Title: Stability of Hidden-Variables Theories: De Broglie Versus Bohm

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Abstract: <p>Hidden-variables theories account for quantum mechanics in terms of a particular 'equilibrium' distribution of underlying parameters corresponding to the Born rule. A natural question to ask is whether the theory is stable under small perturbations away from equilibrium. We compare and contrast two examples: de Broglie's 1927 pilot-wave theory and Bohm's 1952 reformulation thereof. It is well established that in de Broglie's dynamics initial deviations from equilibrium will relax. We show that this is not the case for Bohm's dynamics: initial deviations from equilibrium do not relax and in fact grow with time. On this basis we argue that Bohm's dynamics is untenable as a physical theory (while de Broglie's dynamics remains a viable candidate). We advocate stability as a general selection criterion for hidden-variables theories.</p>

Stability of hidden-variables theories: de Broglie versus Bohm

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A. Sketch of background ideas.

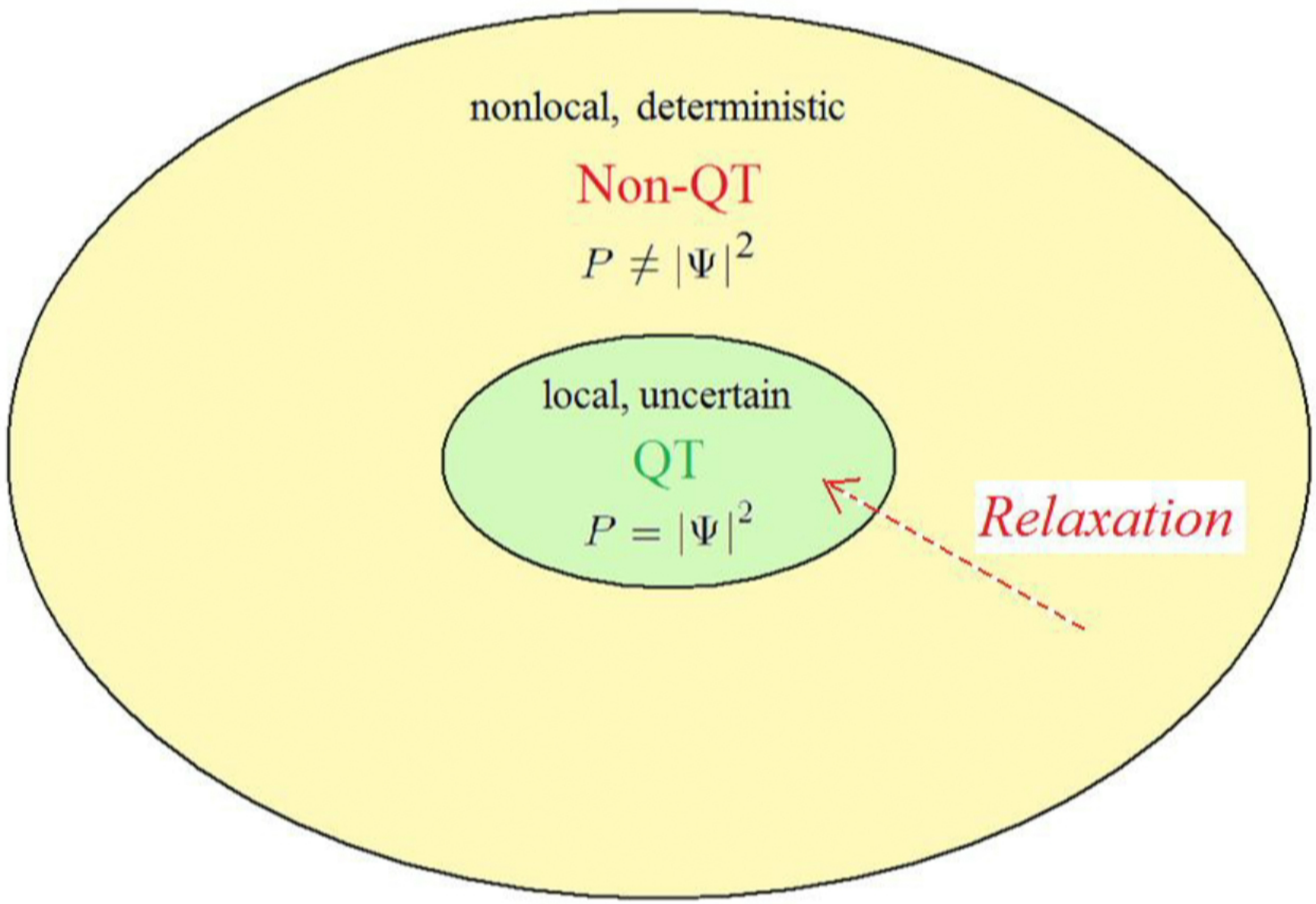
Quantum theory as an "equilibrium" physics (special case of a much wider physics)

B. De Broglie's dynamics (1927) versus Bohm's dynamics (1952)

C. Instability (and untenability) of Bohm's dynamics

D. Stability as a criterion for hidden-variables theories

Quantum theory is a special case of a much wider physics



(Valentini 1991, 1992)

Quantum non-locality: a fine-tuning problem

- Quantum theory is nonlocal
- And yet, we can't use entanglement for nonlocal signalling
- WHY NOT?
- Formally: local statistics, despite Bell's theorem
- Conspiracy in the laws of physics?

Quantum non-locality: a fine-tuning problem

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- Conspiracy in the laws of physics?

Is there a “conspiracy” in the basic laws of physics?

It’s as if something is going on “behind the scenes”, something we cannot control or observe directly, something that violates the standard laws of physics.

Why should these “spooky connections” be hidden?

Quantum theory is a special case of a much wider physics

We cannot send signals faster than light,
and we cannot beat the uncertainty principle.

NOT because these are laws of nature,
but only because we are stuck in “equilibrium”.

And we are stuck in “equilibrium”, because everything we can see has a long and violent astrophysical history.

This can be tested using inflationary cosmology
(colloquium tomorrow)

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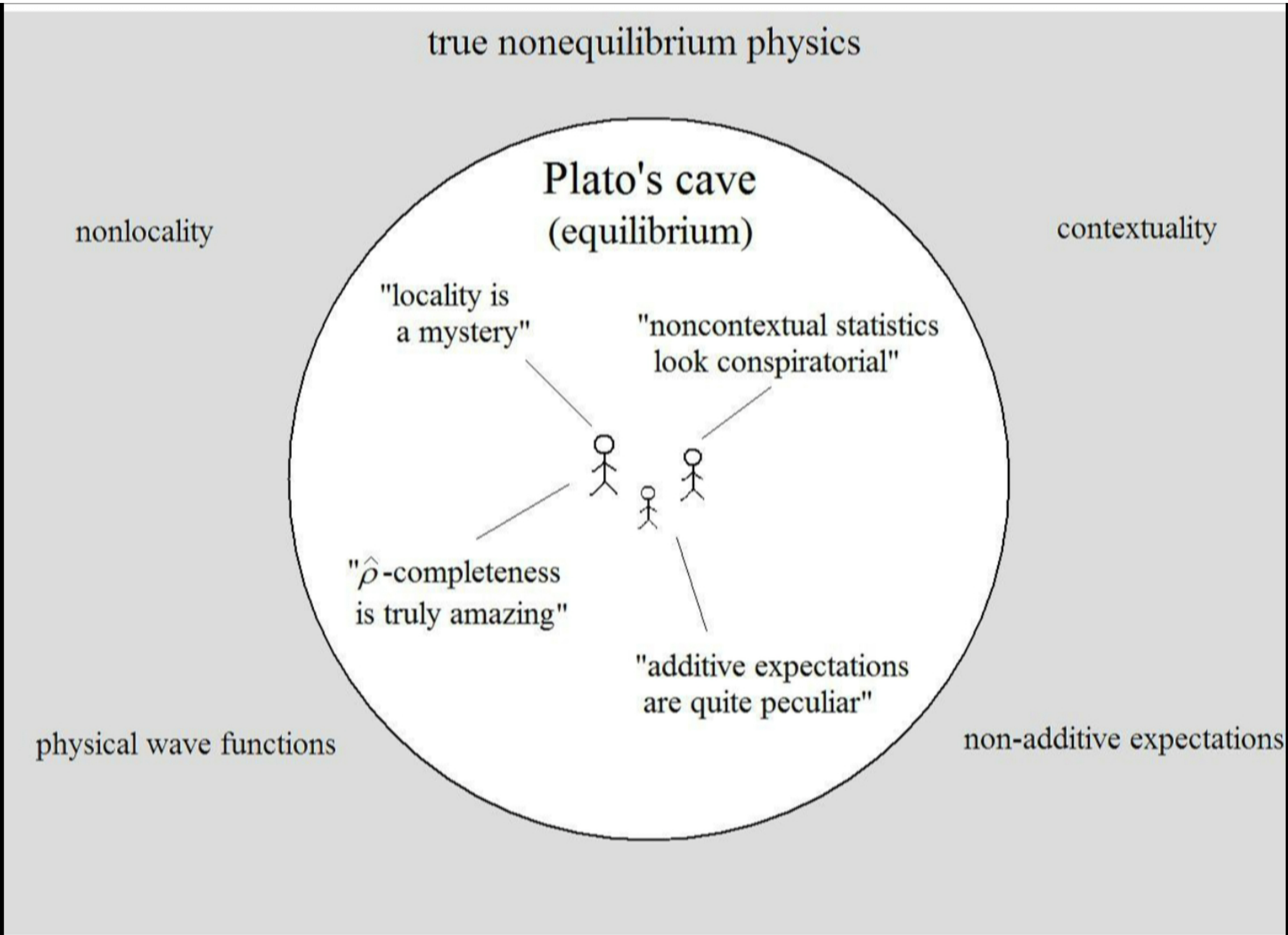
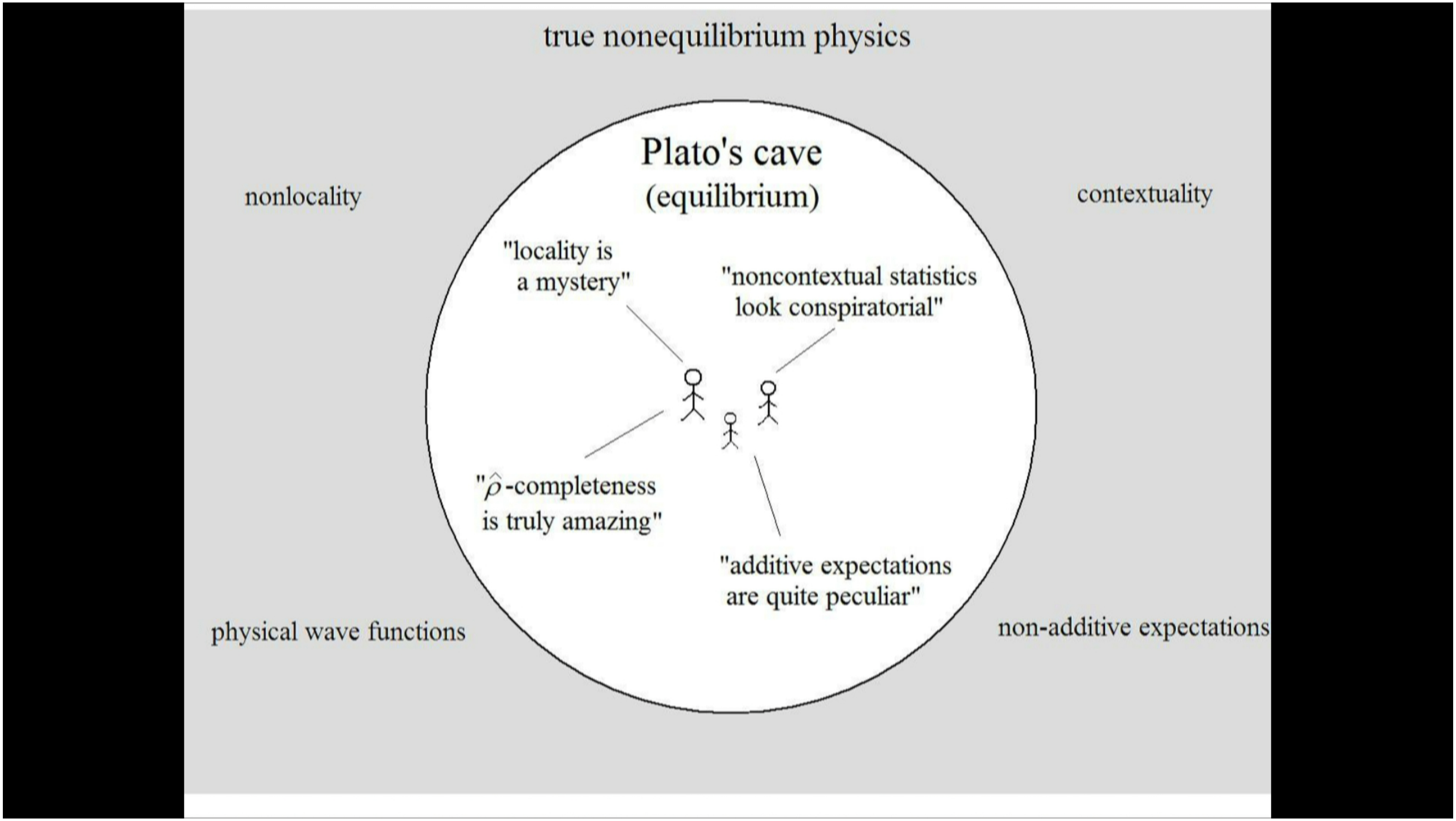
Multiple fine-tuning problems

