Title: PSI 17/18 - Quantum Information - Lecture 1

Date: Feb 20, 2018 11:30 AM

URL: http://pirsa.org/18020044

Abstract:

Quantum Information

- Build a quantum computer
- Properties of QI
  - Quantum algorithms
  - Quantum protocols (QKD, etc)
- Use QI ideas to illuminate physics & CS

computer

Qubit

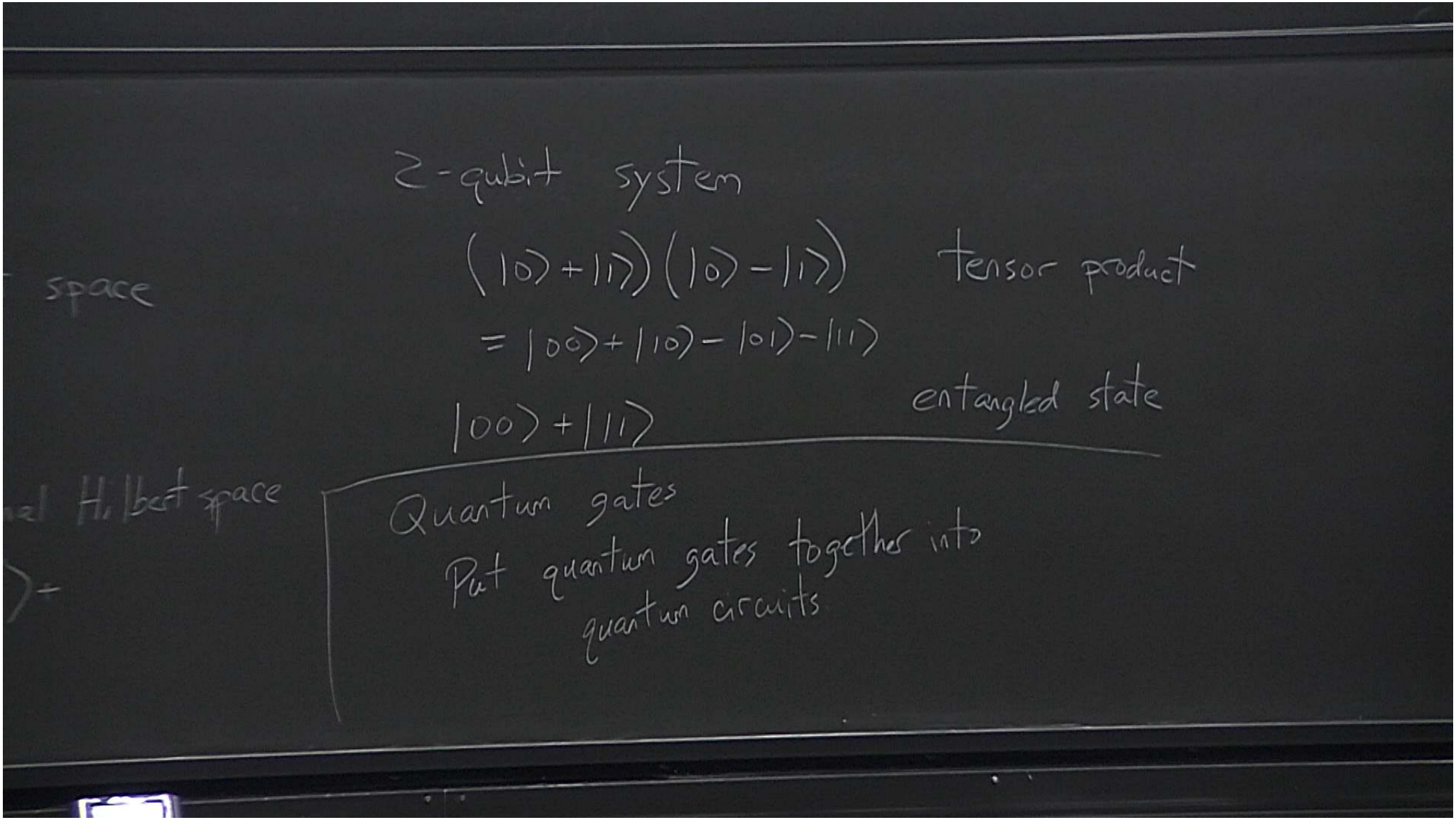
2-dimensional Hilbert space

