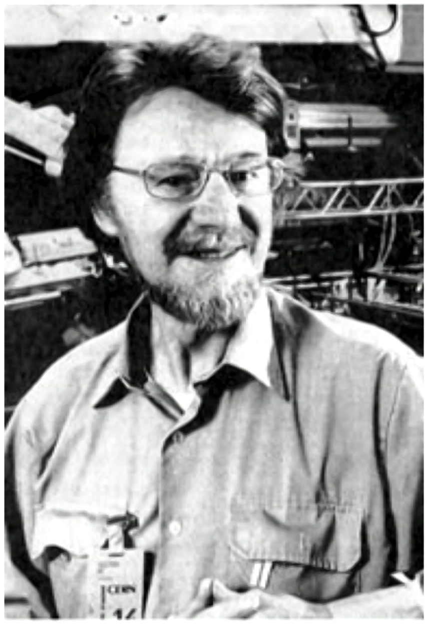
Title: PSI 17/18 - Foundations of Quantum Mechanics - Lecture 12

Date: Feb 13, 2018 10:15 AM

URL: http://pirsa.org/18020071

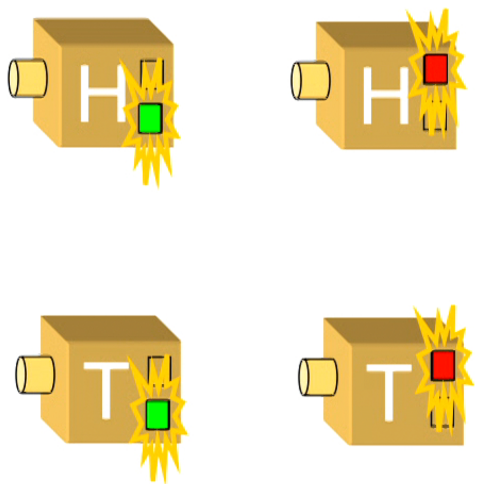
Abstract:

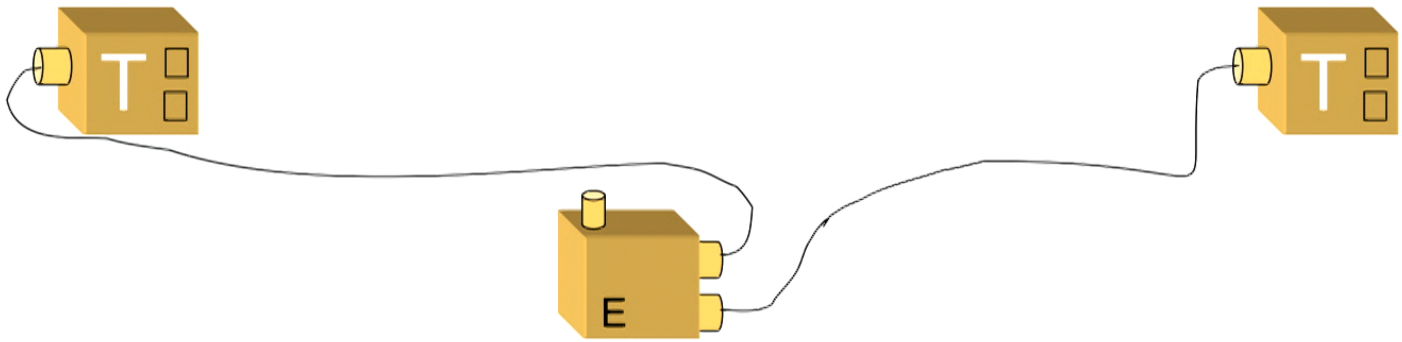
Bell's theorem



John S. Bell
(1928-1990)

A pair of two-outcome measurements




T
T
E

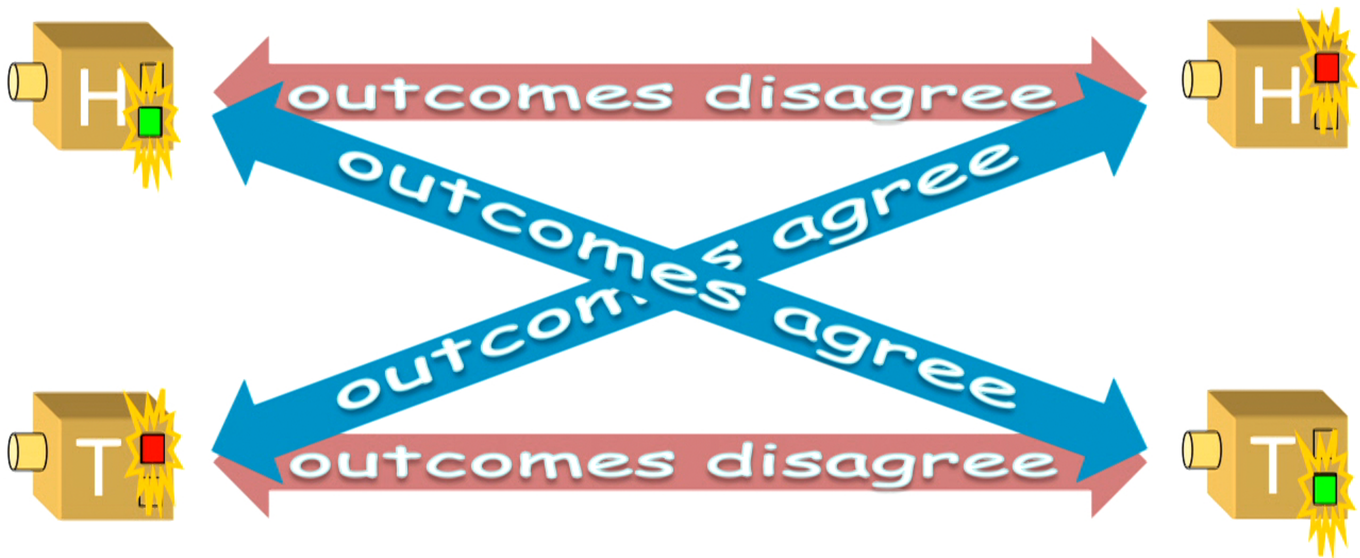
There are two possible measurements, H and T, with two outcomes each: green or red

Suppose which of H or T occurs at each wing is chosen at random

Scenario 1

1. Whenever the same measurement is made on A and B, the outcomes always agree	H and H or T and T
2. Whenever different measurements are made on A and B, the outcomes always disagree	H and T or T and H


