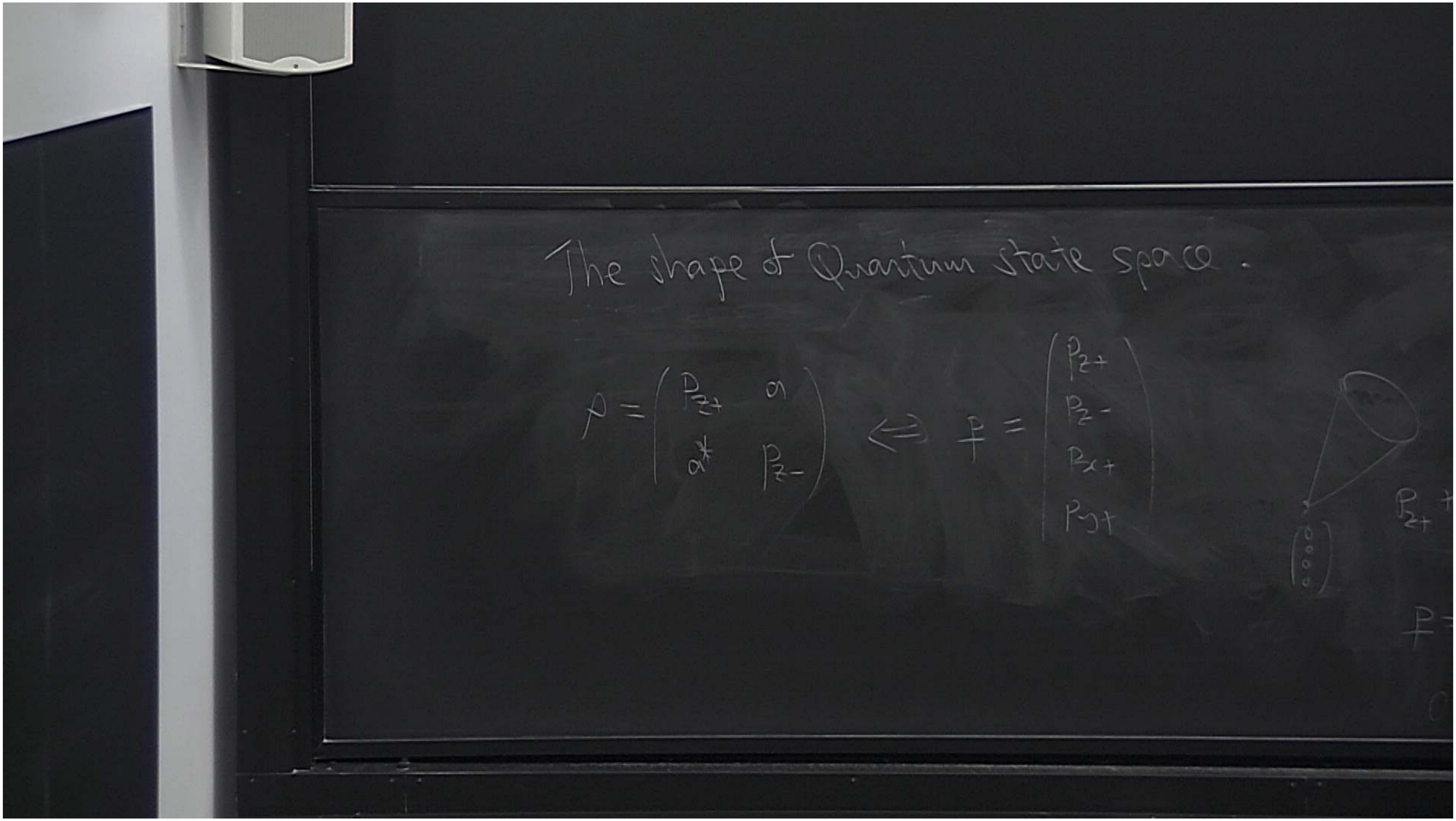
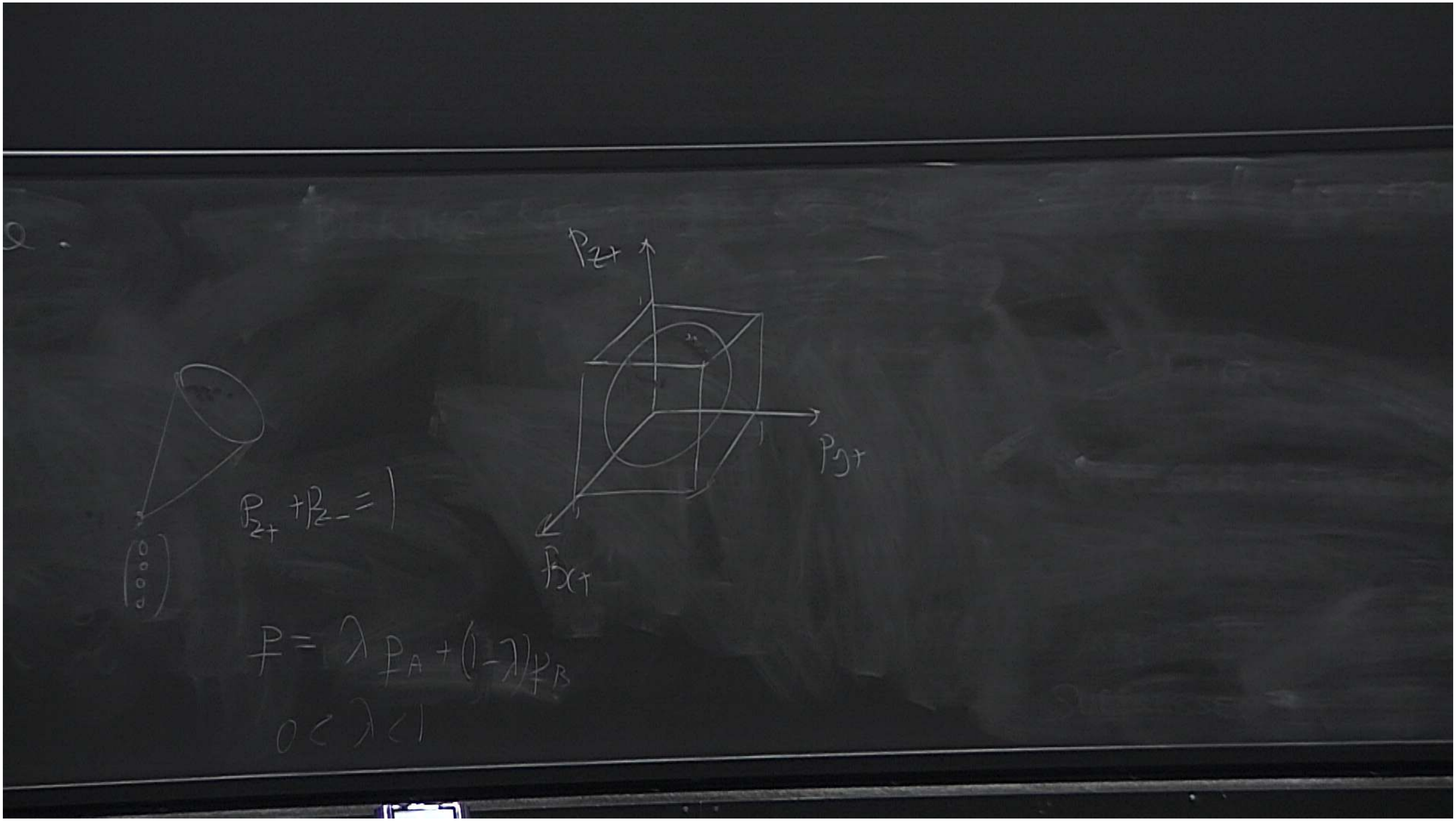
Title: PSI 17/18 - Foundations of Quantum Mechanics - Lecture 6

