Title: PSI 17/18 - Foundations of Quantum Mechanics - Lecture 8

Date: Feb 07, 2018 10:15 AM

URL: http://pirsa.org/18020067

Abstract:

Chris Fuchs.

$$\mathrm{tr}\,\rho^2 = \mathrm{tr}\,\rho^3 = 1$$

ρ Hermitean

$$① \quad \sum_i \lambda_i^2 = 1 \qquad ② \quad \sum_i \lambda_i^3 = 1$$

$$① \Rightarrow \quad |\lambda_i| \leq 1 \quad \Rightarrow \quad 1 - \lambda_i \geq 0$$

$① - ②$

$$\sum_i \underbrace{\lambda_i^2}_{\geq 0} \underbrace{(1 - \lambda_i)}_{\geq 0} = 0 \quad \Rightarrow \quad \lambda_i = 1 \text{ or } 0$$

eg ① now

$\rho = \mathrm{tr}\,\rho$ =

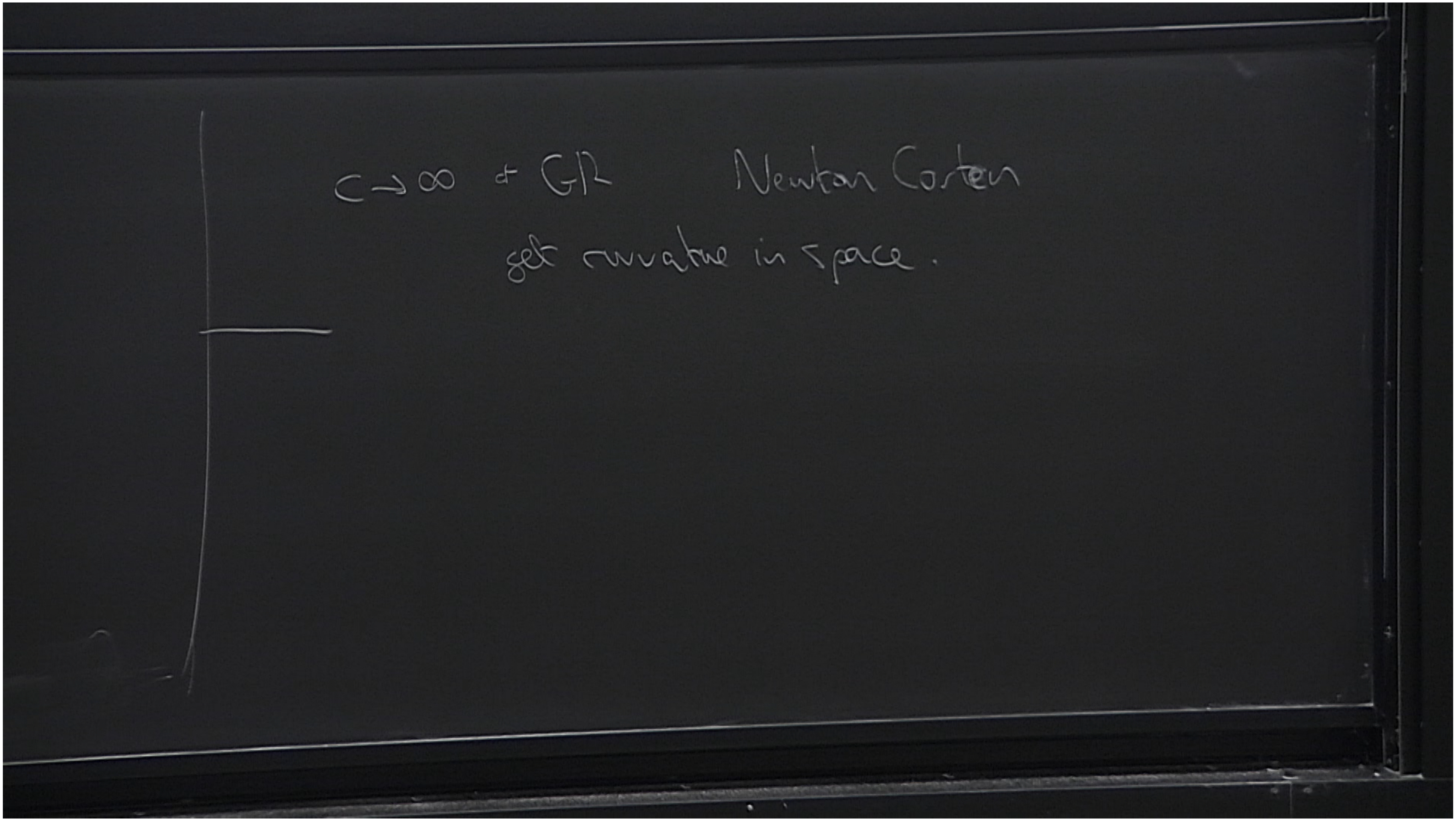
Hemitean

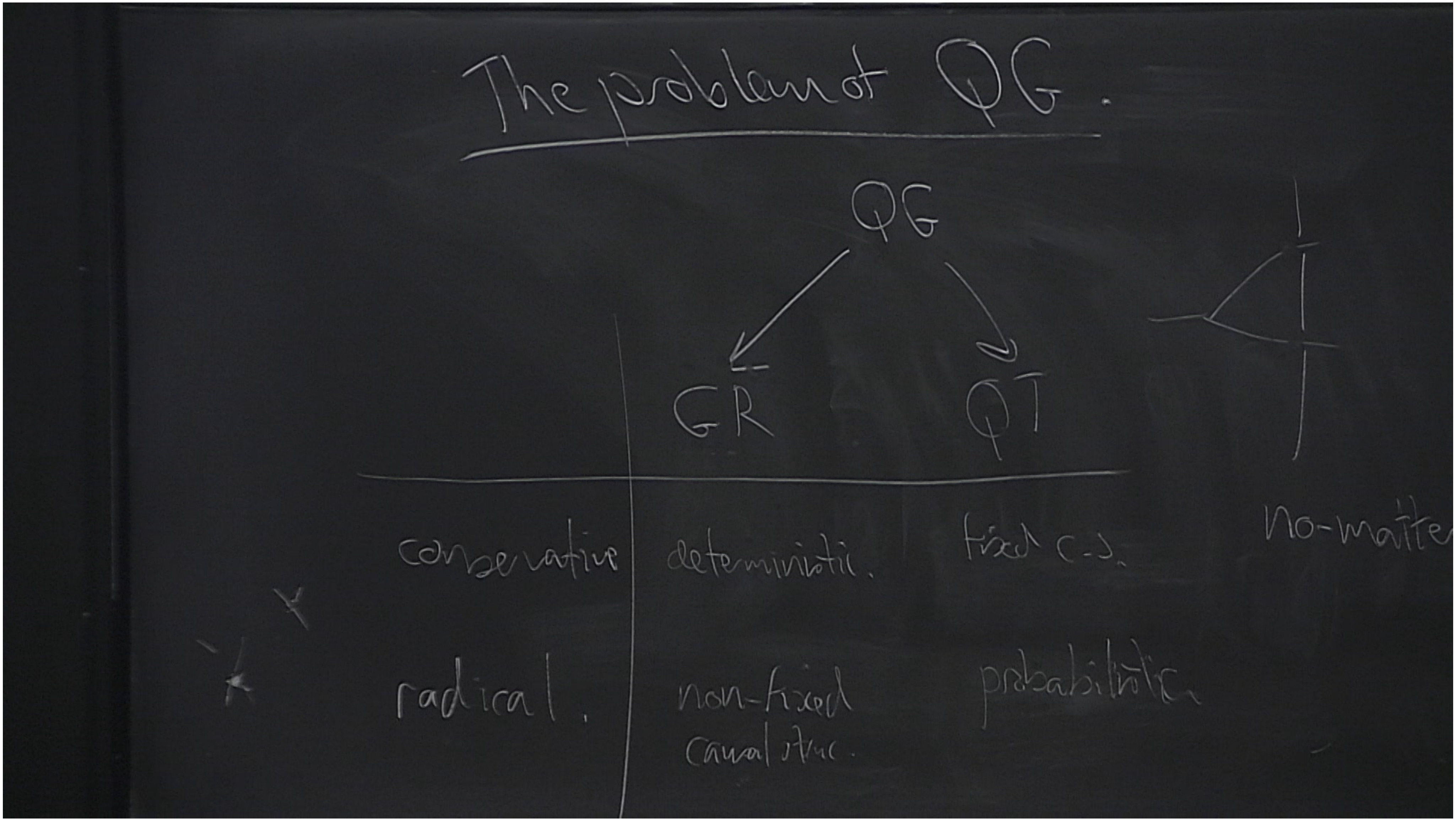
$\lambda_i^2 = 1$ ② $\sum_i \lambda_i^3 = 1$