H
H
T
T
outcomes agree
outcomes disagree
outcomes disagree
outcomes agree

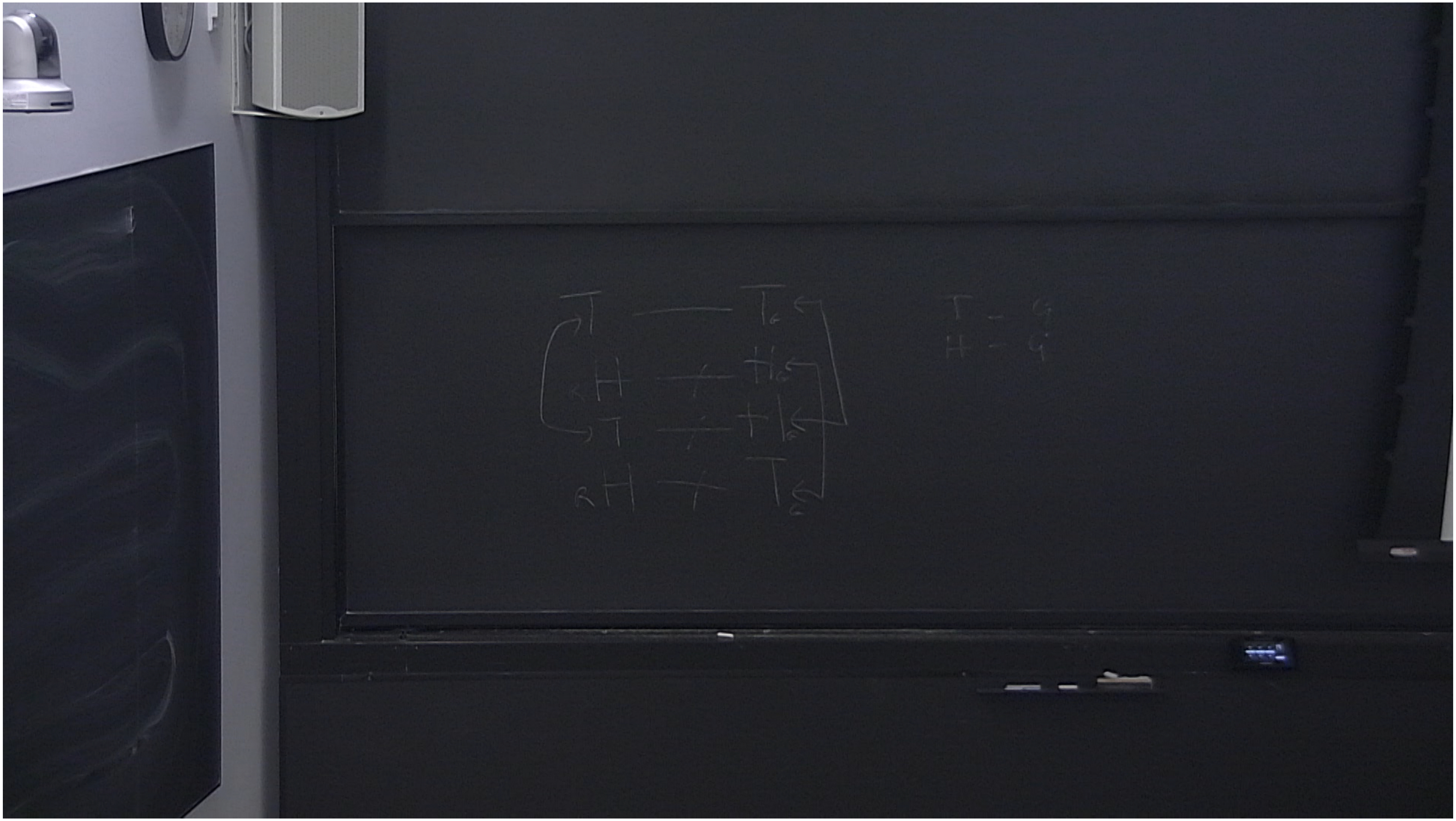

H
H
T
T
outcomes disagree
outcomes agree
outcomes agree
outcomes disagree

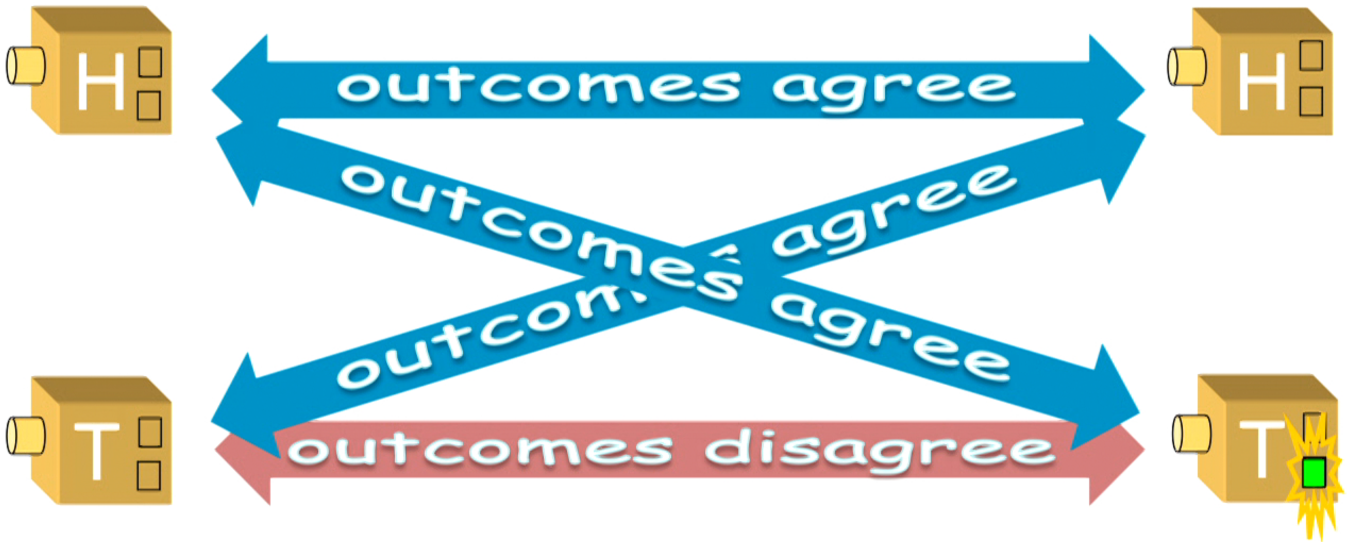
There are two possible measurements, H and T, with two outcomes each: green or red

Suppose which of H or T occurs at each wing is chosen at random

Scenario 3

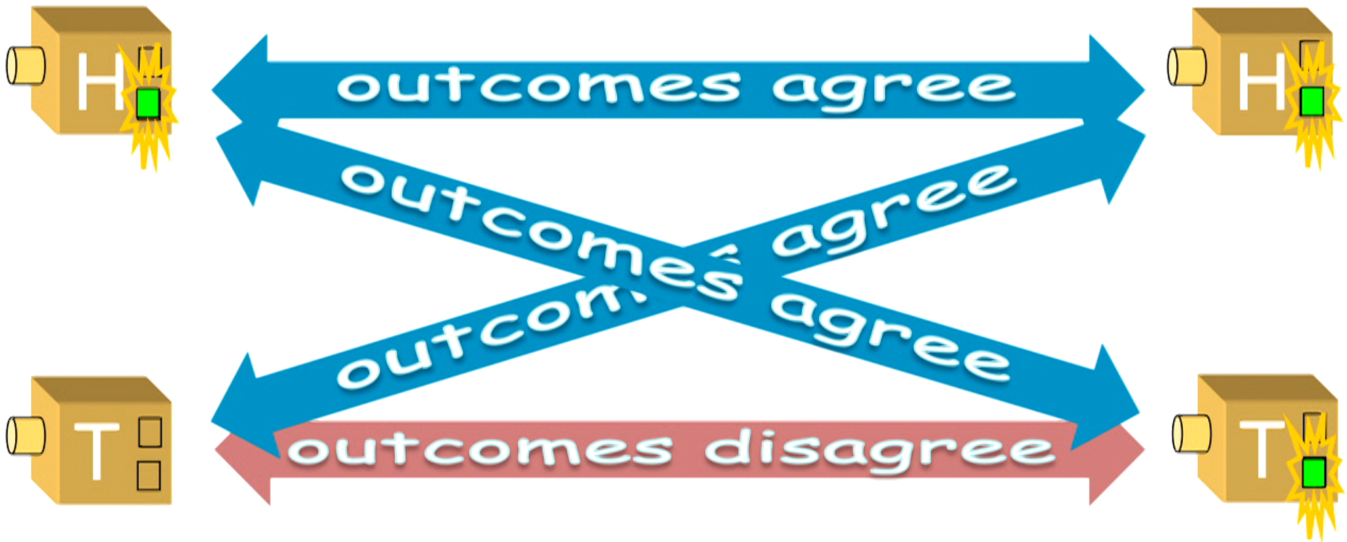
1. Whenever the measurement T is made on both A and B, the outcomes always disagree	T and T
2. Otherwise, the outcomes always agree	H and H or H and T or T and H




H
H
T
T
outcomes agree
outcomes agree
outcomes agree
outcomes disagree

The game can be won at most 75% of the time by local strategies

Q: How could you cheat and win the game all the time?


