Title: Universal Limit on Communication

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Abstract: <p>I derive a universal upper bound on the capacity of any communication channel between two distant systems. The Holevo quantity, and hence the mutual information, is at most of order EÎ”t/â„•, where E the average energy of the signal, and Î”t is the amount of time for which detectors operate. The bound does not depend on the size or mass of the emitting and receiving systems, nor on the nature of the signal. No restrictions on preparing and processing the signal are imposed.
As an example, I consider the encoding of information in the transverse or angular position of a signal emitted and received by systems of arbitrarily large cross-section. In the limit of a large message space, quantum effects become important even if individual signals are classical, and the bound is upheld.</p>

Tensor Network Holography and Deep Learning

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Y.-Z. You, Z. Yang, X.-L. Qi, arXiv: 1709.01223

Perimeter Institute, Nov. 2017

Outline

- Review of two kinds of networks:
  - **Tensor Network - Holographic Duality** (AdS/CFT)
  - **Neural Network - Deep Learning**
- Is there any connection between tensor network and neural network? What about holography and deep learning?
- **Entanglement Feature Learning (EFL)**
  - Holographic spacial geometry emerges from deep learning the entanglement features in a quantum many-body state.
  - Demonstration on 1D free fermion CFT.

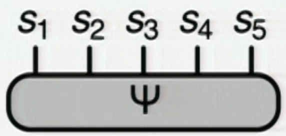
Tensor Network

- Efficient representation of quantum many-body state
  - Quantum states represented by wave functions:

$$|\Psi\rangle = \sum_{[s_i]} \Psi(s_1, s_2, \ldots)\, |s_1\ s_2\ \ldots\rangle$$

← basis states of Hilbert space; ↖ wave function; ↖ for qubits: $s_i = 0, 1$

  - Wave function is also a tensor:



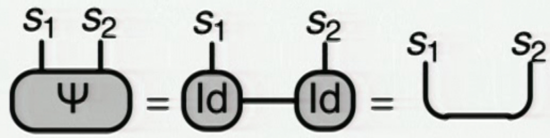
$= \Psi(s_1, s_2, \ldots) \in \mathbb{C}$ 2^N entries!

  - How can we store/represent these data efficiently?
    - Tensor networks are more structural and efficient way to represent a quantum many-body wave function than a single tensor.

Quantum Entanglement

- Example: Einstein-Podolsky-Rosen (EPR) pair

$$|\mathrm{EPR}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



- Quantify entanglement by Entanglement Entropy
  - Reduced Density Matrix $\rho_1 = \mathrm{Tr}_{s_2} |\mathrm{EPR}\rangle \langle \mathrm{EPR}|$
  - (Renyi) Entanglement Entropy

$$S_E^{(n)} = \frac{1}{1-n} \ln \mathrm{Tr}_{s_1} \rho_1^n \leftarrow \text{Renyi index } n$$

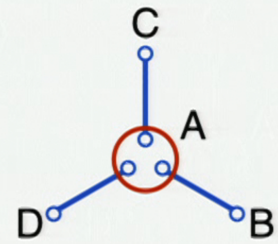
  - For EPR pair, entanglement entropy $S_E^{(n)} = \ln 2 = 1$ bit
  - Mutual Information

$$I^{(n)}(1,2) = S_E^{(n)}(1) + S_E^{(n)}(2) - S_E^{(n)}(1 \cup 2) = 2 \text{ bit}$$

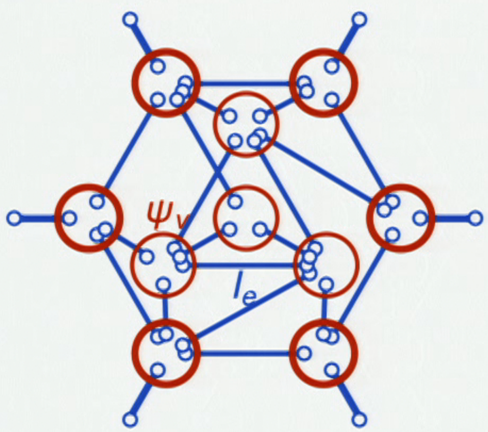
  - Measures the 2×log of the *effective* bond dimension (rank).