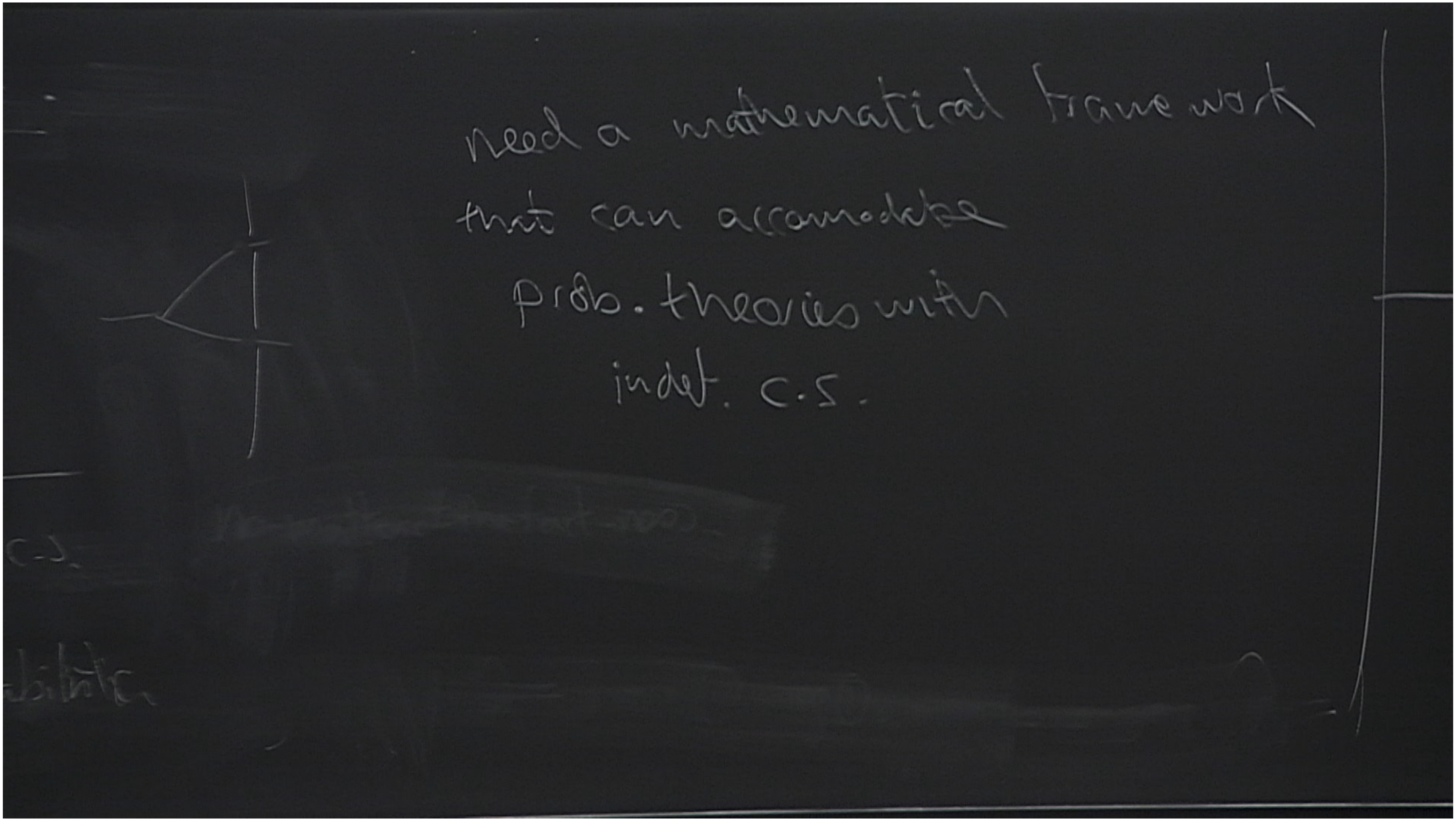
$| \leq 1 \Rightarrow 1 - \lambda_i \geq 0$

