Title: PSI 17/18 - Foundations of Quantum Mechanics - Lecture 11

Date: Feb 12, 2018 10:15 AM

URL: http://pirsa.org/18020070

Abstract:

Some ψ-epistemic ontological models

Classical statistical theory

+

fundamental restriction on statistical distributions

$\Downarrow$

A large part of quantum theory

Classical theory	Statistical theory for the classical theory	Epistemically restricted statistical theory for the classical theory
Mechanics	Liouville mechanics	Restricted Liouville mechanics = Gaussian quantum mechanics

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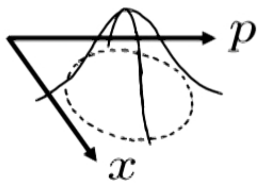
Trits	Statistical theory of trits	Restricted statistical theory of trits = Stabilizer theory for qutrits
Optics	Statistical optics	Restricted statistical optics = linear quantum optics

Epistemically restricted Liouville mechanics
= Gaussian Quantum Mechanics

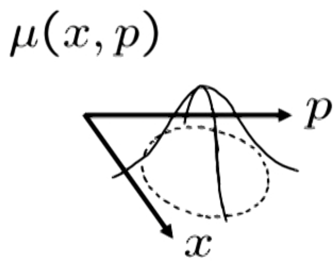
Liouville mechanics

$\mu(x,p)$



What is a good epistemic restriction to apply?
-- look to quantum mechanics

Liouville mechanics



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Quantum mechanics

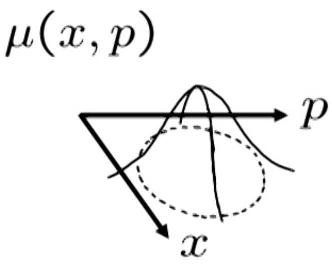
Uncertainty principle:

$$\Delta^2 x \Delta^2 p - C_{x,p}^2 \geq (\hbar/2)^2$$

where

$$\Delta^2 x \equiv \langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2$$
$$C_{x,p} \equiv \frac{1}{2}\langle \hat{x}\hat{p} + \hat{p}\hat{x} \rangle - \langle \hat{x} \rangle \langle \hat{p} \rangle$$
$$\langle \hat{A} \rangle \equiv \mathrm{Tr}(\hat{A}\hat{\rho})$$

Liouville mechanics



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Liouville mechanics with an epistemic restriction

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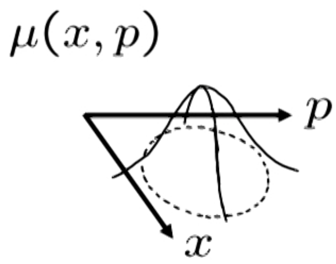

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The classical uncertainty principle (for a single particle in 1D):

The only Liouville distributions that can be prepared are those that satisfy

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and that have maximal entropy for a given set of second-order moments.

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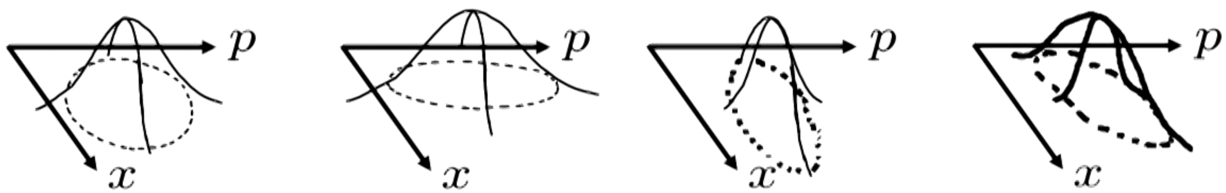
Among $\mu(x,p)$ with a given set of second-order moments, Gaussian distributions maximize the entropy

Valid pure epistemic states for one canonical system



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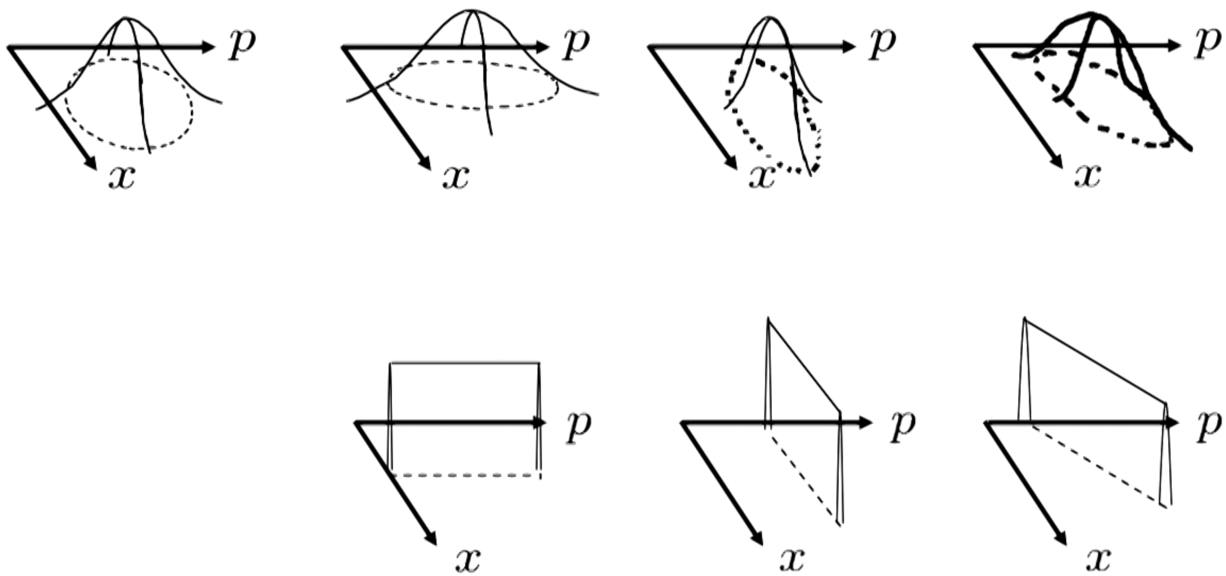

The Wigner representation

Phase point operators

$$A(x,p) = \frac{1}{2\pi\hbar}\int e^{ipy/\hbar}\left|x+\tfrac{1}{2}y\right\rangle\left\langle x-\tfrac{1}{2}y\right|\,\mathrm{d}y.$$

Wigner representation of an operator O

$$W_{\hat{O}}(x,p) = \mathrm{Tr}[\hat{O}\hat{A}(x,p)]$$

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$$\mathrm{Tr}(\hat{\rho}) = 1 \;\rightarrow\; \int \mathrm{d}x\mathrm{d}p W_{\hat{\rho}}(x,p) = 1.$$

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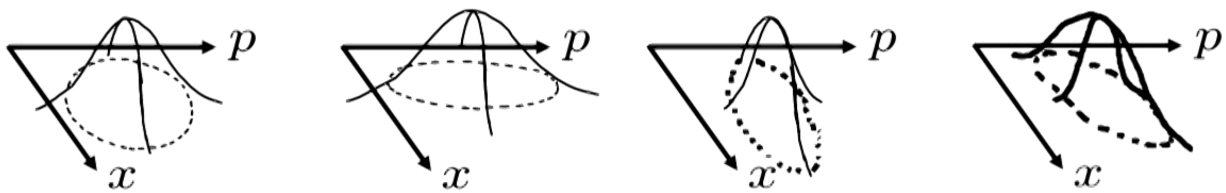
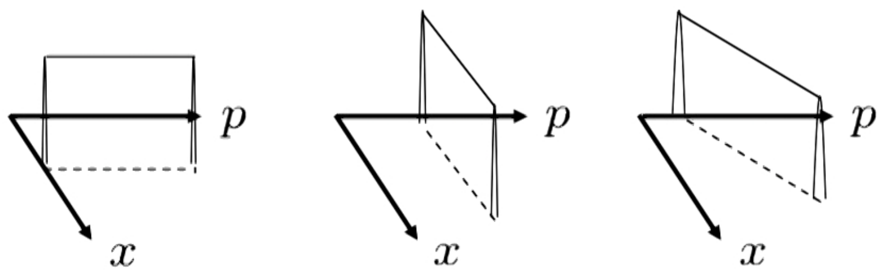
Wigner representation of a map $\mathcal{E}$

$$W_{\mathcal{E}}(x',p'|x,p)=\mathrm{Tr}[\hat{A}(x',p')\mathcal{E}(\hat{A}(x,p))]$$

