z \otimes Z$

$X \otimes Z$ $\quad Z \otimes X$ $\quad Y \otimes Y$

9 binary-outcome measurements associated to these observables

9 binary-outcome sources preparing the mixed states onto the eigenspaces of these observables

$X \otimes I$ $\quad I \otimes X$ $\quad X \otimes X$

$I \otimes Z$ $\quad Z \otimes I$ $\quad Z \otimes Z$

$X \otimes Z$ $\quad Z \otimes X$ $\quad Y \otimes Y$

$$\text{Corr} = \frac{1}{9}\sum_{i=1}^{9}\langle s_i m_i \rangle_{S_i,M_i}$$

Quantum violation

$$\text{Corr} = 1 > \frac{5}{9}$$

$$X \otimes I \quad I \otimes X \quad X \otimes X$$

$$I \otimes Z \quad Z \otimes I \quad Z \otimes Z$$

$$X \otimes Z \quad Z \otimes X \quad Y \otimes Y$$

9 binary-outcome measurements associated to these observables

9 binary-outcome sources preparing the mixed states onto the eigenspaces of these observables

$$X \otimes I \quad I \otimes X \quad X \otimes X$$

$$I \otimes Z \quad Z \otimes I \quad Z \otimes Z$$

$$X \otimes Z \quad Z \otimes X \quad Y \otimes Y$$

$$\text{Corr} = \frac{1}{9}\sum_{i=1}^{9}\langle s_i m_i \rangle_{S_i, M_i}$$

Quantum violation

$$\text{Corr} = 1 > \frac{5}{9}$$

Noise robust

The failure of noncontextuality as a resource

- Preparation noncontextuality powers parity-oblivious multiplexing

This is a cryptographic task for which the probability of success appears in a noncontextuality inequality
RWS, Buzacott, Keehn, Toner, Pryde, PRL 102, 010401 (2009)

- In the state injection model of quantum computation, contextuality is necessary for universal quantum computation

Howard, Wallman, Veitch, Emerson, Nature 510, 351 (2014)

Why this notion of noncontextuality is natural

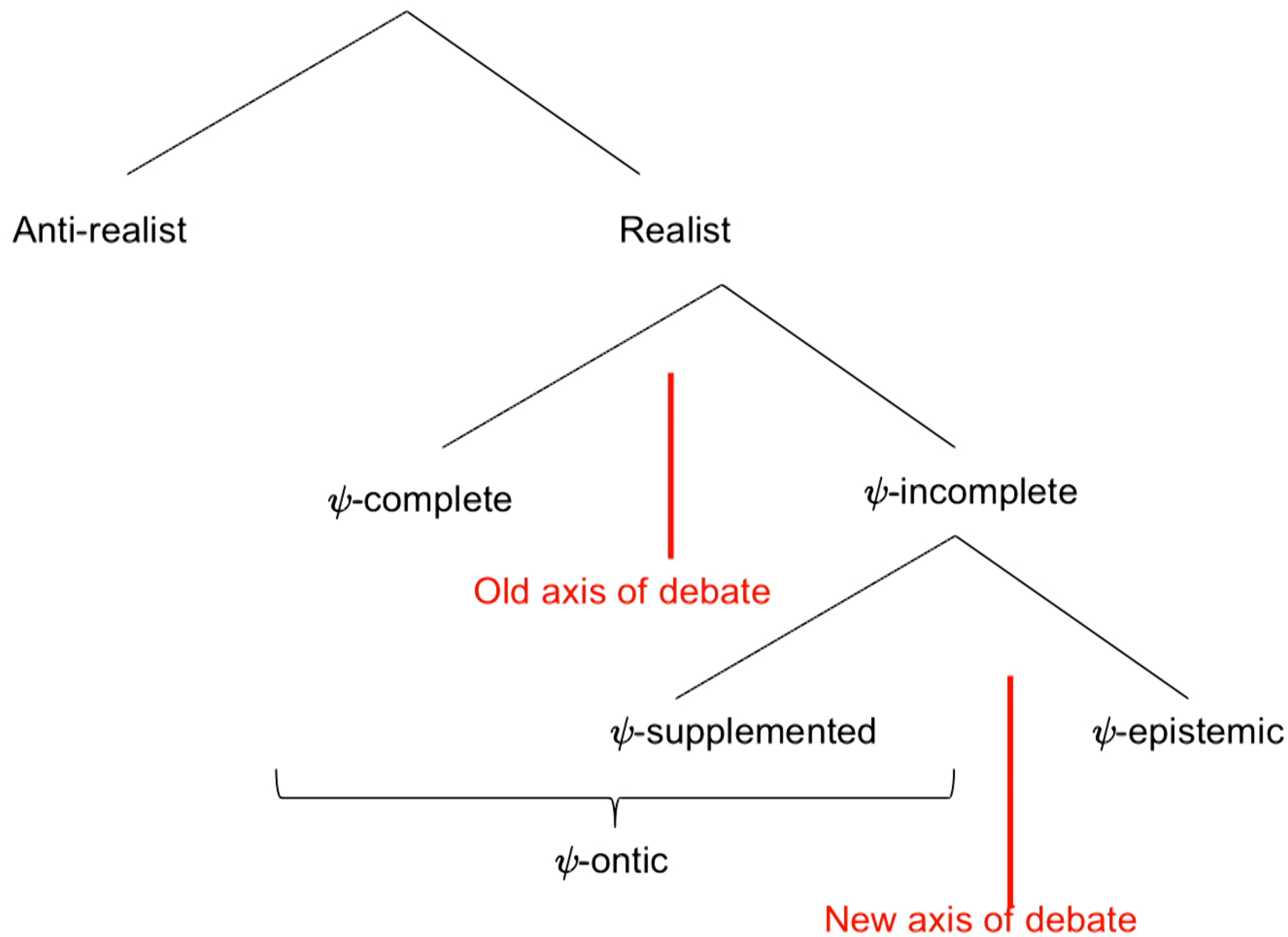
Responses to the measurement problem

1. Deny realism
   - Purely operational account of quantum theory

2. Deny the universality of unitary dynamics
   - Dynamical collapse theories

3. Deny that ψ is a complete representation of reality
   - Hidden variable models
   - Models of reality beyond hidden variables?

4. Deny indeterminism and discontinuity, except as subjective illusions
   - Everett's relative state interpretation, or "many worlds"


Anti-realist
Realist
ψ-complete
ψ-incomplete
Old axis of debate
ψ-supplemented
ψ-epistemic
ψ-ontic
New axis of debate