Title: PSI 17/18 - String Theory - Lecture 5

Date: Feb 26, 2018 10:15 AM

URL: http://pirsa.org/18020041

Abstract:

QFT

T

FERMIONIC SYMMETRY $\quad Q$

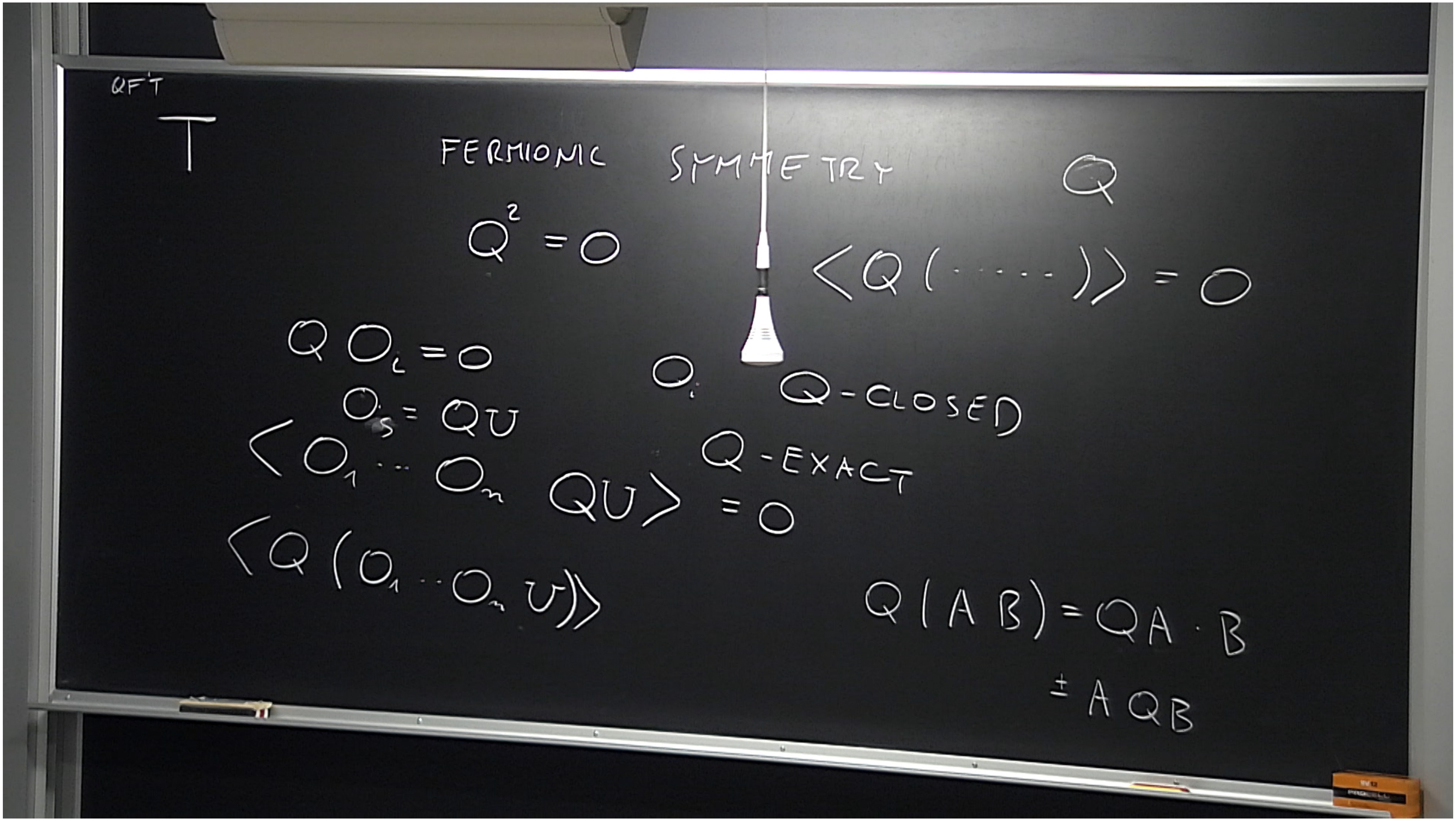
$$Q^2 = 0$$

$$\langle Q(\cdots\cdots)\rangle = 0$$

$$Q\,O_i = 0 \qquad O_i \quad Q\text{-CLOSED}$$

$$\langle O_1 \cdots O_n \, QU \rangle = 0$$

$$\langle Q(O_1 \cdots O_n \, U)\rangle$$


QFT
T
FERMIONIC SYMMETRY Q
$Q^2 = 0$
$\langle Q(\cdots\cdots)\rangle = 0$
$Q\,O_i = 0$
O_i Q-CLOSED
$O_s = QU$
Q-EXACT
$\langle O_1 \cdots O_n\, QU \rangle = 0$
$\langle Q(O_1 \cdots O_n\, U)\rangle$
$Q(AB) = QA \cdot B$
$\pm A\,QB$

$$\langle O_1 \cdots O_n \, QU \rangle = 0$$

