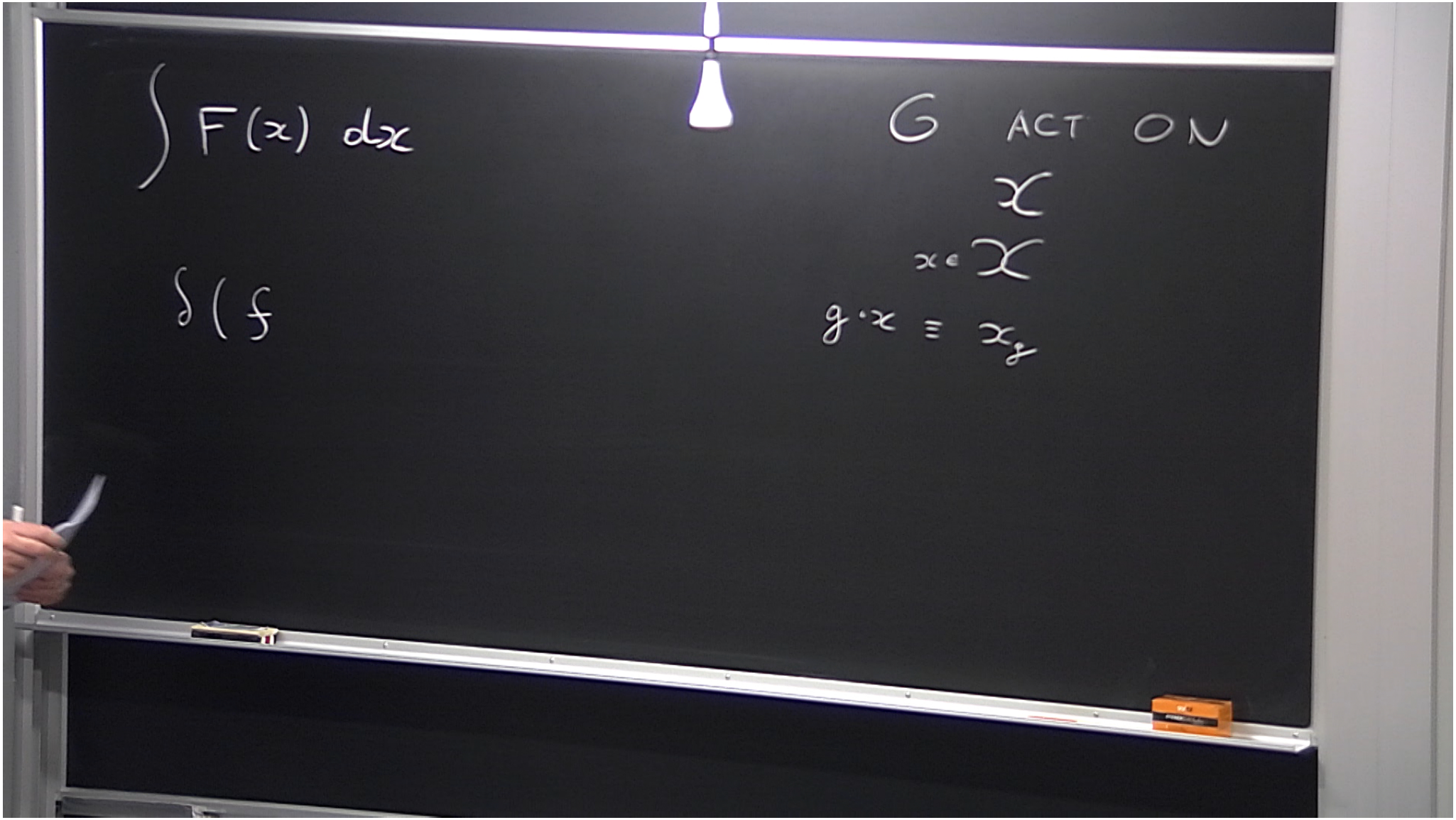
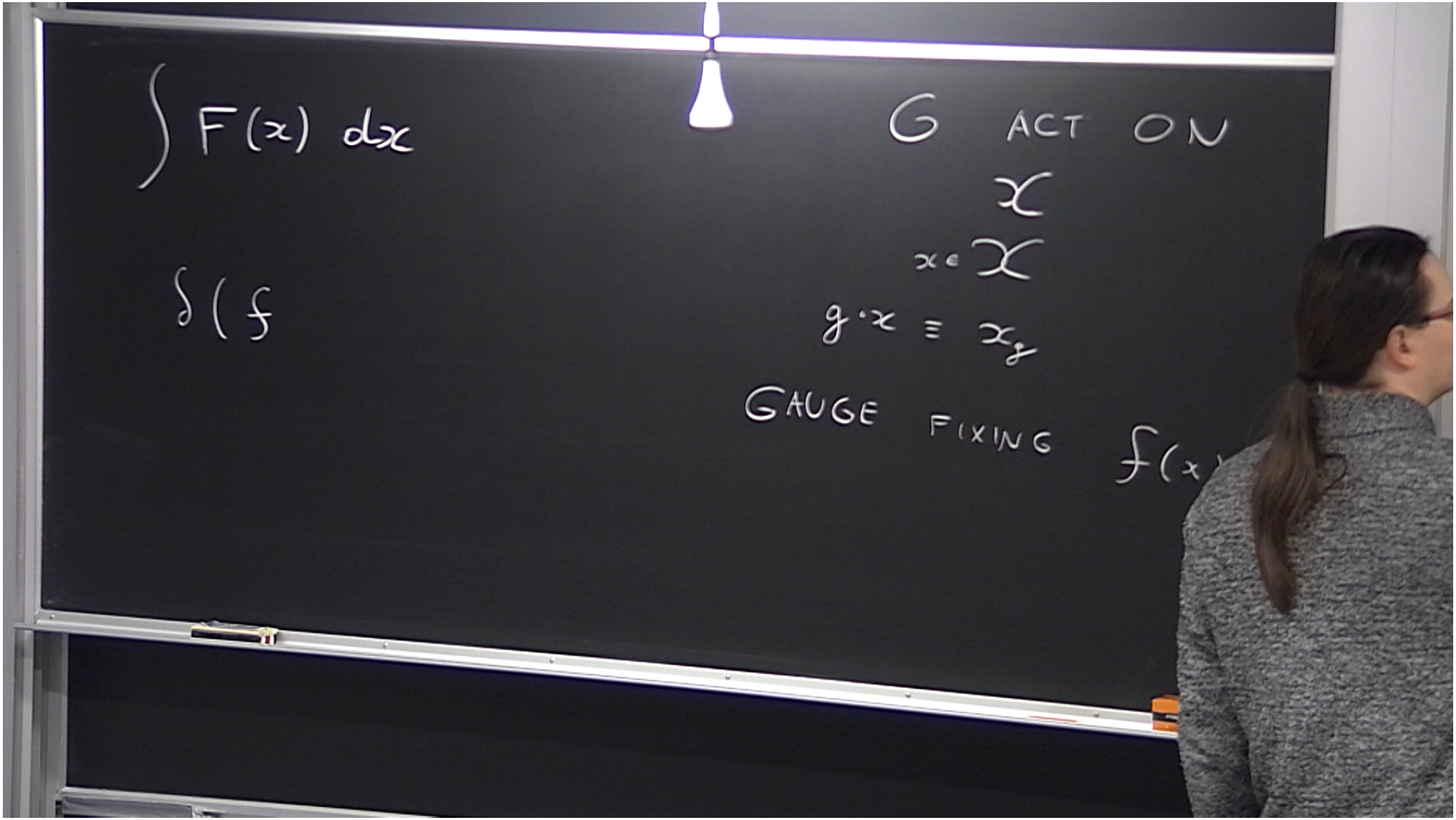
Title: PSI 17/18 - String Theory - Lecture 6

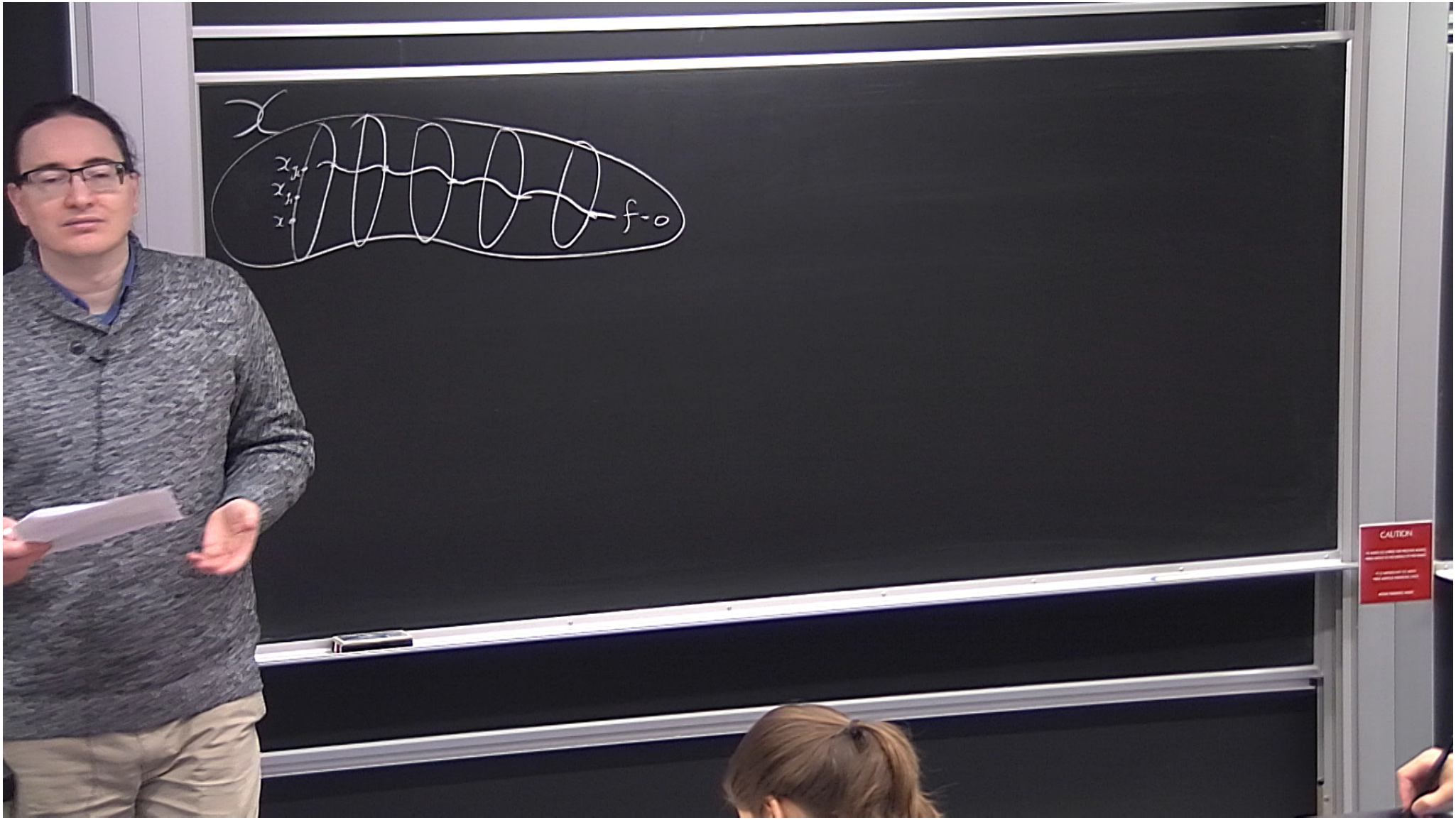
Date: Feb 27, 2018 10:15 AM

URL: http://pirsa.org/18020042

Abstract:


$\int F(x)\,dx$
$\int ( f$
G ACT ON
$X$
$x \in X$
$g \cdot x \equiv x_g$


$\int F(x)\,dx$
G ACT ON
$\mathcal{X}$
$x \in \mathcal{X}$
$g \cdot x \equiv x_g$
GAUGE FIXING
$f(x)$


f-o
CAUTION

$$\int F(x)\,dx$$

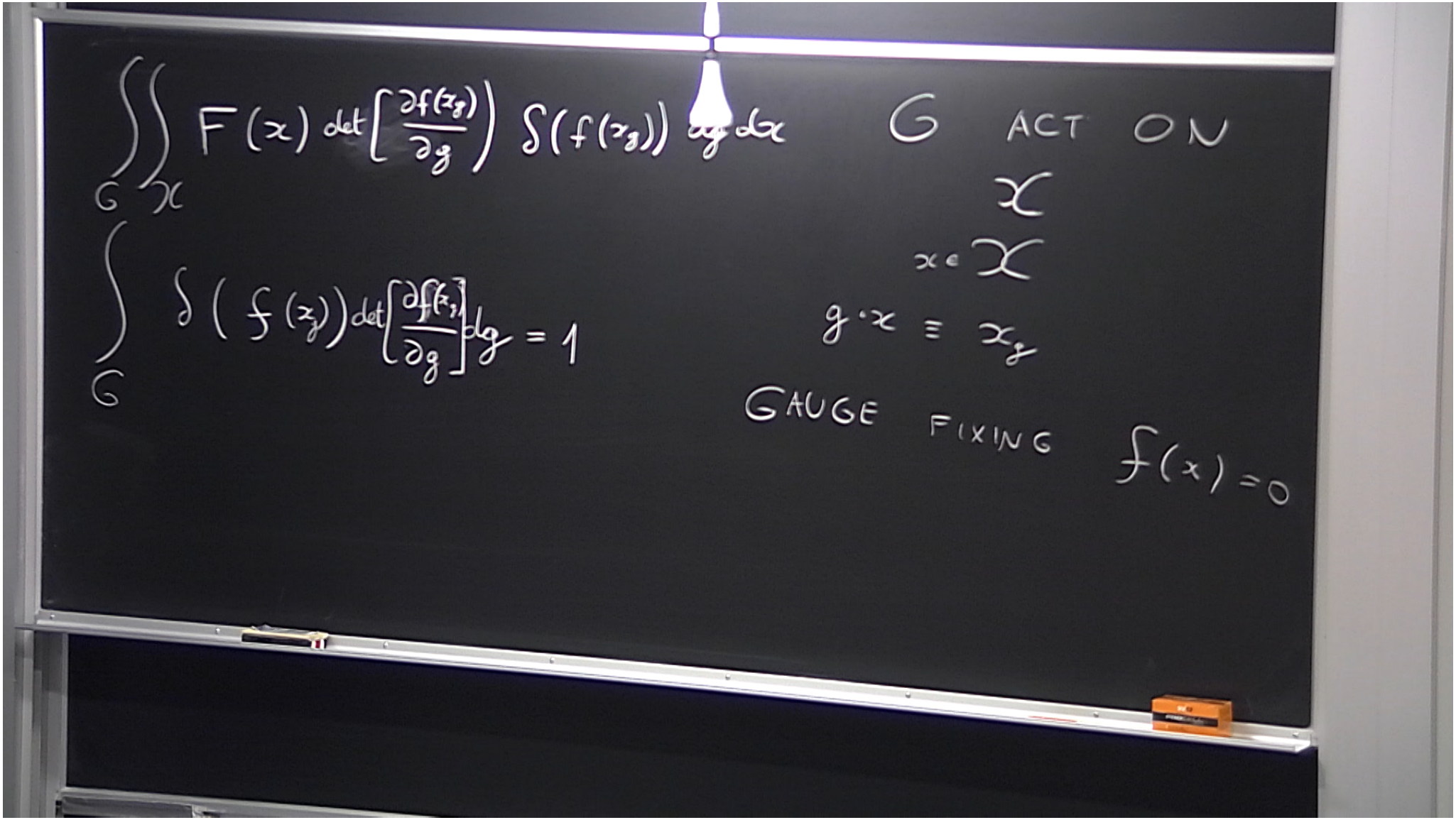
$$\int_G \delta(f(x_g))\det\left[\frac{\partial f(x_g)}{\partial g}\right]dg = 1$$

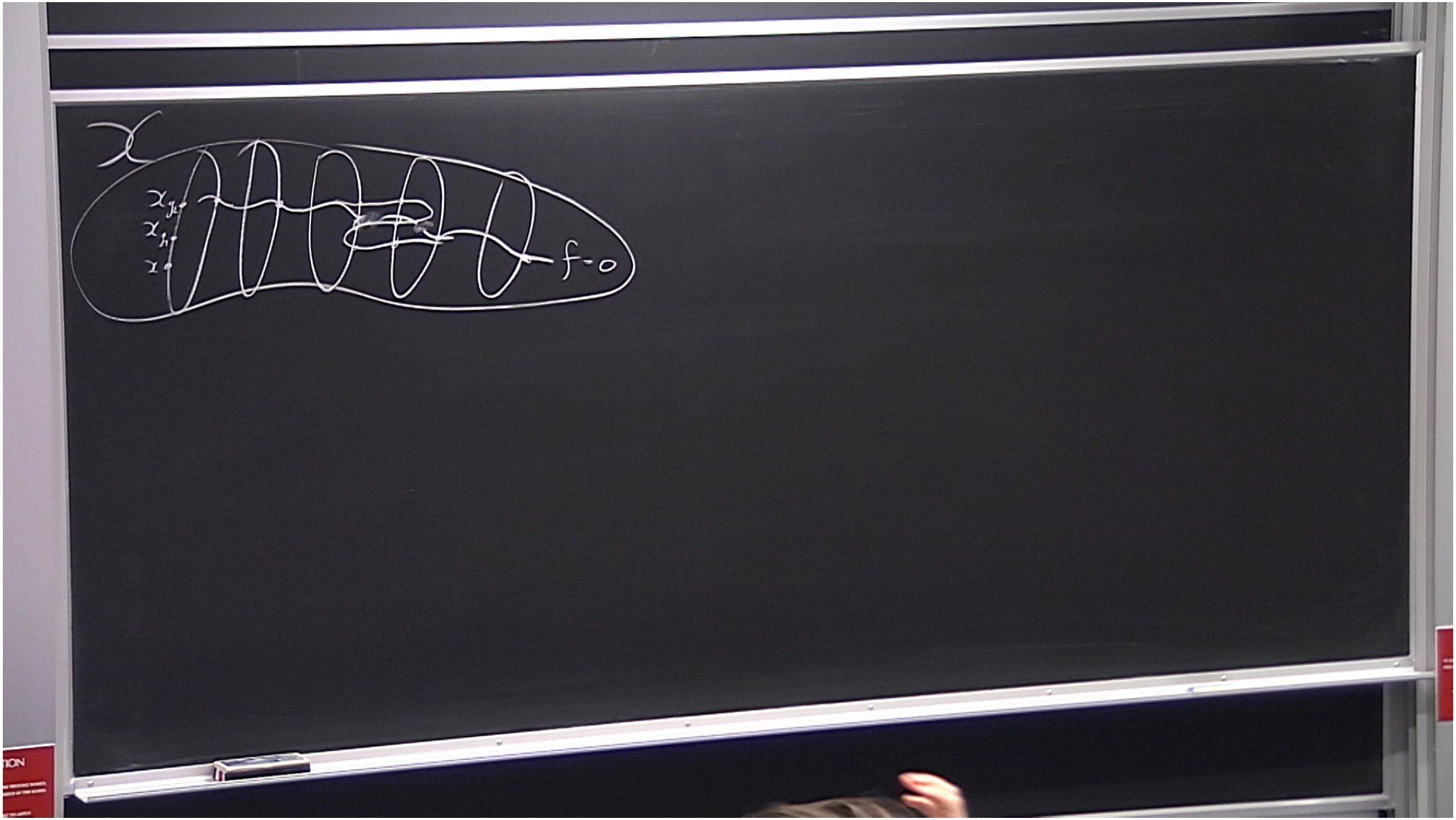
$G$ ACT ON $\mathcal{X}$

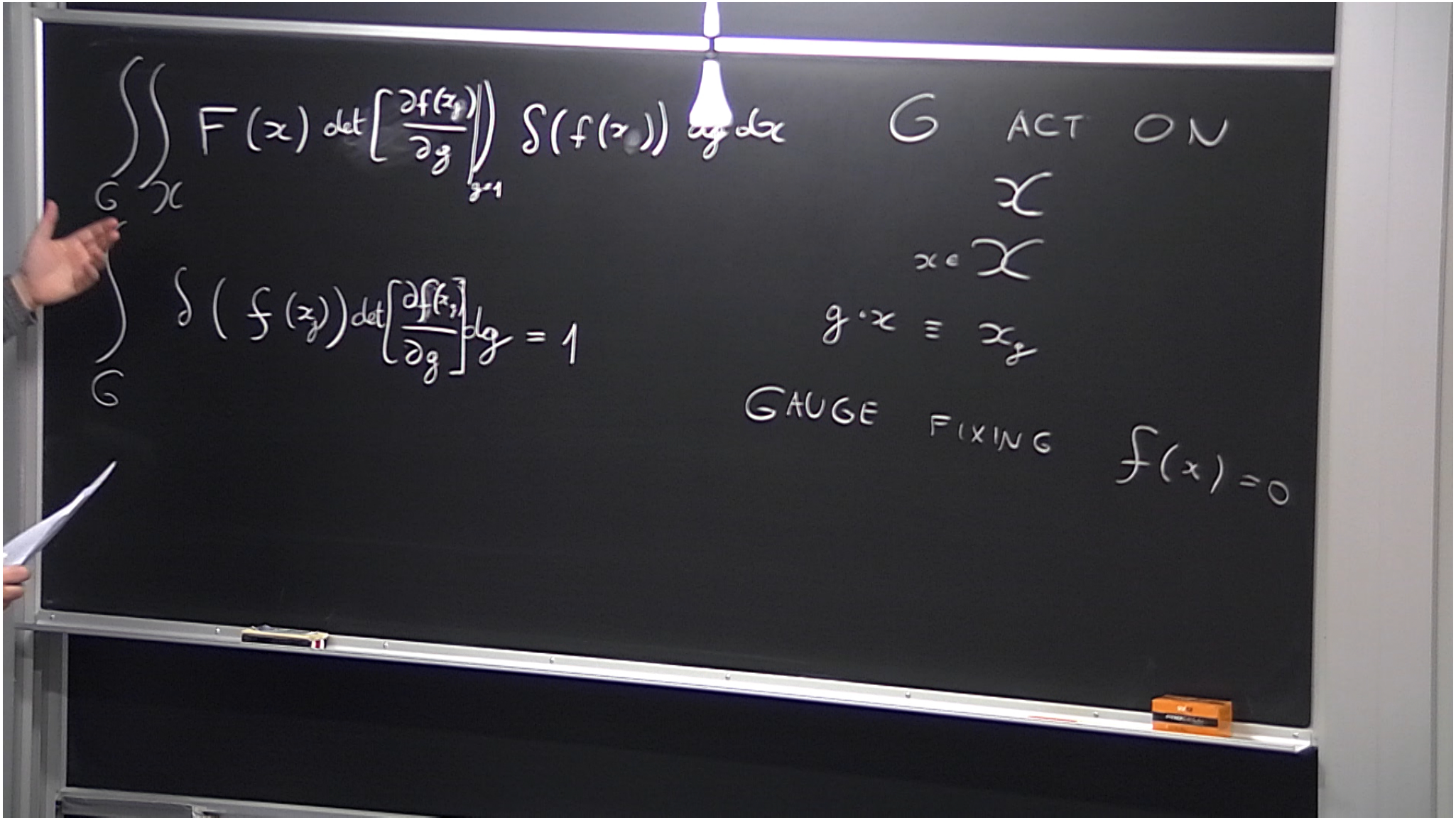
$x \in \mathcal{X}$

$g \cdot x \equiv x_g$

GAUGE FIXING $f(x) = 0$


$\int_G \int_X F(x) \det\left[\frac{\partial f(x_g)}{\partial g}\right] \delta(f(x_g))\, dg\, dx$
$\int_G \delta(f(x_g)) \det\left[\frac{\partial f(x_g)}{\partial g}\right] dg = 1$
G ACT ON X
$x \in X$
$g \cdot x \equiv x_g$
GAUGE FIXING $f(x) = 0$


f-o


$\int_G \int_{\mathcal{X}} F(x) \det\left[\frac{\partial f(x_g)}{\partial g}\right]_{g=1} \delta(f(x_g))\, dg\, dx$
$\int_G \delta(f(x_g)) \det\left[\frac{\partial f(x_g)}{\partial g}\right] dg = 1$
G ACT ON $\mathcal{X}$
$x \in \mathcal{X}$
$g \cdot x \equiv x_g$
GAUGE FIXING $f(x) = 0$

$$\int_{\mathcal{X}} F(x)\, \det\left[\frac{\partial f(x_g)}{\partial g}\right]_{g=1} \delta(f(x))\, dx$$