H
H
T
T
outcomes agree
outcomes agree
outcomes agree
outcomes disagree

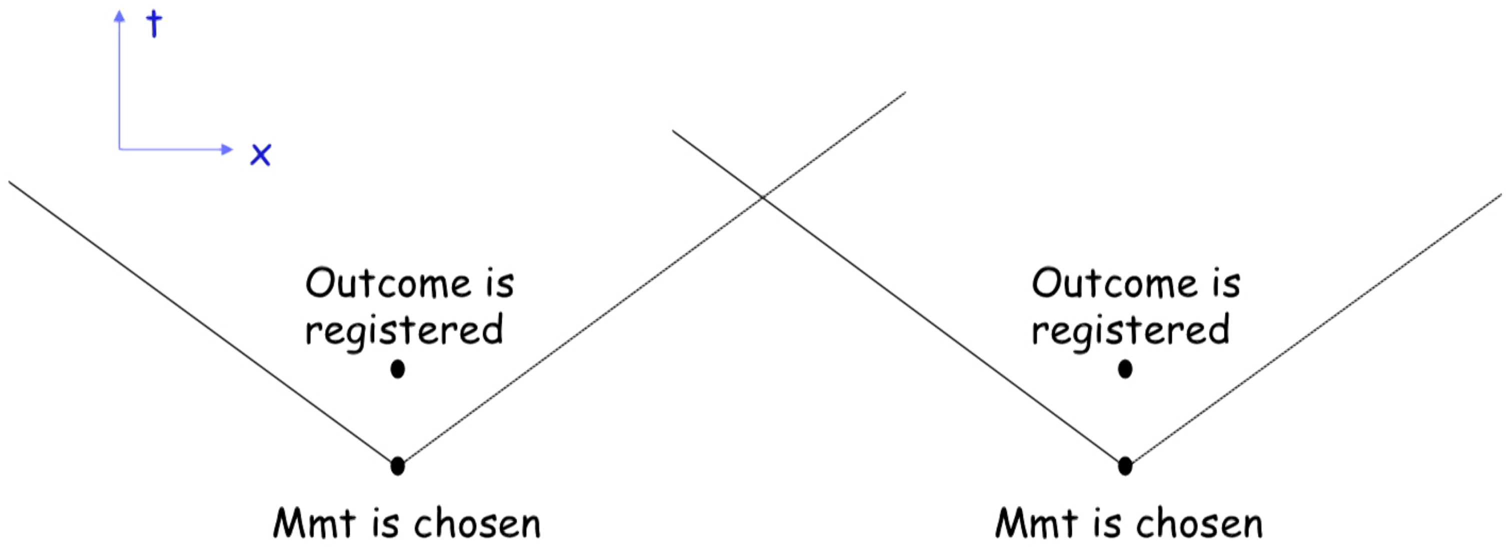
A: Rig the game so that the choices of settings are not random but instead are correlated with the local strategies

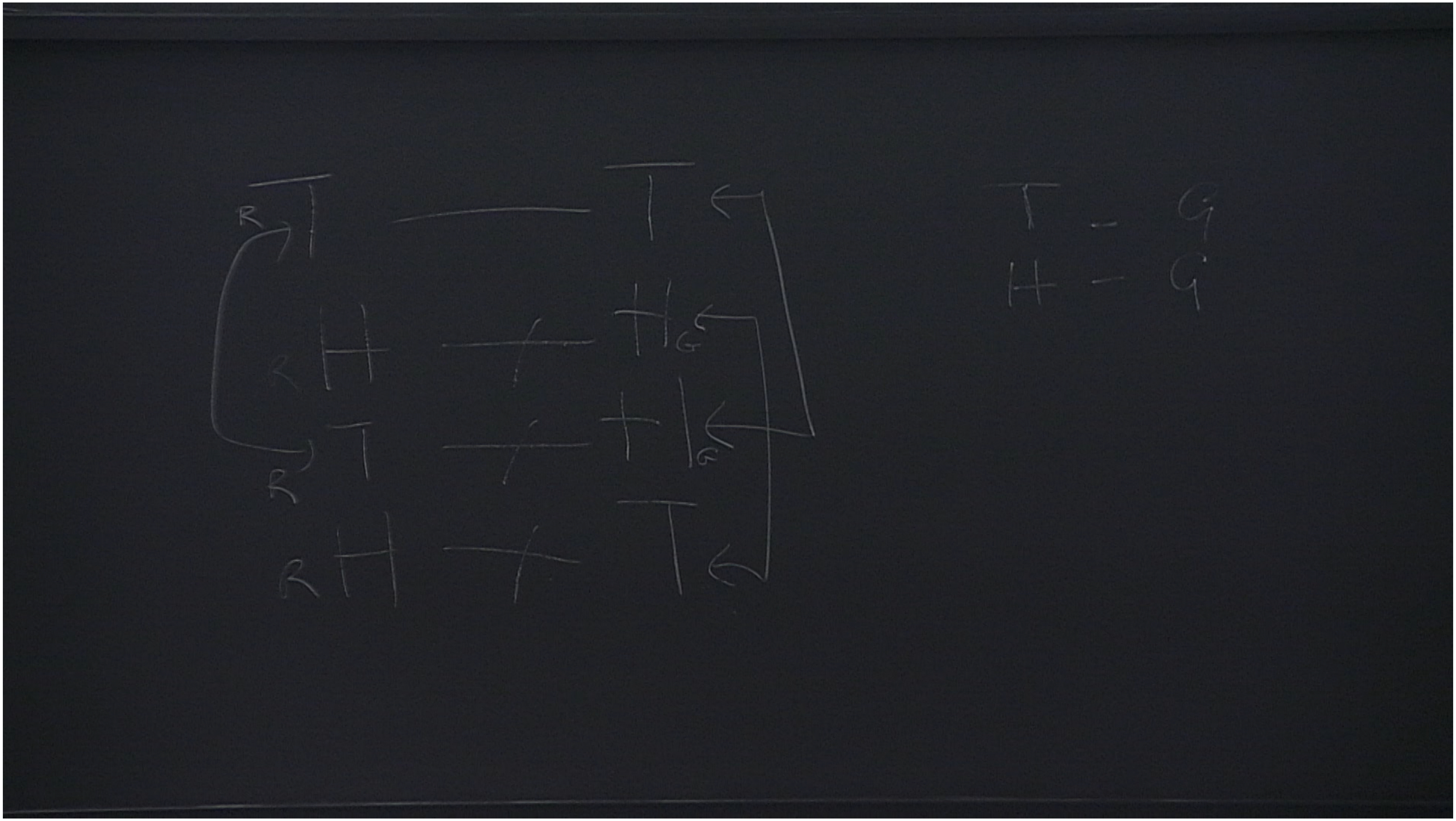
A: Communication of the choice of measurement in one wing to the system in the opposite wing

A: Communication of the choice of measurement in one wing to the system in the opposite wing

But there's a problem...

Tension with the theory of relativity




T – G
H – G

If the particles have access to randomness when deciding on their strategy, can it help them to generate the correlations?

No. The degree of correlation can only be the same or less.

If the particles have access to *local* randomness (i.e. independent randomness at the two wings), can it help them to generate the correlations?

No. The degree of correlation can only be the same or less.

If the detector inefficiencies are sufficiently high (sometimes there is no outcome), can particles obeying local causality simulate the correlations on the detected pairs?

Yes. This is the detector loophole.

Is there a problem if the choice of measurement is made too early?

Yes. This is the locality loophole.