- Local statistics, despite Bell's theorem (AV 2002) (cf. Wood and Spekkens 2015)
- Noncontextual statistics, despite the Kochen-Specker theorem (similar, more general)
- Additive expectations, despite failure of additivity for single systems (AV 2004)
- Completeness of the density matrix, despite the Pusey-Barrett-Rudolph theorem

'Conspiracies' in the laws of physics?

Explain by "information loss" in "equilibrium"


true nonequilibrium physics
Plato's cave
(equilibrium)
nonlocality
contextuality
"locality is a mystery"
"noncontextual statistics look conspiratorial"
"$\hat{\rho}$-completeness is truly amazing"
"additive expectations are quite peculiar"
physical wave functions
non-additive expectations



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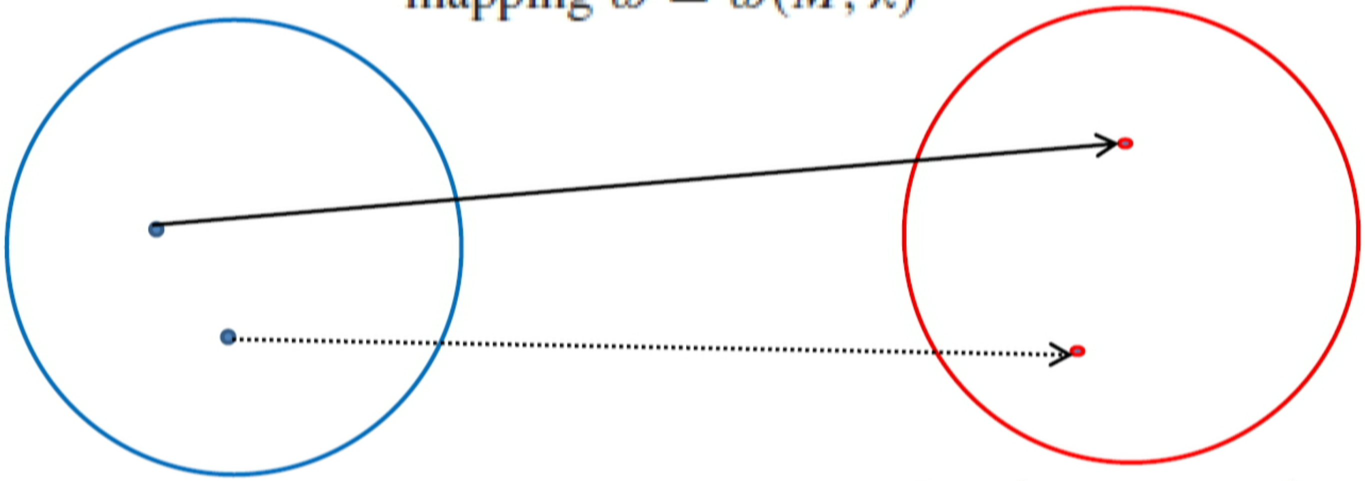
General deterministic hidden-variables theories

mapping $\omega = \omega(M, \lambda)$



Set of hidden variables

Set of outcomes of quantum experiments

(apparatus settings *M*)

distribution $\rho(\lambda)$

$$\langle \omega \rangle = \int \mathrm{d}\lambda \, \rho(\lambda) \omega(M, \lambda)$$