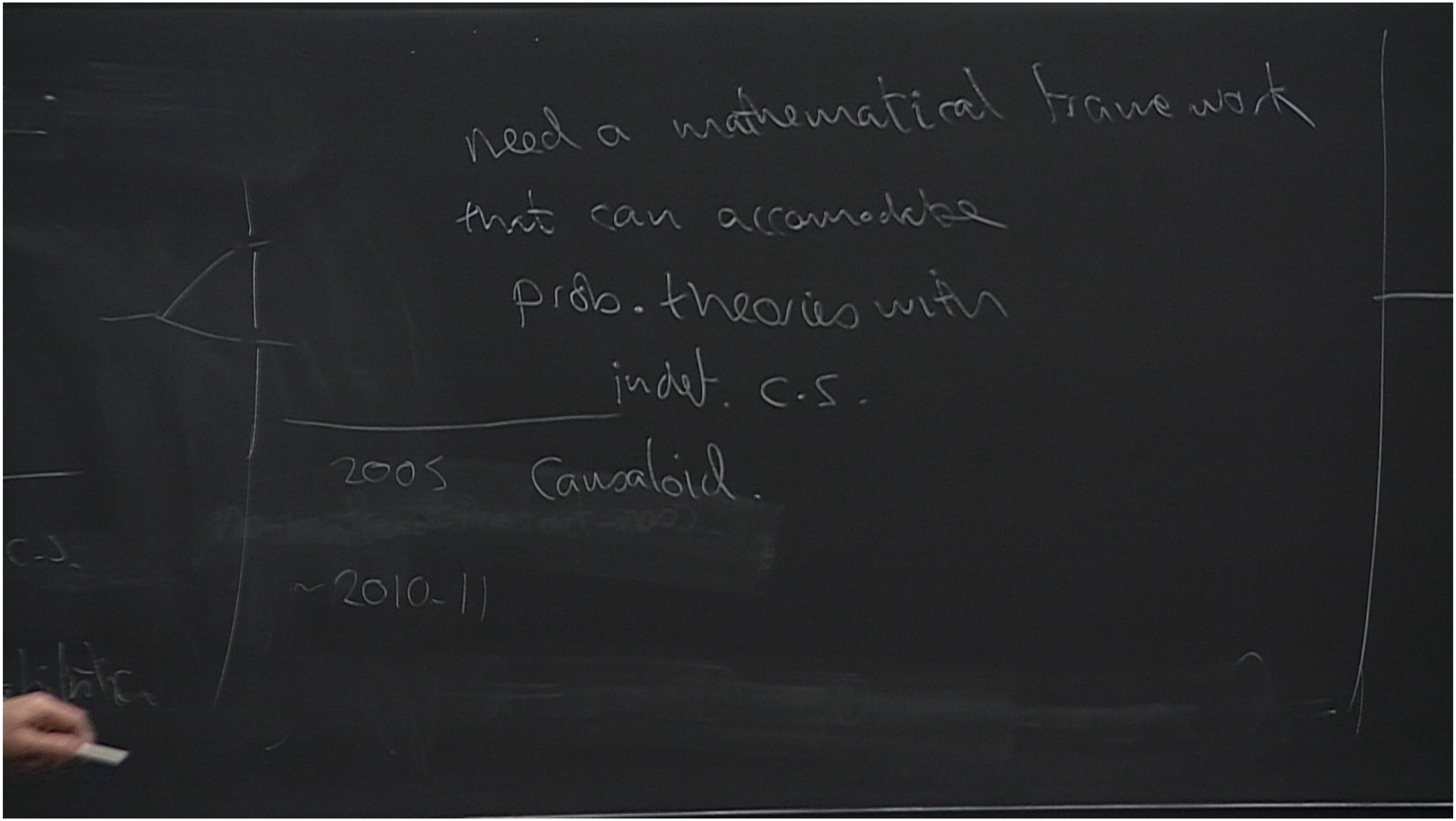
$(1-\lambda_i) = 0 \Rightarrow \lambda_i = 1$ or 0 eq ① now gives only one $\lambda_i = 1$