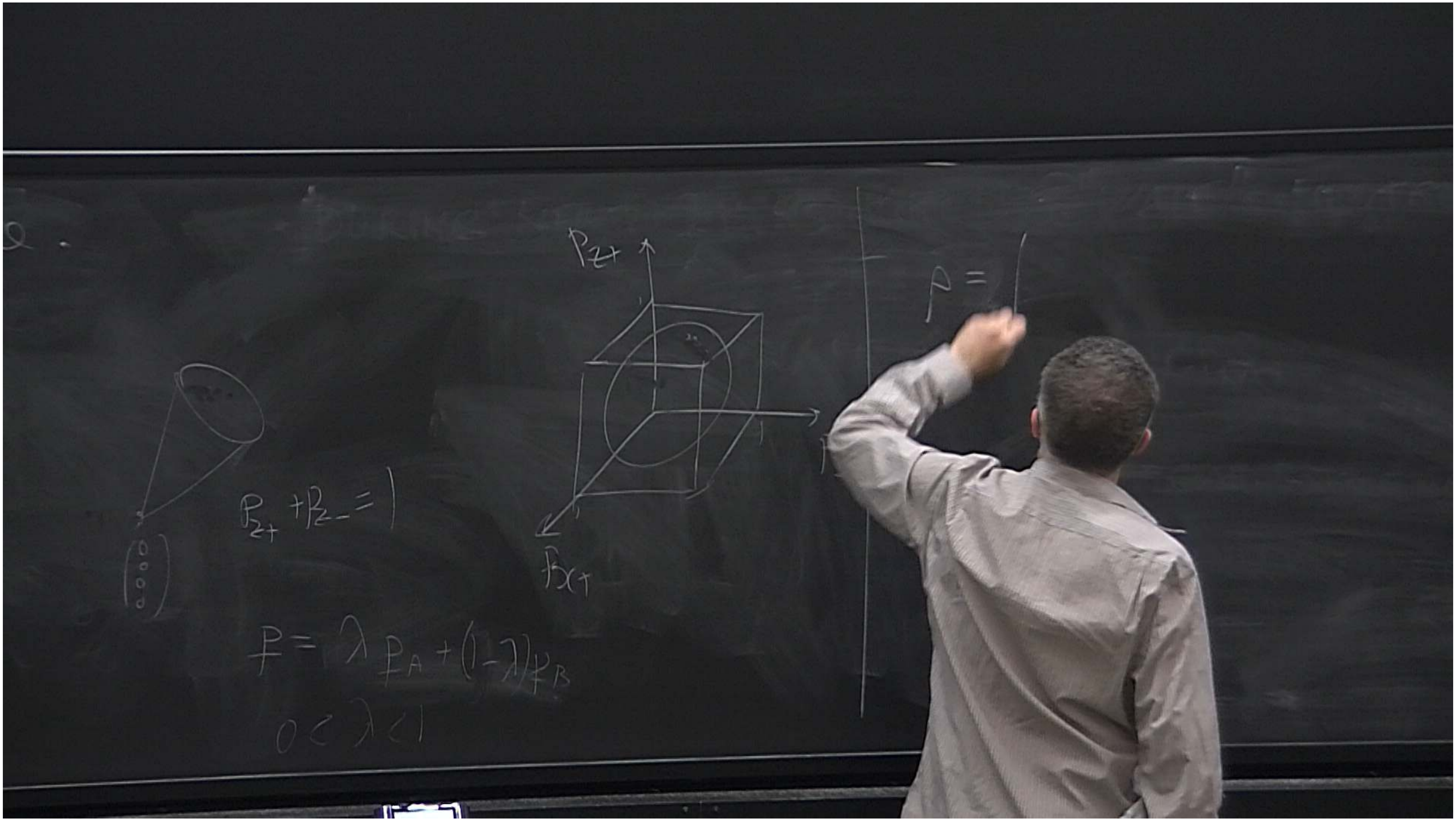
Date: Feb 05, 2018 10:15 AM

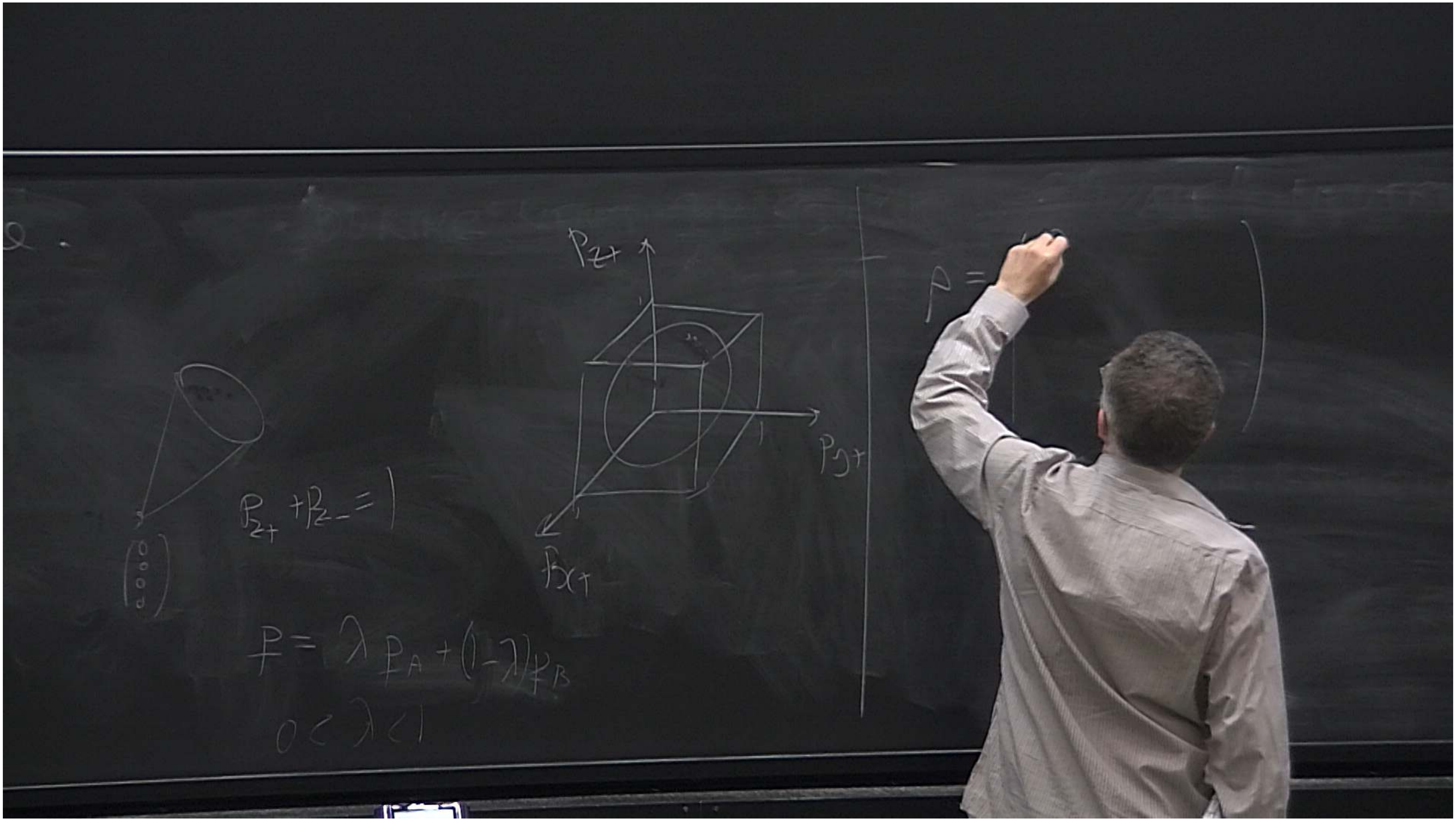
URL: http://pirsa.org/18020065

Abstract:

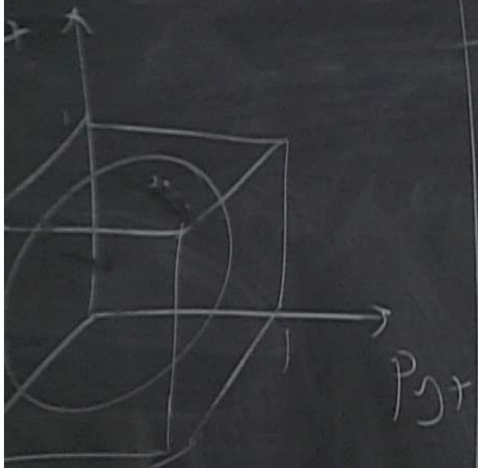

The shape of Quantum state space.


Pz+
Py+
Px+
Pz+ + Pz- = 1
ρ = λ ρA + (1-λ) ρB
0 < λ < 1


P_{z+}
P_{x+}
P_{z+} + P_{z-} = 1
ρ = λ ρ_A + (1-λ) ρ_B
0 < λ < 1
ρ =

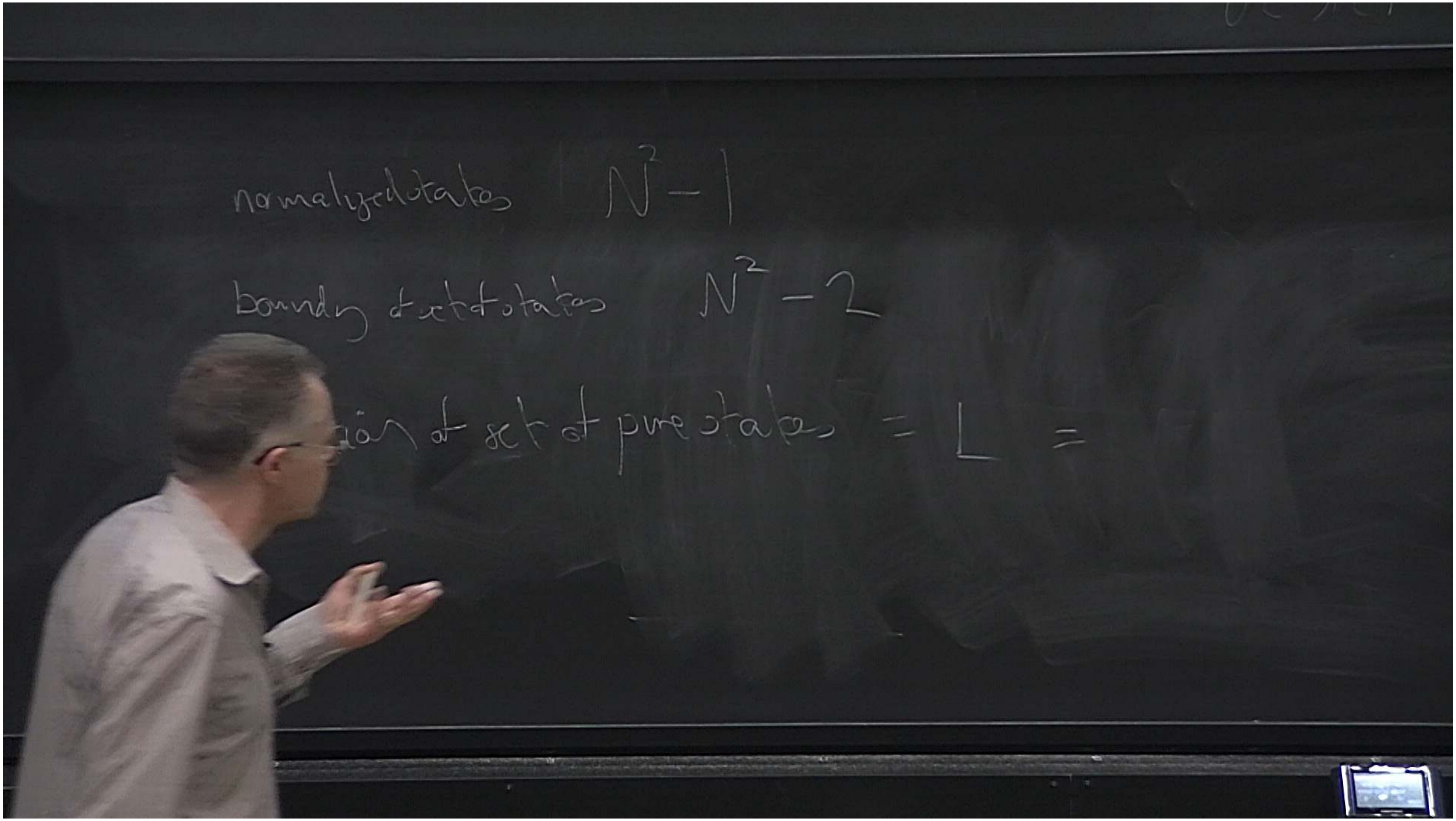


Pz+
Py+
Px+
Pz+ + Pz- = 1
P = λ PA + (1-λ) PB
0 < λ < 1
ρ =



$$\rho = \begin{pmatrix} \rho_1 & \cdots & a_{nm} \\ & \ddots & \\ a^*_{nm} & \cdots & \rho_N \end{pmatrix} \iff \underline{r} = \begin{pmatrix} \rho_1 \\ \vdots \end{pmatrix}$$

$$K = N + 2 \cdot \frac{N(N-1)}{2} = N^2$$

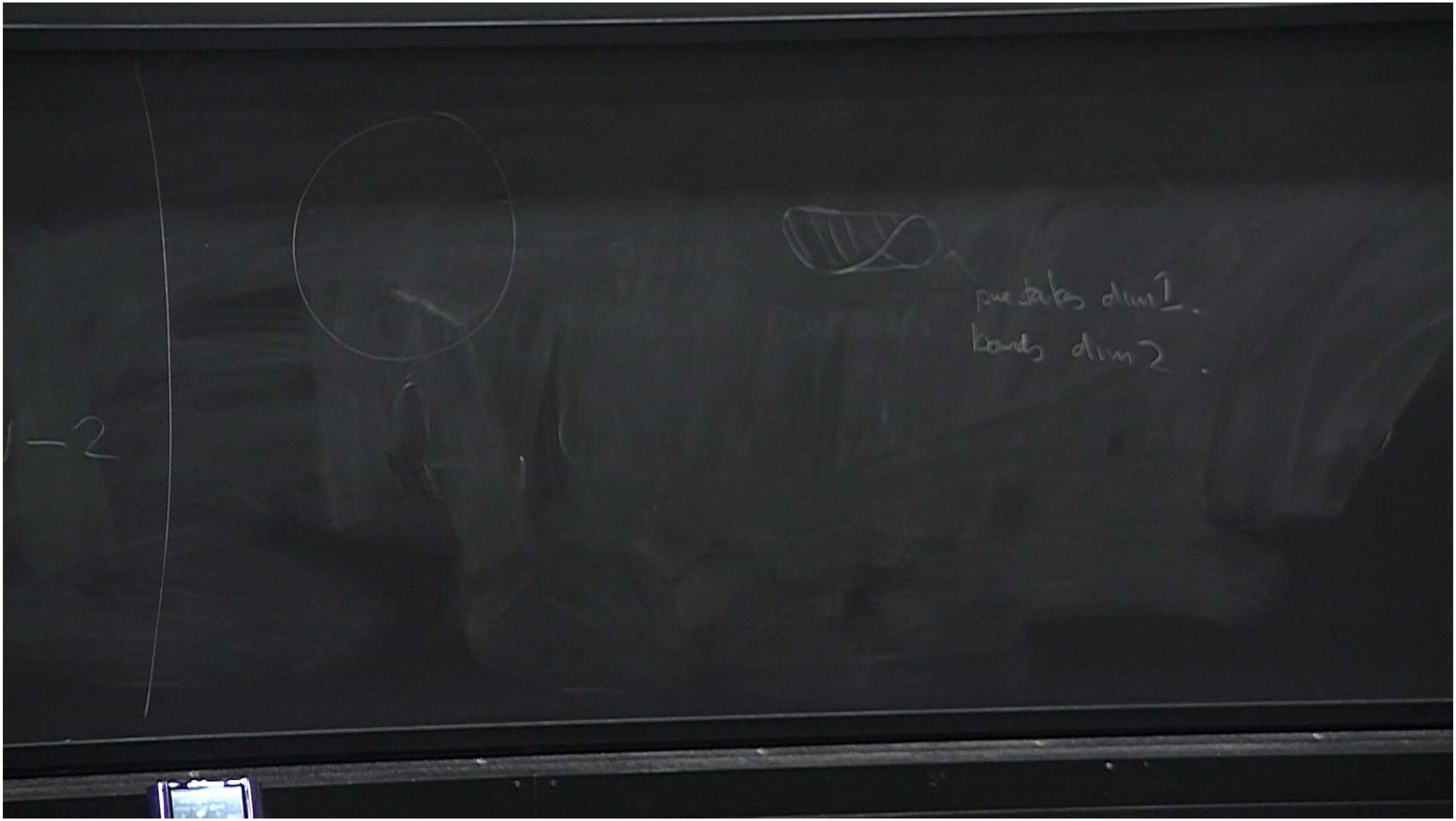

normalized states N^2-1
boundary of set of states N^2-2
of set of pure states $= L =$

normalized states $N^2 - 1$

boundary of set of states $N^2 - 2$

dimension of set of pure states $= L = 2N - 2$

$$|\psi\rangle = \sum_{n=1}^{N} a_n |n\rangle$$



pure states dim 1.
bands dim 2.

pure states dim 1.

