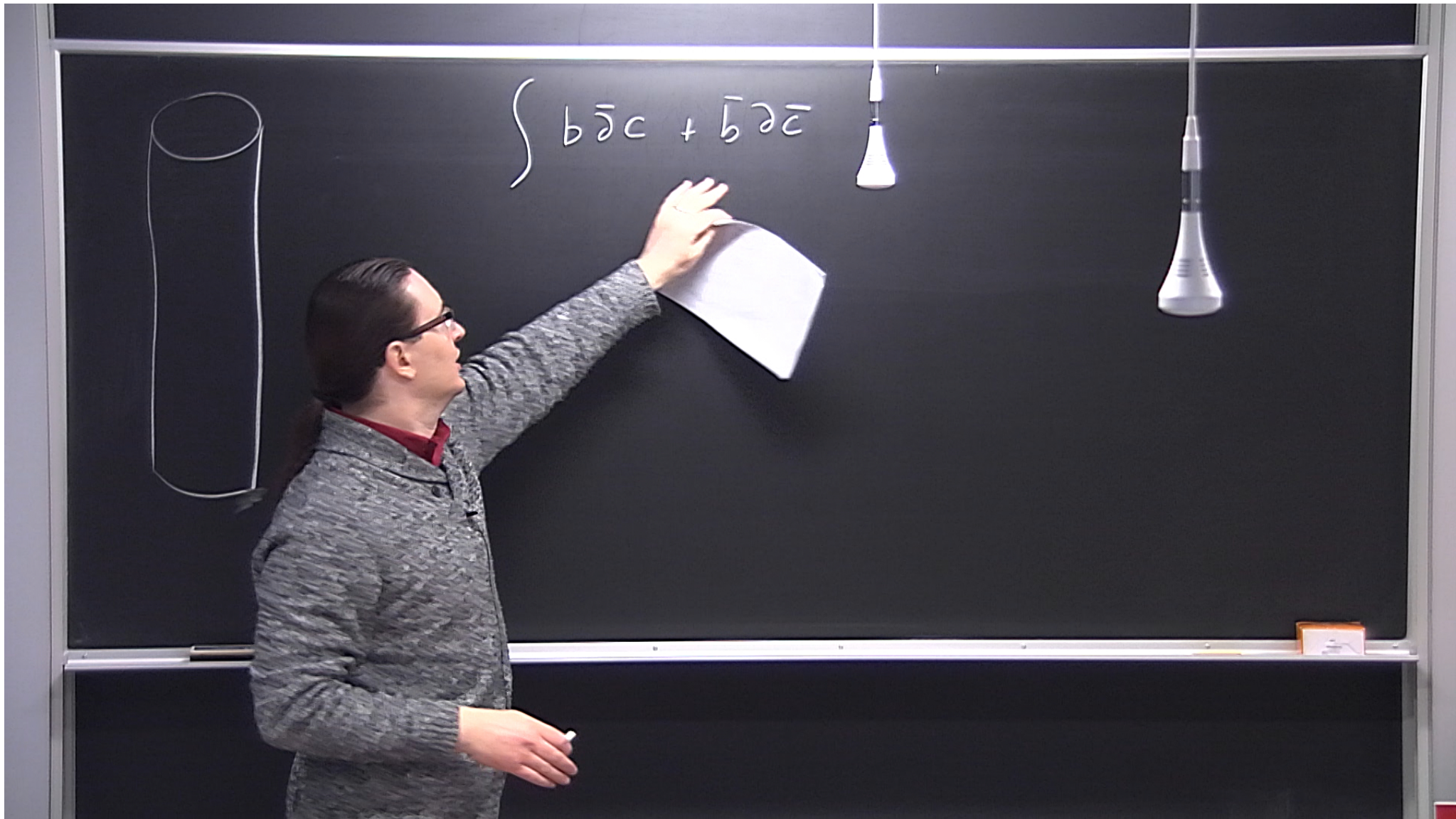


Title: PSI 17/18 - String Theory - Lecture 7

Date: Feb 28, 2018 10:15 AM

URL: <http://pirsa.org/18020043>

Abstract:





$$S_{\text{ga}} = \int b \bar{\partial} c + \bar{b} \partial \bar{c}$$



$$T_{gh}^{cl} = c \partial b + 2 \partial_c b$$

$$T_{gh}^{cl} = c b$$

$$S_{gh} = \int b \bar{\partial} c + \bar{b} \partial \bar{c}$$

$$b(s) = b(s') \left(\frac{\partial s'}{\partial s} \right)^{1-\lambda}$$

$$c(s) = c(s') \left(\frac{\partial s'}{\partial s} \right)^{\lambda}$$

$$\partial_s X(s) = \partial_{s'} X(s') \frac{\partial s'}{\partial s}$$



$$S_{gh} = \int (b \bar{\partial} c + \bar{b} \partial \bar{c}) ds d\bar{s}$$

$$b(s) \quad b(s') \left(\frac{\partial s'}{\partial s} \right)^{1-\lambda}$$

$$c(s) \quad c(s') \left(\frac{\partial s'}{\partial s} \right)^{\lambda}$$

$$\partial_s X(s) = \partial_{s'} X(s') \frac{\partial s'}{\partial s}$$



$$S_{gh} = \int (b \bar{\partial} c + \bar{b} \partial \bar{c}) ds d\bar{s}$$

$$b(s) = b(s') \left(\frac{\partial s'}{\partial s} \right)^2$$

$$c(s) = c(s') \left(\frac{\partial s'}{\partial s} \right)^{-1}$$

$$\partial_s X(s) = \partial_{s'} X(s') \frac{\partial s'}{\partial s}$$

$$S_{gh} = \int (b \bar{\partial} c + \bar{b} \partial \bar{c}) ds d\bar{s}$$

$$b(s) = \sum_n b_n e^{-ns}$$

$$c(s) = \sum_n c_n e^{-ns}$$

$$\{b_n, c_m\} = \delta_{n+m,0}$$

$$\{c_n, c_m\} = 0$$

$$\{b_n, b_m\} = 0$$

$$b(s) = b(s') \left(\frac{\partial s'}{\partial s} \right)^2$$

$$c(s) = c(s') \left(\frac{\partial s'}{\partial s} \right)^{-1}$$

$$\partial_s X(s) = \partial_{s'} X(s') \frac{\partial s'}{\partial s}$$



$$S = \tau - i\sigma$$

$$S_{gh} = \int (b \bar{\partial} c + \bar{b} \partial \bar{c}) ds d\bar{s}$$

$$b(s) = \sum_n b_n e^{-ns}$$

$$c(s) = \sum_n c_n e^{-ns}$$

$$\{b_n, c_m\} = \delta_{n+m,0}$$

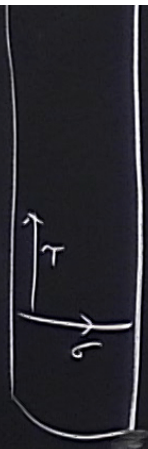
$$\{c_n, c_m\} = 0$$

$$\{b_n, b_m\} = 0$$

$$b(s) = b(s') \left(\frac{\partial s'}{\partial s} \right)^2$$

$$c(s) = c(s') \left(\frac{\partial s'}{\partial s} \right)^{-1}$$

$$\partial_s X(s) = \partial_{s'} X(s') \frac{\partial s'}{\partial s}$$



$$s = \tau - i\sigma$$

$$b(s) = \sum_n b_n e^{-ns}$$

$$c(s) = \sum_n c_n e^{-ns}$$

$$\{b_n, c_m\} = \delta_{m+n, 0}$$

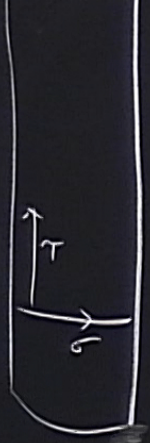
$$\{b_0, c_0\} = 1$$

$$c(s) = c(s') \left(\frac{\partial s}{\partial s'} \right)$$

$$\partial_s X(s) = \partial_{s'} X(s') \frac{\partial s'}{\partial s}$$

$$\{c_n, c_m\} = 0$$

$$\{b_n, b_m\} = 0$$



$$S = T - i\sigma$$

$$b(s) = \sum_n b_n e^{-ns}$$

$$c(s) = \sum_n c_n e^{-ns}$$

$$\{b_n, c_m\} = \delta_{m+n,0}$$

$$\{b_0, c_0\} = 1$$

$$c_0^2 = 0$$

$$\{c_n, c_m\} = 0$$

$$b_0^2 = 0$$

$$\{b_n, b_m\} = 0$$

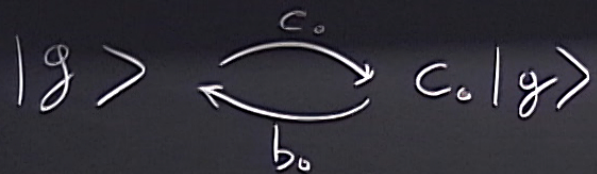
$$c(s) = c(s') \left(\frac{\partial s'}{\partial s} \right)$$