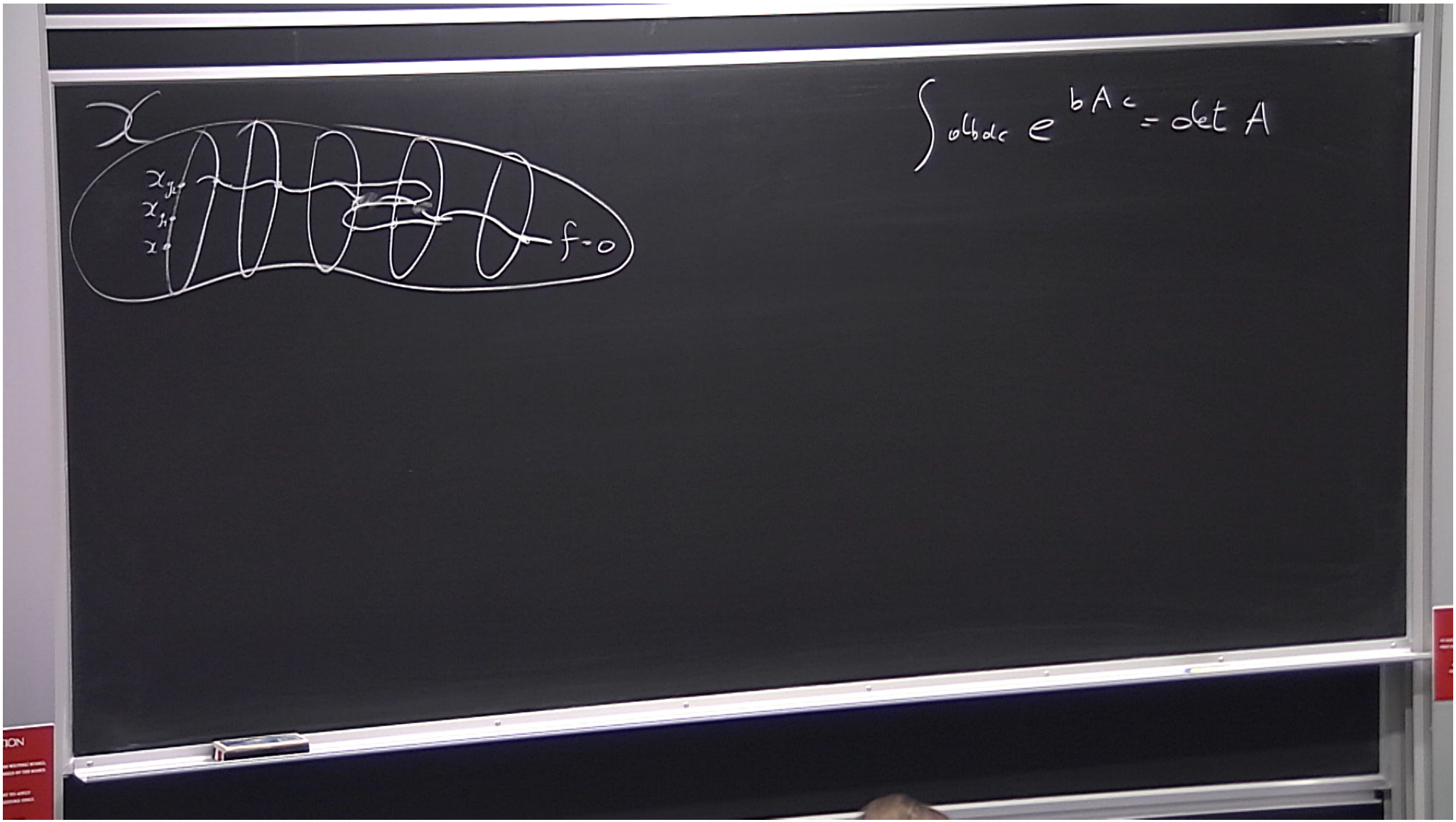
$$\int_G \delta(f(x_g)) \det\left[\frac{\partial f(x_g)}{\partial g}\right] dg = 1$$