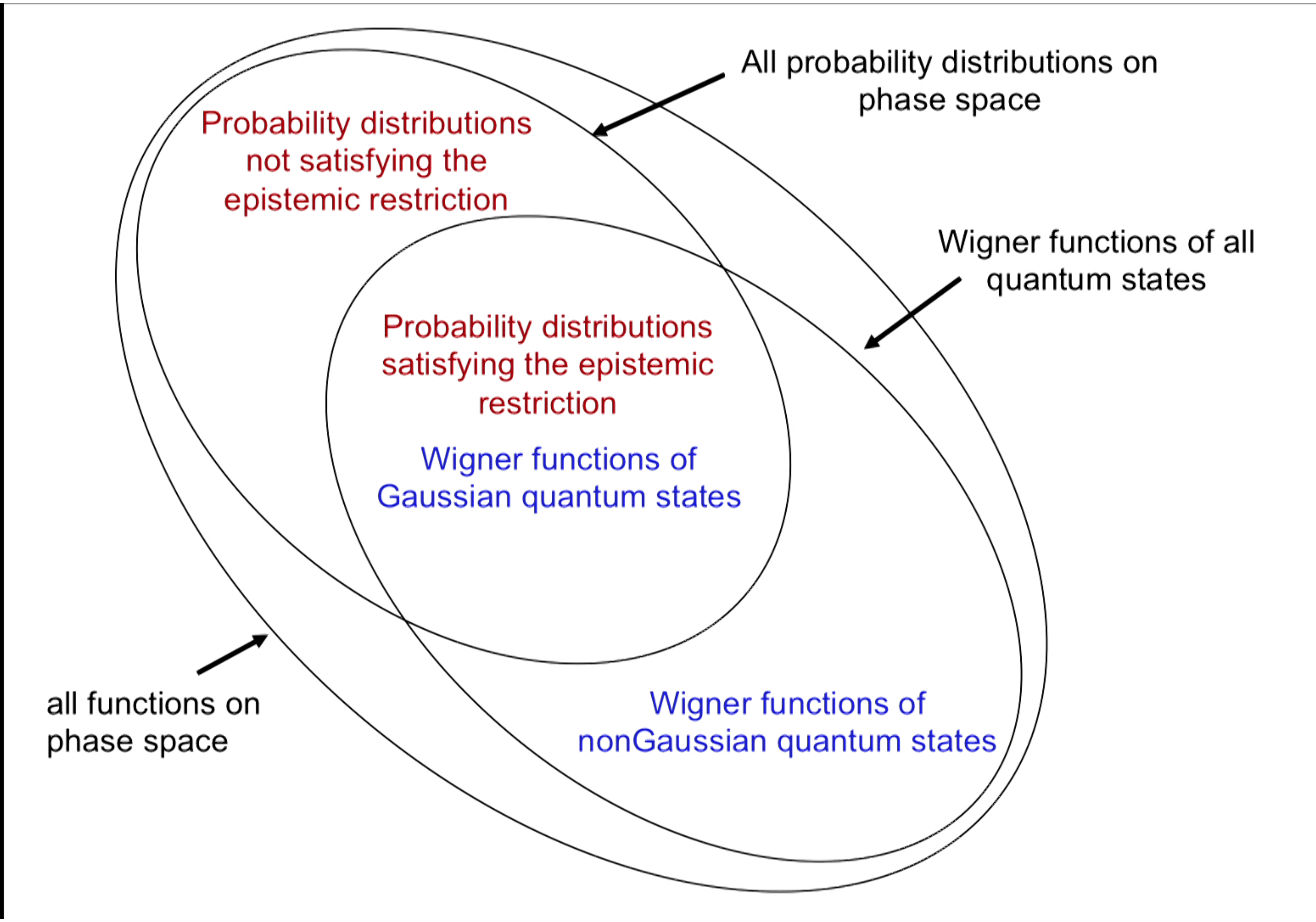
and $W_{\mathcal{E}(\rho)}(x,p)=\int \mathrm{d}x'\mathrm{d}p' W_{\mathcal{E}}(x,p|x',p')W_{\hat{\rho}}(x',p')$

Valid pure epistemic states for one canonical system





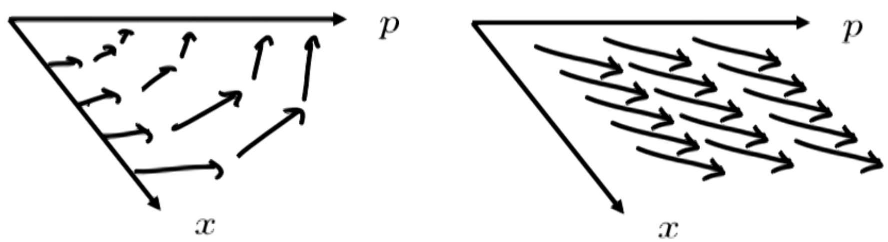
These correspond to the Wigner representations of the pure squeezed states in quantum mechanics


All probability distributions on phase space
Probability distributions not satisfying the epistemic restriction
Wigner functions of all quantum states
Probability distributions satisfying the epistemic restriction
Wigner functions of Gaussian quantum states
all functions on phase space
Wigner functions of nonGaussian quantum states

Valid deterministic transformations

The group of canonical transformations with quadratic Hamiltonian

Only canonical transformations preserve the uncertainty principle
Only quadratic Hamiltonians preserve the gaussianity



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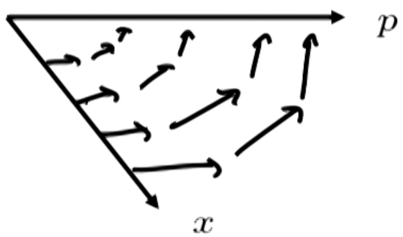
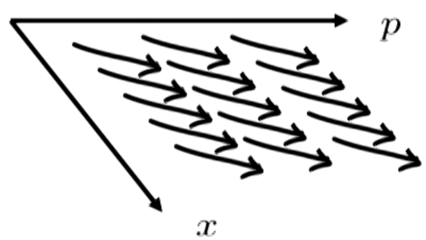


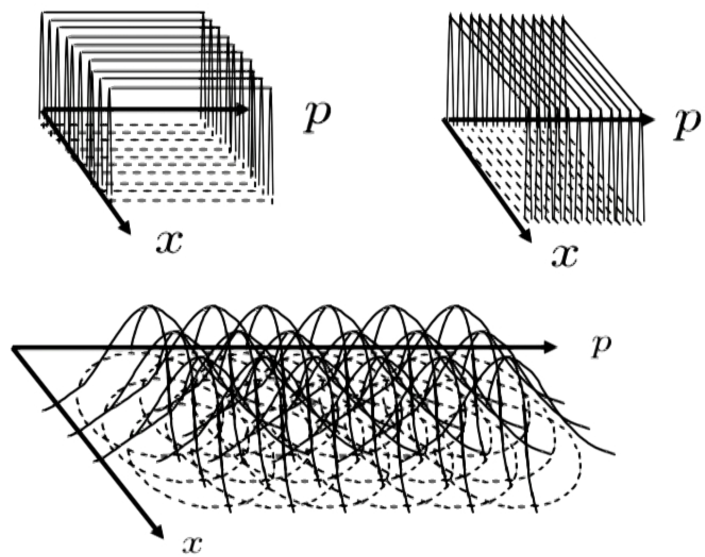


→ These correspond to the Wigner representations of the unitaries associated with these Hamiltonians

e.g. $D_{x_0,p_0} = e^{-ix_0\hat{P} - ip_0\hat{X}}$

$$\rho' = D_{x_0,p_0}\rho D^{\dagger}_{x_0,p_0} \quad \rightarrow \quad W_{\rho'}(x,p) = W_\rho(x+x_0, p+p_0)$$

Valid measurements



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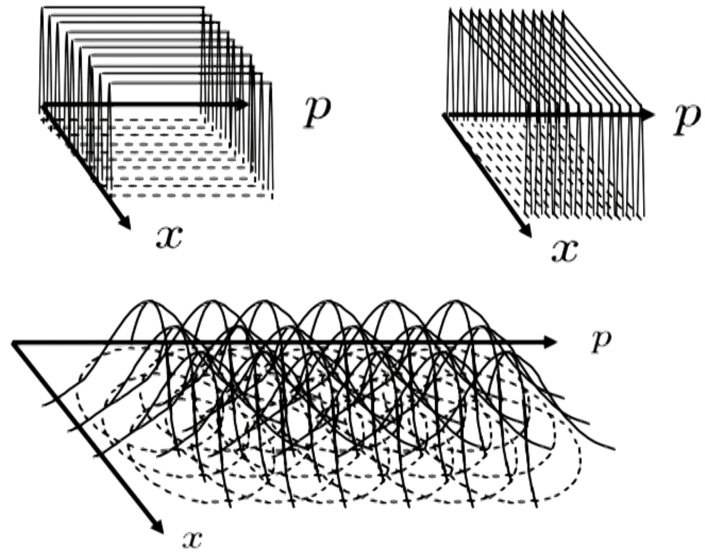

→ These also correspond to the Wigner representations of quantum measurements

Extension to multiple systems

Use a generalization of the uncertainty principle for multiple systems
or:
Allow all products of valid epistemic states
Allow canonical transformations with quadratic Hamiltonians on these

E.g. $x_A \to x_A - x_B$ $\quad$ $x_B \to x_A + x_B$

$p_A \to p_A - p_B$ $\quad$ $p_B \to p_A + p_B$

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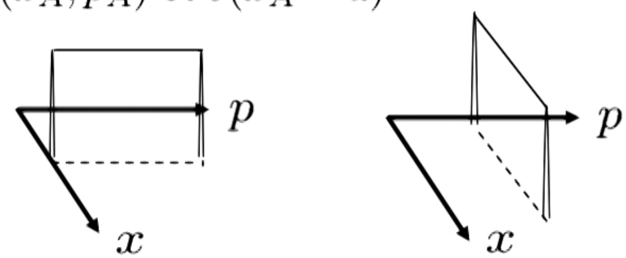
know X_A and P_B

$\mu(x_A, p_A) \propto \delta(x_A - a)$



$\mu(x_B, p_B) \propto \delta(p_B - b)$

Extension to multiple systems

Use a generalization of the uncertainty principle for multiple systems
or:
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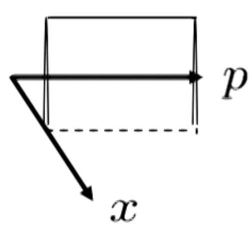

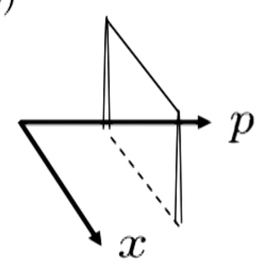

$\mu(x_B, p_B) \propto \delta(p_B - b)$

$\longrightarrow$

know $X_A - X_B$ and $P_A + P_B$

$$\mu(x_A, p_A, x_B, p_B) \propto \delta(x_A - x_B - a)\delta(p_A + p_B - b)$$

corresponds to EPR state

Q: How can one characterize the set of variables that can be jointly known?