$$\partial_s X(s) = \partial_{s'} X(s') \frac{\partial s'}{\partial s}$$

$$\begin{array}{c}
 |g\rangle \xrightleftharpoons[b_0]{c_0} c_0 |g\rangle \\
 b_0 |g\rangle = 0 \qquad b_n |g\rangle = 0 \quad n > 0 \\
 c_n |g\rangle = 0
 \end{array}$$

$$\begin{array}{c}
 |g\rangle \xrightleftharpoons[b_0]{c_0} c_0 |g\rangle \\
 b_0 |g\rangle = 0 \qquad b_n |g\rangle = 0 \quad n > 0 \\
 c_n |g\rangle = 0
 \end{array}$$

$$\begin{array}{l}
 |0\rangle = b_- |g\rangle \\
 |g\rangle = c_+ |0\rangle
 \end{array}$$



$$b_0 |g\rangle = 0$$

$$b_n |g\rangle = 0 \quad n > 0$$

$$c_n |g\rangle = 0$$

$$b_n |0\rangle = 0 \quad n \geq -1$$

$$c_n |0\rangle = 0 \quad n \geq 2$$

$$|0\rangle = b_{-1} |g\rangle$$

$$|g\rangle = c_1 |0\rangle$$

$$L_{-1}|p\rangle = a_{-1}a_0|p\rangle$$



$$|g\rangle \begin{array}{c} \xrightarrow{c_0} \\ \xleftarrow{b_0} \end{array} c_0 |g\rangle$$

$$b_n |g\rangle = 0$$

$$b_n |g\rangle = 0 \quad n > 0$$

$$c_n |g\rangle = 0$$

$$b_n |0\rangle = 0$$

$$n \geq -1$$

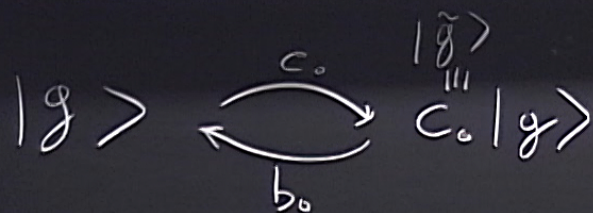
$$c_n |0\rangle = 0$$

$$n \geq 2$$

$$|0\rangle = b_1 |g\rangle$$

$$|g\rangle = c_1 |0\rangle$$

$$\langle \tilde{g} | g \rangle = 1$$



$$b_0 |g\rangle = 0$$

$$b_n |g\rangle = 0 \quad n > 0$$

$$c_n |g\rangle = 0$$

$$b_n |0\rangle = 0 \quad n \geq -1$$

$$c_n |0\rangle = 0 \quad n \geq 2$$

$$|0\rangle = b_1 |g\rangle$$

$$|g\rangle = c_1 |0\rangle$$

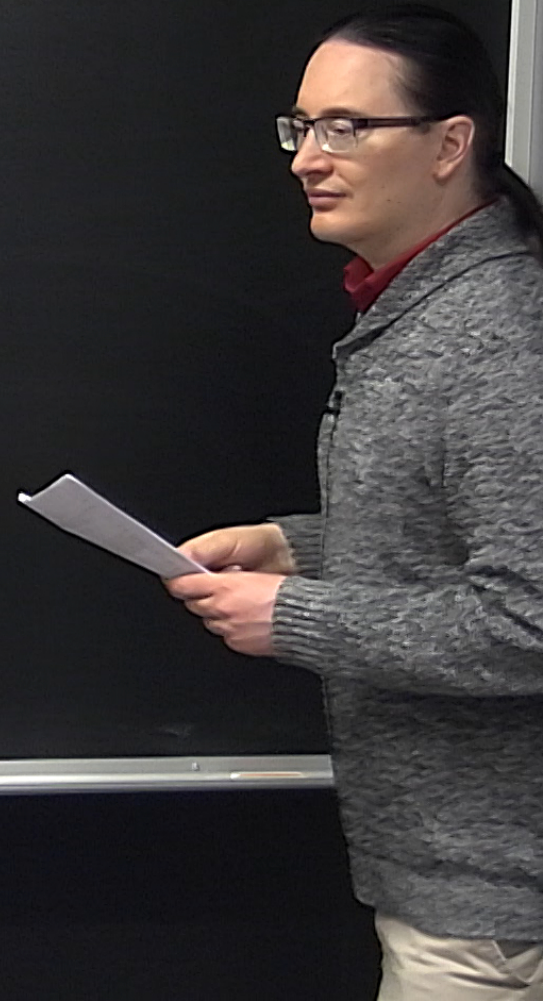
$$\langle \tilde{g} | g \rangle = 1$$

$$\langle \tilde{g} | c_0 = 0$$

$$\langle \tilde{g} | c_n = 0 \quad n < 0$$

$$\langle \tilde{g} | b_n = 0 \quad n < 0$$

$$\langle g | c_0 \bar{c}_0 | g \rangle = 1$$



$$\langle \tilde{g} | c(s) b(s) | g \rangle =$$



$$\begin{aligned} \langle \tilde{g} | c(s) b(s') | g \rangle &= \sum_{n,m} \langle \tilde{g} | c_n b_m | g \rangle e^{-ns - ms'} \\ &= \sum \end{aligned}$$

$$\begin{aligned}
 \langle \tilde{g} | c(s) b(s') | g \rangle &= \sum_{n,m} \langle \tilde{g} | c_n b_m | g \rangle e^{-ns - ms'} \\
 &= \sum e^{s-s'} + e^{2s-2s'} + e^{3s-3s'} \dots
 \end{aligned}$$

$$\begin{aligned}
 \langle \tilde{g} | c(s) b(s') | g \rangle &= \sum_{n,m} \langle \tilde{g} | c_n b_m | g \rangle e^{-ns - ms'} \\
 &= \sum_{n,m} \langle \tilde{g} | c_n b_m | g \rangle e^{s' + s} + e^{2s' - 2s} + e^{3s' - 3s} \dots \\
 &=
 \end{aligned}$$

$$\langle g | c_0 \bar{c}_0 | g \rangle = 1$$

$$\begin{aligned} \langle \tilde{g} | c(s) b(s') | g \rangle &= \sum_{n,m} \langle \tilde{g} | c_n b_m | g \rangle e^{-ns - ms'} \\ &= \sum_{n,m} \langle g | c_{-n} b_{-m} | g \rangle e^{s' + s} + e^{2s' - 2s} + e^{3s' - 3s} \dots \\ &= \frac{e^{s' - s}}{1 - e^{s' - s}} = \frac{e^{s'}}{e^s - e^{s'}} \end{aligned}$$