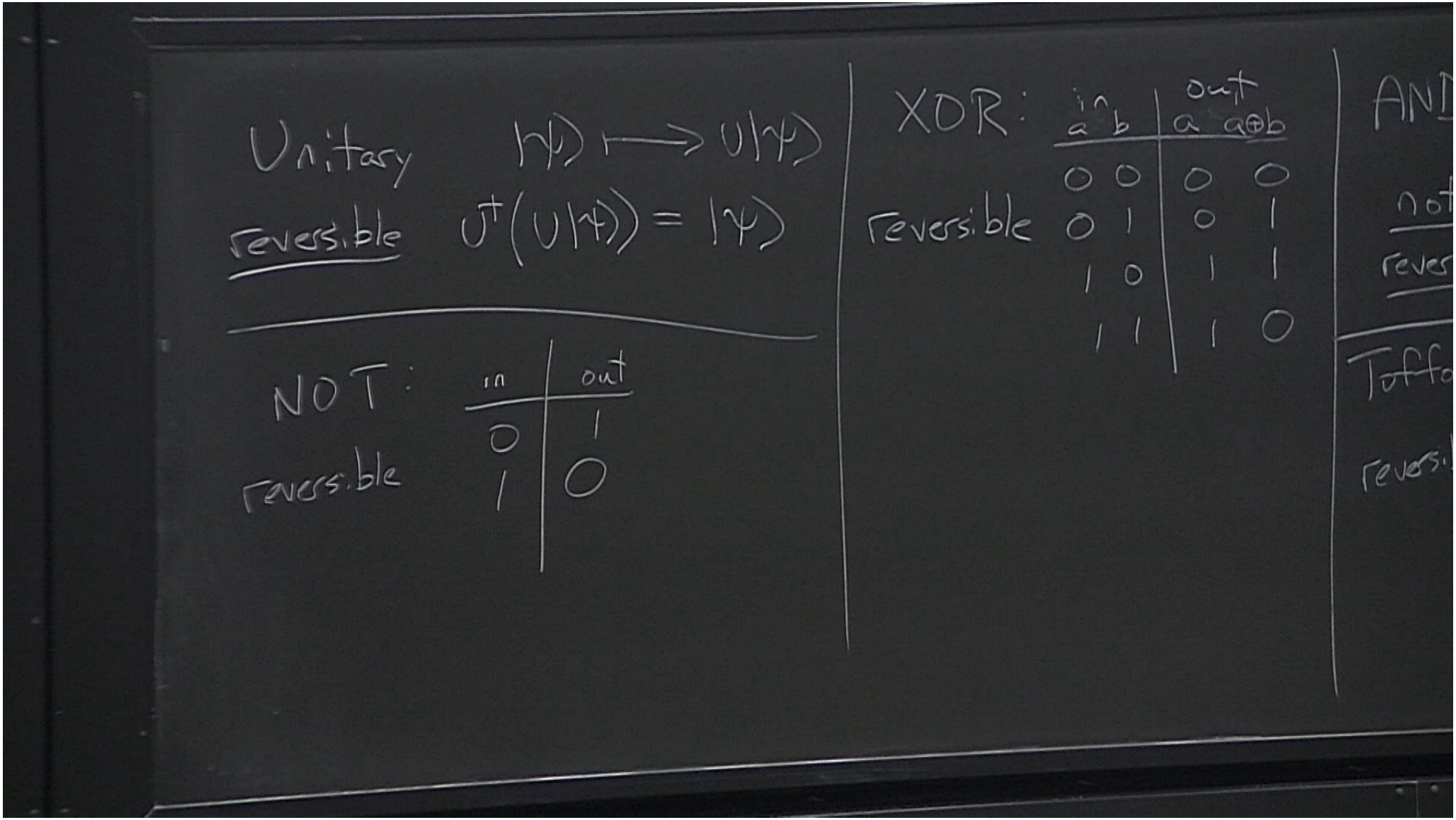
$|0\rangle, |1\rangle$

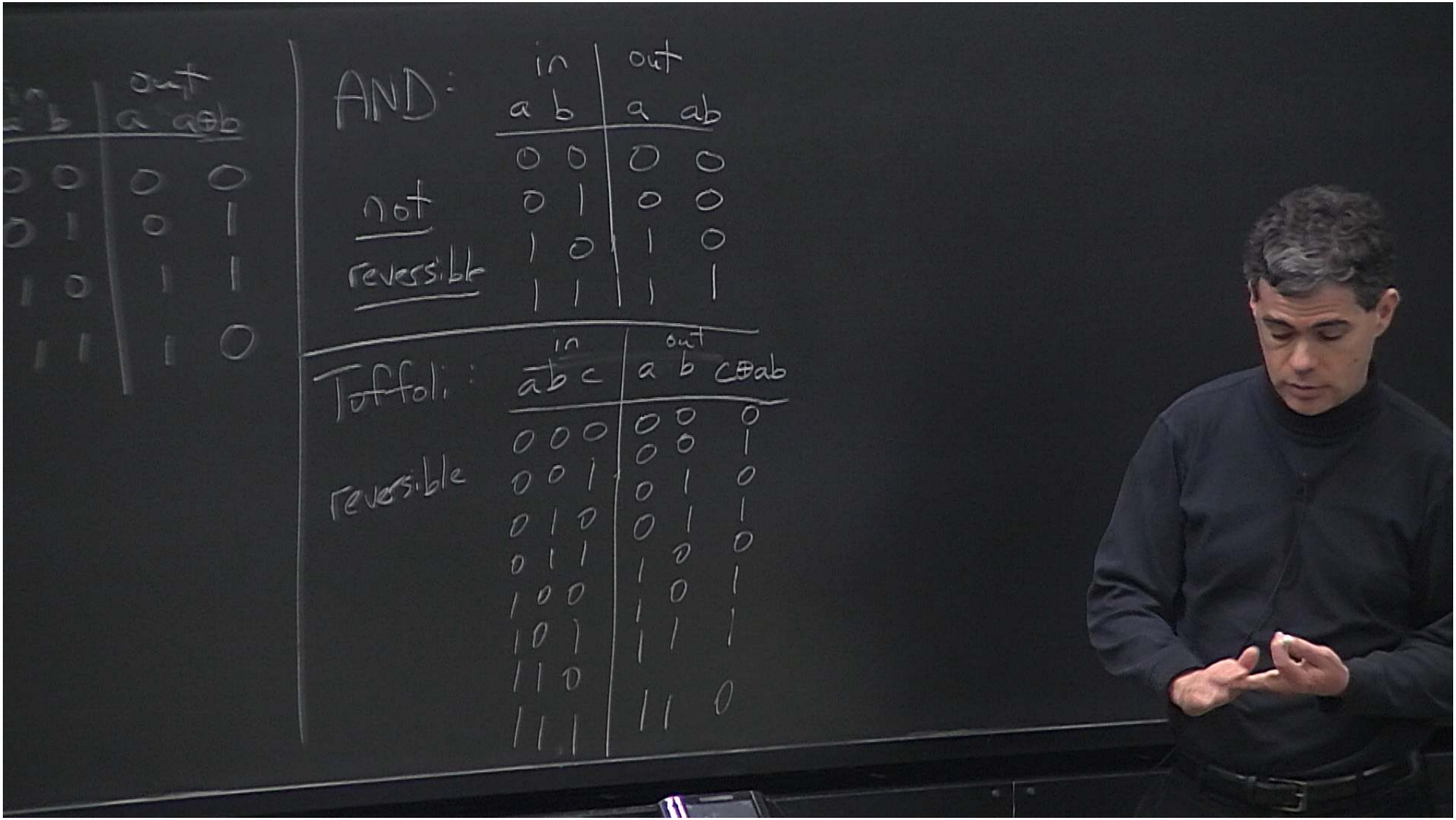
$\alpha|0\rangle + \beta|1\rangle$

n qubits - 2^n-dimensional Hilbert space

$\alpha|0110\rangle + \beta|1100\rangle + \gamma|1001\rangle +$


2-qubit system
$(|0\rangle + |1\rangle)(|0\rangle - |1\rangle)$
tensor product
$= |00\rangle + |10\rangle - |01\rangle - |11\rangle$
$|00\rangle + |11\rangle$
entangled state
Quantum gates
Put quantum gates together into quantum circuits


Unitary
$|\psi\rangle \longmapsto U|\psi\rangle$
reversible
$U^\dagger(U|\psi\rangle) = |\psi\rangle$
NOT:
in | out
0 | 1
1 | 0
reversible
XOR:
in: a ^ b | out: a | a⊕b
0 0 | 0 0
0 1 | 0 1
1 0 | 1 1
1 1 | 1 0
reversible
AND
not
reversible
Toffo
reversi