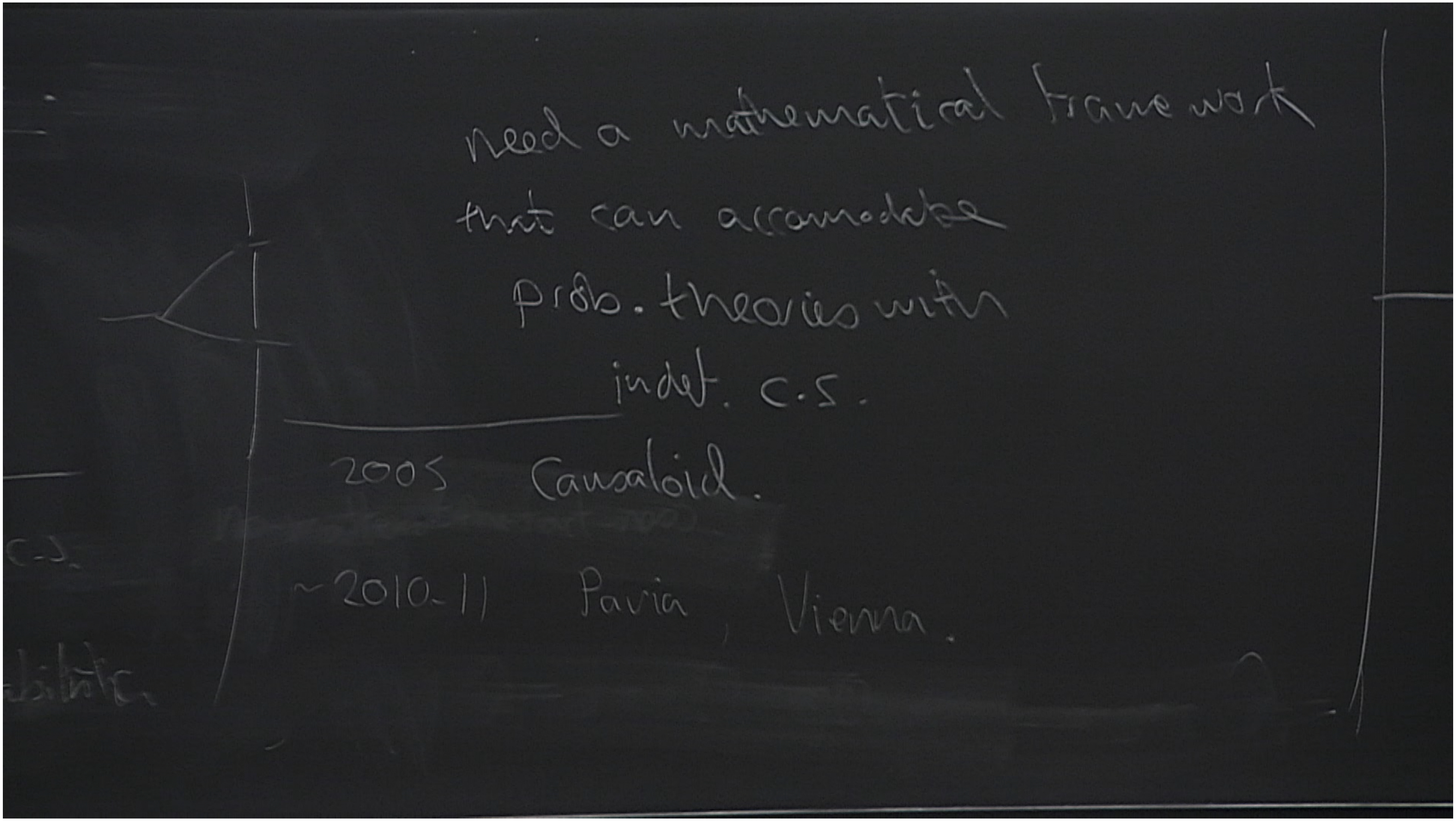
≥ 0

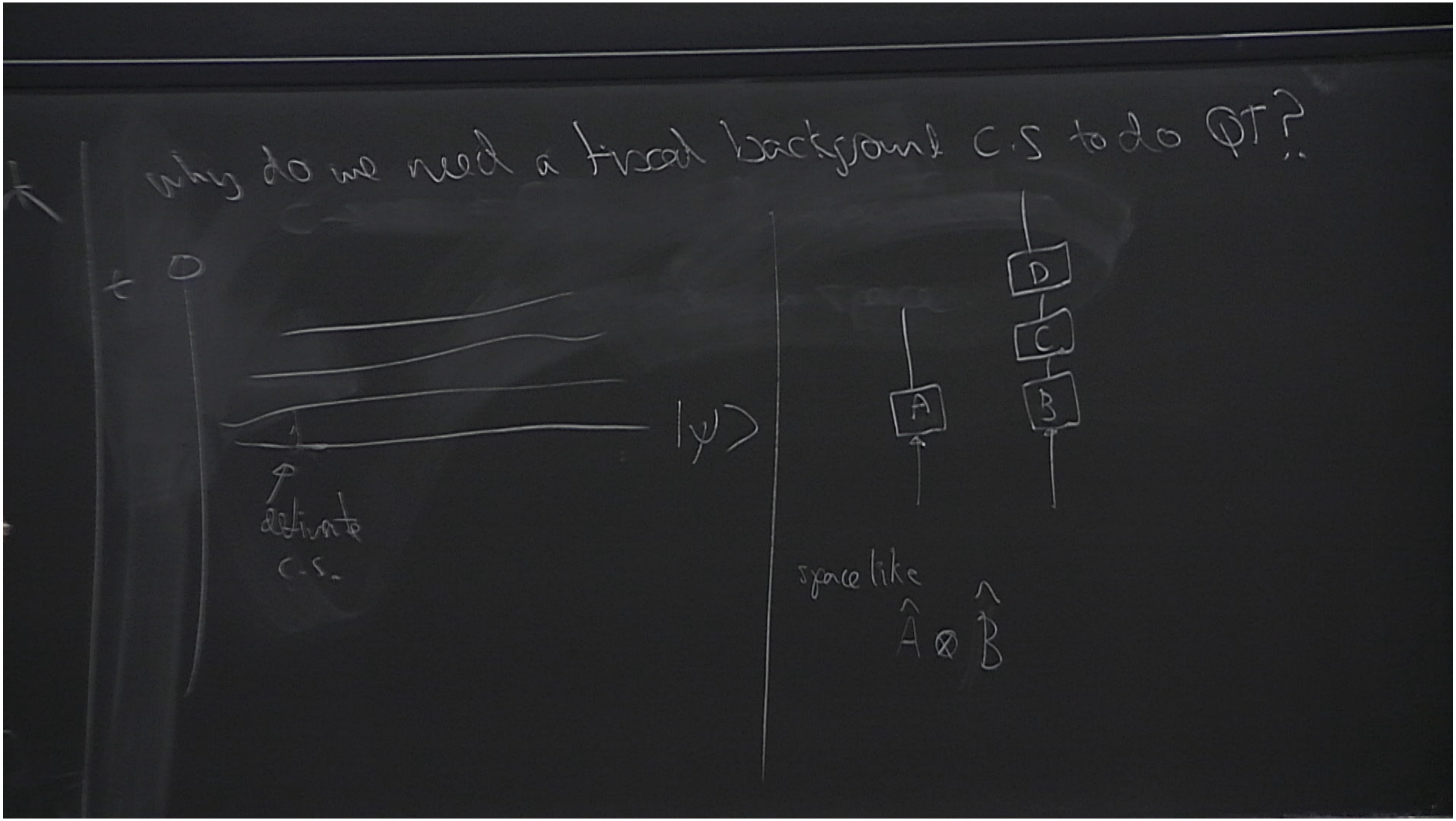

c → ∞ of GR
Newton Cartan
get curvature in space.

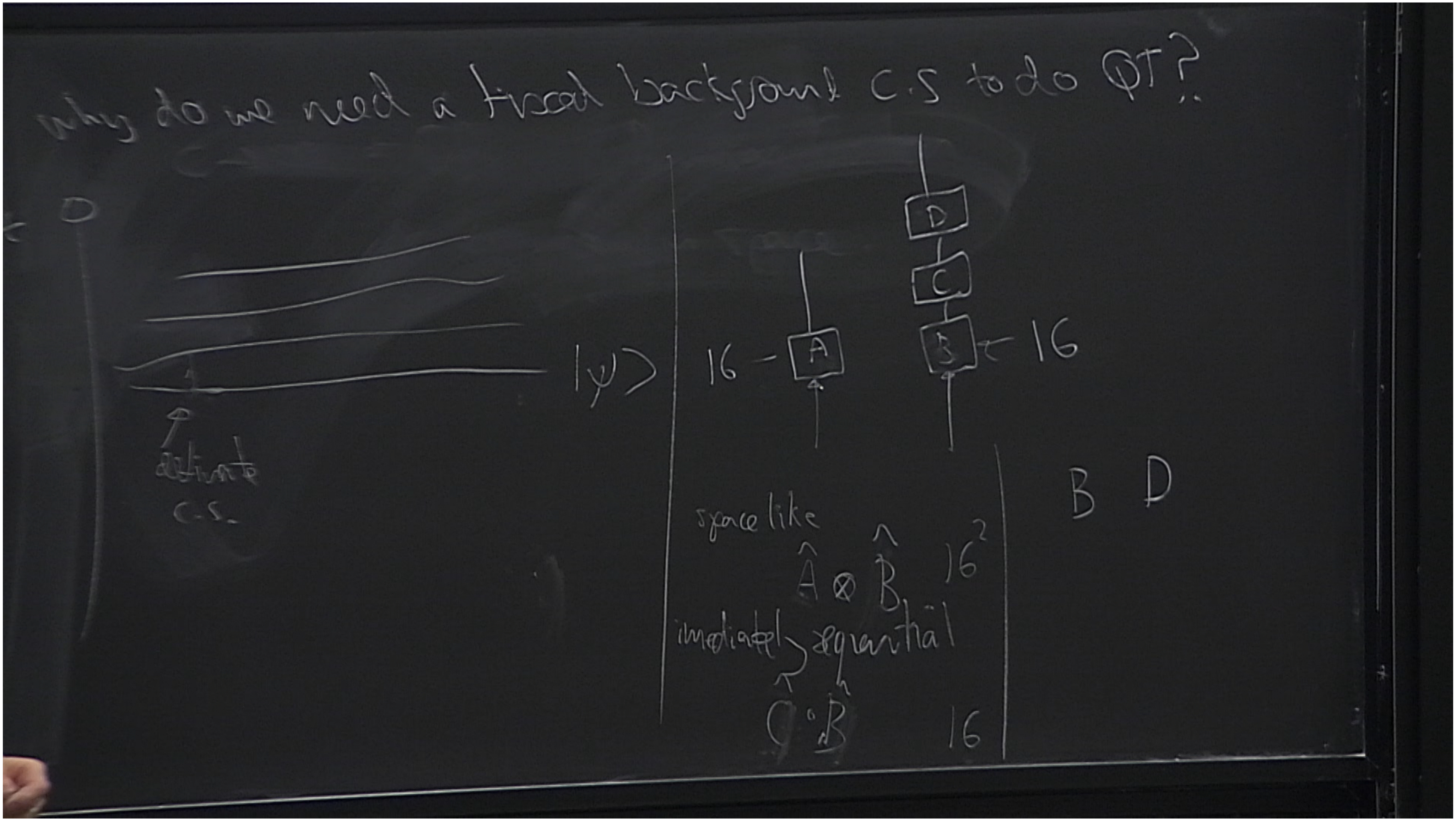

The problem of QG.
QG
GR
QT
conservative
deterministic.
fixed c.s.
no-matte
radical.
non-fixed
causal struc.
probabilistic