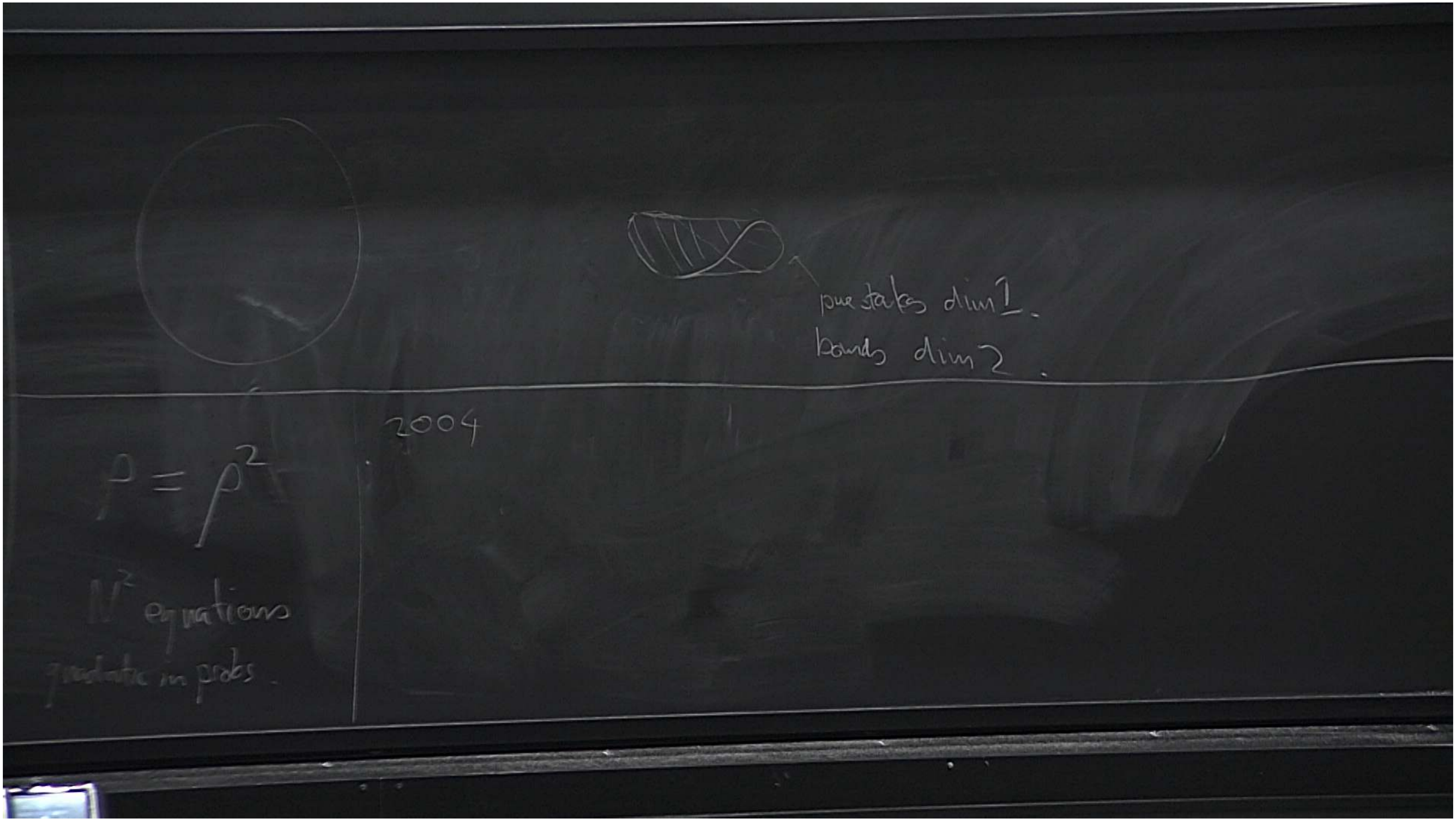
bands dim 2.

-2

$\rho = \rho^2$

N^2 equations

quadratic in probs.


pure states dim 1.
bounds dim 2.
2004
ρ = ρ²
N² equations
quadratic in probs.

pure states dim 1.

bounds dim 2.

2004 Jones Linden!

Fuchs

ρ (Hermitean) iff

$$\mathrm{tr}(\rho^2) = \mathrm{tr}(\rho^3) = 1$$

pure states dim 1.

bounds dim 2.