A: They commute relative to the Poisson bracket!

Configuration space: $\mathbb{R} \ni x$

Phase space: $\Omega \equiv \mathbb{R}^2 \ni (x, p) \equiv m$

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Functionals on phase space: $F : \Omega \to \mathbb{R}$

$$X(m) = x$$
$$P(m) = p$$

Poisson bracket of functionals:

$$[F, G](m) \equiv (\frac{\partial F}{\partial X}\frac{\partial G}{\partial P} - \frac{\partial F}{\partial P}\frac{\partial G}{\partial X})(m)$$

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Recall:

A canonically conjugate pair $[F,G] = 1$

e.g. $\{X_1, P_1\}, \{X_2, P_2\}$, and $\{X_1 + X_2, P_1 + P_2\}$

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A commuting pair $[F,G] = 0$

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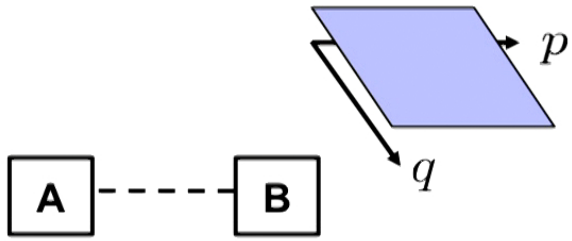
Canonical transformations preserve the Poisson bracket

Gaussian Quantum Mechanics
= States with Gaussian Wigner rep'ns (a.k.a. squeezed states)
+ Measurements with Gaussian Wigner rep'ns
+ Transformations that preserve Gaussianity of states

Epistemically restricted Liouville mechanics
= Gaussian quantum mechanics in the Wigner representation

EPR effect in Epistemically Restricted Liouville mechanics

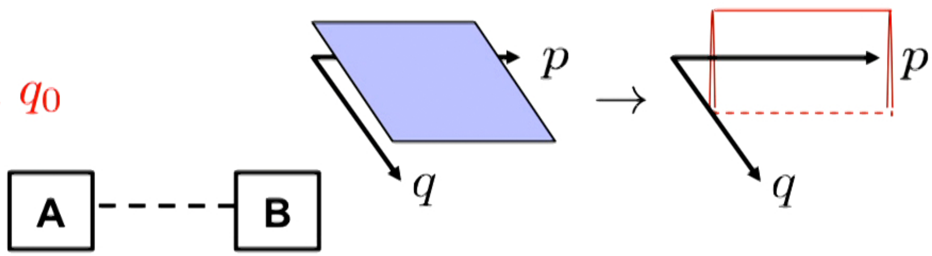


$$P_{\mathrm{EPR}}(q_A, p_A, q_B, p_B) \propto \delta(q_A - q_B)\delta(p_A + p_B)$$

$$Q_B - Q_A = 0$$
$$P_B + P_A = 0$$

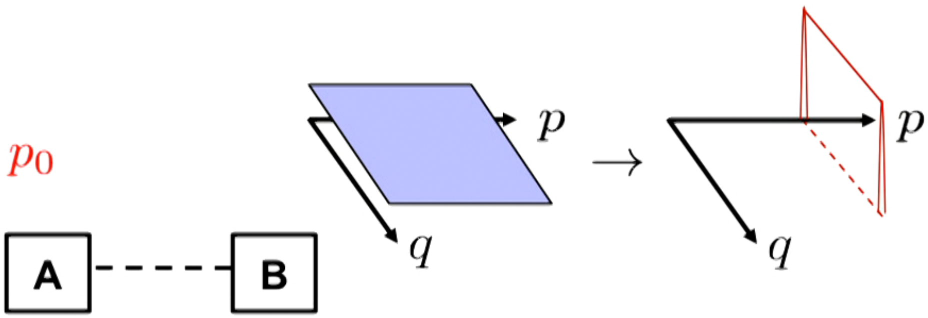
EPR effect in Epistemically Restricted Liouville mechanics

Measure Q_A find q_0



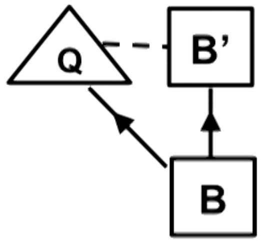
EPR effect in Epistemically Restricted Liouville mechanics

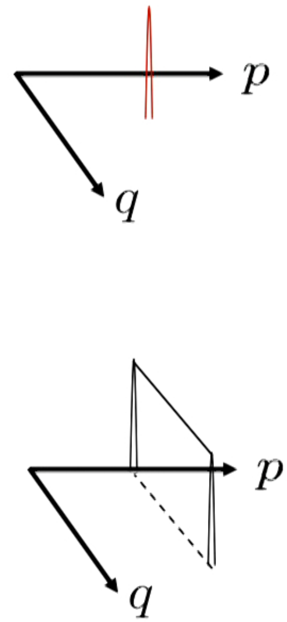
Measure P_A find p_0



Collapse Rule in Epistemically Restricted Liouville mechanics

Measure Q_B find q_0

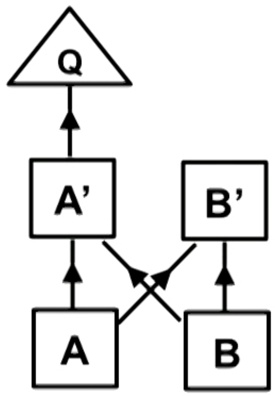




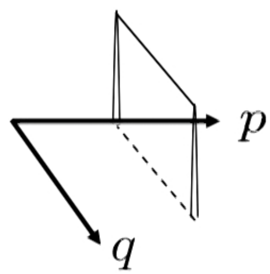
But this would violate the epistemic restriction!

Collapse Rule in Epistemically Restricted Liouville mechanics

Measure $Q_{A'}$

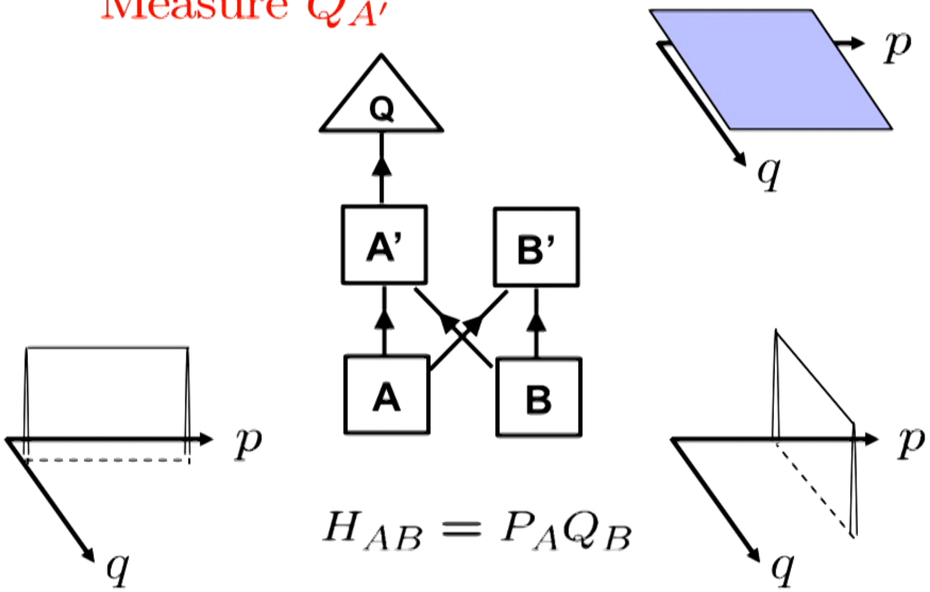


$$H_{AB} = P_A Q_B$$



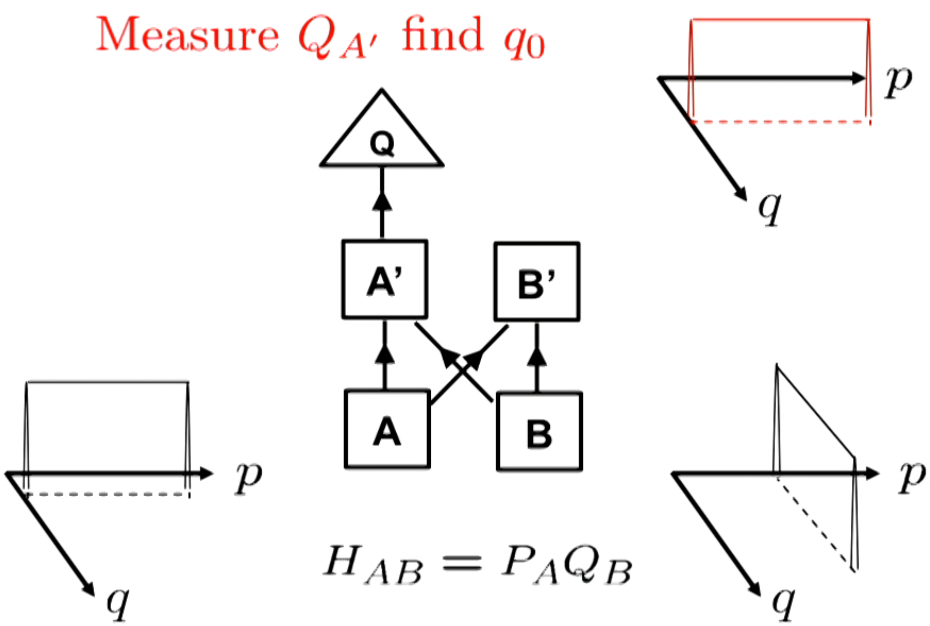
Collapse Rule in Epistemically Restricted Liouville mechanics