particular distribution $\rho_{\mathrm{QT}}(\lambda)$

$$\langle \omega \rangle_{\mathrm{QT}} = \mathrm{Tr}(\hat{\rho}\hat{\Omega})$$

General distributions of hidden-variables violate the Born rule nonequilibrium

$$\rho(\lambda) \neq \rho_{\mathrm{QT}}(\lambda)$$

equilibrium

$$\rho(\lambda) = \rho_{\mathrm{QT}}(\lambda)$$

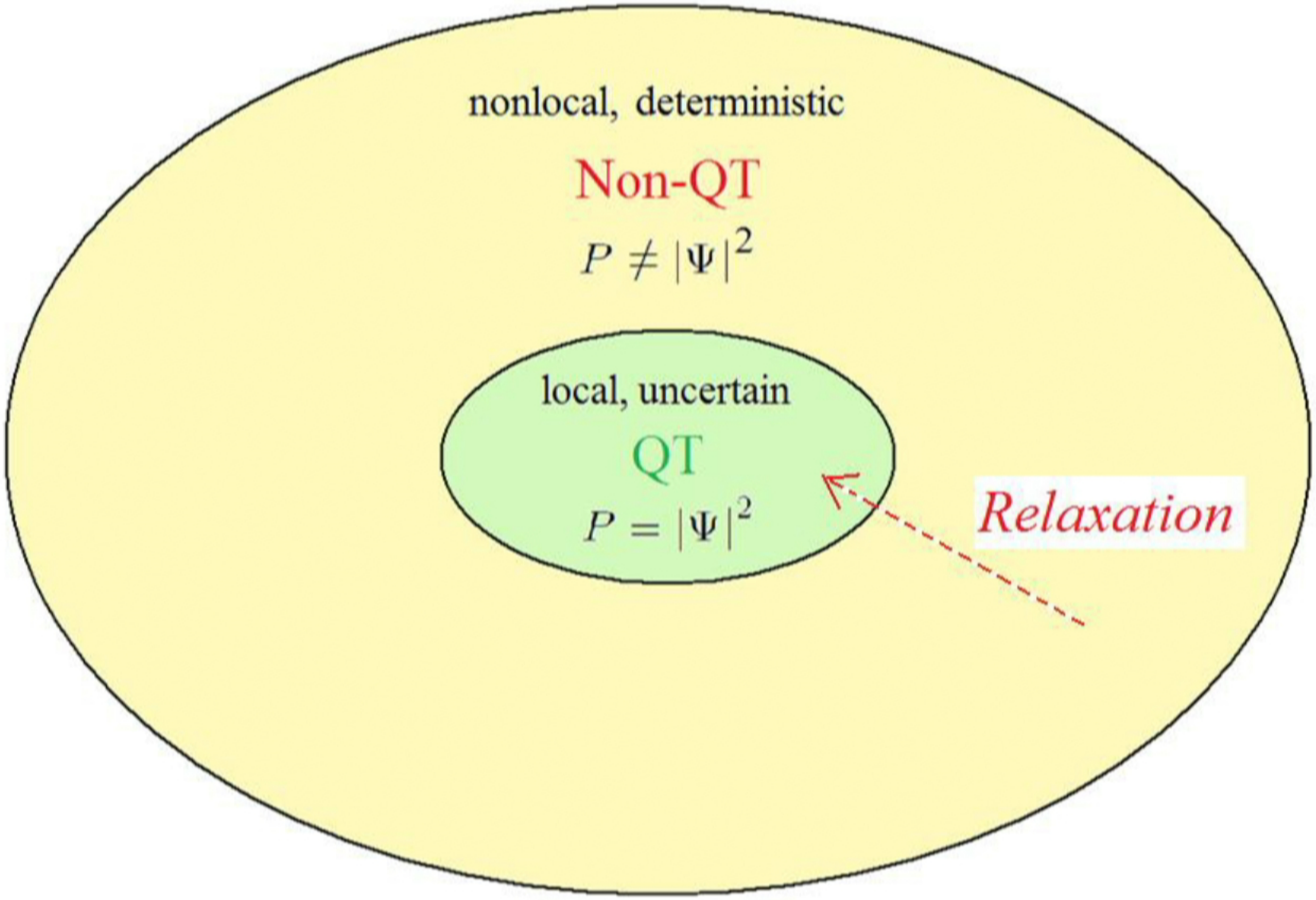
Born rule

$$\langle \omega \rangle = \langle \omega \rangle_{\mathrm{QT}}$$

Born rule violated

$$\langle \omega \rangle \neq \langle \omega \rangle_{\mathrm{QT}}$$

Quantum theory is a special case of a much wider physics

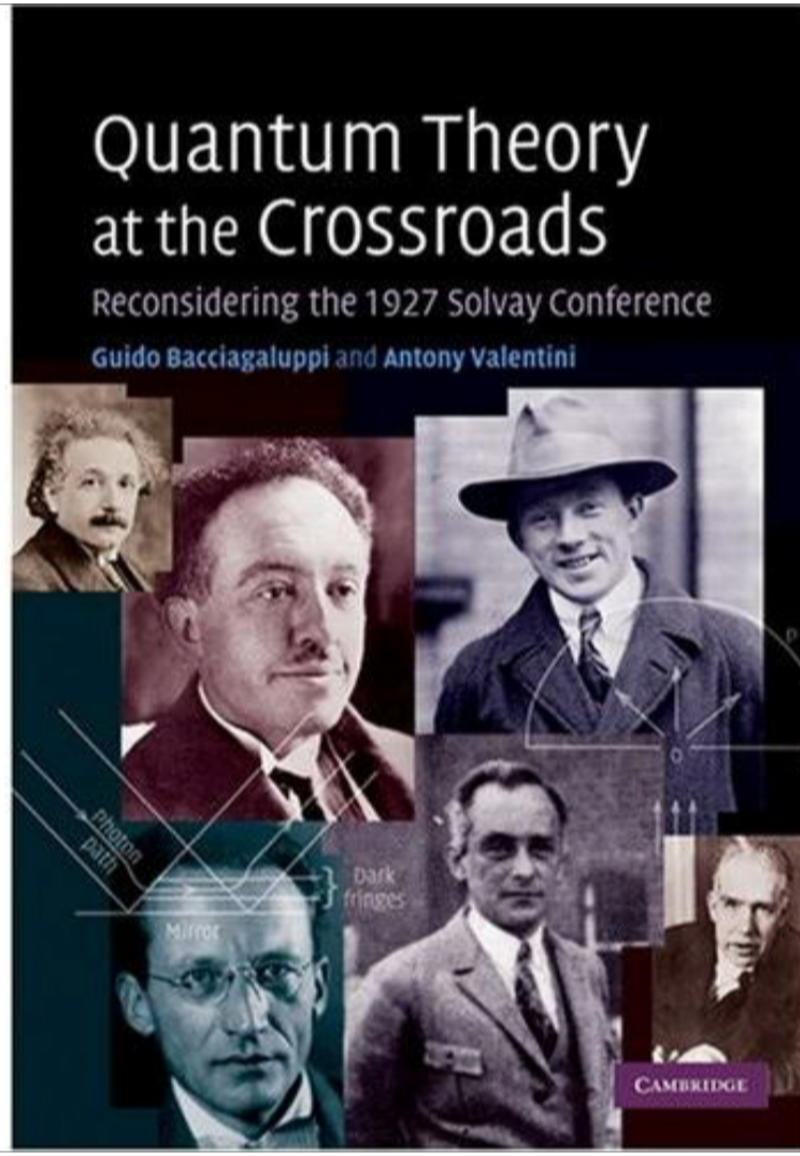


(Valentini 1991, 1992)

G. Bacciagaluppi and A. Valentini, *Quantum Theory at the Crossroads* (Cambridge University Press, 2009) [quant-ph/0609184]

De Broglie's largely-unknown but major contributions:

1923—27: de Broglie developed a new, non-Newtonian form of dynamics

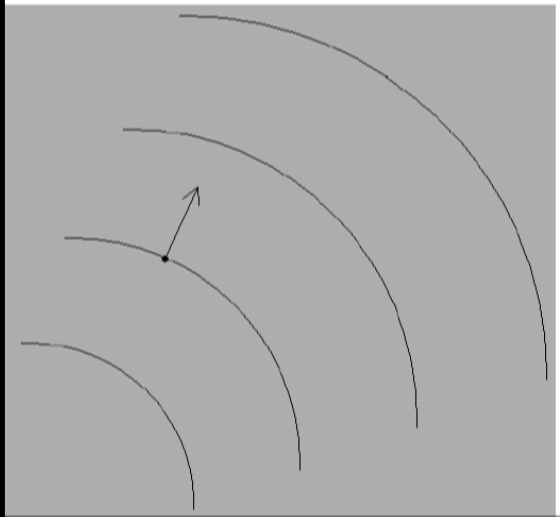


De Broglie's Pilot-Wave Dynamics (1927)

$$i\frac{\partial \Psi}{\partial t} = \sum_{i=1}^{N} -\frac{1}{2m_i}\nabla_i^2\Psi + V\Psi$$

$$(\ \Psi = |\Psi| e^{iS} \ ) \qquad m_i \frac{d\mathbf{x}_i}{dt} = \nabla_i S$$

(cf. Bell 1987)



(Generalise: configuration $q(t)$)

Get QM if *assume* initial Born-rule distribution, $P = |\Psi|^2$ (preserved in time by the dynamics)

(shown fully by Bohm in 1952)

Illustration for one particle

$$i\frac{\partial \Psi}{\partial t} = -\frac{1}{2m}\nabla^2\Psi + V\Psi \Longrightarrow \frac{\partial |\Psi|^2}{\partial t} + \nabla \cdot \left(|\Psi|^2 \frac{\nabla S}{m} \right) = 0$$

Guidance equation $m\frac{d\mathbf{x}}{dt} = \nabla S$ applied to an *ensemble* (same Ψ, different $\mathbf{x}_0$'s).

Distribution $P(\mathbf{x}, t)$ obeys $\frac{\partial P}{\partial t} + \nabla \cdot \left(P \frac{\nabla S}{m} \right) = 0$

THEOREM (quantum equilibrium):

If $P(\mathbf{x}, 0) = |\Psi(\mathbf{x}, 0)|^2$, then $P(\mathbf{x}, t) = |\Psi(\mathbf{x}, t)|^2$ for all *t*

(Generalisation: replace $\mathbf{x}(t)$ by general configuration $q(t)$)

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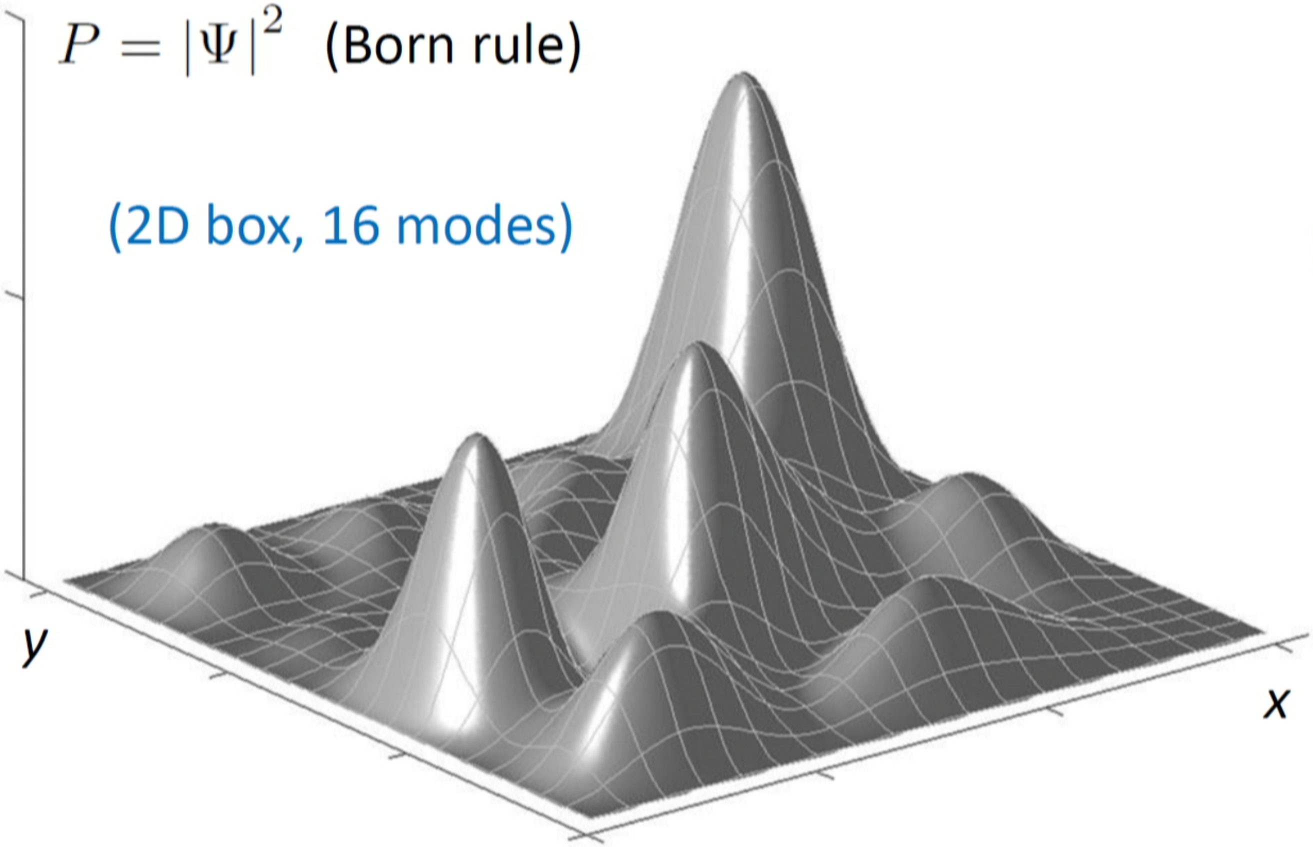
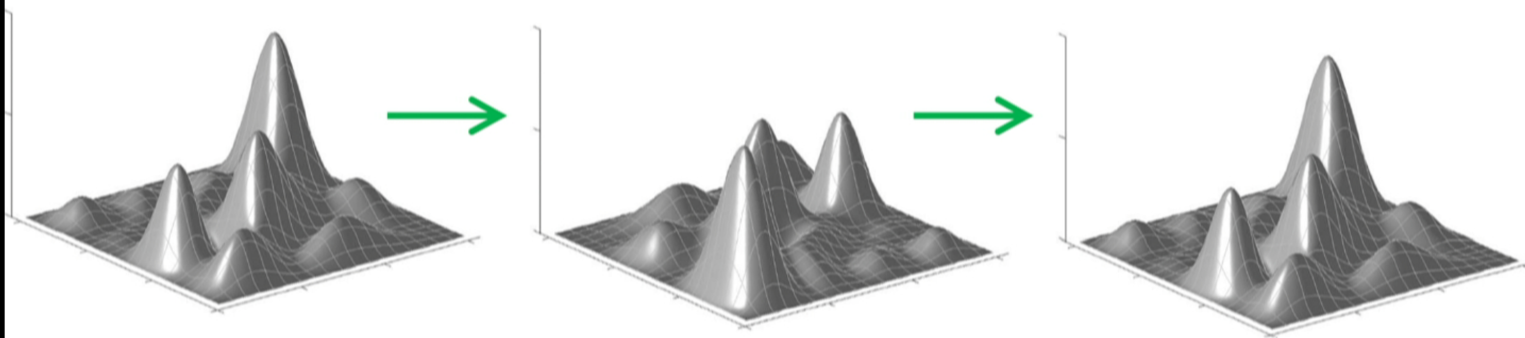
BUT: *experimentally* quantum dof's are always found to have the "quantum equilibrium" distribution:



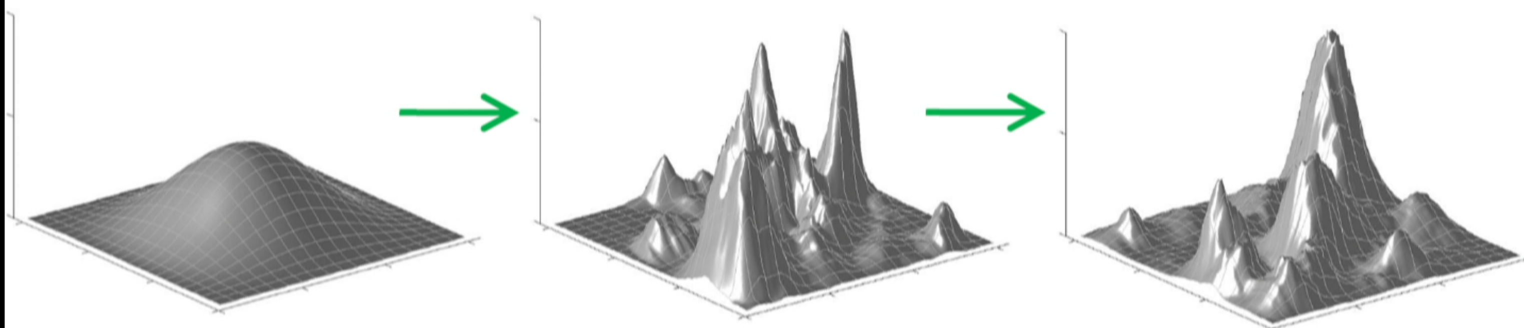
Why?

Quantum relaxation (cf. thermal relaxation)

Equilibrium ($P = |\Psi|^2$) changes with time

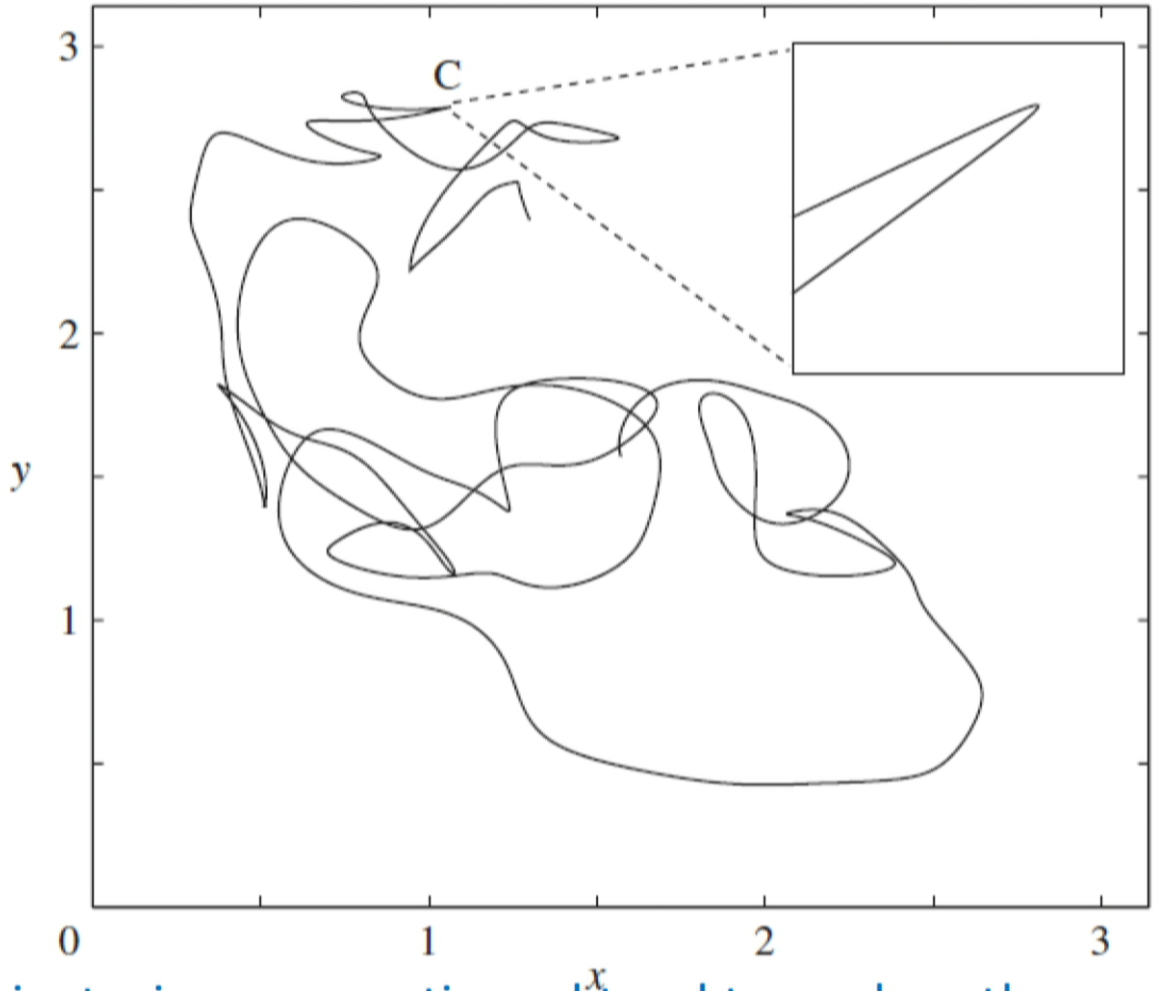


Non-equilibrium ($P \neq |\Psi|^2$) relaxes to equilibrium



(Valentini and Westman, Proc. Roy. Soc. A 2005)

Superposed energies give rapidly-varying velocity fields



Trajectories are erratic and tend to explore the region

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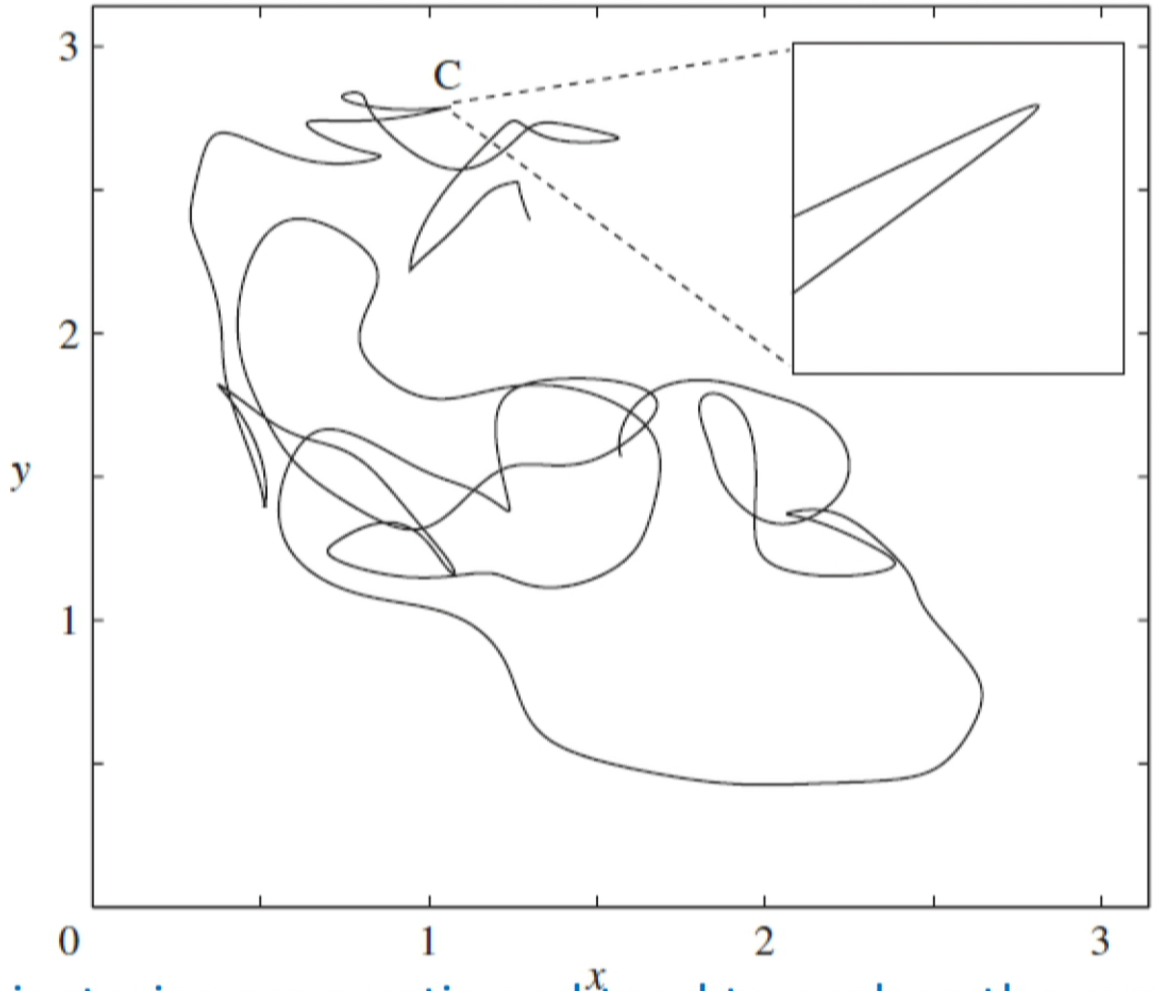