$$\langle g | c_0 \bar{c}_0 | g \rangle = 1$$

$$\begin{aligned} \langle \tilde{g} | c(s) b(s') | g \rangle &= \sum_{n,m} \langle \tilde{g} | c_n b_m | g \rangle e^{-ns - ms'} \\ &= \sum_{n,m} \langle g | c_{-n} b_{-m} | g \rangle e^{s' + s} + e^{2s' - 2s} + e^{3s' - 3s} \dots \\ &= \frac{e^{s' - s}}{1 - e^{s' - s}} = \frac{e^{s'}}{e^s - e^{s'}} \quad \sim \frac{1}{s + s'} \frac{1}{s - s'} \end{aligned}$$

$$T_{gh}^{cl} = c \partial b + 2 \partial c b$$

$$T_{gh}^{cl} = c b$$

$$T_{gh} = \lim_{s \rightarrow s'} \left[c(s) \partial_{s'} b(s') + 2 (\partial_s c(s)) b(s') \right] + \frac{1}{(s-s')^2}$$

$$T_{gh}^d = c \partial b + 2 \partial c b$$

$$T_{gh}^{cl} = c b$$

$$T_{gh} = \lim_{s \rightarrow s'} \left[c(s) \partial_{s'} b(s') + 2 (\partial_s c(s)) b(s') + \frac{1}{(s-s')^2} \right]$$

$$L_n^{gh} = \sum_k (k+n) : c_k b_{n-k} : \quad \begin{matrix} \vdots \\ \vdots \\ \vdots \end{matrix} \text{BASED}$$

$$T_{gh}^{cl} = c \partial b + \dots$$

$$T_{gh}^{cl} = c b$$

$$\left[c(s) \partial_{s'} b(s') + 2 (\partial_s c(s)) b(s') + \frac{1}{(s-s')^2} \right]$$

$$L_n^{gh} = - \sum_k (k+n) : c_k b_{n-k} :$$

BASED
ON VACUUM
 $|0\rangle$

$$T = \sum L_n^{gh}$$

$$c_n |g\rangle$$

$$b_n |0\rangle = 0$$

$$n \geq -1$$

$$c_n |0\rangle = 0$$

$$n \geq 2$$

$$g = 1$$

$$\langle g | c_0 = 0$$

$$\langle \tilde{g} | c_n = 0 \quad n < 0$$

$$\langle \tilde{g} | \begin{array}{c} b_0 \\ \leftarrow \\ c_0 \end{array} \langle g | \langle \tilde{g} | b_n = 0 \quad n < 0$$

$$T_{gh} = c \partial b + 2 \partial c b$$

$$T_{gh}^{cl} = c b$$

$$L_n^{gh} = - \sum_k (k+n) : c_k b_{n-k} :$$

$$T^{gh} = \sum L_n^{gh} e^{-ns} + \frac{13}{12}$$

∴
BASED
ON VACUUM
|0>

$$c_n |g\rangle = 0$$

$$\langle \tilde{g} | g \rangle = 1$$

$$\langle \tilde{g} | c_0 = 0$$

$$\langle \tilde{g} | c_n = 0 \quad n < 0$$

$$\langle \tilde{g} | b_0 = 0 \quad \langle \tilde{g} | b_n = 0 \quad n < 0$$

$$J_{gh}^{cl} = c b$$

$$+ \frac{1}{(s-s')^2}$$

$$L_n^{gh} = - \sum_k (k+n) : c_k b_{n-k} :$$

$\therefore$
BASED
ON VACUUM
 $|0\rangle$

$$T = \sum L_n^{gh} e^{-ns} + \frac{13}{12}$$

$$b_n |g\rangle = 0$$

$$b_n |g\rangle = 0 \quad n > 0$$

$$c_n |g\rangle = 0$$

$$\langle \tilde{g} | g \rangle = 1$$

$$\langle \tilde{g} | c_0 = 0$$

$$\langle \tilde{g} | c_n = 0 \quad n < 0$$

$$b_n |0\rangle = 0$$

$$n \geq -1$$

$$c_n |0\rangle = 0$$

$$n \geq 2$$

$$\langle \tilde{g} | \begin{array}{c} b_1 \\ \leftarrow \\ c_0 \end{array} | g \rangle = \langle \tilde{g} | b_1 = 0 \quad n < 0$$

$$T_{gh}^{\text{cl}} = c \partial b + 2 \partial c b$$

$$T_{gh}^{\text{cl}} = c b$$

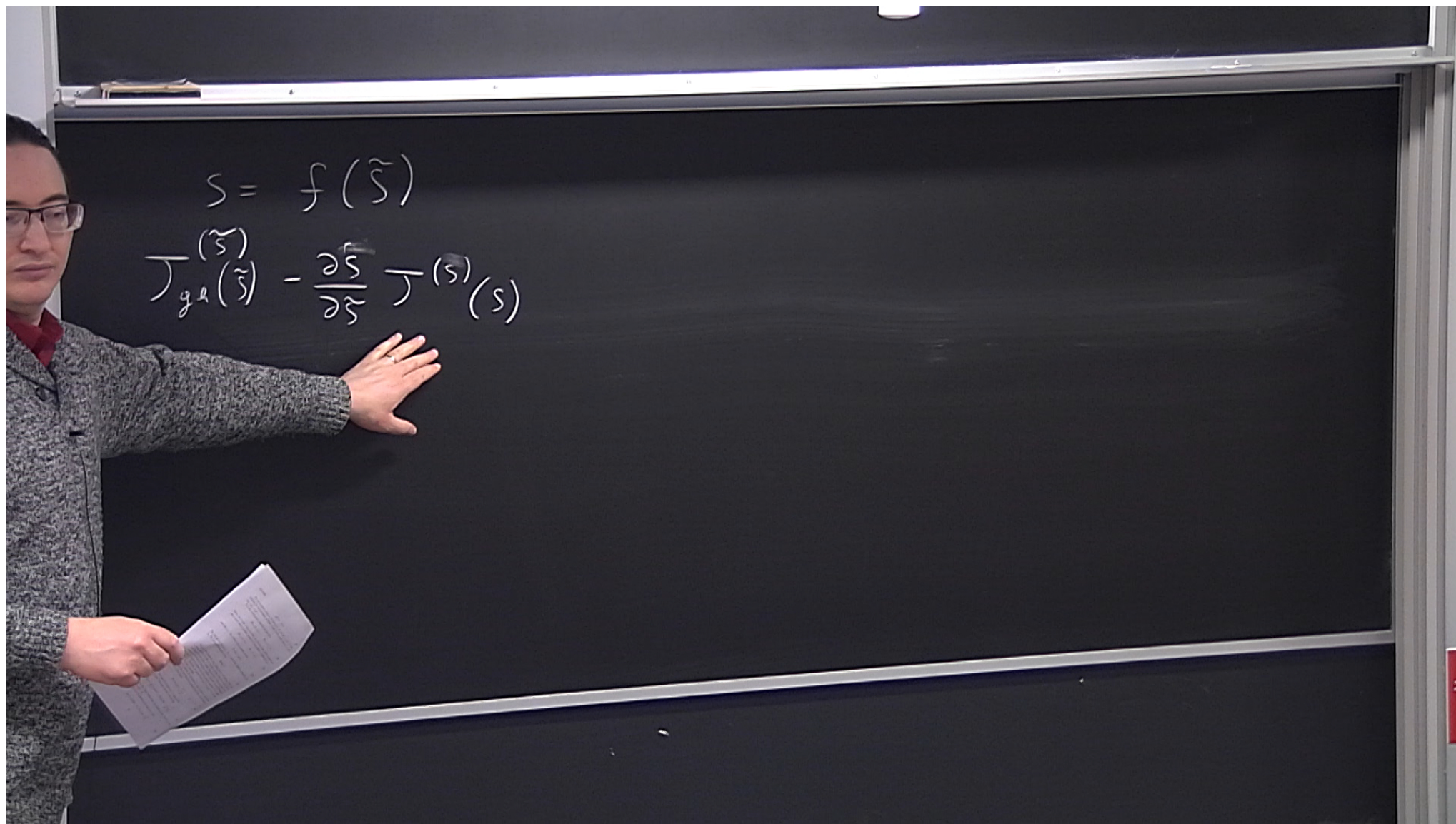
$$T_{gh} = \lim_{s \rightarrow s'} \left[c(s) b(s') - \frac{1}{s-s'} \right]$$