$$0 = \langle Q(O_1 \cdots O_n U) \rangle = \langle QO_1 O_2 \cdots U \rangle + \cdots$$

$$Q(AB) = QA \cdot B \pm AQB$$

$$T' = (T, Q)$$

$$\text{OPERATORS} = \frac{Q\text{-CLOSED}}{Q\text{-EXACT}}$$

$x^i \, , \theta^i$

$Qx^i = \theta^i$

$Q\theta^i = 0$

$$\int \mathcal{O}_1(x,\theta)\,\mathcal{O}_2(x,\theta) \cdots dx\, d\theta$$

$\Pi T X$

$\tilde{x}^i = \tilde{x}^i(x^j)$

X

$Q\tilde{x}^i = \dfrac{\partial \tilde{x}^i}{\partial x^j}\,\theta^j = \tilde{\theta}^i$

$(x^i, \theta^i) \in \Pi T X$

$dx\,d\theta = d\tilde{x}\,d\tilde{\theta}$

$Q\,\mathcal{O}(x,\theta) = 0$?

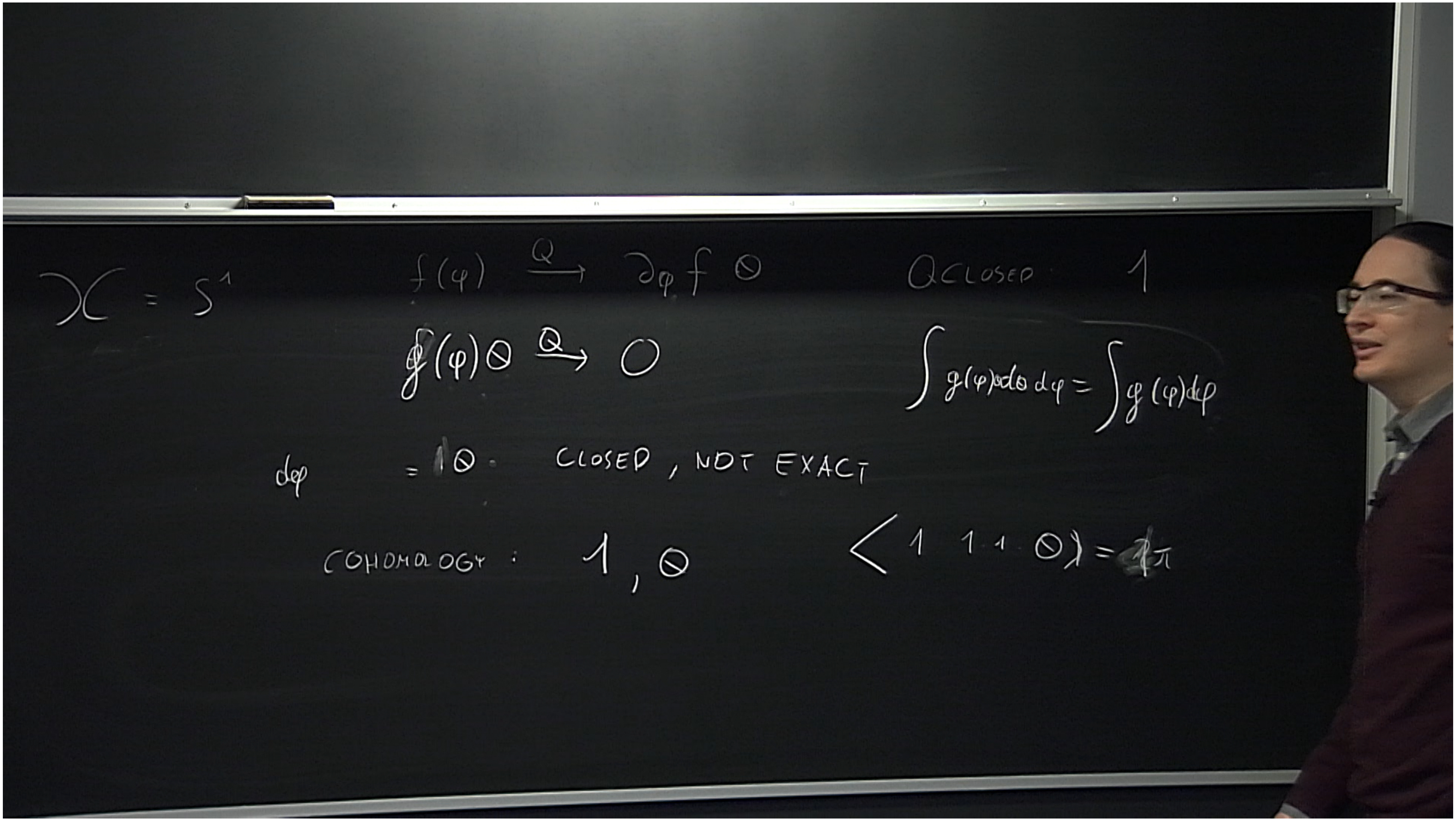
$$\mathcal{O}(x,\theta) = \sum \mathcal{O}^{(n)}_{i_1 \cdots i_n}(x)\,\theta^{i_1} \cdots \theta^{i_n}$$