Tensor Network

- Building entangled many-body state from EPR pairs
  - Starting from $|\mathrm{EPR_{AB}}\rangle \otimes |\mathrm{EPR_{AC}}\rangle \otimes |\mathrm{EPR_{AD}}\rangle$
  - A measures the qubits $\rho_A \rightarrow |\psi_A\rangle\langle\psi_A|$
  - Now B, C, D are entangled



- Distribute EPR pairs + measurement → Entanglement formed among unmeasured qubits



- Tensor Network State (PEPS)
  - Given a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$
  - Pure states $|\psi_v\rangle$ $(\forall\, v \in \mathcal{V})$
  - Entangled pairs $|I_e\rangle$ $(\forall\, e \in \mathcal{E})$

$$|\Psi_{\mathrm{TN}}\rangle = \bigotimes_{v,e \in \mathcal{G}} \langle \psi_v\, | I_e \rangle$$

Verstraete, Cirac, Murg (2009)

Universal Limit on Communication

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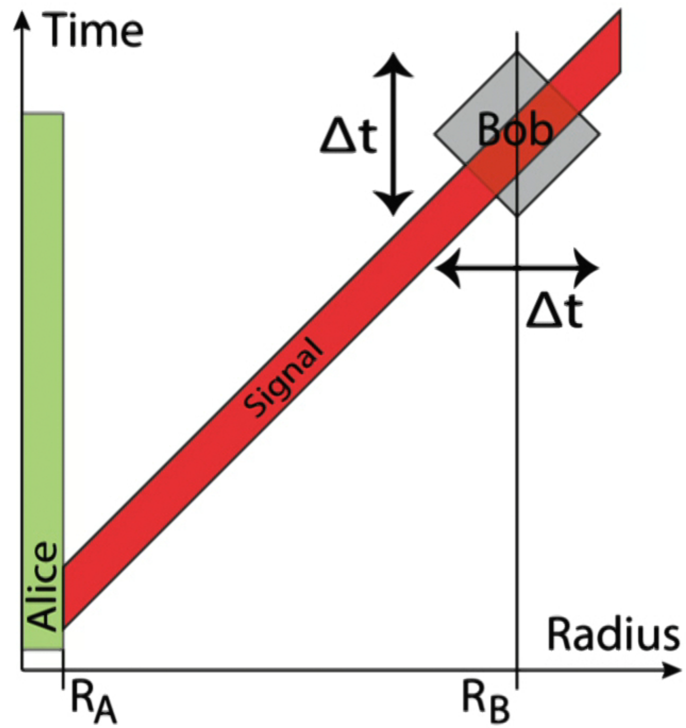
PI Seminar
February 21, 2018

Alice has a lot to say

But Bob is very far away.
Fortunately, they planned ahead and agreed upon a message set:
$\mathcal{N}$ messages to choose from,
each a signal of energy E.
When he finally gets the news from Alice, Bob only has time Δt.

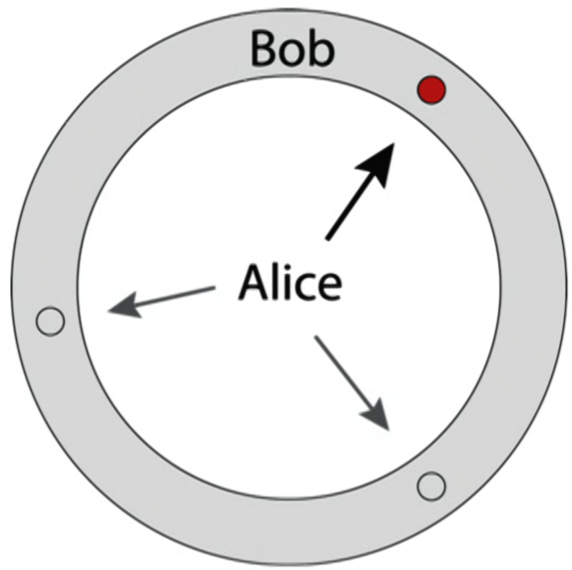
(We require $R_B \gg R_A$,
and weak gravity at R_B.)



