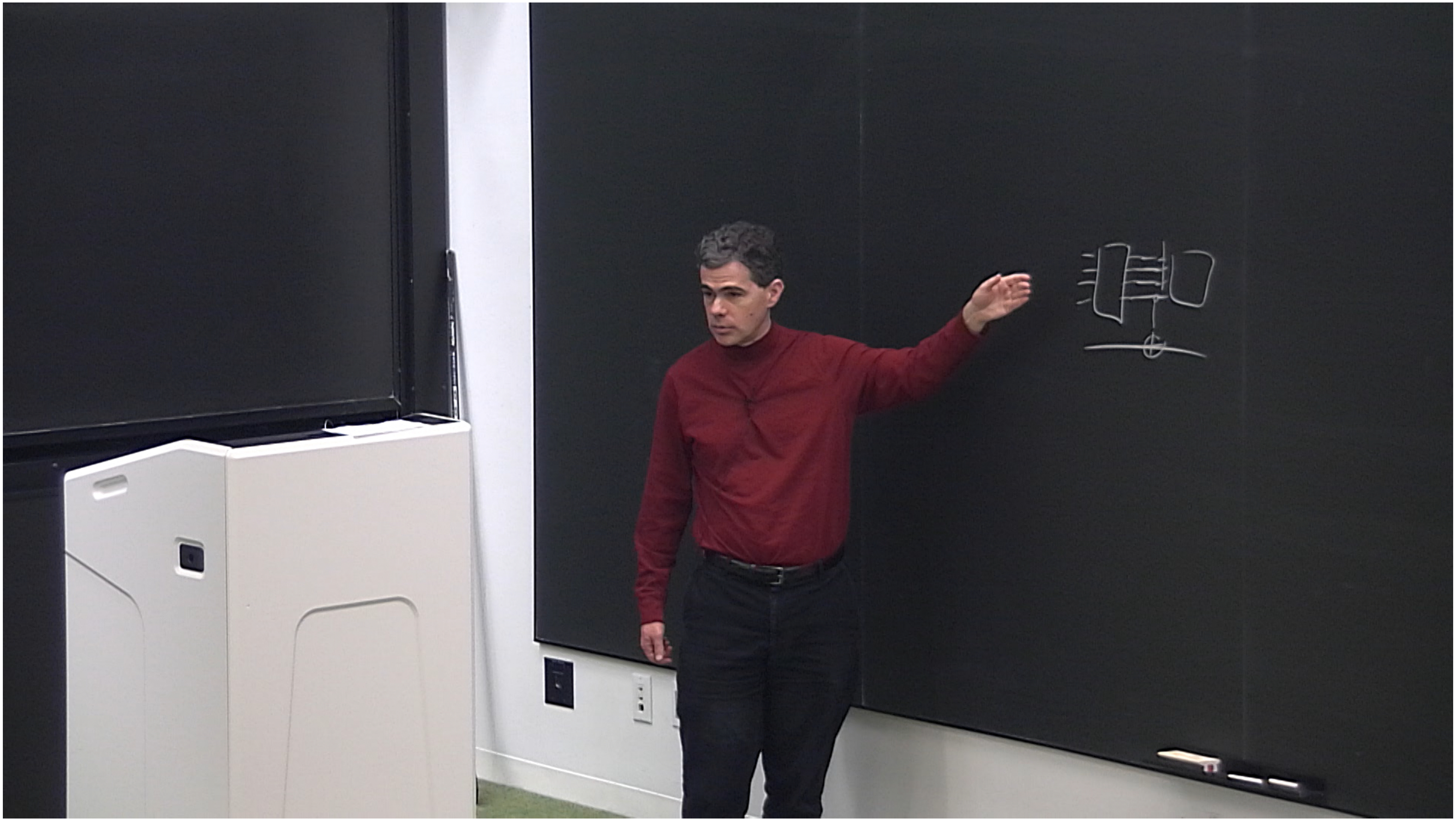
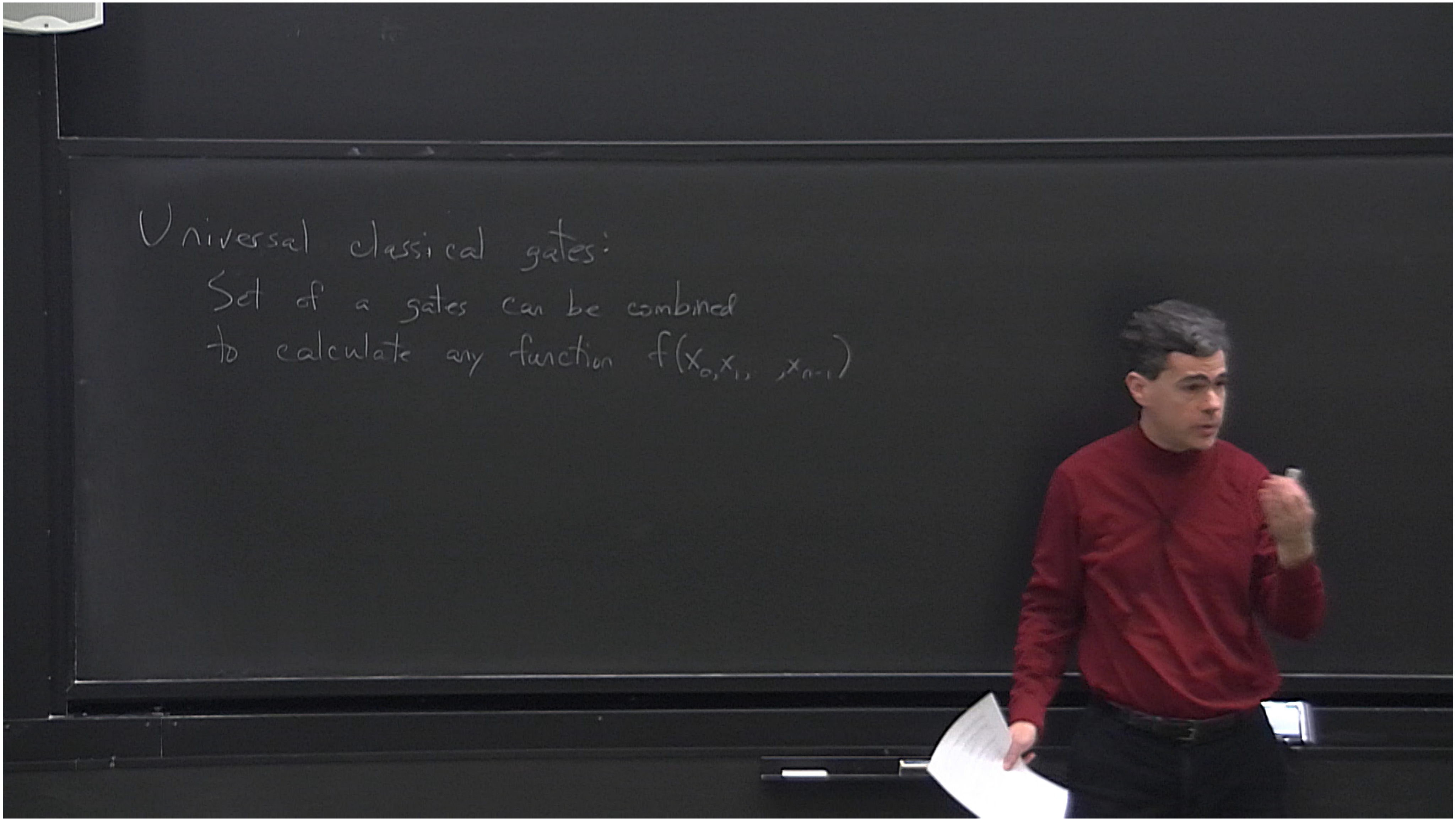
Title: PSI 17/18 - Quantum Information - Lecture 2

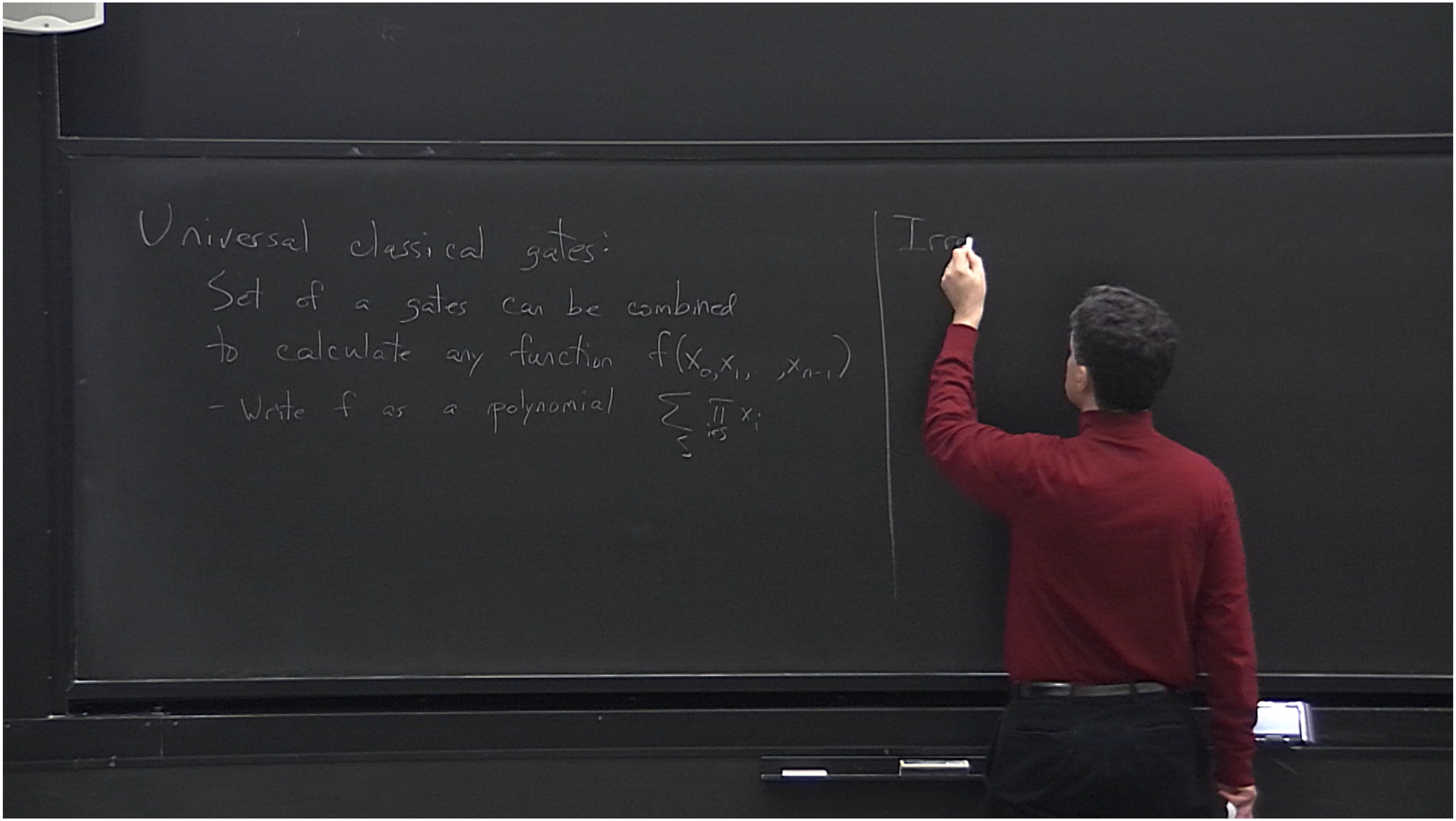
Date: Feb 21, 2018 11:30 AM

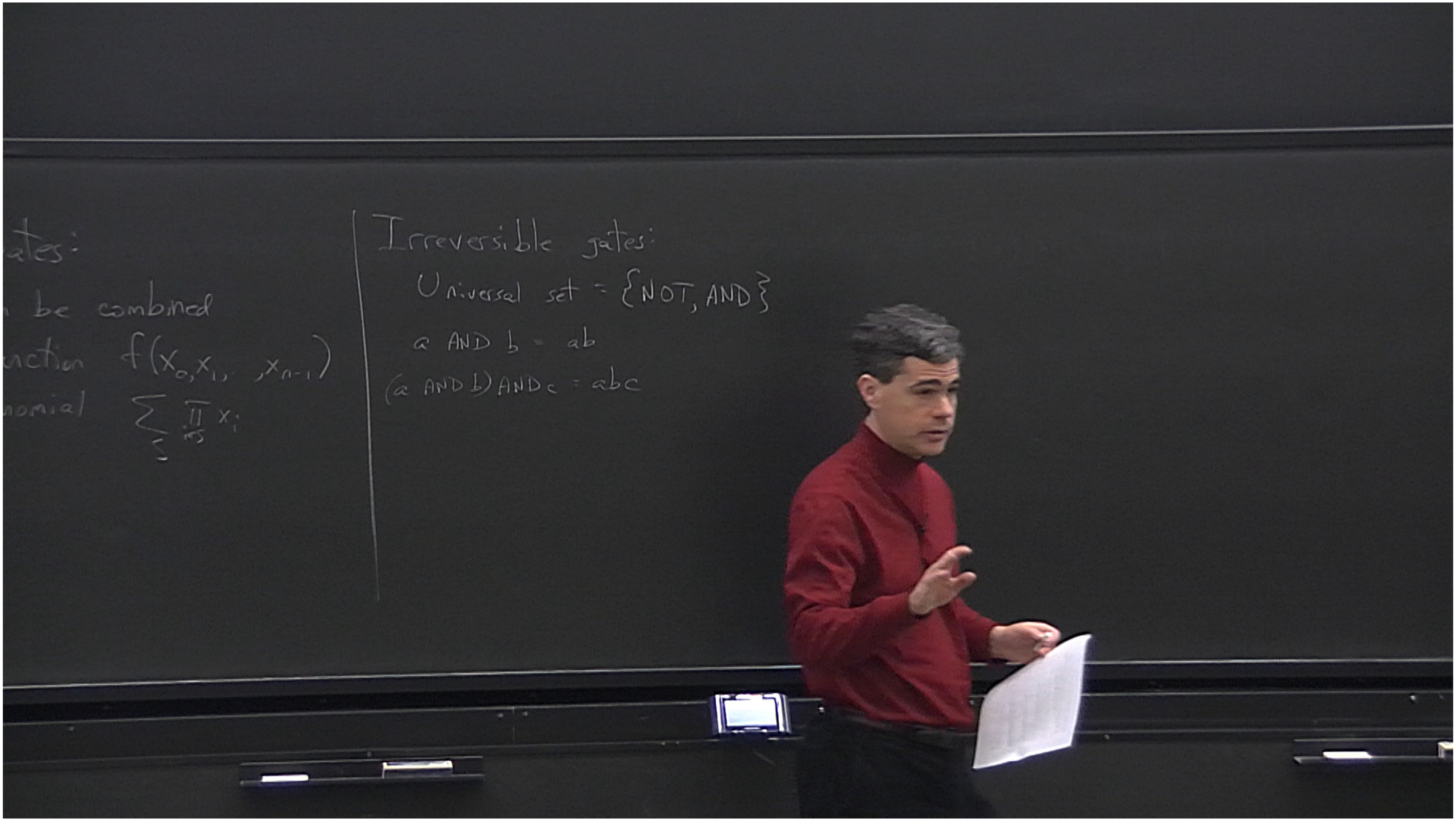
URL: http://pirsa.org/18020045

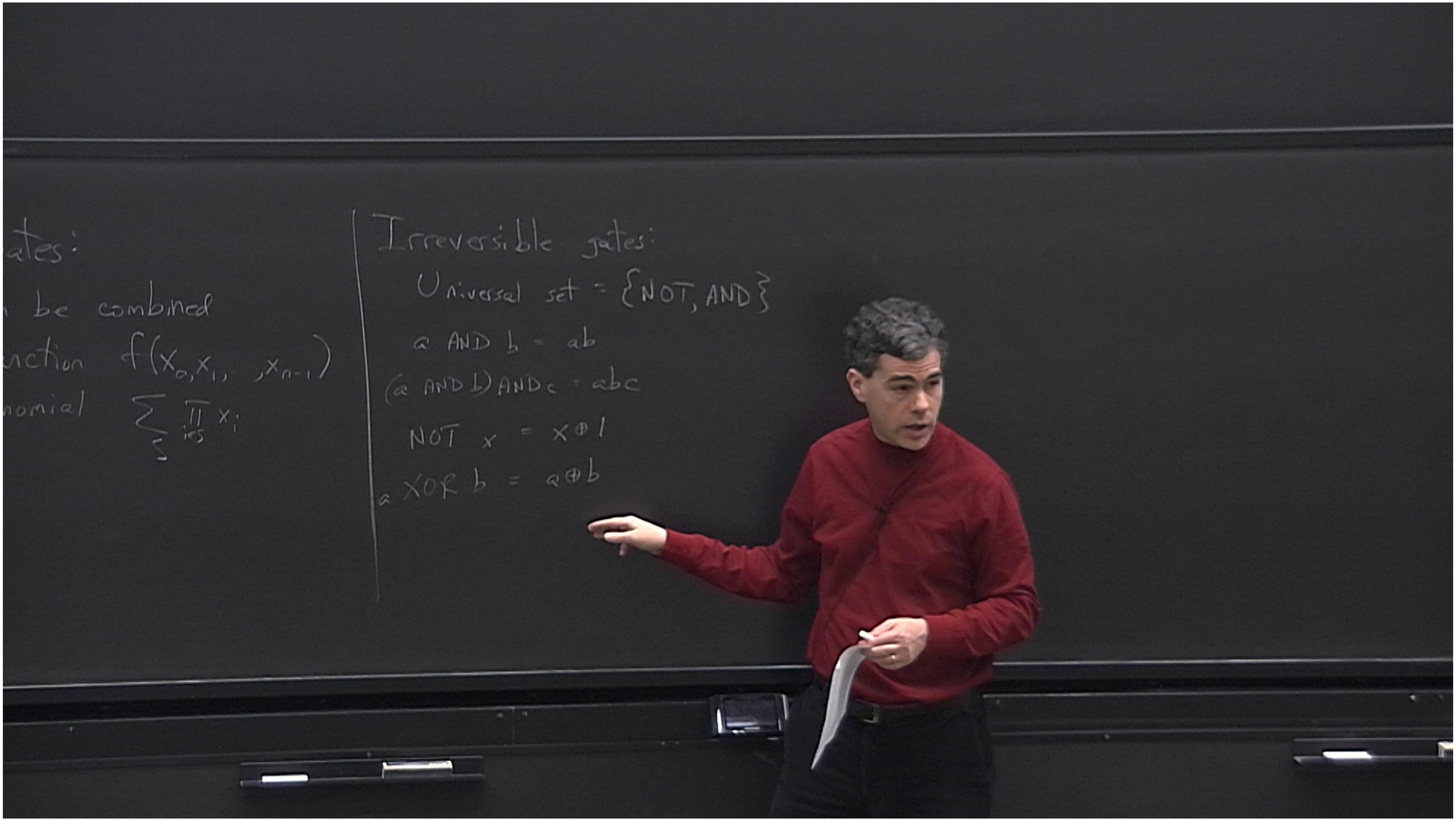
Abstract:

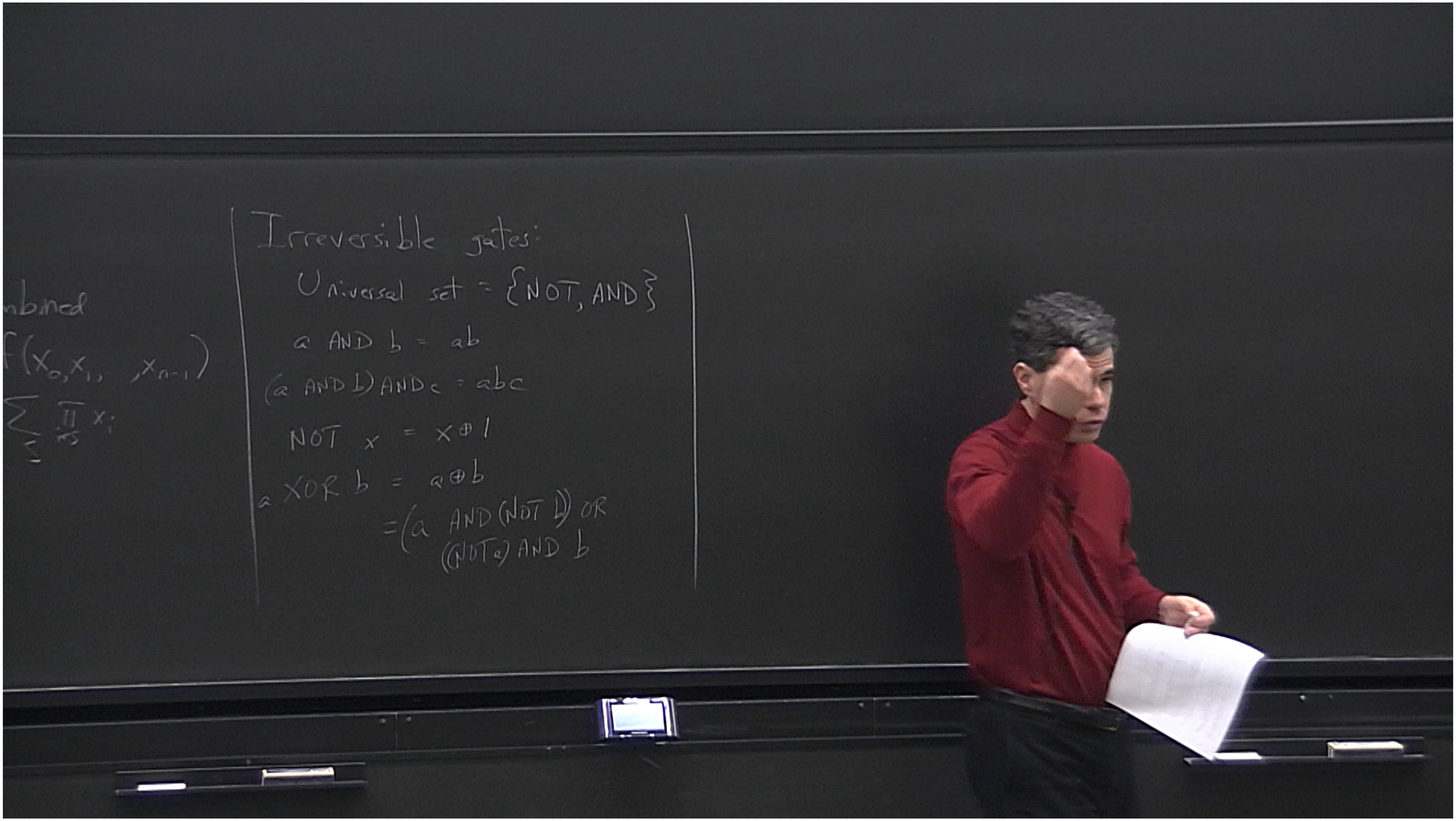


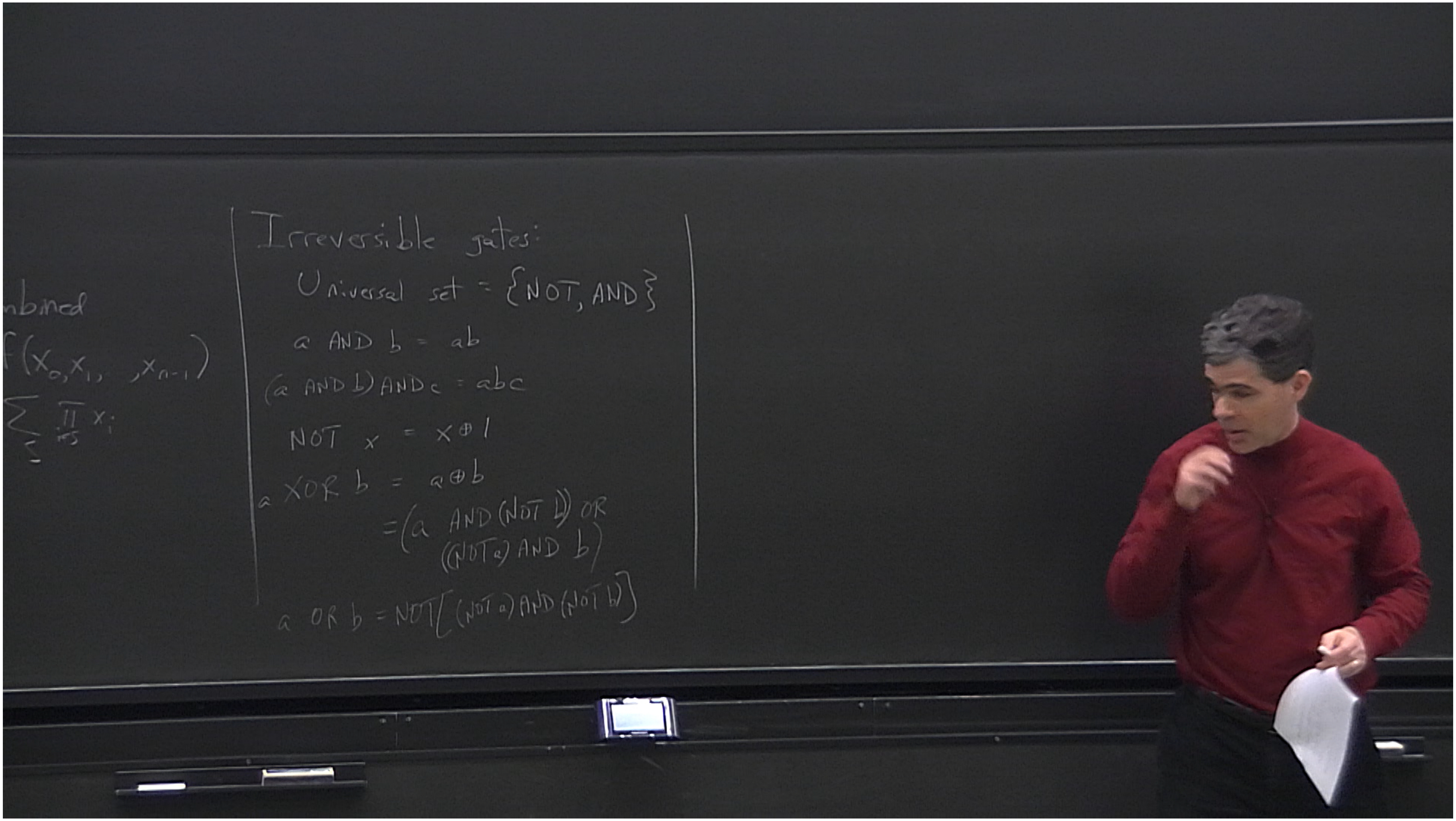

Universal classical gates:
Set of a gates can be combined
to calculate any function $f(x_0, x_1, \ldots, x_{n-1})$

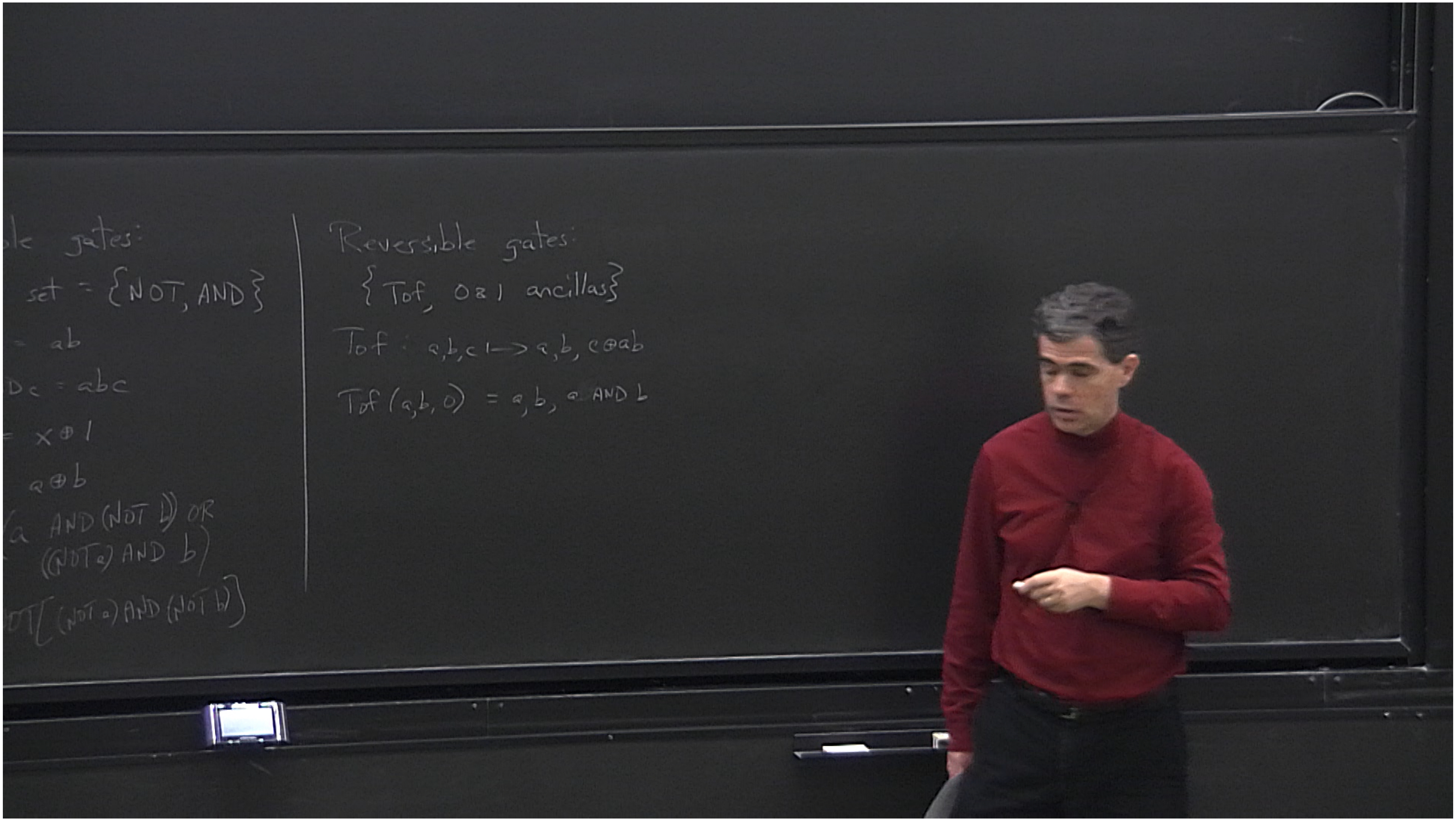

Universal classical gates:
Set of a gates can be combined
to calculate any function f(x_0, x_1, ..., x_{n-1})
- Write f as a polynomial ∑_S ∏_{i∈S} x_i


be combined
Irreversible gates:
Universal set = {NOT, AND}
a AND b = ab
(a AND b) AND c = abc

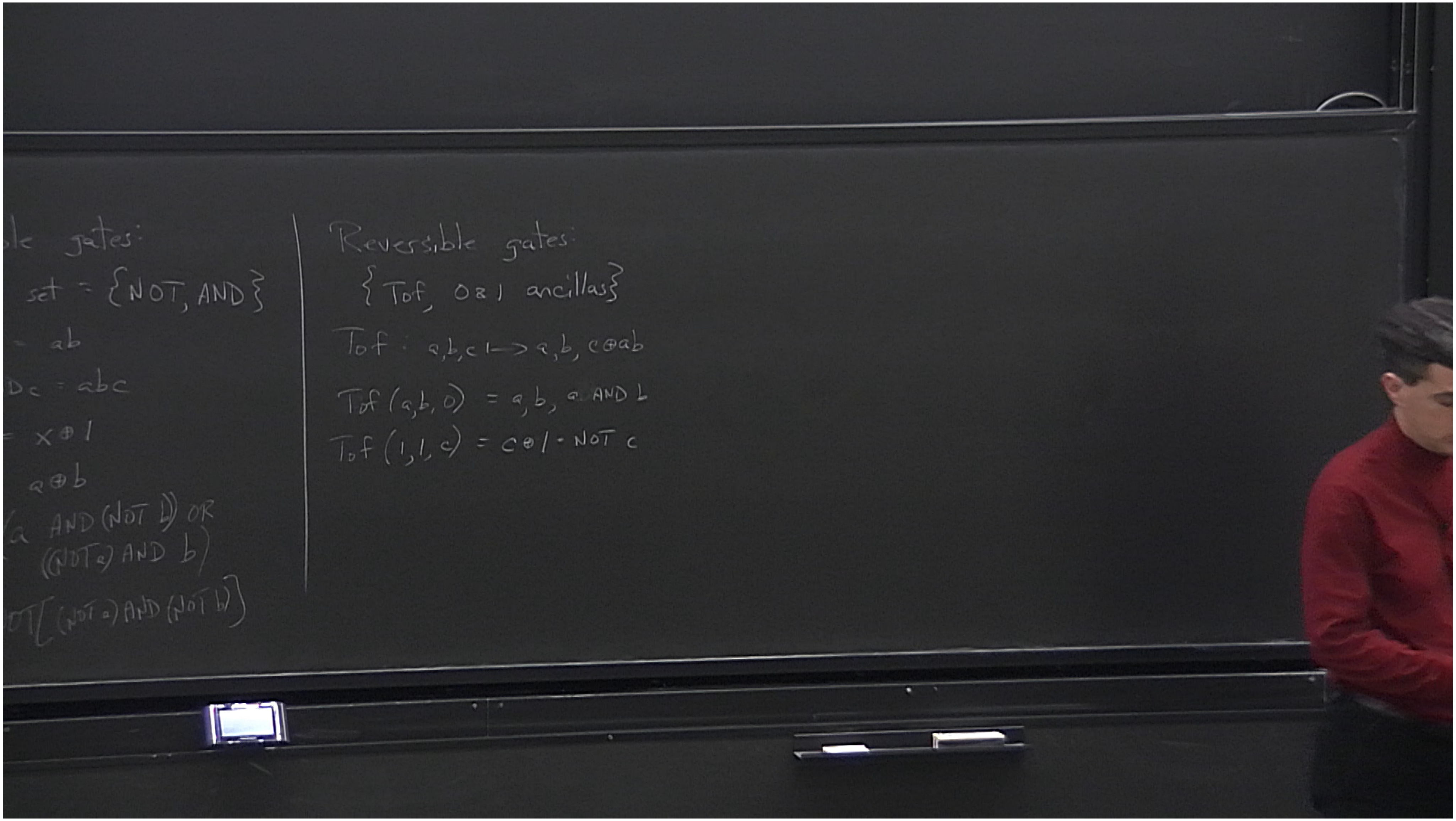

Irreversible gates:
Universal set = {NOT, AND}
a AND b = ab
(a AND b) AND c = abc
NOT x = x ⊕ 1
a XOR b = a ⊕ b
be combined