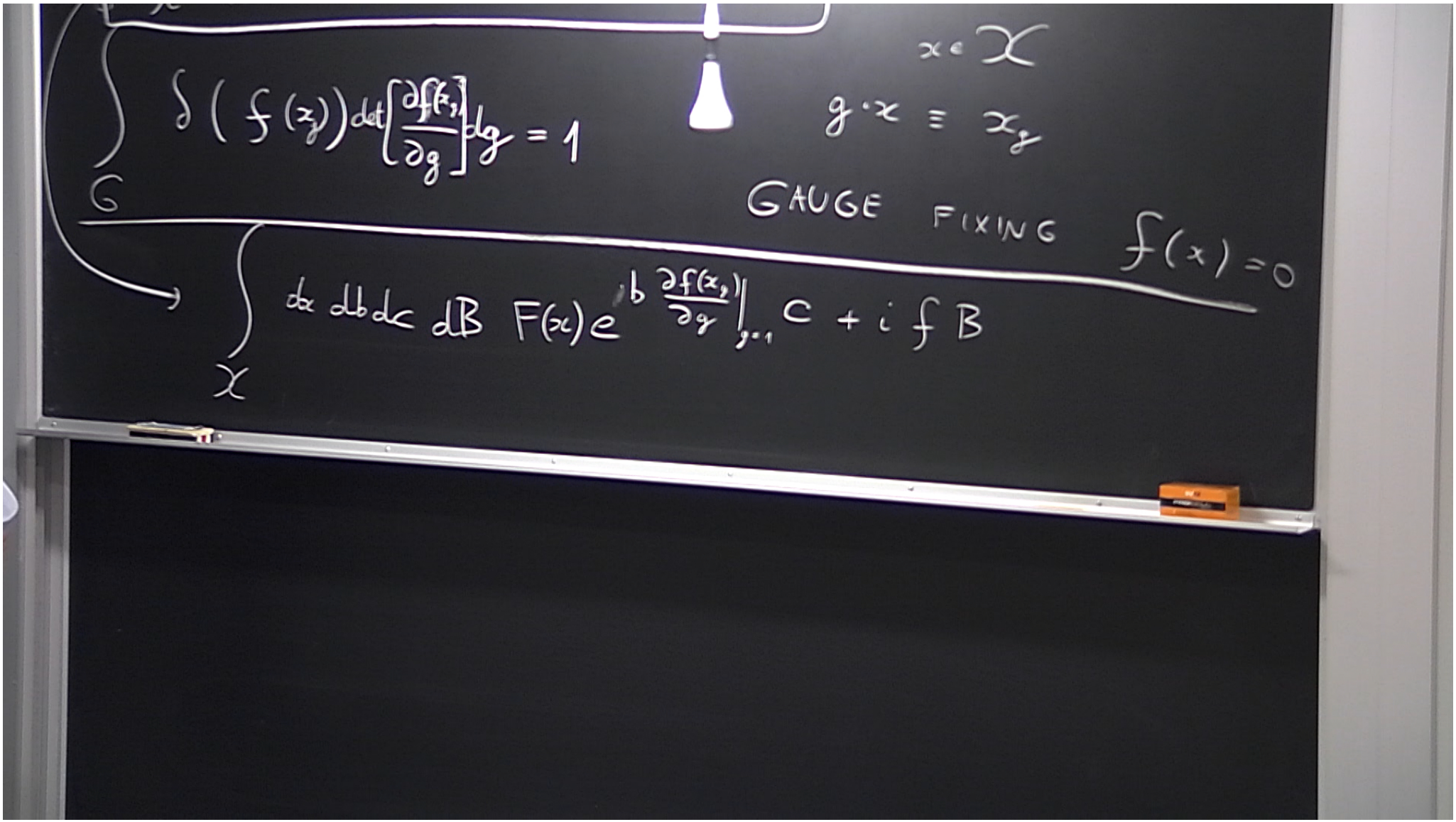
$G$ ACT ON $\mathcal{X}$

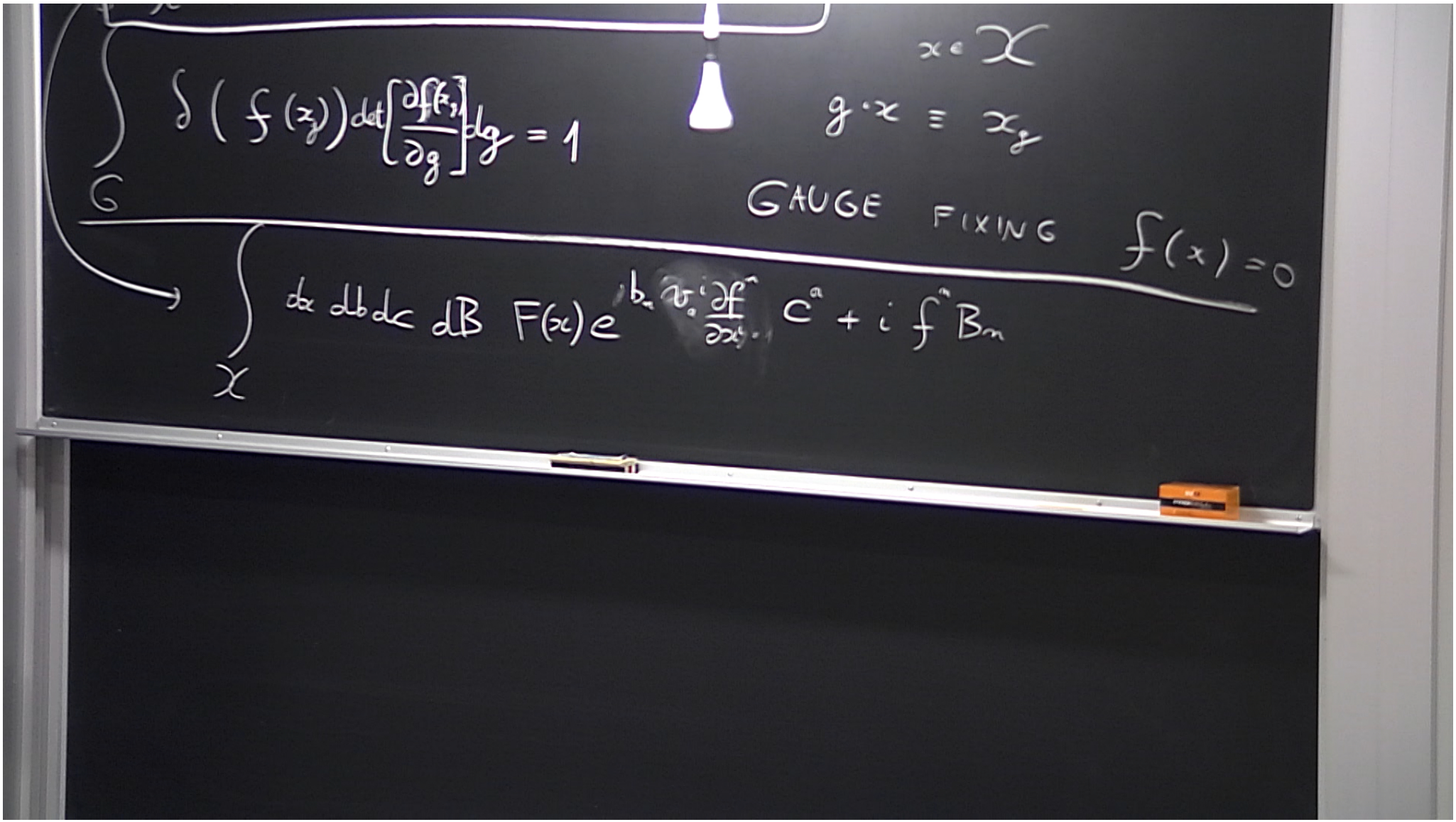
$x \in \mathcal{X}$

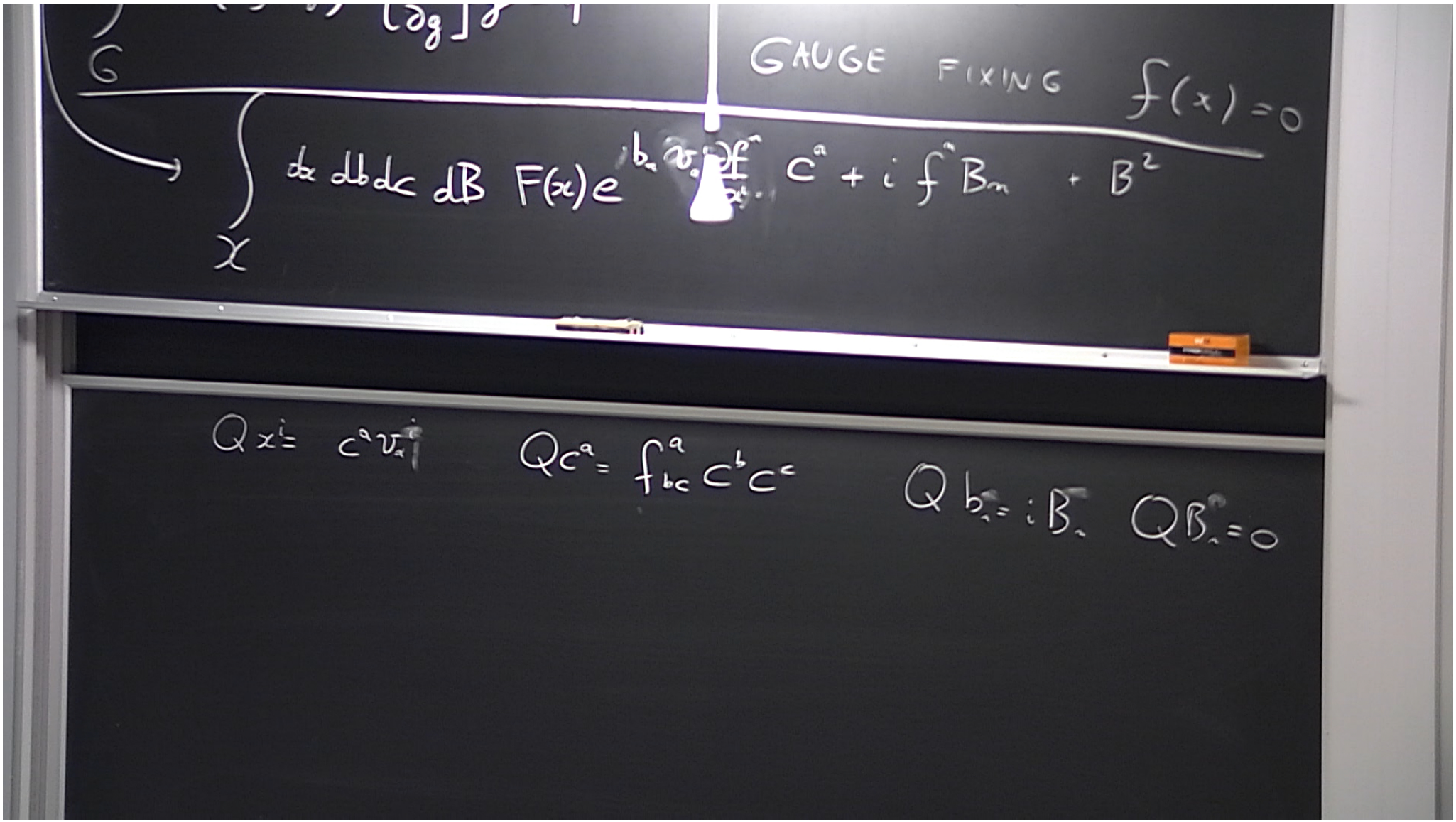
$g \cdot x \equiv x_g$

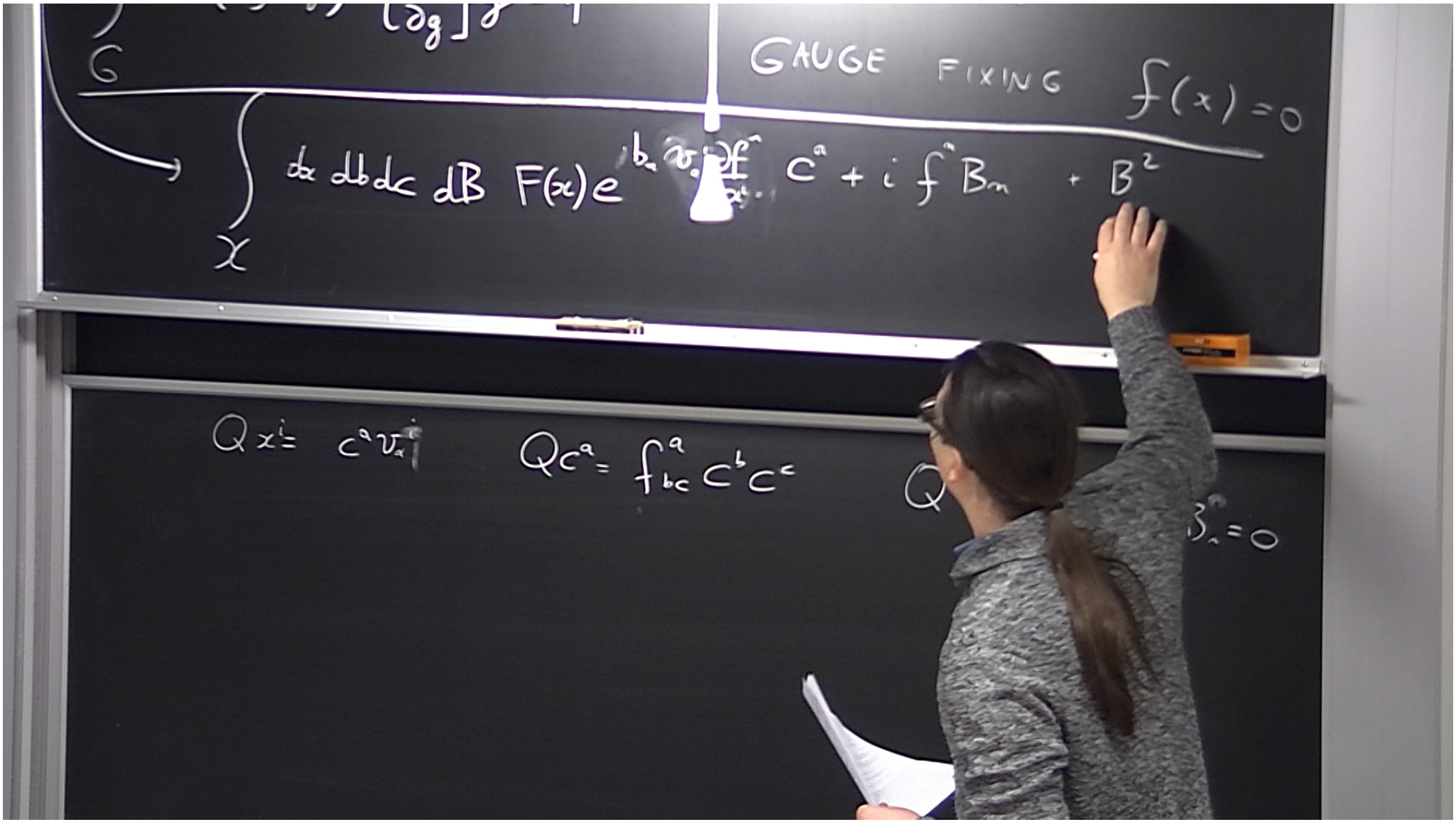
GAUGE FIXING $f(x) = 0$

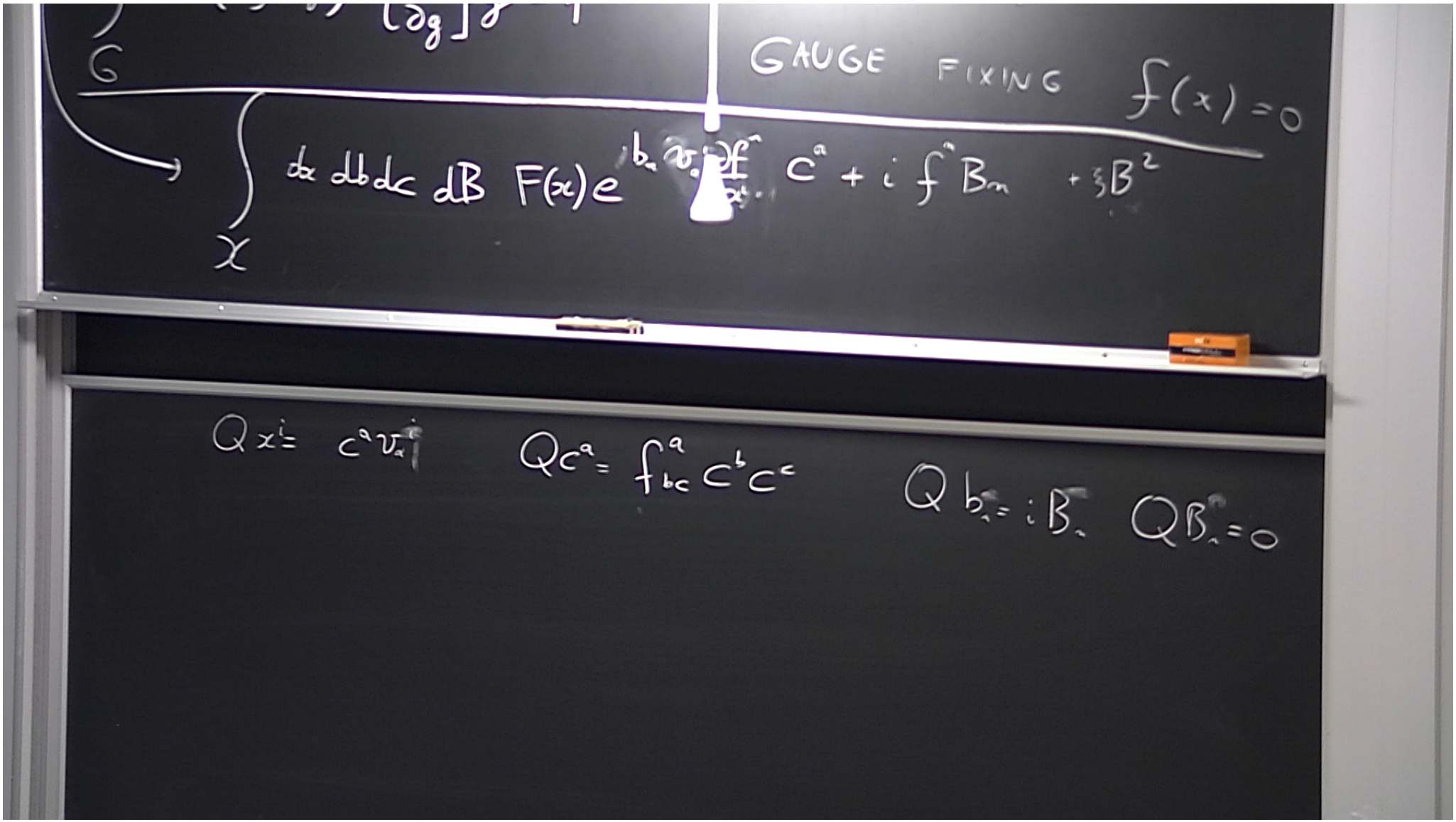

$\int d\bar{b}\,db\,dc\; e^{\bar{b}Ac} = \det A$
$f=0$

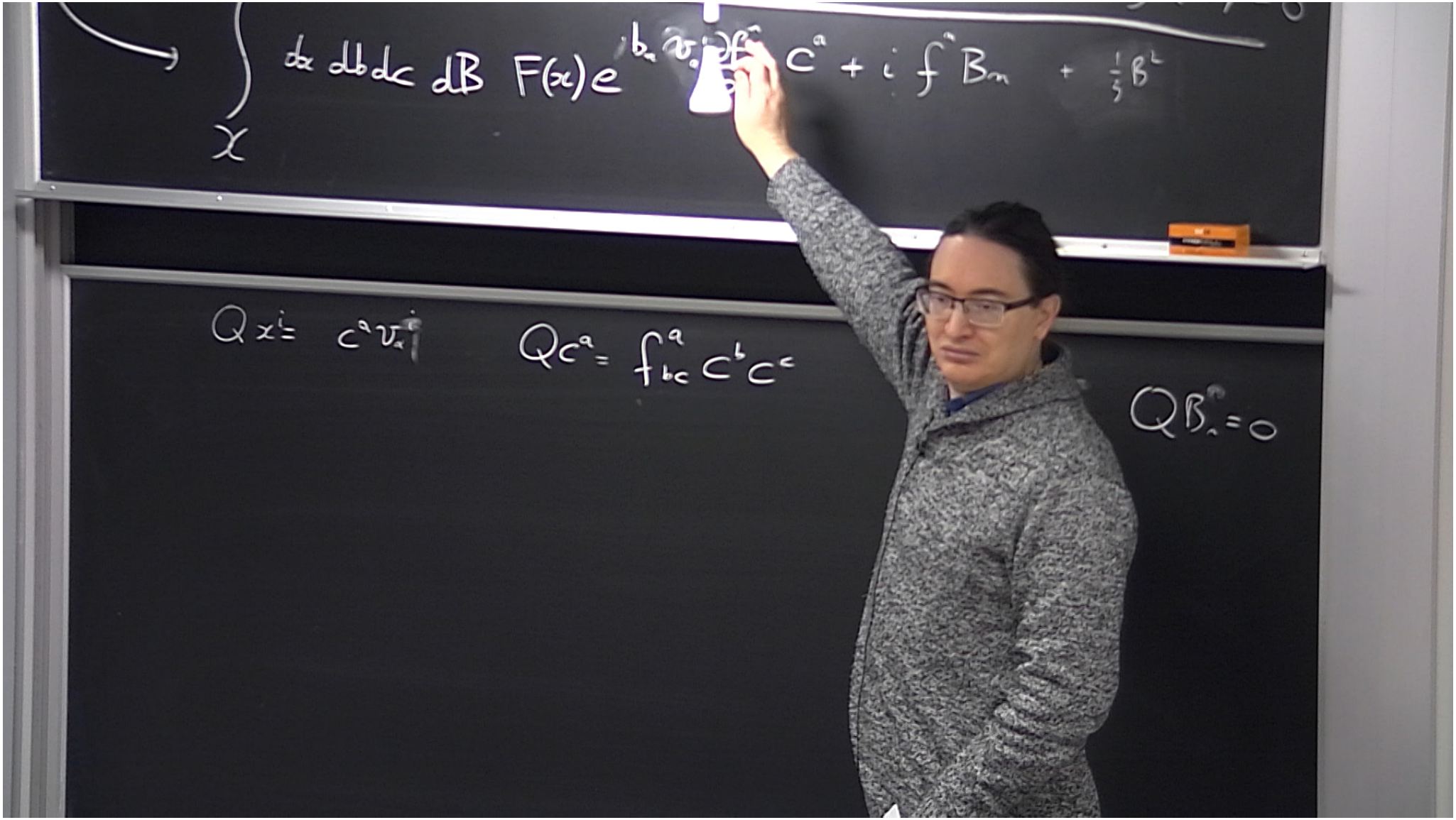

$x \in X$
$g \cdot x \equiv x_g$
$\int \delta(f(x_g)) \det\left[\frac{\partial f(x_g)}{\partial g}\right] dg = 1$
G
GAUGE FIXING
$f(x) = 0$
$\int_X dx\, db\, dc\, dB\, F(x) e^{ib \frac{\partial f(x_g)}{\partial g}\Big|_{g=1}} c + i f B$

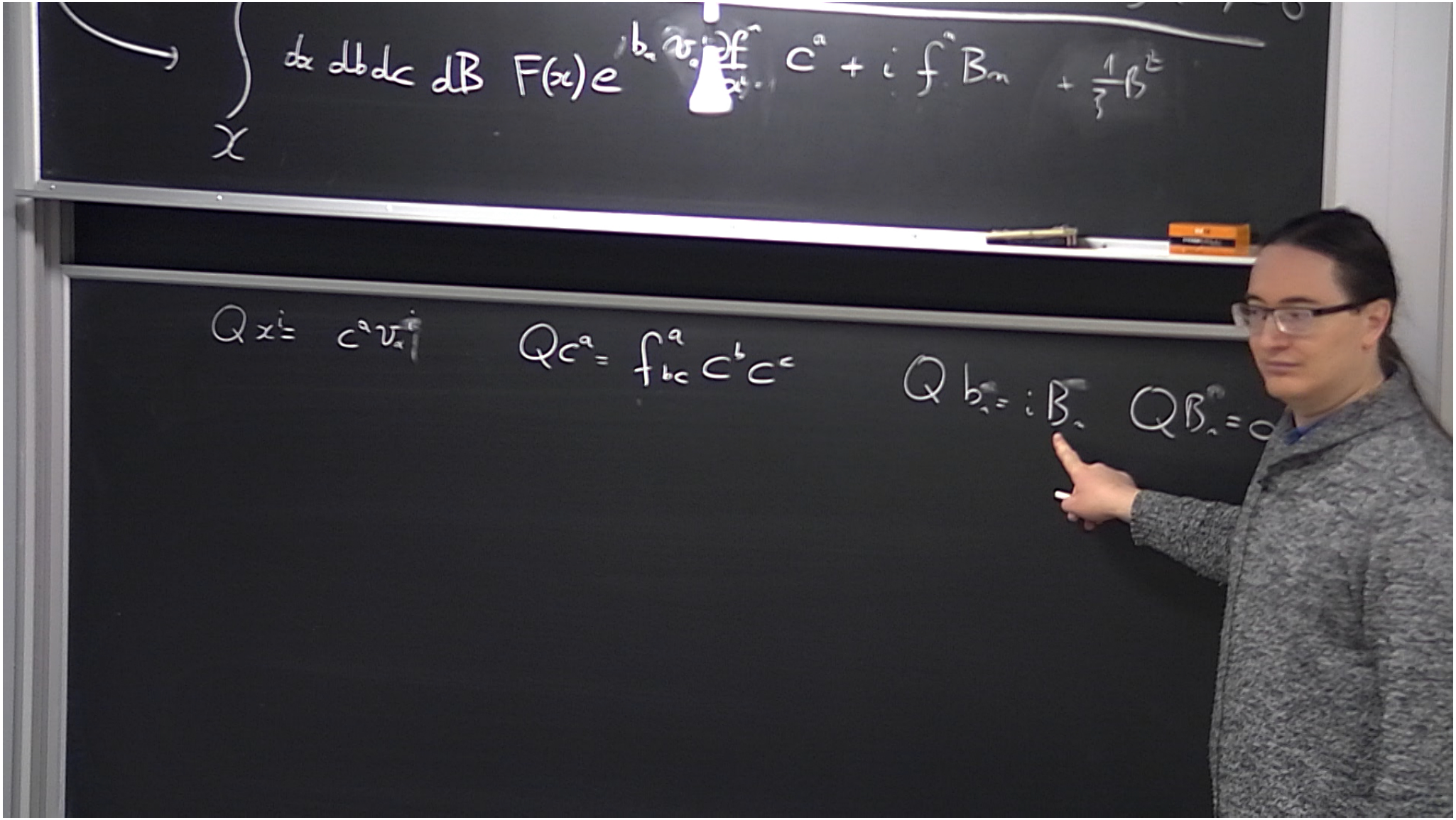

$x \in X$
$g \cdot x \equiv x_g$
$\int \delta(f(x_g)) \det\left[\frac{\partial f(x_g)}{\partial g}\right] dg = 1$
GAUGE FIXING
$f(x) = 0$
G
X
$dx\, da\, db\, dc\, dB\, F(x) e^{i b_n x_\mu^i \frac{\partial f^n}{\partial x_\mu^i} c^a + i f^n B_n}$

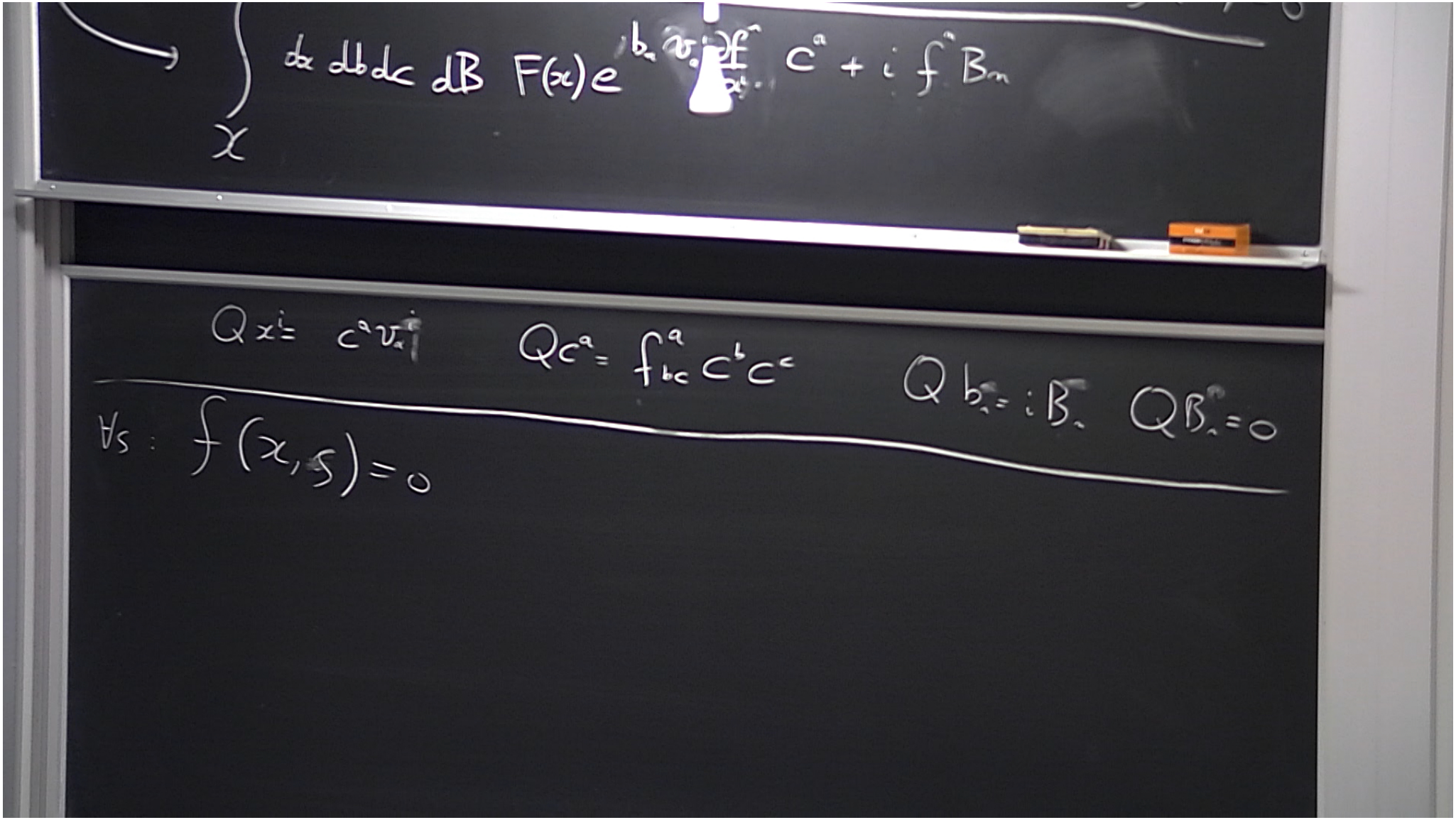

GAUGE FIXING $f(x)=0$
$\int dx\, db\, dc\, dB\, F(x) e^{i b_a v_i^a \frac{\partial f^a}{\partial x^i} c^a + i f^a B_a + B^2}$
$Q x^i = c^a v_a^i$
$Q c^a = f^a_{bc} c^b c^c$
$Q b_a = i B_a$
$Q B_a = 0$

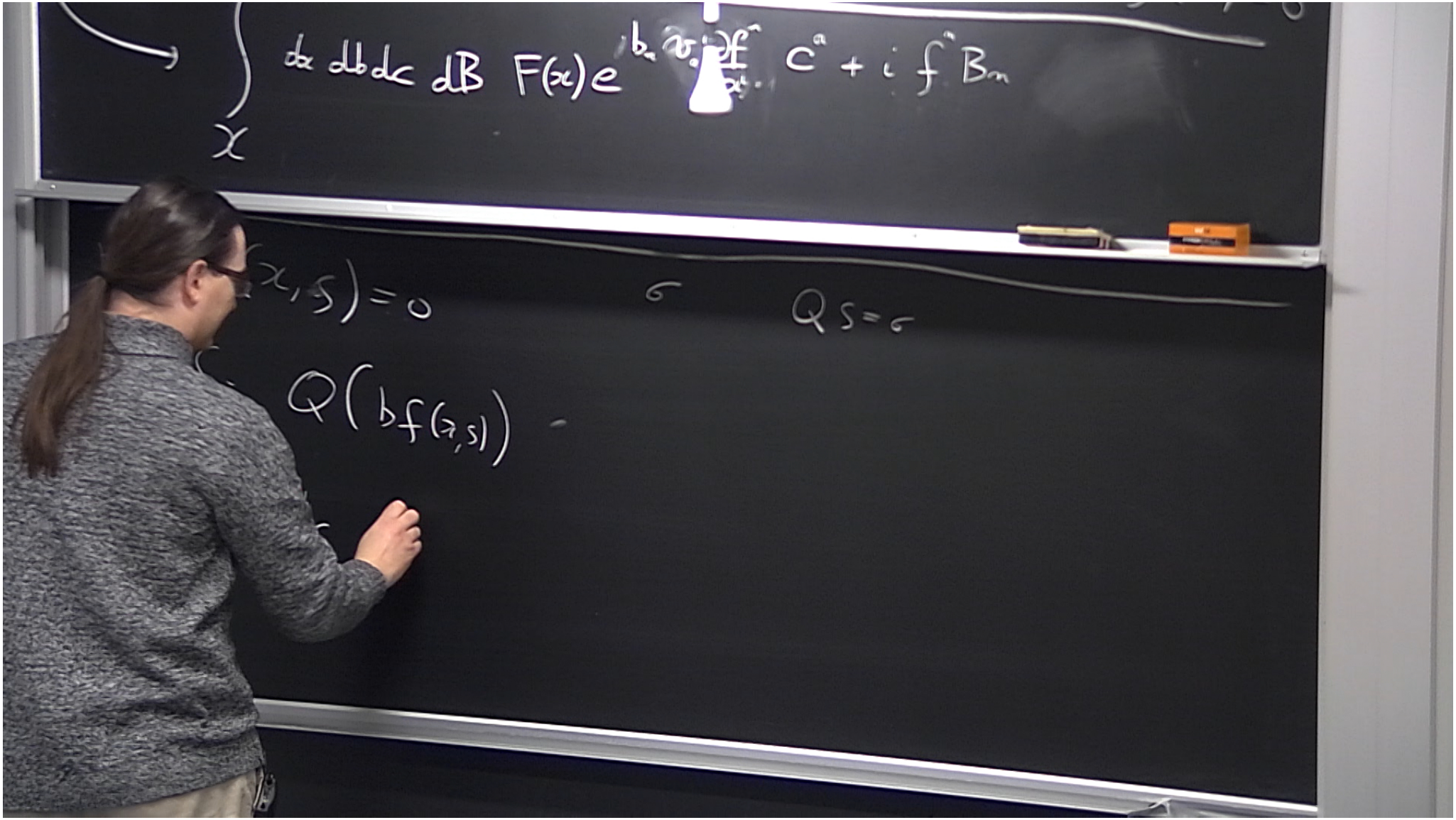

GAUGE FIXING
f(x) = 0
G
X
Qx^i = c^a v_a^i
Qc^a = f_bc^a c^b c^c

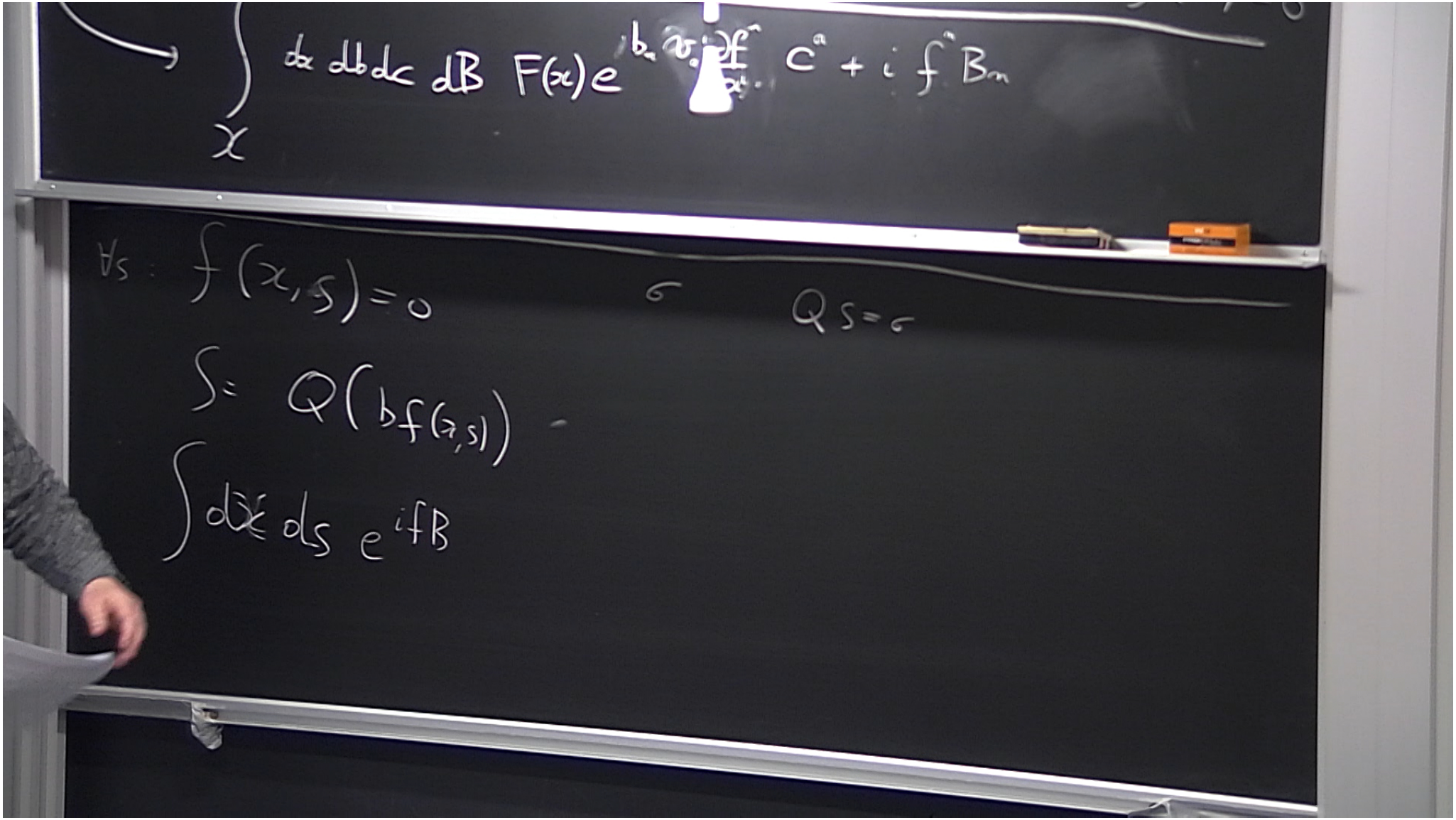

GAUGE FIXING $f(x)=0$
$\int_G \to \int_X$
$dx\,db\,dc\,dB\;F(x)\,e^{\,ib_a \partial_\mu v_\nu^a \frac{\partial f^a}{\partial x^\nu} c^a + i f^a B_a + \frac{\alpha}{2} B^2}$
$Q x^i = c^a v_a^i$
$Q c^a = f^a_{bc} c^b c^c$
$Q b_a = i B_a$
$Q B_a = 0$

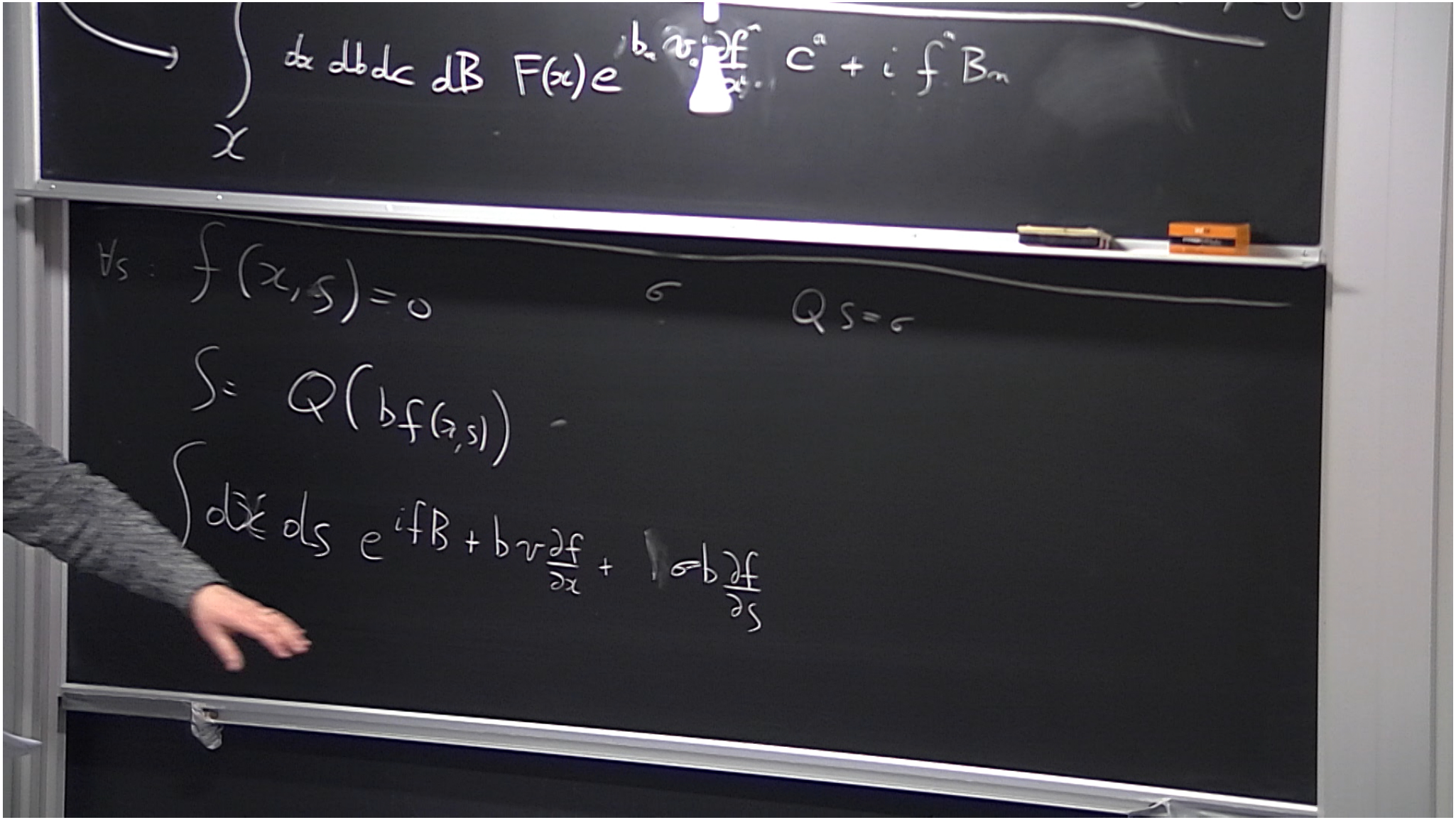

$\int_X da\, db\, dc\, dB\, F(a) e^{i b_a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D_\mu c^a + i f^{abc} B_m + \frac{1}{2} B^2}$
$Q x^i = c^a v_a^i$
$Q c^a = f^a_{bc} c^b c^c$
$Q B^a = 0$