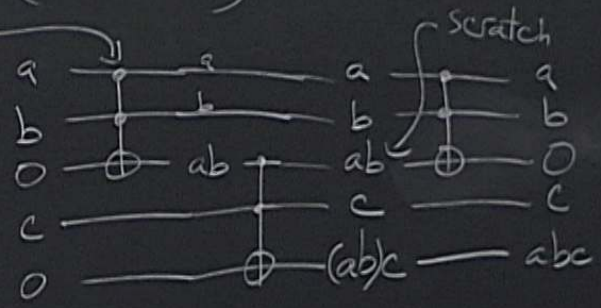

out
a a⊕b
0 0
0 1
1 1
1 0
AND:
in
a b
out
a ab
0 0 0 0
0 1 0 0
1 0 1 0
1 1 1 1
not
reversible
Toffoli:
in
a b c
out
a b c⊕ab
0 0 0 0 0 0
0 0 1 0 0 1
0 1 0 0 1 0
0 1 1 0 1 1
1 0 0 1 0 0
1 0 1 1 0 1
1 1 0 1 1 1
1 1 1 1 1 0
reversible

AND:

in		out
a	b	a ab
0	0	0 0
0	1	0 0
1	0	1 0
1	1	1 1

not reversible

Toffoli: (a AND b) AND c = abc

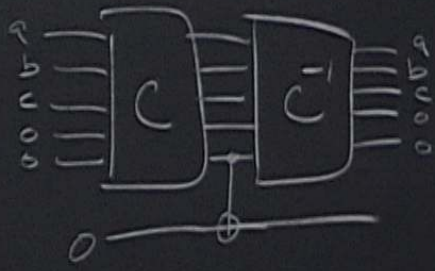


Circuit diagram

Toffoli:

in			out		
a	b	c	a	b	c⊕ab
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	0	1
1	1	0	1	1	1
1	1	1	1	1	0

reversible



CNOT (Controlled-NOT):

$$|a,b\rangle \longmapsto |a, a\oplus b\rangle$$

$$\mathrm{CNOT}(\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\delta|11\rangle)$$

$$= \alpha|00\rangle+\beta|01\rangle+\gamma|11\rangle+\delta|10\rangle$$

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

NOT $|a\rangle = |\bar{a}\rangle$

Toffoli: $|a,b,c\rangle = |a,b,c \oplus ab\rangle$

Hadamard transform: $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$|0\rangle \longmapsto |0\rangle + |1\rangle$

$|1\rangle \longmapsto |0\rangle - |1\rangle$

Phase rotation: $R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$ $\quad |0\rangle \longmapsto |0\rangle$, $\quad |1\rangle \longmapsto e^{i\theta}|1\rangle$

2-qub

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|00

Quant

Put

NOT $|a\rangle = |\bar{a}\rangle$

Toffoli: $|a,b,c\rangle = |a,b,c \oplus ab\rangle$

Hadamard transform: $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

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Phase rotation: $R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$

$|0\rangle \longmapsto |0\rangle$

$|1\rangle \longmapsto e^{i\theta}|1\rangle$

$\alpha|0\rangle + \beta|1\rangle \longmapsto \alpha|0\rangle + \beta e^{i\theta}|1\rangle$

2-qub

Non-unitary:

introduce $|0\rangle$ qubit $|0\rangle$ —

measurement (eg. in standard basis)

$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ — H —

$|0\rangle \mapsto |0\rangle$

$|1\rangle \mapsto e^{i\theta}|1\rangle$ — R_θ —

$\alpha|0\rangle + \beta|1\rangle \mapsto \alpha|0\rangle + \beta e^{i\theta}|1\rangle$

Hadamard is self-inverse

$H^2 = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

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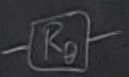