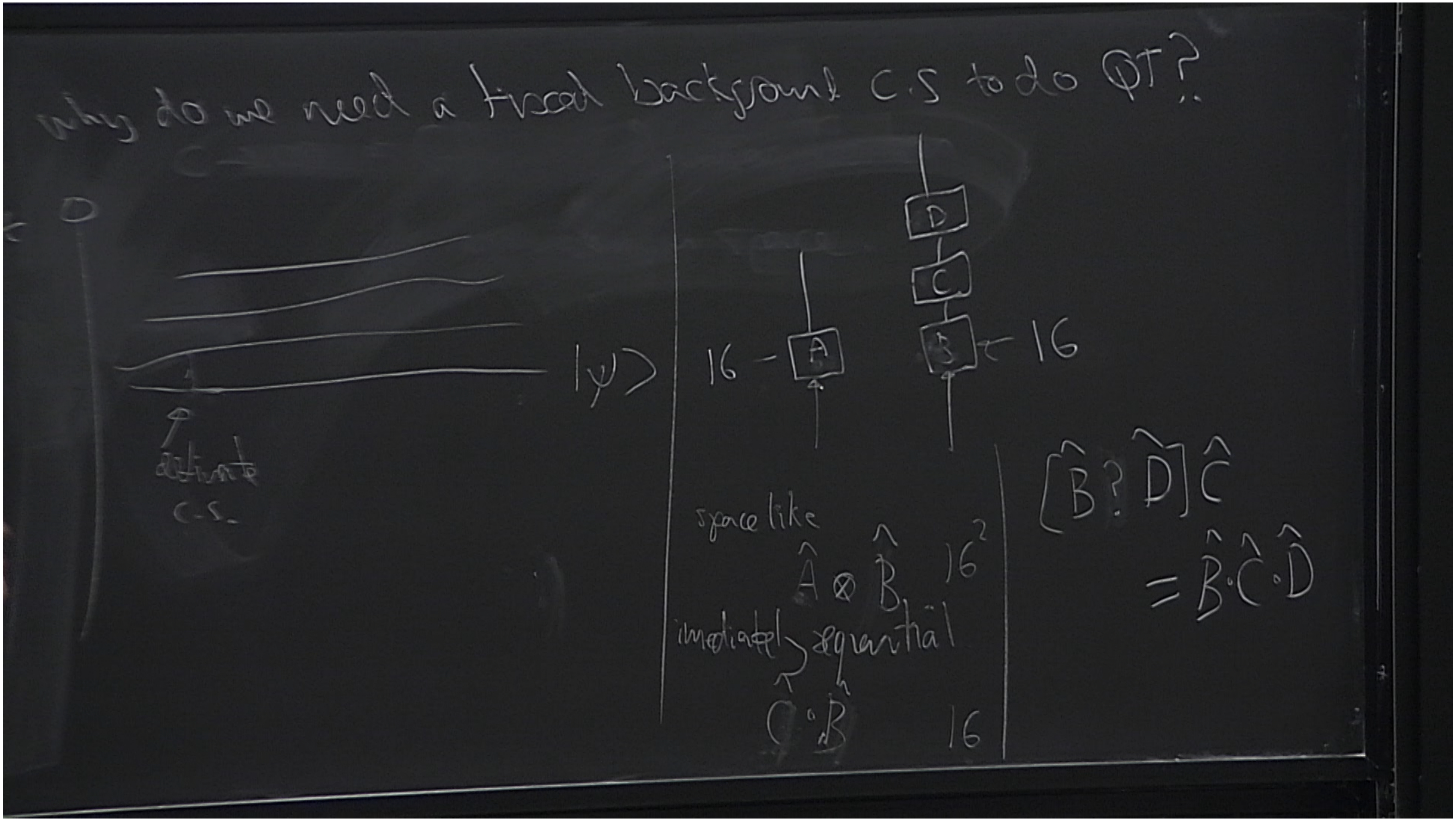

need a mathematical framework
that can accomodate
prob. theories with
indet. c.s.


need a mathematical framework
that can accomodate
prob. theories with
indef. c.s.
2005 Causaloid.
~2010-11

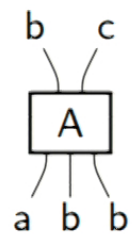

need a mathematical framework
that can accomodate
prob. theories with
indef. c.s.
2005 Causaloid.
~2010-11 Pavia, Vienna.


why do we need a fixed background C.S to do QT?
t
|ψ⟩
activate
C.S.
D
C
A
B
space like
Â ⊗ B̂


why do we need a fixed background C.S to do QT?
|ψ>
16
A
D
C
B
16
B D
C.S.
space like
Â ⊗ B̂
16²
sequential
16


why do we need a fixed background C.S to do QT?
|ψ⟩
16
16
space like
Â ⊗ B̂
16²
immediately sequential
Ĉ ∘ B̂
16
[B̂ ? D̂] Ĉ
= B̂ ∘ Ĉ ∘ D̂
C.S.

Operations - take 1



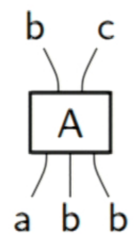
$$\Longleftrightarrow \qquad A^{b_4 c_5}_{a_1 b_2 b_3}$$

An operation, A, corresponds to one use of an apparatus and has the following features.

- *Inputs and outputs.* Come in various types, a, b, ... (e.g. electrons, photons, ...).
- *An outcome*, x_A.

If the outcome is x_A then we say operation A "happened".

Operations - take 2 (with coarse graining)



$$\Longleftrightarrow \qquad A_{a_1 b_2 b_3}^{b_4 c_5}$$

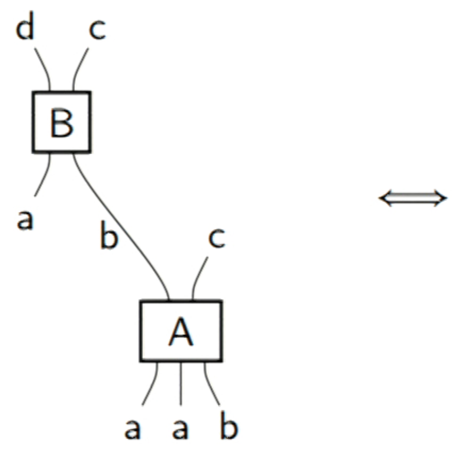
An operation, A, corresponds to one use of an apparatus and has the following features.

- *Inputs and outputs.* Come in various types, a, b, ... (e.g. electrons, photons, ...).
- *An outcome set*, o(A).

If $x_A \in o(A)$ then we say operation A "happened".

Wires

Outputs can be connected to inputs by wires.



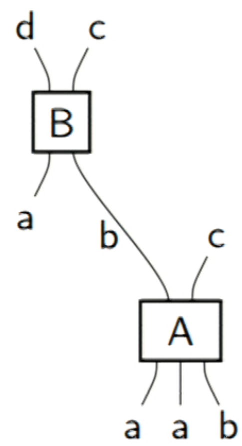
$$A^{b_4 c_5}_{a_1 a_2 b_3} B^{d_6 c_7}_{a_5 b_4}$$

Wiring rules.

- *One wire:* At most one wire can be connected to any given input or output.
- *Type matching:* Wires can connect inputs and outputs of the same type.
- *No closed loops.*

Wires

Outputs can be connected to inputs by wires.