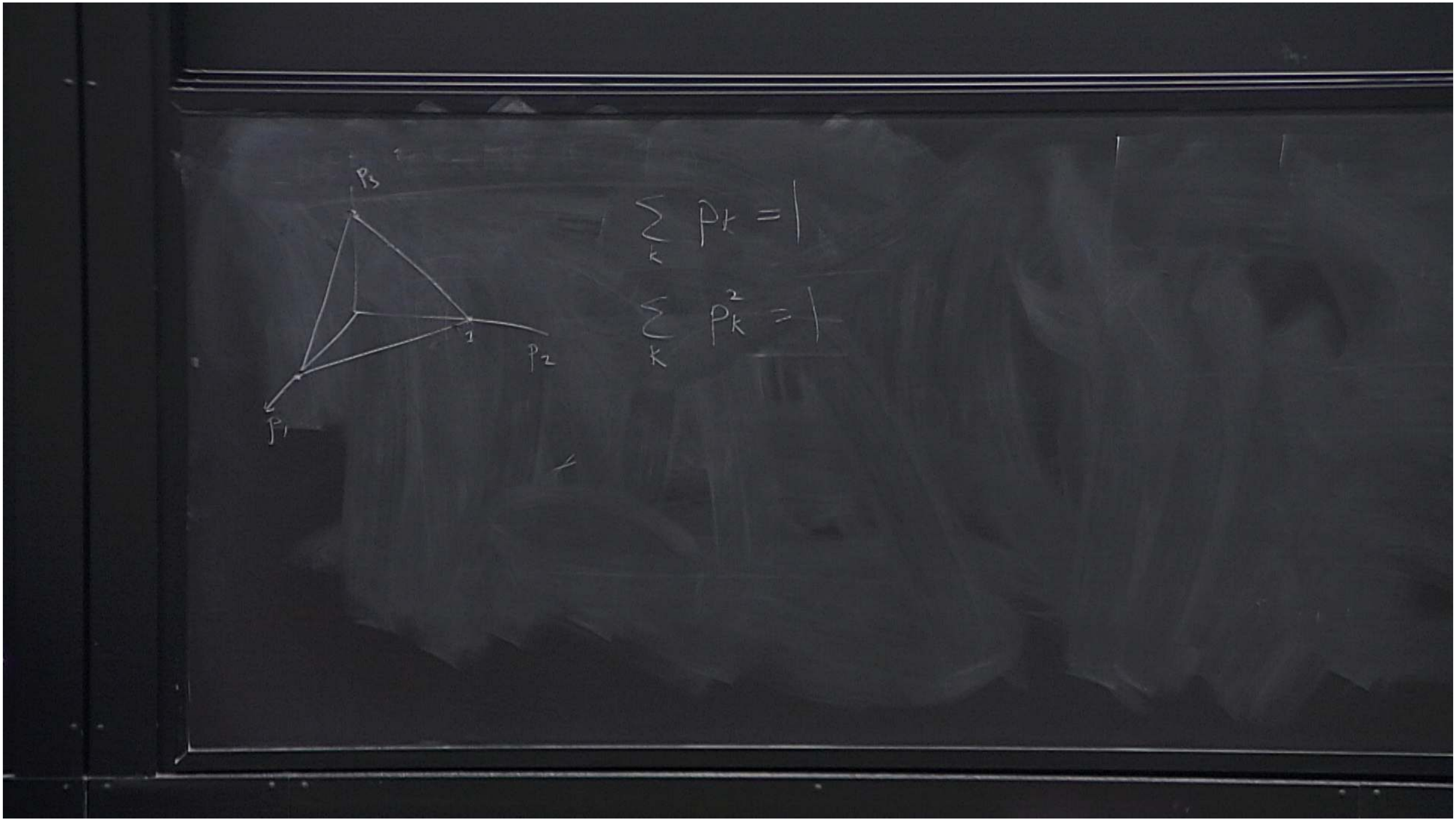
2004 Jones Linden /
Fuchs

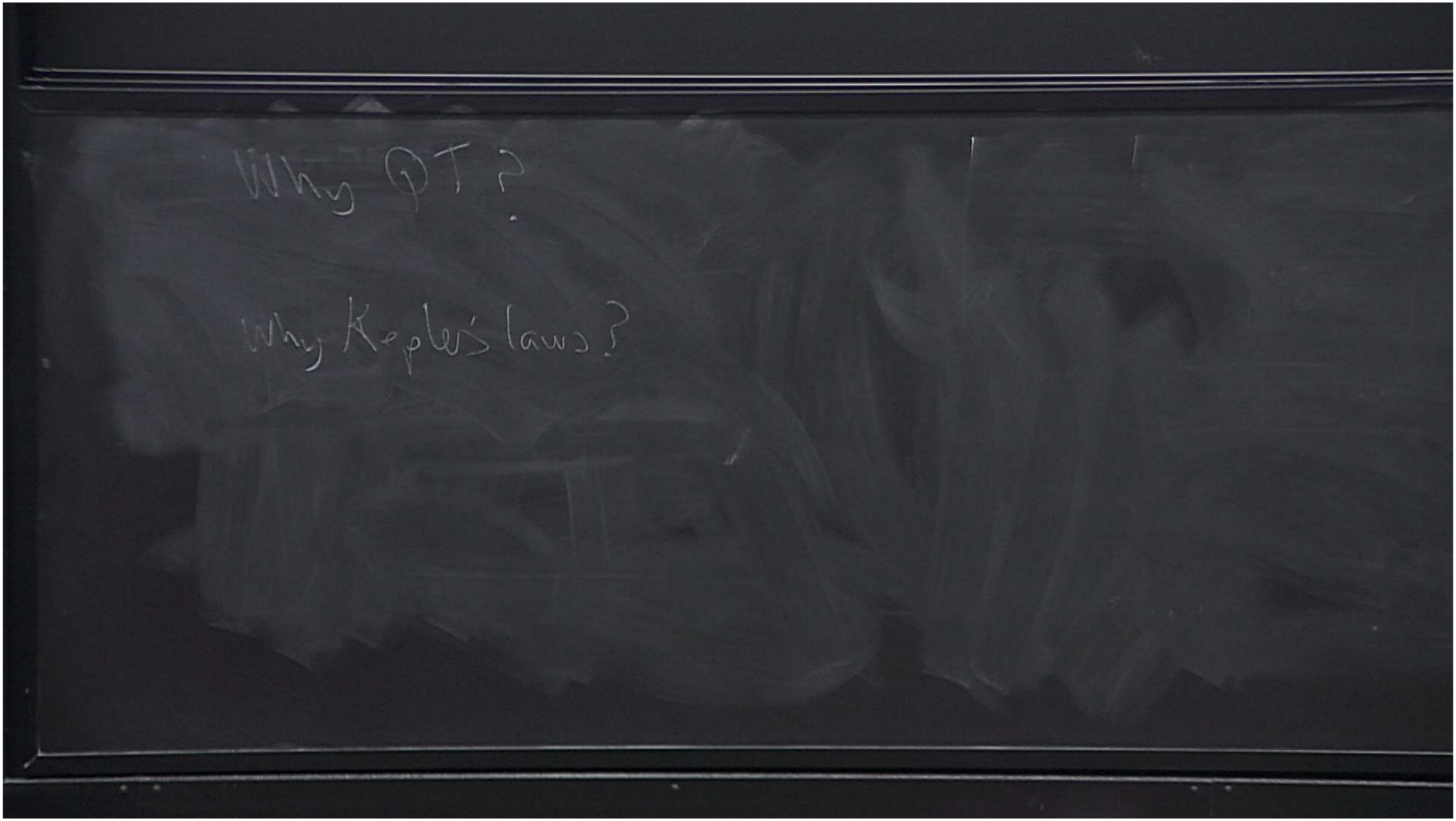
ρ (Hermitean) iff

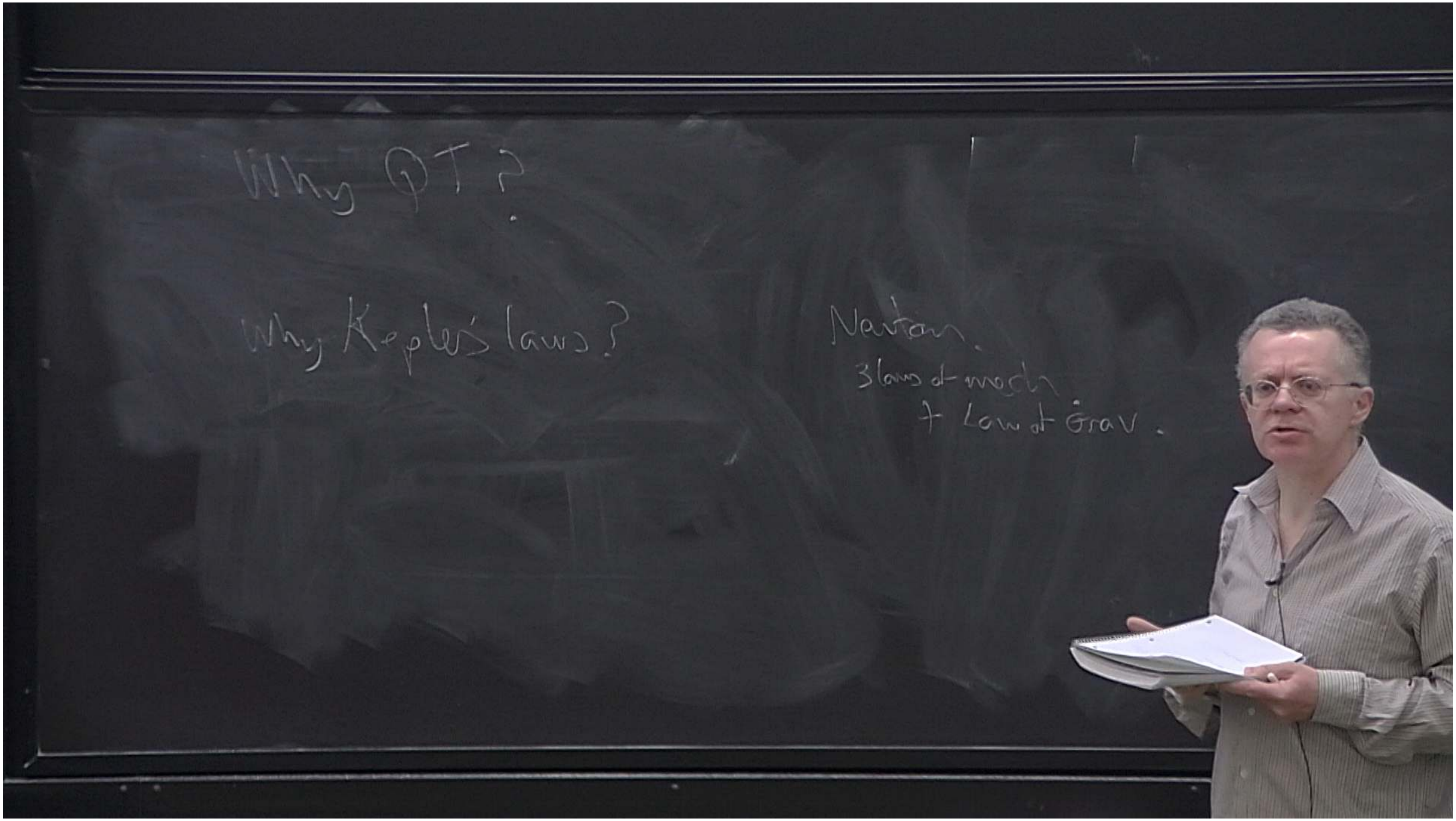
$$\mathrm{tr}(\rho^2) = \mathrm{tr}(\rho^3) = 1$$

1 cubic eqn

1 quadratic eqn


p_3
$\sum_k p_k = 1$
$\sum_k p_k^2 = 1$
1
p_2
p_1


Why QT?
Why Kepler's laws?


Why QT ?
Why Kepler's laws ?
Newton.
3 laws of mech
+ Law of Grav.

Why QT?

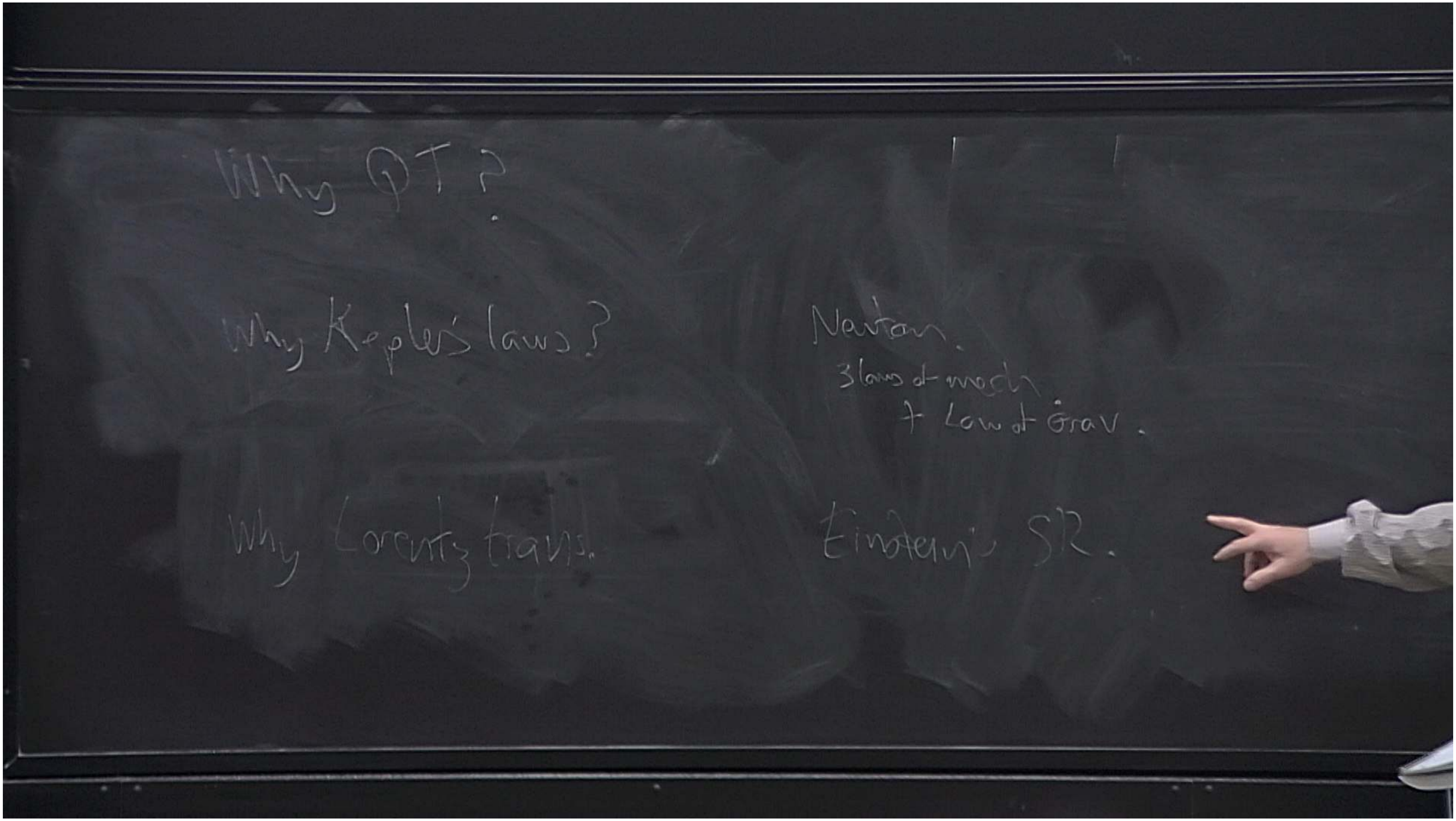
Why Kepler's laws?

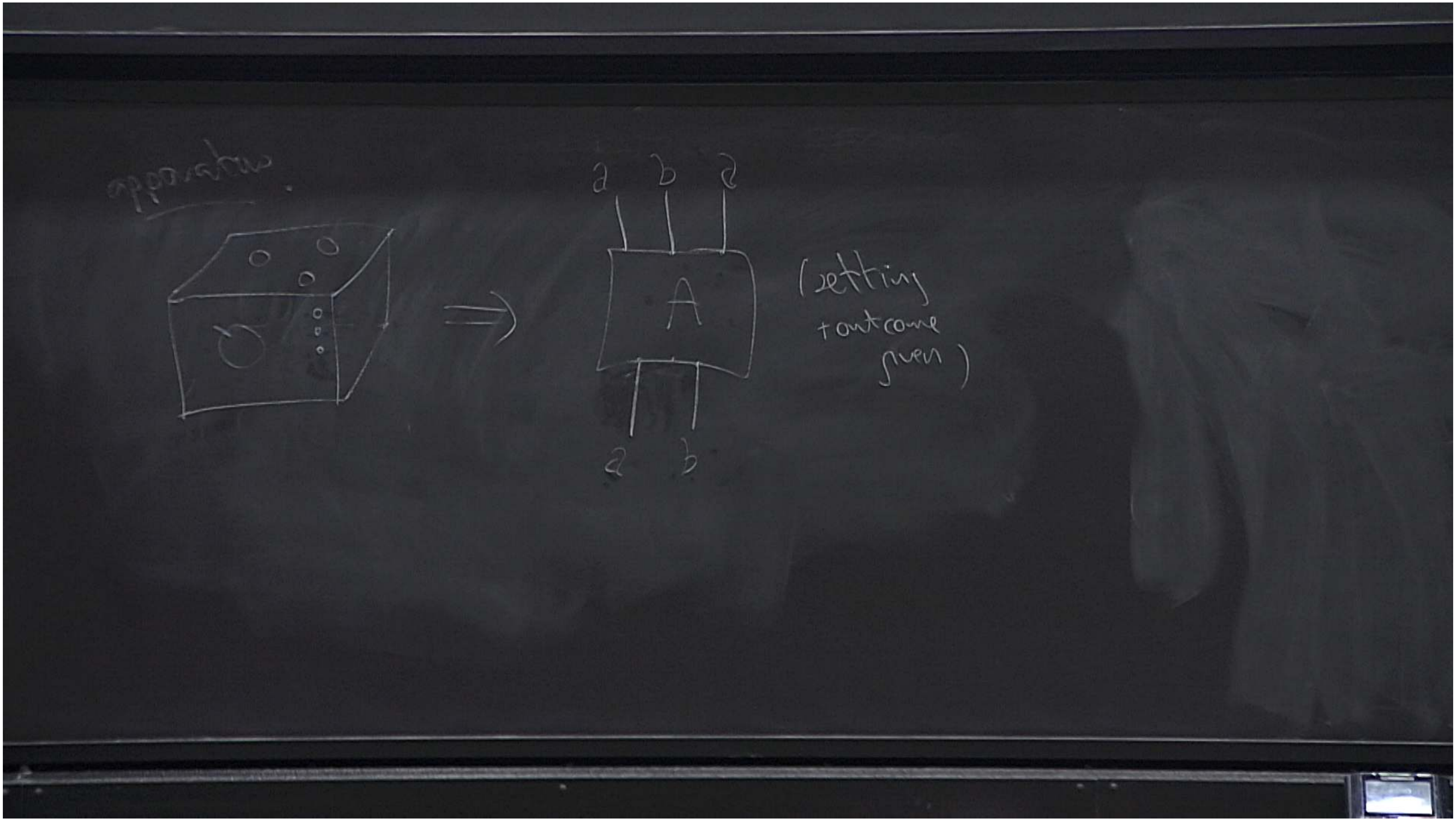
Newton.