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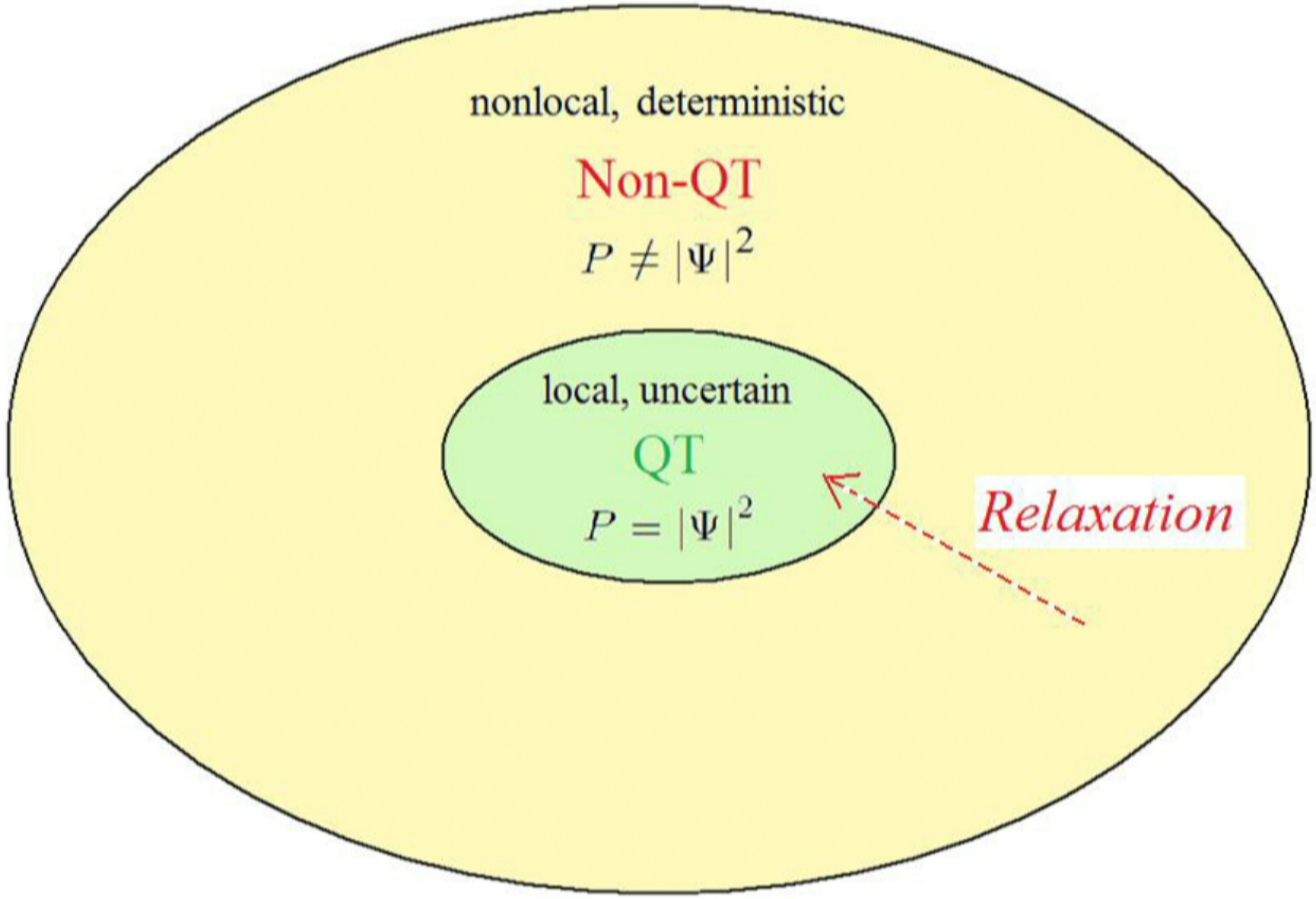
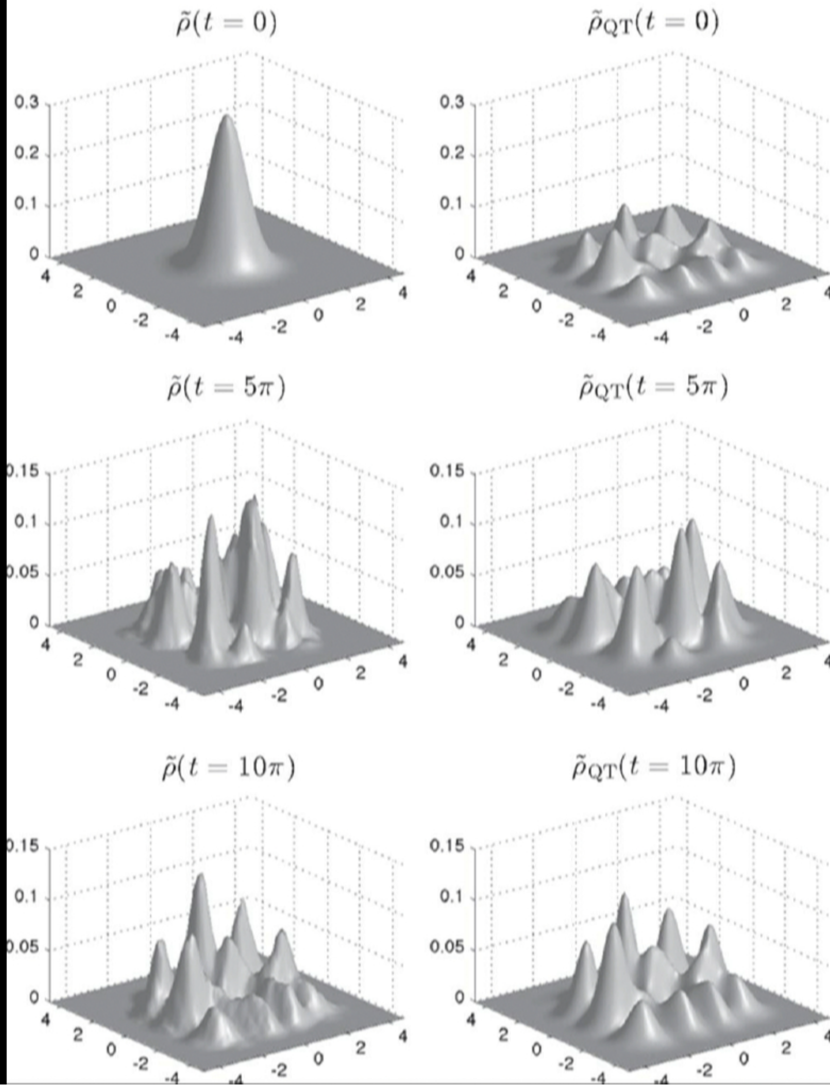
Quantum theory is a special case of a much wider physics



(Valentini 1991, 1992)

Confirmed and extended by many independent simulations

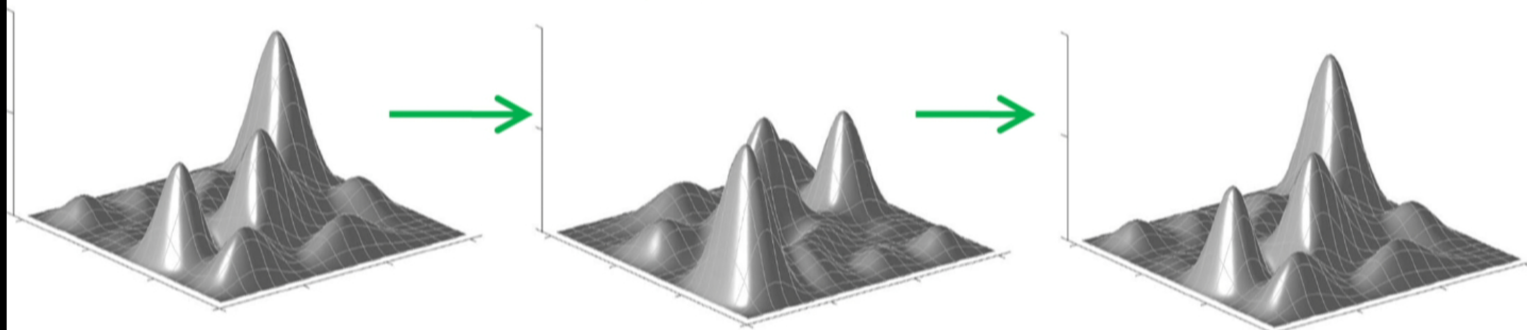


2D oscillator, 25 modes in superposition (Abraham, Colin and Valentini, J. Phys. A 2014)

Main point for purposes of this talk:

in de Broglie's dynamics, quantum equilibrium is stable under (even large) perturbations away from that state

Equilibrium ($P = |\Psi|^2$) changes with time



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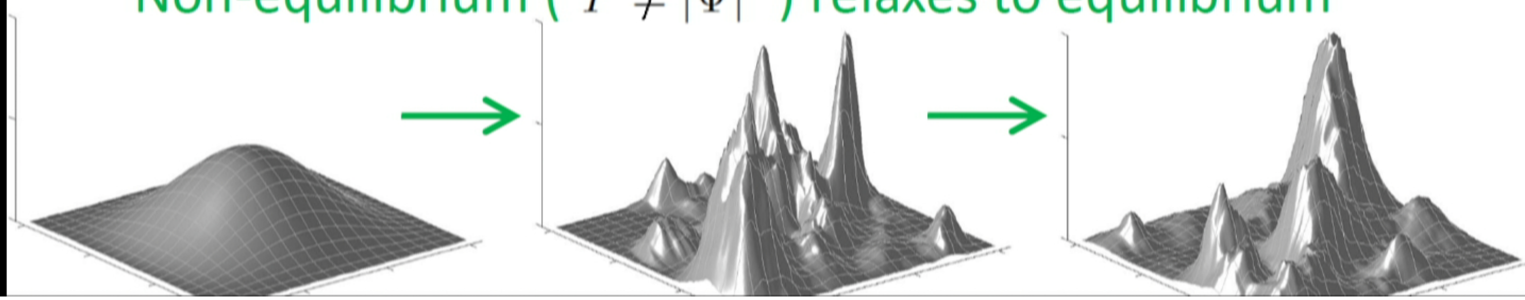
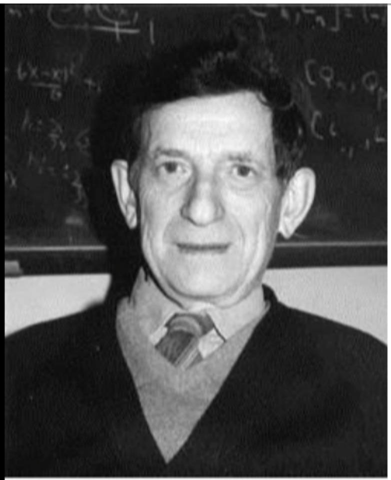


What about something similar in Bohm's dynamics?