Irreversible gates:
Universal set = {NOT, AND}
a AND b = ab
(a AND b) AND c = abc
NOT x = x ⊕ 1
a XOR b = a ⊕ b
= (a AND (NOT b)) OR
((NOT a) AND b
mbined
f(x0, x1, , xn-1)


Irreversible gates:
Universal set = {NOT, AND}
a AND b = ab
(a AND b) AND c = abc
NOT x = x ⊕ 1
a XOR b = a ⊕ b
= (a AND (NOT b)) OR
((NOT a) AND b)
a OR b = NOT[(NOT a) AND (NOT b)]



set = {NOT, AND}
= ab
= abc
= x ⊕ 1
a ⊕ b
(a AND (NOT b)) OR
((NOT a) AND b)
[(NOT a) AND (NOT b)]
Reversible gates:
{Tof, 0 & 1 ancillas}
Tof: a,b,c ↦ a,b, c⊕ab
Tof(a,b,0) = a,b, a AND b


gates:
set = {NOT, AND}
= ab
= abc
= x ⊕ 1
a ⊕ b
a AND (NOT b) OR
((NOT a) AND b)
(NOT a) AND (NOT b)]
Reversible gates:
{Tof, 0 & 1 ancillas}
Tof: a,b,c ⟼ a,b, c⊕ab
Tof(a,b,0) = a,b, a AND b
Tof(1,1,c) = c⊕1 = NOT c

le gates:

set = {NOT, AND}

= ab

D c = abc

= $x \oplus 1$

$a \oplus b$

(a AND (NOT b)) OR ((NOT a) AND b)

OT[(NOT a) AND (NOT b)]

Reversible gates:

{Tof, 0 & 1 ancillas}

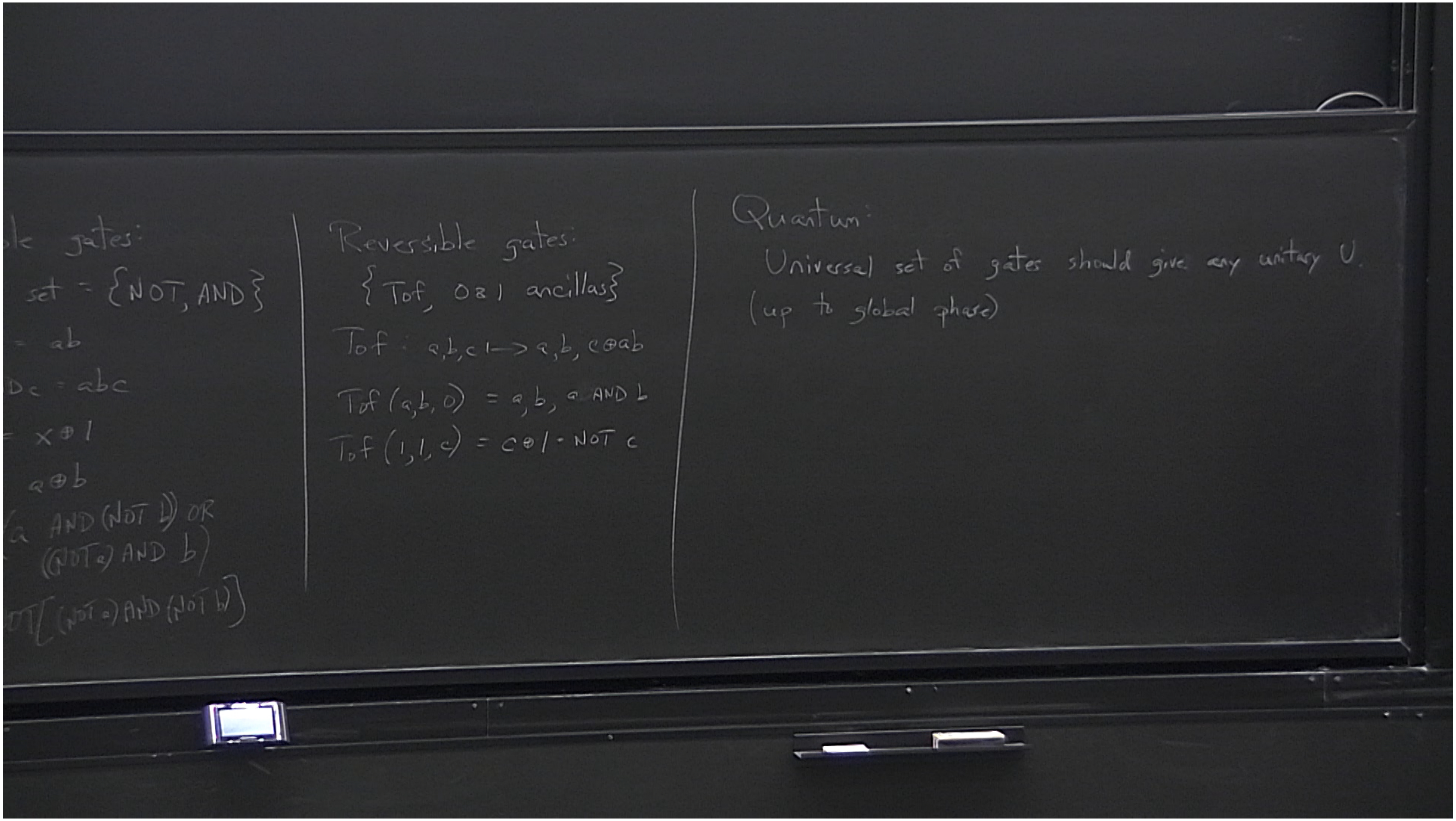
Tof: $a, b, c \mapsto a, b, c \oplus ab$

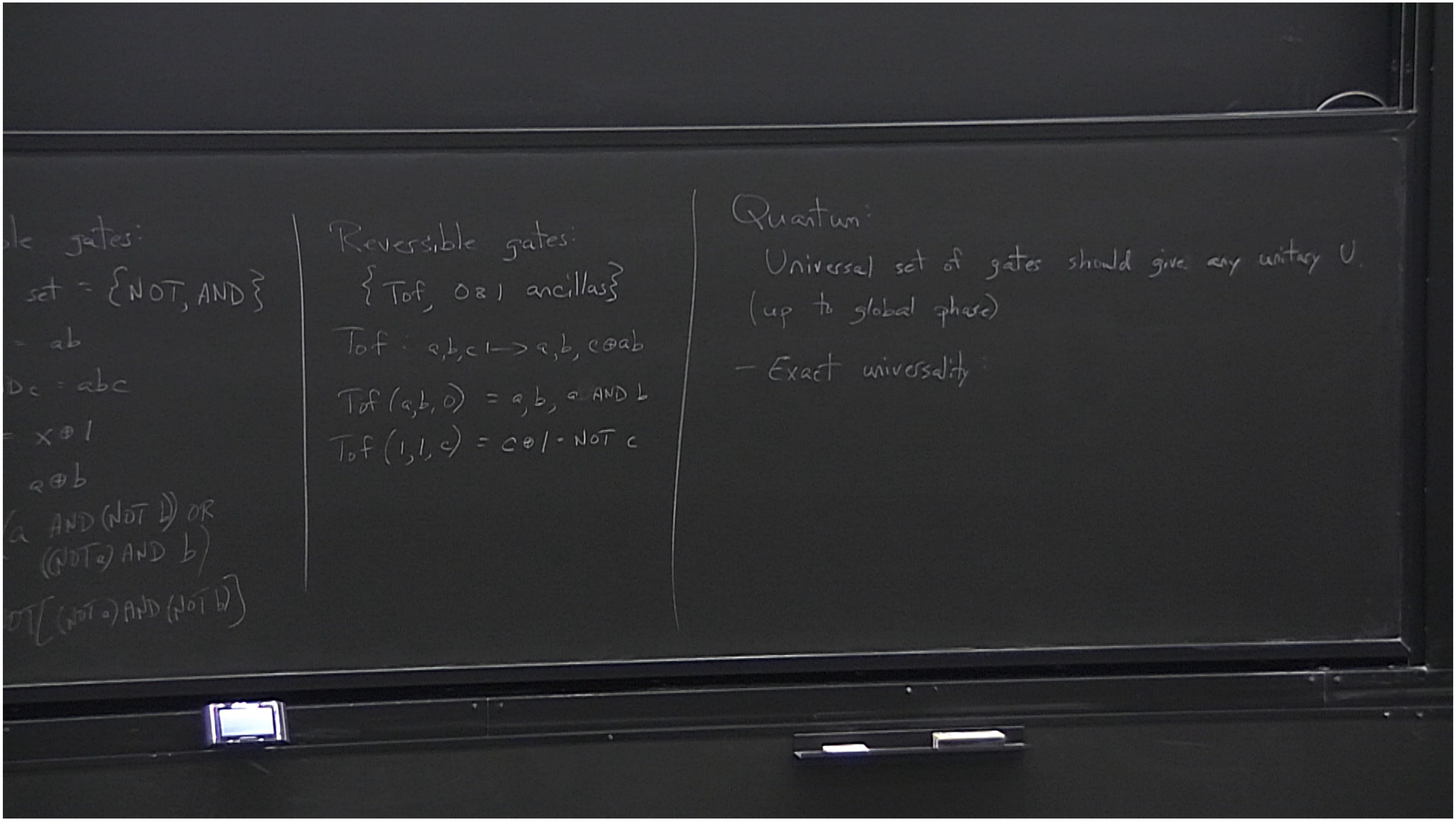
Tof$(a,b,0) = a, b,$ a AND b

Tof$(1,1,c) = c \oplus 1$ = NOT c

Quantum:

Universal set of gates should give any unitary U.


le gates:
set = {NOT, AND}
= ab
Dc = abc
= x ⊕ 1
a ⊕ b
(a AND (NOT b)) OR
((NOT a) AND b)
OT[(NOT a) AND (NOT b)]
Reversible gates:
{Tof, 0 & 1 ancillas}
Tof : a,b,c ↦ a,b, c⊕ab
Tof (a,b,0) = a,b, a AND b
Tof (1,1,c) = c⊕1 = NOT c
Quantum:
Universal set of gates should give any unitary U.
(up to global phase)


le gates:
set = {NOT, AND}
= ab
D c = abc
= x ⊕ 1
a ⊕ b
(a AND (NOT b)) OR
((NOT a) AND b)
OT[(NOT a) AND (NOT b)]
Reversible gates:
{Tof, 0 & 1 ancillas}
Tof: a,b,c ⟼ a,b, c⊕ab
Tof (a,b,0) = a,b, a AND b
Tof (1,1,c) = c⊕1 = NOT c
Quantum:
Universal set of gates should give any unitary U.
(up to global phase)
– Exact universality:

...le gates:

set = {NOT, AND}

= ab

...D c = abc

= $x \oplus 1$

$a \oplus b$

(a AND (NOT b)) OR ((NOT a) AND b)

...OT[(NOT a) AND (NOT b)]

Reversible gates:

{Tof, 0 & 1 ancillas}

Tof: $a,b,c \longmapsto a,b,c \oplus ab$

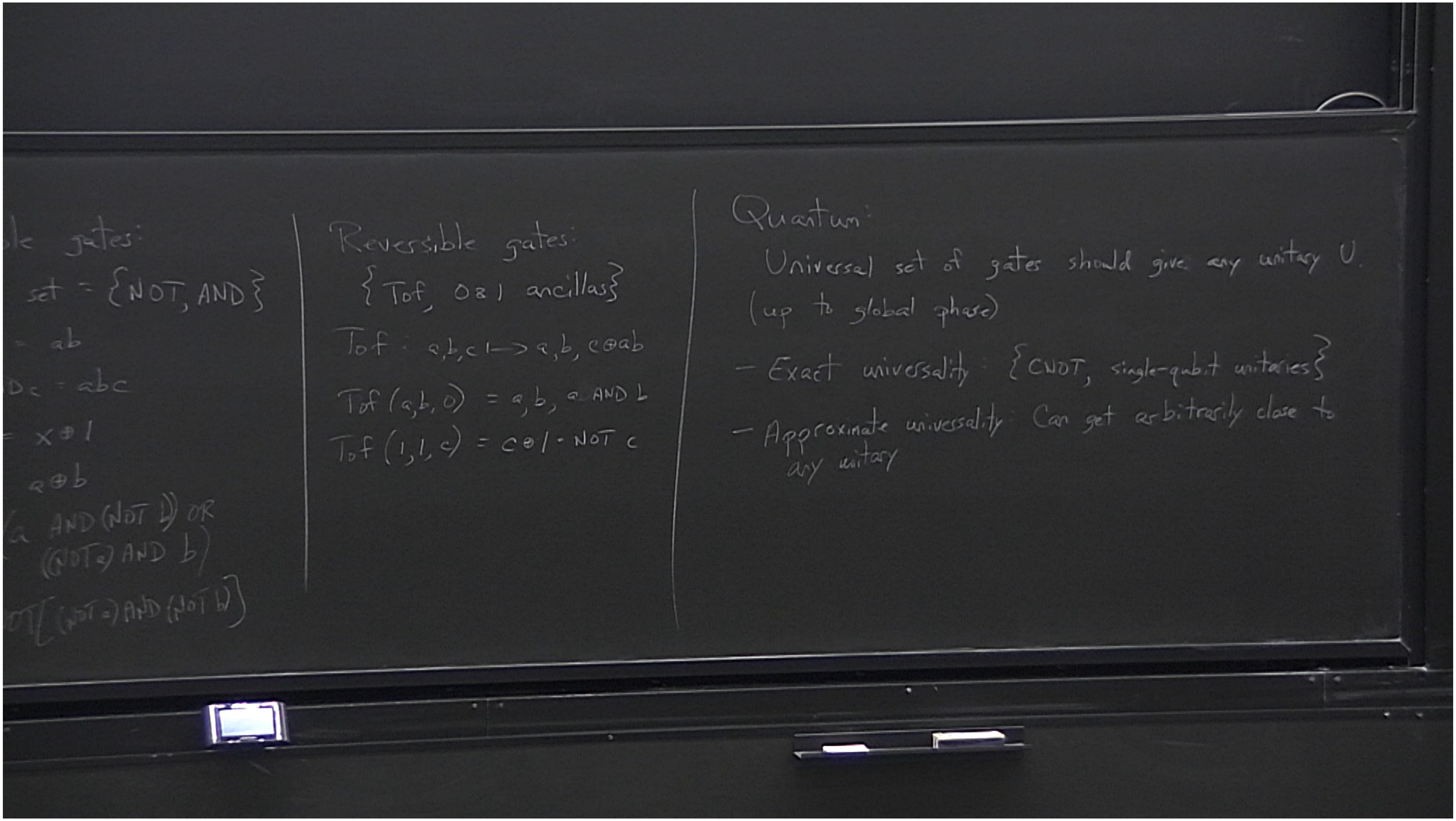
Tof$(a,b,0) = a,b,$ a AND b

Tof$(1,1,c) = c \oplus 1 =$ NOT c

Quantum:

Universal set of gates should give any unitary U.
(up to global phase)

- Exact universality: {CNOT, single-qubit unitaries}


le gates:
set = {NOT, AND}
= ab
Dc = abc
= x ⊕ 1
a ⊕ b
(a AND (NOT b)) OR ((NOT a) AND b)
OT[(NOT a) AND (NOT b)]
Reversible gates:
{Tof, 0 & 1 ancillas}
Tof: a,b,c ⟼ a,b, c⊕ab
Tof(a,b,0) = a,b, a AND b
Tof(1,1,c) = c⊕1 = NOT c
Quantum:
Universal set of gates should give any unitary U. (up to global phase)
– Exact universality: {CNOT, single-qubit unitaries}
– Approximate universality: Can get arbitrarily close to any unitary

...le gates:

... set = {NOT, AND}

... = ab

...D c = abc

... = $x \oplus 1$

... $a \oplus b$

... (a AND (NOT b)) OR ((NOT a) AND b)

...OT[(NOT a) AND (NOT b)]

Reversible gates:

{Tof, 0 & 1 ancillas}

Tof: $a, b, c \longmapsto a, b, c \oplus ab$

Tof$(a, b, 0) = a, b,$ a AND b

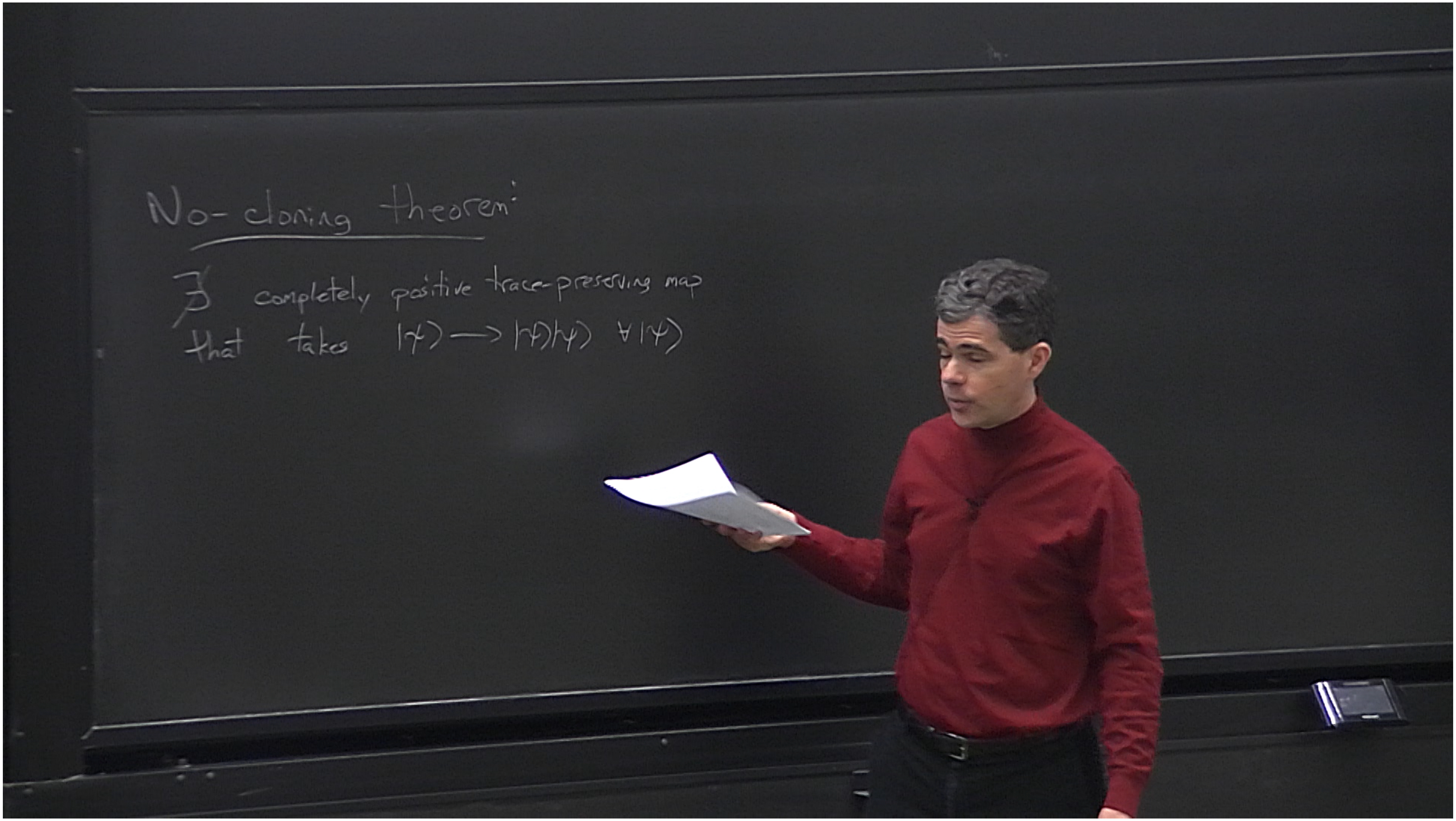
Tof$(1, 1, c) = c \oplus 1$ = NOT c

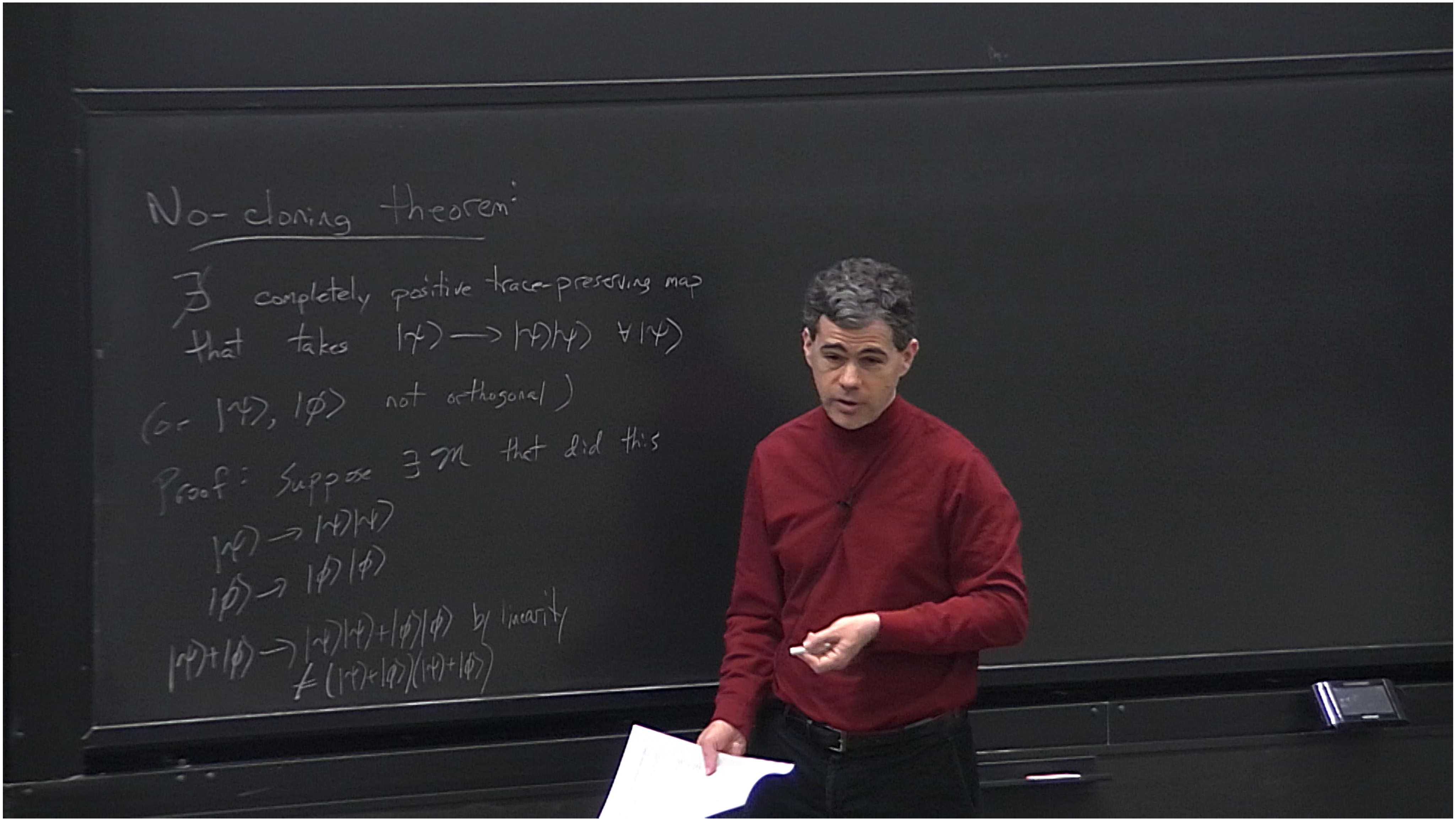
Quantum:

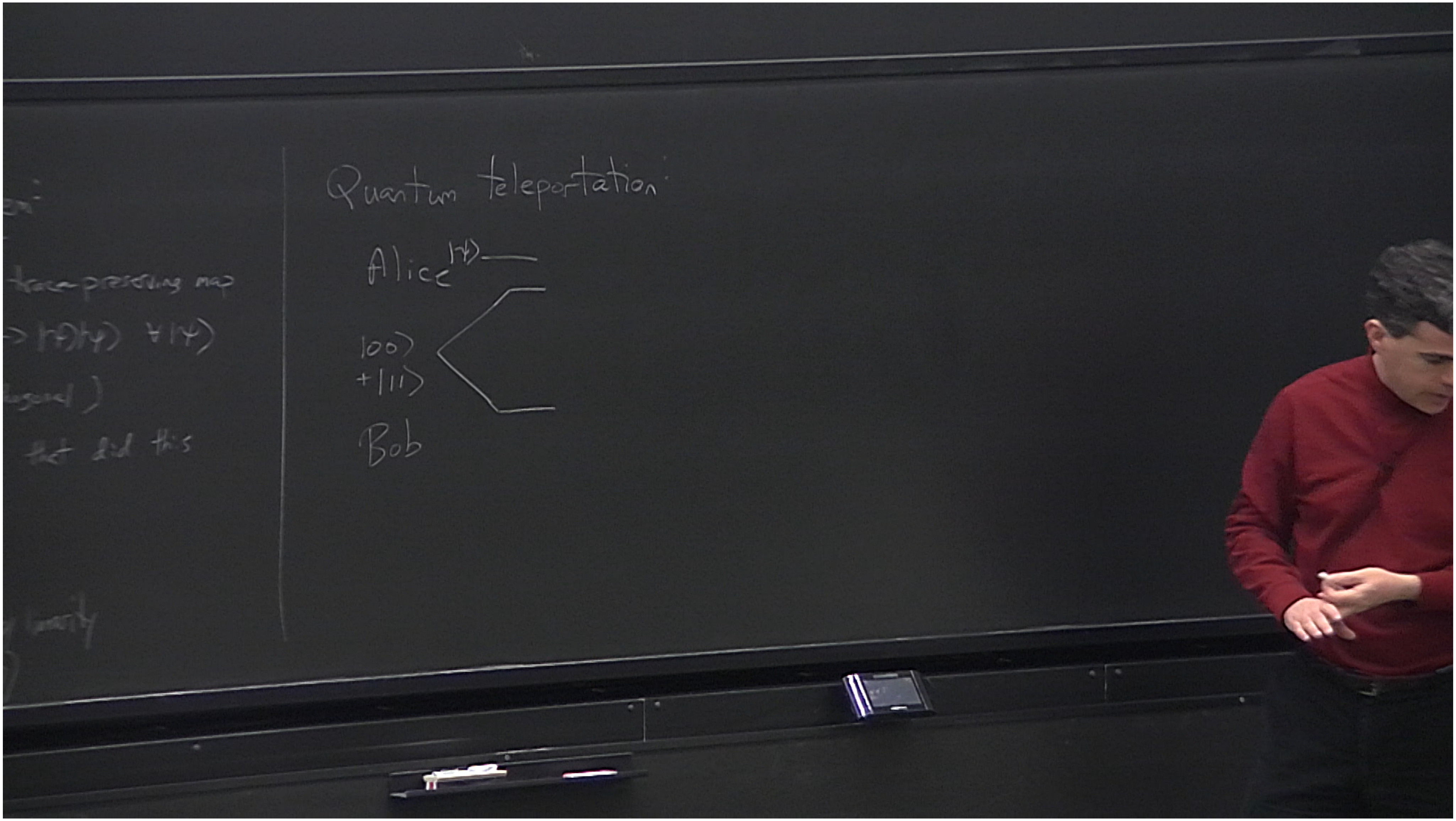
Universal set of gates should give any unitary U. (up to global phase)

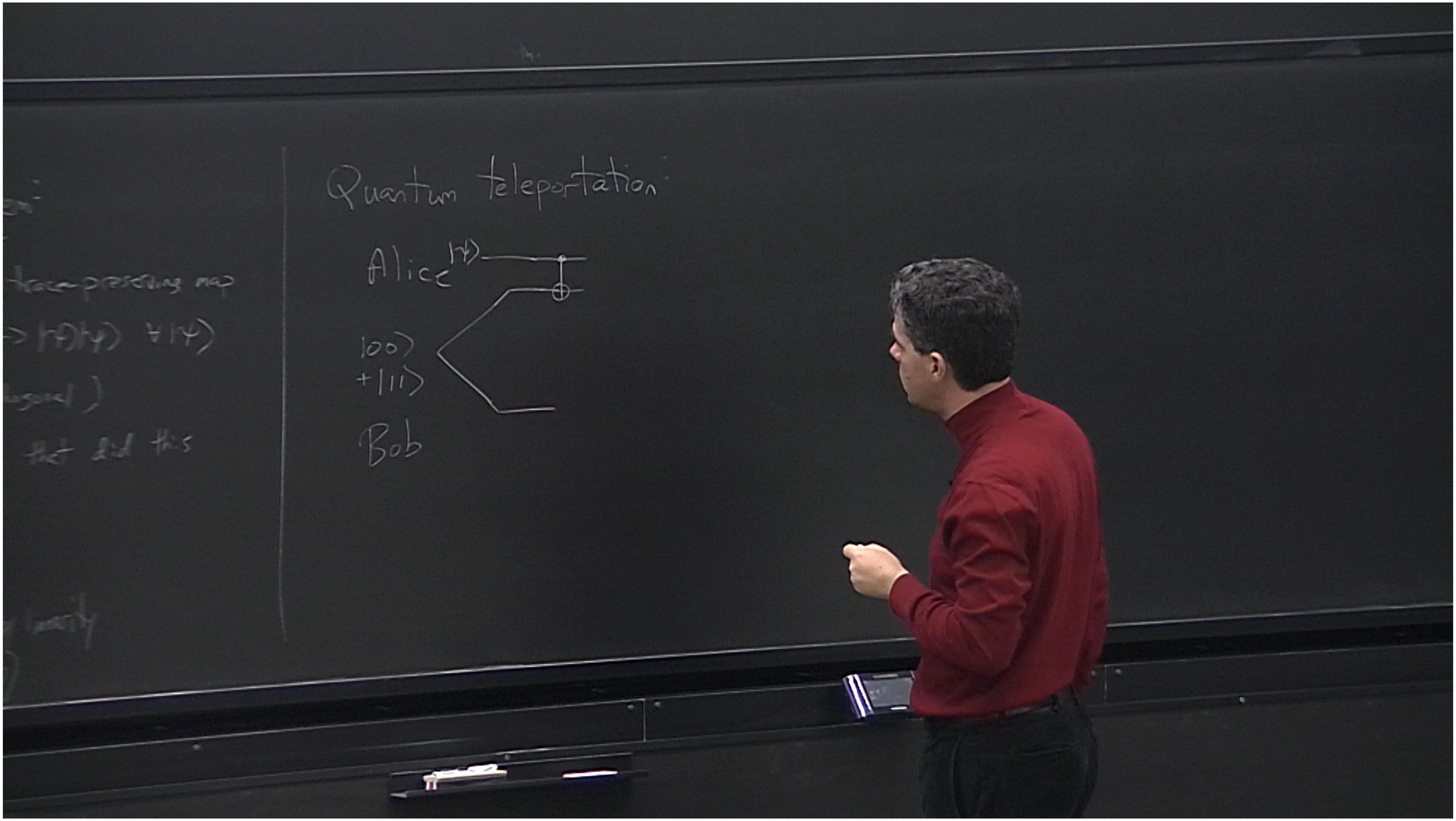
- Exact universality: {CNOT, single-qubit unitaries}
- Approximate universality: Can get arbitrarily close to any unitary