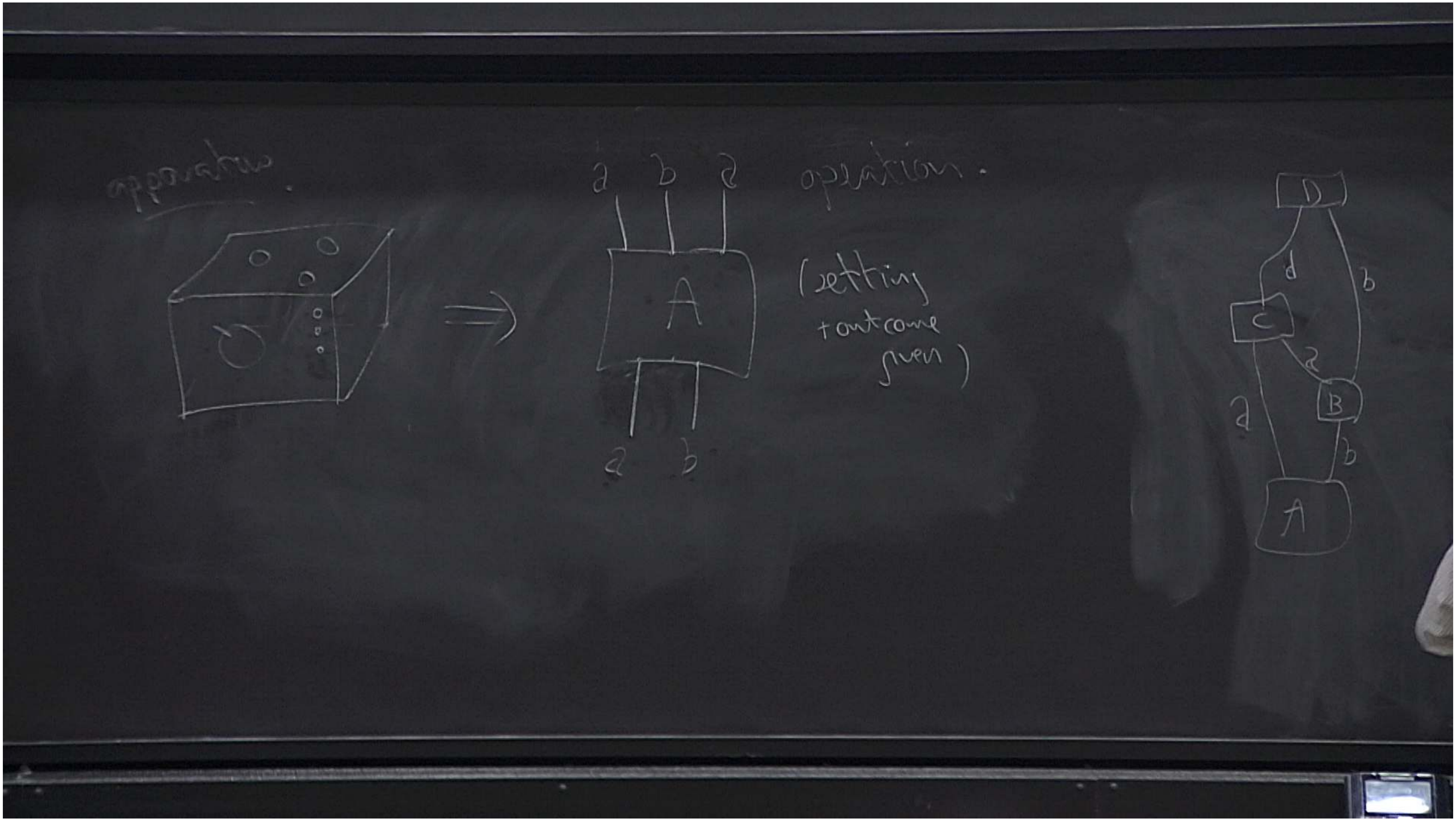
3 laws of mech

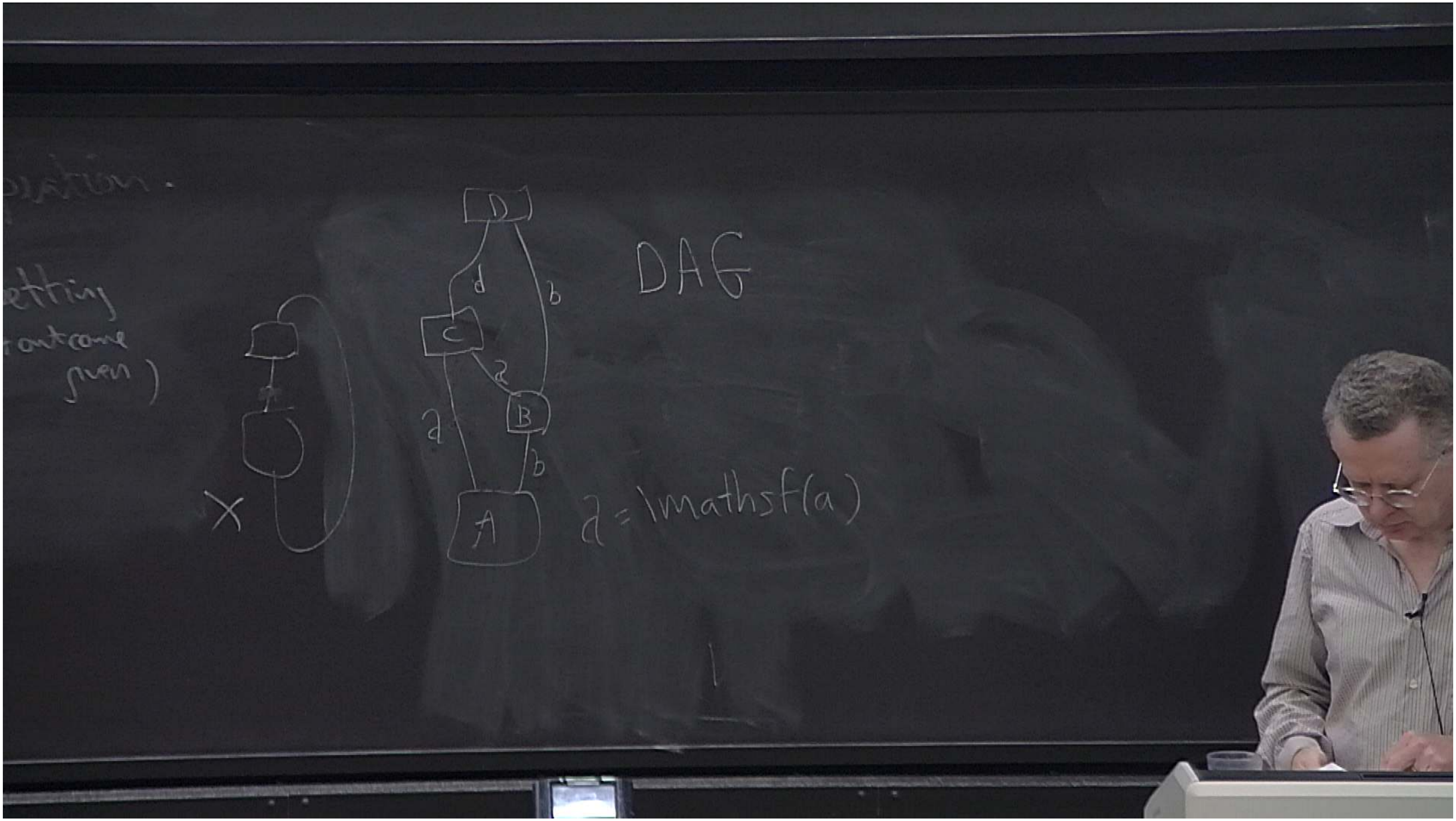
+ Law of Grav.

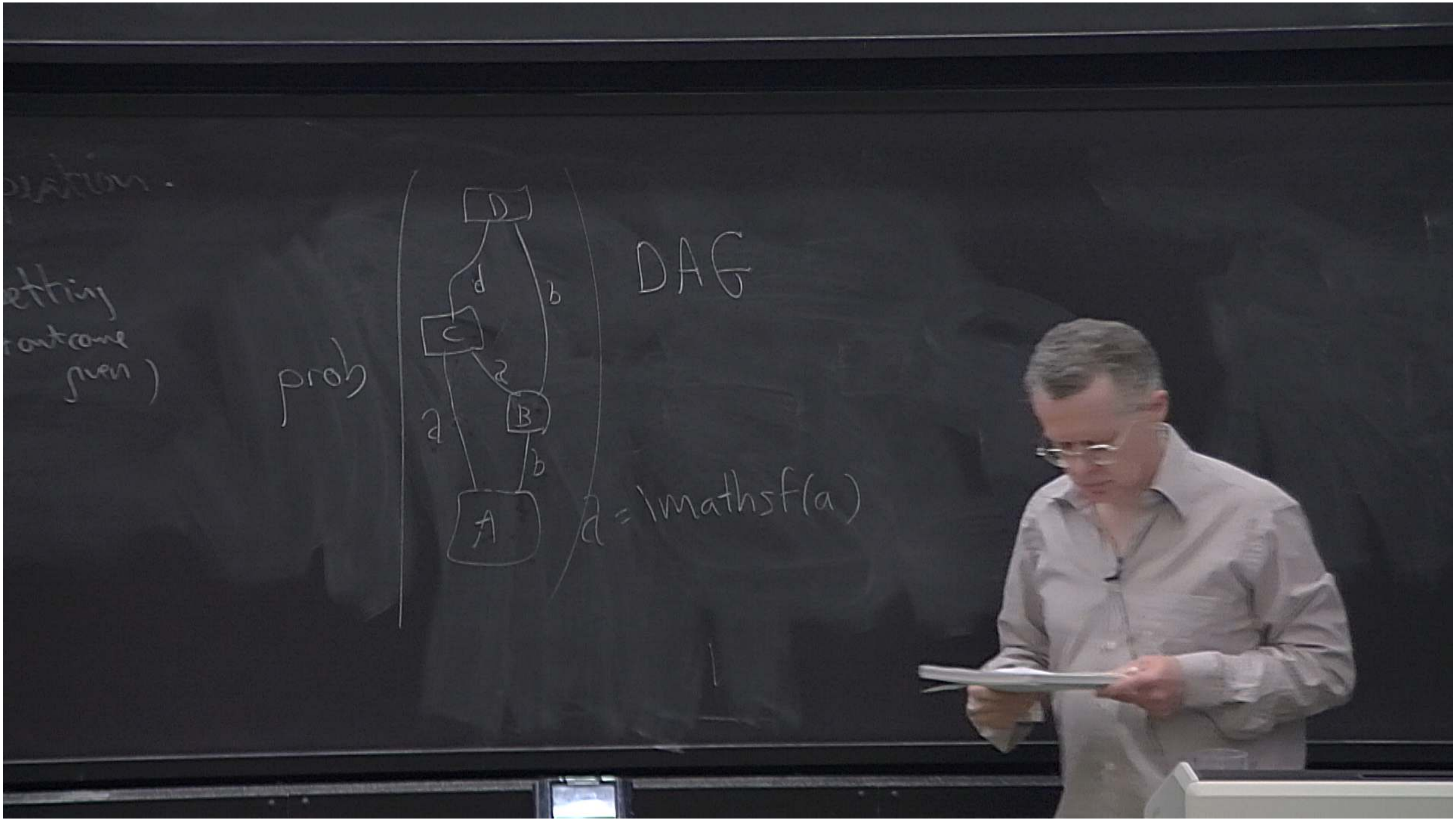
Why Lorentz trans.