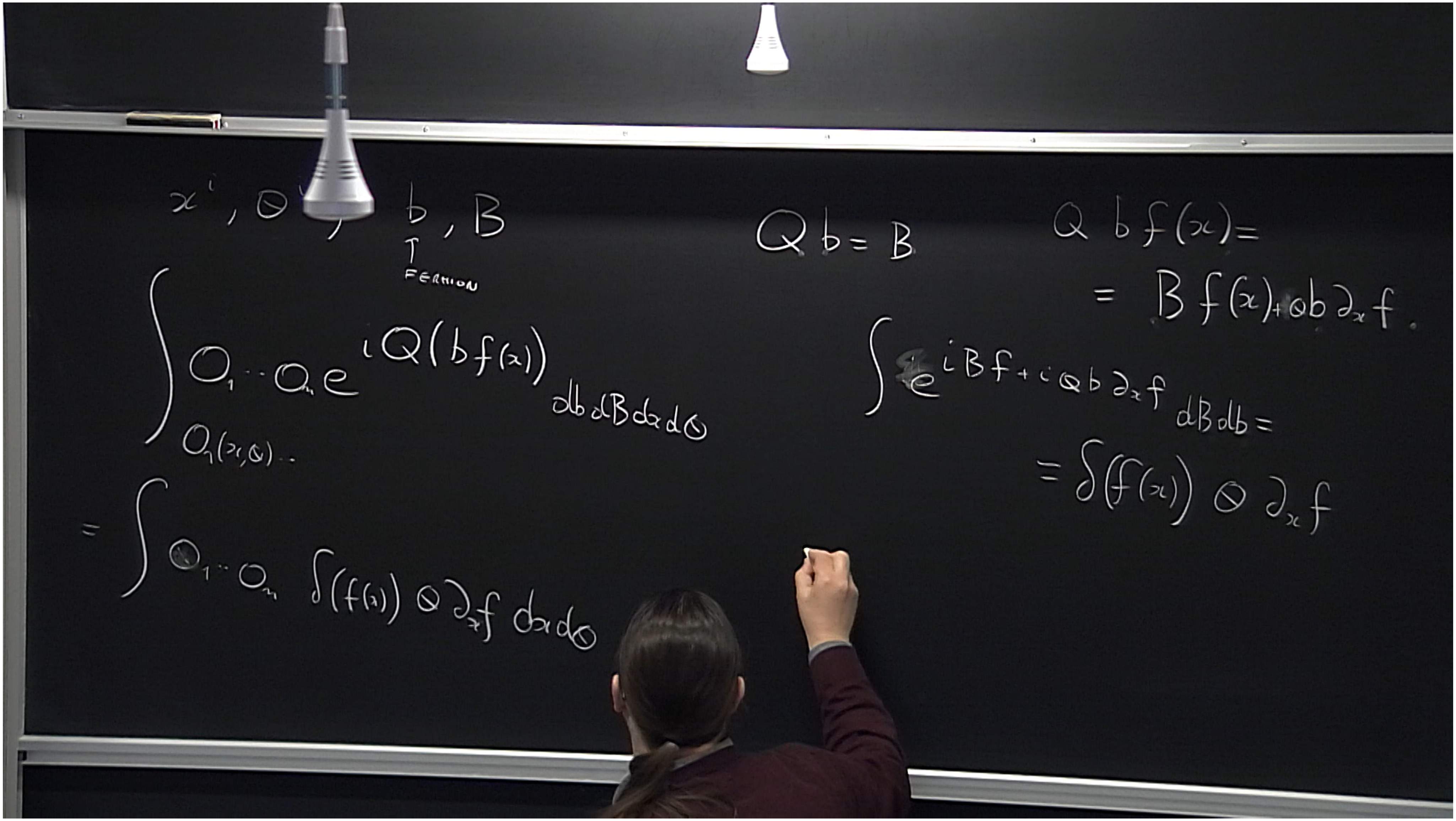
$$Q\,\mathcal{O}(x,\theta) = \sum \partial_{x^{i_0}} \mathcal{O}^{(n)}_{i_1 \cdots i_n}\,\theta^{i_0}\theta^{i_1} \cdots \theta^{i_n}$$