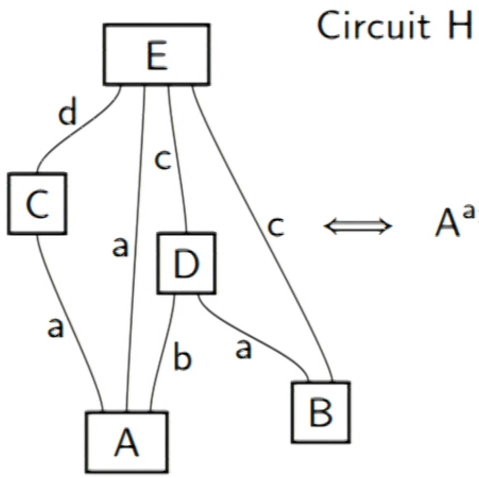
$$\iff \qquad A^{b_4 c_5}_{a_1 a_2 b_3} B^{d_6 c_7}_{a_5 b_4}$$

Wiring rules.

- *One wire:* At most one wire can be connected to any given input or output.
- *Type matching:* Wires can connect inputs and outputs of the same type.
- *No closed loops.*

Circuits

Circuits have no open inputs or outputs.



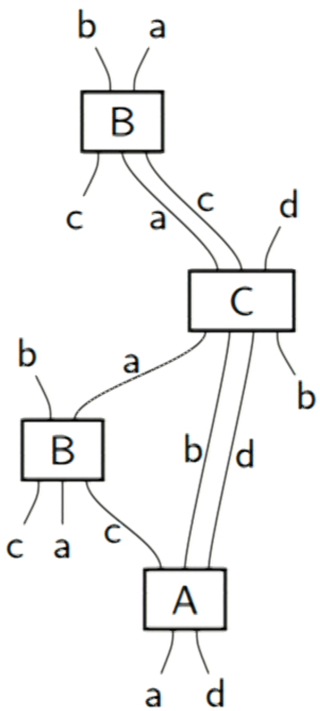
$$\iff A^{a_1 a_2 b_3} B^{a_4 c_7} C^{d_5}_{a_1} D^{c_6}_{b_3 a_4} E_{d_5 a_2 c_6 c_7}$$

Have outcome set $o(A) \times o(B) \times o(C) \times o(D) \times o(E)$

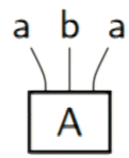
We say the circuit "happens" if the outcome is in outcome set.

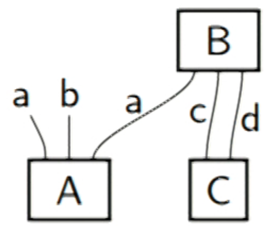
Fragments of a circuit

Fragments can have open inputs and outputs



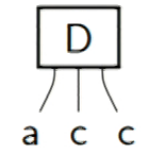
Preparations have only open outputs

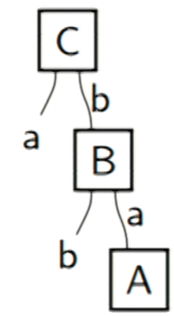




Associated with *states*.

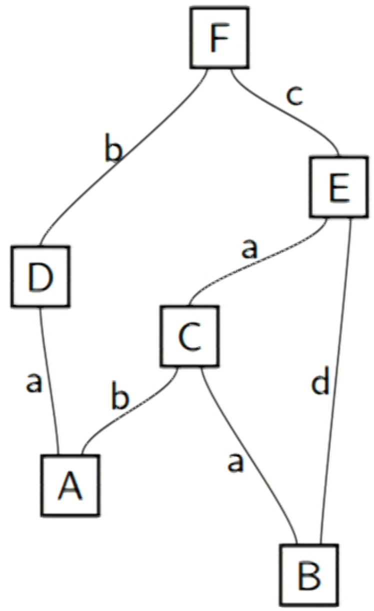
Results have only open inputs





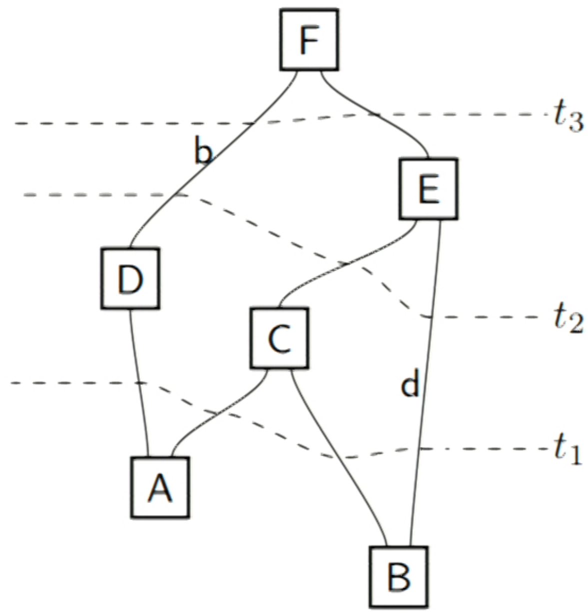
Associated with *effects*.

Consider a circuit



How do we calculate the probability for this circuit in standard framework of QT?

We foliate



then

$$\mathsf{Prob}(\mathsf{A}^{\mathsf{a}_1\mathsf{b}_2}\mathsf{B}^{\mathsf{a}_3\mathsf{d}_4}\mathsf{C}^{\mathsf{a}_5}_{\mathsf{b}_2\mathsf{a}_3}\mathsf{D}^{\mathsf{b}_6}_{\mathsf{a}_1}\mathsf{E}^{\mathsf{c}_7}_{\mathsf{a}_5\mathsf{d}_4}\mathsf{F}_{\mathsf{b}_6\mathsf{c}_7}) = \mathsf{Trace}\Big(\hat{P}_{\mathsf{F}}[I_{\mathsf{b}}\otimes\$_{\mathsf{E}}]\circ[\$_{\mathsf{D}}\otimes\$_{\mathsf{C}}\otimes I_{\mathsf{d}}](\hat{\rho}_{\mathsf{A}}\otimes\hat{\rho}_{\mathsf{B}})\Big)$$

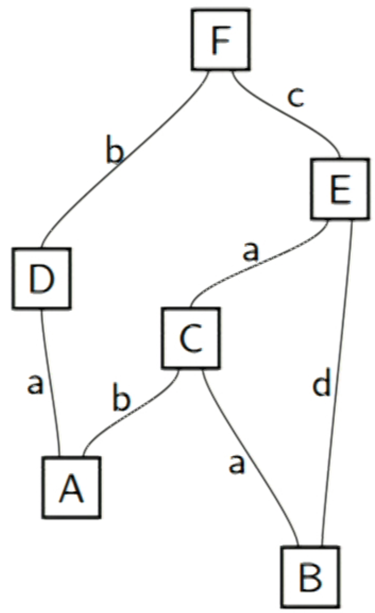
$$\text{Prob}(A^{a_1 b_2} B^{a_3 d_4} C^{a_5}_{b_2 a_3} D^{b_6}_{a_1} E^{c_7}_{a_5 d_4} F_{b_6 c_7}) = \text{Trace}\Big(\hat{P}_F [I_b \otimes \$_E] \circ [\$_D \otimes \$_C \otimes I_d](\hat{\rho}_A \otimes \hat{\rho}_B)\Big)$$

Problems

1. We need to choose an arbitrary foliation even though this is not part of the physics.
2. We have to pad the calculation with identity maps (such as I_d) whenever two or more foliation lines intersect a given wire.
3. Our treatment is disunified in that we use different types of mathematical object for preparations (positive operator having trace less than or equal to one), transformations (trace non-increasing completely positive map), and results (positive operator less than or equal to the identity).