How much can Alice say?

With no restriction on the size of the emitter and receiver, $\mathcal{N}$ is unlimited.

For example, consider the case where $E\Delta t \gg \hbar$. Then Alice can send a classical flash of light from one of $\mathcal{N}$ pixels tiling her planet. Bob has surrounded her with (at least) $\mathcal{N}$ detectors tiling his much larger sphere. The solid angle where the detector triggers tells Bob the message.



How much information can Bob receive?

However, recently derived asymptotic entropy bounds imply that the amount of information Bob can receive is at most

$$\frac{E\Delta t}{\hbar},$$

which does not increase with $\mathcal{N}$!

This is a surprising quantum effect that applies even if individual signals are classical. It is associated neither directly with the uncertainty principle, nor with gravitational collapse.

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Remark

Alternative interpretation: bound on the information content of the output of a scattering experiment.

Therefore, of potential relevance to the information paradox.
(I will briefly return to this at the end.)

Quantum description

Alice prepares a signal state ρ_a^A with probability $p(a)$. The average signal state is

$$\rho_{\mathrm{av}}^A = \sum_{a=1}^{N} p(a)\, \rho_a^A \,,$$

We allow Alice unlimited access and power so we can treat ρ^A as a global state.

Channel Capacity

The infomation communicated by Alice to Bob is quantified by the classical mutual information

$$
\begin{aligned}
H(A:B) &\equiv H(A) + H(B) - H(A,B) \\
&\leq \log \mathcal{N} \,,
\end{aligned}
$$

where $H(X)$ is the Shannon entropy of the probability distribution $p(x)$.

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(Naively one expects that Bob can decode classical signals perfectly, no matter how large the message space: $p(b|a) = \delta_{ab}$. With $p(a) = 1/\mathcal{N}$, this would achieve $H(A) = \log \mathcal{N}$, for any $\mathcal{N}$.)

Entropy Bound

Positivity of the relative entropy $S(\rho^B_{\mathrm{av}} || \sigma^B)$ implies

$$\Delta S(\rho^B_{\mathrm{av}}) \lesssim \Delta K(\rho^B_{\mathrm{av}}) \, ,$$

where ΔK is the modular energy of the reduced state:

$$\Delta K(\rho^B) \equiv \mathrm{tr}_B \, \hat{K} \rho^B - \mathrm{tr}_B \, \hat{K} \sigma^B \, ,$$

and $\hat{K}$ is the modular Hamiltonian, defined by

$$\sigma^B = \frac{e^{-\hat{K}}}{\mathrm{tr}_B \, e^{-\hat{K}}} \, .$$

Marolf *et al.* 2004, Casini 2008

Asymptotic Entropy Bound

For a shell of width Δt,

$$\Delta K(\rho^B) \lesssim E(\rho^B)\Delta t/\hbar \, .$$

(Precise expressions for ΔK are given in

RB 2016
RB, Halpern & Koeller 2016

building on

RB, Casini, Fisher & Maldacena 2014a,b
RB, Fisher, Leichenauer & Wall 2015
RB, Fisher, Koeller, Leichenauer & Wall 2015.)

Bound on the Channel Capacity

$$H(A:B) \le \chi \le \Delta K(\rho^B_{\mathrm{av}}) - \sum_a p(a)\,\Delta S(\rho^B_a)\ .$$

This upper bound does not increase with the size of the message space, if the average energy and average ΔS of the individual signals are fixed.

