Title: PSI 17/18 - String Theory - Lecture 3

Date: Feb 22, 2018 10:15 AM

URL: http://pirsa.org/18020039

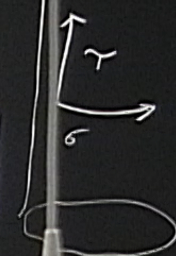
Abstract:

$$\frac{1}{p^2+m^2-i\epsilon} = \int_0^\infty d\tau \, e^{-\tau(p^2+m^2-i\epsilon)}$$

$$\tau = it$$

$$\frac{1}{p^2+m^2-i\epsilon} = \int_0^\infty dt \, e^{-it(p^2+m^2-i\epsilon)}$$

$$X(\sigma,\tau) = x - \frac{2i}{L} p\tau + \sum_{n\neq 0}\left[\frac{i}{n} a_n e^{i\frac{n}{L}(\sigma+i\tau)} + \frac{i}{n}\bar{a}_n e^{-i\frac{n}{L}(\sigma-i\tau)} \right]$$



$$[x, p] = i \qquad [a_n, a_{-n}] = n$$

$$[\bar{a}_n, \bar{a}_{-n}] = n$$

$$[X(\sigma,0),\ i\partial_\tau X(\sigma',0)] = \frac{2i}{L}\sum e^{i\frac{n}{L}(\sigma-\sigma')} = 4\pi i\,\delta(\sigma-\sigma')$$

$$\langle p | p' \rangle = \delta(p - p')$$

$$\langle p | a_{-1} | p' \rangle = 0$$

$$\langle p | a_1 \; a_{-1} | p' \rangle = \langle p | [a_1, a_{-1}] | p' \rangle = \delta(p - p)$$

$$\frac{\;}{p^2+m^2-i\epsilon} = \int_0 dt\; e$$

$$X \to X + \epsilon(z) + \bar{\epsilon}(\bar{z})$$

$$J_a = \partial_a X$$

$$\partial^a J_a = 0 = \partial^a \partial_a X = 0$$

$$\partial \bar{\partial} X = 0$$

$$\bar{\partial}\left(f(z)\partial X\right) = 0$$

$$f(z)\,\partial X$$

$$\bar{f}(\bar{z})\,\bar{\partial} X$$

$$\int d^2 z\; \partial X\, \bar{\partial} X$$

$$\frac{1}{p^2+m^2-i\epsilon} = \int_0 dt\ e$$

$$X \to X + \epsilon(s) + \bar\epsilon(\bar s)$$

$$J_a = \partial_a X$$

$$\partial^a J_a = 0 = \partial^a \partial_a X = 0$$

$$i\partial_s X = \frac{1}{L} p + \sum_{n\neq 0} \frac{1}{L} a_n e^{-\frac{n}{L}s}$$

$$i\partial_{\bar s} X = \frac{1}{L} p - \sum_{n\neq 0} \frac{1}{L} \bar a_n e^{-\frac{n}{L}\bar s}$$

$$a_0 = -\bar a_0 = p$$

$$\langle 0|\, \partial_s X(s)\, \partial_{s'} X(s') \,|0\rangle = -\frac{1}{l^2}\sum_{n>0} n\, e^{\frac{n}{l}(s'-s)} = -\frac{1}{l^2}\frac{e^{\frac{1}{l}(s+s')}}{\left(e^{\frac{s}{l}}-e^{\frac{s'}{l}}\right)^2}$$

$$\frac{\quad}{p^2+m^2-i\epsilon} = \int_0 dt \;\; e$$

$$X \to X + c(s) + \bar{c}(\bar{s}) \qquad J_a = \partial_a X$$

$$\partial^a J_a = 0 = \partial^a \partial_a X = 0$$

$$T = -\frac{1}{2} \lim_{s' \to s} \left[\partial_s X(s) \partial_{s'} X(s') + \frac{1}{(s-s')^2} \right]$$

$$\frac{1}{p^2+m^2-i\epsilon} = \int_0 dt \; e$$

$$X \to X + \epsilon(s) + \bar\epsilon(\bar s) \qquad J_a = \partial_a X$$

$$\partial^a J_a = 0 = \partial^a \partial_a X = 0$$

$$T = -\frac{1}{2} \lim_{s'\to s} \left[\partial_s X(s)\, \partial_{s'} X(s') + \frac{1}{(s-s')^2} \right]$$

$$= -\frac{1}{2} :\partial_s X \partial_s X: \; - \frac{1}{2} \lim_{s'\to s} \left[-\frac{1}{L^2} \frac{e^{\frac{s+s'}{L}}}{\left(e^{\frac{s}{L}} - e^{\frac{s'}{L}}\right)^2} + \frac{1}{(s-s')^2} \right]$$

$$= -\frac{1}{2} :\partial X \partial X: \; - \frac{1}{24 L^2}$$

$$X \to X + c(s) + \bar{c}(\bar{s}) \qquad J_a = \partial_a X$$

$$\partial^a J_a = 0 = \partial^a \partial_a X = 0$$

$$T = -\frac{1}{2} \lim_{s' \to s} \left[\partial_s X(s) \partial_{s'} X(s') + \frac{1}{(s-s')^2} \right]$$

$$= -\frac{1}{2} :\partial_s X \partial_s X: - \frac{1}{2} \lim_{s' \to s} \left[-\frac{1}{L^2} \frac{e^{\frac{s+s'}{L}}}{\left(e^{\frac{s}{L}} - e^{\frac{s'}{L}}\right)^2} + \frac{1}{(s-s')^2} \right]$$