Measure $Q_{A'}$



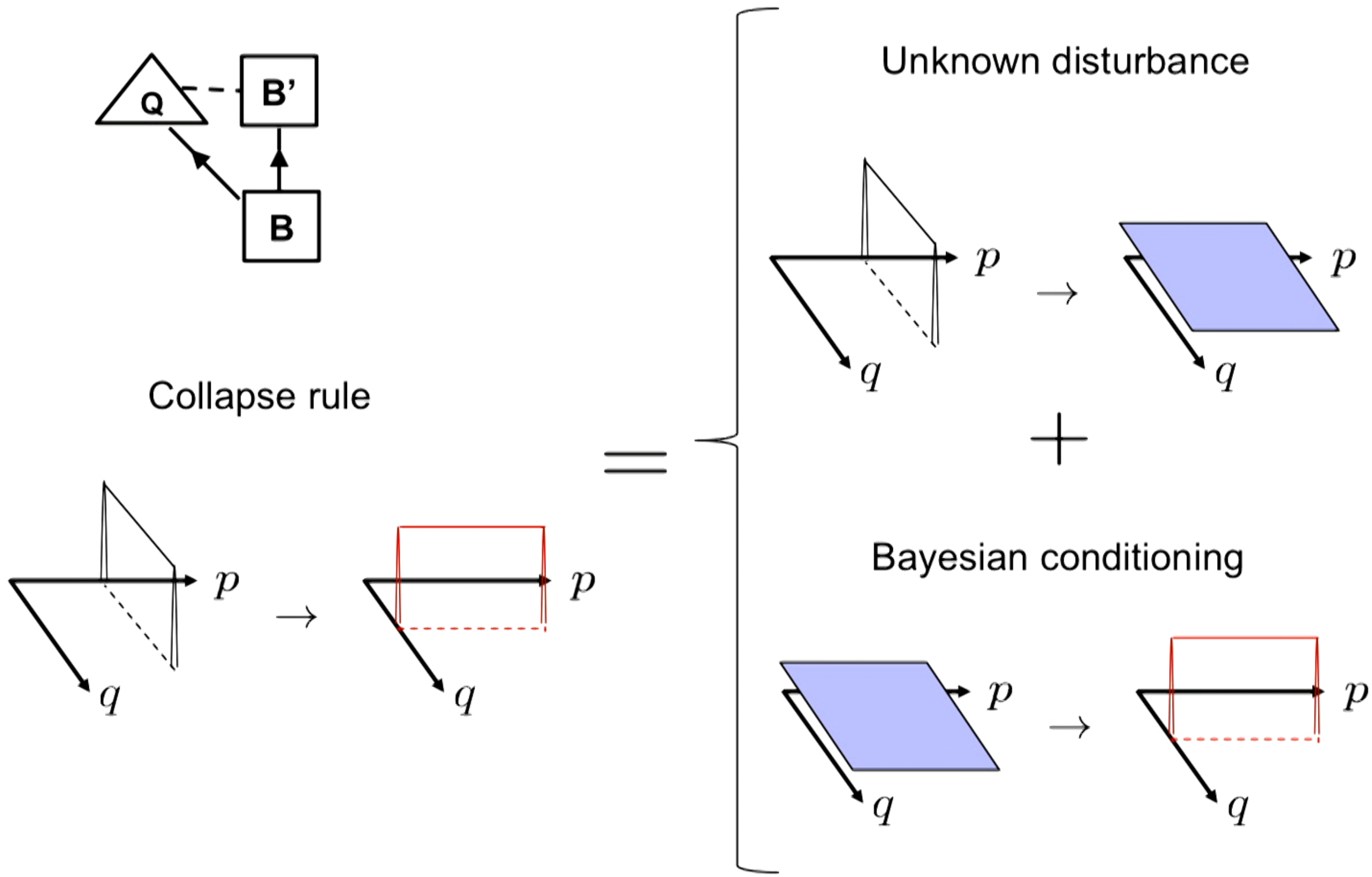
$H_{AB} = P_A Q_B$

Collapse Rule in Epistemically Restricted Liouville mechanics



$H_{AB} = P_A Q_B$

Collapse Rule in Epistemically Restricted Liouville mechanics



Epistemically restricted statistical theory of bits

Recall: particle mechanics

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Phase space: $\Omega \equiv \mathsf{R}^{2n} \ni (x_1, p_1, x_2, p_2, \ldots, x_n, p_n) \equiv m$

Functionals on phase space: $F : \Omega \to \mathsf{R}$

$$X_i(m) = x_i$$
$$P_i(m) = p_i$$

Recall: particle mechanics

Configuration space: $\mathsf{R}^n \ni (x_1, x_2, \ldots, x_n)$

Phase space: $\Omega \equiv \mathsf{R}^{2n} \ni (x_1, p_1, x_2, p_2, \ldots, x_n, p_n) \equiv m$

Functionals on phase space: $F : \Omega \to \mathsf{R}$

$$X_i(m) = x_i$$
$$P_i(m) = p_i$$

Poisson bracket of functionals:

$$[F, G](m) \equiv \sum_{i=1}^{n} \left(\frac{\partial F}{\partial X_i}\frac{\partial G}{\partial P_i} - \frac{\partial F}{\partial P_i}\frac{\partial G}{\partial X_i}\right)(m)$$

"bit mechanics" $Z_2 = \{0, 1\}$

Configuration space: $(Z_2)^n \ni (x_1, x_2, \ldots, x_n)$

Phase space: $\Omega \equiv (Z_2)^{2n} \ni (x_1, p_1, x_2, p_2, \ldots, x_n, p_n) \equiv m$

Functionals on phase space: $F : \Omega \to Z_2$

$$X_k(m) = x_k$$
$$P_k(m) = p_k$$

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Functionals on phase space: $F : \Omega \to Z_2$

$$X_k(m) = x_k$$
$$P_k(m) = p_k$$

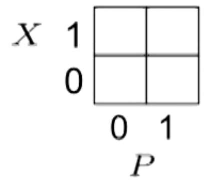
Poisson bracket of functionals:

$$[F,G](m) \equiv \sum_{i=1}^{n} (F[m+e_{x_i}] - F[m])(G[m+e_{p_i}] - G[m]) \\ -(F[m+e_{p_i}] - F[m])(G[m+e_{q_i}] - G[m])$$

The epistemic restriction:

An observer can only have knowledge of the values of a set of canonical variables that commute relative to the Poisson bracket and is maximally ignorant otherwise.