In particular, for classical signal states, $\Delta S(\rho^B_a) \ge 0$. Hence

$$H(A:B) \lesssim E_{\mathrm{av}}\Delta t/\hbar\ .$$

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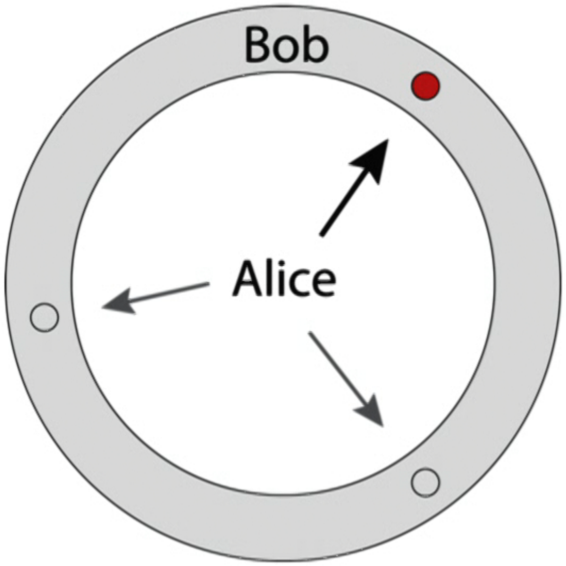
Irreducible noise



The expected total number of false detections is $\mathcal{N}P$.

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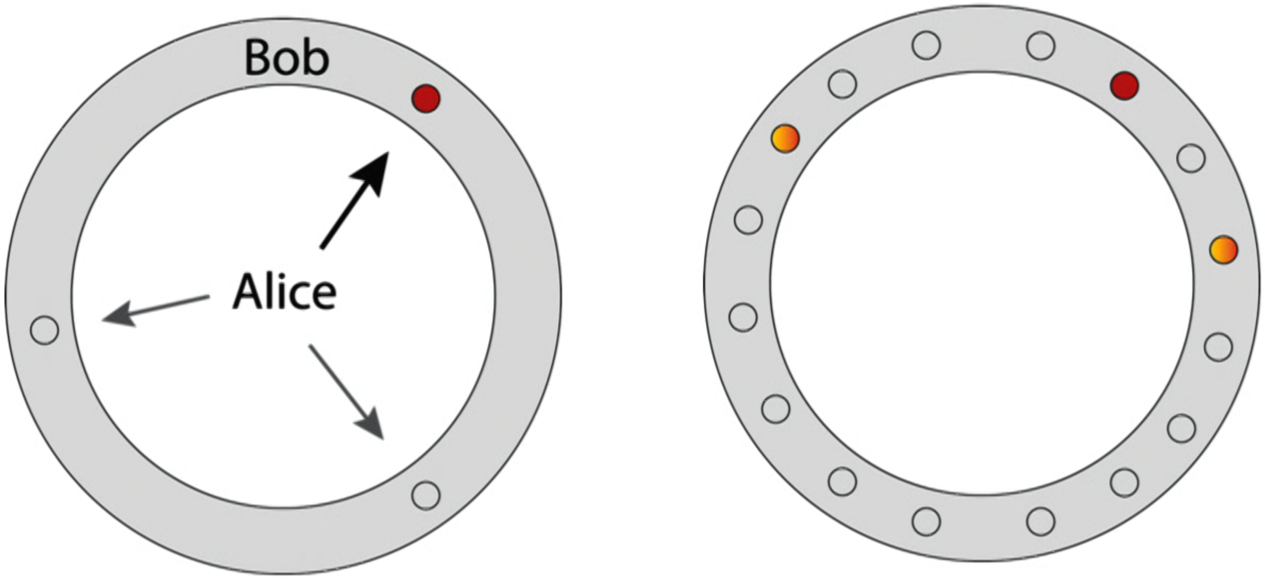

For $\log \mathcal{N} \gg E\Delta t/\hbar$, this becomes greater than one!