We will remove all these problems in the operator tensor formulation.

Consider again the circuit



$$A^{a_1 b_2} B^{a_3 d_4} C^{a_5}_{b_2 a_3} D^{b_6}_{a_1} E^{c_7}_{a_5 d_4} F_{b_6 c_7}$$

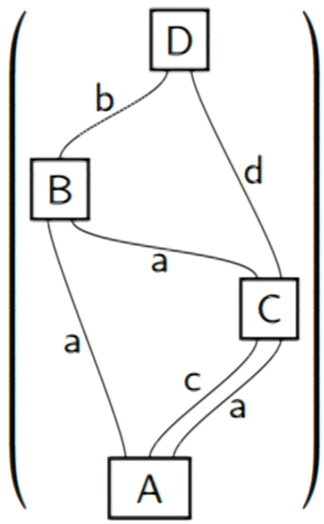
In the operator tensor formulation

$$\mathrm{Prob}(A^{a_1 b_2} B^{a_3 d_4} C^{a_5}_{b_2 a_3} D^{b_6}_{a_1} E^{c_7}_{a_5 d_4} F_{b_6 c_7}) = \hat{A}^{a_1 b_2} \hat{B}^{a_3 d_4} \hat{C}^{a_5}_{b_2 a_3} \hat{D}^{b_6}_{a_1} \hat{E}^{c_7}_{a_5 d_4} \hat{F}_{b_7 c_7}$$

We will explain what the RHS means later.

Introducing probabilities

Assump 1 *We can associate a probability with any given circuit (the probability that the circuit "happens"), and this probability depends only on the specification of the given circuit (the knob settings and outcome sets at the operations, and the wiring).*

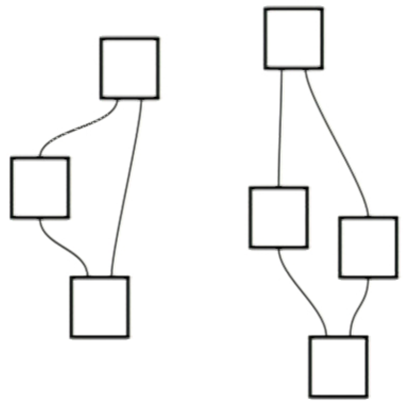
Prob  is well conditioned

Probabilities factorize over circuits

It follows from Assumption 1 that

$$\text{Prob}(AB) = \text{Prob}(A)\text{Prob}(B)$$

for circuits A and B



The $p(\cdot)$ function

We define the function $p(\cdot)$ as follows

$$p(\alpha\mathsf{A} + \beta\mathsf{B} + \dots) := \alpha\mathsf{Prob}(\mathsf{A}) + \beta\mathsf{Prob}(\mathsf{B}) + \dots$$

for *circuits* A, B, ... and real numbers α, β, . . . (these can be negative).

Will use this to define a notion of equivalence.

Example of equivalence

Have

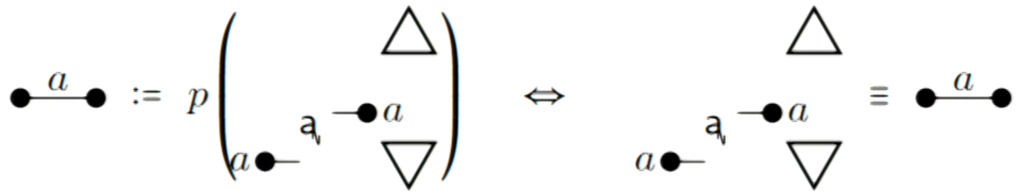
$$\alpha A^{a_1} + \beta B^{a_1} \equiv \gamma C^{a_1} + \delta D^{a_1}$$

if

$$p([\alpha A^{a_1} + \beta B^{a_1}]E_{a_1}) = p([\gamma C^{a_1} + \delta D^{a_1}]E_{a_1}) \quad \text{for all } E_{a_1}$$

The hopping metric

We define *the hopping metric*



The $p(\cdot)$ function

We define the function $p(\cdot)$ as follows

$$p(\alpha \mathsf{A} + \beta \mathsf{B} + \dots) := \alpha \mathrm{Prob}(\mathsf{A}) + \beta \mathrm{Prob}(\mathsf{B}) + \dots$$

for *circuits* A, B, ... and real numbers α, β, . . . (these can be negative).

Will use this to define a notion of equivalence.

Example of equivalence

Have

$$\alpha A^{a_1} + \beta B^{a_1} \equiv \gamma C^{a_1} + \delta D^{a_1}$$

if

$$p([\alpha A^{a_1} + \beta B^{a_1}]E_{a_1}) = p([\gamma C^{a_1} + \delta D^{a_1}]E_{a_1}) \quad \text{for all} \quad E_{a_1}$$

General definition of equivalence

We consider expressions like

$$\text{expression} = \alpha + \beta \mathsf{A} + \gamma \mathsf{B} + \ldots$$

where A, B, . . . are fragments.

Equivalence: *We write*

$$\textit{expression}_1 \equiv \textit{expression}_2$$

if

$$p(\textit{expression}_1 \mathsf{E}) \equiv p(\textit{expression}_2 \mathsf{E})$$

for any fragment E *that makes the contents of the argument on both sides of this equation into a linear sum of circuits.*

Equivalence is a weaker notion than equality.

Another example of equivalence

In general, we have

$$\mathsf{A} \equiv \mathsf{Prob}(\mathsf{A}) \quad \text{for any circuit} \quad \mathsf{A}$$

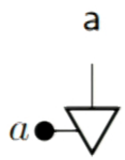
Proof: For any circuit E

$$p(\mathsf{AE}) = p(\mathsf{A})p(\mathsf{E}) = p(\mathsf{Prob}(\mathsf{A})\mathsf{E})$$

This example illustrates how equivalence is a weaker notion than equality.

