

Title: PSI 17/18 - Foundations of Quantum Mechanics - Lecture 7

Date: Feb 06, 2018 10:15 AM

URL: <http://pirsa.org/18020066>

Abstract:

$$P = \begin{pmatrix} P_{z+} \\ P_{z-} \\ P_{x+} \\ P_{y+} \end{pmatrix}$$

$$P_{z+} = \begin{pmatrix} 1 \\ 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$P_{z-} = \begin{pmatrix} 0 \\ 1 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$p_{\text{obs}} = \underbrace{\Gamma}_{\text{effect}} \cdot \underbrace{\frac{P}{\mathcal{N}}}_{\text{state}}$$

$$\Gamma_{z+} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Gamma_{z-} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

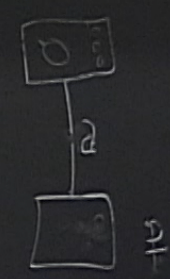
$$\begin{pmatrix} 0 \\ 1 \\ 2 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 6 \end{pmatrix}$$

$$\underline{\Sigma}_I = \underline{\Sigma}_{z+} + \underline{\Sigma}_{z-} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\underline{\Sigma}_I \cdot \underline{p} = p_{z+} + p_{z-}$$

$$\underline{\Sigma}_{x+} + \underline{\Sigma}_{x-} = \underline{\Sigma}_I$$

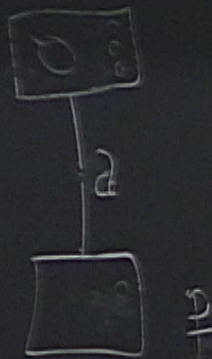


$$\underline{\Sigma}_I = \sum_i \underline{\Sigma}_i$$

labels outside

$$\sum_j \underline{\Sigma}'_j = \underline{\Sigma}_I$$



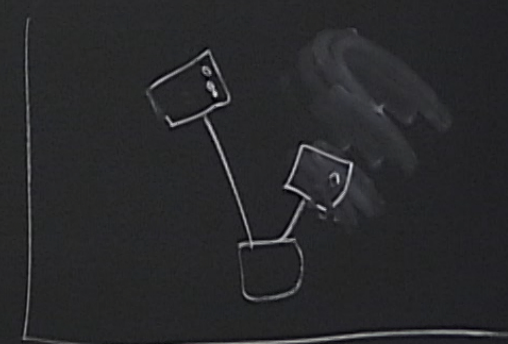


$$\Sigma_I = \sum_i \Sigma_i$$

labels outcomes.

$$\sum_j \Sigma_j = \Sigma_I$$

$$\Sigma_I \cdot P$$



Causality (future choices don't influence past)

The deterministic effect is unique

Pavia  
D'Ariano  
Chiribella  
Renno

$$P = \begin{pmatrix} P_{z+} \\ P_{z-} \\ P_{x+} \\ P_{x-} \end{pmatrix}$$

$$P_{z+} = \begin{pmatrix} 1 \\ 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$P_{z-} = \begin{pmatrix} 0 \\ 1 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$prob = \underbrace{\Gamma}_{\text{object}} \cdot \underbrace{D}_{\text{state}} = \underbrace{\Gamma}_{\text{object}} \cdot \underbrace{D}_{\text{state}}$$

$$\Gamma_{z+} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Gamma_{z-} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$P_{x+} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$P_{x-} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$\Gamma_{x+} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$\Gamma_{x-} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$\Gamma_{x+} + \Gamma_{x-} = \hat{I}$$