$$f_{i_1\cdots i_n}(x)\, dx^{i_1}\cdots dx^{i_n} \longleftrightarrow f_{i_1\cdots i_n}\,\theta^{i_1}\cdots\theta^{i_n} = \mathcal{O}_f$$

$$df \qquad \mathcal{O}_{df} = Q\,\mathcal{O}_f$$

$$\int_{\Pi TX} \mathcal{O}_{f_1}\mathcal{O}_{f_2}\cdots\mathcal{O}_{f_n}\, dx\, d\theta = \int_X f_1\wedge f_2\wedge\cdots f_n$$


X = S¹
f(φ) →Q ∂φ f θ
Q CLOSED: 1
g(φ)θ →Q 0
∫ g(φ) dθ dφ = ∫ g(φ) dφ
dφ = 1θ. CLOSED, NOT EXACT
COHOMOLOGY: 1, θ
⟨1 1·1·θ⟩ = 2π



x^i, θ^i, b, B
FERMION
$\int O_1 \cdots O_n e^{iQ(bf(x))} \, db \, dB \, dx \, d\theta$
$O_1(x,\theta) \cdots$
$= \int O_1 \cdots O_n \, \delta(f(x)) \, \theta \, \partial_x f \, dx \, d\theta$
$Qb = B$
$Q \, b f(x) =$
$= B f(x) + \theta b \partial_x f .$
$\int e^{iBf + i\theta b \partial_x f} \, dB \, db =$
$= \delta(f(x)) \, \theta \, \partial_x f$