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Bob cannot tell which is the real signal, so the protocol fails.

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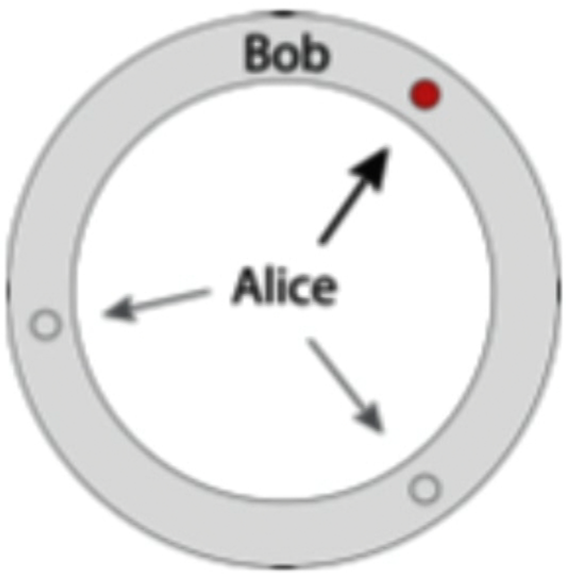

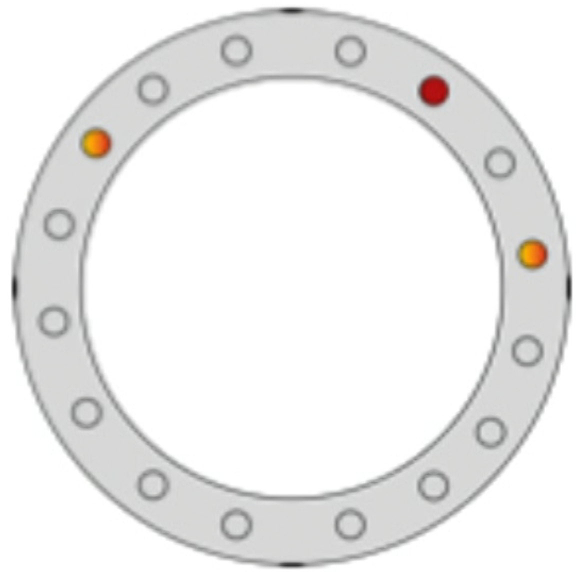

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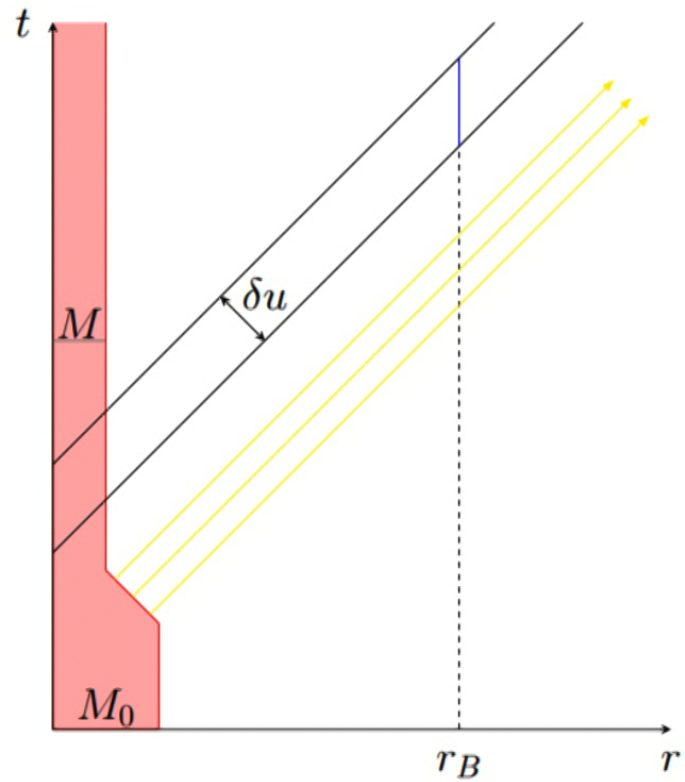
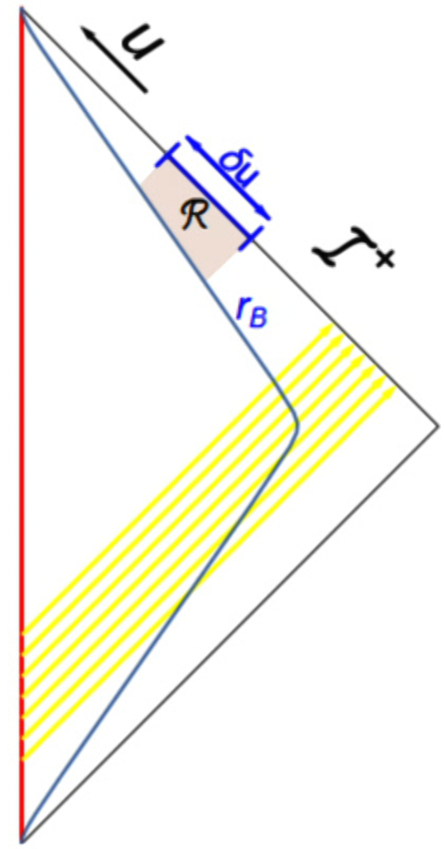
Another Apparent Contradiction: Conserved Charges

Bob can measure the Bondi mass (or total electric charge). In principle this allows Bob to receive information without receiving energy.



Another Apparent Contradiction: Conserved Charges

If Bob could do this in finite time δu, at arbitrarily large radius r, then this would violate the communication bound!