A single bit

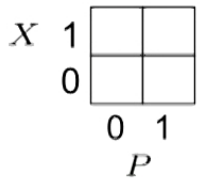


Canonical variables

$$aX + bP \qquad a, b \in \mathbb{Z}_2$$ Addition is mod2

$$X, \quad P, \quad X + P$$

A single bit



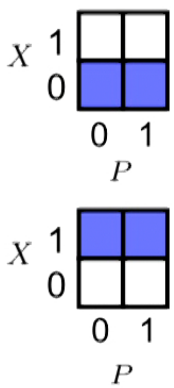
Canonical variables

$$aX + bP \qquad a, b \in \mathrm{Z}_2$$ Addition is mod2

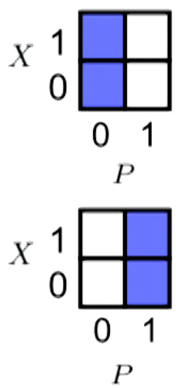
$$X, \quad P, \quad X + P$$

Statistical distributions

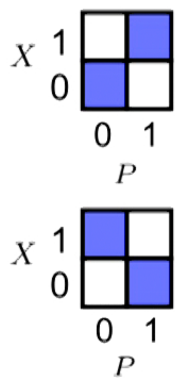
X known



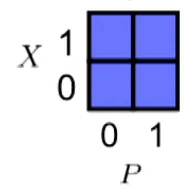
P known



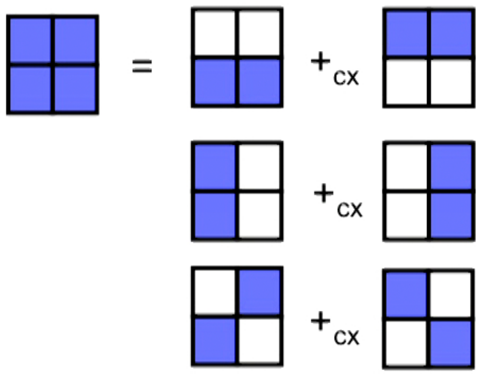
$X + P$ known



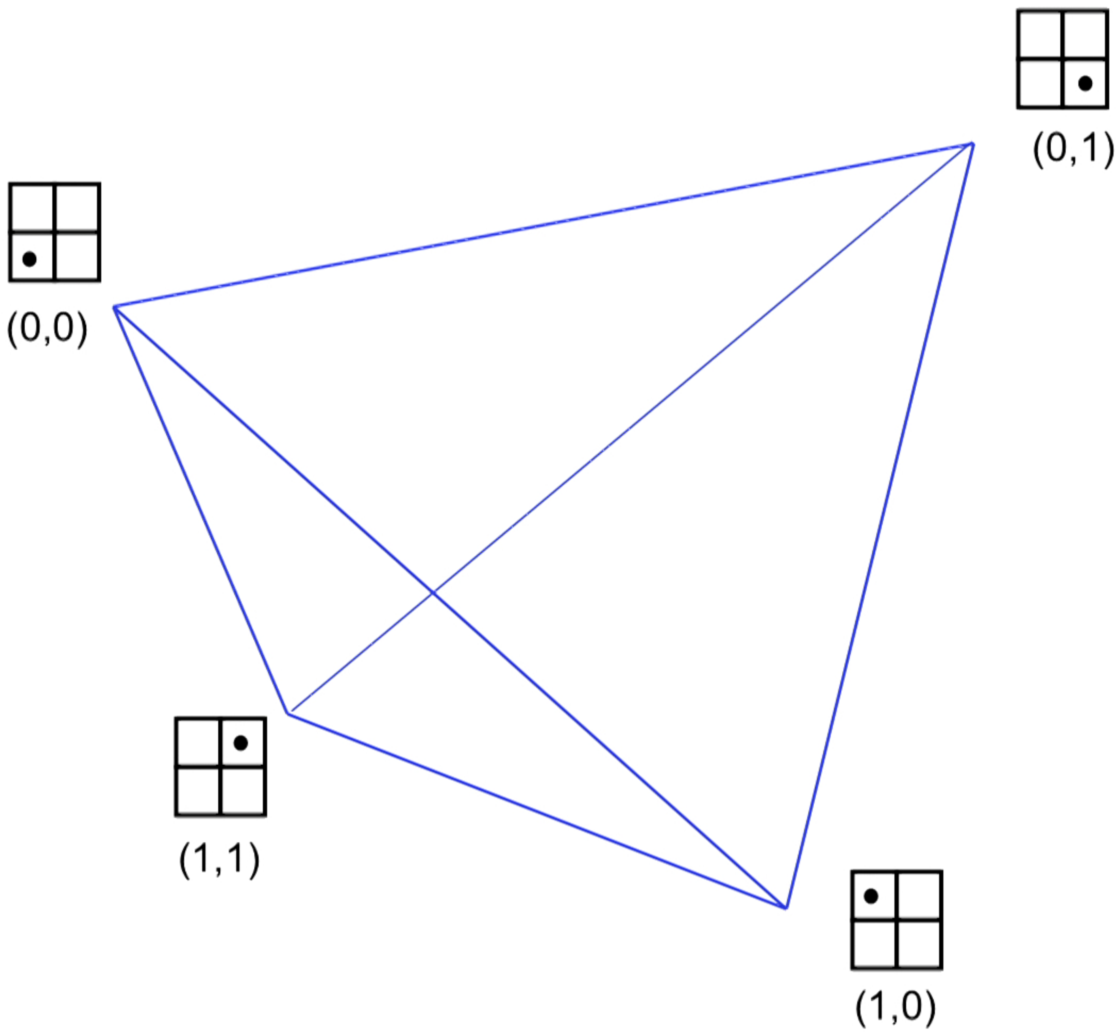
Nothing known

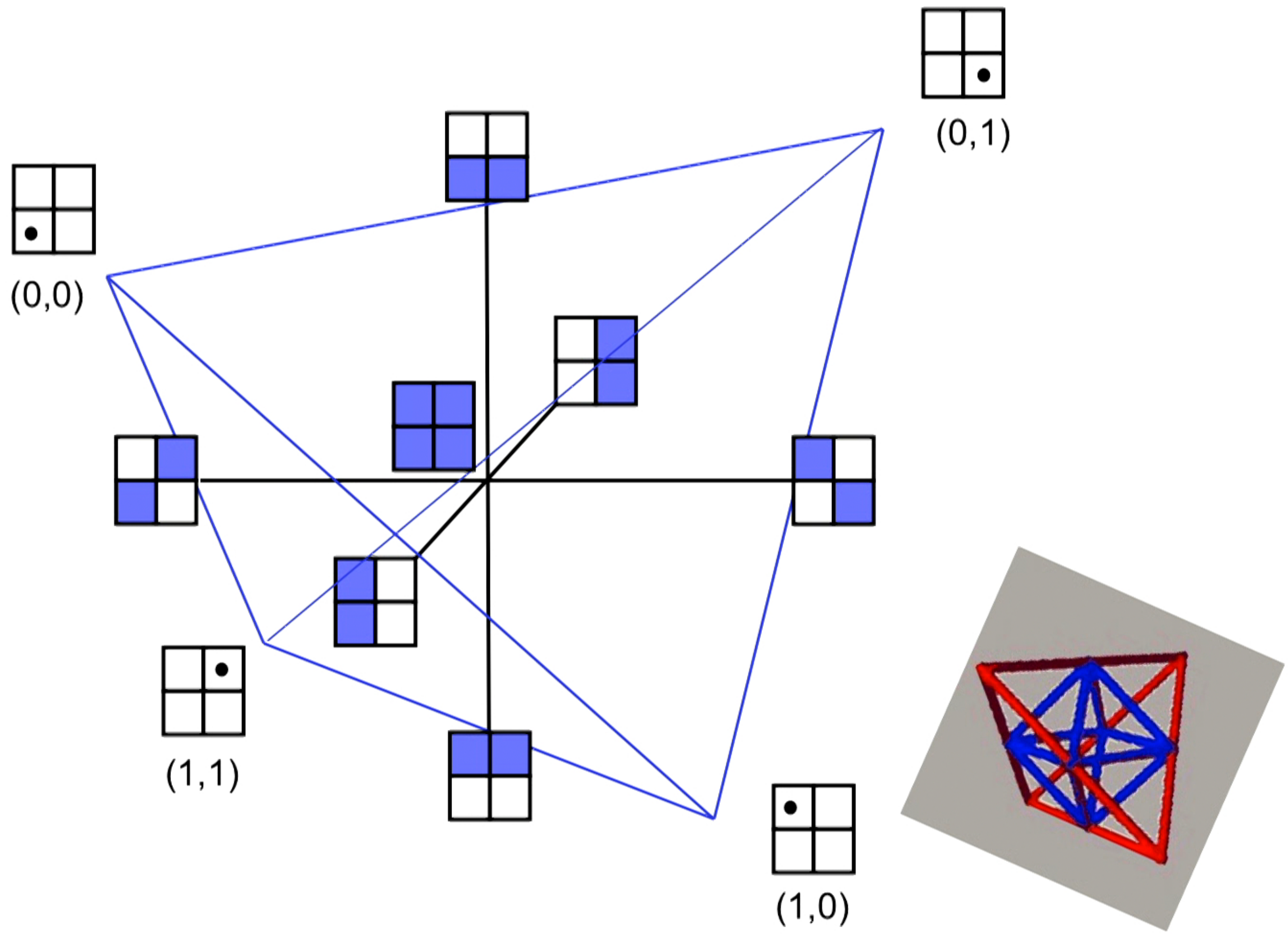


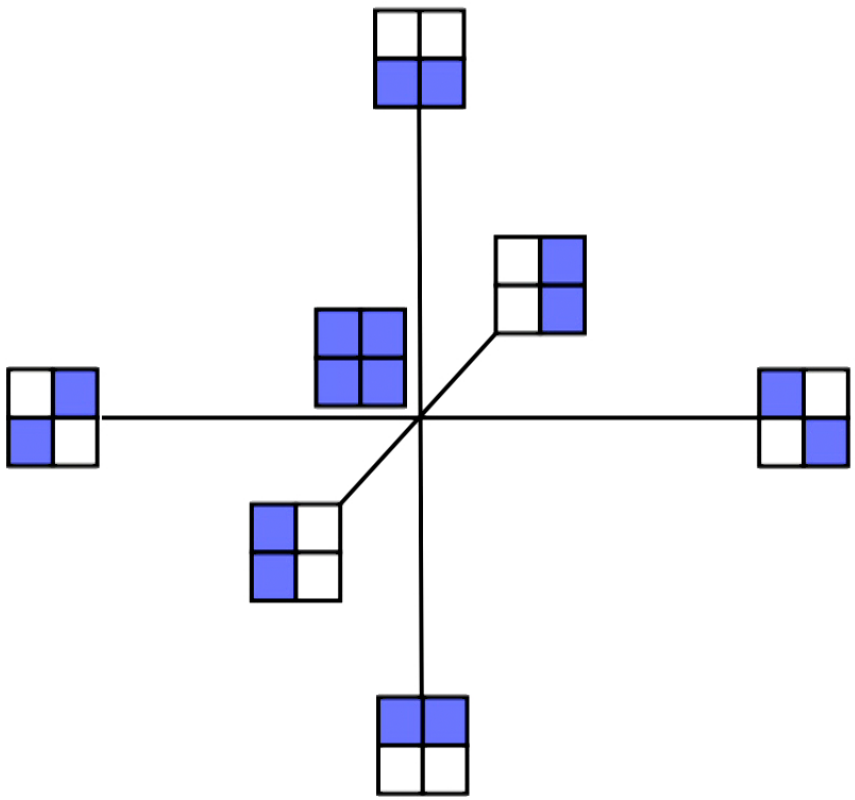
Convex combination

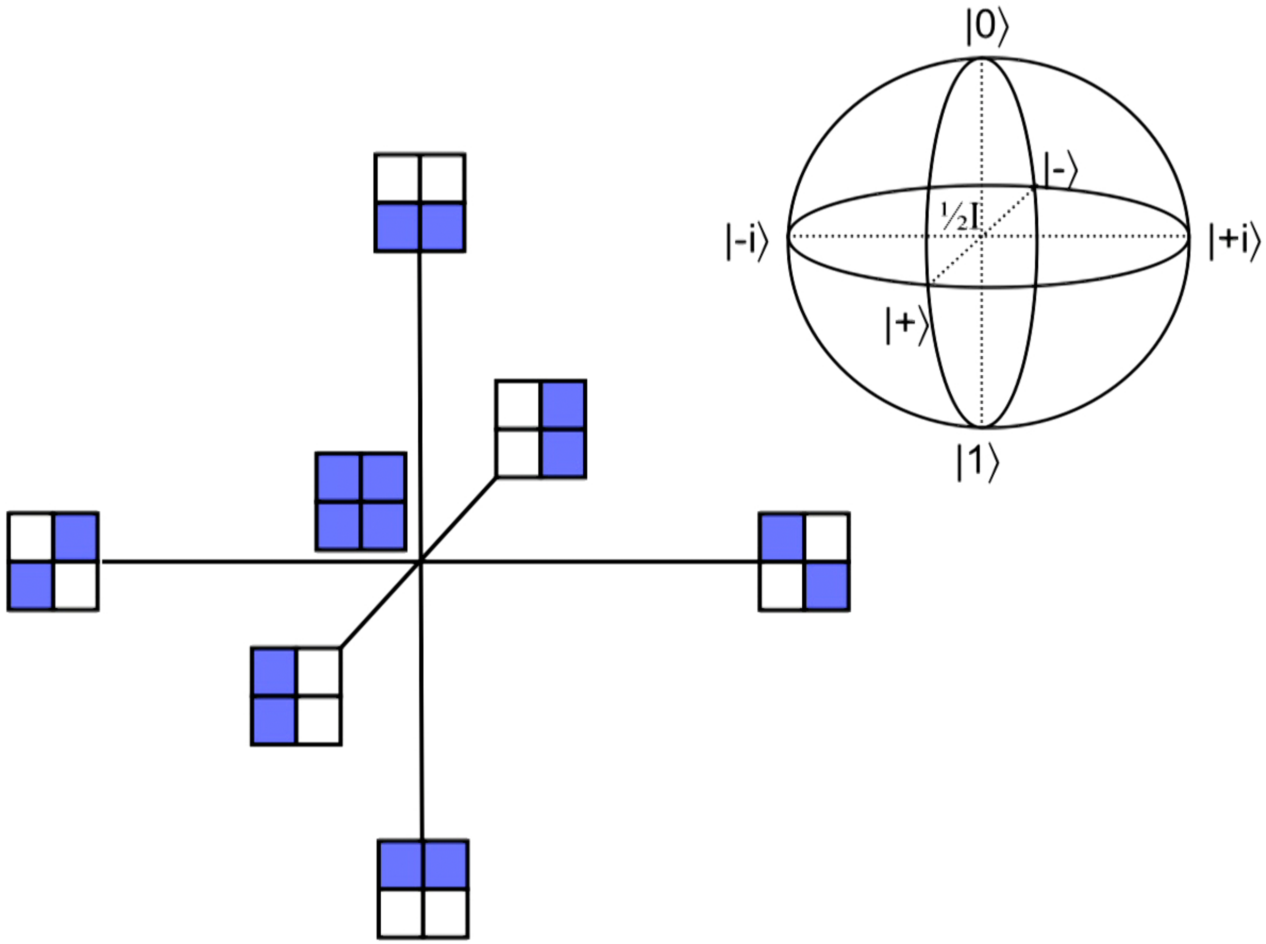


$$\frac{1}{2}I = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$