$$T_{gh} = \lim_{s \rightarrow s'} \left[c(s) \partial_{s'} b(s') + 2 (\partial_s c(s)) b(s') + \frac{1}{(s-s')^2} \right]$$

$$L_n^{gh} = - \sum_k (k+n) : c_k b_{n-k} :$$

∴
BASED
ON VACUUM
|0>

$$T^{gh} = \sum L_n^{gh} e^{-ns} + \frac{13}{12}$$



$$s = f(\tilde{s})$$

$$\mathcal{J}_{g_h}^{(\tilde{s})}(\tilde{s}) - \frac{\partial \tilde{s}}{\partial s} \mathcal{J}^{(s)}(s) = \ln \left[\frac{\partial s}{\partial \tilde{s}} \frac{1}{s-s'} - \frac{1}{\tilde{s}-\tilde{s}'} \right] = \ln \left[\frac{\partial f}{\partial \tilde{s}} \frac{1}{f(\tilde{s})-f(\tilde{s}')} - \frac{1}{\tilde{s}-\tilde{s}'} \right]$$

$$\gamma(s) = \lim_{\tilde{s} \rightarrow s} \left[\frac{\partial s}{\partial \tilde{s}} \frac{1}{s - s'} - \frac{1}{\tilde{s} - \tilde{s}'} \right] = \lim_{\tilde{s} \rightarrow s} \left[\frac{\partial f}{\partial \tilde{s}} \right]$$

$$s = f(\tilde{s})$$

$$\begin{aligned} \gamma_{g_1}^{(\tilde{s})} - \frac{\partial \tilde{s}}{\partial s} \gamma^{(s)}(s) &= \ln \left[\frac{\partial s}{\partial \tilde{s}} \frac{1}{s-s'} - \frac{1}{\tilde{s}-\tilde{s}'} \right] = \ln \left[\frac{\frac{\partial f}{\partial \tilde{s}}}{f(\tilde{s})-f(\tilde{s}')} - \frac{1}{\tilde{s}-\tilde{s}'} \right] \\ &\quad \parallel \\ &\quad \frac{\frac{\partial f}{\partial \tilde{s}}}{(\tilde{s}-\tilde{s}') \frac{\partial f}{\partial \tilde{s}} + \frac{1}{2}(\tilde{s}-\tilde{s}')^2 \frac{\partial^2 f}{\partial \tilde{s}^2} \dots} \end{aligned}$$

$$s = f(\tilde{s})$$

$$\begin{aligned} J_{g_1}(\tilde{s}) - \frac{\partial \tilde{s}}{\partial s} J^{(s)}(s) &= \ln \left[\frac{\partial s}{\partial \tilde{s}} \frac{1}{s-s'} - \frac{1}{\tilde{s}-\tilde{s}'} \right] = \ln \left[\frac{\frac{\partial f}{\partial \tilde{s}}}{f(\tilde{s})-f(\tilde{s}')} - \frac{1}{\tilde{s}-\tilde{s}'} \right] \\ &= - \frac{3}{2} \frac{\partial^2 f}{\partial \tilde{s}^2} \end{aligned}$$

$\parallel$
 $\frac{\partial f}{\partial \tilde{s}}$
 $\frac{(\tilde{s}-\tilde{s}') \frac{\partial f}{\partial \tilde{s}} + \frac{1}{2} (\tilde{s}-\tilde{s}')^2 \frac{\partial^2 f}{\partial \tilde{s}^2} \dots}{\partial \tilde{s}}$

$$\partial_a \bar{T}_{44}^a = \frac{3}{2} R[h]$$

$$s = f(\tilde{s})$$

$$\bar{T}_{44}^{(s)}(\tilde{s}) - \frac{\partial \tilde{s}}{\partial s} \bar{T}^{(s)}(s) = \ln \left[\frac{\partial s}{\partial \tilde{s}} \frac{1}{s - s'} \right]$$

$$= -\frac{3}{2} \frac{\partial^2 f}{\partial f^2}$$

$$\left[\frac{\partial f}{\partial \tilde{s}} \frac{1}{f(\tilde{s}) - f(\tilde{s}')} - \frac{1}{\tilde{s} - \tilde{s}'} \right]$$

$$\parallel \frac{\partial f}{\partial \tilde{s}}$$

$$\left(\frac{\partial f}{\partial \tilde{s}} + \frac{1}{2} (\tilde{s} - \tilde{s}')^2 \frac{\partial^2 f}{\partial \tilde{s}^2} \dots \right)$$

$$\partial_a \bar{T}_{gh}^{\hat{a}} = \frac{3}{2} R[h]$$

$$s = f(\tilde{s})$$

$$\begin{aligned} \bar{T}_{gh}^{(\tilde{s})} - \frac{\partial \tilde{s}}{\partial s} \bar{T}^{(s)}(s) &= \ln \left[\frac{\partial s}{\partial \tilde{s}} \frac{1}{s-s'} - \frac{1}{\tilde{s}-\tilde{s}'} \right] = \ln \left[\frac{\partial f}{\partial \tilde{s}} \frac{1}{f(\tilde{s})-f(\tilde{s}')} - \frac{1}{\tilde{s}-\tilde{s}'} \right] \\ &= -\frac{3}{2} \frac{\partial^2 f}{\partial \tilde{s}^2} = -\frac{3}{2} \frac{\frac{\partial^2 f}{\partial \tilde{s}^2}}{\frac{\partial f}{\partial \tilde{s}}} \end{aligned}$$

$$\frac{\frac{\partial f}{\partial \tilde{s}}}{\left(\tilde{s}-\tilde{s}' \right) \frac{\partial f}{\partial \tilde{s}} + \frac{1}{2} (\tilde{s}-\tilde{s})^2 \frac{\partial^2 f}{\partial \tilde{s}^2} \dots}$$

$$Q = \frac{1}{2\pi} \oint$$

$$\partial_a \mathcal{T}_{gh} = \frac{3}{2} R[h]$$

$$T^0 = \sum L_n e^{-ns} + \frac{15}{12}$$

$$= \ln \left[\frac{\partial s}{\partial \tilde{s}} \frac{1}{s-s'} - \frac{1}{\tilde{s}-\tilde{s}'} \right] = \ln \left[\frac{\partial f}{\partial \tilde{s}} \frac{1}{f(\tilde{s})-f(\tilde{s}')} - \frac{1}{\tilde{s}-\tilde{s}'} \right]$$

$$\frac{\partial^2 f}{\partial \tilde{s}^2} = -\frac{3}{2} \frac{\frac{\partial^2 f}{\partial \tilde{s}^2}}{\frac{\partial f}{\partial \tilde{s}}}$$

$$\frac{\frac{\partial f}{\partial \tilde{s}}}{(\tilde{s}-\tilde{s}') \frac{\partial f}{\partial \tilde{s}} + \frac{1}{2} (\tilde{s}-\tilde{s}')^2 \frac{\partial^2 f}{\partial \tilde{s}^2} \dots}$$

$$Q = \frac{1}{2\pi} \oint \mathcal{T}_{\text{BRST}}$$

$$\partial_a \mathcal{T}_{\text{gh}}^a = \frac{3}{2} R[h]$$

$$T^{\text{gh}} = \sum L_n^{\text{gh}} e^{-ns} + \frac{13}{12}$$

$$\mathcal{T}_{\text{BRST}} = c T_X + bc \partial c$$

$$\mathcal{T}_{\text{gh}}^{(s)}(\tilde{s}) - \frac{\partial \tilde{s}}{\partial s} \mathcal{T}^{(s)}(s) = \ln \left[\frac{\partial s}{\partial \tilde{s}} \frac{1}{s-s'} - \frac{1}{\tilde{s}-\tilde{s}'} \right]$$