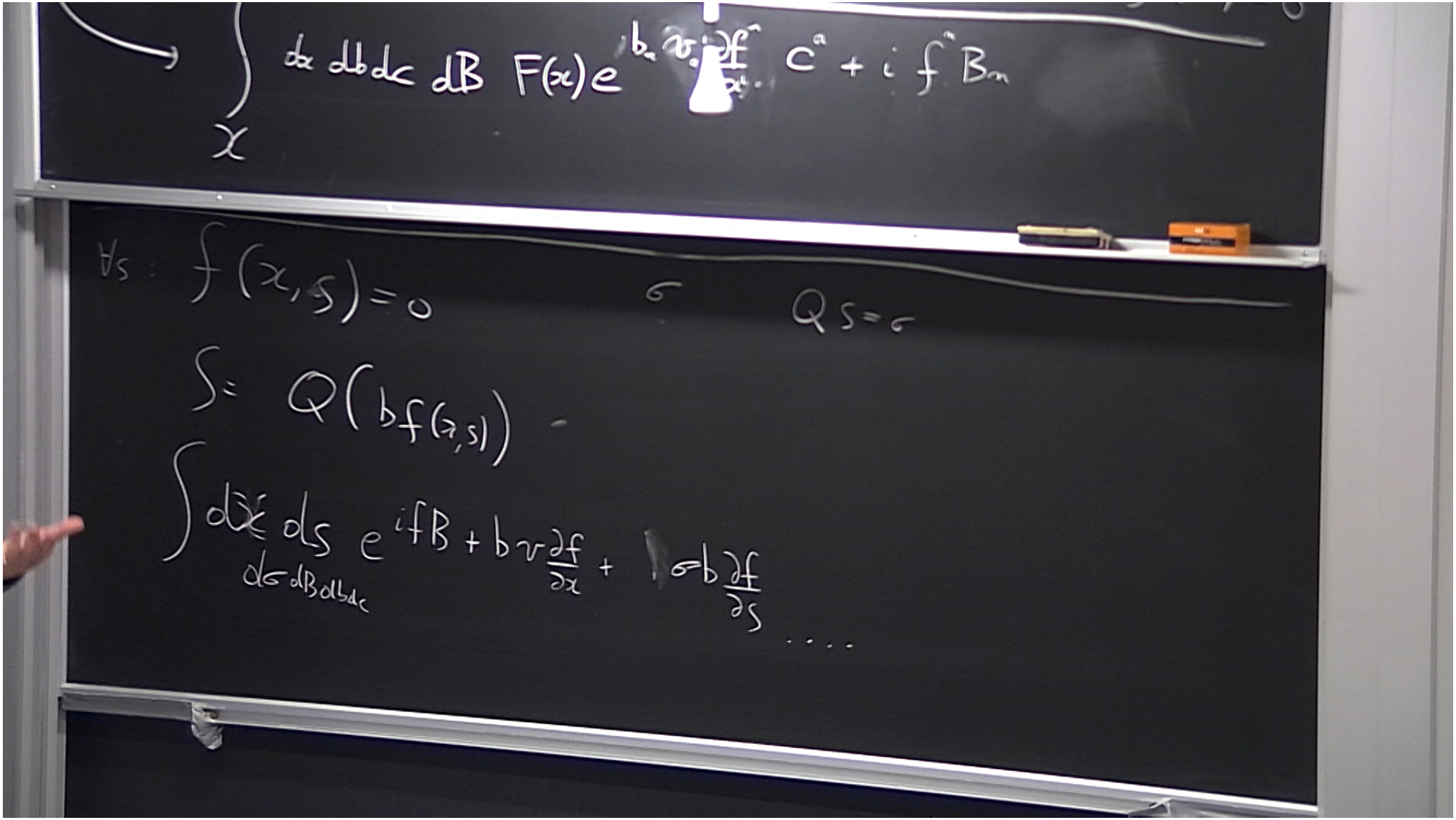

$\int_X da\, db\, dc\, dB\, F(\alpha) e^{i b_a v_i^a \frac{\partial f^a}{\partial x^i} c^a + i f^a B_m + \frac{1}{3} B^2}$
$Q x^i = c^a v_a^i$
$Q c^a = f^a_{bc} c^b c^c$
$Q b_a = i B_a$
$Q B_a = 0$

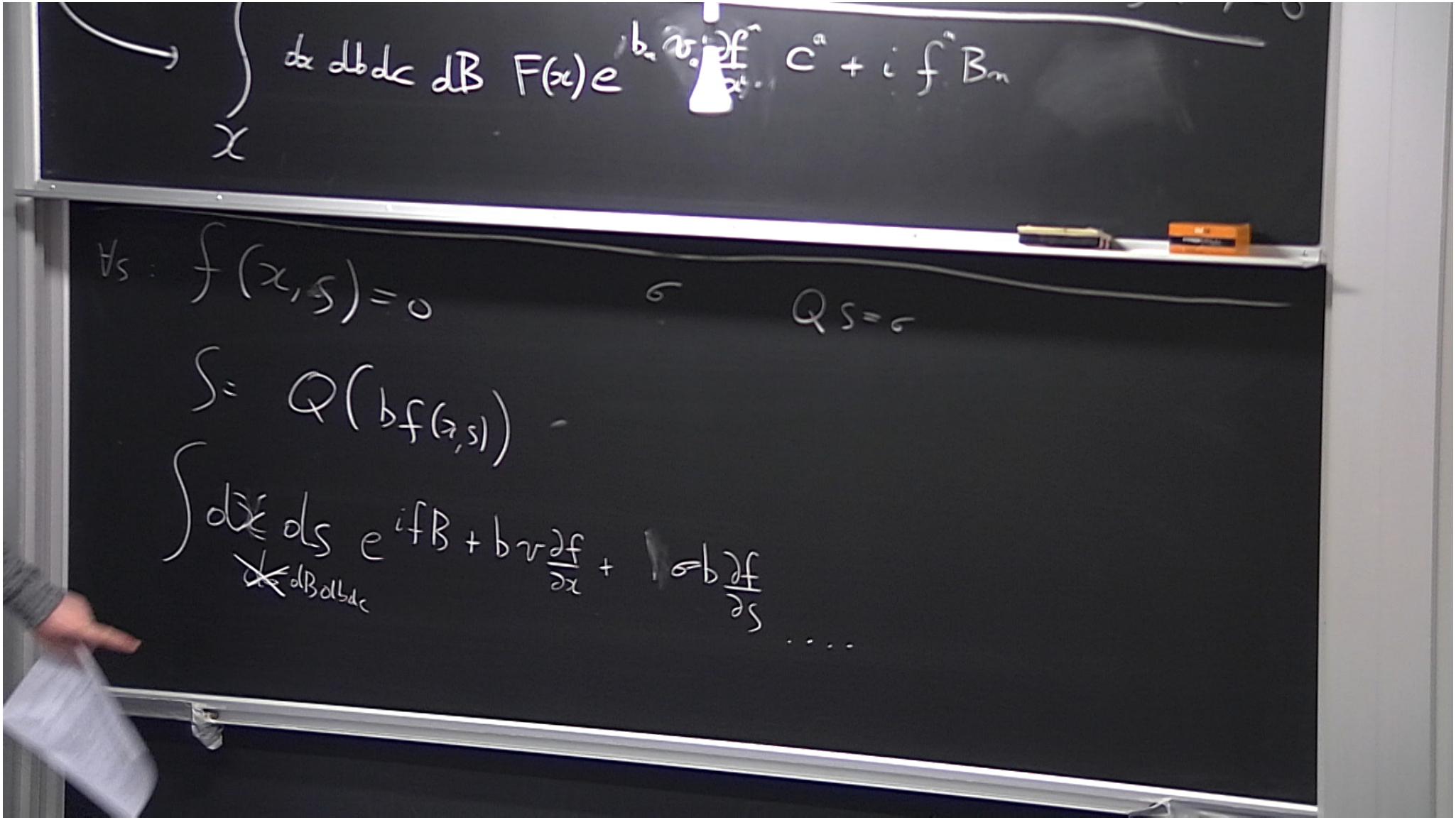

$\int_X dx\, db\, dc\, dB\, F(x) e^{i b_a v_i^a \frac{\partial f^a}{\partial x^i} c^a + i f^a B_a}$
$Q x^i = c^a v_a^i$
$Q c^a = f^a_{bc} c^b c^c$
$Q b_a = i B_a$
$Q B_a = 0$
$\forall s : f(x, s) = 0$

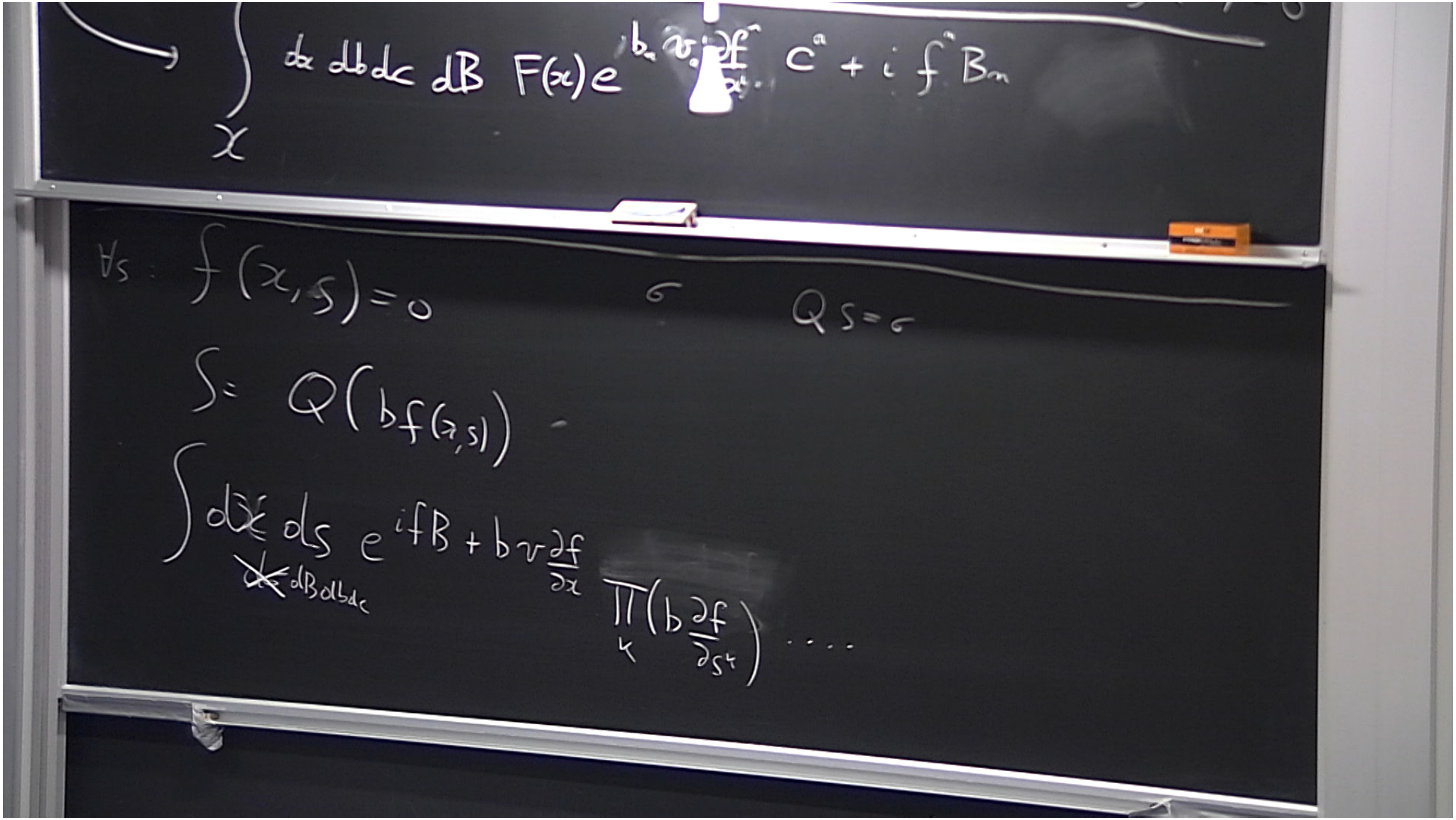

dx db dc dB F(x) e
c^a + i f^ B_m
X
Q(bf(a,s))
Qs=

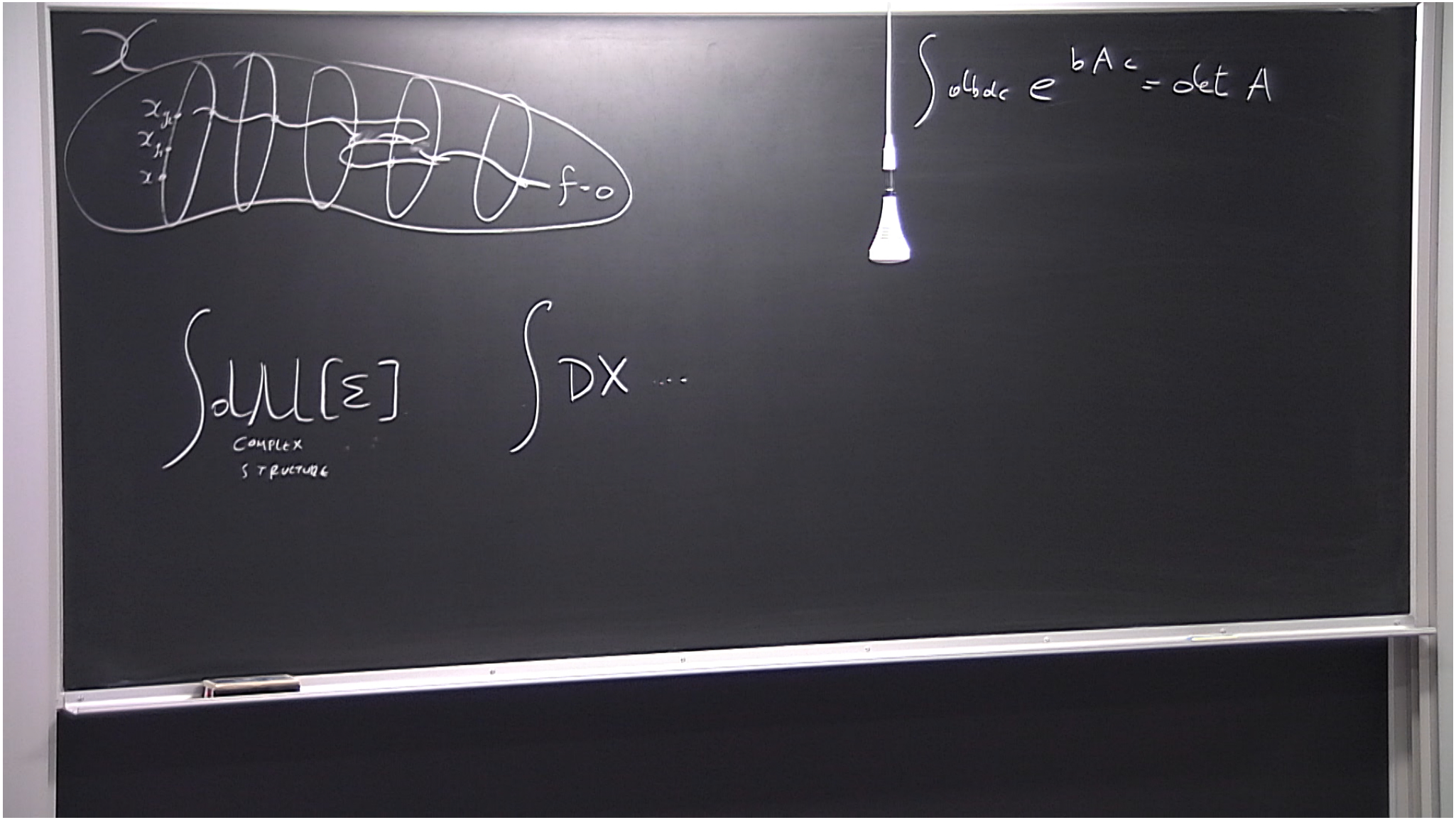

$\int_X dx\, db\, dc\, dB\, F(x) e^{i b_a \partial_\mu \frac{\partial f^a}{\partial x^\mu} c^a + i f^a B_a}$
$\forall s: f(x,s) = 0$
$\sigma$
$Qs = \sigma$
$S = Q(b f(x,s))$
$\int dx\, ds\, e^{i f B}$

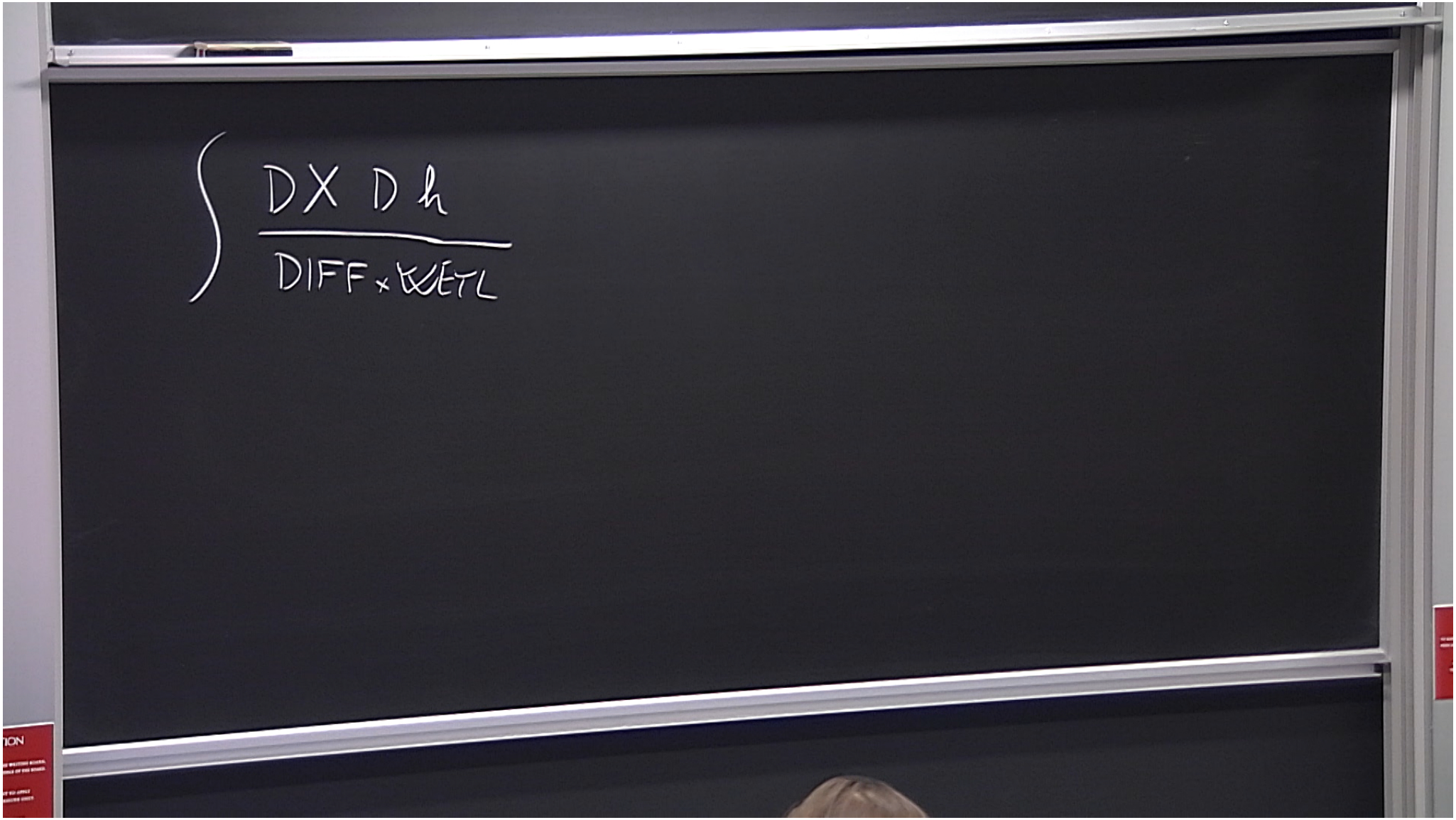

$\int dx\, db\, dc\, dB\, F(x) e^{\dots} c^a + i f^a B_m$
$\forall s : f(x,s) = 0$
$\sigma$
$Qs = \sigma$
$s = Q(bf(x,s))$
$\int dx\, ds\, e^{ifB + b v \frac{\partial f}{\partial x} + \dots \sigma b \frac{\partial f}{\partial s}}$