$x^i, \Theta^i, \quad b, B$

FERMION

$Qb = B$

$Q\, b f(x) =$

$= B f(x) + \Theta b\, \partial_x f.$

$$\int O_1 \cdots O_n e^{iQ(bf(x))} db\, dB\, dx\, d\Theta$$

$O_1(x, \Theta) \ldots$

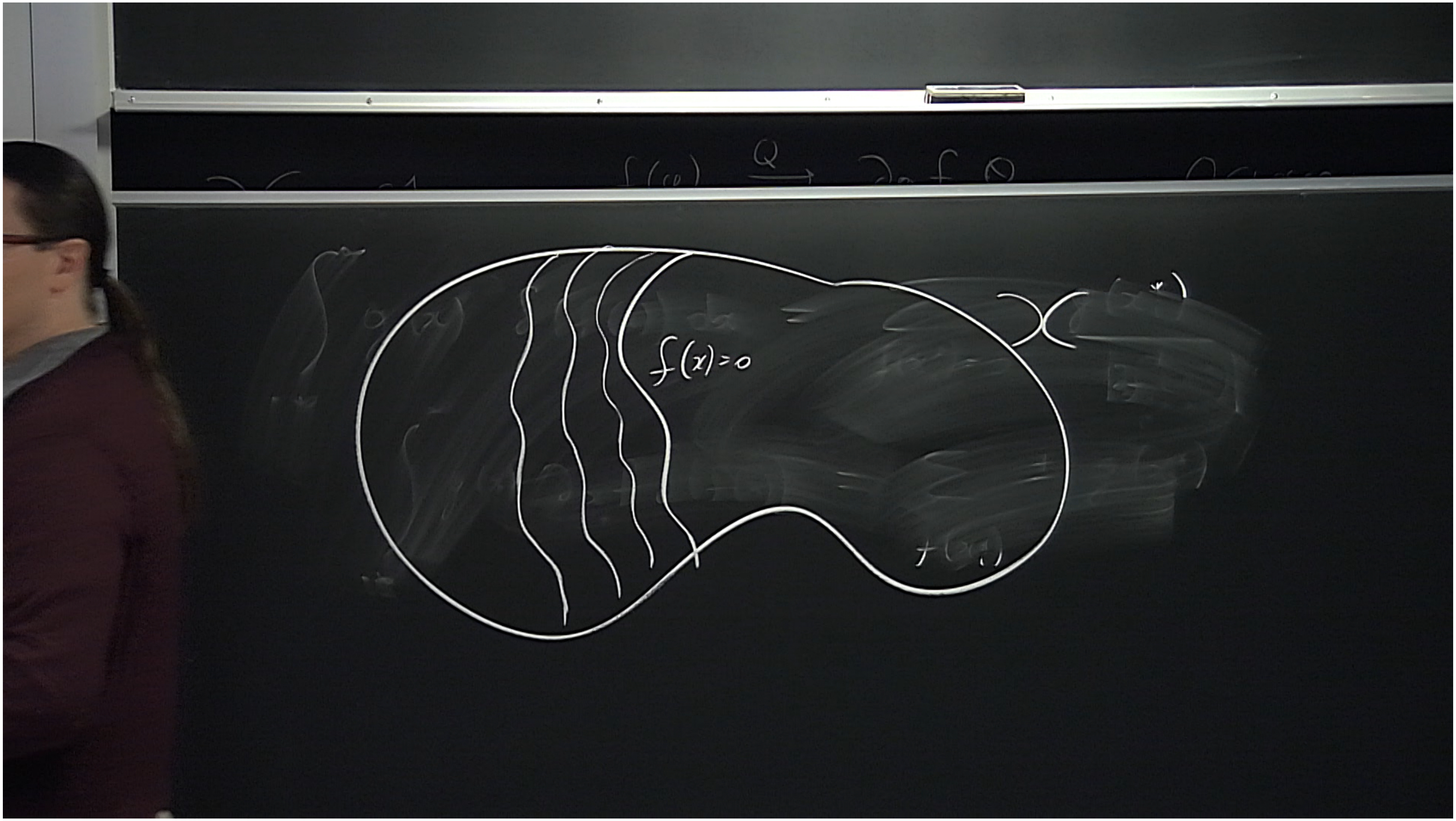
$$\int e^{iBf + i\Theta b \partial_x f} dB\, db =$$