The quantum correlations

$$p(\text{success}) = \frac{1}{4}[\, p(\text{agree}|HH) + p(\text{agree}|HT) + p(\text{agree}|TH) + p(\text{disagree}|TT)]$$

Realist theories that are locally causal predict

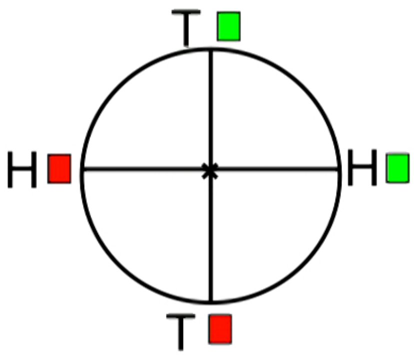
$$p(\text{success}) \leq 0.75$$

A Bell Inequality

Quantum theory predicts that one can achieve

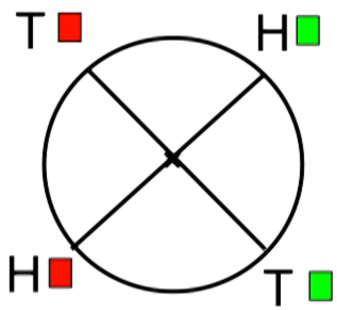
$$p(\text{success}) \simeq 0.85$$

The Bell-inequality violation in quantum theory

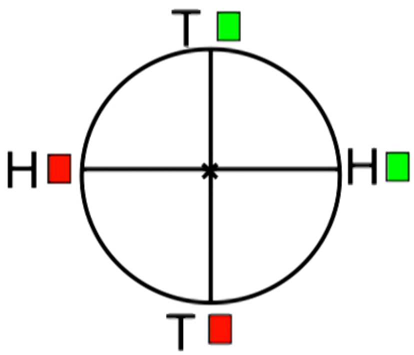


$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

$$\text{p(success)} = \frac{1}{2} + \frac{1}{2\sqrt{2}} \simeq 0.85$$



The Bell-inequality violation in quantum theory



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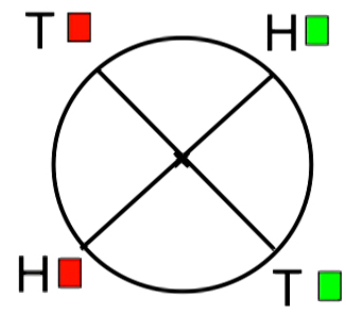
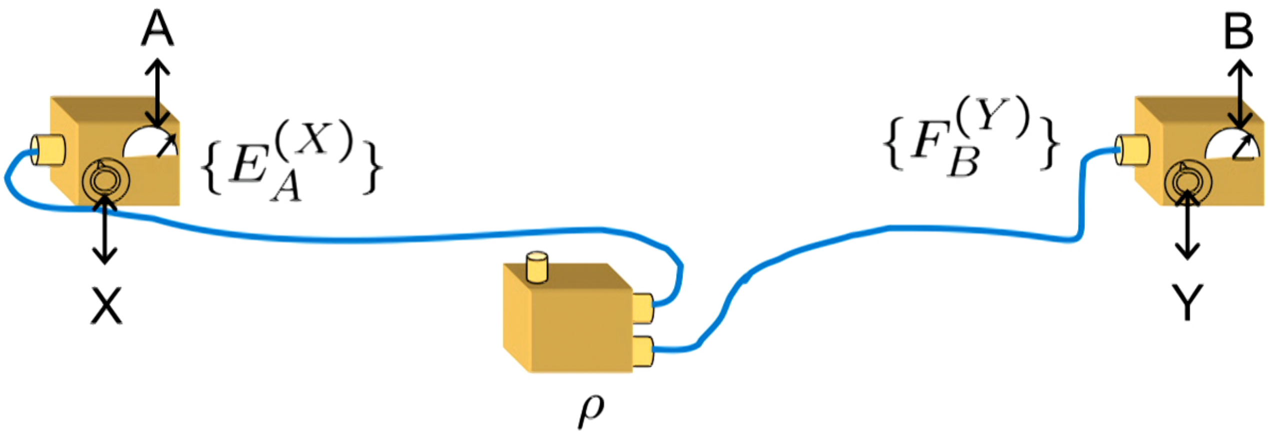


$$_A\langle +\hat{n}|\psi\rangle_{AB} = [\cos(\theta/2)_A\langle 0| + \sin(\theta/2)_A\langle 1|]\frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$
$$= \cos(\theta/2)|0\rangle_B + \sin(\theta/2)|1\rangle_B$$
$$= |+\hat{n}\rangle_B$$

$$|\langle +\hat{n}|_A\langle +\hat{m}|_B|\psi\rangle_{AB}|^2 = |\langle +\hat{m}|+\hat{n}\rangle|^2 = \cos^2(\theta/2)$$

$$p(\text{agree}|HH) = p(\text{agree}|HT) = p(\text{agree}|TH) = p(\text{disagree}|TT)$$
$$= \cos^2(\pi/8) = \frac{1}{2} + \frac{1}{2\sqrt{2}}$$

No signalling in quantum theory



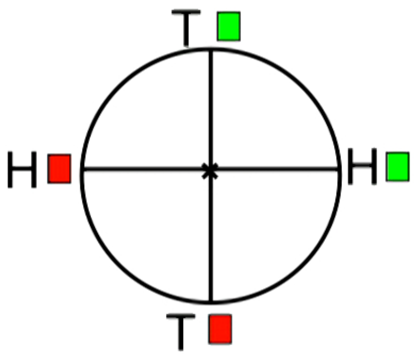
$$\begin{aligned} p(B|XY) &= \sum_A p(AB|XY) \\ &= \sum_A \mathrm{Tr}[\,(E_A^{(X)} \otimes F_B^{(Y)})\,\rho] \\ &= \mathrm{Tr}[\,(I \otimes F_B^{(Y)})\,\rho] \\ &= p(B|Y) \end{aligned}$$

independent of choice of measurement at left

Note $[E_A^{(X)} \otimes I, I \otimes F_B^{(Y)}] = 0$ because left and right are space-like separated

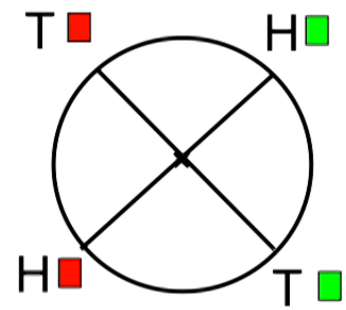
Is the proof robust to imperfection in the state preparation? (the state is mixed rather than pure)