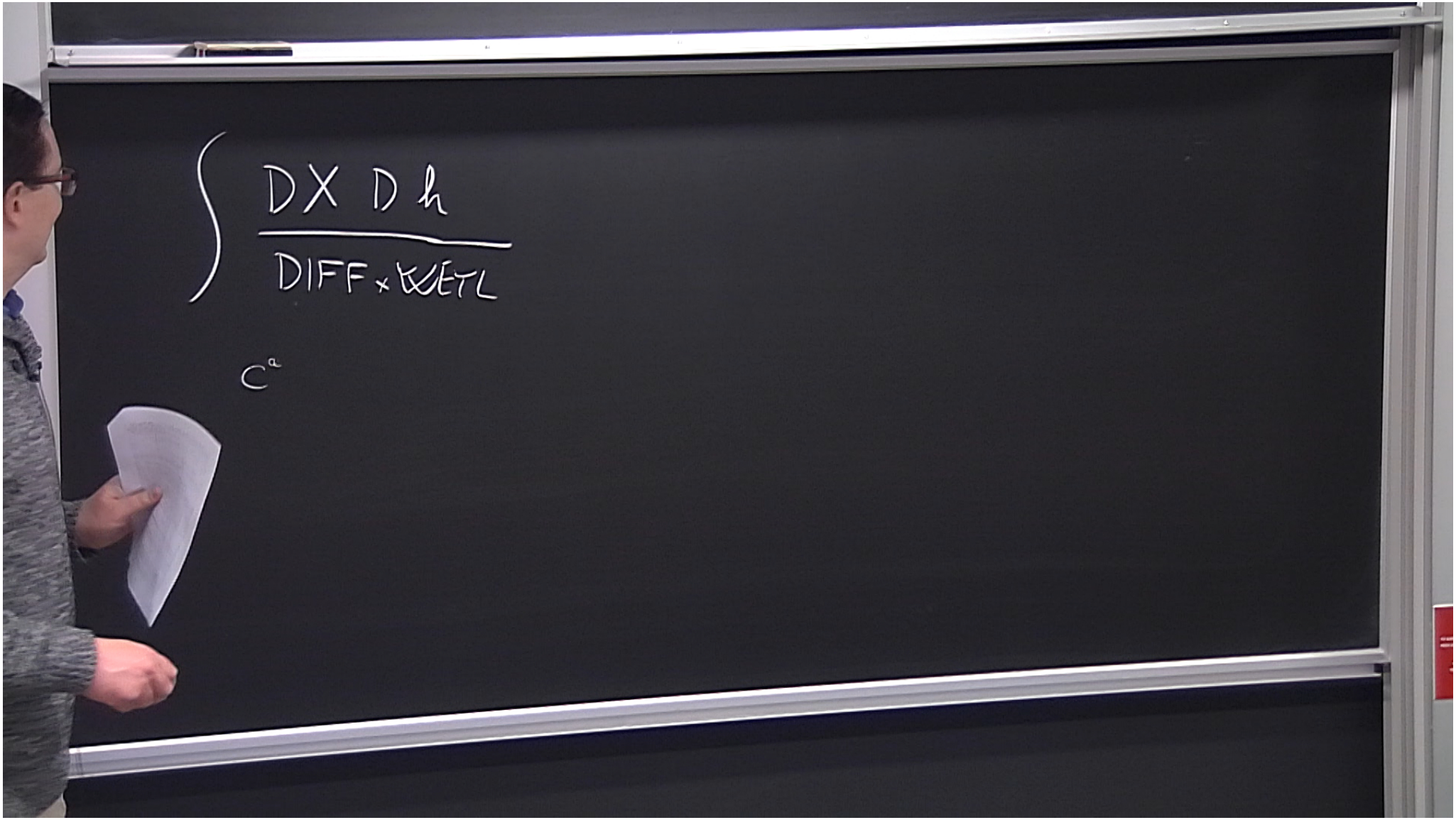

$\int_X dx\, db\, dc\, dB\, F(x) e^{i b_a v_i \frac{\partial f^a}{\partial x^i} c^a + i f^a B_a}$
$\forall s : f(x,s) = 0$
$\sigma$
$Qs = \sigma$
$S = Q(b f(a,s))$
$\int \frac{dx\, ds}{d\sigma\, dB\, db\, dc} e^{i f B + b v \frac{\partial f}{\partial x} + \sigma b \frac{\partial f}{\partial s}} \ldots$

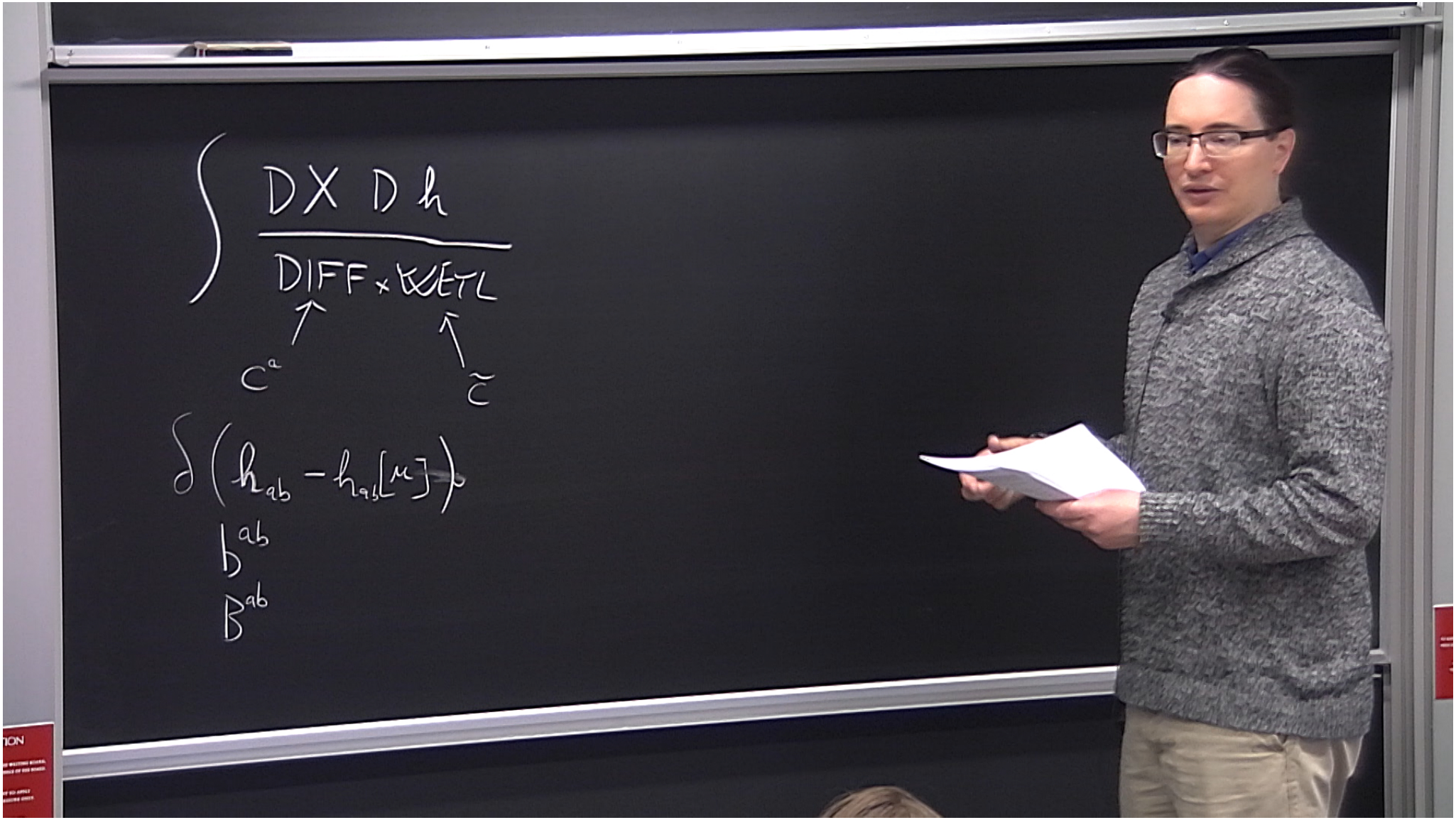

$\int dx\, db\, dc\, dB\, F(x) e^{i b_a v_i \frac{\partial f^a}{\partial x^i} c^a + i f^a B_a}$
$\forall s : f(x,s) = 0$
$\sigma$
$Qs = \sigma$
$S = Q(b f(x,s))$
$\int dx\, ds\, e^{i f B + b v \frac{\partial f}{\partial x} + \sigma b \frac{\partial f}{\partial s}} \ldots$
$dB db dc$

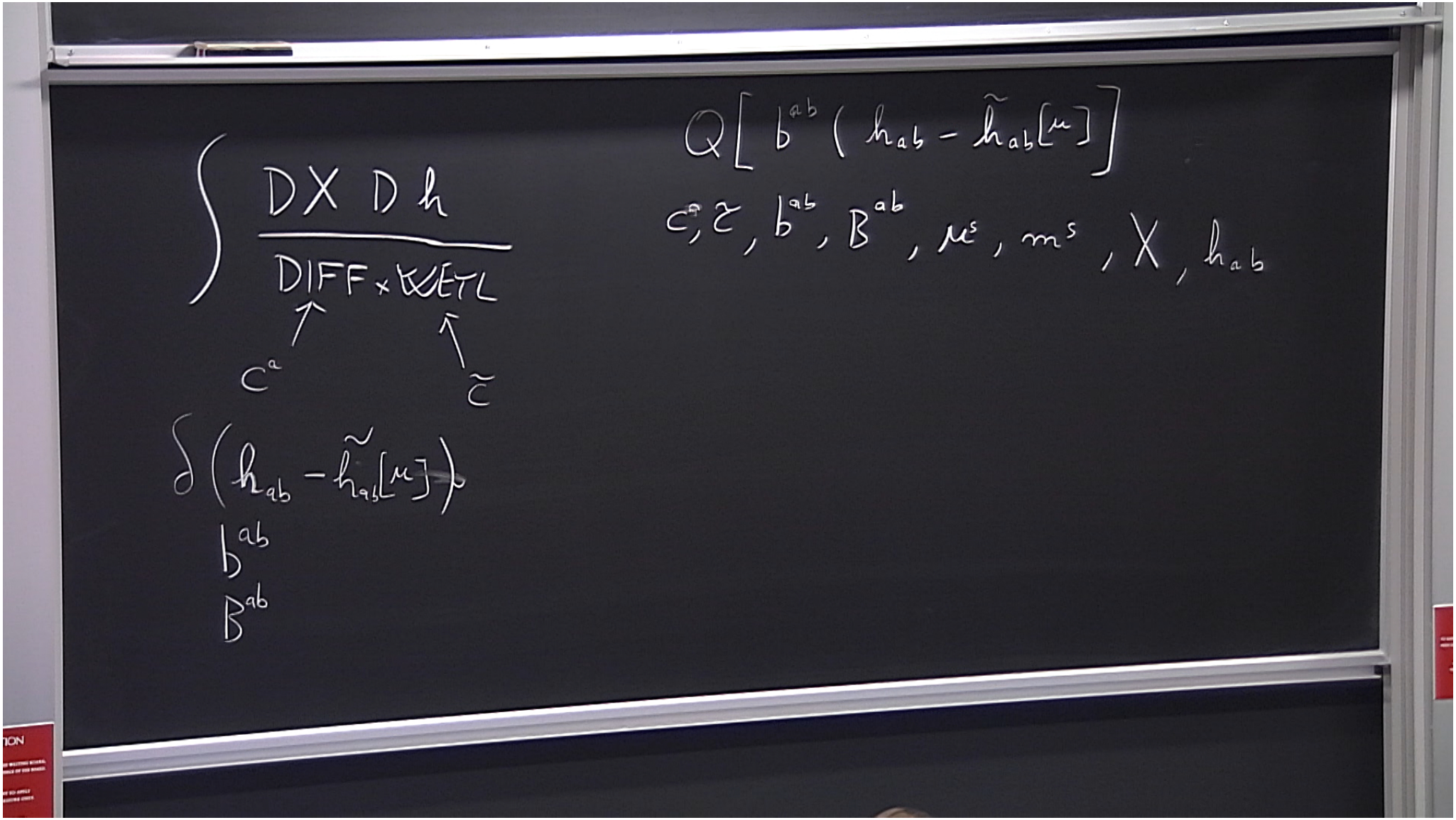

$\int_X dx\, db\, dc\, dB\, F(x) e^{b_a v_i \frac{\partial f^a}{\partial x^i} c^a + i f^a B_a}$
$\forall s : f(x,s) = 0$
$G \qquad Qs = 0$
$S = Q(b f(a,s))$
$\int dx\, ds\, e^{ifB + bv\frac{\partial f}{\partial x}} \prod_k \left(b \frac{\partial f}{\partial s_k}\right) \cdots$
$dB\, db\, dc$

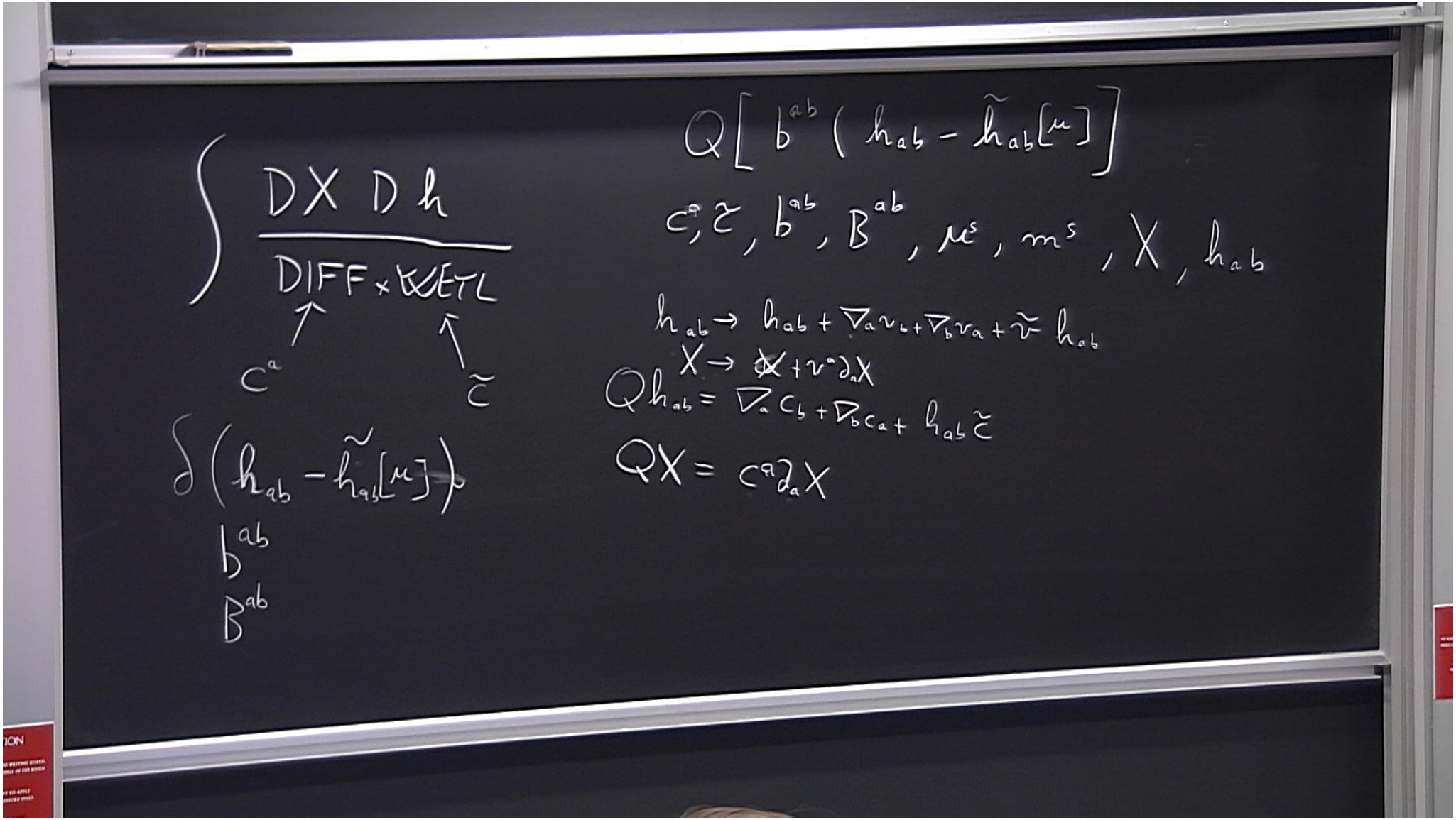

f-o
$\int d\psi_b d\psi_c \, e^{\psi_b A \psi_c} = \det A$
$\int d\mathcal{M}[\varepsilon]$
COMPLEX
STRUCTURE
$\int DX \ldots$

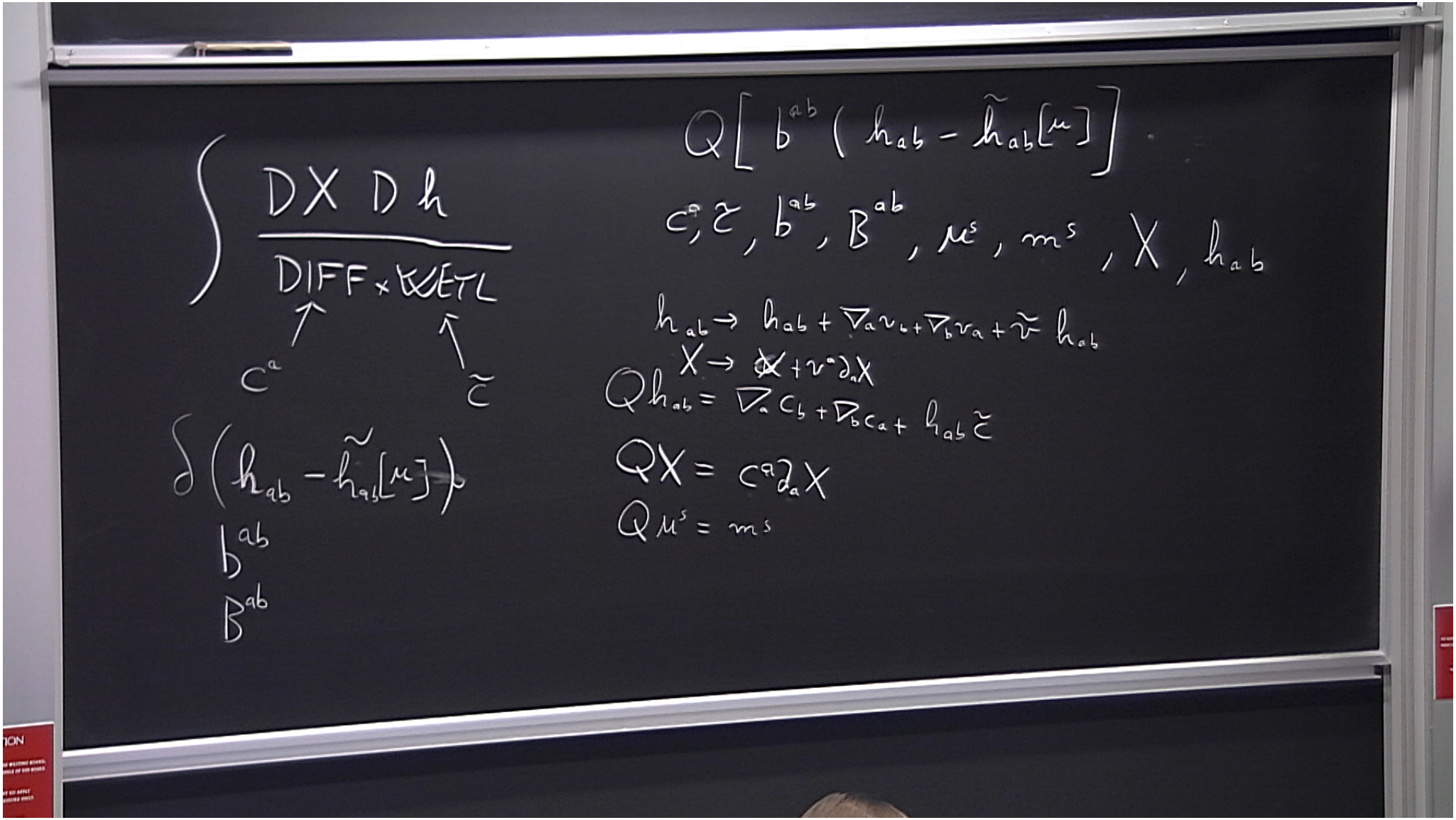

∫ DX Dh
DIFF × WETL


$\int \frac{DX\,Dh}{DIFF \times \mathrm{Vol}_{ETL}}$
$C^a$

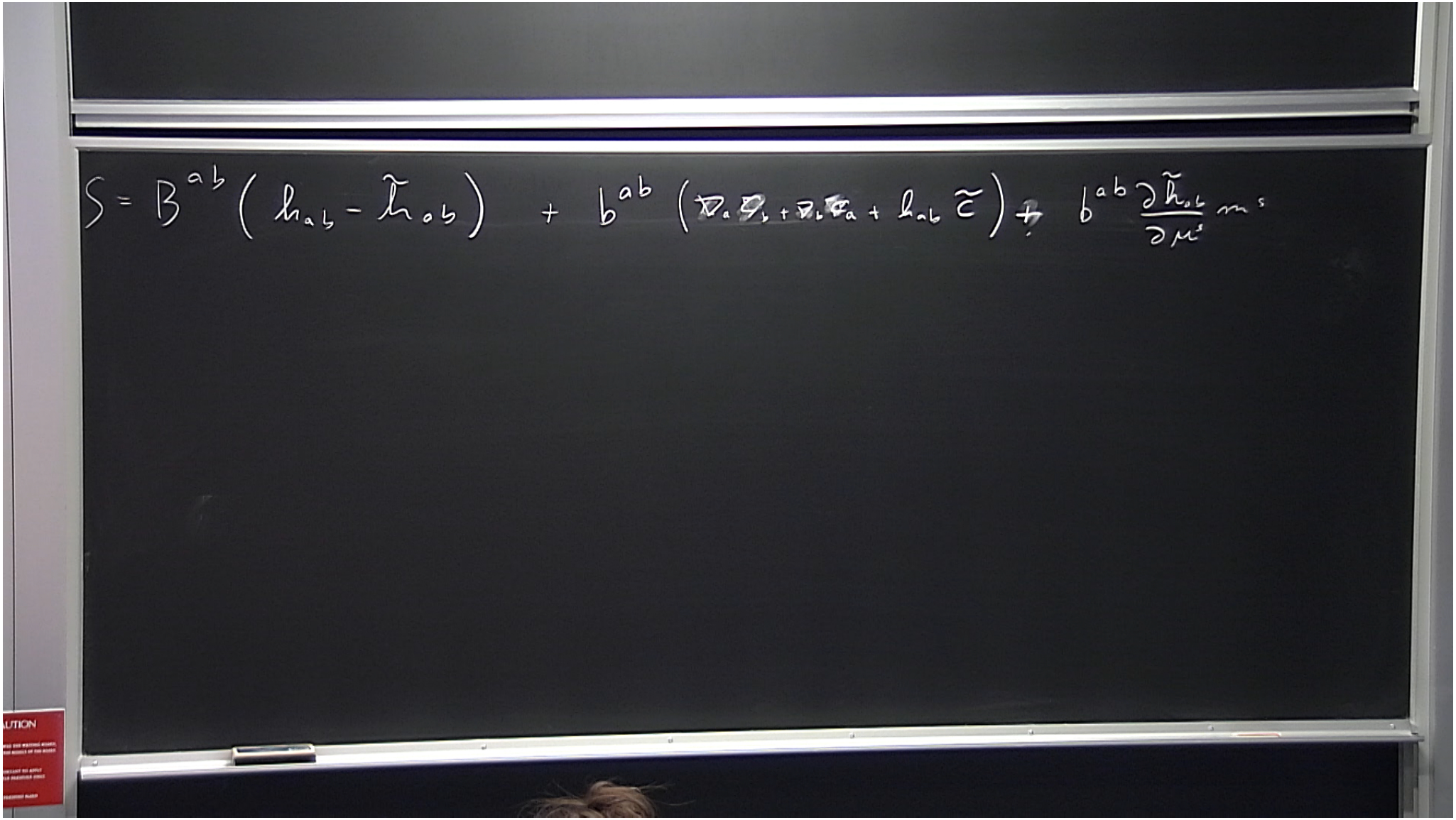

$\int \frac{DX\, Dh}{DIFF \times \mathcal{WETL}}$
$c^a$
$\tilde{c}$
$\delta\left(h_{ab} - h_{ab}[\mu]\right)$
$b^{ab}$
$B^{ab}$