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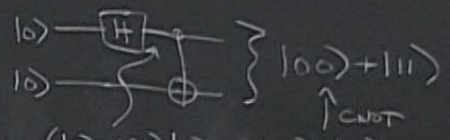
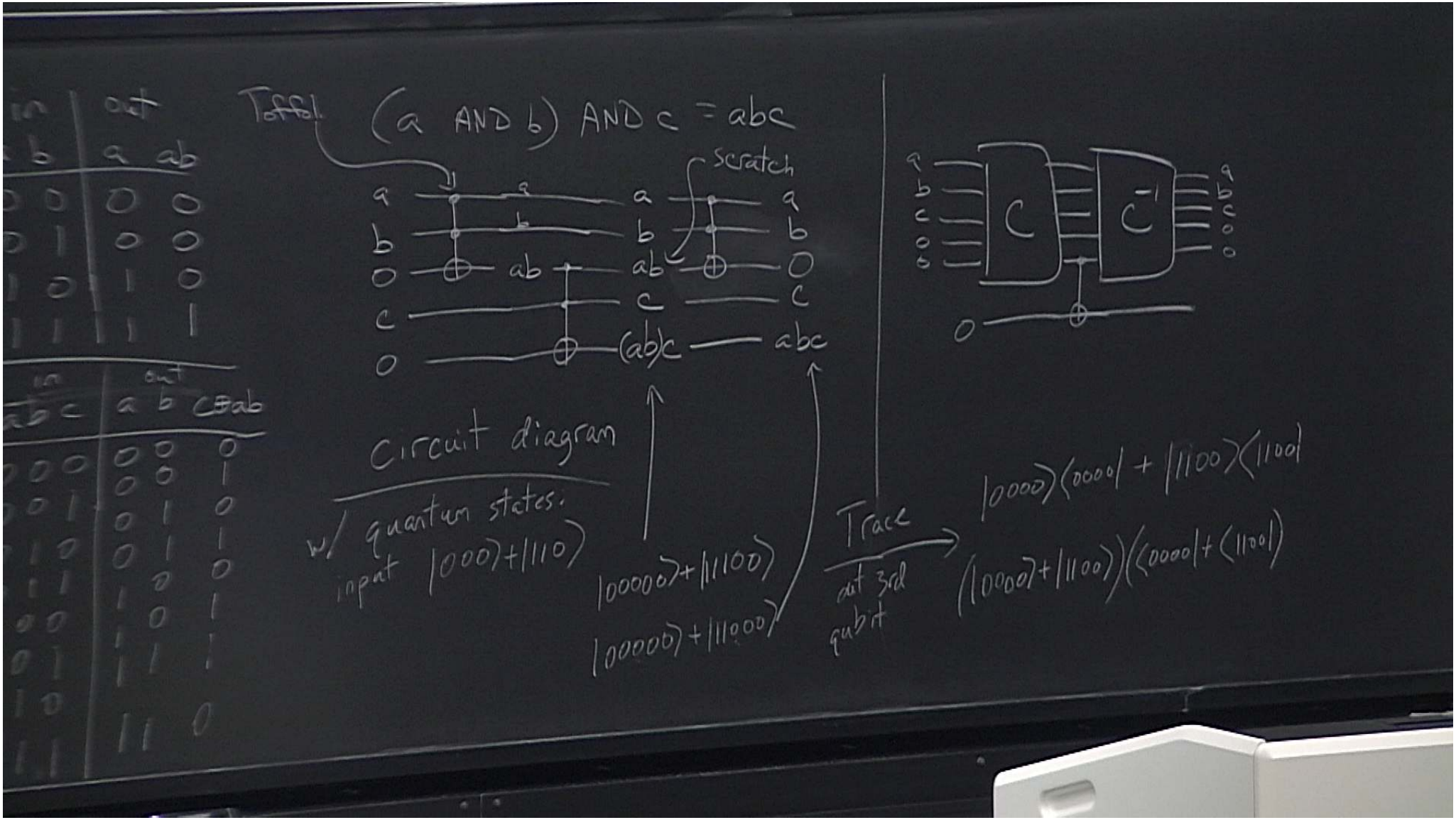


$(|0\rangle+|1\rangle)|0\rangle = |00\rangle+|10\rangle$


Toffoli
(a AND b) AND c = abc
scratch
Circuit diagram
w/ quantum states.
input |000⟩+|110⟩
|00000⟩+|11100⟩
|00000⟩+|11000⟩
Trace
out 3rd qubit
|00000⟩⟨00000| + |11100⟩⟨11100|
(|00000⟩+|11100⟩)(⟨00000|+⟨11100|)

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measurement (eg: in standard basis)

H —

Hadamard is self-inverse

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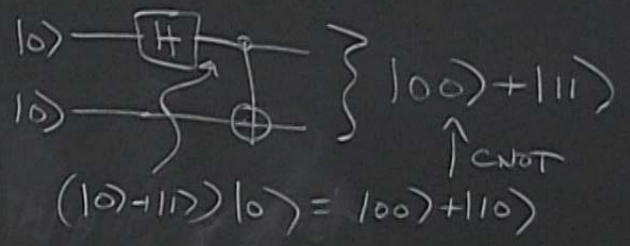
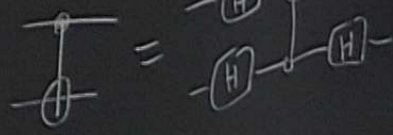


CNOT $(|0\rangle+|1\rangle)(|0\rangle-|1\rangle)$

$= |00\rangle + |11\rangle - |01\rangle - |10\rangle$

$= |0\rangle(|0\rangle-|1\rangle) + |1\rangle(|1\rangle-|0\rangle)$

$= (|0\rangle-|1\rangle)(|0\rangle-|1\rangle)$