$$\Rightarrow -\frac{3}{2} \frac{\partial_s^2 f}{\partial_s f} = -\frac{3}{2} \frac{\frac{\partial^2 f}{\partial \tilde{s}^2}}{\frac{\partial f}{\partial \tilde{s}}}$$

$$Q = \frac{1}{2\pi} \int_{\text{BRST}}$$

$$\partial_a \bar{T}_{\alpha\beta} = \frac{3}{2} R[h]$$

$$T_{\text{BRST}} = c T_X + bc\partial c$$

$$\{Q, b_c\} = T^{\text{total}}$$

$$S = f(s)$$

$$\begin{aligned} \bar{T}_{g^2}^{(\tilde{s})} - \frac{\partial \tilde{s}}{\partial s} \bar{T}^{(s)}(s) &= \lim \left[\frac{\partial s}{\partial \tilde{s}} \frac{1}{s-s'} - \frac{1}{\tilde{s}-\tilde{s}'} \right] = \lim \left[\frac{\partial f}{\partial \tilde{s}} - \frac{1}{\tilde{s}-\tilde{s}'} \right] \\ &= -\frac{3}{2} \frac{\partial^2 f}{\partial \tilde{s}^2} = -\frac{3}{2} \frac{\frac{\partial^2 f}{\partial \tilde{s}^2}}{\frac{\partial f}{\partial \tilde{s}}} \end{aligned}$$

$$\partial_a \hat{T}_{gh} = \frac{3}{2} R[h]$$

$$T_{\text{BEST}} = c T_x + bc \partial c$$

$$\{Q, \dots\} = T^{\text{total}}$$

$$s = f(\tilde{s})$$

$$\begin{aligned} \hat{T}_{gh}^{(\tilde{s})} - \frac{\partial \tilde{s}}{\partial s} T^{(s)}(s) &= \ln \left[\frac{\partial s}{\partial \tilde{s}} \frac{1}{s-s'} - \frac{1}{\tilde{s}-\tilde{s}'} \right] = \ln \left[\frac{\partial f}{\partial \tilde{s}} \frac{1}{f(\tilde{s})-f(\tilde{s}')} - \frac{1}{\tilde{s}-\tilde{s}'} \right] \\ &= -\frac{3}{2} \frac{\partial^2 f}{\partial \tilde{s}^2} = -\frac{3}{2} \frac{\frac{\partial^2 f}{\partial \tilde{s}^2}}{\frac{\partial f}{\partial \tilde{s}}} \end{aligned}$$

$$\frac{\frac{\partial f}{\partial \tilde{s}}}{(\tilde{s}-\tilde{s}') \frac{\partial f}{\partial \tilde{s}} + \frac{1}{2} (\tilde{s}-\tilde{s})^2 \frac{\partial^2 f}{\partial \tilde{s}^2} \dots}$$

$$\langle g | c_0 \bar{c}_0 | g \rangle = 1$$

$$\langle \tilde{g} | c(s) b(s') | g \rangle = \sum_{m,n} \langle \tilde{g} | c_n b_m | g \rangle e^{-ns - ms'}$$

$$\langle \tilde{g} | T^{\text{ge}} | g \rangle$$

$$= \sum_{\langle \tilde{g} | c_n b_m | g \rangle} e^{s' + s} + e^{2s' - 2s} + e^{3s' - 3s} \dots$$

$$= \frac{e^{s' - s}}{1 - e^{s' - s}} = \frac{e^{s'}}{e^s - e^{s'}} \quad \sim \frac{1}{s - s'}$$

$$\langle \tilde{g} | T^{\text{ge}} | g \rangle$$

$$\langle g | c_0 \bar{c}_0 | g \rangle = 1$$

$$\langle \tilde{g} | c(s) b(s') | g \rangle = \sum_{n,m} \langle \tilde{g} | c_n b_m | g \rangle e^{-ns - ms'}$$

$$\langle \tilde{g} | T^{\text{se}} | g \rangle$$

$$= \sum_{n,m} \langle \tilde{g} | c_n b_m | g \rangle e^{-ns - ms'} = e^{s' + s} + e^{2s' - 2s} + e^{3s' - 3s} + \dots$$

$$= \frac{e^{s' - s}}{1 - e^{s' - s}} = \frac{e^{s'}}{e^s - e^{s'}} \quad \sim \frac{1}{s - s'}$$

$$\langle \tilde{g} | T^{\text{se}} | g \rangle$$

$$\langle g | c_0 c_0 | g \rangle = 1$$

$$\langle \tilde{g} | c(s) b(s') | g \rangle = \sum_{n,m} \langle \tilde{g} | c_n b_m | g \rangle e^{-ns - ms'}$$

$$\langle \tilde{g} | T^{\text{ge}} | g \rangle$$

$$= \sum_{n,m} \langle \tilde{g} | c_n b_m | g \rangle e^{-ns - ms'} = e^{s' + s} + e^{2s' - 2s} + e^{3s' - 3s} + \dots$$

$$= \frac{e^{s' - s}}{1 - e^{s' - s}} = \frac{e^{s'}}{e^s - e^{s'}} \quad \sim \frac{1}{s + s'} \frac{1}{s - s'}$$

$$\langle g | T^{\text{ge}} | g \rangle$$

$$L_0^{\text{ge}} | 0 \rangle = 0$$

$$L_0^{\text{ge}} | g \rangle = - | g \rangle$$

$$\langle g | T^{\text{el}} | g \rangle$$

$$\langle g | T^{\text{el}} | g \rangle$$

$$L_0^{\text{el}} | 0 \rangle = 0$$

$$L_0^{\text{el}} | g \rangle = -\frac{1}{2} | g \rangle$$

$$= \frac{e^{s'-s}}{1-e^{s'-s}} = \frac{e^{s'}}{e^s - e^{s'}} \quad \sim \frac{1}{s-s'}$$

$$\langle g | T^x | g \rangle$$

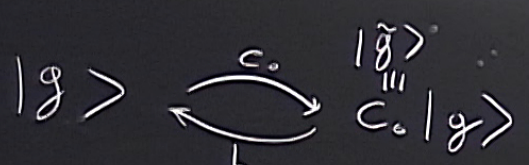
$$L_0^x |0\rangle = 0$$

$$L_0^x |g\rangle = -\frac{1}{2} |g\rangle$$

$$|g\rangle \otimes |\text{matter}\rangle \in H[x^\wedge]$$



$$\{Q, b_c\} = T^{\text{total}}$$



$$b_0 |g\rangle = 0$$

$$b_n |g\rangle = 0 \quad n > 0$$

$$c_n |g\rangle = 0$$

$$b_n |0\rangle = 0$$

$$n \geq -1$$

$$c_n |0\rangle = 0$$

$$n \geq 2$$

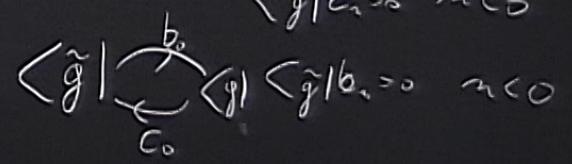
$$|0\rangle = b_{-1} |g\rangle \quad \checkmark^{-\frac{1}{2}}$$