Bohm's Newtonian version (1952)

$$m_i \frac{d^2\mathbf{x}_i}{dt^2} = -\nabla_i (V + Q) \quad \text{(law of motion)}$$

$$Q \equiv -\sum_{i=1}^{N} \frac{1}{2m_i} \frac{\nabla_i^2 |\Psi|}{|\Psi|}$$

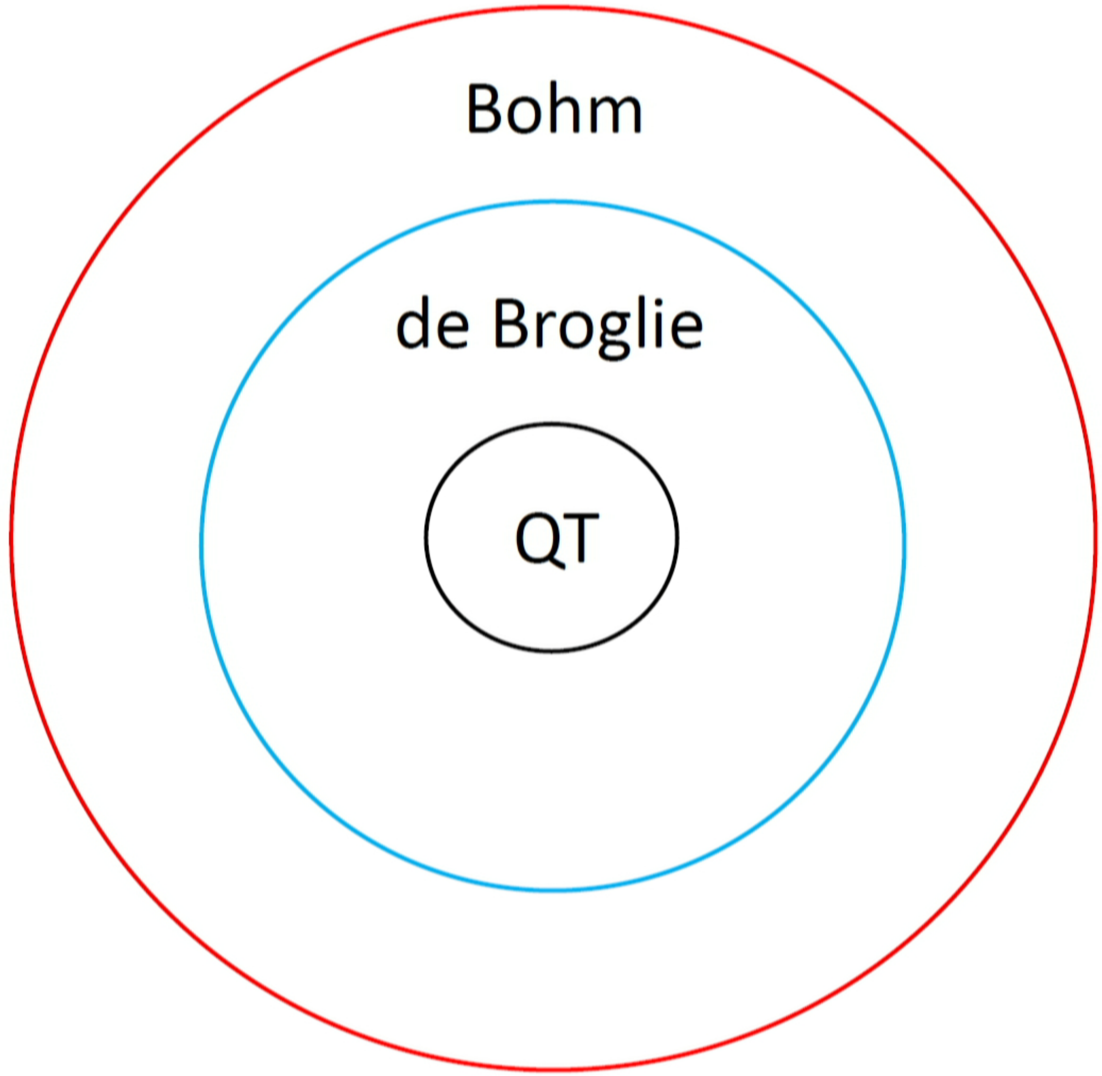
Get QM if assume initial $P = |\Psi|^2$ *and* $\mathbf{p}_i = \nabla_i S$

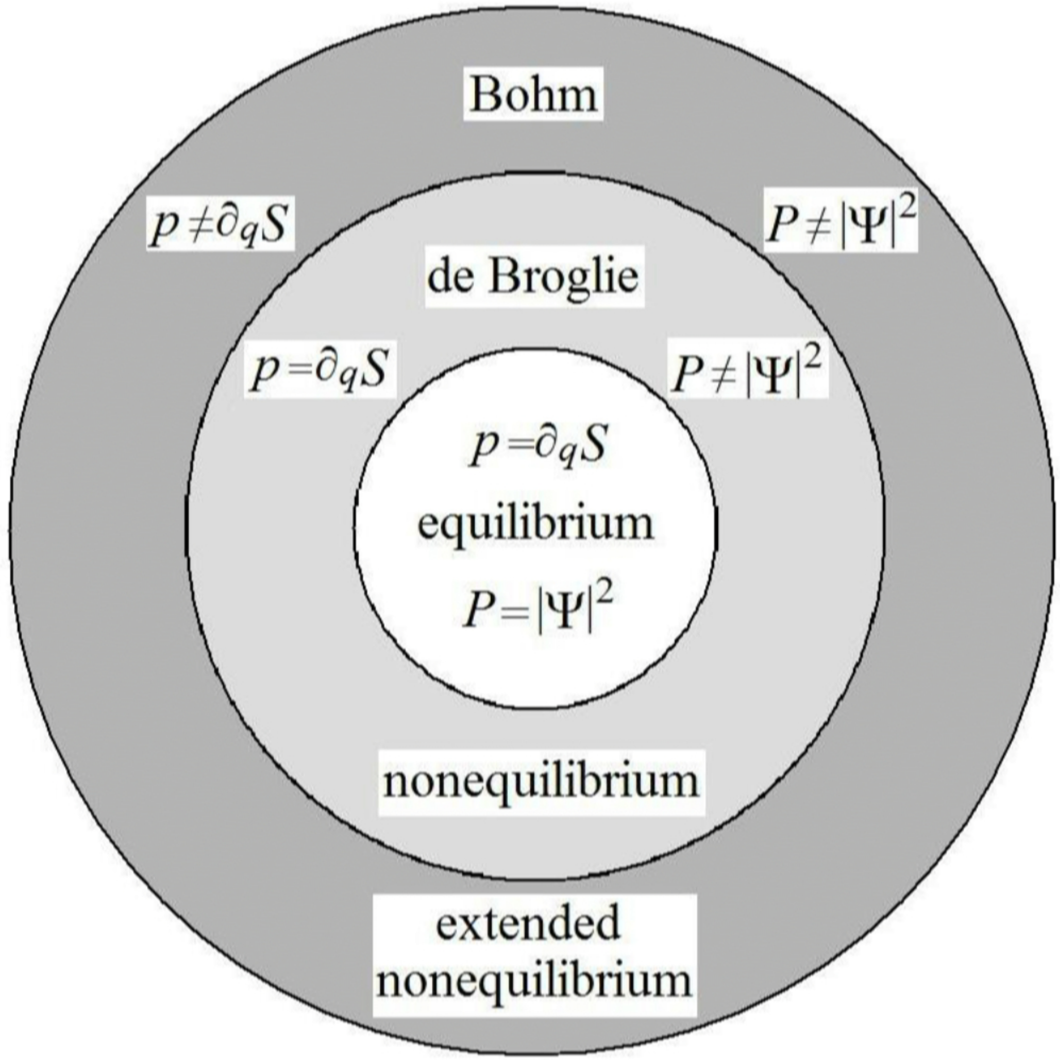
For Bohm, $\mathbf{p}_i = \nabla_i S$ is an initial condition; can drop it.

For de Broglie, $\mathbf{p}_i = \nabla_i S$ is the law of motion.

De Broglie's dynamics and Bohm's dynamics are different.

"Bohmian mechanics" is a misnomer for de Broglie's dynamics.


Bohm
de Broglie
QT



Bohm
$p \neq \partial_q S$
$P \neq |\Psi|^2$
de Broglie
$p = \partial_q S$
$P \neq |\Psi|^2$
$p = \partial_q S$
equilibrium
$P = |\Psi|^2$
nonequilibrium
extended nonequilibrium

Bohm's dynamics (1952)

$$m_i \frac{d^2 \mathbf{x}_i}{dt^2} = -\nabla_i (V + Q) \qquad Q \equiv -\sum_{i=1}^{N} \frac{1}{2m_i} \frac{\nabla_i^2 |\Psi|}{|\Psi|}$$

Get QM if assume initial $P = |\Psi|^2$ *and* $\mathbf{p}_i = \nabla_i S$

For Bohm, $\mathbf{p}_i = \nabla_i S$ is an initial condition; can be relaxed.

In phase space, the equilibrium distribution is

$$\rho_{\mathrm{eq}}(x, p, t) = |\psi(x,t)|^2 \, \delta(p - \nabla S(x,t))$$

[(x, p) multi-dimensional]

Extended nonequilibrium in Bohm's dynamics

Bohm's dynamics allows an arbitrary $\rho(x,p,t)$, satisfying the continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho \dot{x}) + \frac{\partial}{\partial p}(\rho \dot{p}) = 0$$

with the phase-space velocity field

$$\dot{x} = p/m, \qquad \dot{p} = -\nabla(V+Q)$$

(For simplicity, write as if one particle in 1D)

The key question is whether 'reasonable' initial nonequilibrium conditions

$$\rho(x,p,0) \neq |\psi(x,0)|^2 \, \delta(p - \nabla S(x,0))$$

tend to relax to (extended) quantum equilibrium

Answer: they do not

Comparison with classical case

Bohm's dynamics is just Newton's dynamics
with an additional time-dependent potential $Q(x,t)$
added to the usual potential V

Because of the time dependence of Q,
the energy of the system is not conserved.