Why QT?
Why Kepler's laws?
Newton.
3 laws of mech
+ Law of Grav.
Why Lorentz trans.
Einstein's SR.

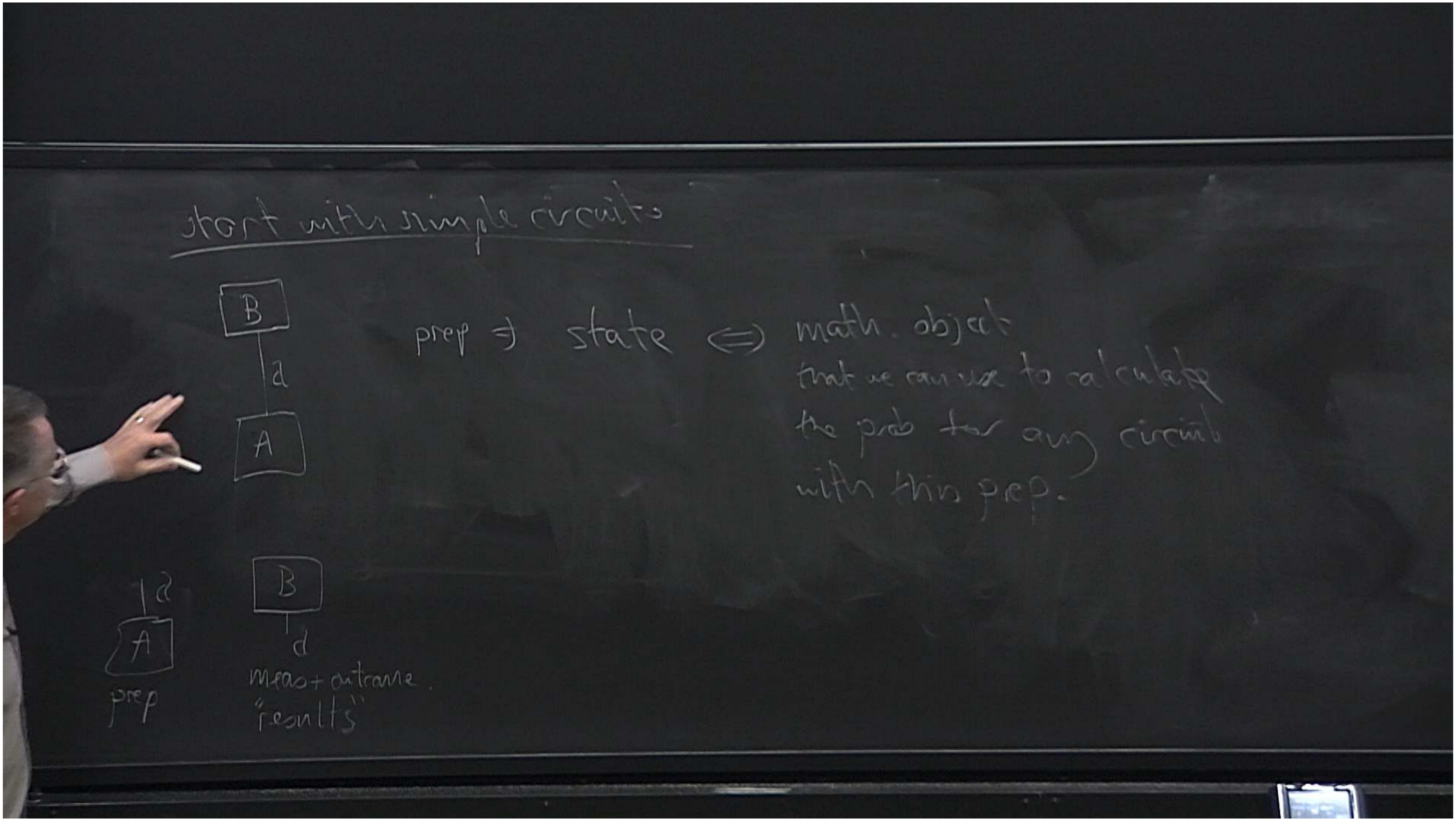

apparatus.
a b a
A
a b
(setting
+ outcome
given)

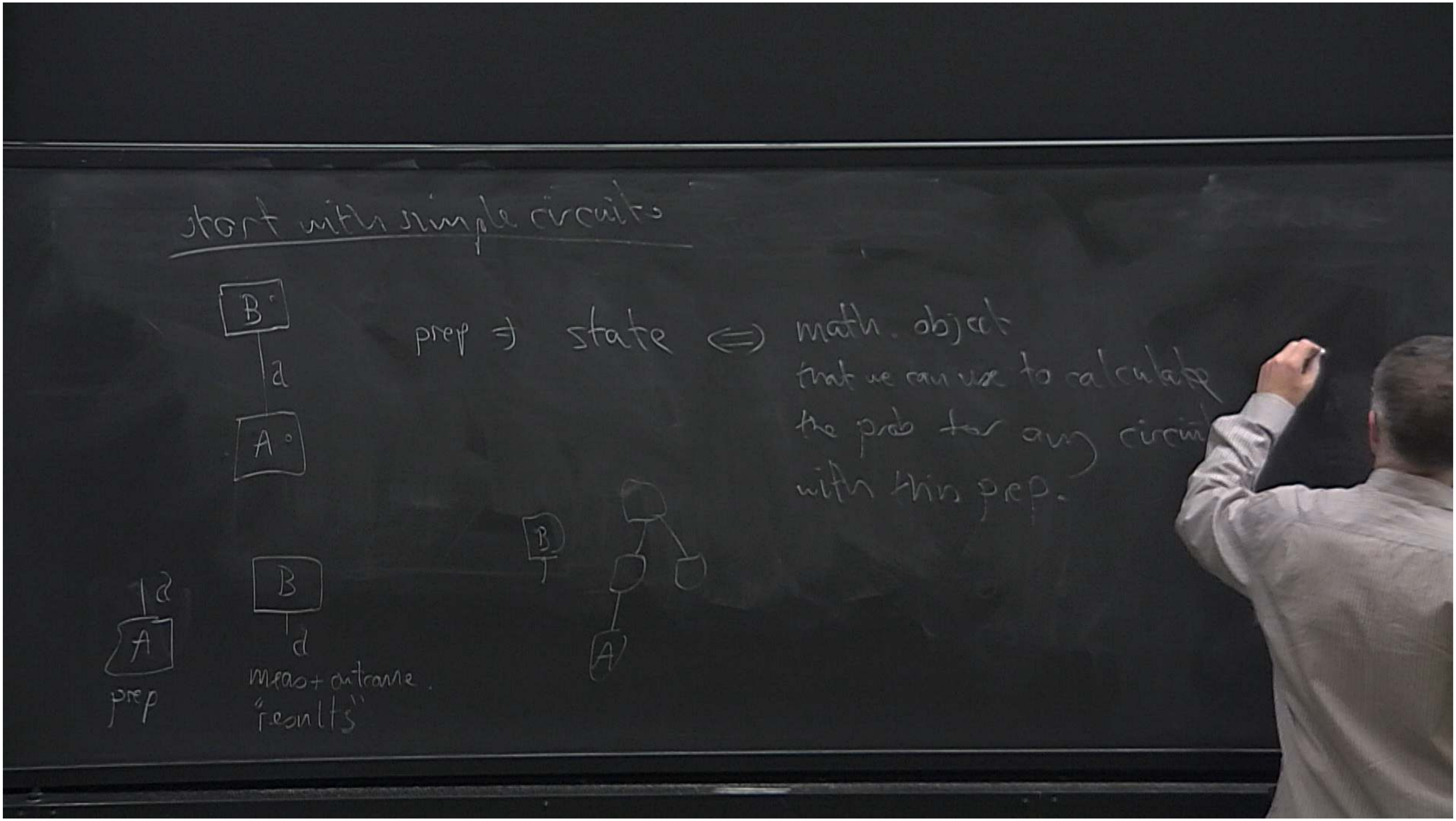

apparatus.
a b a
A
a b
operation.
(setting
+ outcome
given)
D
d
b
C
a
a
B
b
A


D
d
b
C
B
A
DAG



prob
DAG
D
d
b
C
a
B
a
b
A
a = \mathsf(a)


start with simple circuits
B
a
A
prep ⇒ state ⟺ math. object
that we can use to calculate
the prob for any circuit
with this prep.
a
A
prep
B
a
meas + outcome
"results"


start with simple circuits
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prep ⇒ state ⟺ math. object
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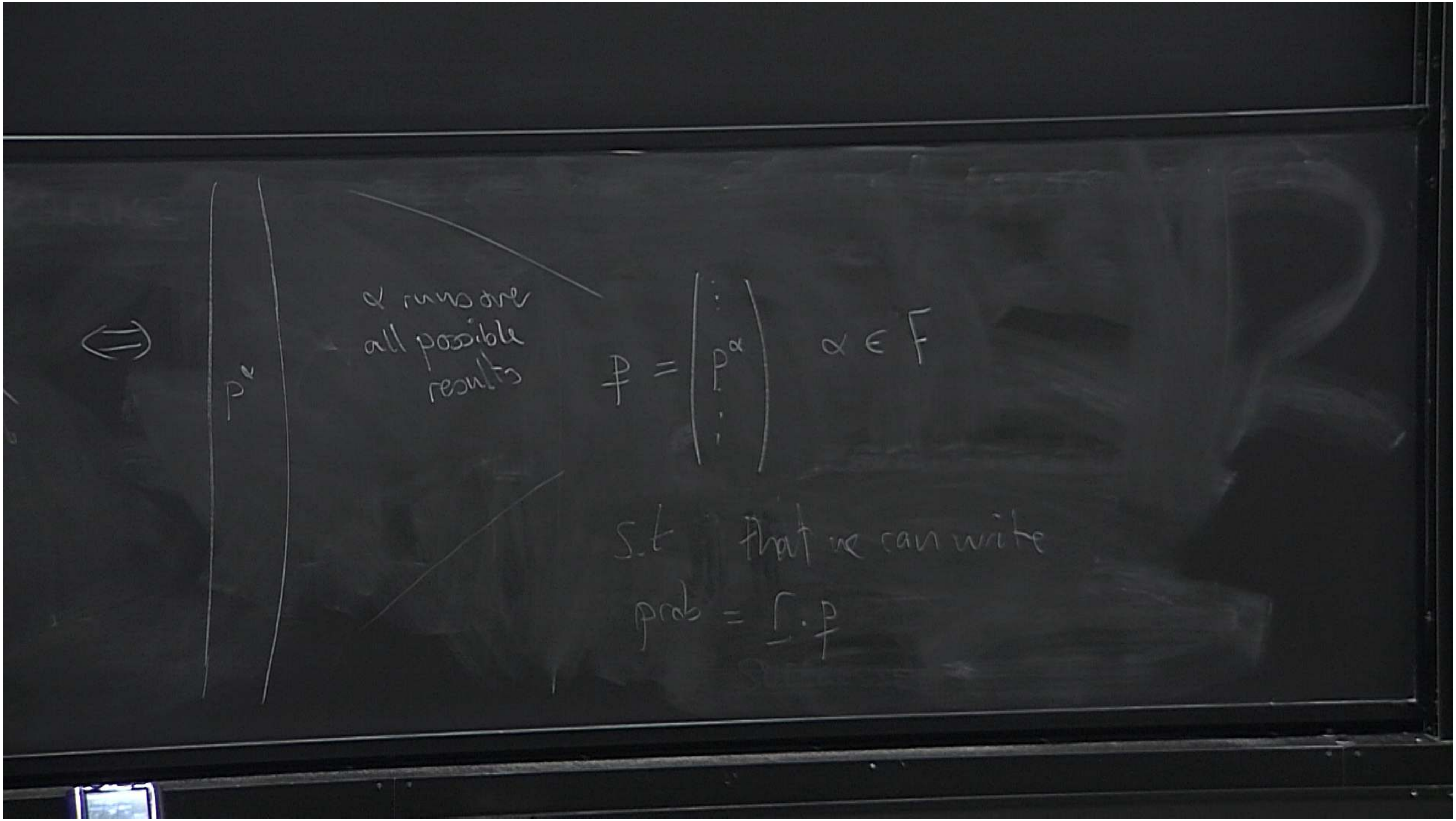
a
A
prep
B
a
meas + outcome
"results"
B
A