The Bell-inequality violation in quantum theory



$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

$$\text{p(success)} = \frac{1}{2} + \frac{1}{2\sqrt{2}}$$
$$\simeq 0.85$$

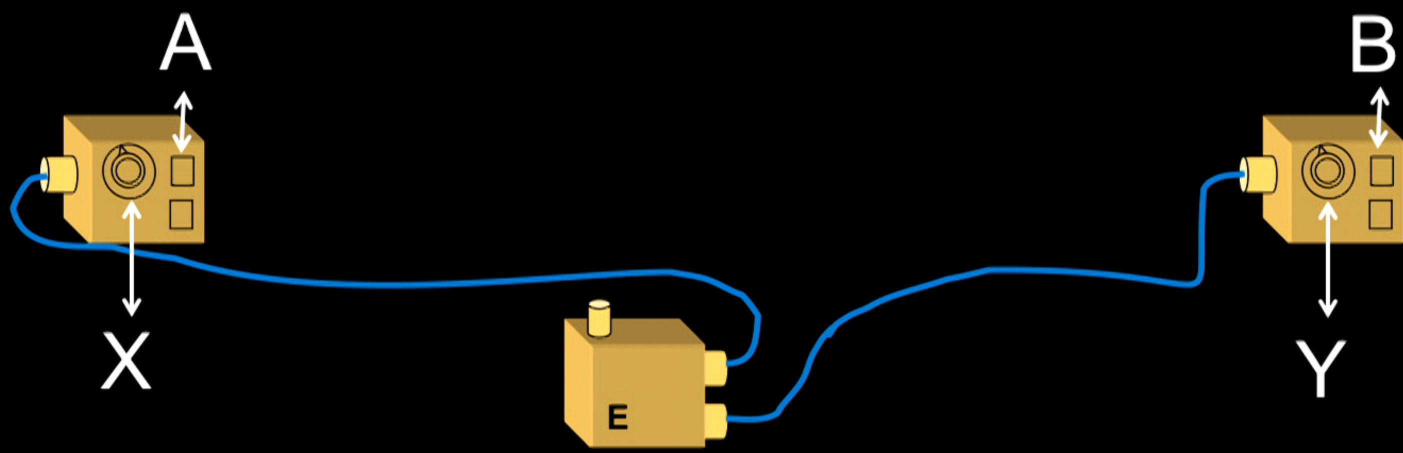


$$_A\langle +\hat{n}|\psi\rangle_{AB} = [\cos(\theta/2)_A\langle 0| + \sin(\theta/2)_A\langle 1|]\frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$
$$= \cos(\theta/2)|0\rangle_B + \sin(\theta/2)|1\rangle_B$$
$$= |+\hat{n}\rangle_B$$

$$|\langle +\hat{n}|_A\langle +\hat{m}|_B|\psi\rangle_{AB}|^2 = |\langle +\hat{m}|+\hat{n}\rangle|^2 = \cos^2(\theta/2)$$

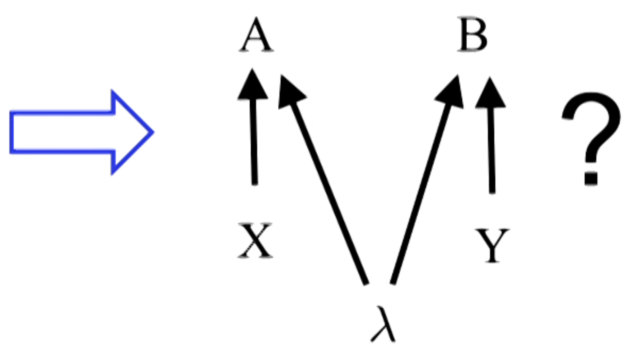
$$p(\text{agree}|HH) = p(\text{agree}|HT) = p(\text{agree}|TH) = p(\text{disagree}|TT)$$
$$= \cos^2(\pi/8) = \frac{1}{2} + \frac{1}{2\sqrt{2}}$$

Causal models perspective on Bell's theorem



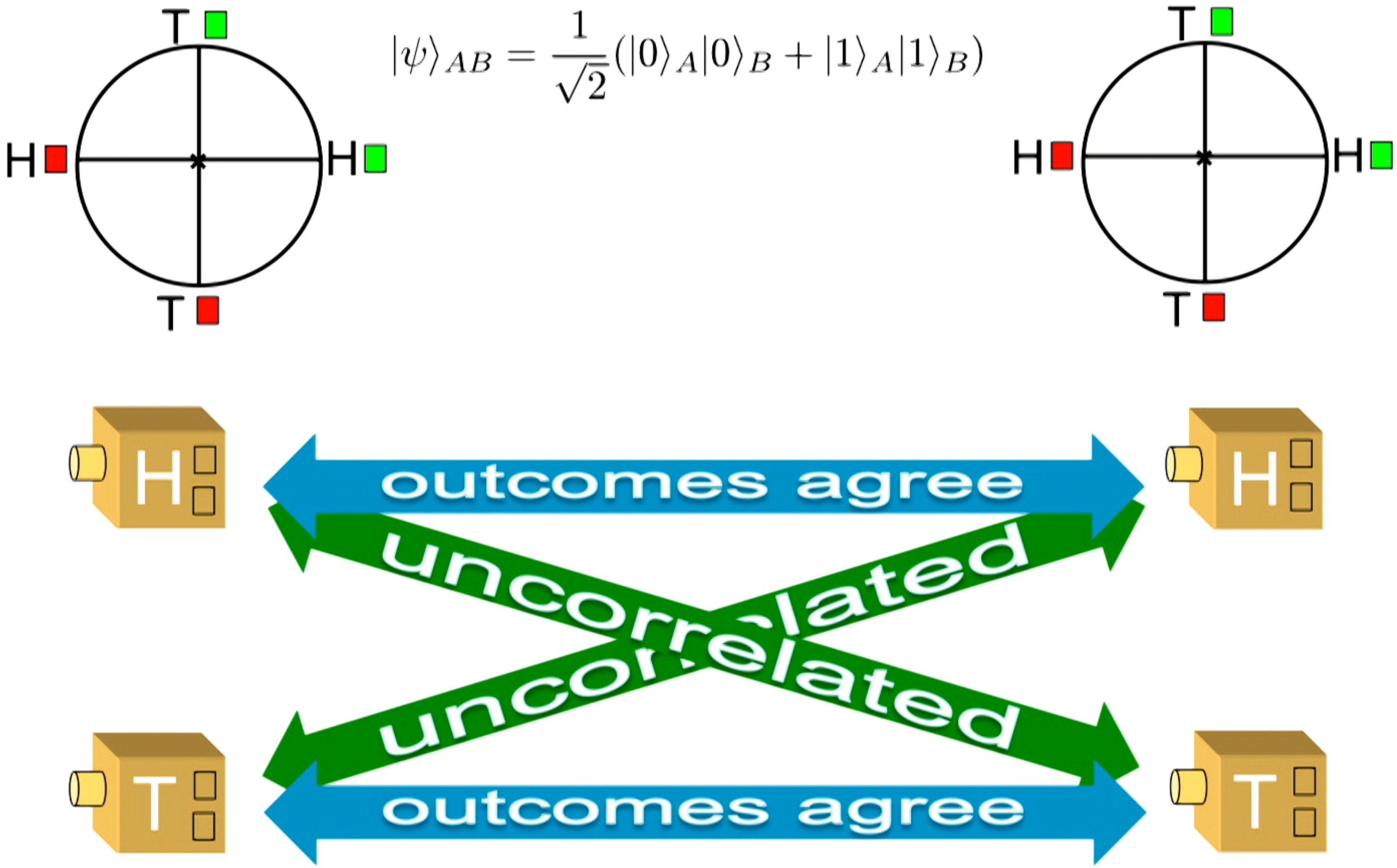
P(A,B|X,Y)

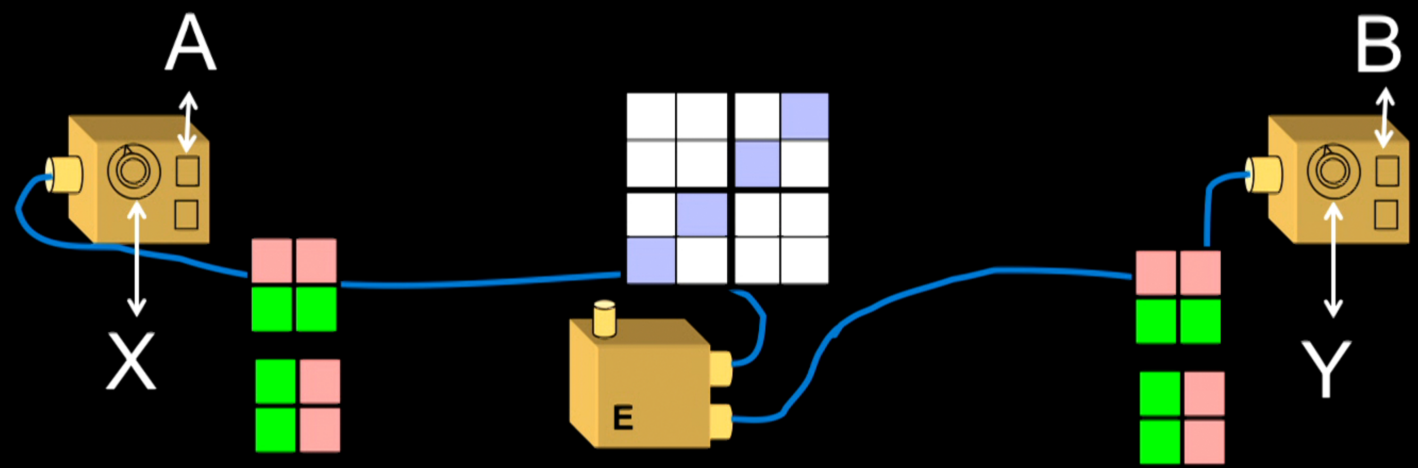
	A=0, B=0	A=0, B=1	A=1, B=0	A=1, B=1
X=0, Y=0	1	0	0	1
X=0, Y=1	0.25	0.25	0.25	0.25
X=1, Y=0	0.25	0.25	0.25	0.25
X=1, Y=1	1	0	0	1