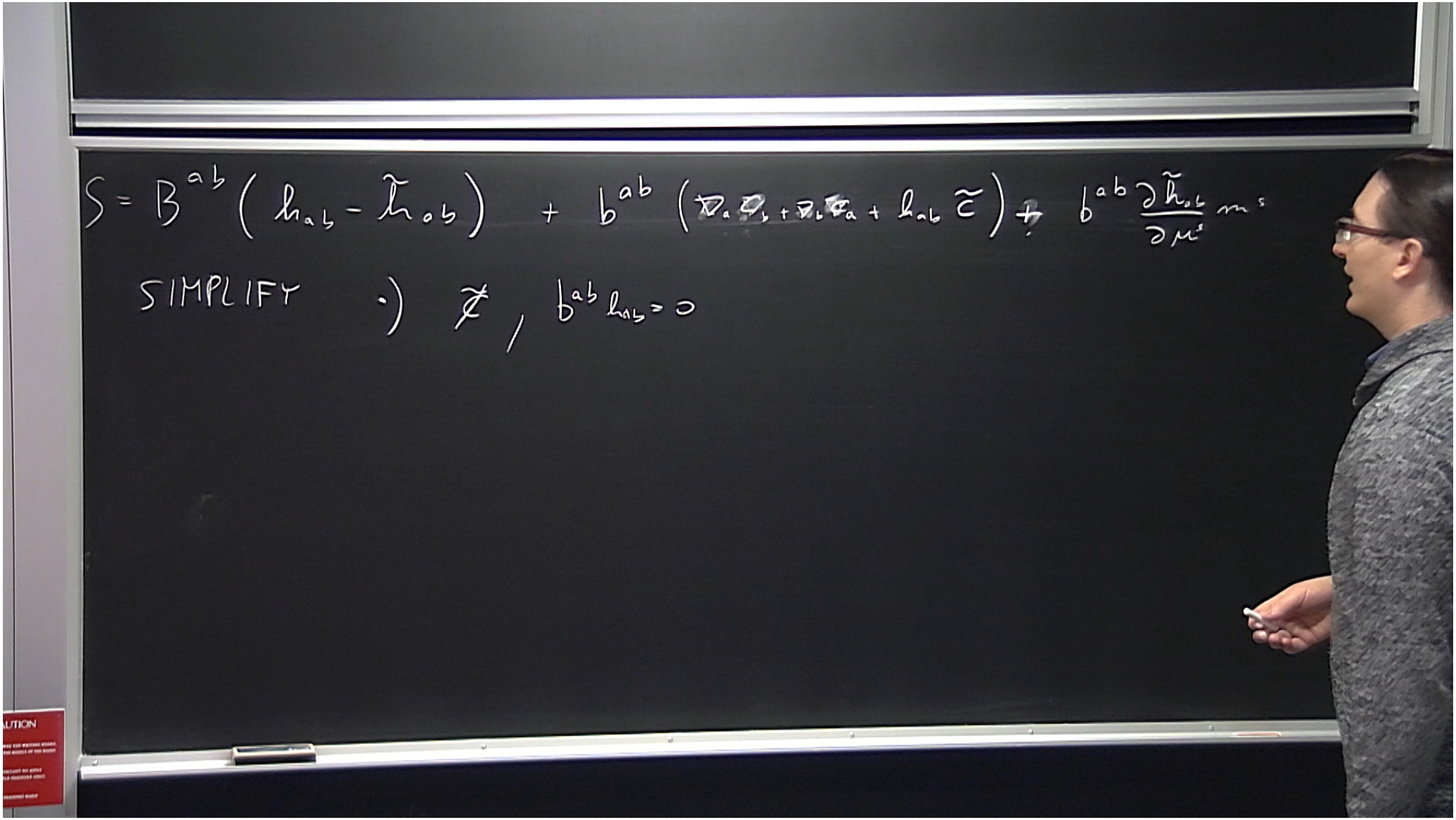

$\int \frac{DX\,Dh}{DIFF \times \cancel{WEYL}}$
$c^a$
$\tilde{c}$
$\delta\left(h_{ab} - \tilde{h}_{ab}[\mu]\right)$
$b^{ab}$
$B^{ab}$
$Q\left[b^{ab}\left(h_{ab} - \tilde{h}_{ab}[\mu]\right)\right]$
$c^a, \tilde{c}, b^{ab}, B^{ab}, \mu^s, m^s, X, h_{ab}$

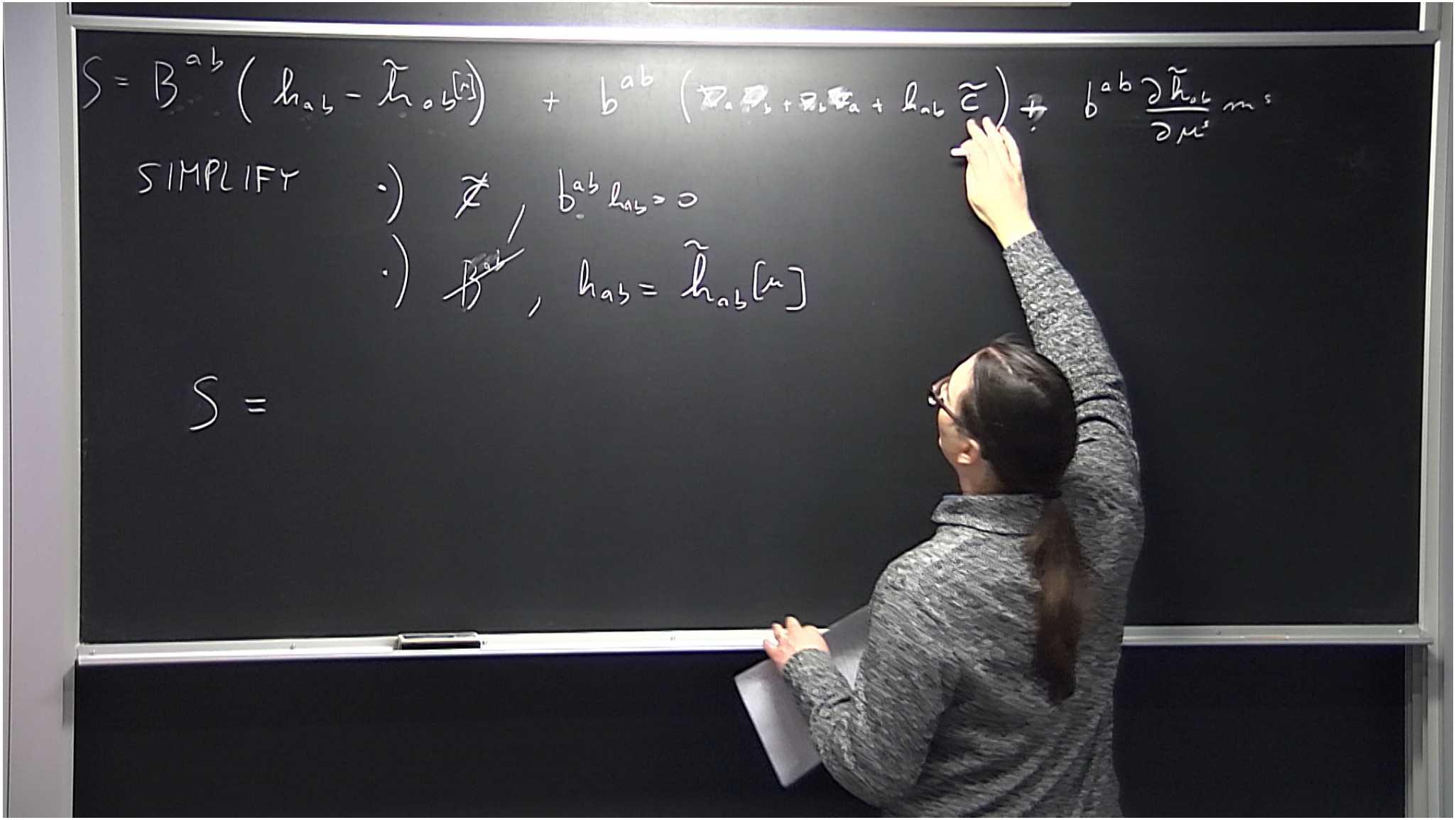

$\int \frac{DX\,Dh}{DIFF \times \cancel{WEYL}}$
$c^a$
$\tilde{c}$
$\delta\left(h_{ab} - \tilde{h}_{ab}[\mu]\right)$
$b^{ab}$
$B^{ab}$
$Q\left[b^{ab}\left(h_{ab} - \tilde{h}_{ab}[\mu]\right)\right]$
$c^a, \tilde{c}, b^{ab}, B^{ab}, \mu^s, m^s, X, h_{ab}$
$h_{ab} \to h_{ab} + \nabla_a v_b + \nabla_b v_a + \tilde{v}\, h_{ab}$
$X \to \cancel{X} + v^a \partial_a X$
$Q h_{ab} = \nabla_a c_b + \nabla_b c_a + h_{ab}\tilde{c}$
$QX = c^a \partial_a X$


$\int \frac{DX\,Dh}{DIFF \times \cancel{DIFF}_{ETL}}$
$c^a$
$\tilde{c}$
$\delta\left(h_{ab} - \tilde{h}_{ab}[\mu]\right)$
$b^{ab}$
$B^{ab}$
$Q\left[\, b^{ab}\left(h_{ab} - \tilde{h}_{ab}[\mu]\right)\right]$
$c^a, \tilde{c}, b^{ab}, B^{ab}, \mu^s, m^s, X, h_{ab}$
$h_{ab} \to h_{ab} + \nabla_a v_b + \nabla_b v_a + \tilde{v}\, h_{ab}$
$X \to \cancel{X} + v^a \partial_a X$
$Q h_{ab} = \nabla_a c_b + \nabla_b c_a + h_{ab}\tilde{c}$
$QX = c^a \partial_a X$
$Q\mu^s = m^s$

$$B^{ab}\left(h_{ab}-\tilde{h}_{ab}\right) + b^{ab}\left(c_a\nabla_b + c_b\nabla_a + h_{ab}\tilde{c}\right) + b^{ab}\frac{\partial\tilde{h}_{ab}}{\partial\mu^s}m^s$$


$S = B^{ab}(h_{ab} - \tilde{h}_{ab}) + b^{ab}(\nabla_a \varphi_b + \nabla_b \varphi_a + h_{ab}\tilde{c}) + b^{ab}\frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s$


$S = B^{ab}(h_{ab} - \tilde{h}_{ab}) + b^{ab}(\nabla_a \sigma_b + \nabla_b \sigma_a + h_{ab}\tilde{c}) + b^{ab}\frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s$
SIMPLIFY
$b^{ab} h_{ab} = 0$


$S = B^{ab}\left(h_{ab} - \tilde{h}_{ab}[\mu]\right) + b^{ab}\left(\nabla_a \xi_b + \nabla_b \xi_a + h_{ab}\tilde{c}\right) + b^{ab}\frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s$
SIMPLIFY
·) $\xi$, $b^{ab}h_{ab} = 0$
·) $B^{ab}$, $h_{ab} = \tilde{h}_{ab}[\mu]$
$S =$

$$S = B^{ab}\left(h_{ab} - \tilde{h}_{ab}[h]\right) + b^{ab}\left(\nabla_a \xi_b + \nabla_b \xi_a + h_{ab}\tilde{c}\right) + b^{ab}\frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s$$

SIMPLIFY

·) $\xi$, $b^{ab} h_{ab} = 0$

·) ~~$B^{ab}$~~, $h_{ab} = \tilde{h}_{ab}[h]$

$$S = \left(b^{ab}\nabla_a c_b + b^{ab}\frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s + \partial_a X^\mu \partial_b X^\nu \tilde{h}^{ab}\right)\sqrt{g}$$

$$S = B^{ab}\left(h_{ab} - \tilde{h}_{ab}[h]\right) + b^{ab}\left(\nabla_a C_b + \nabla_b C_a + h_{ab}\tilde{c}\right) + b^{ab}\frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s$$

SIMPLIFY

·) $\tilde{c}$, $b^{ab} h_{ab} = 0$

·) ~~$B^{ab}$~~, $h_{ab} = \tilde{h}_{ab}[\mu] \Rightarrow Q b^{ab} = T_{\text{extra}}{}^{ab}$

$$S = \left(b^{ab}\nabla_a C_b + b^{ab}\frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s + \partial_a X^\mu \partial_b X^\nu \tilde{h}^{ab}\right)\sqrt{h}$$

$$S = B^{ab}\left(h_{ab} - \tilde{h}_{ab}[\mu]\right) + b^{ab}\left(\nabla_a c_b + \nabla_b c_a + h_{ab}\tilde{c}\right) + b^{ab}\frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s$$

SIMPLIFY

·) $\tilde{c}$, $b^{ab}h_{ab} = 0$ $\qquad QX = c^a \partial_a X$

·) ~~$B^{ab}$~~, $h_{ab} = \tilde{h}_{ab}[\mu] \Rightarrow$ $\qquad Q b^{ab} = T^{ab}$

$Q c^a = c^b \partial_b c^a$

$$S = \left(b^{ab}\nabla_a c_b + b^{ab}\frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s + \partial_a X^\mu \partial_b X^\nu \tilde{h}^{ab}\right)\sqrt{g}$$

$$S = B^{ab}\left(h_{ab} - \tilde{h}_{ab}[\mu]\right) + b^{ab}\left(\nabla_a c_b + \nabla_b c_a + h_{ab}\tilde{c}\right) + b^{ab}\frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s$$

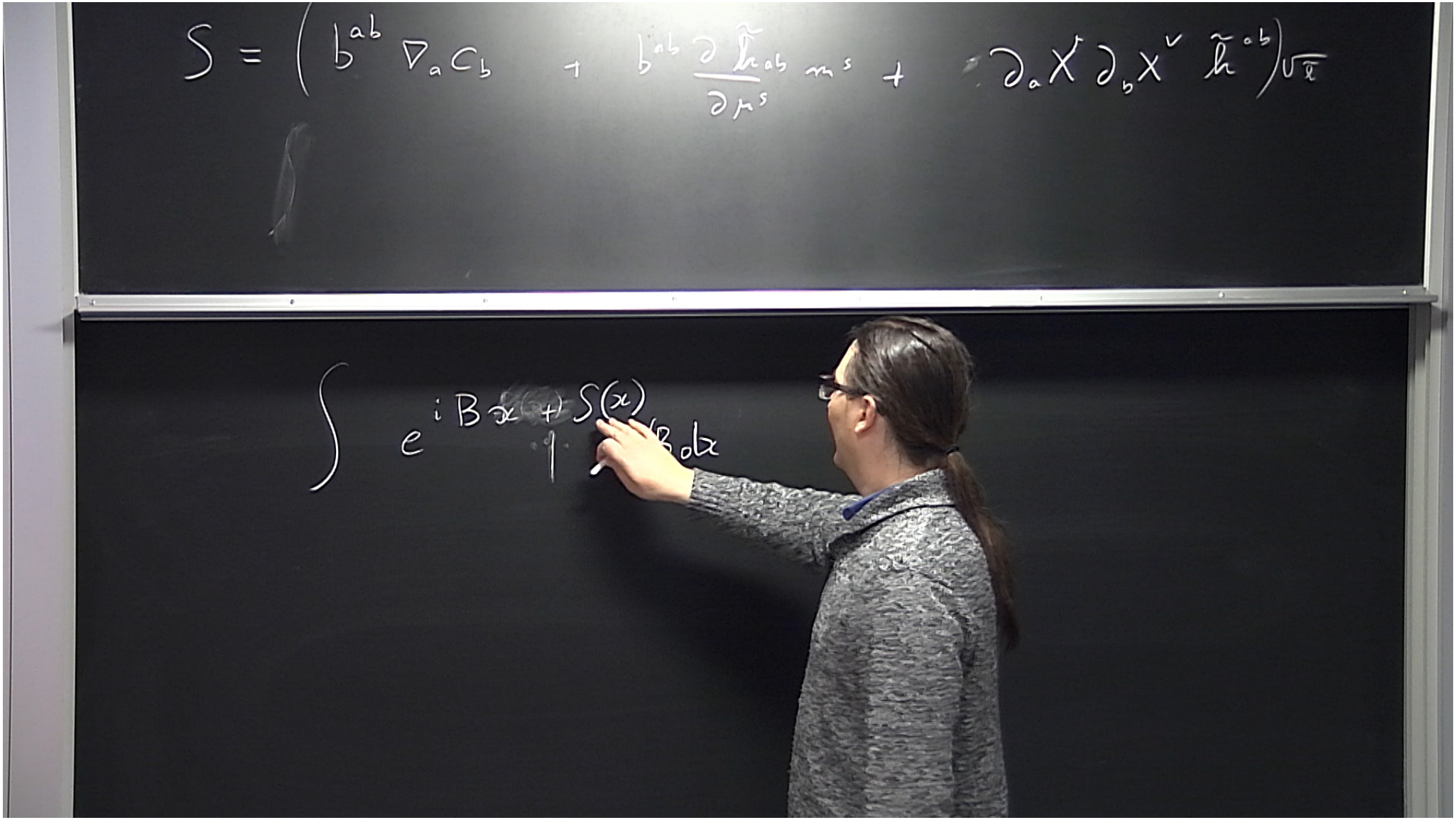
SIMPLIFY

·) $\tilde{c}$, $b^{ab}h_{ab} = 0$ $\qquad QX = c^a \partial_a X$

·) ~~$B^{ab}$~~, $h_{ab} = \tilde{h}_{ab}[\mu] \Rightarrow Q b^{ab} = T_{tot}^{ab}$

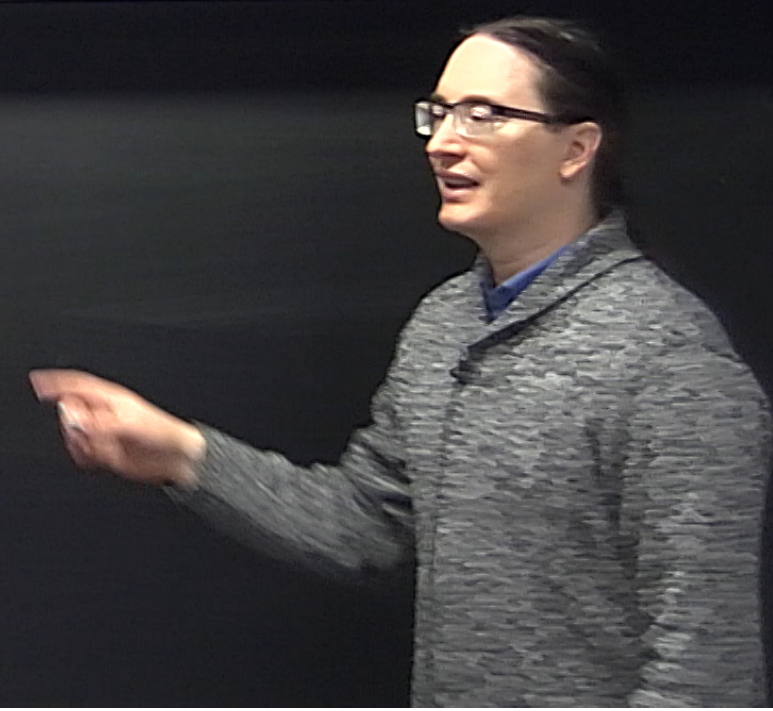
$Q c^a = c^b \partial_b c^a$ ---

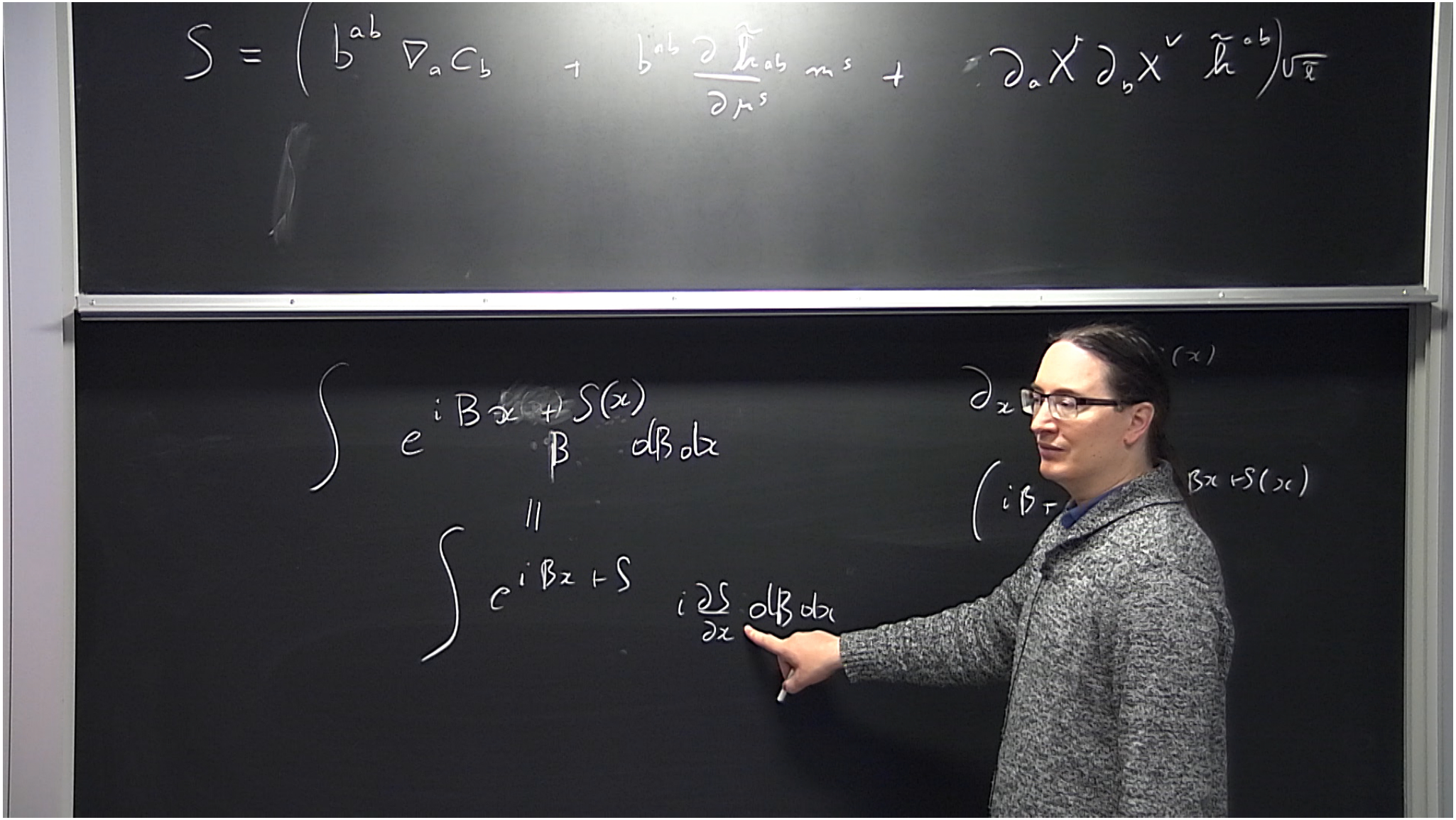
$$S = \left(b^{ab}\nabla_a c_b + b^{ab}\frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s + \partial_a X^\mu \partial_b X^\nu \tilde{h}^{ab}\right)\sqrt{g}$$

