

Title: PSI 17/18 - Quantum Information - Lecture 1

Date: Feb 20, 2018 11:30 AM

URL: <http://pirsa.org/18020044>

Abstract:

Quantum Information

- Build a quantum computer
- Properties of QI
 - Quantum algorithms
 - Quantum protocols (QKD, etc.)
- Use QI ideas to illuminate physics & CS

Qubit

2-dimensional Hilbert space

$|0\rangle, |1\rangle$

$\alpha|0\rangle + \beta|1\rangle$

n qubits - 2^n -dimensional Hilbert space

$\alpha|0110\rangle + \beta|1100\rangle + \gamma|1001\rangle +$

2-qubit system

$$(|0\rangle + |1\rangle)(|0\rangle - |1\rangle) \quad \text{tensor product}$$

$$= |00\rangle + |10\rangle - |01\rangle - |11\rangle$$

$$|00\rangle + |11\rangle$$

entangled state

Quantum gates

Put quantum gates together into
quantum circuits

space

al Hilbert space

$\rangle +$

Unitary $|\psi\rangle \mapsto U|\psi\rangle$
reversible $U^\dagger(U|\psi\rangle) = |\psi\rangle$

NOT:
reversible

in	out
0	1
1	0

XOR:
reversible

in		out	
a	b	a	a ⊕ b
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

AND
not
revers
 Toffoli
 revers.

in		out	
a	b	a	a⊕b
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

AND:

not

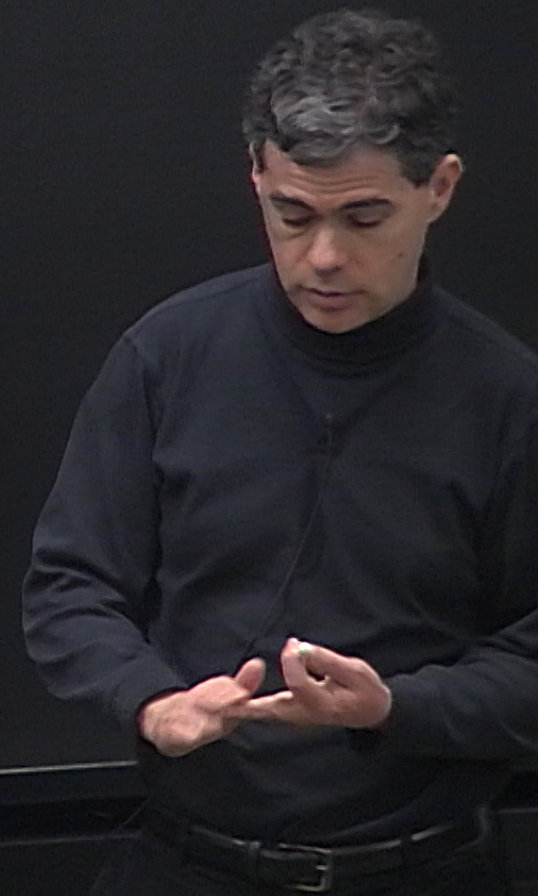
reversible

in		out	
a	b	a	ab
0	0	0	0
0	1	0	0
1	0	1	0
1	1	1	1

Toffoli:

reversible

in			out		
a	b	c	a	b	c⊕ab
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	0
1	0	0	1	0	1
1	0	1	1	1	1
1	1	0	1	1	0
1	1	1	1	1	0



AND:

in		out	
a	b	a	ab
0	0	0	0
0	1	0	0
1	0	1	0
1	1	1	1

not

reversible

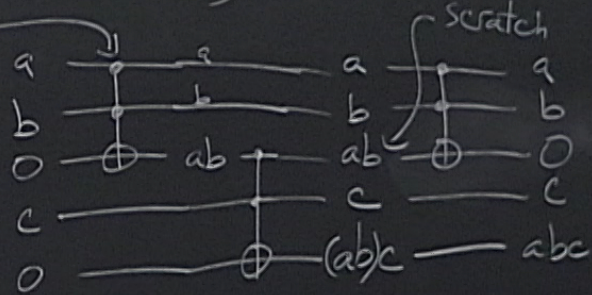
Toffoli:

reversible

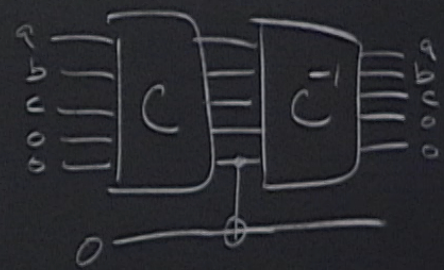
in			out		
a	b	c	a	b	c ⊕ ab
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	0
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	1	1	0
1	1	1	1	1	0

Toffoli:

$$(a \text{ AND } b) \text{ AND } c = abc$$



circuit diagram



CNOT (Controlled-NOT):

$$|a,b\rangle \mapsto |a, a \oplus b\rangle$$

$$\begin{aligned} \text{CNOT}(\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle) \\ = \alpha|00\rangle + \beta|01\rangle + \gamma|11\rangle + \delta|10\rangle \end{aligned}$$