EPR experiment

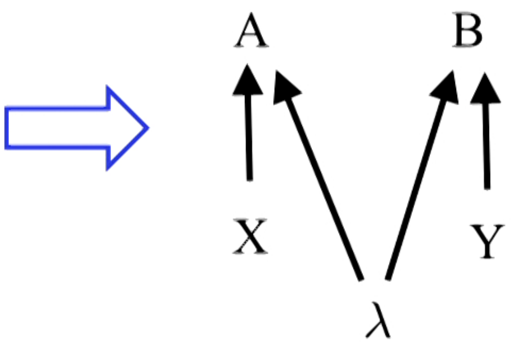
$$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$$

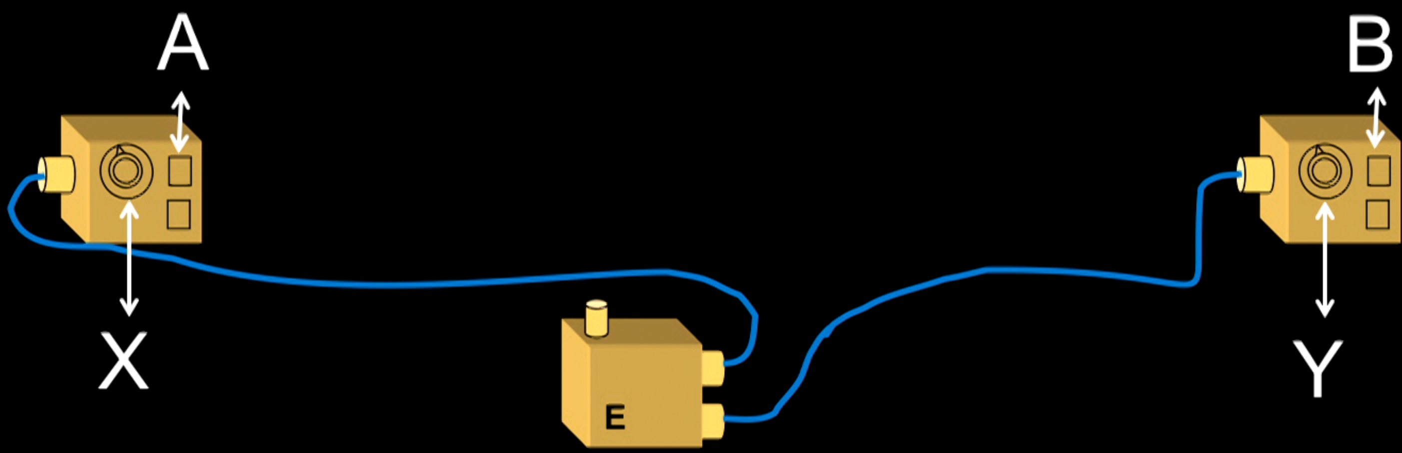




P(A,B|X,Y)

	A=0, B=0	A=0, B=1	A=1, B=0	A=1, B=1
X=0, Y=0	1	0	0	1
X=0, Y=1	0.25	0.25	0.25	0.25
X=1, Y=0	0.25	0.25	0.25	0.25
X=1, Y=1	1	0	0	1





P(A,B|X,Y)

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X=0, Y=0	0.427	0.073	0.073	0.427
X=0, Y=1	0.427	0.073	0.073	0.427
X=1, Y=0	0.427	0.073	0.073	0.427
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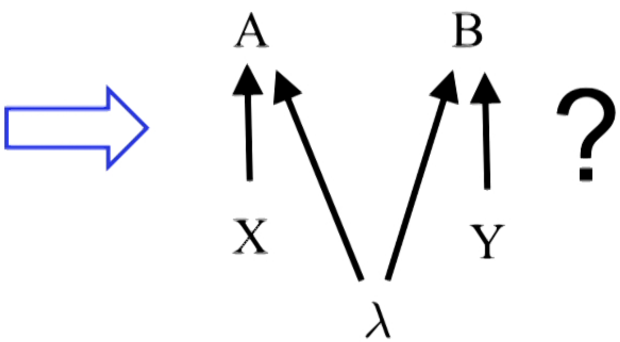

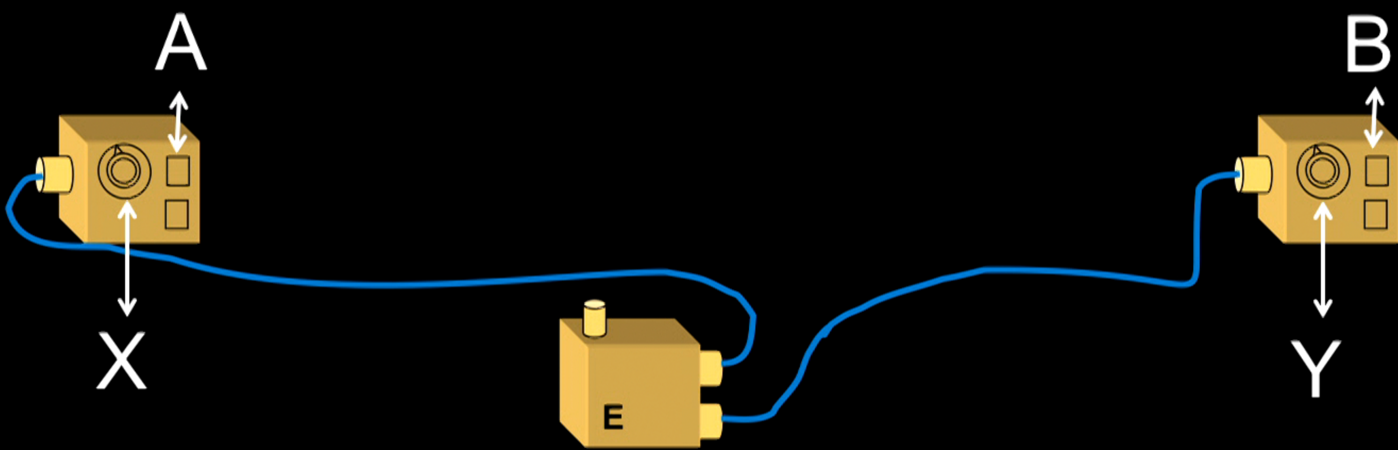

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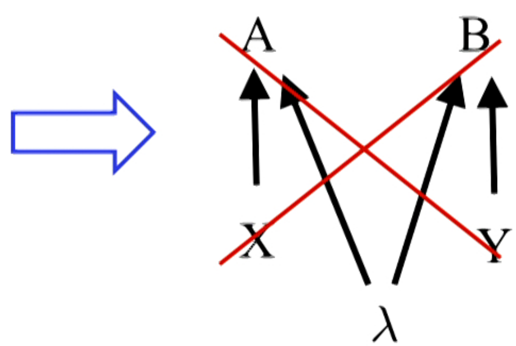
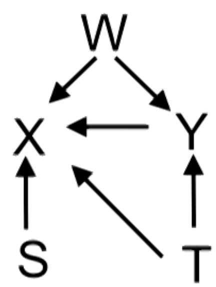