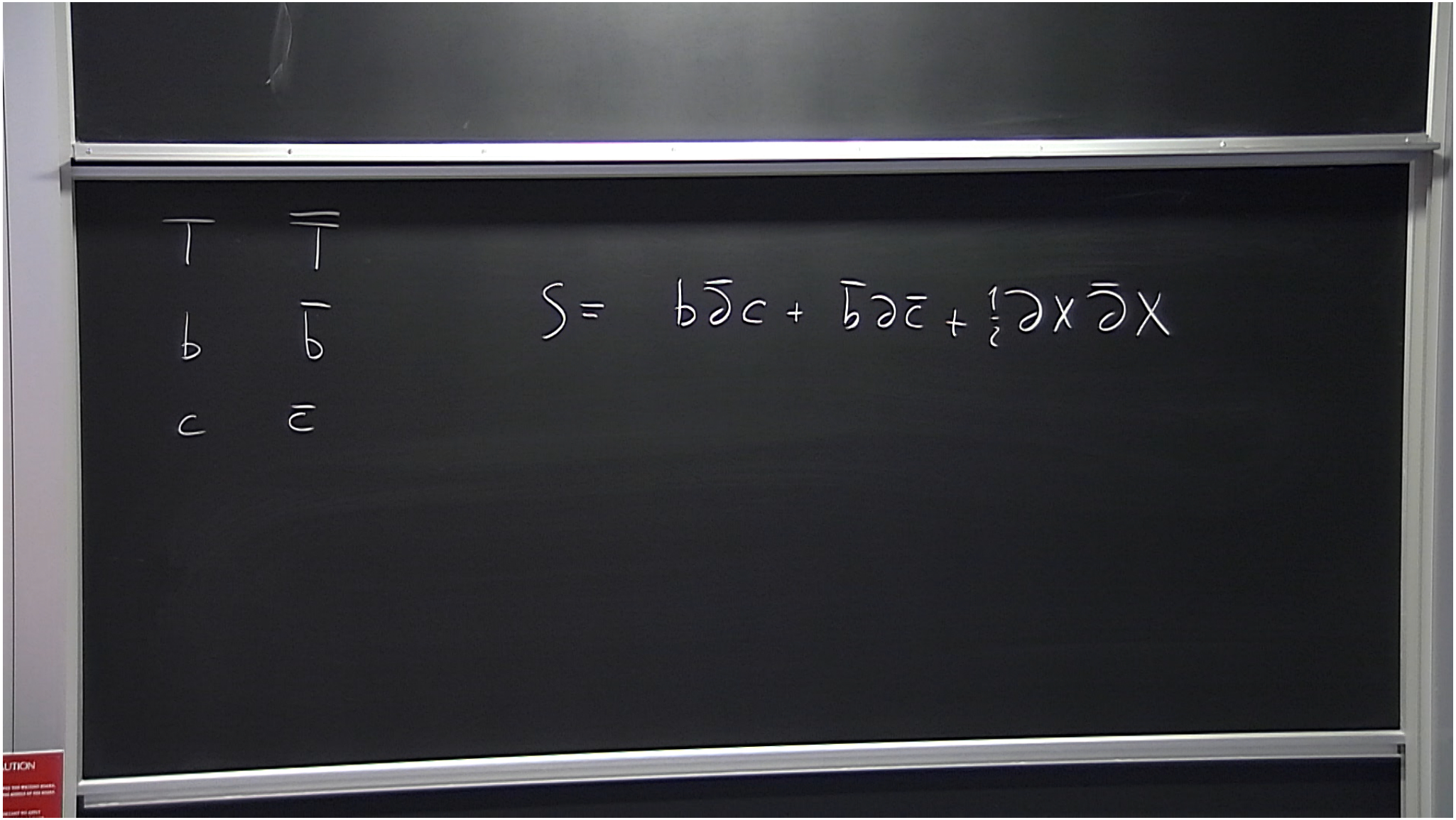

$S = \left( b^{ab} \nabla_a c_b + b^{ab} \frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s + \partial_a X^\mu \partial_b X^\nu \tilde{h}^{ab} \right) \sqrt{g}$
$\int e^{iB[x] + S(x)} \mathcal{D}x$

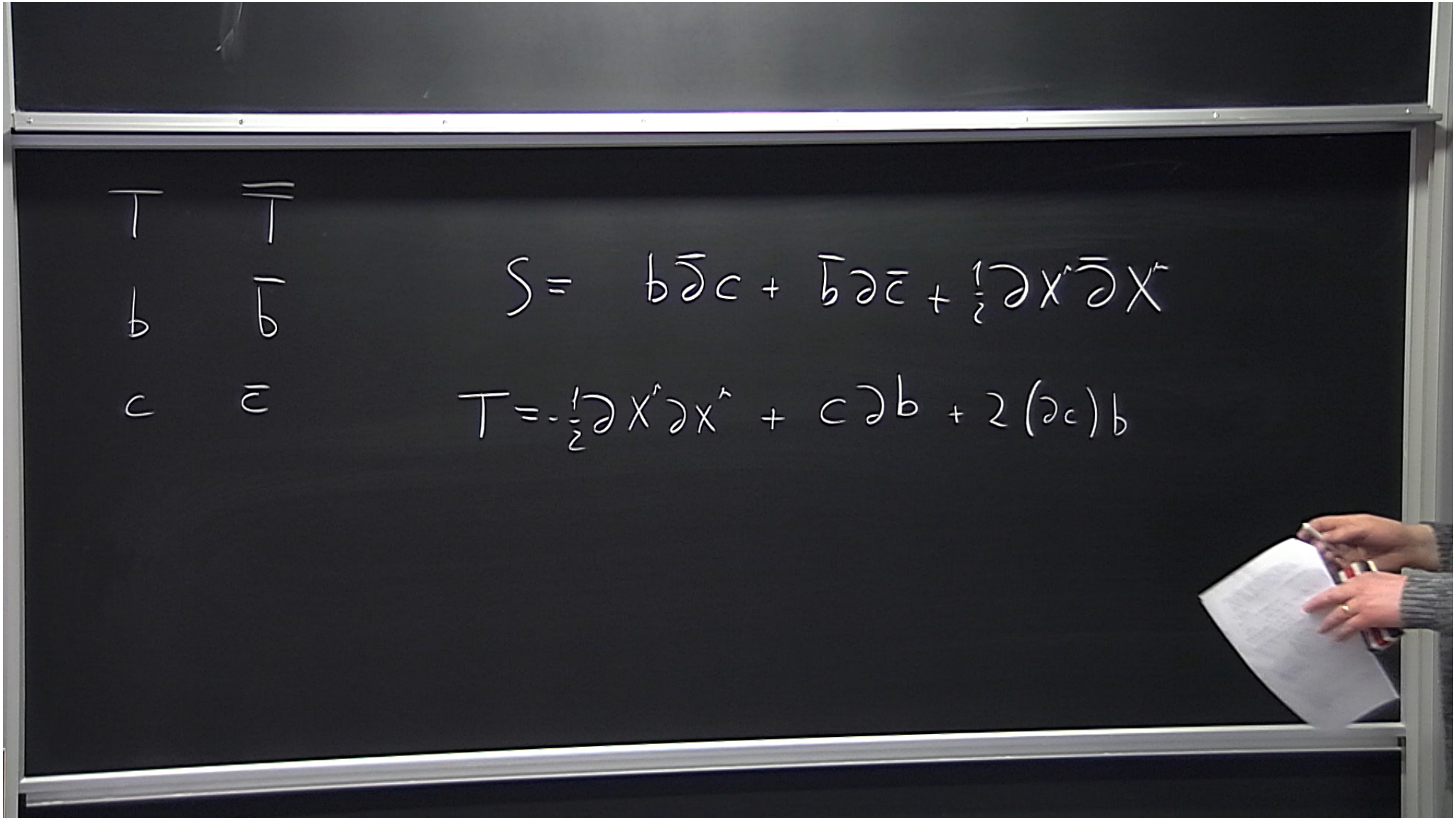
$$S = \left( b^{ab} \nabla_a c_b + b^{ab} \frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s + \partial_a X^\mu \partial_b X^\nu \tilde{h}^{ab} \right) \sqrt{g}$$

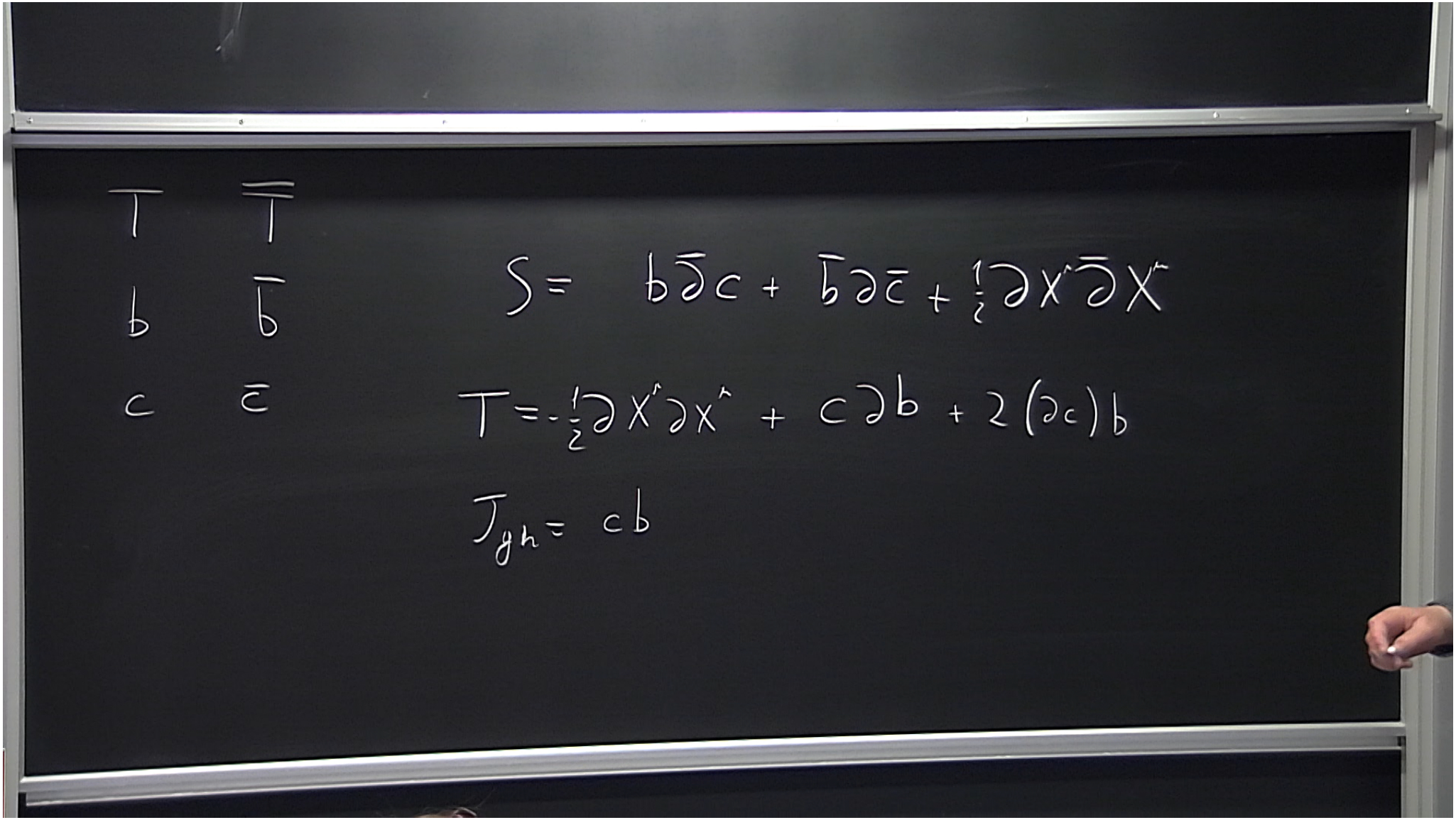
$$\int_\beta e^{i B \alpha + S(\alpha)} d\beta \, d\alpha$$

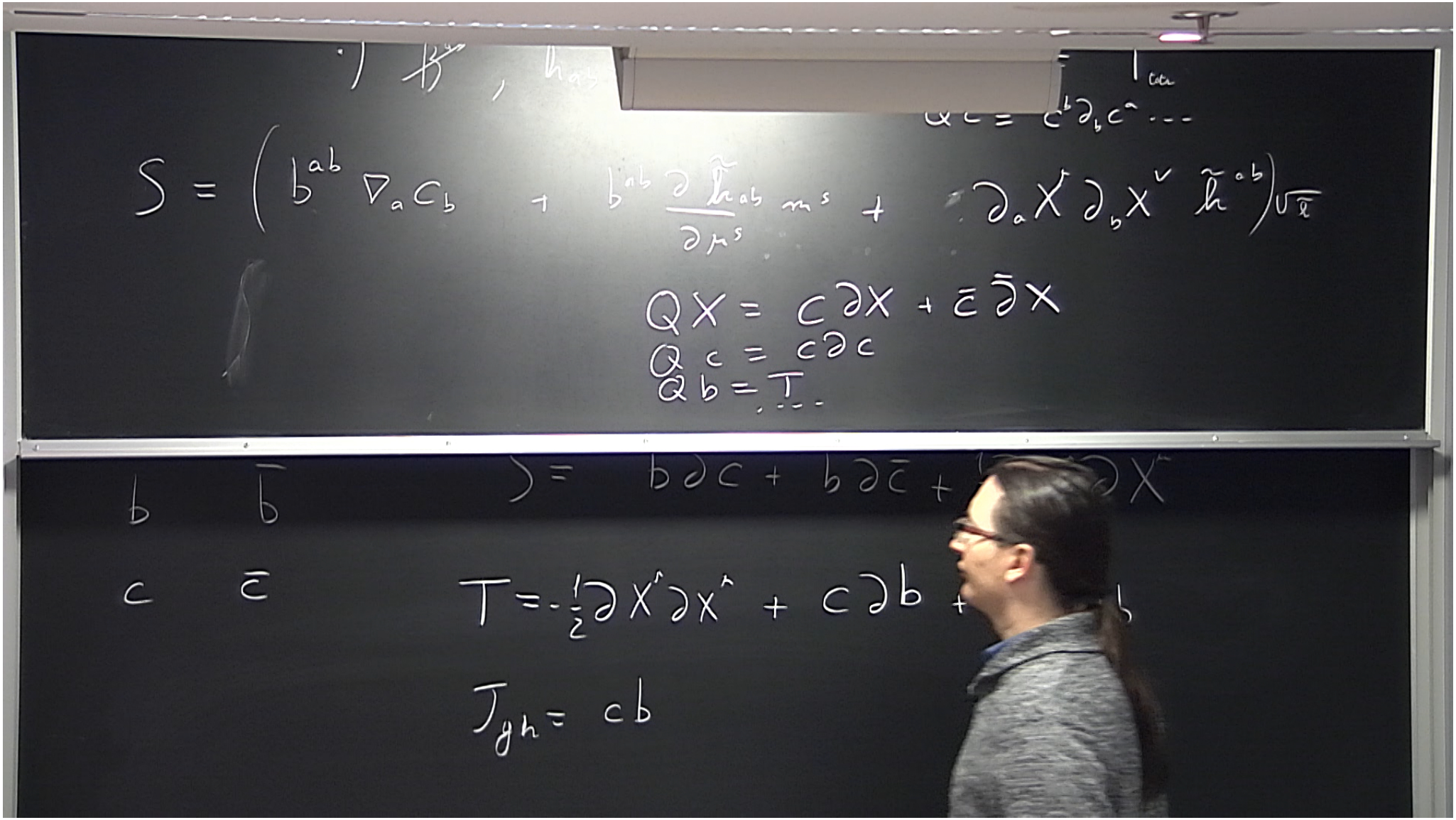



$S = \left( b^{ab} \nabla_a c_b + b^{ab} \frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s + \partial_a X^\mu \partial_b X^\nu \tilde{h}^{ab} \right) \sqrt{g}$
$\int e^{i B x + S(x)} \, dB \, dx$
$\int e^{i B x + S} \, i \frac{\partial S}{\partial x} \, dB \, dx$


T
$\bar{T}$
b
$\bar{b}$
c
$\bar{c}$
$S = b\bar{\partial}c + \bar{b}\partial\bar{c} + \frac{1}{2}\partial X \bar{\partial} X$


T T̄
b b̄
c c̄
S = ∫ b∂̄c + b̄∂c̄ + ½ ∂X^μ ∂̄X^μ
T = -½ ∂X^μ ∂X^μ + c∂b + 2(∂c)b


$T$ $\bar{T}$
$b$ $\bar{b}$
$c$ $\bar{c}$
$S = b\bar{\partial}c + \bar{b}\partial\bar{c} + \frac{1}{\pi}\partial X^{\mu}\bar{\partial}X^{\mu}$
$T = -\frac{1}{2}\partial X^{\mu}\partial X^{\mu} + c\partial b + 2(\partial c)b$
$J_{gh} = cb$


$S = \left( b^{ab} \nabla_a c_b + b^{ab} \frac{\partial \tilde{h}_{ab}}{\partial \rho^s} m^s + \partial_a X^\mu \partial_b X^\nu \tilde{h}^{ab} \right) \sqrt{g}$
$Q X = c \partial X + \bar{c} \bar{\partial} X$
$Q c = c \partial c$
$Q b = T$
$T = -\frac{1}{2} \partial X^\mu \partial X^\mu + c \partial b$
$J_{gh} = cb$

$Q\, c = c\partial c$

$Q\, b = T$

$T \quad \bar{T}$

$b \quad \bar{b}$

$c \quad \bar{c}$

$\partial(c\partial x \bar{\partial} x) + \partial x \partial x \bar{\partial} c \epsilon = \partial(c \partial x \bar{\partial} x \epsilon) + c.c$

$S = b\bar{\partial} c + \bar{b}\partial\bar{c} + \frac{1}{2}\partial X^{\mu}\bar{\partial} X^{\mu}$

$T = -\frac{1}{2}\partial X^{\mu}\partial X^{\mu} + c\partial b + 2(\partial c) b$

$J_{gh} = cb$

$Q_c = c\partial c$

$Q_b = T$

$T \quad \bar{T}$

$b = b^{ss} \quad \bar{b} = b^{\bar{s}\bar{s}}$

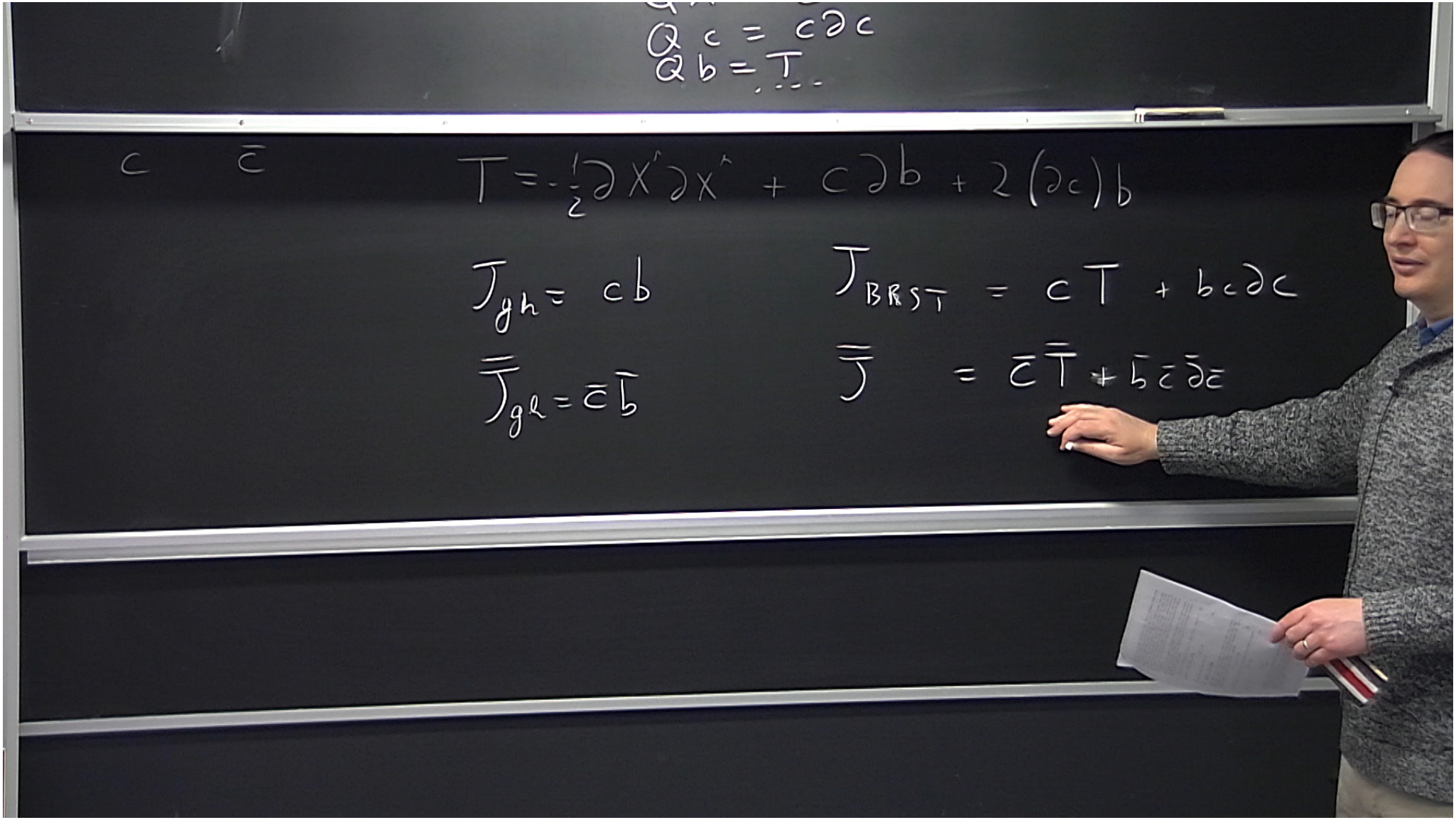
$c \quad \bar{c}$

$\partial(c\partial x\bar{\partial}x) + \partial x\partial x\bar{\partial}c + c.c = \partial(c\partial x + \bar{c}\bar{\partial}x)\bar{\partial}x + c.c$

$S = b\bar{\partial}c + \bar{b}\partial\bar{c} + \frac{1}{2}\partial x^{\mu}\bar{\partial}x^{\mu}$

$T = -\frac{1}{2}\partial x^{\mu}\partial x^{\mu} + c\partial b + 2(\partial c)b$

$J_{gh} = cb$


$Q_c = c\partial c$
$Qb = T$
$c \quad \bar{c}$
$T = -\frac{1}{2}\partial X^\mu \partial X_\mu + c\partial b + 2(\partial c)b$
$J_{gh} = cb$
$J_{BRST} = cT + bc\partial c$
$\bar{J}_{gh} = \bar{c}\bar{b}$
$\bar{J} = \bar{c}\bar{T} + \bar{b}\bar{c}\bar{\partial}\bar{c}$