$$= \delta(f(x))\, \Theta\, \partial_x f$$

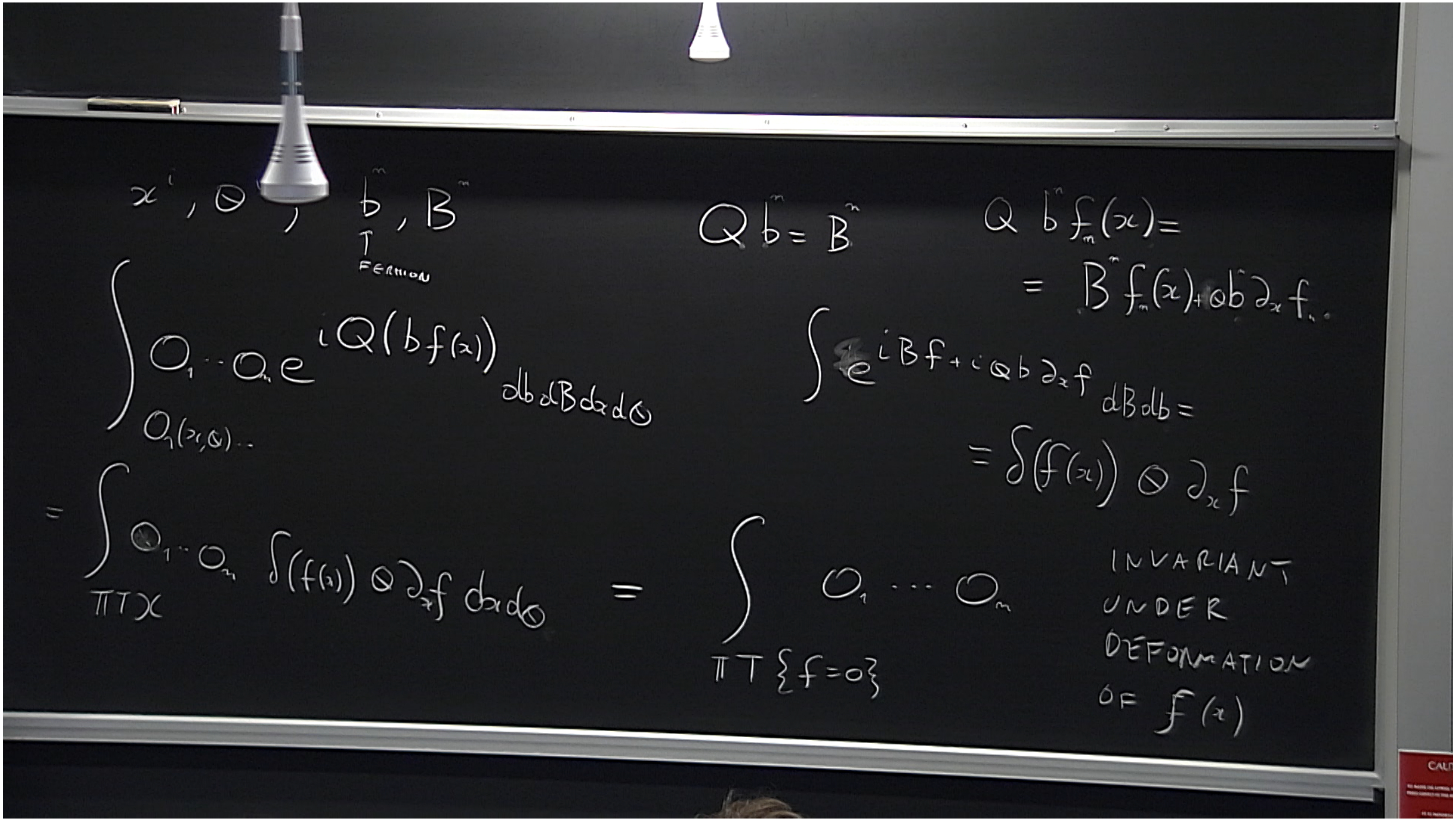
$$= \int_{\Pi T X} O_1 \cdots O_n\, \delta(f(x))\, \Theta\, \partial_x f\, dx\, d\Theta = \int_{\{f=0\}}$$

$$\int_{-\infty}^{\infty} g(x)\;\delta(f(x))\,dx = \sum_{f(x_i^*)=0} \frac{g(x_i^*)}{|\partial f(x_i^*)|}$$

$$\int_{-\infty}^{\infty} g(x)\;\partial_x f\;\delta(f(x)) = \sum_{f(x_i^*)} \pm\, g(x_i^*)$$