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\text{NOT } |a\rangle = |\bar{a}\rangle$$

$$\text{Toffoli } |a, b, c\rangle = |a, b, c \oplus ab\rangle$$

$$\text{Hadamard transform: } H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|0\rangle \mapsto |0\rangle + |1\rangle$$

$$|1\rangle \mapsto |0\rangle - |1\rangle$$

$$\text{Phase rotation: } R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

$$\begin{aligned} |0\rangle &\mapsto |0\rangle \\ |1\rangle &\mapsto e^{i\theta} |1\rangle \end{aligned}$$

2-qubit

(1)

=

100

Quantum

Put

$$\text{NOT } |a\rangle = |\bar{a}\rangle$$

$$\text{Toffoli } |a, b, c\rangle = |a, b, c \oplus ab\rangle$$

$$\text{Hadamard transform: } H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|0\rangle \mapsto |0\rangle + |1\rangle$$

$$|1\rangle \mapsto |0\rangle - |1\rangle$$

$$\text{Phase rotation: } R_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \quad \begin{array}{l} |0\rangle \mapsto |0\rangle \\ |1\rangle \mapsto e^{i\theta} |1\rangle \end{array}$$

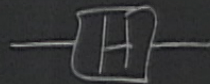
$$\alpha|0\rangle + \beta|1\rangle \mapsto \alpha|0\rangle + \beta e^{i\theta} |1\rangle$$

Non-unitary:

introduce $|0\rangle$ qubit $|0\rangle$ —
 measurement (eg in standard basis)

b, c, d, a, b

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

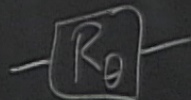


$+|1\rangle$

$-|1\rangle$

$$|0\rangle \mapsto |0\rangle$$

$$|1\rangle \mapsto e^{i\theta} |1\rangle$$



$$|1\rangle \mapsto \alpha |0\rangle + \beta e^{i\theta} |1\rangle$$

Hadamard is self-inverse

$$H^2 = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Non-unitary

introduce $|0\rangle$ qubit $|0\rangle$ —
measurement (e.g. in standard basis)

Hadamard is self-inverse

$$H^2 = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{array}{c} |0\rangle \text{---} \boxed{H} \text{---} \\ |0\rangle \text{---} \text{---} \oplus \text{---} \end{array} \left. \vphantom{\begin{array}{c} |0\rangle \\ |0\rangle \end{array}} \right\} |00\rangle + |11\rangle$$

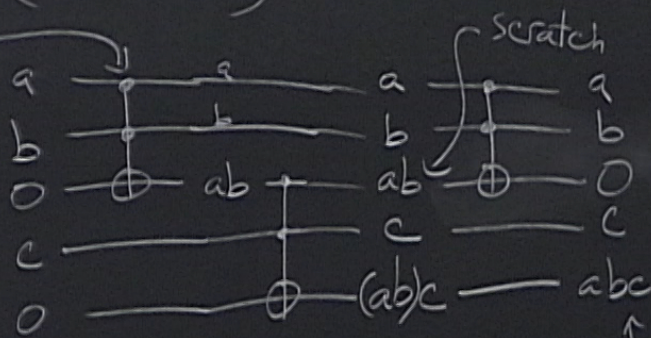
$\uparrow$ CNOT

$$(|0\rangle + |1\rangle)|0\rangle = |00\rangle + |10\rangle$$

in	out
a b	a ab
0 0	0 0
0 1	0 0
1 0	1 0
1 1	1 1

in	out
abc	a b c⊕ab
0 0 0	0 0 0
0 0 1	0 0 0
0 1 0	0 1 0
0 1 1	0 1 0
1 0 0	1 0 1
1 0 1	1 0 1
1 1 0	1 1 0
1 1 1	1 1 0

Total: $(a \text{ AND } b) \text{ AND } c = abc$



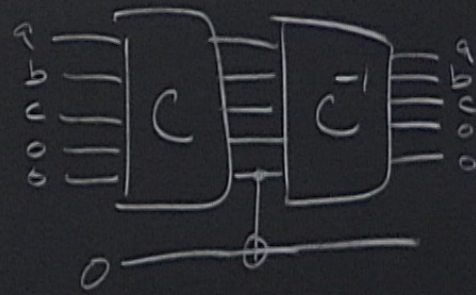
Circuit diagram

w/ quantum states:
input $|000\rangle + |110\rangle$

$|00000\rangle + |11100\rangle$
 $|00000\rangle + |11000\rangle$

Trace
out 3rd
qubit

$|0000\rangle\langle 0000| + |1100\rangle\langle 1100|$
 $(|0000\rangle + |1100\rangle)(\langle 0000| + \langle 1100|)$



Non-unitary:

introduce $|0\rangle$ qubit $|0\rangle$ —
 measurement (e.g. in standard basis)



Hadamard is self-inverse

$$H^2 = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{array}{c} |0\rangle \text{---} [H] \text{---} \oplus \\ |0\rangle \text{---} \oplus \end{array} \Bigg\} |00\rangle + |11\rangle$$

$\uparrow$ CNOT

$$(|0\rangle + |1\rangle)|0\rangle = |00\rangle + |10\rangle$$

$$\begin{aligned} & \text{CNOT}(|0\rangle + |1\rangle)(|0\rangle - |1\rangle) \\ &= |00\rangle + |11\rangle - |01\rangle - |10\rangle \\ &= |0\rangle(|0\rangle - |1\rangle) + |1\rangle(|1\rangle - |0\rangle) \\ &= (|0\rangle - |1\rangle)(|0\rangle - |1\rangle) \end{aligned}$$