$$= \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -|$$
$$= \frac{1}{2}|+i\rangle\langle +i| + \frac{1}{2}|-i\rangle\langle -i|$$


(0,1)
(0,0)
(1,1)
(1,0)

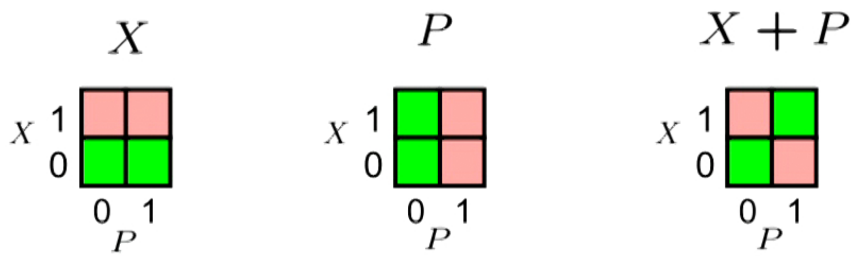

(0,1)
(0,0)
(1,1)
(1,0)

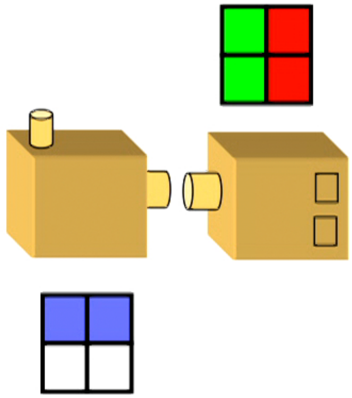


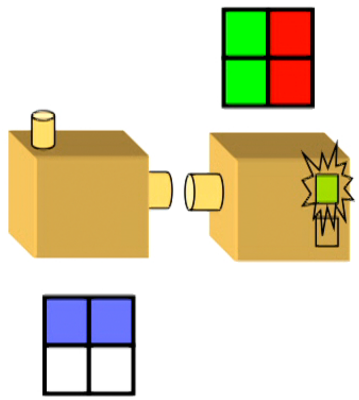

|0⟩
|-⟩
½I
|-i⟩
|+i⟩
|+⟩
|1⟩

Valid reproducible measurements:

Any commuting set of canonical variables







½ of the time

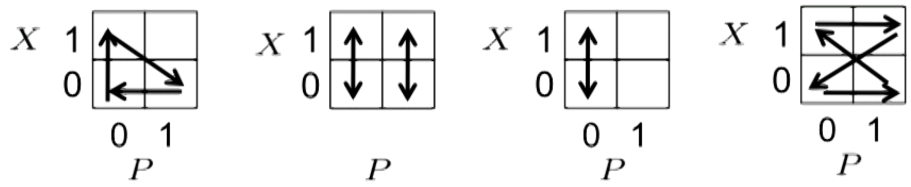
Valid reversible transformations:

The discrete canonical transformations
(those that preserve the discrete Poisson bracket)

Valid reversible transformations:

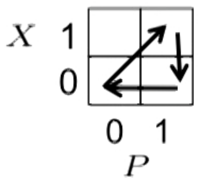
The discrete canonical transformations
(those that preserve the discrete Poisson bracket)

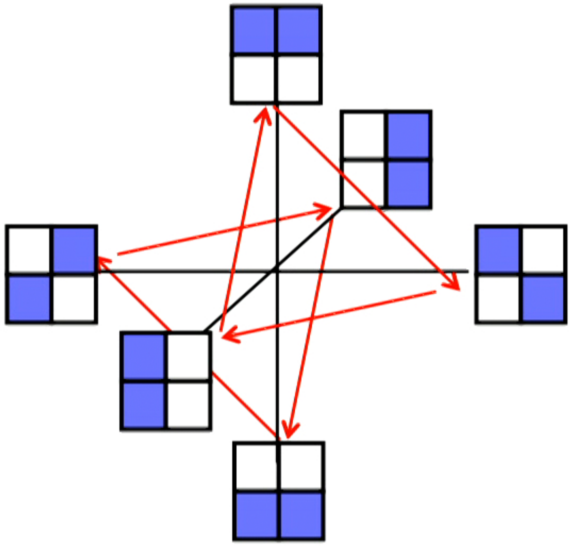
→ All 24 permutations of the four ontic states