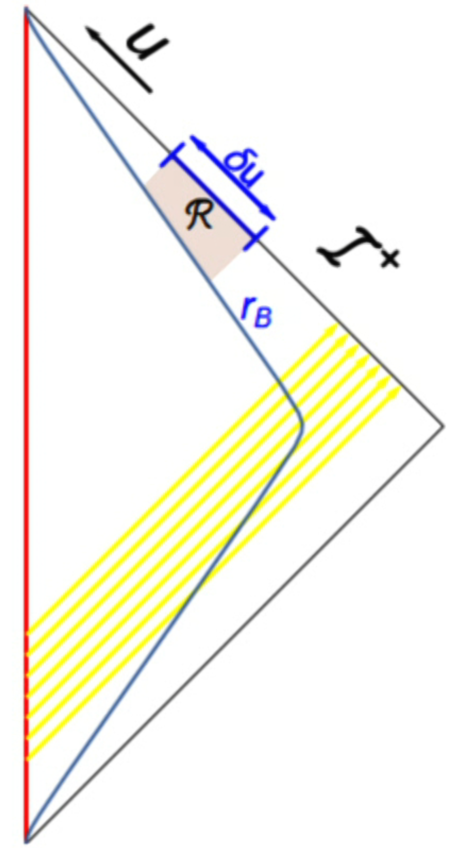


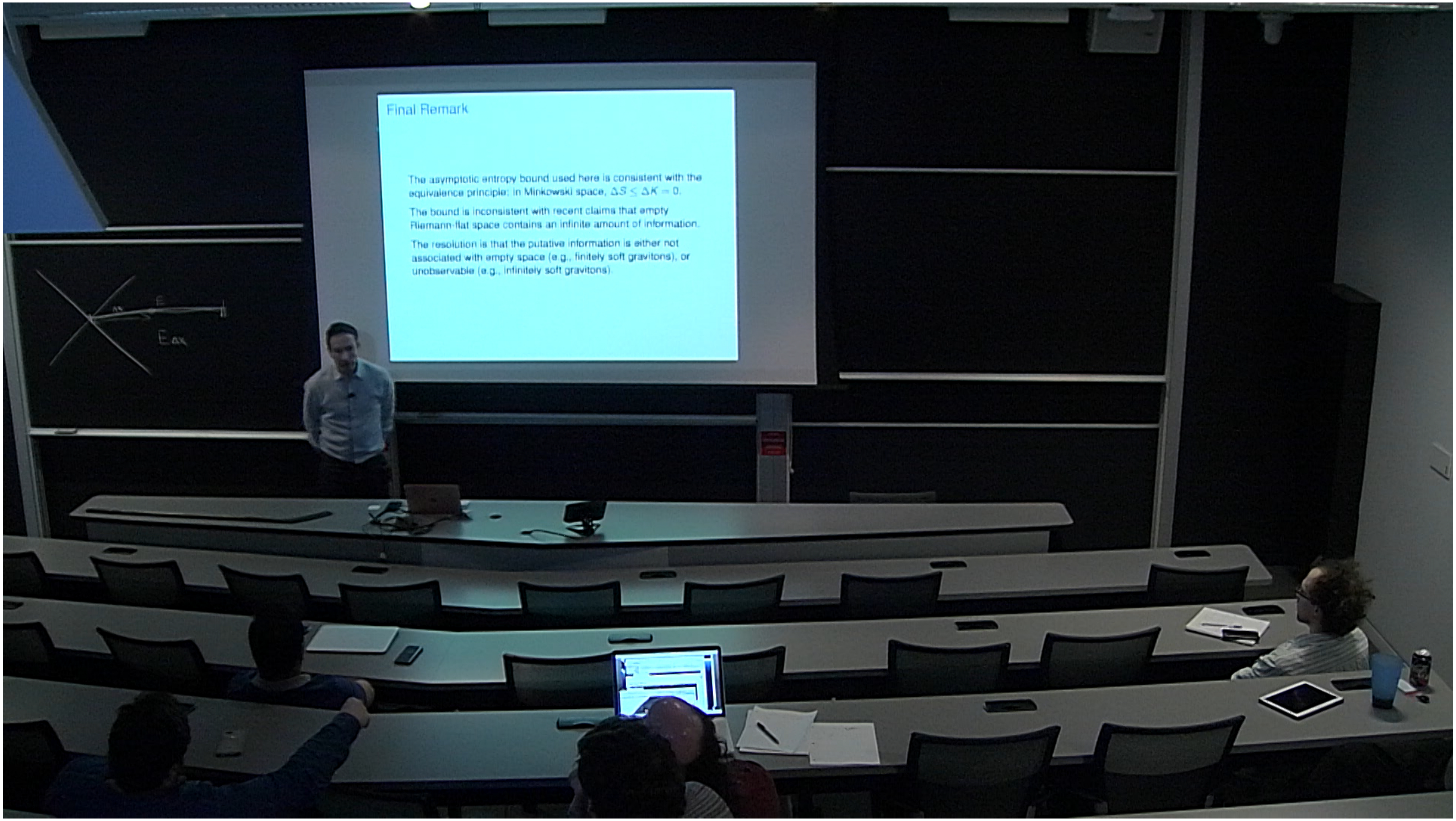
Another Apparent Contradiction: Conserved Charges

Resolution of the Paradox: quantum fluctuations!

$$\langle Q^2 \rangle \sim r^2 \delta u^{-2}$$

RB, Chandrasekharan, Halpern, Wall 2017





Final Remark
The asymptotic entropy bound used here is consistent with the equivalence principle: in Minkowski space, $\Delta S \leq \Delta K = 0$.
The bound is inconsistent with recent claims that empty Riemann-flat space contains an infinite amount of information.
The resolution is that the putative information is either not associated with empty space (e.g., finitely soft gravitons), or unobservable (e.g., infinitely soft gravitons).

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