$\Longleftrightarrow$

p^{α}

α runs over all possible results


$\Longleftrightarrow$
p^α
α runs over all possible results
$p = \begin{pmatrix} \vdots \\ p^\alpha \\ \vdots \end{pmatrix}$
$\alpha \in F$
s.t that we can write
prob = $r \cdot p$

$\Longleftrightarrow$ $\begin{pmatrix} p^\alpha \end{pmatrix}$

α runs over all possible results

$$\underset{\sim}{p} = \begin{pmatrix} \vdots \\ p^\alpha \\ \vdots \end{pmatrix} \quad \alpha \in F$$

s.t. that we can write

$$\text{prob} = \underset{\sim}{r}_{B} \cdot \underset{\sim}{p}_{A}$$

$\Longleftrightarrow$ $\left(p^\alpha \right)$

α runs over all possible results

$$\underline{p} = \begin{pmatrix} \vdots \\ p^\alpha \\ \vdots \end{pmatrix} \quad \alpha \in F$$

s.t. that we can write

$$\text{prob} = \underline{r}_{B} \cdot \underline{p}_{A}$$

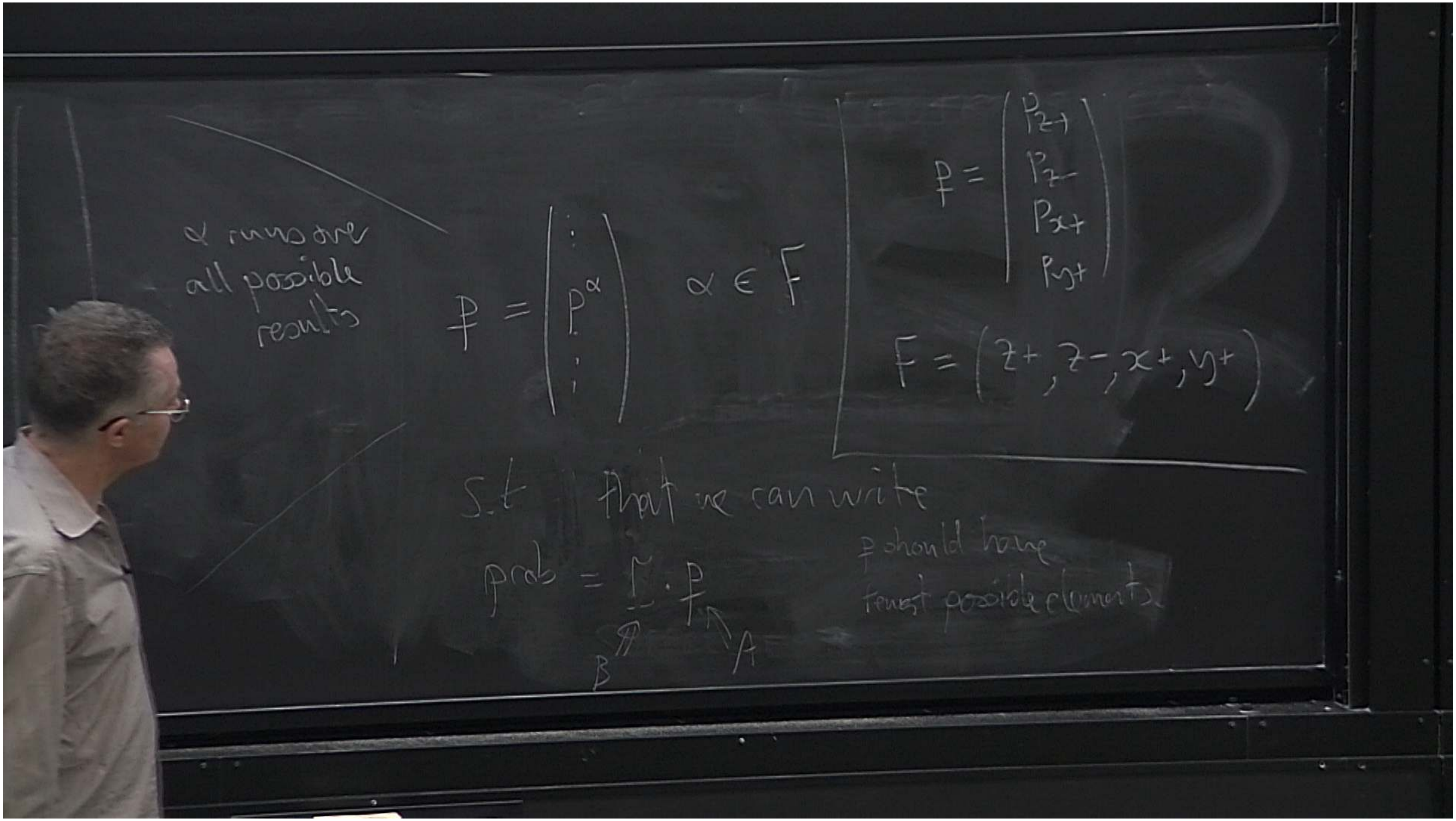
$\underline{p}$ should have fewest possible elements

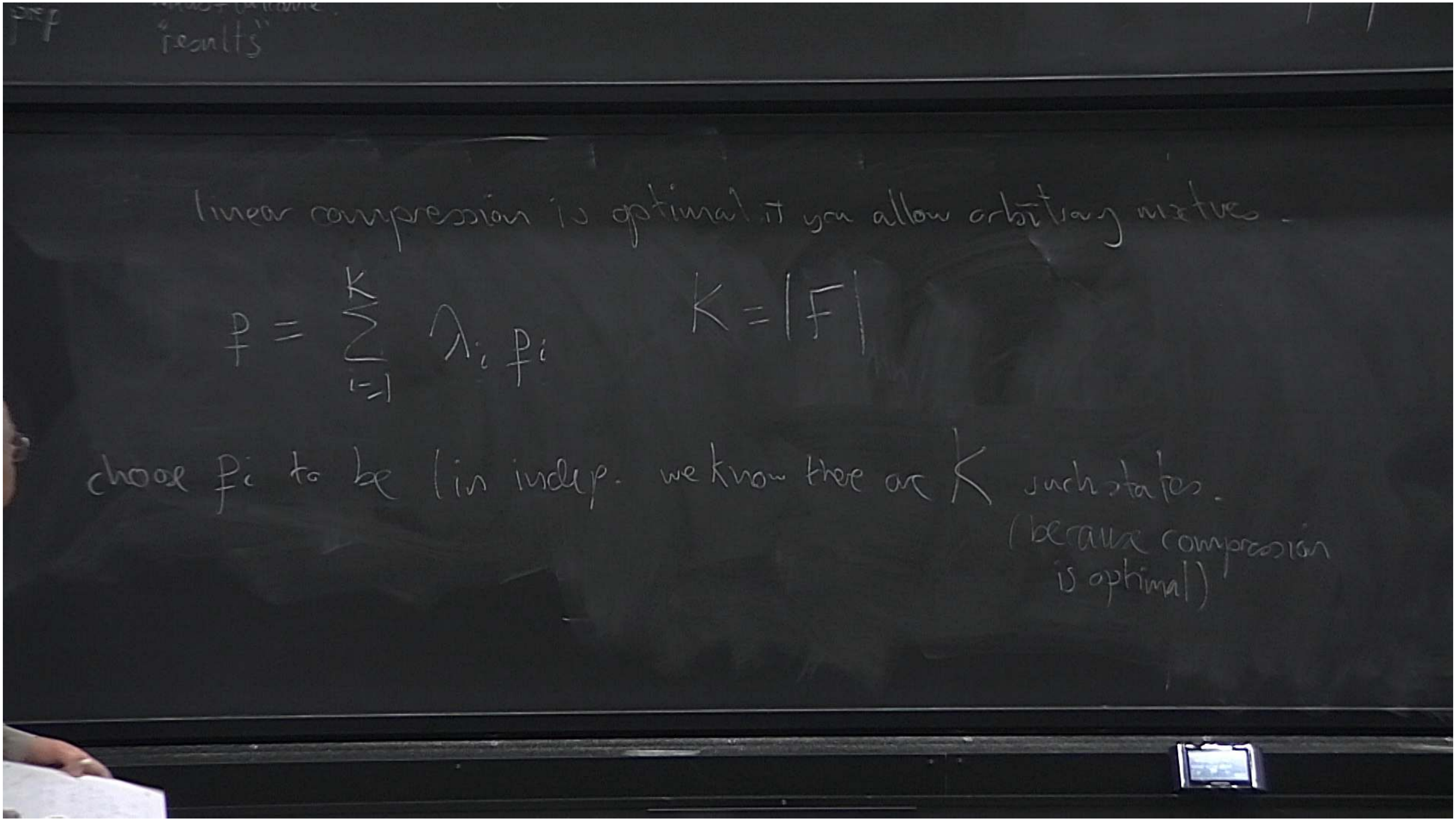
$$p = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$