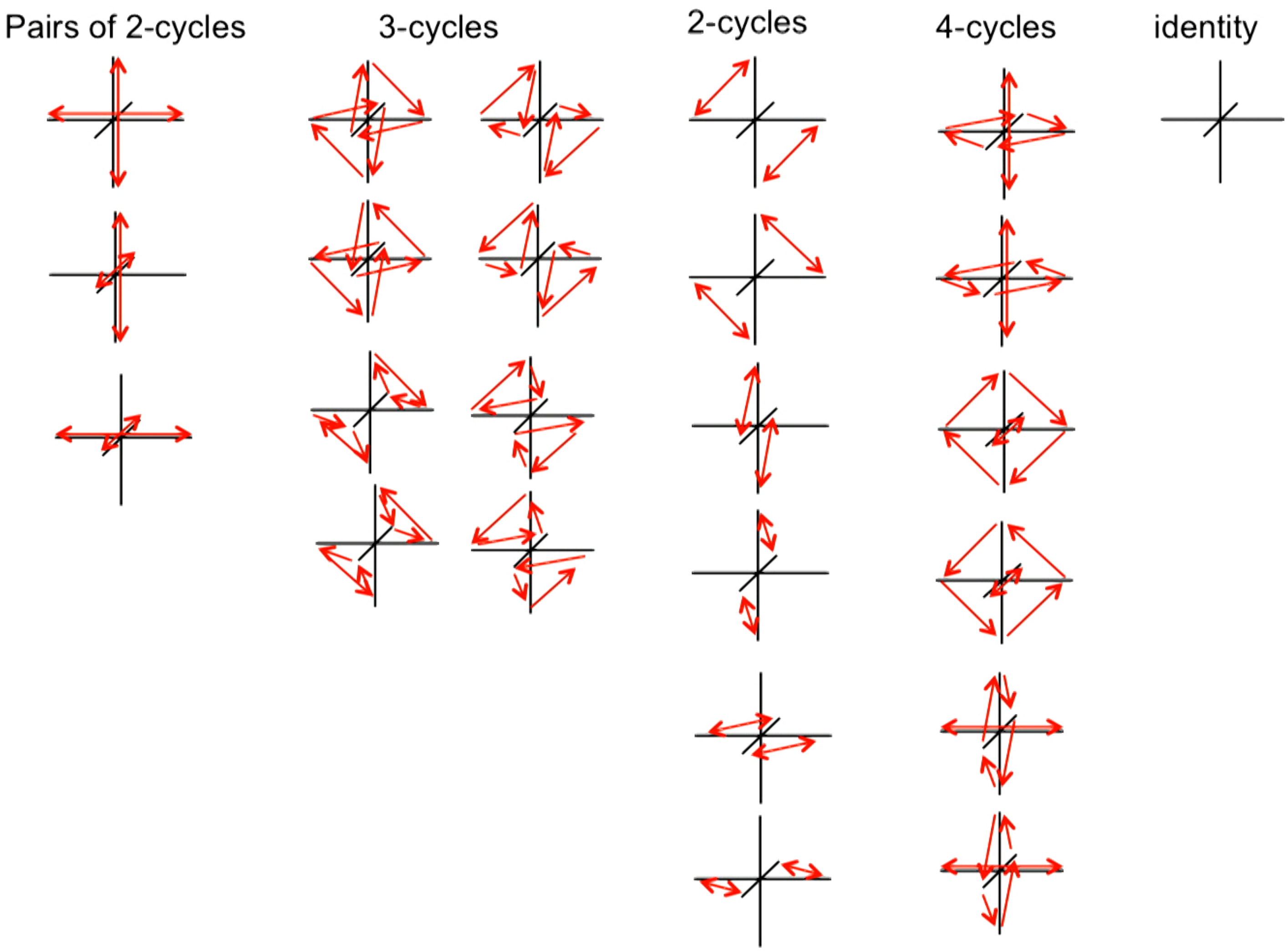
etcetera

A 3-cycle



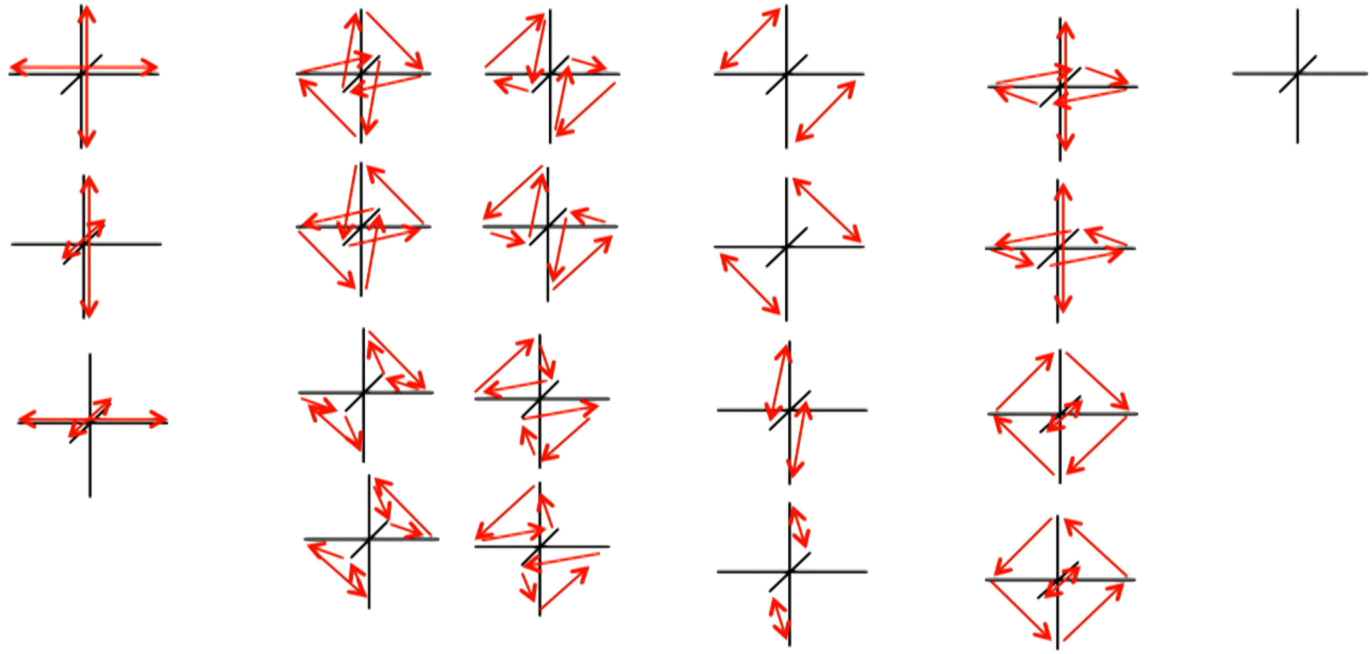


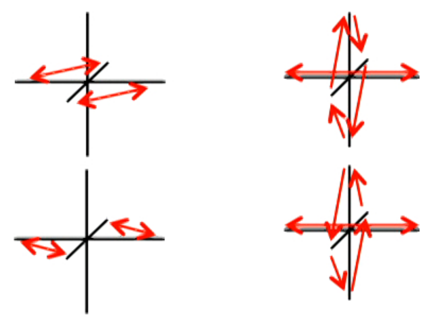
Reversible transformations:

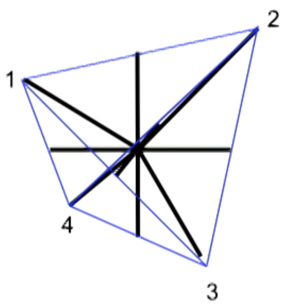


Reversible transformations:

Pairs of 2-cycles | 3-cycles | 2-cycles | 4-cycles | identity







Symmetries of the tetrahedron under rotations and reflections

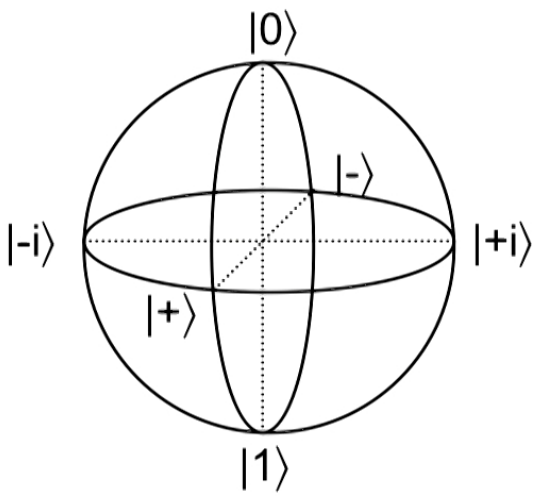
Coherent superposition

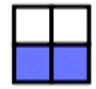
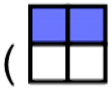
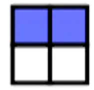
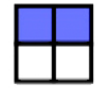
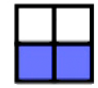
Find U such that $|1\rangle = U|0\rangle$
Find all U' such that $(U')^2 = U$
$U'|0\rangle$ = superpos'n of $|0\rangle$ and $|1\rangle$.