Causal Model

Causal Structure	Causal-Statistical Parameters



$P(W)$

$P(S)$

$P(T)$

$P(X|S,T,W,Y)$

$P(Y|T,W)$

$$P(X,Y,W,S,T) = P(X|S,T,W,Y)P(Y|T,W)P(W)P(S)P(T)$$

Causal inference algorithms seek to solve the inverse problem

Principle #1
Statistical dependences need to be explained causally

$$P(A,B,C)$$

A is independent of B

$$P(A,B) = P(A)P(B)$$

no other independence relations

A ? B

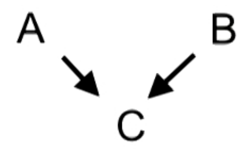
C

$$P(A,B,C)$$

A is independent of B

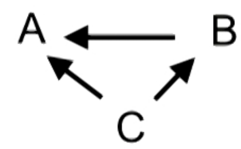
$$P(A,B) = P(A)P(B)$$

no other independence relations



$P(A)$
$P(B)$
$P(C|A,B)$
✓

$$P(A,B,C) = P(C|A,B)P(A)P(B)$$

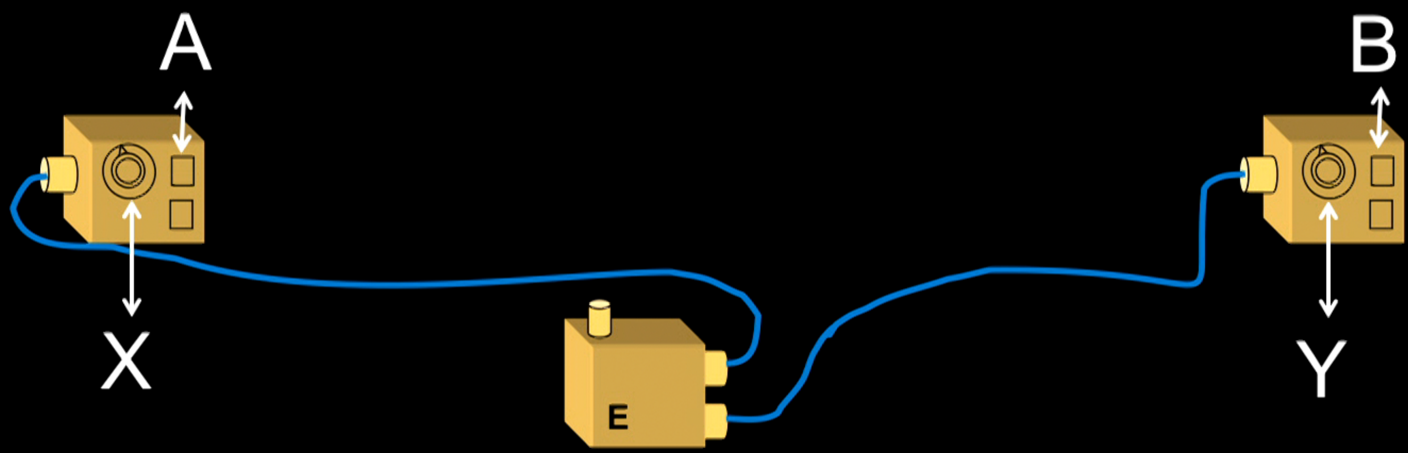


$P(C)$
$P(B|C)$
$P(A|B,C)$
✗

$$P(A,B,C) = P(A|B,C)P(B|C)P(C)$$

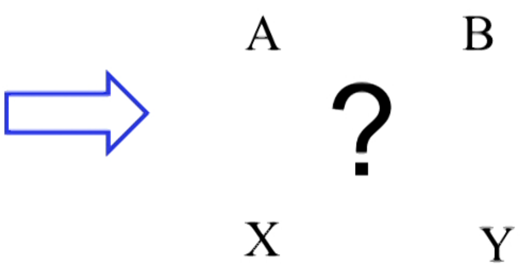
This model is fine-tuned

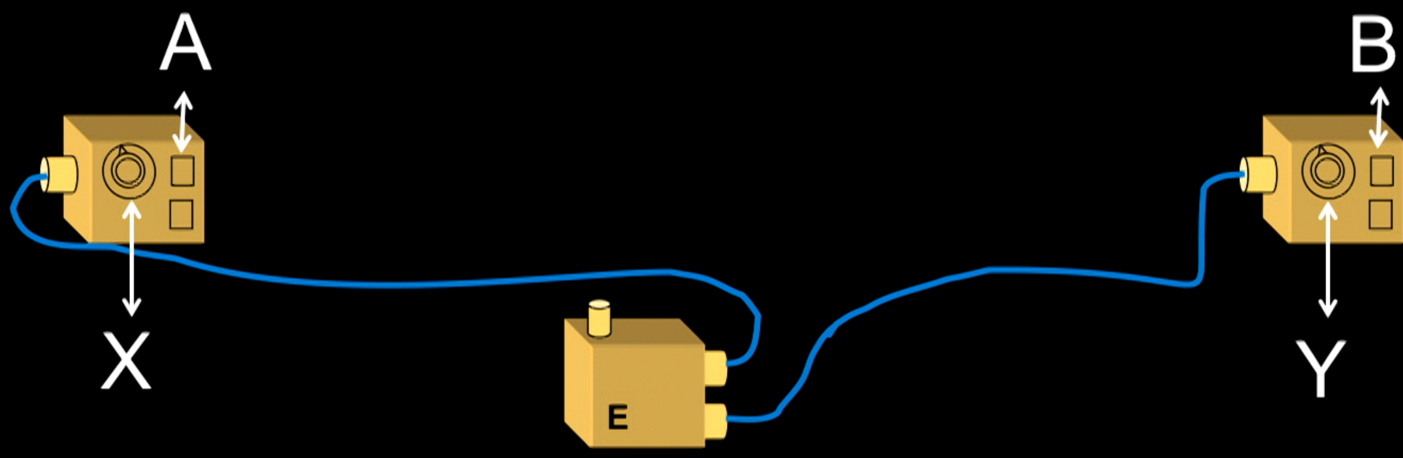
Principle #2
No fine-tuning



P(A,B|X,Y)

	A=0, B=0	A=0, B=1	A=1, B=0	A=1, B=1
X=0, Y=0	0.427	0.073	0.073	0.427
X=0, Y=1	0.427	0.073	0.073	0.427
X=1, Y=0	0.427	0.073	0.073	0.427
X=1, Y=1	0.073	0.427	0.427	0.073





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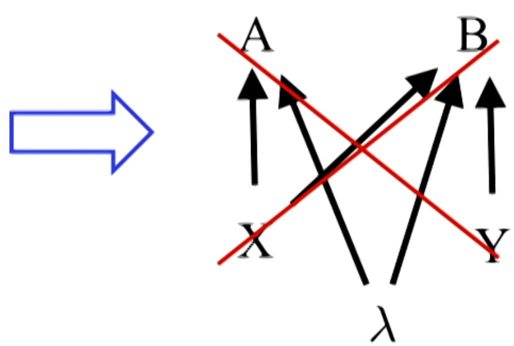