$Q\, c = c\partial c$

$Q\, b = T$

$T \qquad \bar{T}$

$b = b^{ss} \qquad \bar{b} = b^{\bar{s}\bar{s}}$

$c \qquad \bar{c}$

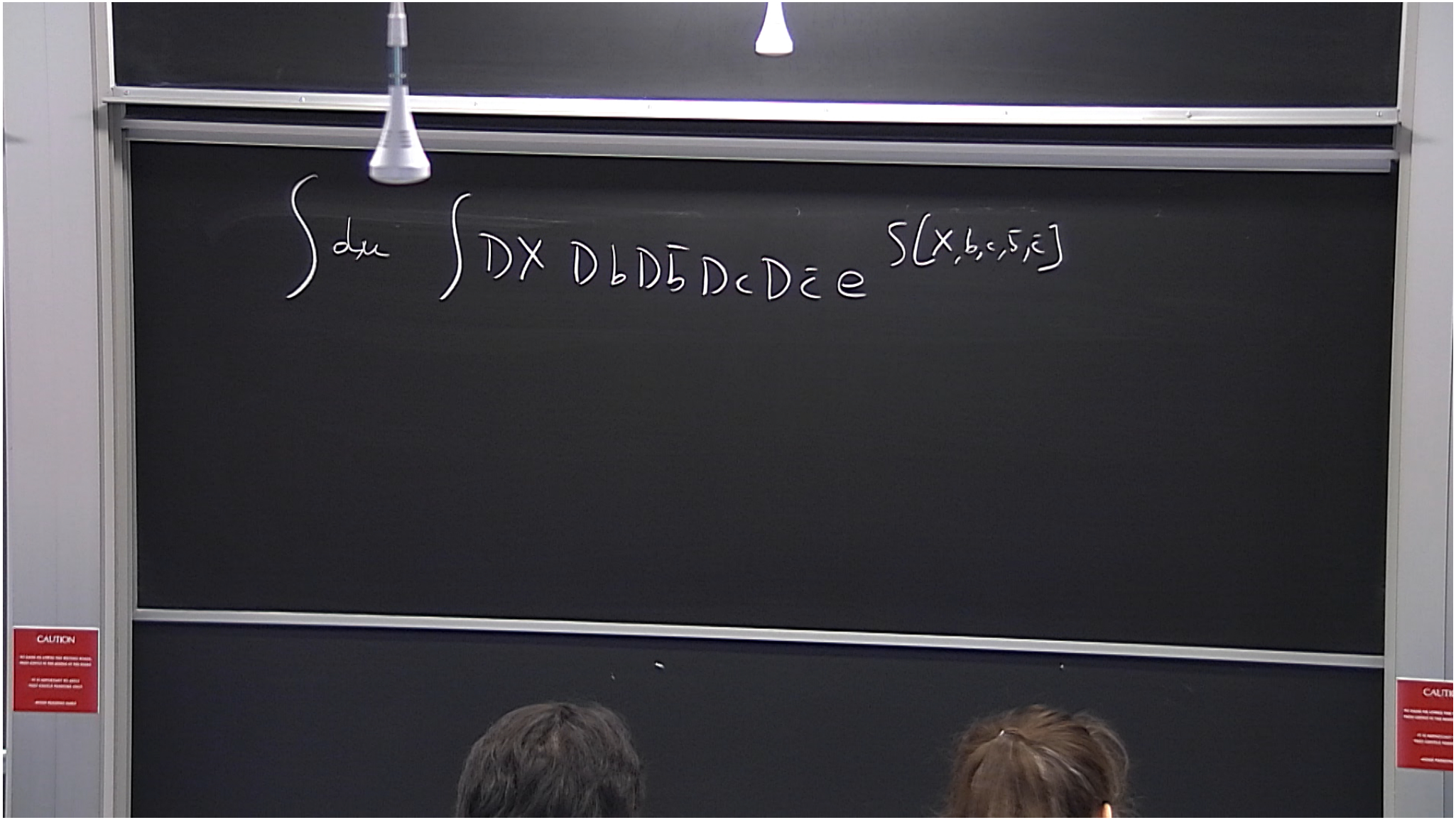
$\partial(c\partial x\bar{\partial}x) + \partial x\bar{\partial}x\partial c \underset{+c.c.}{=} \partial\left(c\partial x + c\bar{\partial}x\right)\bar{\partial}x + c.c$

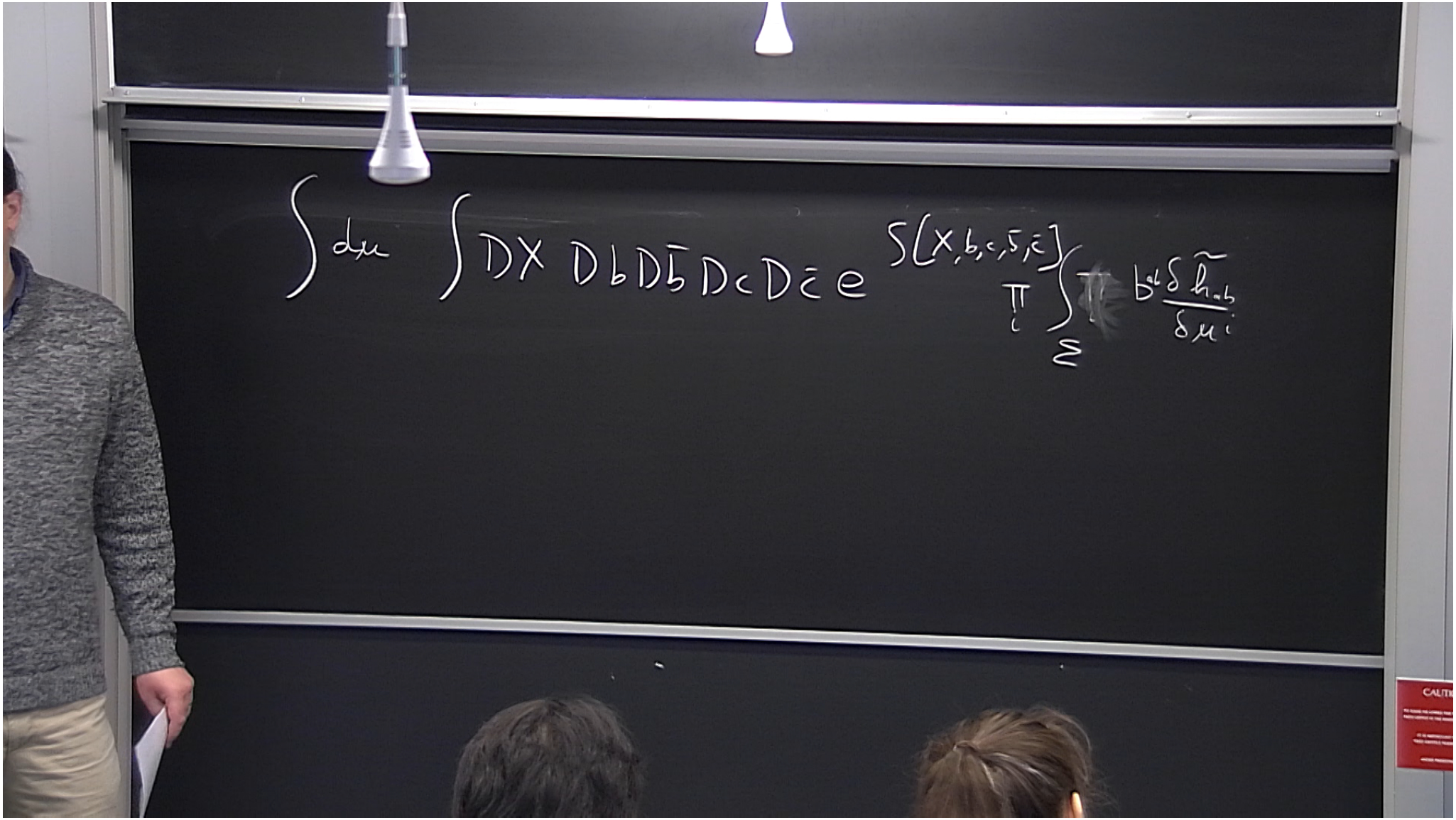
$S = b\bar{\partial}c + \bar{b}\partial\bar{c} + \frac{1}{2}\partial x^{\hat{\mu}}\bar{\partial}x^{\hat{\mu}}$

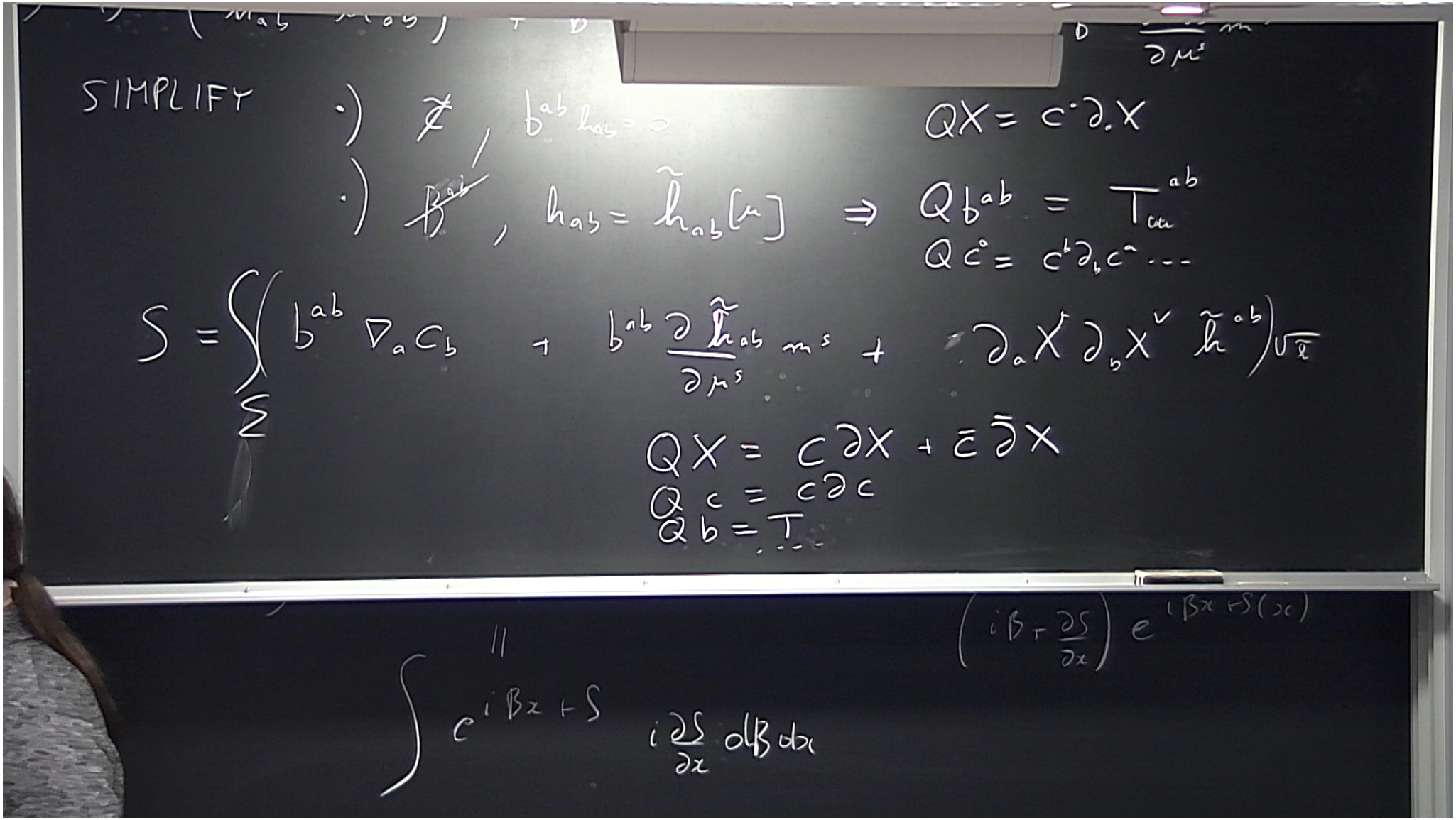
$T = -\frac{1}{2}\partial x^{\hat{\mu}}\partial x^{\hat{\mu}} + c\partial b + 2(\partial c)b$

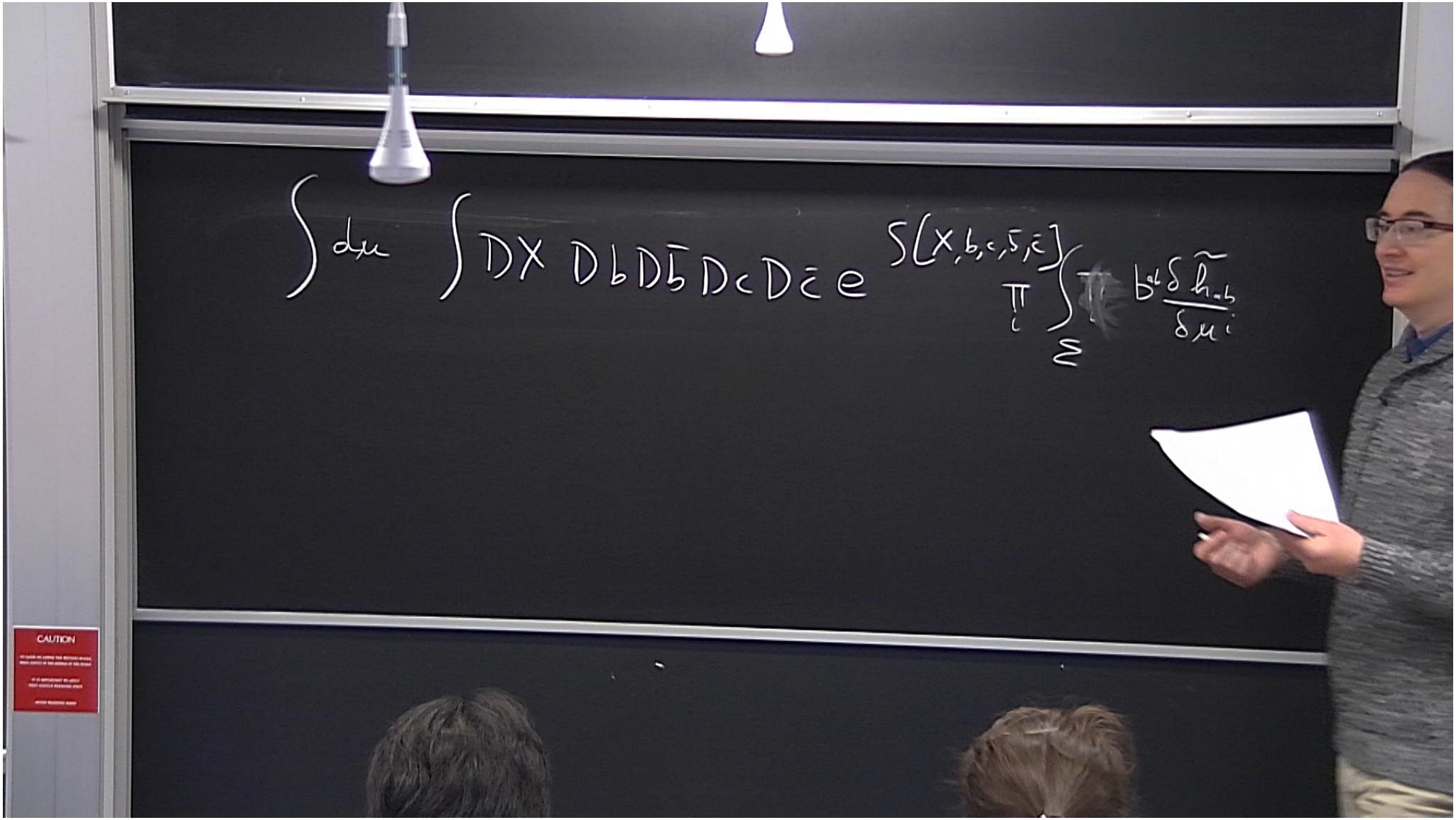
$J_{gh} = cb \qquad J_{BRST} = cT + bc\partial c$

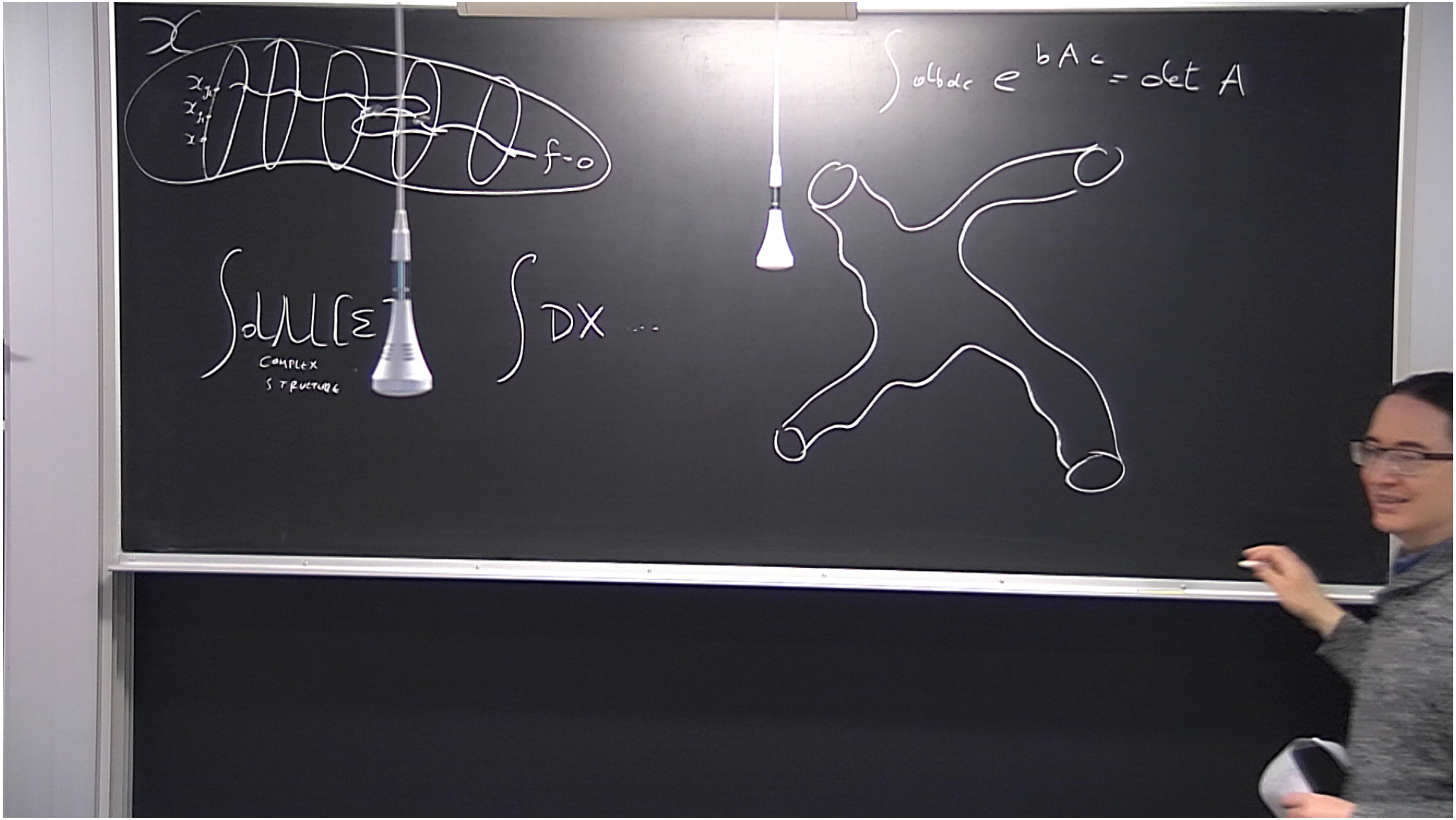
$\bar{J}_{gh} = \bar{c}\bar{b} \qquad \bar{J}_{BRST} = \bar{c}\bar{T} + \bar{b}\bar{c}\bar{\partial}\bar{c}$

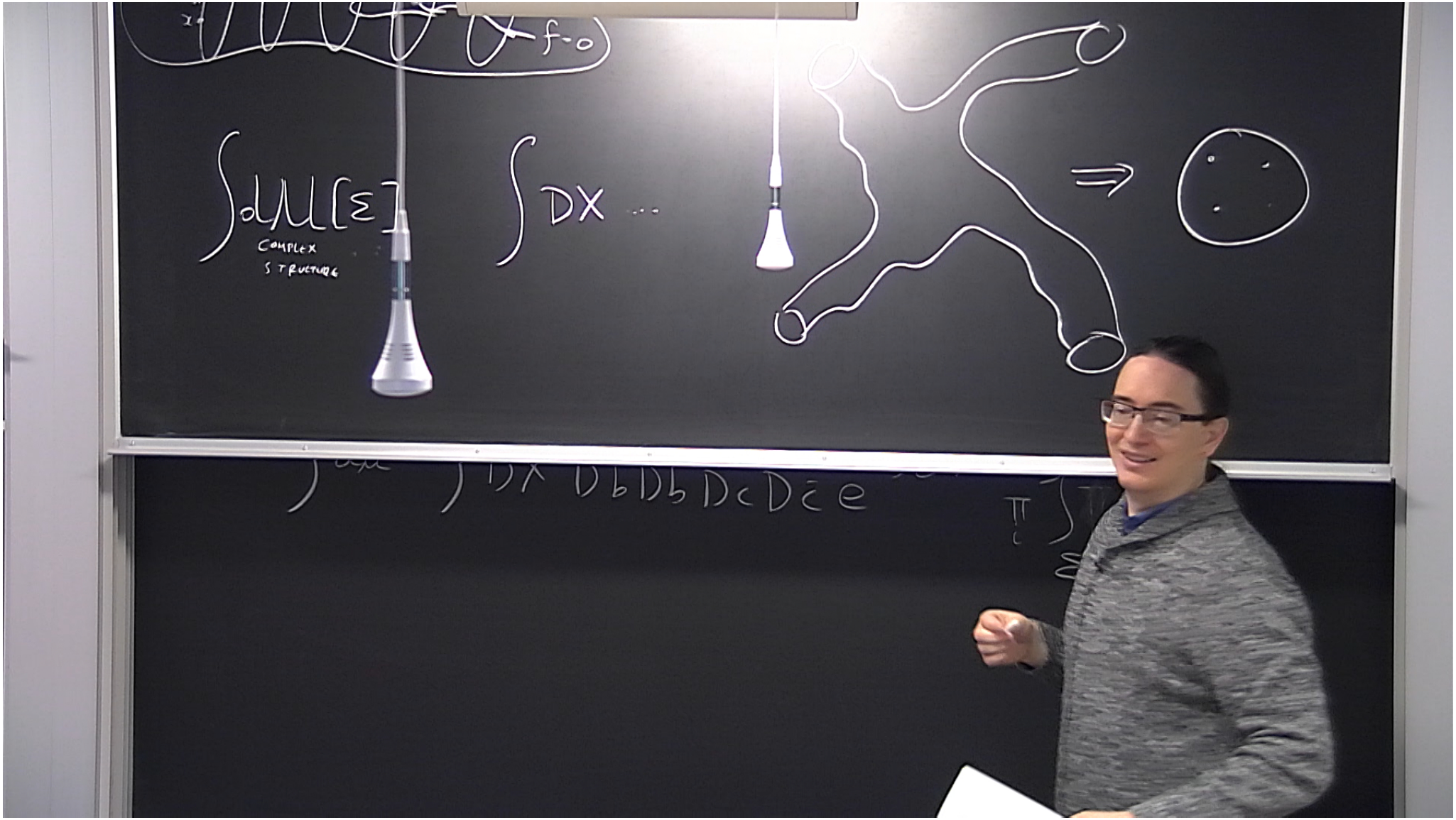

$\int d\mu \int DX\, Db\, D\bar{b}\, Dc\, D\bar{c}\, e^{S[X,b,c,5,\bar{c}]}$
CAUTION


$\int d\mu \int DX\, Db Db̄ Dc Dc̄\, e^{S[X,b,c,5,c̄]} \prod_i \int_\Sigma \cdots b^{ab} \frac{\delta \tilde{h}_{ab}}{\delta \mu_i}$


SIMPLIFY
$b^{ab} h_{ab} = 0$
$h_{ab} = \tilde{h}_{ab}[\mu]$
$QX = c\cdot\partial X$
$Q b^{ab} = T^{ab}$
$Q c^a = c^b \partial_b c^a \cdots$
$S = \int_\Sigma \left( b^{ab} \nabla_a c_b + b^{ab} \frac{\partial \tilde{h}_{ab}}{\partial \mu^s} m^s + \partial_a X^\mu \partial_b X^\nu \tilde{h}^{ab} \right) \sqrt{h}$
$QX = c\partial X + \bar{c}\bar{\partial} X$
$Q c = c \partial c$
$Q b = T \cdots$
$\int e^{i\beta x + S} \, i\frac{\partial S}{\partial x} \, d\beta \, dx$
$\left( i\beta + \frac{\partial S}{\partial x} \right) e^{i\beta x + S(x)}$


$\int d\mu \int DX\, Db\, D\bar{b}\, Dc\, D\bar{c}\, e^{S[X,b,c,\bar{b},\bar{c}]} \prod_i \int_\Sigma \prod b^{\alpha} \frac{\delta \tilde{h}_{\alpha b}}{\delta \mu_i}$
CAUTION


f-o
COMPLEX
STRUCTURE
∫ DX ...


f-o
∫DX ...
COMPLEX
STRUCTURE
⇒