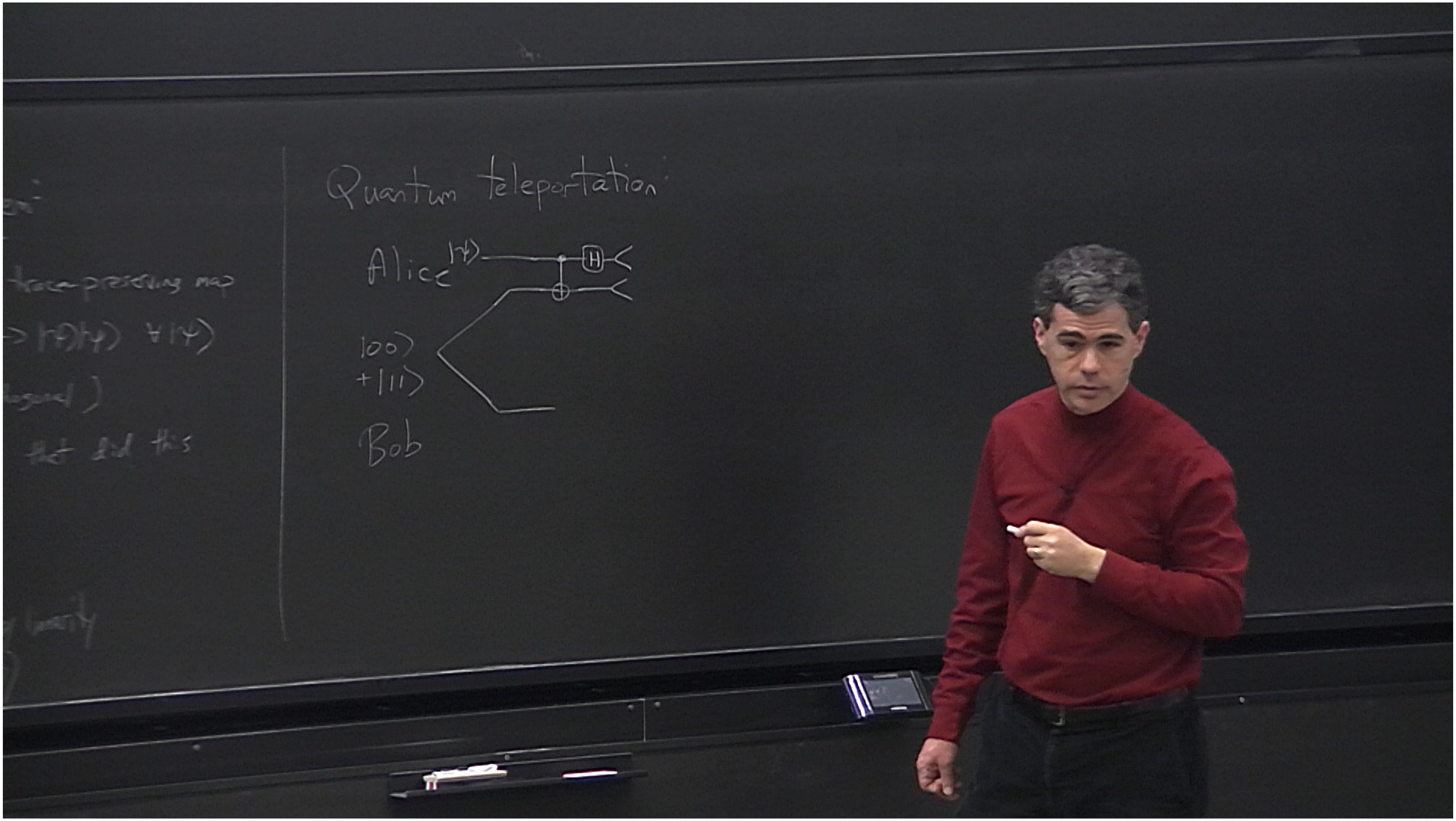
$\{H, R_{\pi/2}, \text{Tof}\}$, $\{\text{CNOT}, H, R_{\pi/4}\}$


No-cloning theorem:
$\nexists$ completely positive trace-preserving map
that takes $|\psi\rangle \longrightarrow |\psi\rangle|\psi\rangle \; \forall |\psi\rangle$

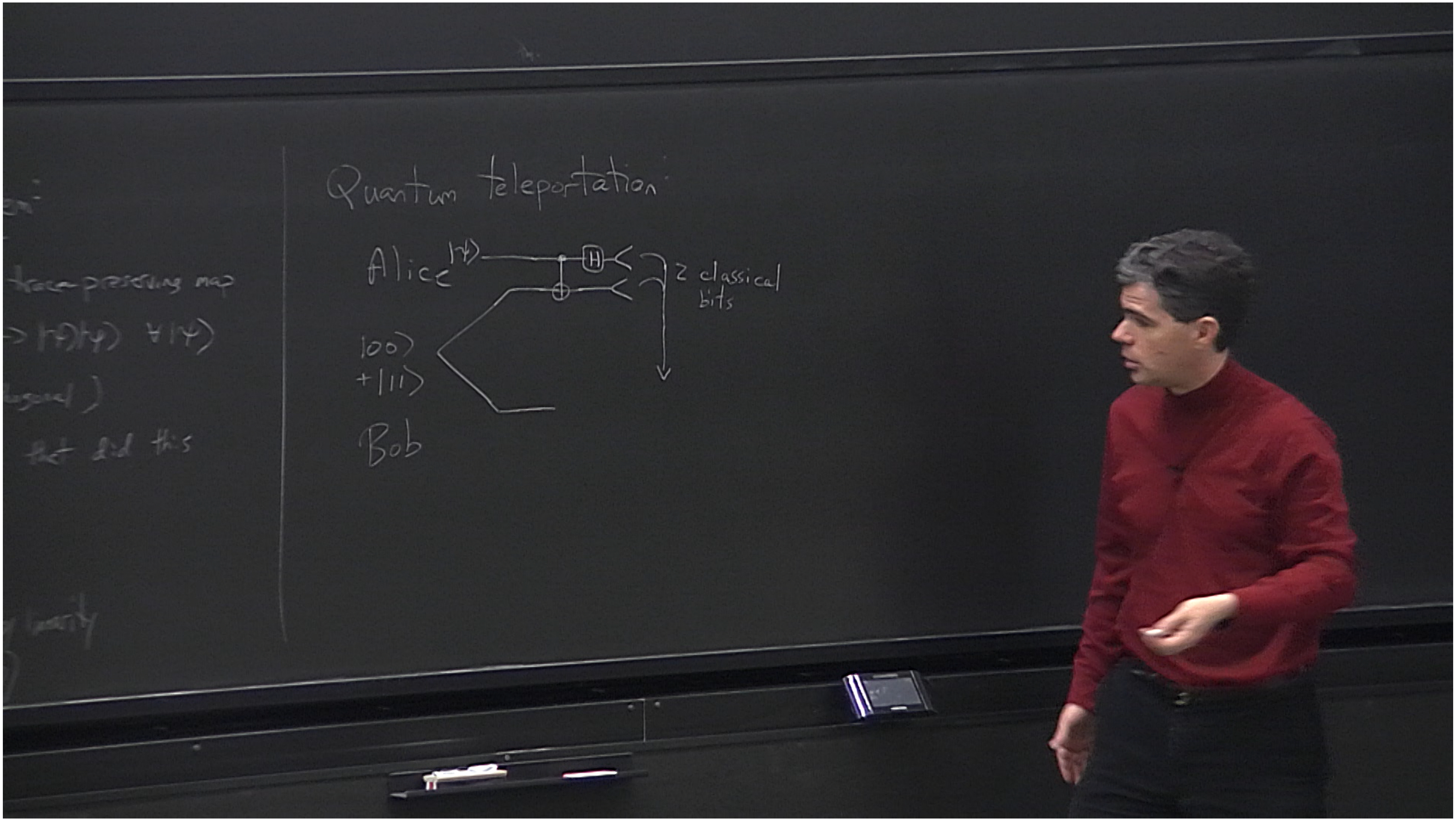

No-cloning theorem:
$\nexists$ completely positive trace-preserving map
that takes $|\psi\rangle \longrightarrow |\psi\rangle|\psi\rangle \; \forall |\psi\rangle$
(or $|\psi\rangle, |\phi\rangle$ not orthogonal)
Proof: Suppose $\exists \mathcal{M}$ that did this
$|\psi\rangle \rightarrow |\psi\rangle|\psi\rangle$
$|\phi\rangle \rightarrow |\phi\rangle|\phi\rangle$
$|\psi\rangle+|\phi\rangle \rightarrow |\psi\rangle|\psi\rangle+|\phi\rangle|\phi\rangle$ by linearity
$\neq (|\psi\rangle+|\phi\rangle)(|\psi\rangle+|\phi\rangle)$

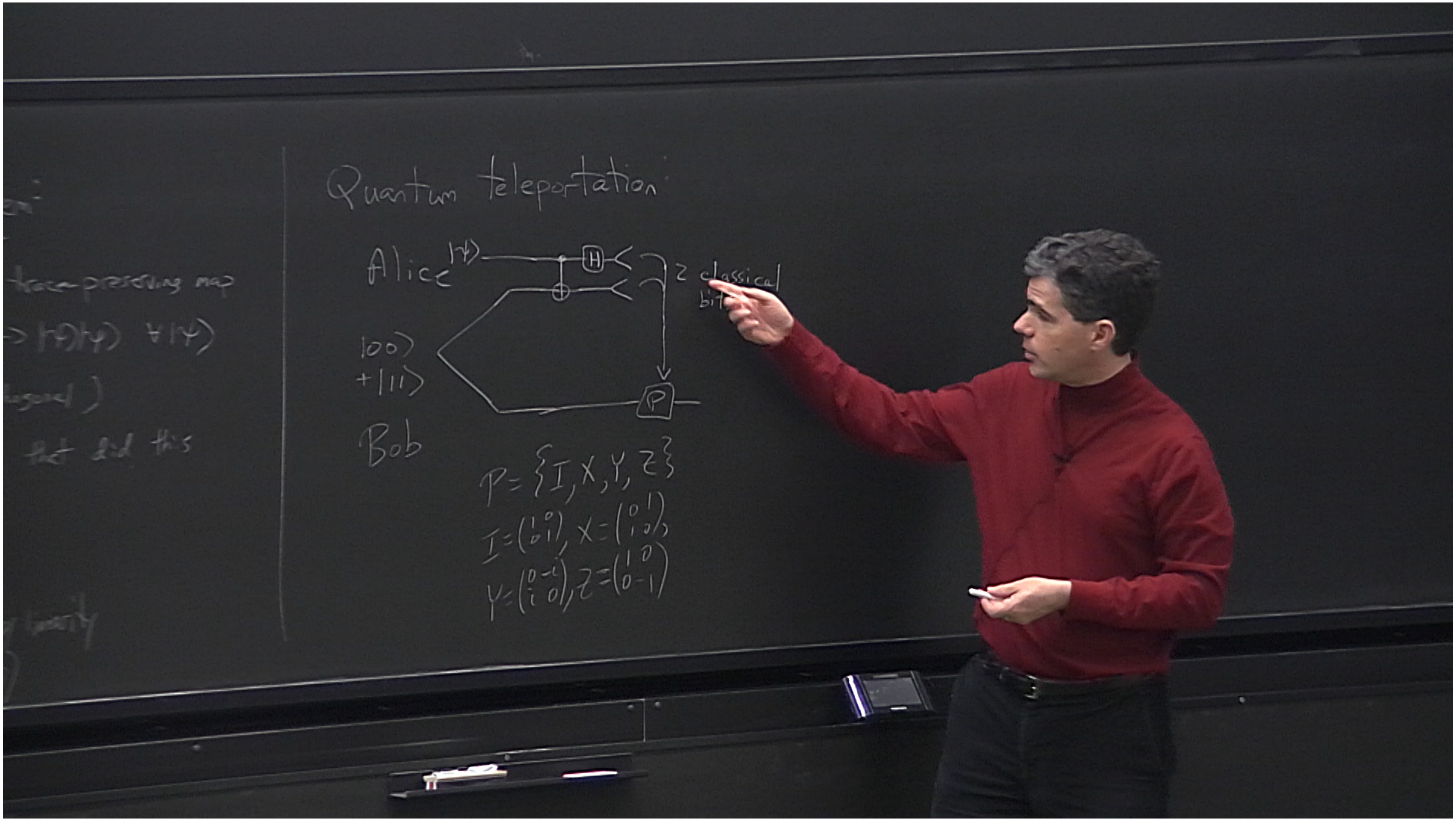

Quantum teleportation:
Alice
|ψ⟩
|00⟩
+|11⟩
Bob
trace-preserving map
that did this

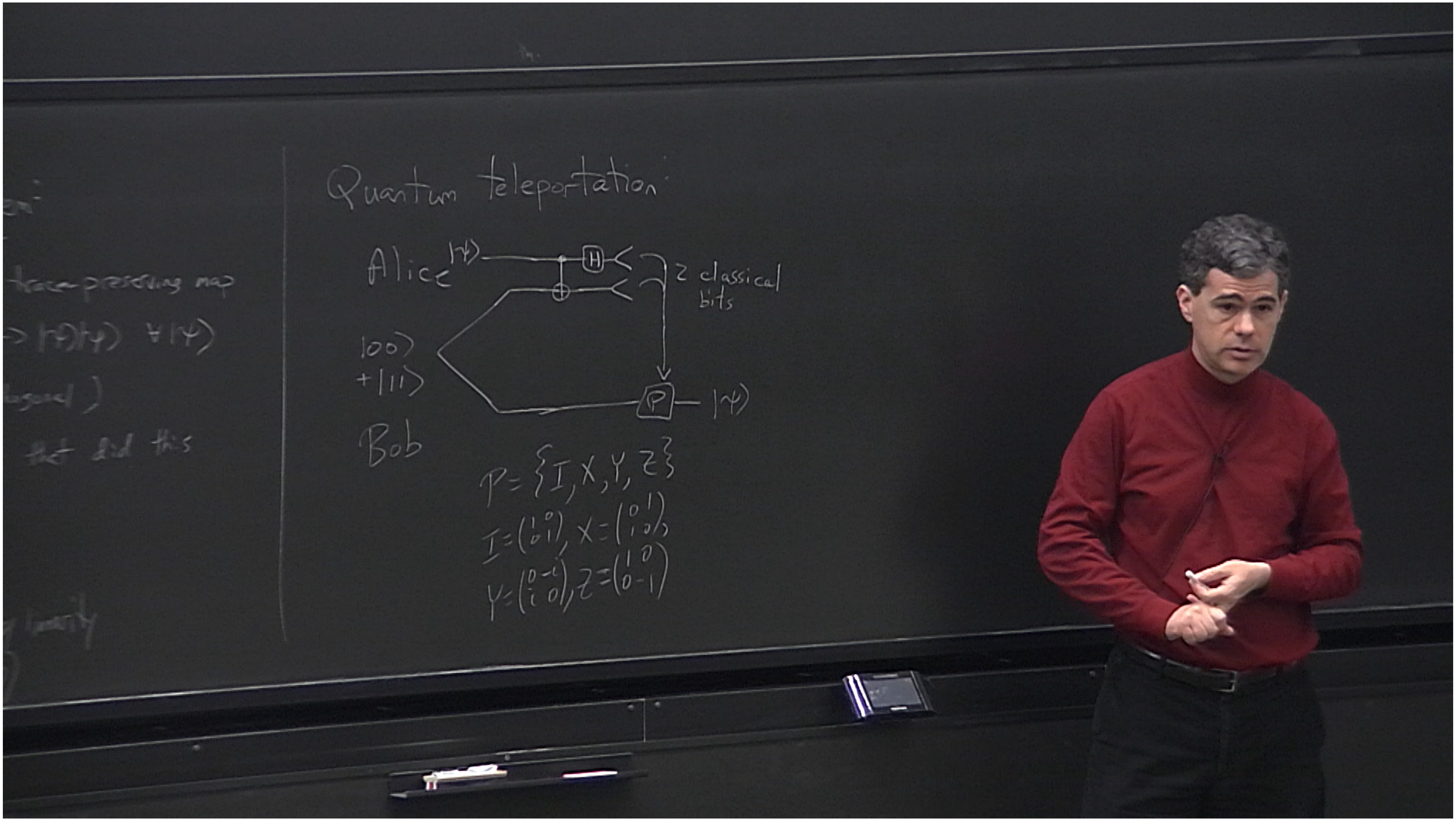

Quantum teleportation:
Alice |ψ⟩
|00⟩
+|11⟩
Bob
trace-preserving map
that did this