$f(x)=0$


x^i, Θ^i, b^m, B^m
FERMION
$Q b^m = B^m$
$Q b^m f_m(x) =$
$= B^m f_m(x) + \Theta b^m \partial_x f_m$
$\int O_1 \cdots O_n e^{iQ(b f(x))} db\, dB\, dx\, d\Theta$
$O_1(x,\Theta) \cdots$
$\int e^{iBf + i\Theta b \partial_x f} dB\, db =$
$= \delta(f(x))\, \Theta\, \partial_x f$
$= \int_{\Pi T X} O_1 \cdots O_n\, \delta(f(x))\, \Theta\, \partial_x f\, dx\, d\Theta = \int_{\Pi T\{f=0\}} O_1 \cdots O_n$
INVARIANT
UNDER
DEFORMATION
OF $f(x)$

x^i, Θ, b^m, B^m

FERMION

$f \to f + \delta f$

$Q b^m = B^m$

$Q\, b^m f_m(x) =$

$= B^m f_m(x) + \Theta b^m \partial_x f_m$

$$\int \mathcal{O}_1 \cdots \mathcal{O}_n\, e^{iQ(bf(x))}\, db\, dB\, dx\, d\Theta$$

$\mathcal{O}_1(x,\Theta)\ldots$

$$\int e^{iBf + i\Theta b \partial_x f}\, dB\, db =$$

$$= \delta(f(x))\, \Theta\, \partial_x f$$

$$= \int_{\Pi T X} \mathcal{O}_1 \cdots \mathcal{O}_n\, \delta(f(x))\, \Theta\, \partial_x f\, dx\, d\Theta = \int_{\Pi T\{f=0\}} \mathcal{O}_1 \cdots \mathcal{O}_n$$

INVARIANT UNDER DEFORMATION OF $f(x)$

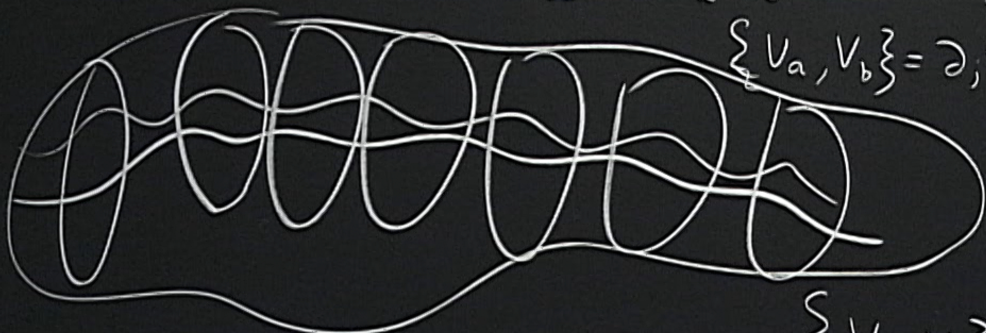
$$T' = (T, Q)$$

$$\text{OPERATORS} = \frac{Q\text{-CLOSE}}{Q\text{-EXACT}}$$

$$\boxed{\begin{aligned} Q c^a &= f^a_{bc}\, c^b c^c \\ Q x^i &= v^i_a(x)\, c^a \end{aligned}}$$

$$V_a = v_a^i \partial_i \qquad x^i, c^a$$

$$\{V_a, V_b\} = \partial_j v^i_{[a} v^j_{b]} \partial_i$$



$$Q^2 x = [\partial_j v^i_a v^j_b] c^b c^a + v^i_a Q c^a$$

$$= v^i_c f^c_{ab} c^b c^a + v^i_a Q c^a$$

$$\{V_a, V_b\} = f^c_{ab} V_c$$

$$Q^2 c^a = f f c c c = 0$$