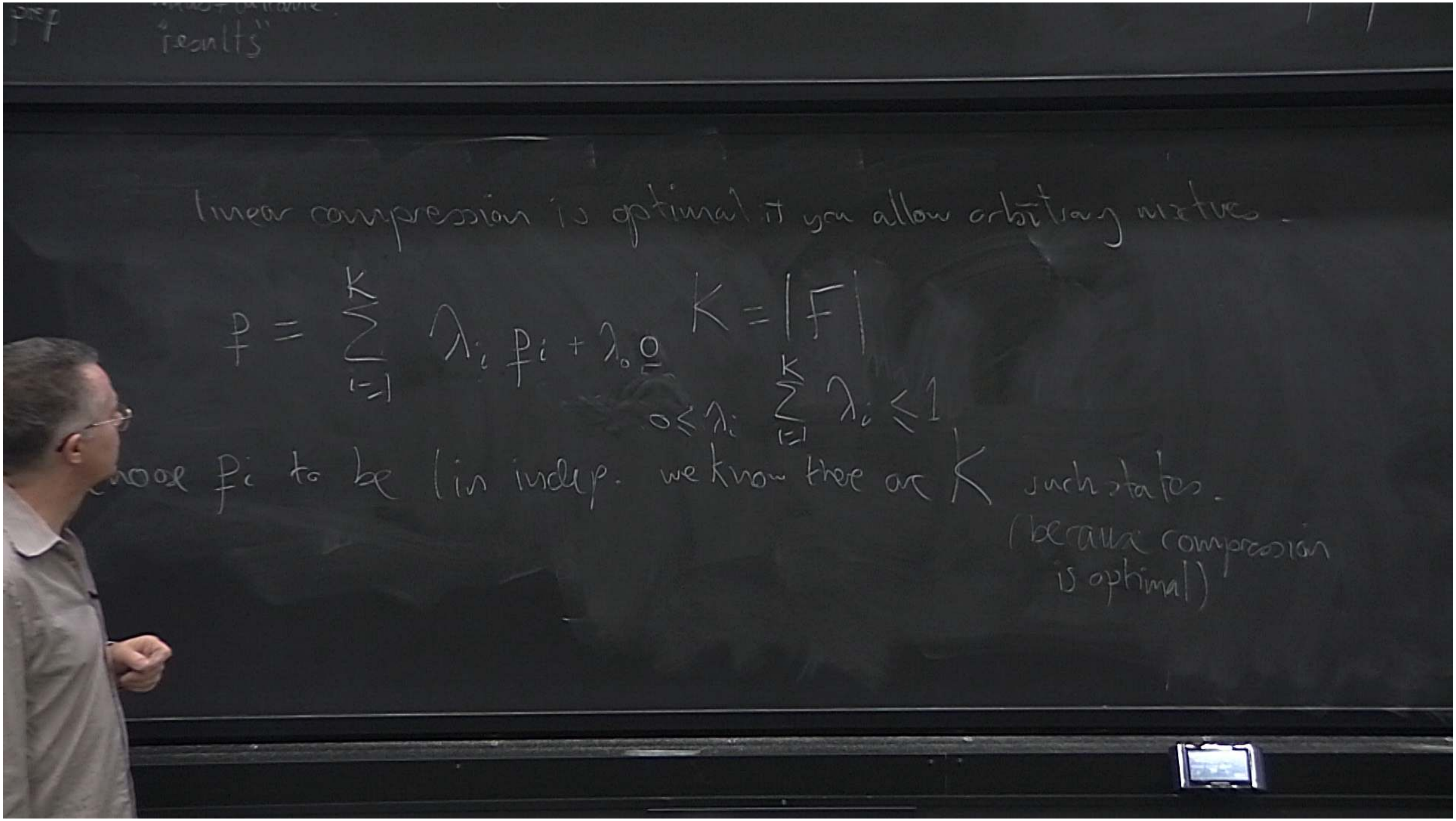
$$= \begin{pmatrix} \vdots \\ p^{\alpha} \\ \vdots \end{pmatrix} \qquad \alpha \in F$$

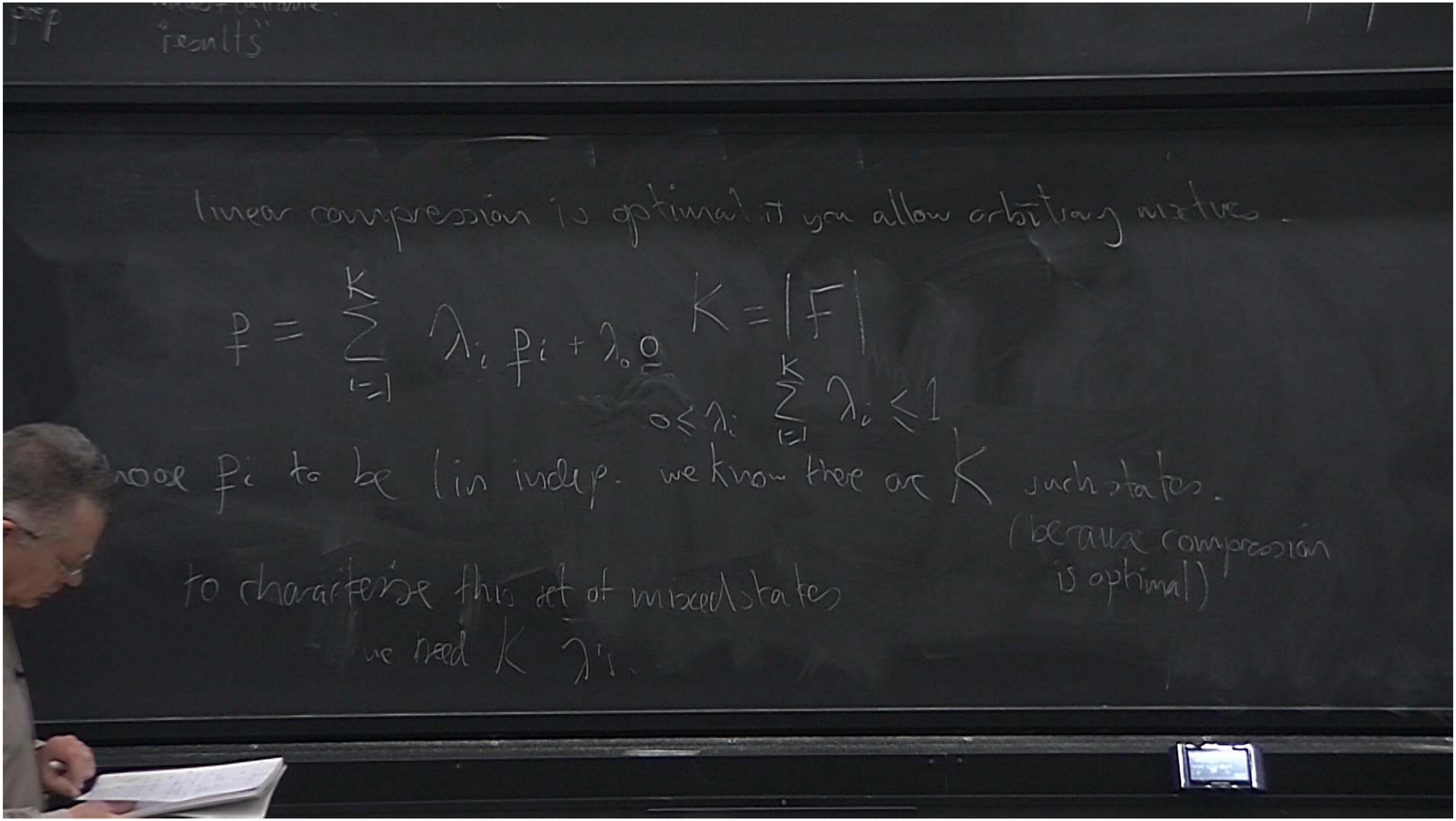
$$F = (z+, z-, x+, y+)$$

that we can write

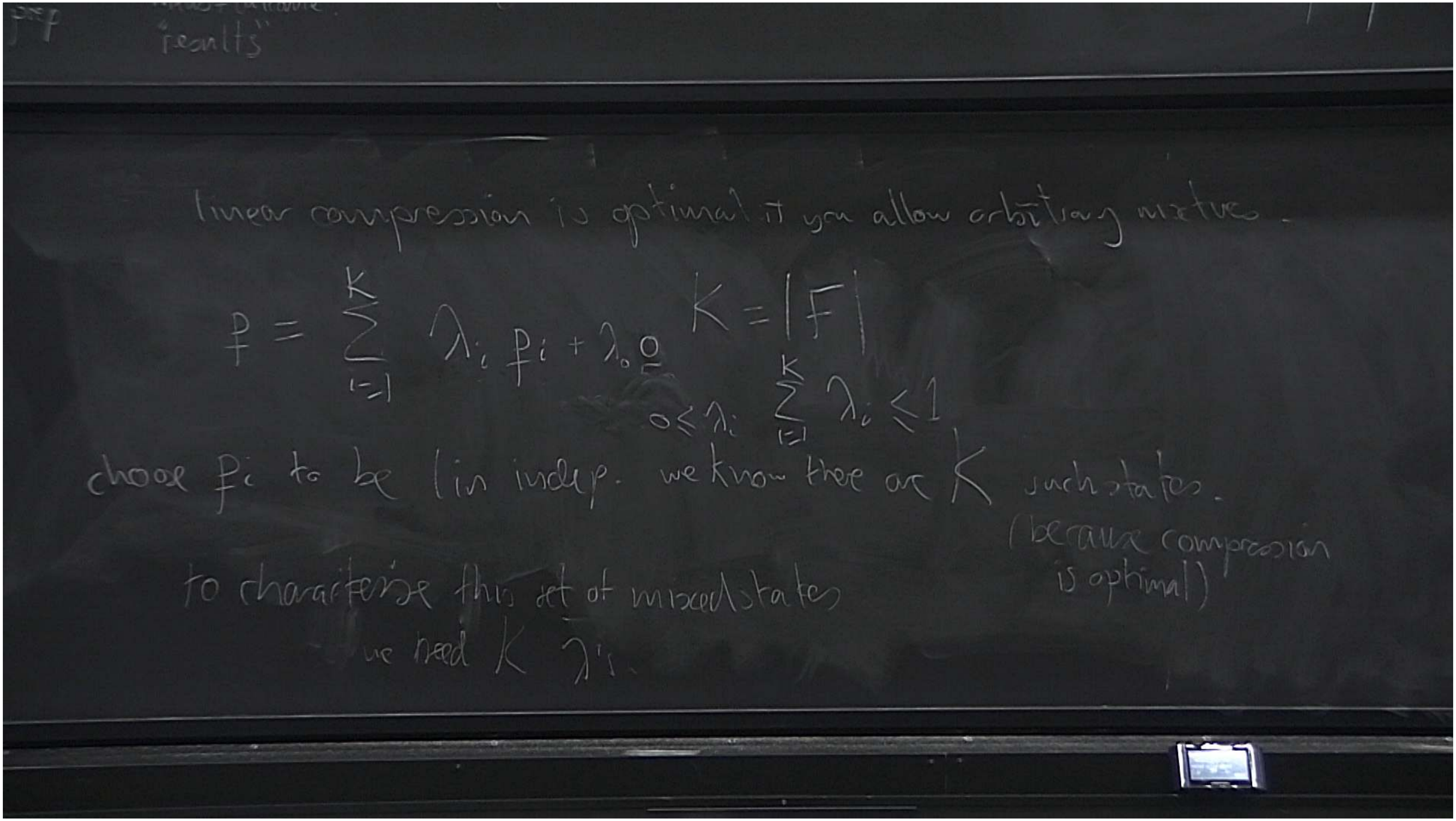

α runs over
all possible
results
p = (⋮ p^α ⋮)
α ∈ F
p = (p_z+ p_z- p_x+ p_y+)
F = (z+, z-, x+, y+)
s.t that we can write
prob = r · p
B
A
p should have
fewest possible elements


"results"
linear compression is optimal if you allow arbitrary mixtures.
$\rho = \sum_{i=1}^{K} \lambda_i \rho_i$
$K = |F|$
choose ρ_i to be lin indep. we know there are K such states.
(because compression is optimal)


"results"
linear compression is optimal if you allow arbitrary mixtures.
ρ = Σ_{i=1}^{K} λ_i ρ_i + λ_0 0
K = |F|
0 ≤ λ_i   Σ_{i=1}^{K} λ_i ≤ 1
choose ρ_i to be lin indep. we know there are K such states.
(because compression is optimal)



"results"
linear compression is optimal if you allow arbitrary mixtures.
$\rho = \sum_{i=1}^{K} \lambda_i \rho_i + \lambda_0 \underline{0}$
$K = |F|$
$0 \leq \lambda_i \quad \sum_{i=1}^{K} \lambda_i \leq 1$
hoose ρ_i to be lin indep. we know there are K such states.
(because compression is optimal)
to characterise this set of mixed states
we need K λ_i.


"results"
linear compression is optimal if you allow arbitrary mixtures.
$\rho = \sum_{i=1}^{K} \lambda_i \rho_i + \lambda_0 0$ $K = |F|$
$0 \leq \lambda_i$ $\sum_{i=1}^{K} \lambda_i \leq 1$
choose ρ_i to be lin indep. we know there are K such states.
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