(No fixed energy surface in phase space.)

But we still have Liouville's theorem, as for
any Hamiltonian system:

$$\begin{aligned}\frac{d\rho}{dt} &= \frac{\partial \rho}{\partial x}\dot{x} + \frac{\partial \rho}{\partial p}\dot{p} + \frac{\partial \rho}{\partial t} \\ &= \frac{\partial \rho}{\partial x}\dot{x} + \frac{\partial \rho}{\partial p}\dot{p} - \frac{\partial}{\partial x}(\rho\dot{x}) - \frac{\partial}{\partial p}(\rho\dot{p})\end{aligned}$$

and so (along a trajectory)

$$\frac{d\rho}{dt} = -\rho\left(\frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{p}}{\partial p}\right) = 0$$

Because dρ/dt = 0 , we have TWO equilibrium distributions:

(1) If $\rho(x,p,0) = c$ (for some constant c) over the available region of phase space, then

$$\rho(x,p,t) = c$$

at all times t (the classical equilibrium distribution)

(2) The quantum equilibrium distribution

$$\rho_{\mathbf{eq}}(x,p,t) = |\psi(x,t)|^2\,\delta(p - \nabla S(x,t))$$

(also conserved by Bohm's dynamics)

This is unusual

Some comments and queries:

1. Will an arbitrary initial state relax to one of these equilibrium states? Or to neither? Might guess that the existence of two equilibrium measures will 'confuse' the system

2. If we appeal to 'typicality' wrt to an equilibrium measure, which one should we choose?

3. Bohm's dynamics is an unusual dynamical system. Beware of standard intuitions and expectations.

Our central claims (Bohm's dynamics)

We have extensive evidence that:

(1) there is no tendency to relax to quantum equilibrium

(2) the quantum equilibrium state is in fact unstable

(3) if the universe started in nonequilibrium,
we would not see equilibrium today:
-- there would be no bound states (atoms etc)
-- vacuum fields would be arbitrarily large

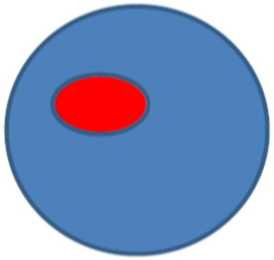
Instability of Bohm's dynamics (simple example)

ground state of a bound system,
e.g. a hydrogen atom or a simple harmonic oscillator (SHO)

Contrast with de Broglie's dynamics:

p = grad S = 0 , *velocities* vanish

Therefore, an initial small deviation from $P = |\Psi|^2$
stays small (and indeed static)



For superpositions, initial small deviations relax (see before)

Whereas in Bohm's dynamics:

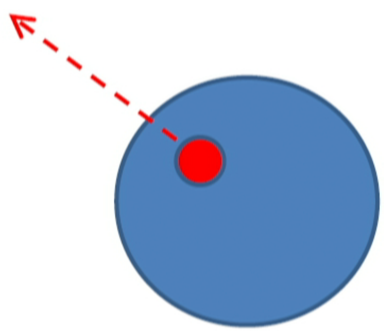
for a ground state,

$$-\text{grad } V - \text{grad } Q = 0 \quad , \qquad \textit{accelerations} \text{ vanish}$$

Therefore, an initial small deviation of p from grad S (= 0) remains small (and indeed static)

But: causes a GROWTH in non-equilibrium wrt *position*.

Bound states become UNBOUND



And for superpositions, do NOT get relaxation ...

Nonequilibrium *superpositions* in Bohm's dynamics

(relevant for the history of our universe)

-- instability is not an artifact of simple ground state

-- states with superpositions are also unstable

-- show this numerically and analytically

Numerical results for the harmonic oscillator I

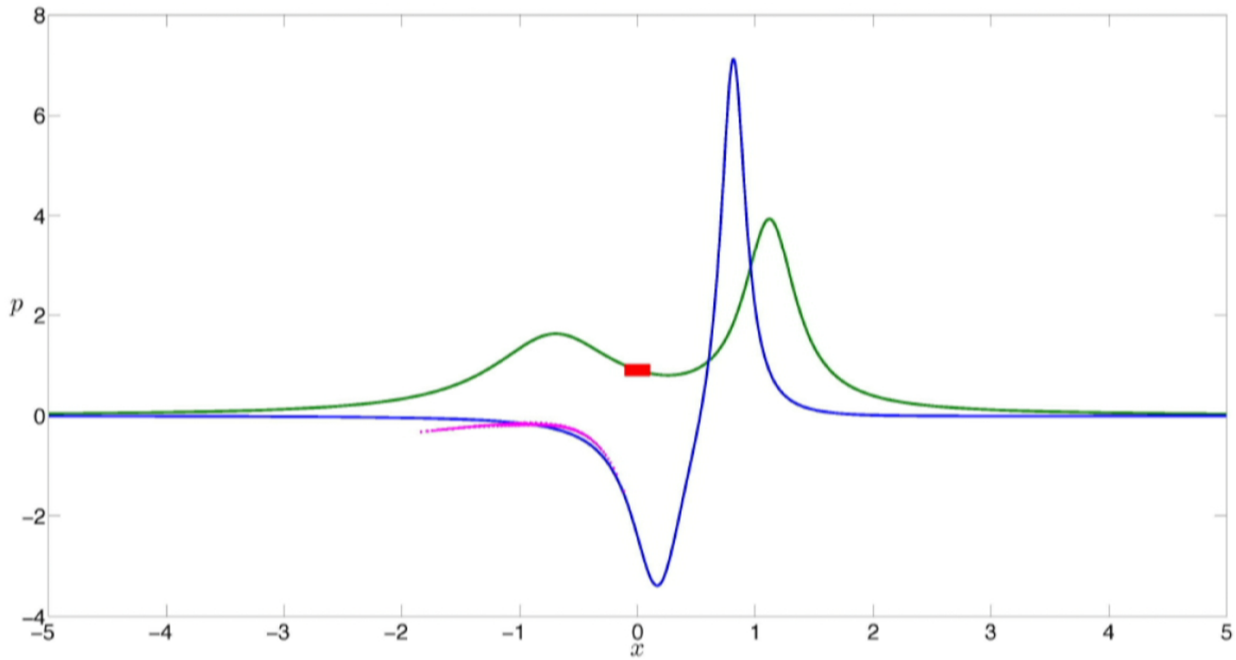


Figure 3: Partial relaxation with almost-de Broglian initial momenta. The distribution at $t = 0$ (red) is concentrated in a small region centred on a point of the curve $p = \partial S(x,0)/\partial x$ (green). The distribution at $t = 5$ (magenta) is found to be bunched around the curve $p = \partial S(x,5)/\partial x$ (blue).

$$\psi(x,t) = \frac{1}{\sqrt{3}}(\phi_0\, e^{-it/2} + e^{i\theta_1}\phi_1\, e^{-i3t/2} + e^{i\theta_2}\phi_2\, e^{-i5t/2})$$

Numerical results for the hydrogen atom

$$\psi(\mathbf{x},0) = \frac{1}{\sqrt{3}}[\phi_{100}(\mathbf{x}) + e^{i}\phi_{211}(\mathbf{x}) + e^{2i}\phi_{32-1}(\mathbf{x})]$$

Initial velocities increasingly perturbed from those of de Broglie

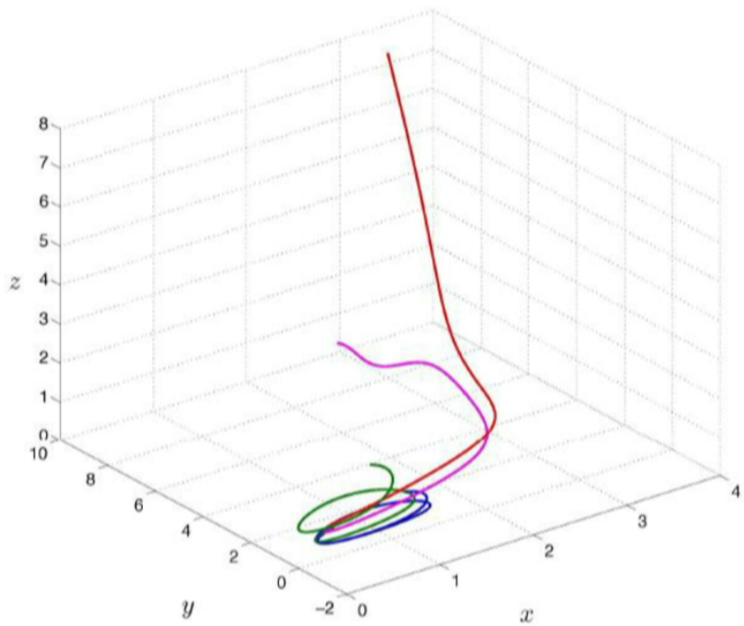


Figure 5: Instability of quantum equilibrium for the hydrogen atom, illustrated by five trajectories with the same initial position $\mathbf{x}_0$ and varying initial momenta $\mathbf{p}_0$ (see main text).

Analytical proof for a harmonic oscillator

Arbitrary superposition of energy states (bounded spectrum):

$$\psi(x,t) = \sum_{m=0}^{M} c_m(0)\, \mathrm{e}^{-\mathrm{i}(m+1/2)t}\, \phi_m(x)$$

Find lower bound on acceleration (for large x)

$a = (-\,\mathrm{grad}\, V - \mathrm{grad}\, Q)/m \; > \; -\, b/x^2$ (some constant b)

Implies *escapes to infinity* for initial velocities above a small critical value

$$v_{\text{escape}} = \sqrt{2b/x_0}$$

(cf. escape velocity from earth, except critical velocity is small)
(escape velocity is small for large x, where de Broglie velocity ≈ 0)

Expect no bound states today?