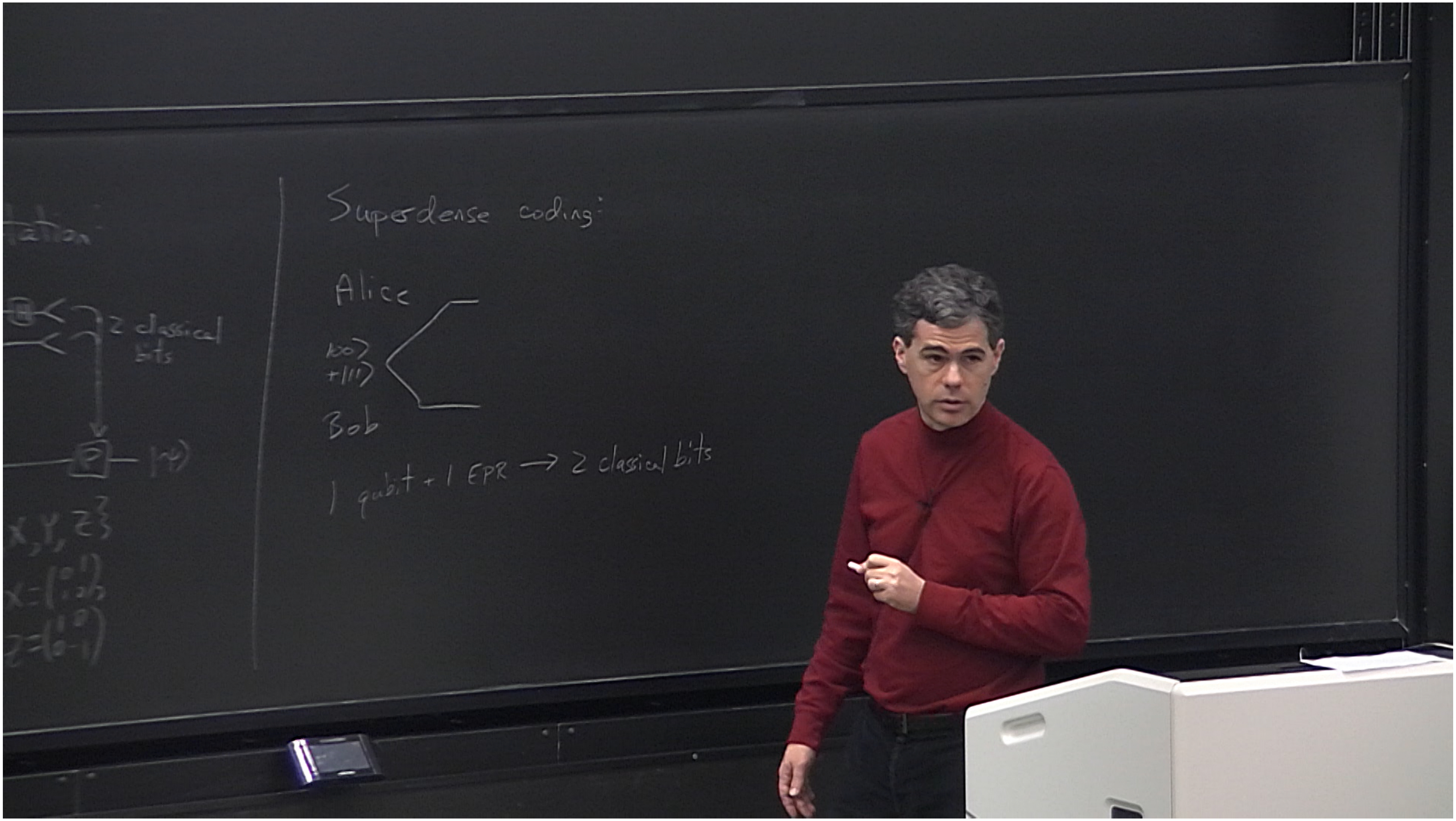


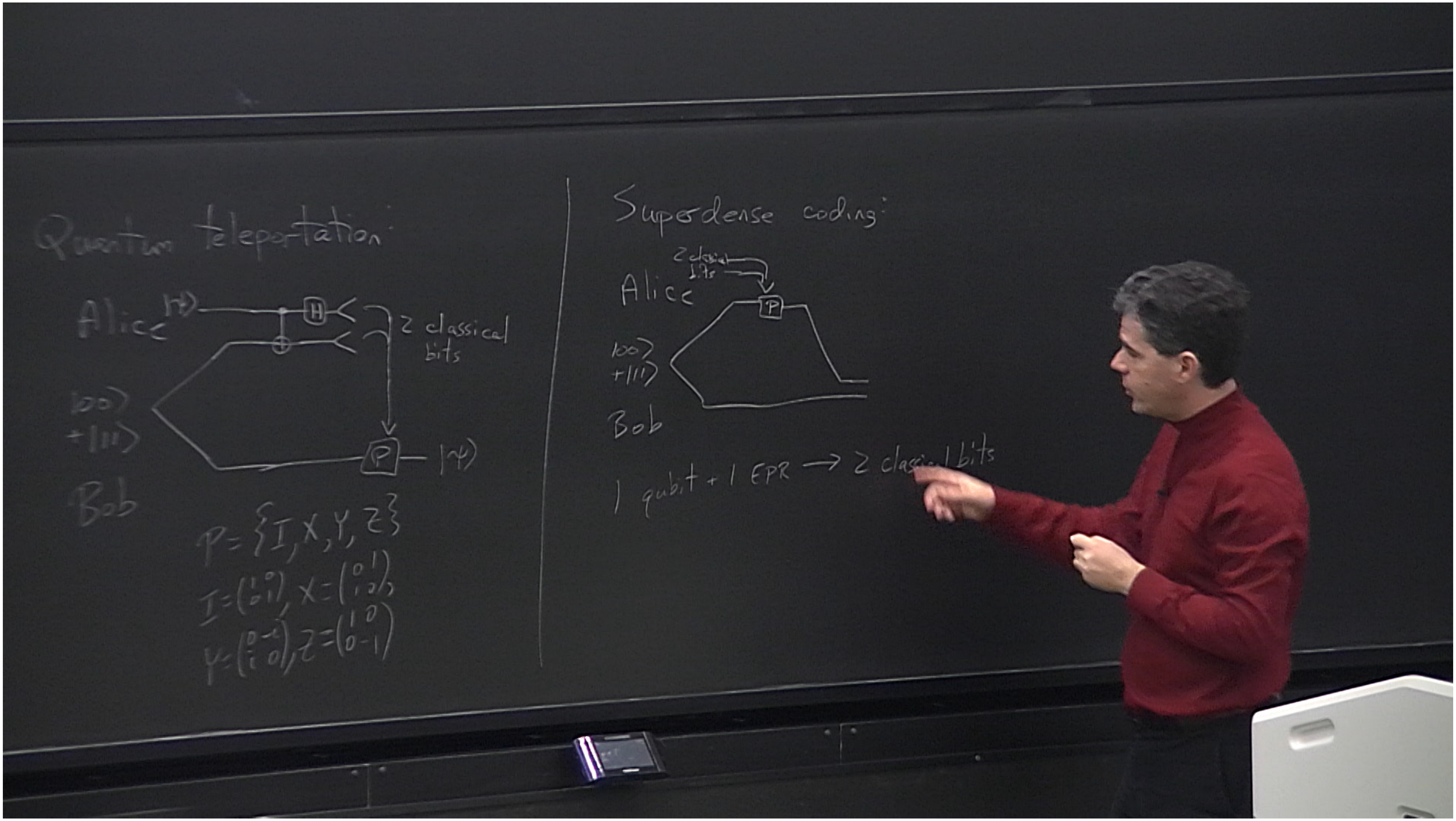
Quantum teleportation:
Alice
|ψ⟩
H
|00⟩
+|11⟩
Bob
trace-preserving map
that did this

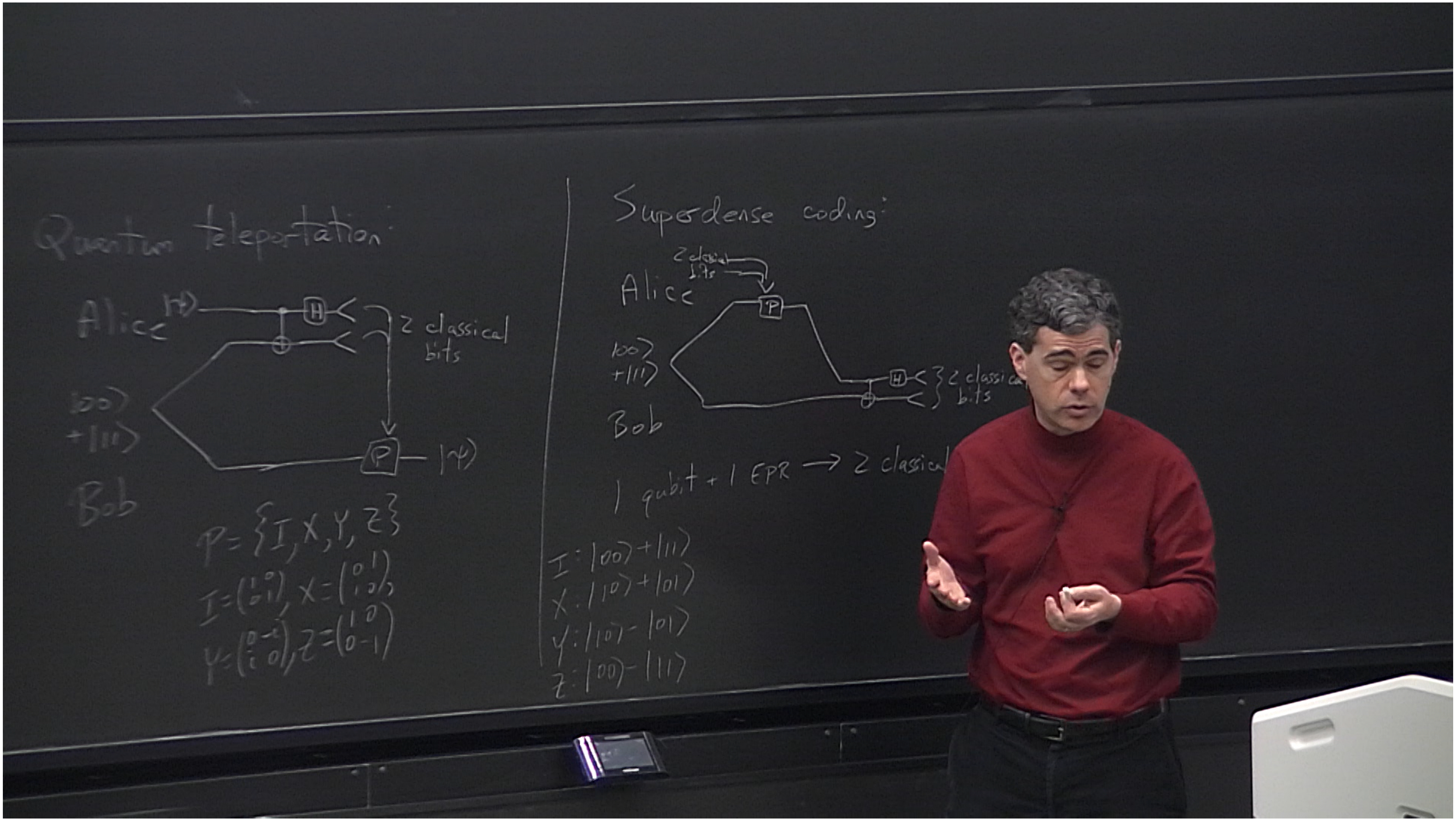

Quantum teleportation:
Alice
|ψ⟩
H
2 classical bits
|00⟩
+|11⟩
Bob
trace-preserving map
that did this

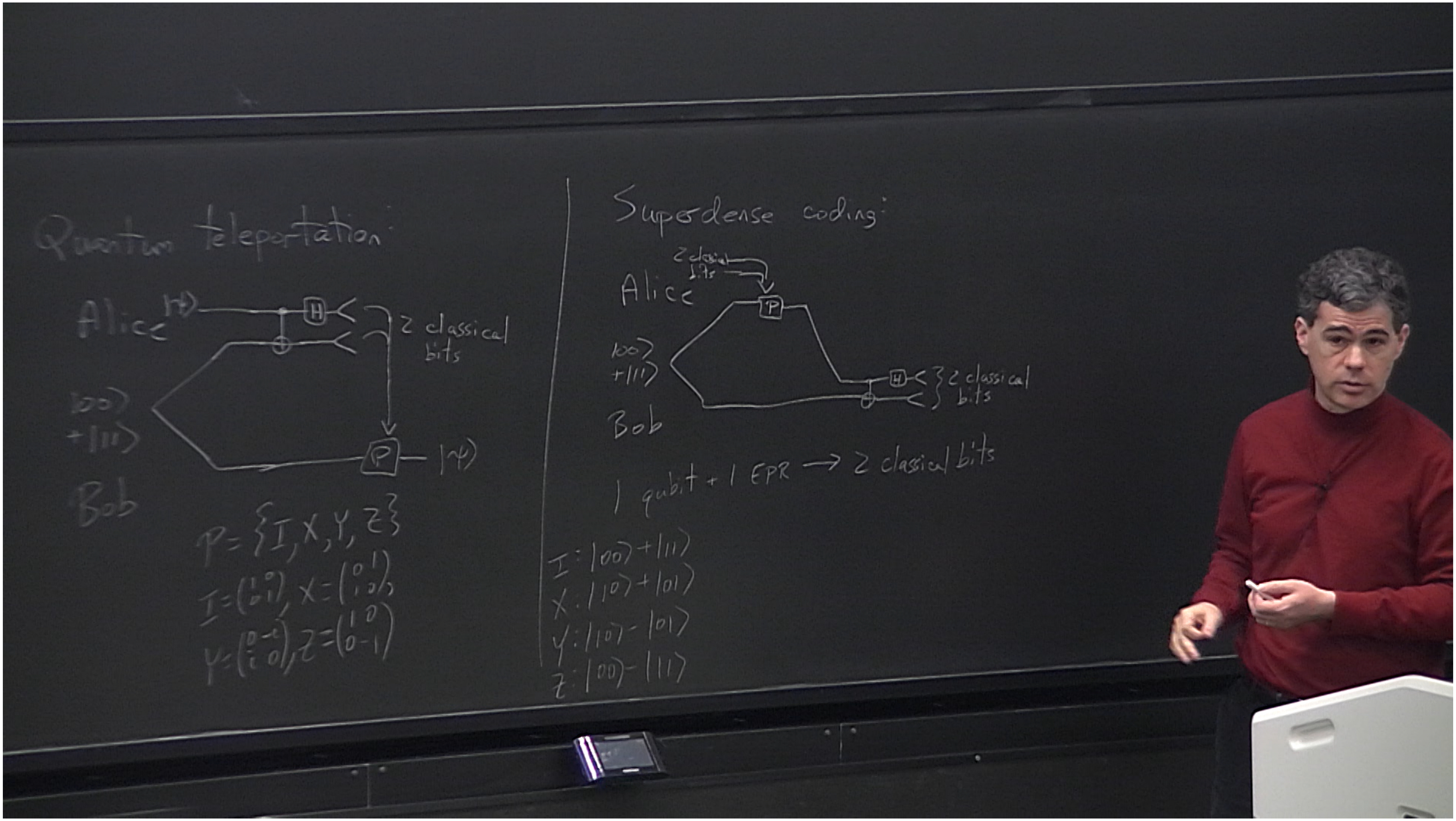

Quantum teleportation:
Alice
Bob
2 classical
bits
P = {I, X, Y, Z}
trace-preserving map
that did this

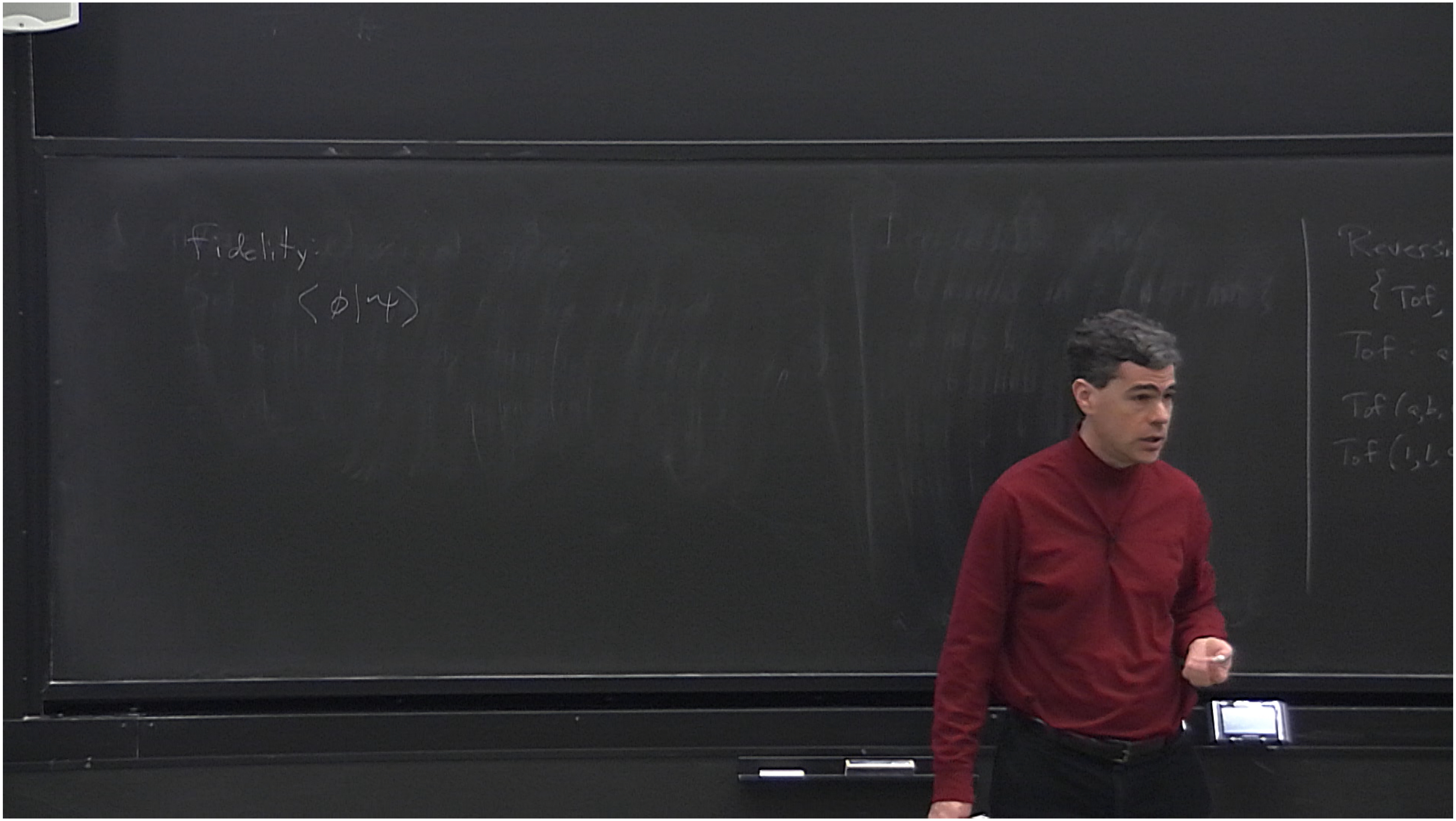

Quantum teleportation:
Alice
|ψ⟩
H
2 classical bits
|00⟩ + |11⟩
Bob
P
|ψ⟩
P = {I, X, Y, Z}
I = (1 0 / 0 1), X = (0 1 / 1 0),
Y = (0 -i / i 0), Z = (1 0 / 0 -1)
trace-preserving map
that did this