$$|g\rangle = c_1 |0\rangle$$

$$\langle \tilde{g} | g \rangle = 1$$

$$\langle \tilde{g} | c_0 = 0$$

$$\langle \tilde{g} | c_n = 0 \quad n < 0$$



$$\langle \tilde{g} | b_n = 0 \quad n < 0$$

$$L_0 |0\rangle = 0$$

$$L_0 |0\rangle = -18$$

$$|g\rangle \otimes |matter\rangle \in H[X^\wedge]$$

BRST COHOMOLOGY

$$L_0 |0\rangle = 0$$

$$L_0 |0\rangle = -18$$

$$|g\rangle \otimes |matter\rangle \in H[x^\wedge]$$

BRST COHOMOLOGY

$$b_m |w\rangle = 0 \quad n > 0$$

$$\{Q, b_n\} |w\rangle$$

$$L_0 |0\rangle = 0$$

$$L_0 |0\rangle = -18$$

$$|g\rangle \otimes |matter\rangle \in H[X^\wedge]$$

BRST COHOMOLOGY

$$b_n |v\rangle = 0 \quad n \geq 0$$

$$\{Q, b_n\} |v\rangle = 0$$

$$L_n |v\rangle = 0$$

$$L_0^{\text{gh}} |0\rangle = 0$$

$$L_0^{\text{gh}} |g\rangle = -\frac{1}{2} |g\rangle$$

$$|g\rangle \otimes |\text{matter}\rangle \in H[X^{\mu}]$$

BRST COHOMOLOGY

$$b_n |n\rangle = 0 \quad n \geq 0$$

$$\{Q, b_n\} |n\rangle = 0$$

$$L_n^x |\text{matter}\rangle = 0 \quad n > 0$$

$$L_n |n\rangle = 0$$

$$(L_0^x - 1) |\text{matter}\rangle = 0$$

$$L_0^g |0\rangle = 0$$

$$L_0^g |g\rangle = -\frac{1}{2} |g\rangle$$

$$|matter\rangle = \sum_n L_n^x |w\rangle$$

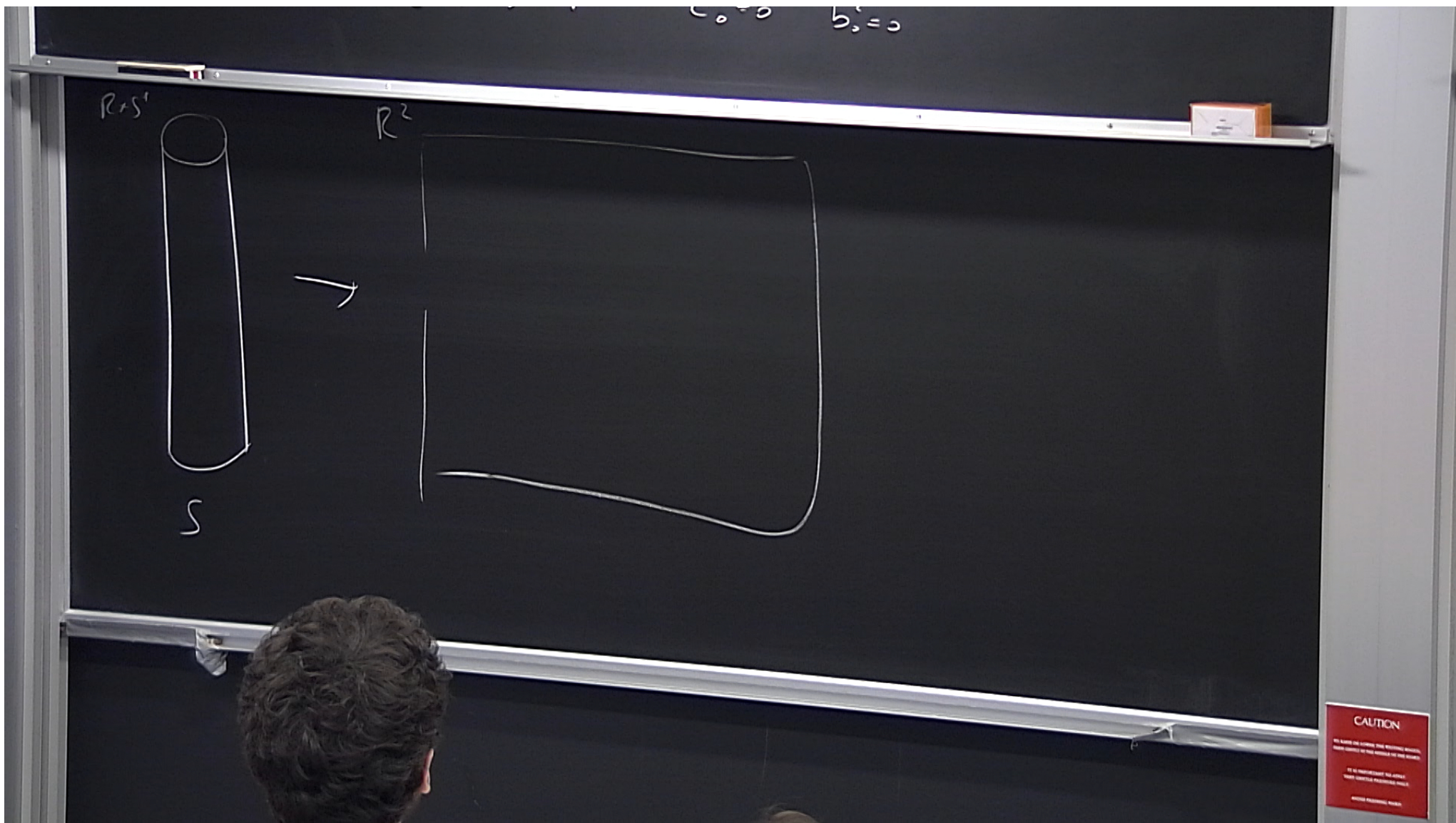
$$Q \left[b_{-n} |g\rangle \otimes |w\rangle \right] =$$