Fiducial preparations

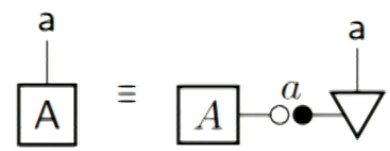
Fiducial preparations

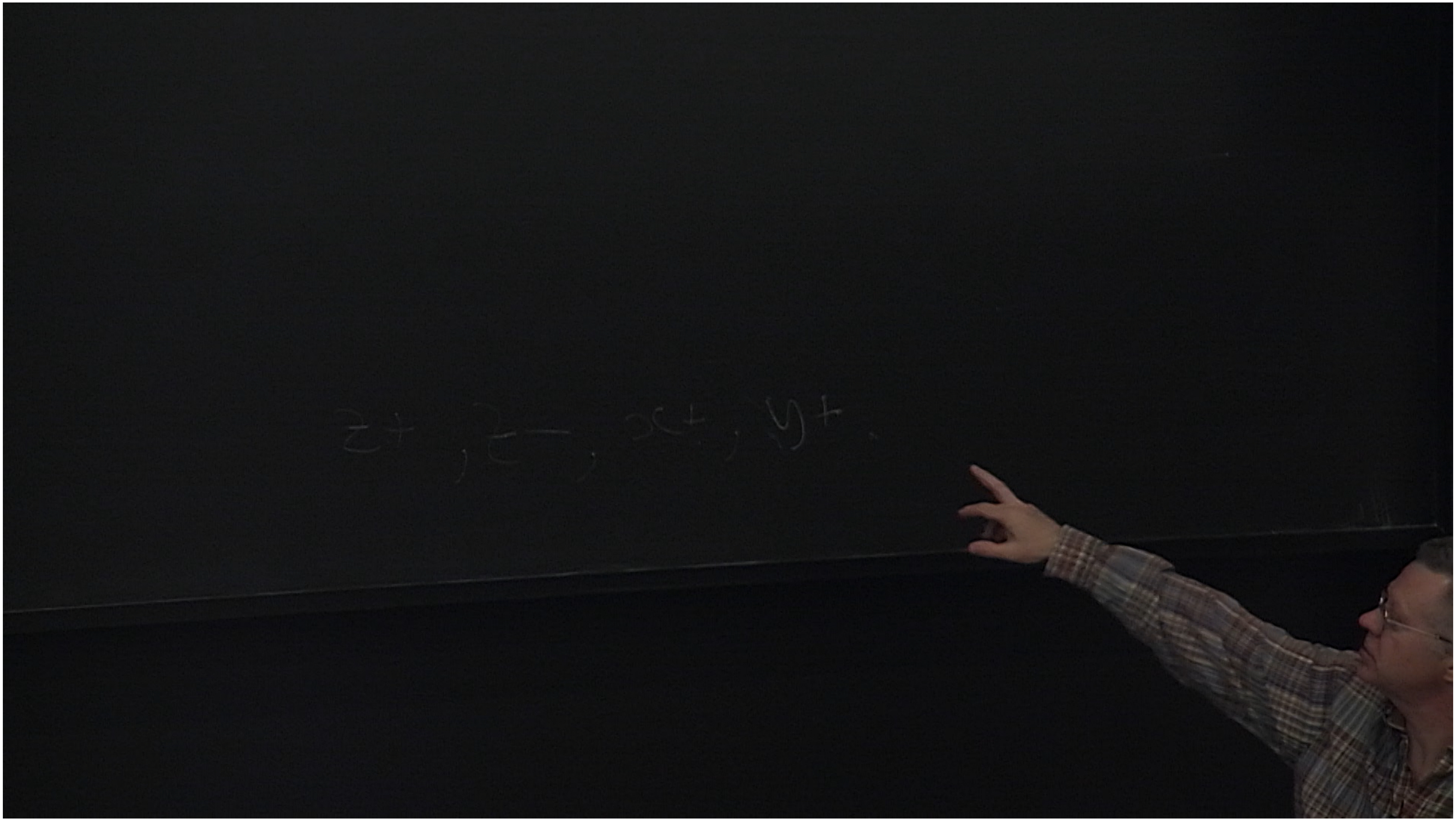


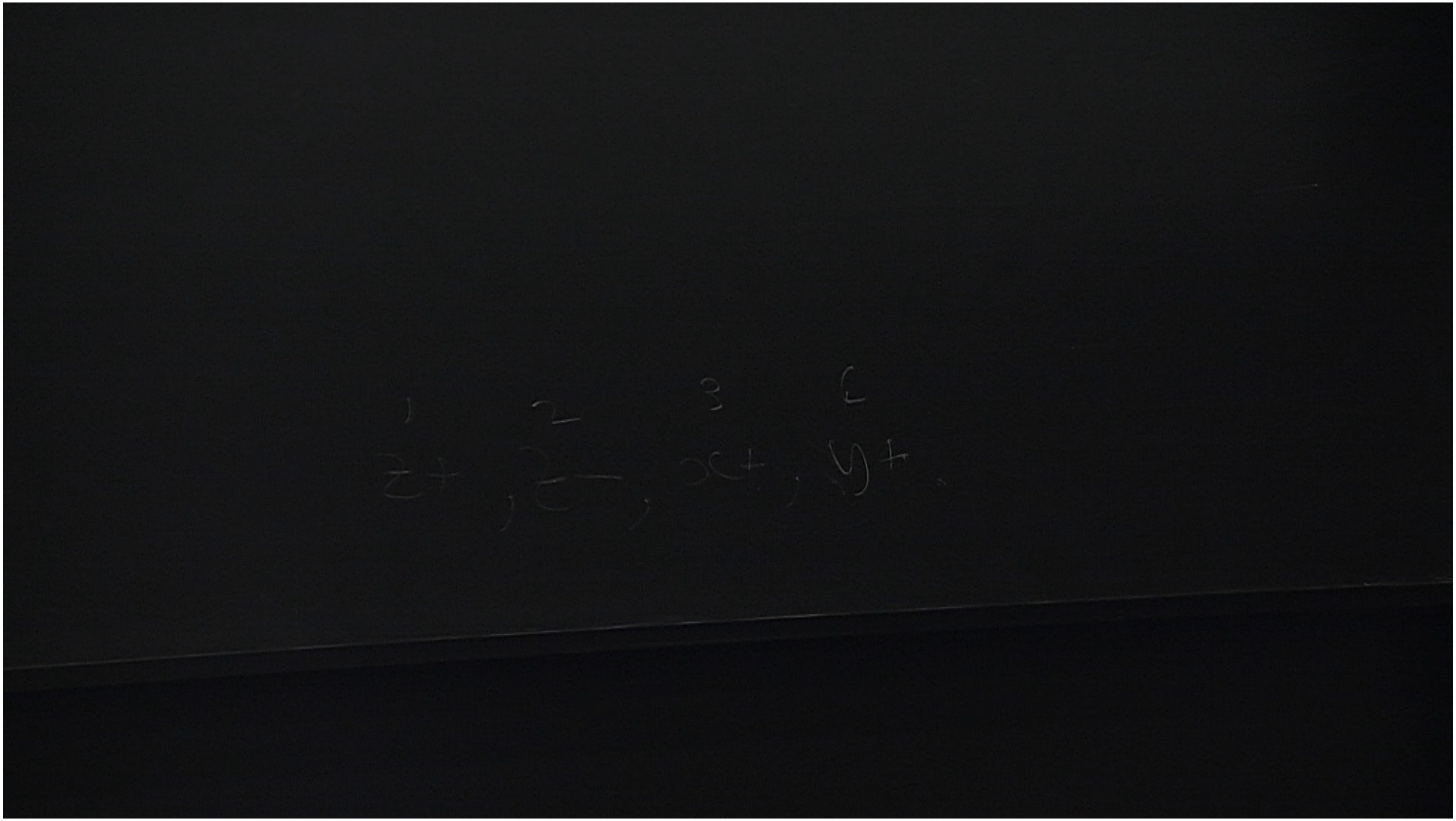
$\Longleftrightarrow \quad {}_{a_1}\mathsf{X}^{\mathsf{a}_1}$ where $a_1 = 1$ to K_a

For any preparation $\mathsf{A}^{\mathsf{a}_1}$ (summation over a_1 implicit below)

$\mathsf{A}^{\mathsf{a}_1} \equiv {}^{a_1}A \; {}_{a_1}\mathsf{X}^{\mathsf{a}_1} \quad \Longleftrightarrow$




z+ , z− , x+ , y+ .


1 2 3
z+ , z− , x+ , y+