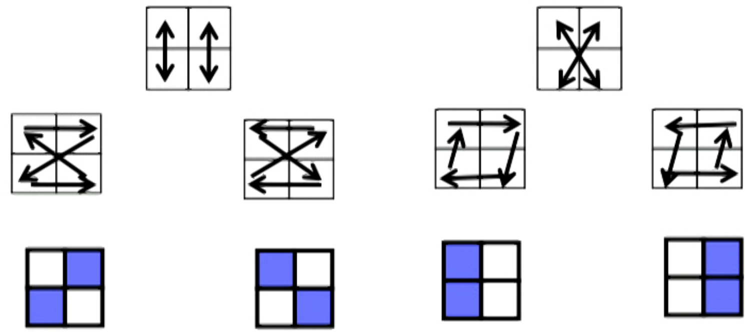
$R_y(\pi)$ $R_x(\pi)$

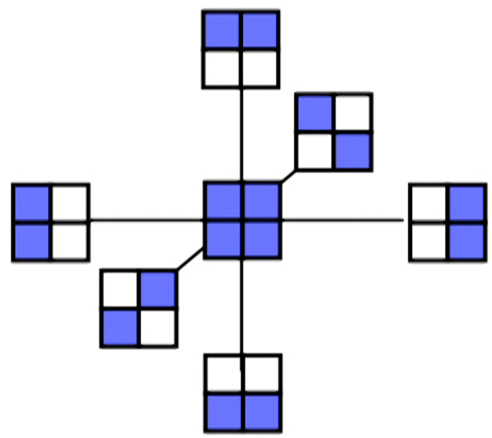
$R_y(-\pi/2)$ $R_y(\pi/2)$ $R_x(\pi/2)$ $R_x(-\pi/2)$

$|+\rangle$ $|-\rangle$ $|+i\rangle$ $|-i\rangle$



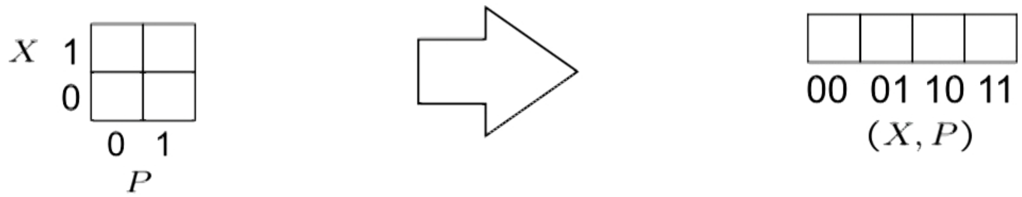
Find P such that  = P()
Find all P' such that $(P')^2 = P$
P'() = superpos'n of  and 

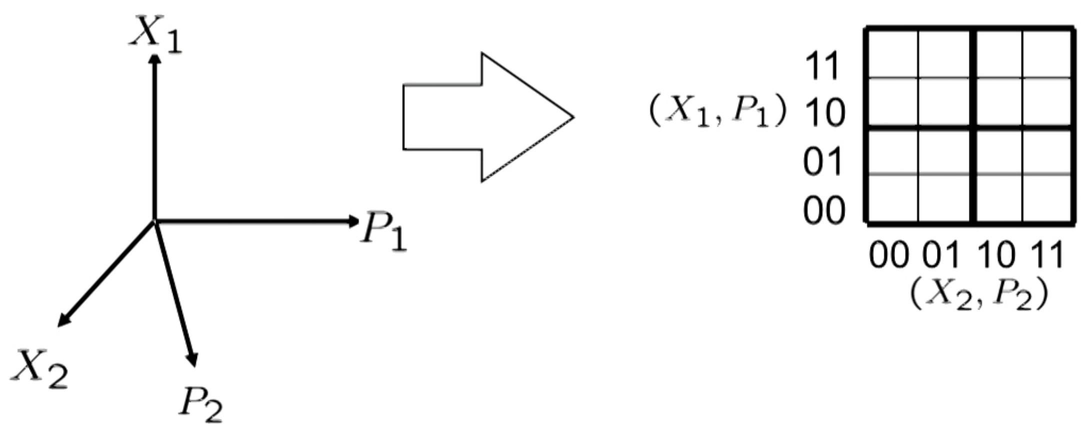




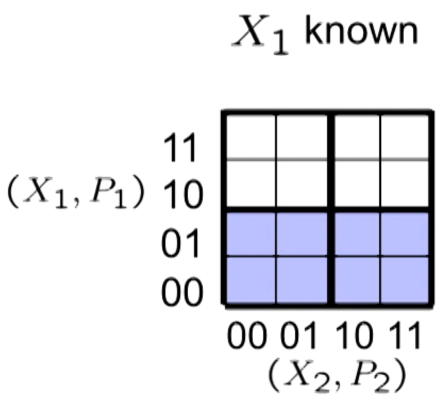
A pair of bits

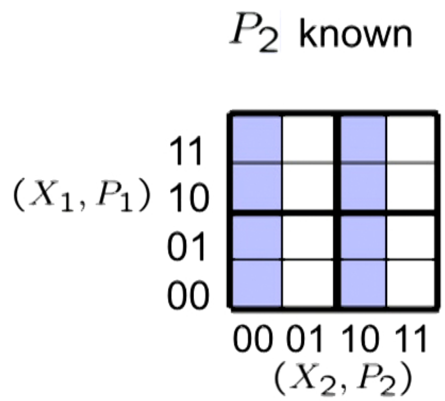
Canonical variables $a_1X_1 + b_1P_1 + a_2X_2 + b_2P_2$ $\quad a_1, b_1, a_2, b_2 \in \mathbb{Z}_2$



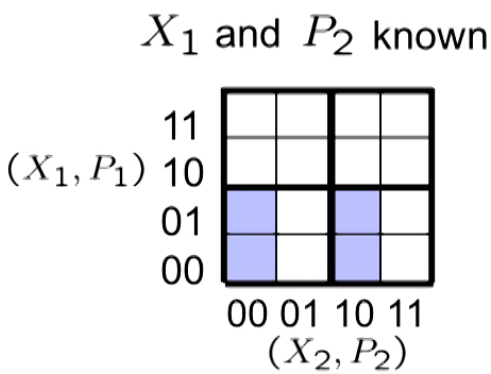


1 variable known



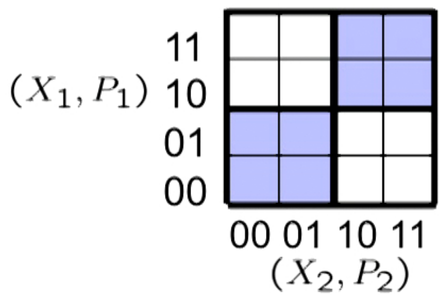


2 variables known

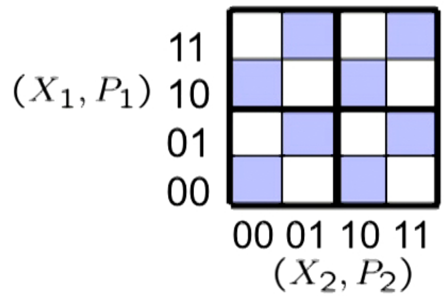


1 variable known

$X_1 + X_2$ known

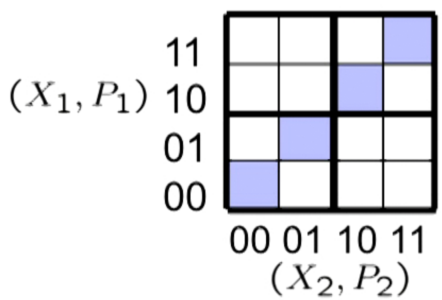


$P_1 + P_2$ known



2 variables known

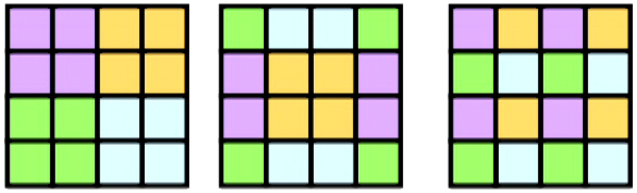
$X_1 + X_2$ and $P_1 + P_2$ known



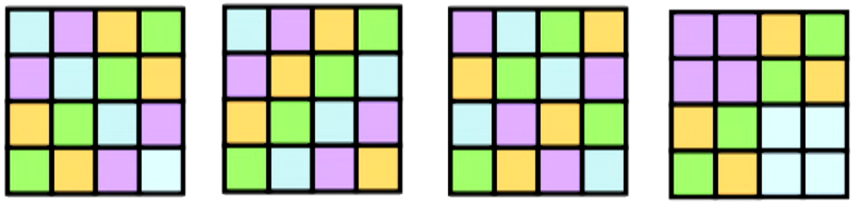
$$\frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$$

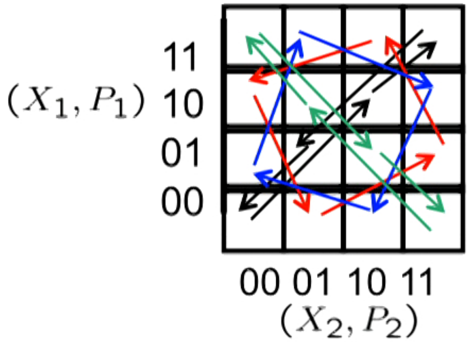
Maximally informative reproducible measurements

Product basis Measurements



Entangled basis Measurements




11
(X_1, P_1) 10
01
00
00 01 10 11
(X_2, P_2)

The way to understand EPR steering and the collapse rule are precisely analogous to how it is done in epistemically restricted Liouville mechanics

Teleportation

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle \pm |1\rangle|1\rangle)$$
$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle \pm |1\rangle|0\rangle)$$