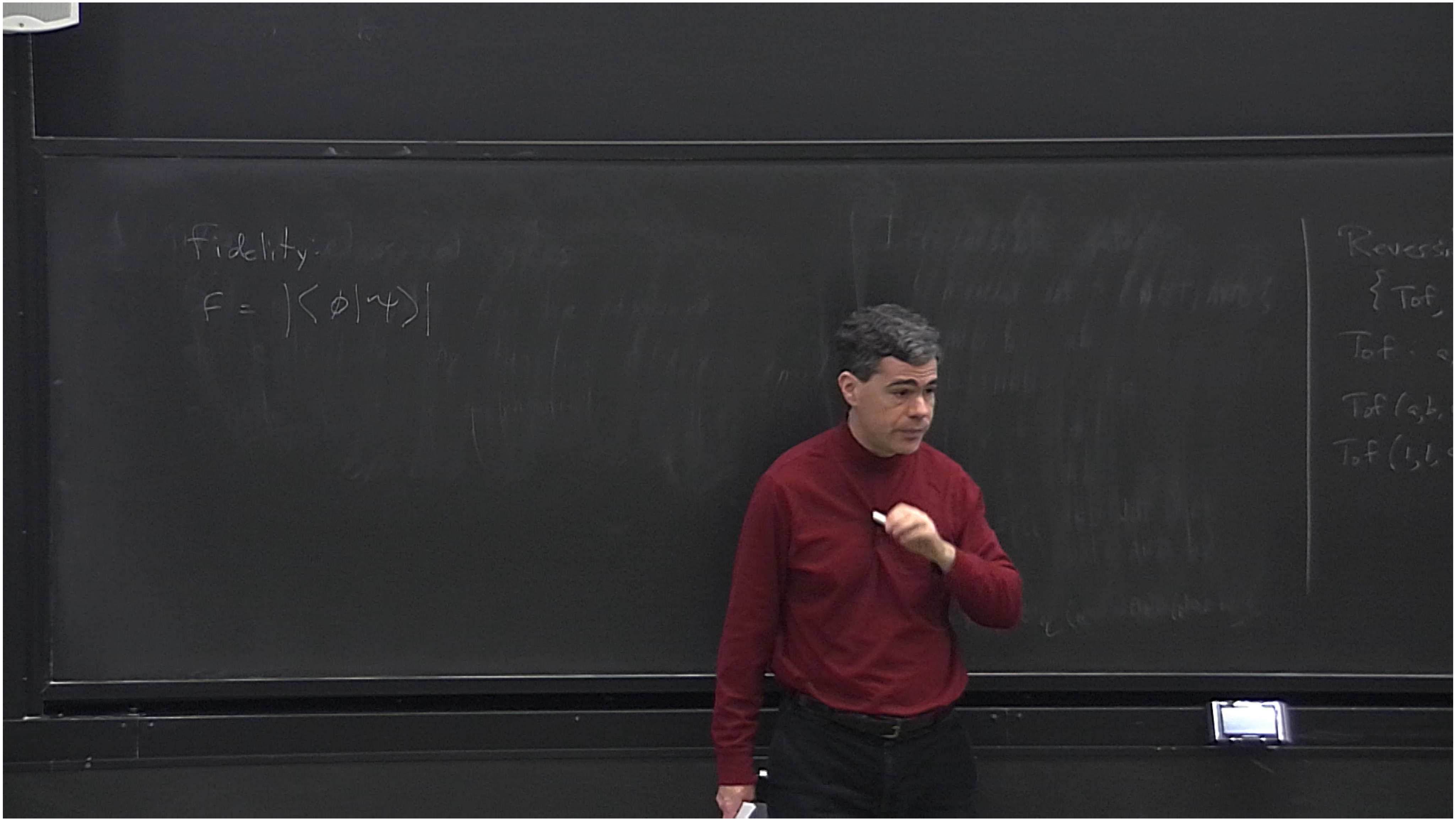

Superdense coding:
Alice
|00⟩ +|11⟩
Bob
1 qubit + 1 EPR → 2 classical bits
2 classical bits

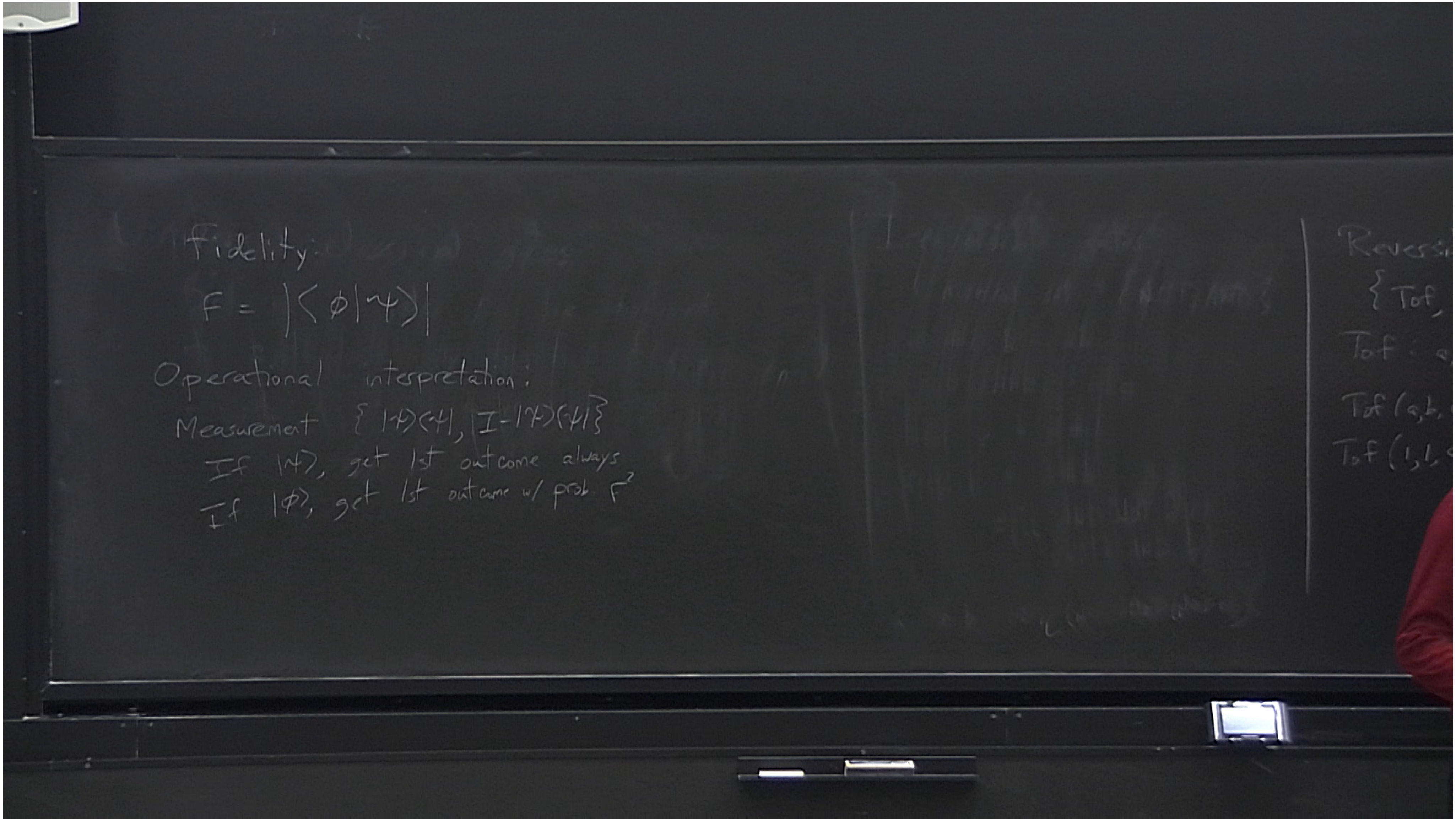

Quantum teleportation:
Alice
Bob
2 classical bits
P = {I, X, Y, Z}
Superdense coding:
1 qubit + 1 EPR → 2 classical bits


Quantum teleportation:
Alice
Bob
2 classical bits
P = {I, X, Y, Z}
Superdense coding:
Alice
Bob
2 classical bits
1 qubit + 1 EPR → 2 classical
I: |00⟩+|11⟩
X: |10⟩+|01⟩
Y: |10⟩−|01⟩
Z: |00⟩−|11⟩

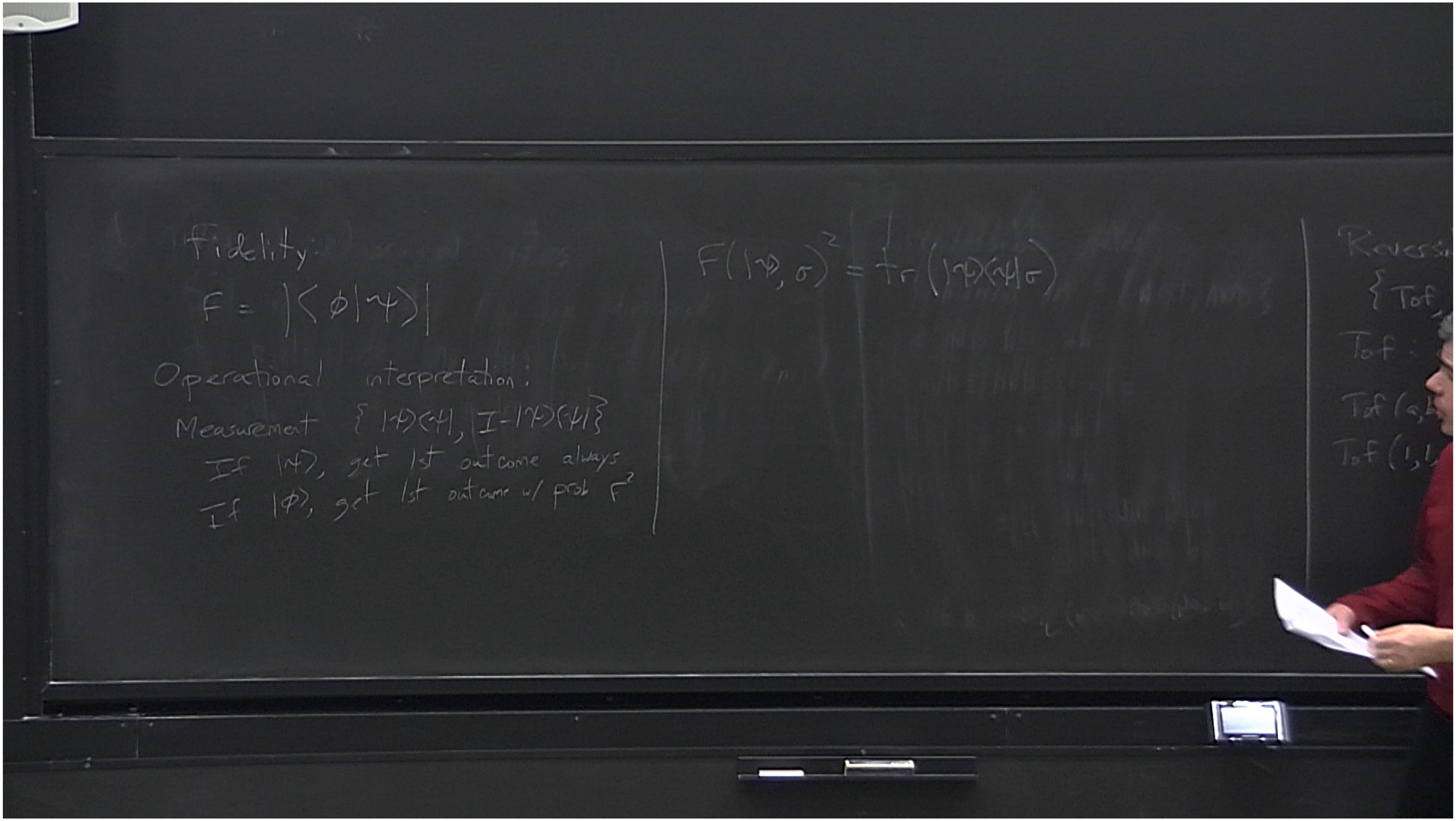

Quantum teleportation:
Alice
|ψ⟩
H
2 classical bits
|00⟩ + |11⟩
P
|ψ⟩
Bob
P = {I, X, Y, Z}
Superdense coding:
2 classical bits
Alice
P
|00⟩ + |11⟩
H
2 classical bits
Bob
1 qubit + 1 EPR → 2 classical bits
I: |00⟩ + |11⟩
X: |10⟩ + |01⟩
Y: |10⟩ − |01⟩
Z: |00⟩ − |11⟩


Fidelity:


Fidelity:
F = |⟨φ|ψ⟩|



Fidelity:
$F = |\langle \phi | \psi \rangle|$
Operational interpretation:
Measurement $\{ |\psi\rangle\langle\psi|, I - |\psi\rangle\langle\psi| \}$
If $|\psi\rangle$, get 1st outcome always
If $|\phi\rangle$, get 1st outcome w/ prob. F^2


Fidelity:
F = |⟨φ|ψ⟩|
Operational interpretation:
Measurement { |ψ⟩⟨ψ|, I - |ψ⟩⟨ψ| }
If |ψ⟩, get 1st outcome always
If |φ⟩, get 1st outcome w/ prob F²
F(|ψ⟩, σ)² = tr(|ψ⟩⟨ψ|σ)

Fidelity:

$F = |\langle \phi | \psi \rangle|$

Operational interpretation:

Measurement $\{ |\psi\rangle\langle\psi|, I - |\psi\rangle\langle\psi| \}$

If $|\psi\rangle$, get 1st outcome always

If $|\phi\rangle$, get 1st outcome w/ prob. F^2

$F(|\psi\rangle, \sigma)^2 = \mathrm{tr}(|\psi\rangle\langle\psi|\sigma)$

$F(\rho, \sigma) = \mathrm{tr}\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$

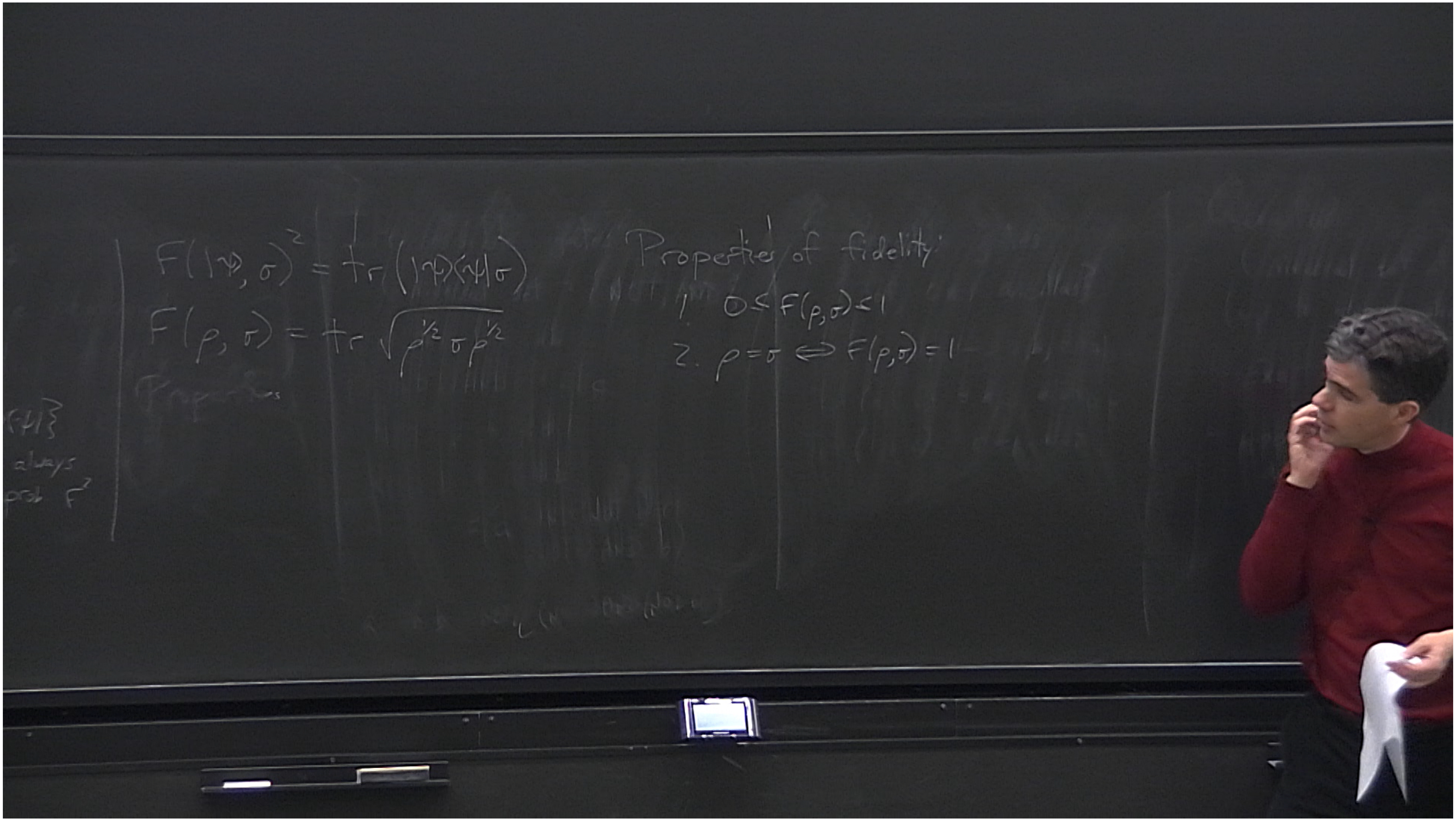
Reversi

{ Tof,

Tof :