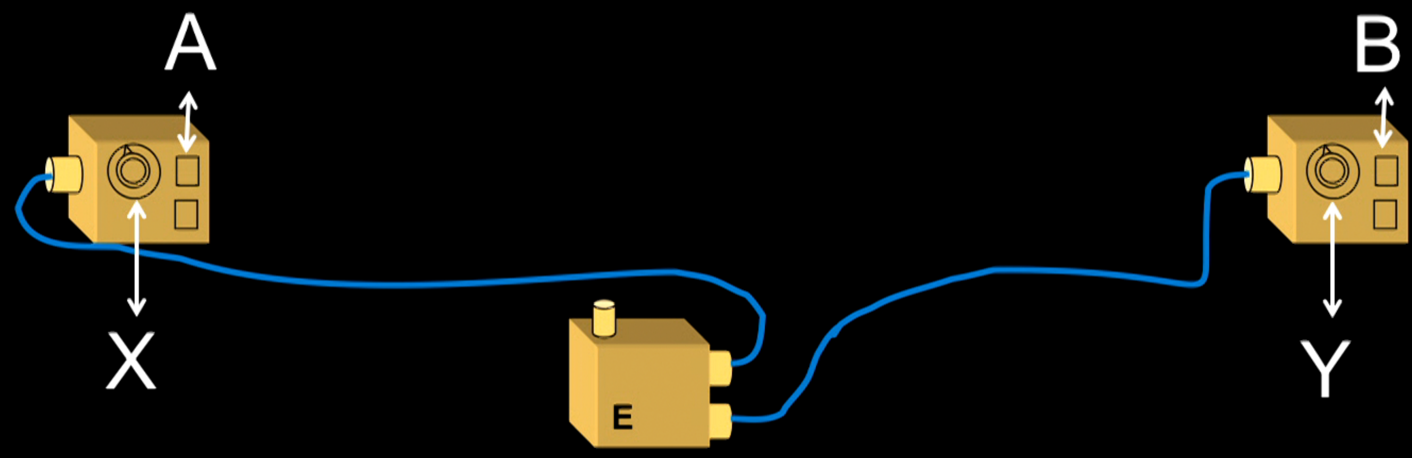

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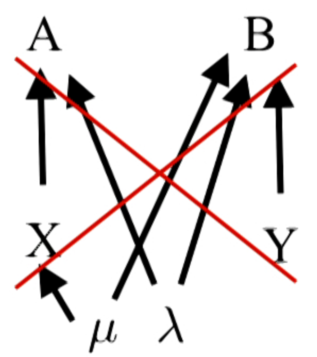
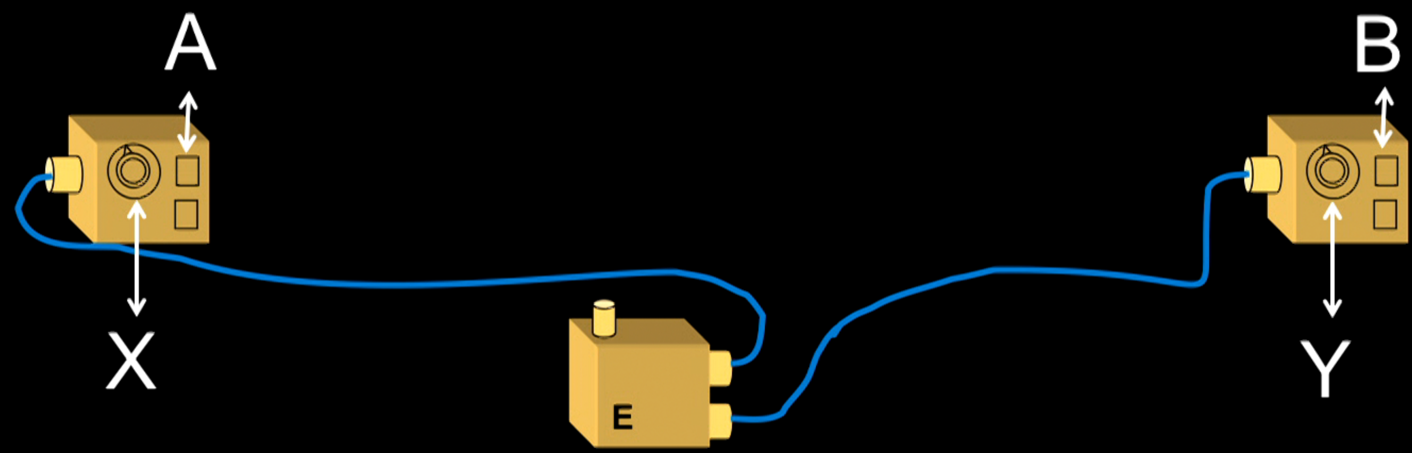
X=1, Y=1	0.073	0.427	0.427	0.073





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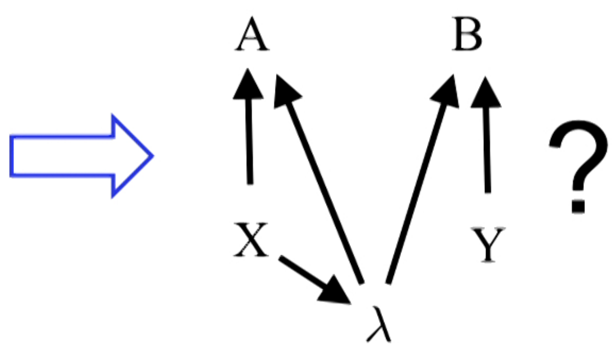

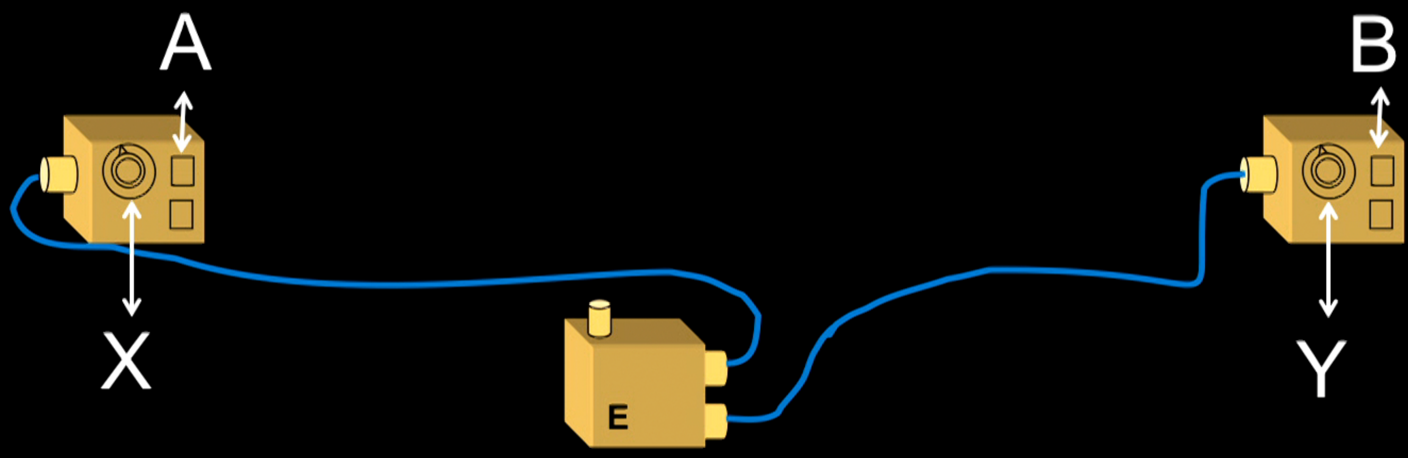

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X=1, Y=1	0.073	0.427	0.427	0.073

⇒ Nothing works!

- Statistical dependences need to be explained causally
    - No fine-tuning

⇩

Contradiction with quantum theory and experiment

- Realism
- Locality (no superluminal causal influences)
- No superdeterminism (free will)
- No retrocausation

⇩

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- Statistical dependences need to be explained causally
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