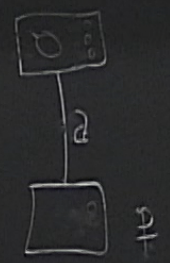
Paola  
D'Ariano  
Chiribella  
Perinotti

$$P_{x+} = \begin{pmatrix} 1/2 \\ 1/2 \\ \vdots \\ 1/2 \end{pmatrix} \quad P_{x-} = \begin{pmatrix} 1/2 \\ 0 \\ \vdots \\ 1/2 \end{pmatrix}$$

$$\Sigma_{x+} = \begin{pmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \end{pmatrix} \quad \Sigma_{x-} = \begin{pmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \end{pmatrix}$$

$$\hat{I} \quad \hat{I} \quad \hat{I}$$

$$\Sigma_{x+} + \Sigma_{x-} = \Sigma_I$$

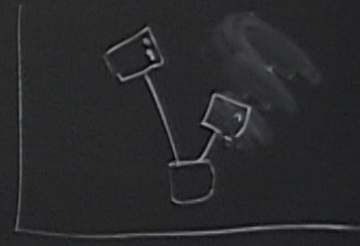


$$\Sigma_I = \sum_i \Sigma_i$$

$$\sum_j \Sigma_j = \Sigma_I$$

$$\Sigma_I \cdot P$$

labels outcomes



Pavia  
D'Ariano  
Chiribella  
Rennothi

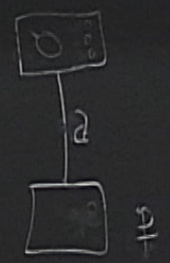
Causality (future choices don't influence past)

The deterministic effect is unique

$$P_{x+} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \vdots \\ \frac{1}{2} \end{pmatrix} \quad P_{x-} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \vdots \\ \frac{1}{2} \\ 0 \\ \vdots \\ 0 \\ \frac{1}{2} \end{pmatrix}$$

$$\Sigma_{x+} = \begin{pmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{pmatrix} \quad \Sigma_{x-} = \begin{pmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{pmatrix}$$

$$\hat{I} \quad \hat{I} \quad \hat{I} \\ \Sigma_{x+} + \Sigma_{x-} = \Sigma_I$$

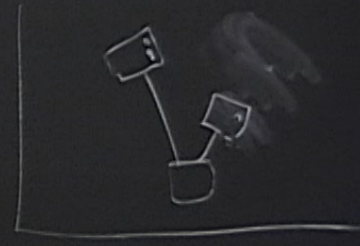


$$\Sigma_I = \sum_i \Sigma_i$$

$$\sum_j \Sigma_j = \Sigma_I$$

$$\Sigma_I \cdot P$$

labels outcomes



Pavia  
D'Ariano  
Chiribella  
Rennothi

Causality (future choices don't influence past)

The deterministic effect is unique

effect state

Maximal measurement

$\{P_n : n=1 \text{ to } N\}$  distinguishable set of states

$\{E_n : n=1 \text{ to } N\}$



$$E_{n'} \cdot E_n = \delta_{n'n}$$



Effect State

# Maximal measurement

$$\{ \rho_n : n=1 \text{ to } N \}$$

$$\{ \Gamma_n \quad n=1 \text{ to } N \}$$



$$\Gamma_{n'} \cdot \Gamma_n = \delta_{n'n}$$

distinguishable set of states  
 maximal if there doesn't  
 exist a set with more elements (for given  
 system type)

← this is a maximal measurement.

$\rho_n$  maximal states.

$\Gamma_n$  maximal effects.

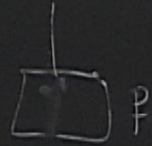
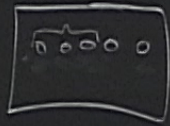
$N$  tells us what the information capacity is.

5 postulates for QT.

( quant-ph/0101012 0912.4740)

Information systems having, or constrained to have, the same information carrying capacity have the same properties.

info. carrying cap. =  $\log_2 N$ .



$|1\rangle, |2\rangle, |3\rangle, |4\rangle, |5\rangle$

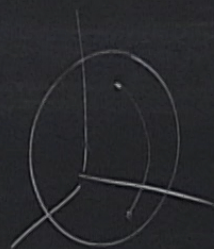
5 postulates for QT.

(quant-ph/0101012 0912.4740)

Information systems having, or constrained to have, the same information carrying capacity have the same properties.