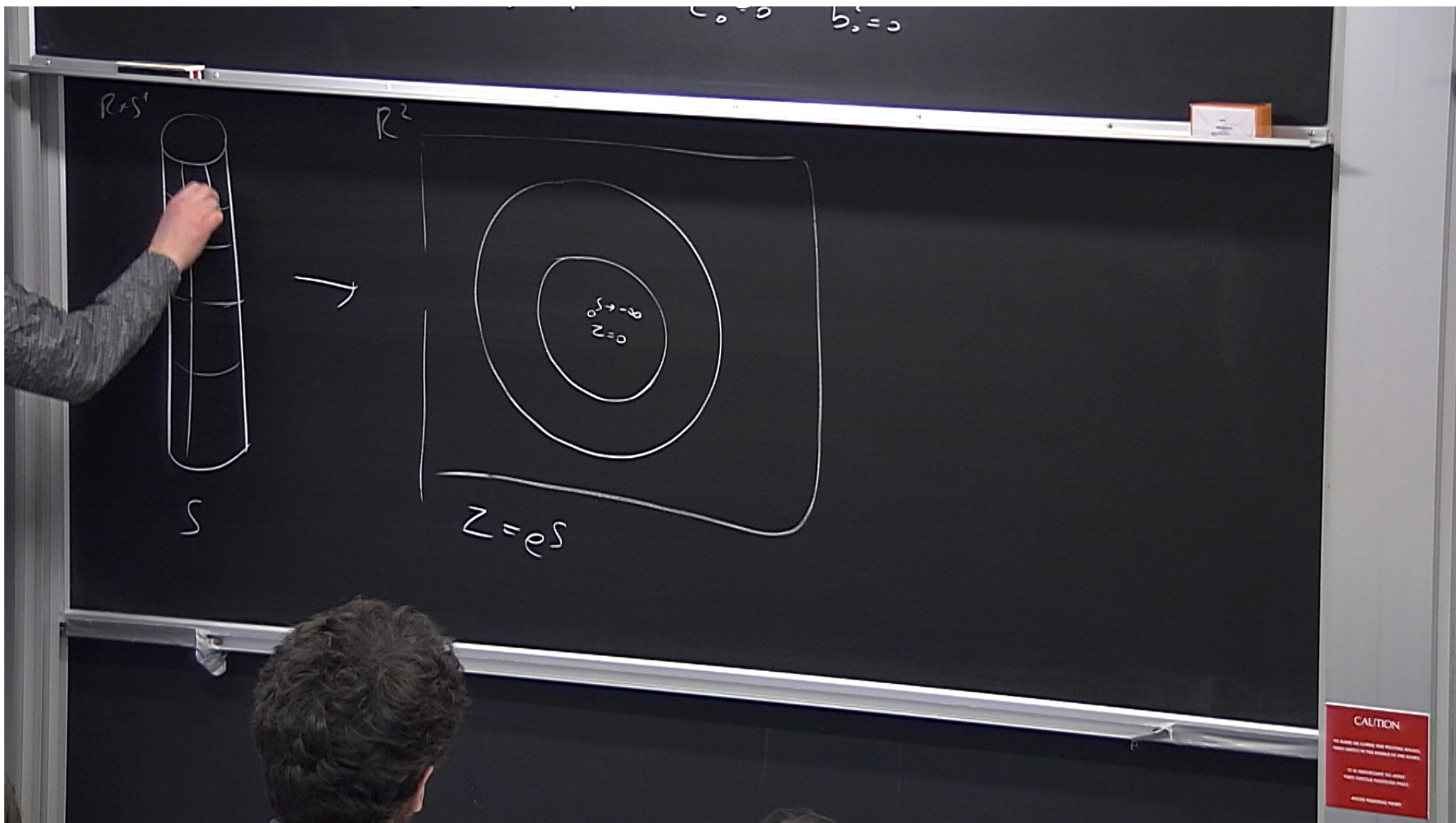
$$L_0^g |0\rangle = 0$$

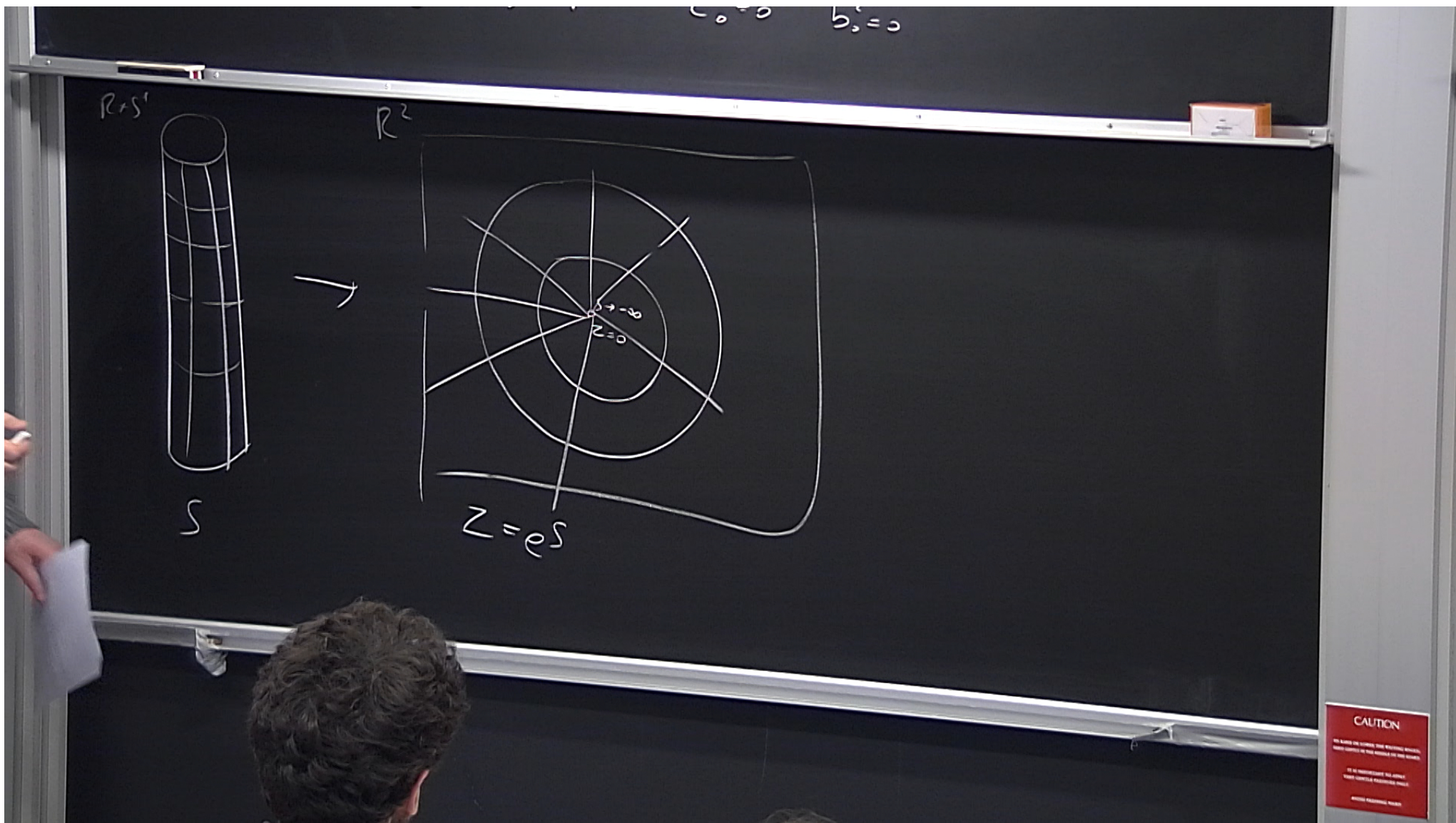
$$L_0^g |g\rangle = -\frac{1}{2} |g\rangle$$

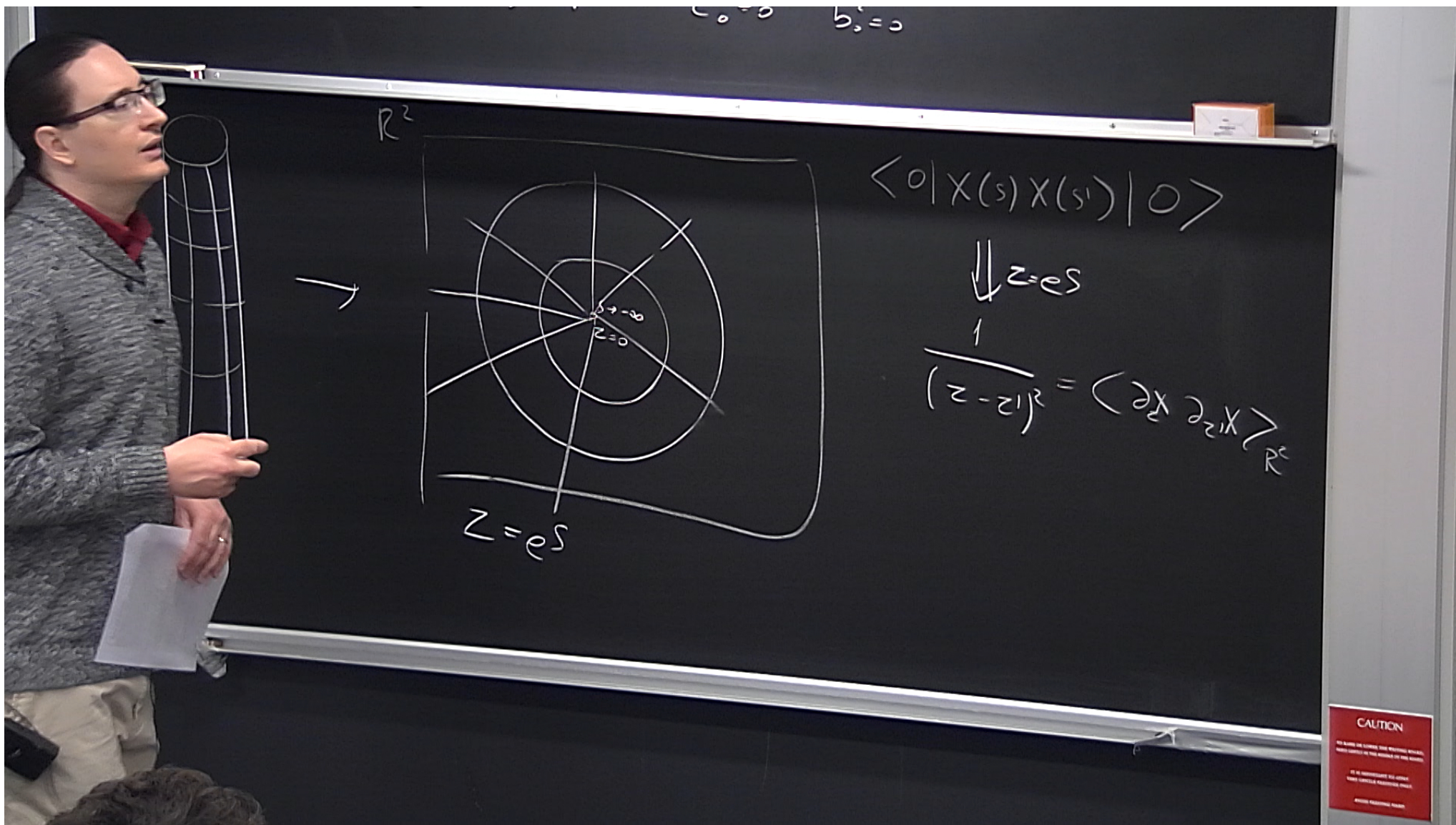
$$|matter\rangle = \sum_n L_n^x |w\rangle$$