Might claim that early universe will reach equilibrium long before atoms form (about 400,000 years after the big bang)

If so, would still form bound states later

But: *early fields will show the same instability*

For example, a decoupled scalar field mode is mathematically the same as a 2D simple harmonic oscillator

$$\phi_{\mathbf{k}} = \frac{\sqrt{V}}{(2\pi)^{3/2}}(q_{\mathbf{k}1} + \mathrm{i}q_{\mathbf{k}2}) \qquad \mathrm{i}\frac{\partial \psi_{\mathbf{k}}}{\partial t} = -\frac{1}{2a^3}\left(\frac{\partial^2}{\partial q_{\mathbf{k}1}^2} + \frac{\partial^2}{\partial q_{\mathbf{k}2}^2}\right)\psi_{\mathbf{k}} + \frac{1}{2}ak^2(q_{\mathbf{k}1}^2 + q_{\mathbf{k}2}^2)\psi_{\mathbf{k}},$$

$$a^3\ddot{q}_{\mathbf{k}1} = -\frac{\partial}{\partial q_{\mathbf{k}1}}(V+Q), \quad a^3\ddot{q}_{\mathbf{k}2} = -\frac{\partial}{\partial q_{\mathbf{k}2}}(V+Q) \qquad \dot{q}_{\mathbf{k}1} \neq \frac{1}{a^3}\frac{\partial s_{\mathbf{k}}}{\partial q_{\mathbf{k}1}}, \quad \dot{q}_{\mathbf{k}2} \neq \frac{1}{a^3}\frac{\partial s_{\mathbf{k}}}{\partial q_{\mathbf{k}2}}$$

Would expect to see arbitrarily large field strengths, even in the vacuum, in sharp conflict with observation

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Would expect to see arbitrarily large field strengths, even in the vacuum, in sharp conflict with observation

Bohm's dynamics agrees with quantum theory *only if we assume the special initial conditions*

$$\rho_{\mathbf{eq}}(x,p,t) = |\psi(x,t)|^2\,\delta(p - \nabla S(x,t))$$

Bohm's dynamics agrees with quantum theory *only if we assume the special initial conditions*

$$\rho_{\text{eq}}(x, p, t) = |\psi(x, t)|^2 \, \delta(p - \nabla S(x, t))$$

If Bohm's dynamics is correct:

we should *not* expect to see effective quantum theory today, in contradiction with observation

Conclusion: Bohm's dynamics is untenable

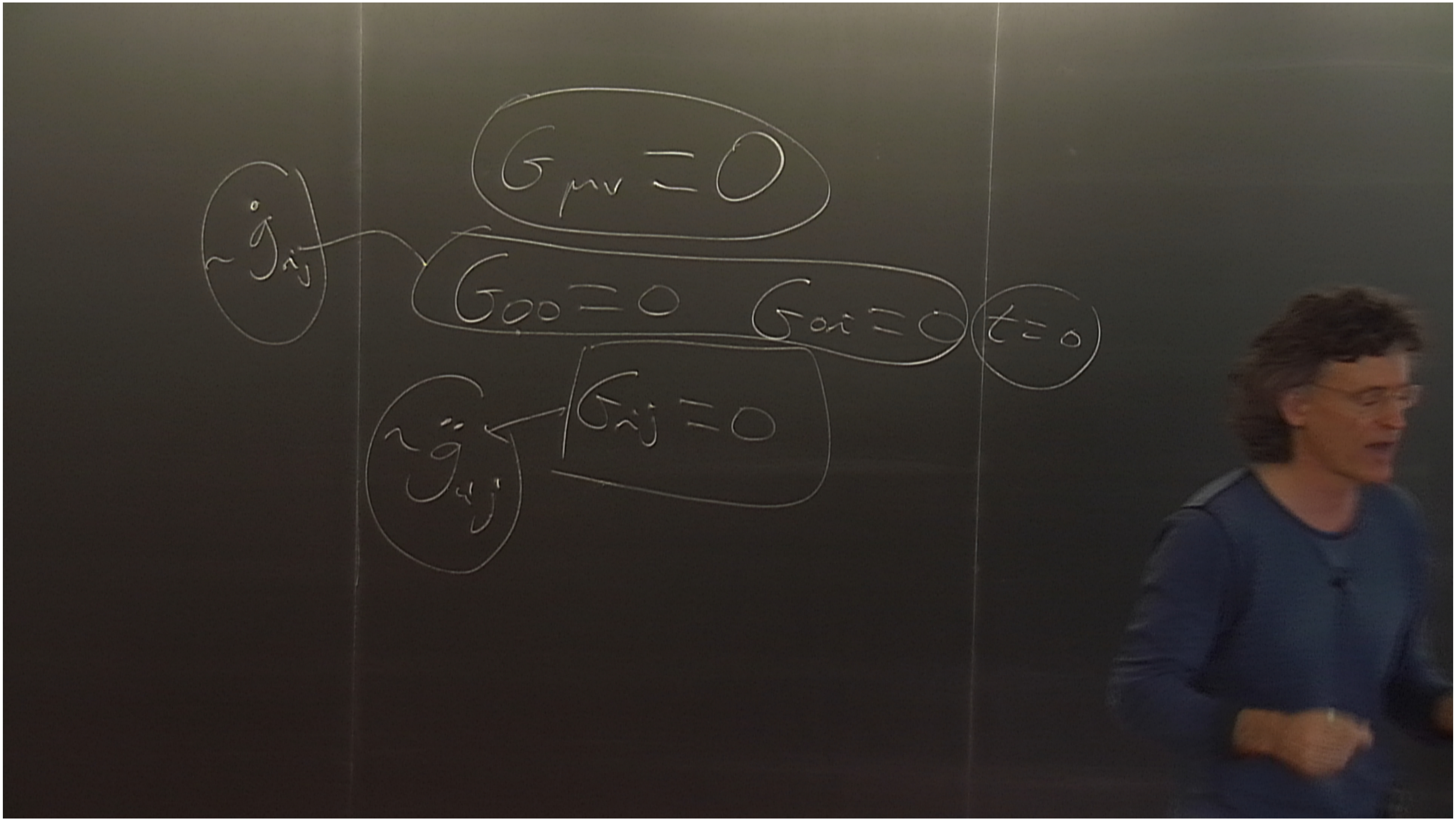
Bohm's (1952) pseudo-Newtonian reformulation of de Broglie's dynamics was a mistake.
It is not a viable candidate for a physical theory.

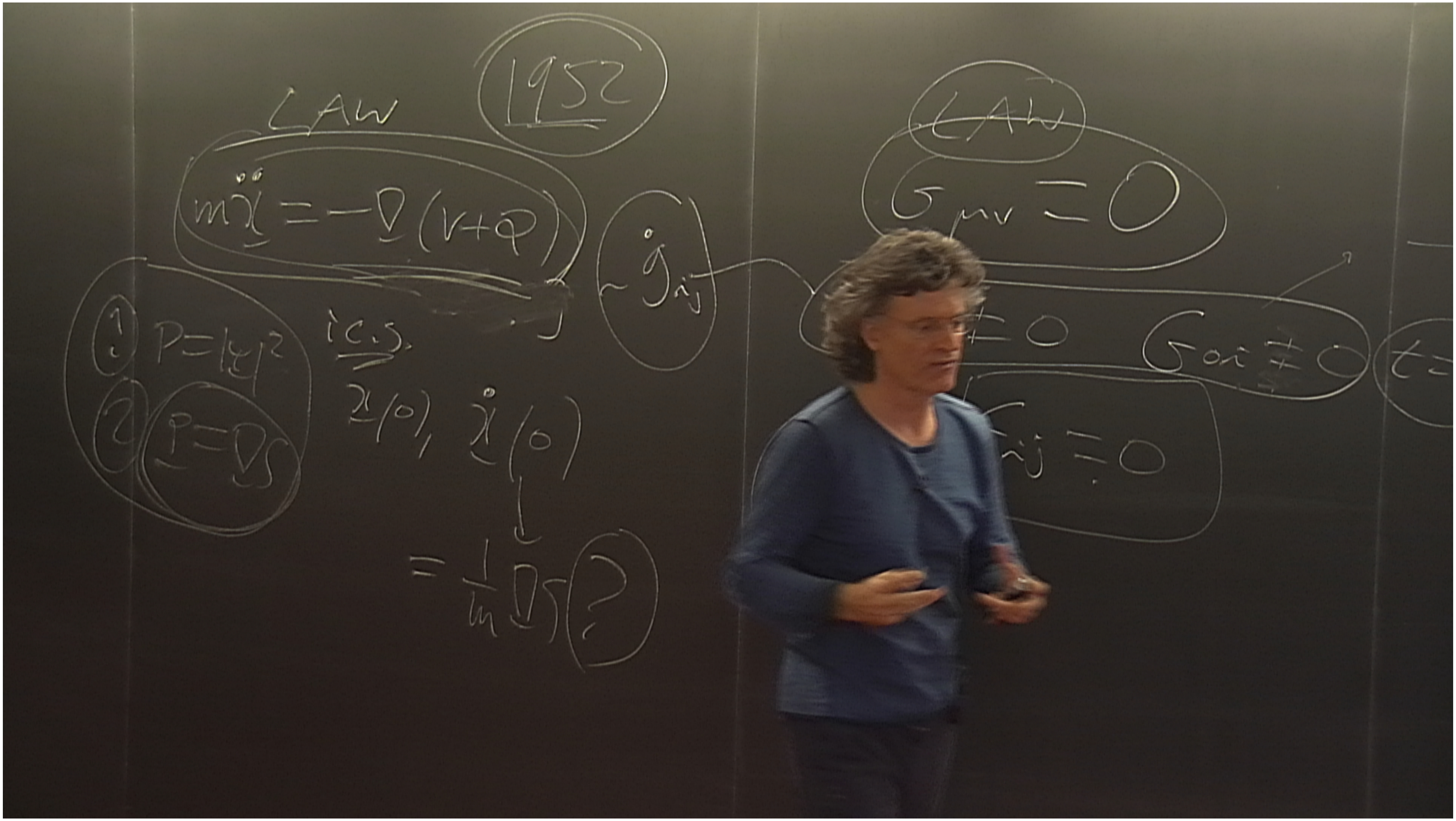
De Broglie's original (1927) formulation was the correct one.
It remains a viable candidate for a physical theory.

Stability as a criterion for hidden-variables theories

-- proper interpretation of a (deterministic) hidden-variables theory requires that we consider quantum nonequilibrium, in order to see the true and general physics of the theory (AV, since 1991)

-- to account convincingly for what we see, the theory should be stable under (at least) small perturbations away from equilibrium, otherwise we replace one conspiracy with another


Gμν = 0
G00 = 0
G0i = 0
t = 0
Gij = 0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