I

0912.4740



systems having, or constrained to have,  
the same information carrying capacity  
the same properties.

Information locality

$$N_{ab} = N_a N_b$$

Tomographic locality

$$K_{ab} = K_a K_b$$

Continuity

There exists a continuous reversible transition  
between any pair of pure states.

$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ 
 $\Sigma_{t-} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

$\Sigma_{x+} = \begin{pmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \end{pmatrix}$ 
 $\Sigma_{x-} = \begin{pmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \end{pmatrix}$

$\Sigma_{x+} + \Sigma_{x-} = \Sigma_I$

$\hat{I} \quad \frac{1}{N}$

Pavia  
 D'Ariano  
 Chiribella  
 Reniotti

$\Sigma_1 \cdot P$

causality (future choices don't influence past)  
 The deterministic effect is unique

distinguishable set of states  
 maximal if there doesn't  
 exist a set with more elements (for given  
 system type)

then is a  
 maximal measurement.

$P_N$  maximal states.

$\Sigma_N$  maximal effects.

$N$  tells us what the information capacity is.

info. carrying cap. =  $\log_2 N$ .



$|1\rangle, |2\rangle, |3\rangle, |4\rangle, |5\rangle$



$P = \lambda P_A + (1-\lambda) P_B$



5 postulates  
 (quant-ph)

Information

Information locality

$$N_{ab} = N_a N_b$$

Tomographic locality

$$K_{ab} = K_a K_b$$

Continuity There exists a continuous reversible transformation between any pair of pure states.

Simplicity Systems are described by the smallest number of probabilities consistent with the other postulates.

$$K(N)$$

$$N = 1, 2, 3, 4, \dots$$

$$K(N+1) > K(N)$$

$$K(N_a N_b) = K(N_a) K(N_b)$$

comp. multiplicative fn.



$$N) \quad N = 1, 2, 3, 4, \dots$$

$$(N+1) > K(N)$$

$$K(N_a N_b) = K(N_a) K(N_b)$$

comp. multiplicative fn.

$$K = N^r \quad r = 1, 2, 3, \dots$$

$r=1$  classical.  
 $r=2$  quantum.

$r=3$  ?

first consider  $N=2$  case.

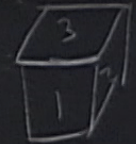
$K = N^2 = 4$  - continuity of Bloch ball.

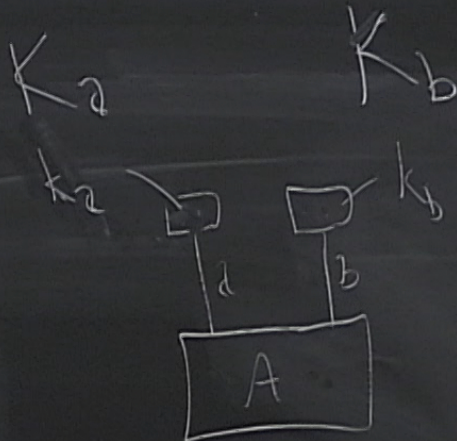


Comp. multiplicative in,

First consider  $N=2$  case.

$K = N^2 = 4$  continuity  $\neq$  Bloch ball.



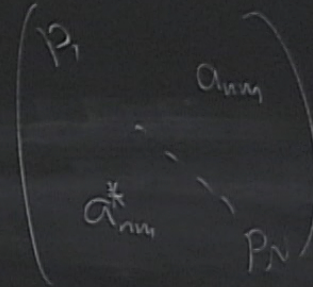


$$K_{ab} = K_a K_b$$

margin capacity is.

$$K = N + 2 \frac{N(N-1)}{2!} = N^2$$

$$N_a=2, N_b=2, N_{ab}=4$$



$N$	$K_{\text{quantum}}$	$K_{\text{real QT}}$	$K_{\text{quaternionic QT}}$
2	4	3	6
4	$16=4^2$	$10>3^2$	$28<5^2$

$$K_{ab} \geq K_a K_b$$