Tof (a,b,

Tof (1,1,



$F(|\psi\rangle, \sigma)^2 = \mathrm{tr}(|\psi\rangle\langle\psi|\sigma)$
$F(\rho, \sigma) = \mathrm{tr}\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$
Properties
Properties of fidelity:
1. $0 \leq F(\rho,\sigma) \leq 1$
2. $\rho = \sigma \Leftrightarrow F(\rho,\sigma) = 1$
always
prob F^2

$\{\psi|\}$
always
prob F^2

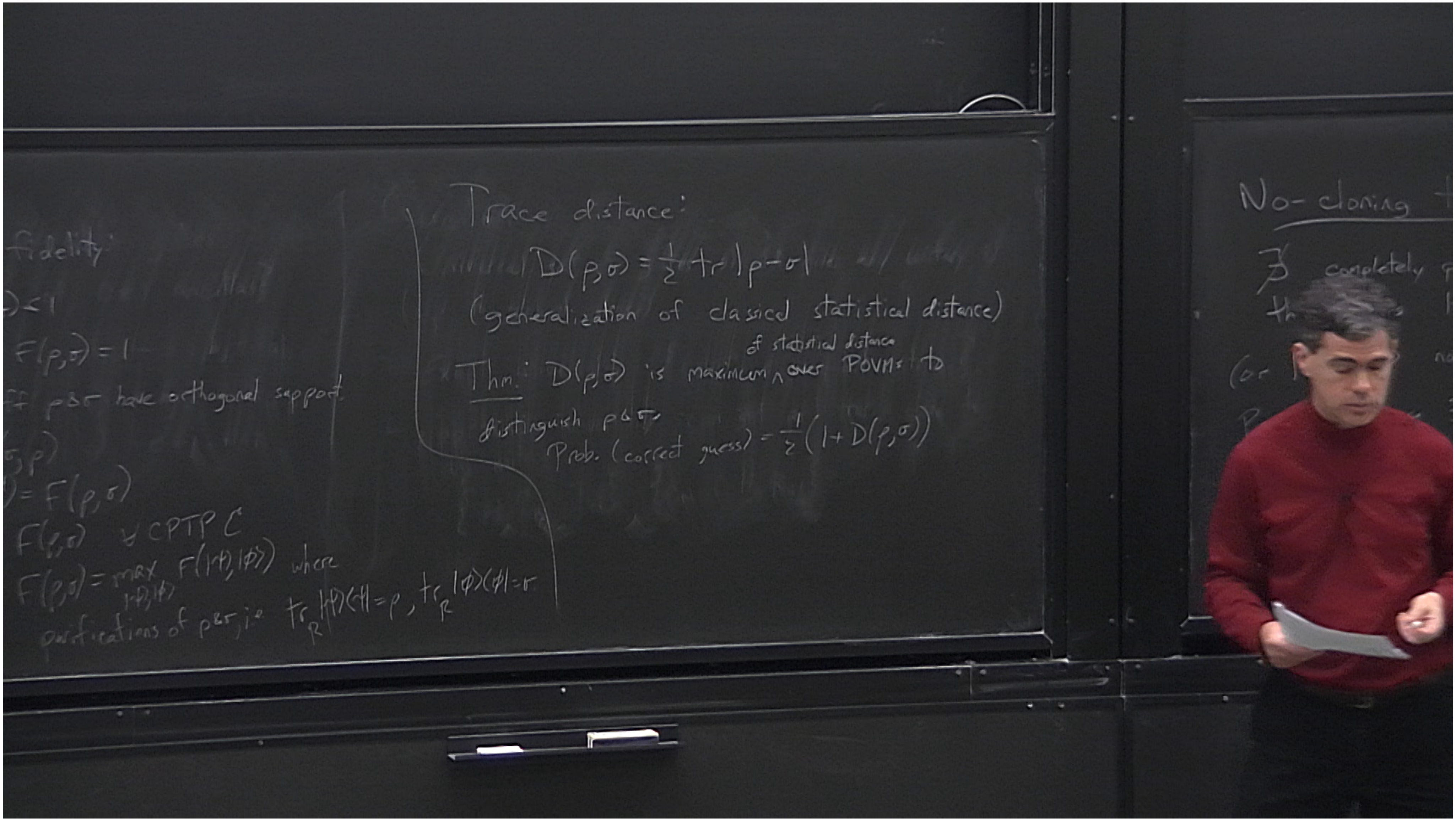
$$F(|\psi\rangle, \sigma)^2 = \mathrm{tr}(|\psi\rangle\langle\psi|\sigma)$$

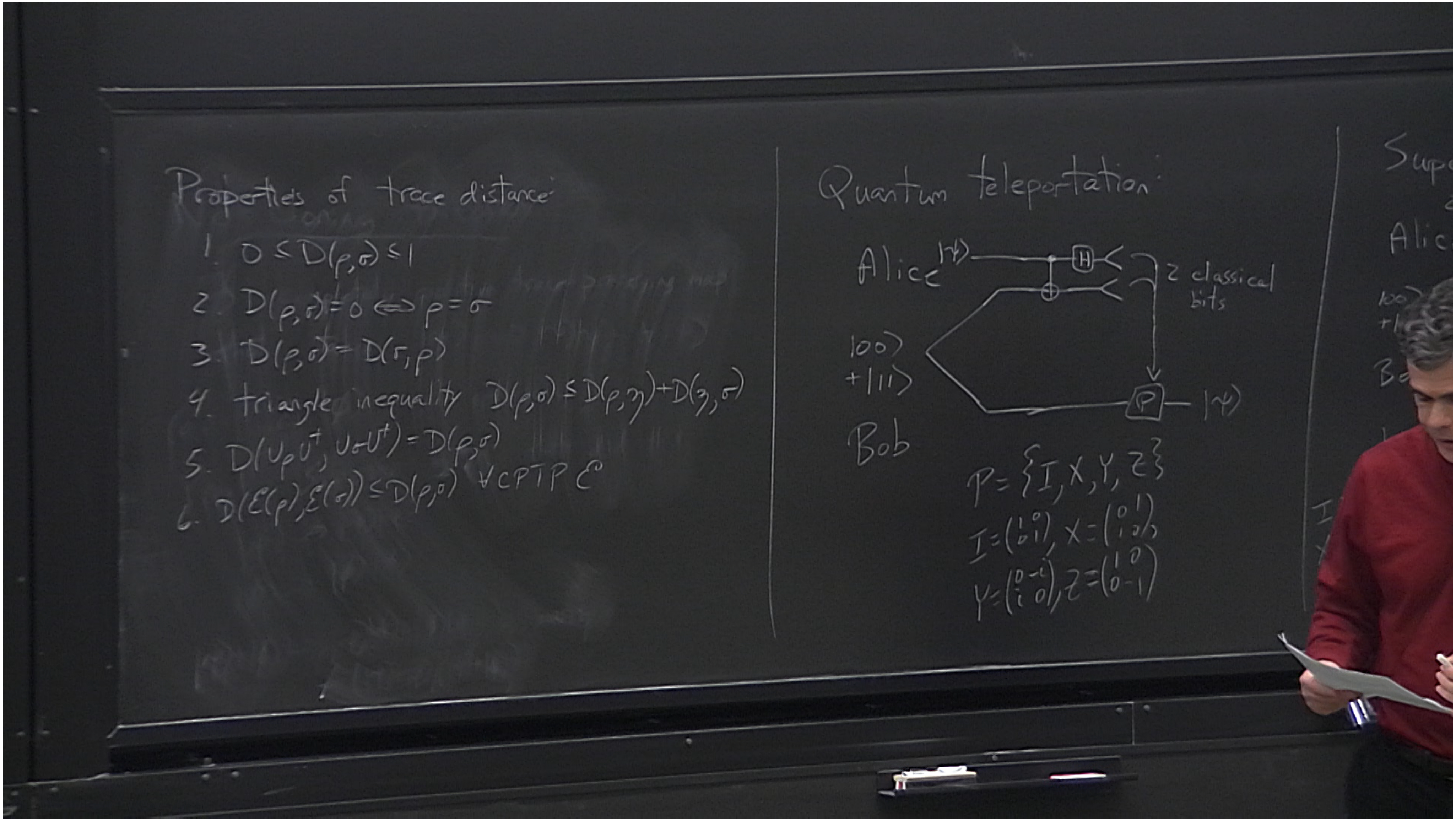
$$F(\rho, \sigma) = \mathrm{tr}\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$$

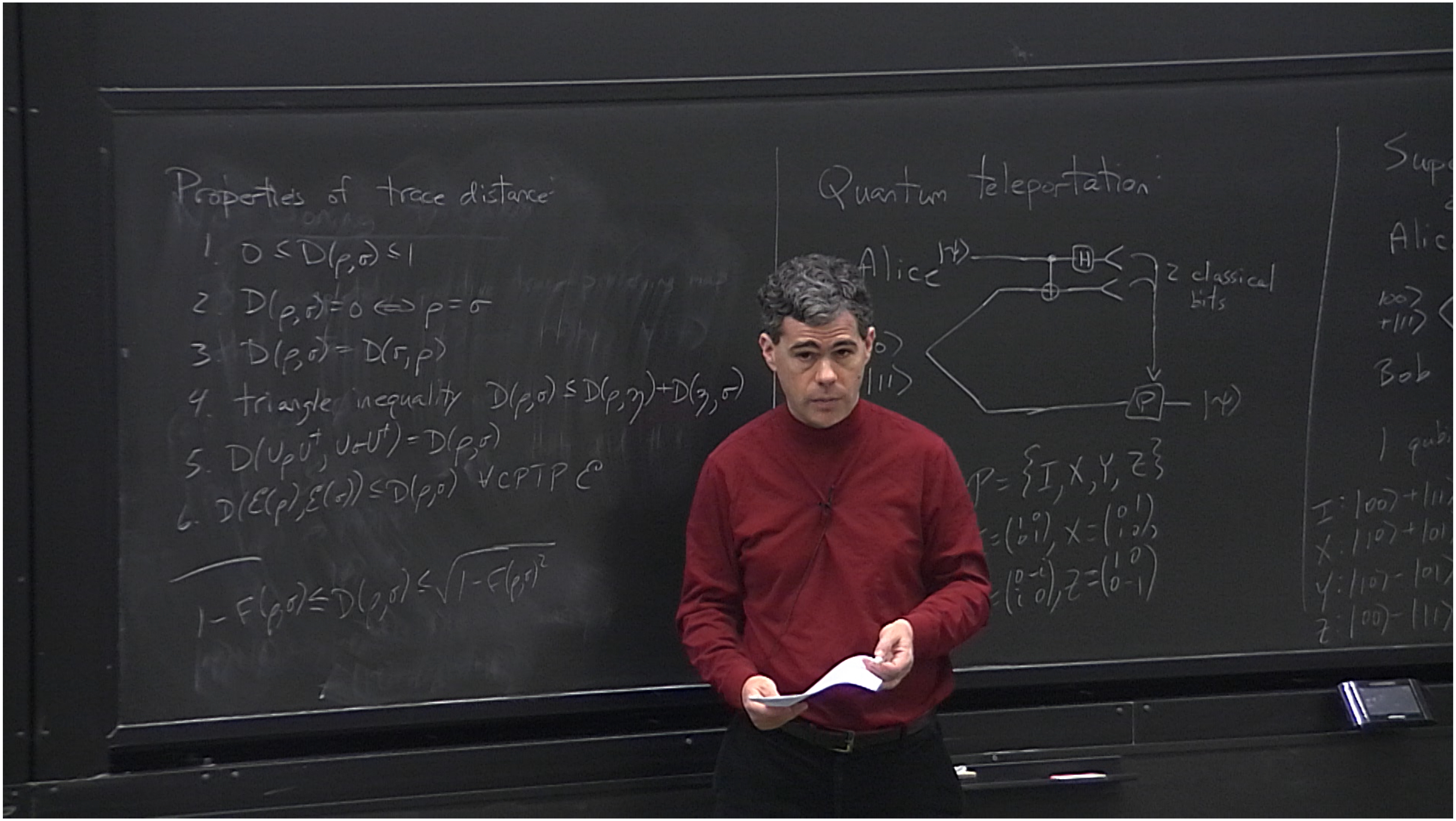
Properties

Properties of fidelity:

1. $0 \le F(\rho,\sigma) \le 1$
2. $\rho = \sigma \iff F(\rho,\sigma) = 1$
3. $F(\rho,\sigma) = 0$ iff ρ & σ have orthogonal support
4. $F(\rho,\sigma) = F(\sigma,\rho)$
5. $F(U\rho U^\dagger, U\sigma U^\dagger) = F(\rho,\sigma)$
6. $F(\mathcal{E}(\rho),\mathcal{E}(\sigma)) \ge F(\rho,\sigma)$ $\forall$ CPTP $\mathcal{E}$
7. (Uhlmann's thm) $F(\rho,\sigma) = \max_{|\psi\rangle,|\phi\rangle} F(|\psi\rangle,|\phi\rangle)$ where purifications of ρ & σ, i.e. $\mathrm{tr}_R|\psi\rangle\langle\psi| = \rho$, $\mathrm{tr}_R|\phi\rangle\langle\phi| = \sigma$


Trace distance:
$D(\rho,\sigma) = \frac{1}{2}\,\mathrm{tr}\,|\rho-\sigma|$
(generalization of classical statistical distance)
Thm: $D(\rho,\sigma)$ is maximum of statistical distance over POVMs to distinguish ρ & σ
Prob. (correct guess) $= \frac{1}{2}(1 + D(\rho,\sigma))$
fidelity:
$F(\rho,\sigma) = 1$
ff ρ & σ have orthogonal support
$= F(\rho,\sigma)$
$F(\rho,\sigma)$ $\forall$ CPTP $\mathcal{C}$
$F(\rho,\sigma) = \max_{|\psi\rangle,|\phi\rangle} F(|\psi\rangle,|\phi\rangle)$ where
purifications of ρ & σ, i.e. $\mathrm{tr}_R |\psi\rangle\langle\psi| = \rho$, $\mathrm{tr}_R |\phi\rangle\langle\phi| = \sigma$
No-cloning
completely


Properties of trace distance:
1. 0 ≤ D(ρ,σ) ≤ 1
2. D(ρ,σ) = 0 ⟺ ρ = σ
3. D(ρ,σ) = D(σ,ρ)
4. triangle inequality D(ρ,σ) ≤ D(ρ,τ) + D(τ,σ)
5. D(UρU†, UσU†) = D(ρ,σ)
6. D(E(ρ), E(σ)) ≤ D(ρ,σ) ∀ CPTP E
Quantum teleportation:
Alice
|ψ⟩
H
2 classical bits
|00⟩ + |11⟩
Bob
P
|ψ⟩
P = {I, X, Y, Z}
I = (1 0; 0 1), X = (0 1; 1 0),
Y = (0 −i; i 0), Z = (1 0; 0 −1)



Properties of trace distance
1. $0 \le D(\rho,\sigma) \le 1$
2. $D(\rho,\sigma)=0 \iff \rho=\sigma$
3. $D(\rho,\sigma)=D(\sigma,\rho)$
4. triangle inequality $D(\rho,\sigma) \le D(\rho,\eta)+D(\eta,\sigma)$
5. $D(U\rho U^\dagger, U\sigma U^\dagger)=D(\rho,\sigma)$
6. $D(\mathcal{E}(\rho),\mathcal{E}(\sigma)) \le D(\rho,\sigma)$ $\forall$ CPTP $\mathcal{E}$
$1-F(\rho,\sigma) \le D(\rho,\sigma) \le \sqrt{1-F(\rho,\sigma)^2}$
Quantum teleportation
Alice
2 classical bits
Bob