$$Q \left[b_{-n} |g\rangle \otimes |w\rangle + \dots \right] = |g\rangle \otimes \sum_n L_n^x |w\rangle$$









$$\langle g | \psi^0 | g \rangle$$

$$\langle g | T^x | g \rangle$$

$$L_0^{\text{gr}} |0\rangle = 0$$

$$= \frac{e^{s'-s}}{1-e^{s'-s}} = \frac{e^{s'}}{e^s - e^{s'}} \quad \sim \frac{1}{s+s'} \frac{1}{s-s'}$$

$$\langle \tilde{0} | c(s) b(s') | 0 \rangle = \frac{e^{2s'-s}}{e^s - e^{s'}}$$

$$|g\rangle \otimes |\text{matter}\rangle \in H[X^*]$$

$$b_m |w\rangle = 0 \quad m \geq 0$$

$$w\rangle = 0$$

$$|\text{matter}\rangle = L_{-n}^x |w\rangle$$

$$Q \left[b_{-n} |g\rangle \otimes |w\rangle + \dots \right] = L_{-n}^x |w\rangle$$

$$\langle \tilde{g} | \dots | g \rangle$$

$$\langle \tilde{g} | T^x | g \rangle$$

$$L_0^{\text{gh}} |0\rangle = 0$$

$$= \frac{e^{s'-s}}{1-e^{s'-s}} = \frac{e^{s'}}{e^s - e^{s'}} \quad \sim \frac{1}{s+s'} \frac{1}{s-s'}$$

$$\langle \tilde{0} | c(s) b(s') | 0 \rangle = \frac{e^{2s'-s}}{e^s - e^{s'}}$$

$$\Downarrow \\ \langle c(z) b(z') \rangle$$

$$|g\rangle \otimes |matter\rangle \in H[X^\wedge]$$

BRST COHOMOLOGY

$$b_n |w\rangle = 0 \quad n \geq 0$$

$$\{Q, b_n\} |w\rangle = 0$$

$$|matter\rangle = L_{-n}^x |w\rangle$$

$$Q [b_{-n} |g\rangle \otimes |w\rangle + \dots] = |g\rangle \otimes L_{-n}^x |w\rangle$$



$$S = \tau - i\sigma$$

$$S_{gh} = \int (b \bar{\partial} c + \bar{b} \partial \bar{c}) ds d\bar{s}$$

$$b(s) = \sum_n b_n e^{-ns}$$

$$c(s) = \sum_n c_n e^{-ns}$$

$$\{b_n, c_m\} = \delta_{n+m, 0}$$

$$\{b_0, c_0\} = 1$$

$$c_0^2 = 0$$

$$\{c_n, c_m\} = 0$$

$$b_0^2 = 0$$

$$b(s) = b(s') \left(\frac{\partial s'}{\partial s} \right)^2$$

$$c(s) = c(s') \left(\frac{\partial s'}{\partial s} \right)^{-1}$$

$$\partial_s X(s) = \partial_{s'} X(s') \frac{\partial s'}{\partial s}$$

$$\{b_n, b_m\} = 0$$

$$\langle \tilde{g} | T^{\mu} | g \rangle$$

$$\langle \tilde{g} | T^{\mu} | g \rangle$$

$$L_0 |0\rangle = 0$$

$$= \frac{e^{s'-s}}{1-e^{s'-s}} = \frac{e^{s'}}{e^s - e^{s'}} \quad \sim \frac{1}{s-s'}$$

$$\langle \tilde{0} | c(s) b(s') | 0 \rangle = \frac{e^{2s'-s}}{e^s - e^{s'}}$$

$$\langle c(z) b(z') \rangle$$

$$|g\rangle \otimes |matter\rangle \in H[X^+]$$

BRST cohomology

$$b_n |w\rangle = 0 \quad n \geq 0$$

$$\{Q, b_n\} |w\rangle$$

$$|matter\rangle = \sum_n L_n^- |w\rangle$$

$$Q \left[b_n |g\rangle \otimes |w\rangle + \dots \right] = |g\rangle \otimes |w\rangle$$

$$\langle \tilde{g} | T^{\mu\nu} | g \rangle$$

$$\langle \tilde{g} | T^{\mu\nu} | g \rangle$$

$$L_0^{\mu\nu} |0\rangle = 0$$

$$= \frac{e^{s'-s}}{1-e^{s'-s}} = \frac{e^{s'}}{e^s - e^{s'}} \quad \sim \frac{1}{s-s'}$$

$$\langle \tilde{0} | c(s) b(s') | 0 \rangle = \frac{e^{2s'-s}}{e^s - e^{s'}}$$

$$\Downarrow$$

$$\langle c(z) b(z') \rangle = \frac{1}{z-z'}$$

$$|g\rangle \otimes |matter\rangle \in H[X^*]$$

BRST COHOMOLOGY

$$b_n |w\rangle = 0 \quad n \geq 0$$

$$\{Q, b_n\} |w\rangle = 0$$

$$|matter\rangle = L_{-n}^x |w\rangle$$

$$Q \left[b_{-n} |g\rangle \otimes |w\rangle + \dots \right] = |g\rangle \otimes L_{-n}^x |w\rangle$$

$$\langle \tilde{g} | c(s) b(s') | g \rangle = \sum_{n,m} \langle \tilde{g} | c_n b_m | g \rangle e^{-ns - ms'}$$

$$\langle \tilde{g} | T^{\otimes} | g \rangle$$

$$= \sum_{n,m} \langle \tilde{g} | c_n b_m | g \rangle e^{-ns - ms'} = e^{s'-s} + e^{2s'-2s} + e^{3s'-3s} + \dots$$

$$= \frac{e^{s'-s}}{1 - e^{s'-s}} = \frac{e^{s'}}{e^s - e^{s'}} \Rightarrow \frac{z}{z'(z-z')} \underset{s+s'}{\sim} \frac{1}{s-s'}$$

$$\langle \tilde{g} | T^{\otimes} | g \rangle$$

$$L_0 |0\rangle = 0$$

$$\langle \tilde{0} | c(s) b(s') | 0 \rangle = \frac{e^{2s'-s}}{e^s - e^{s'}}$$

$$\Downarrow$$

$$\langle c(z) b(z') \rangle = \frac{1}{z - z'}$$

$$|m, l\rangle = L_{-n}^x |w\rangle$$

$$\left[b_{-n} |g\rangle \otimes |w\rangle + \dots \right] = |g\rangle \otimes L_{-n}^x |w\rangle$$