$$= -\frac{1}{2} :\partial X \partial X: - \frac{1}{24 L^2} \qquad L = 1$$

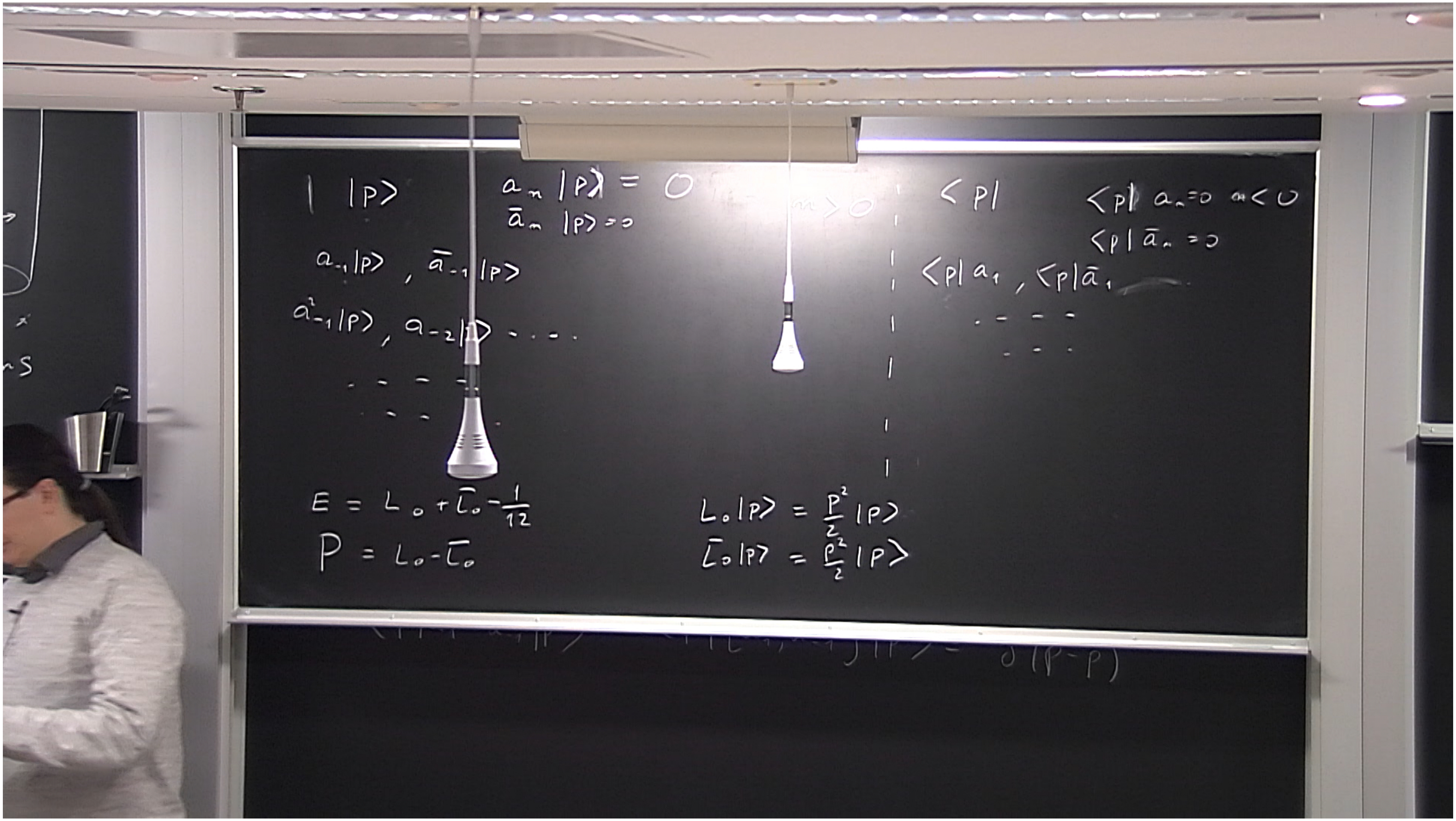
$$[x,p]=i \qquad [a_m, a_{-m}] = m \qquad -\frac{i}{n}\bar{a}_n e^{-i\frac{n}{l}(\sigma - i\tau)}\Big]$$

$$[\bar{a}_n, \bar{a}_{-n}] = n$$

$$T(s) = -\frac{1}{24} + \frac{1}{2}\sum_{m,n} :a_n a_m: e^{-(n+m)s} = -\frac{1}{24} + \sum_n L_n e^{-ns}$$

$$\langle 0 | \partial_s X(s)\, \partial_{s'} X(s') | 0 \rangle = -\frac{1}{l^2}\sum_{n>0} n\, e^{\frac{n}{l}(s'-s)} = -\frac{1}{l^2}\frac{e^{\frac{1}{l}(s+s')}}{\left(e^{\frac{s}{l}} - e^{\frac{s'}{l}}\right)^2}$$

$$L_n = \frac{1}{2}\sum_m :a_{n-m} a_m:$$


|p>
a_n |p> = 0
ā_m |p> = 0
<p|
<p| a_n = 0 n < 0
<p| ā_n = 0
a_-1 |p> , ā_-1 |p>
<p| a_1 , <p| ā_1
a²_-1 |p> , a_-2 |p> ....
E = L_0 + L̄_0 - 1/12
P = L_0 - L̄_0
L_0 |p> = p²/2 |p>
L̄_0 |p> = p²/2 |p>

$$[\bar{a}_n, \bar{a}_{-n}] = n$$

$$T(s) = -\frac{1}{24} + \frac{1}{2}\sum_{n,m} :a_n a_m: e^{-(n+m)s} = -\frac{1}{24} + \sum_n L_n e^{-ns}$$

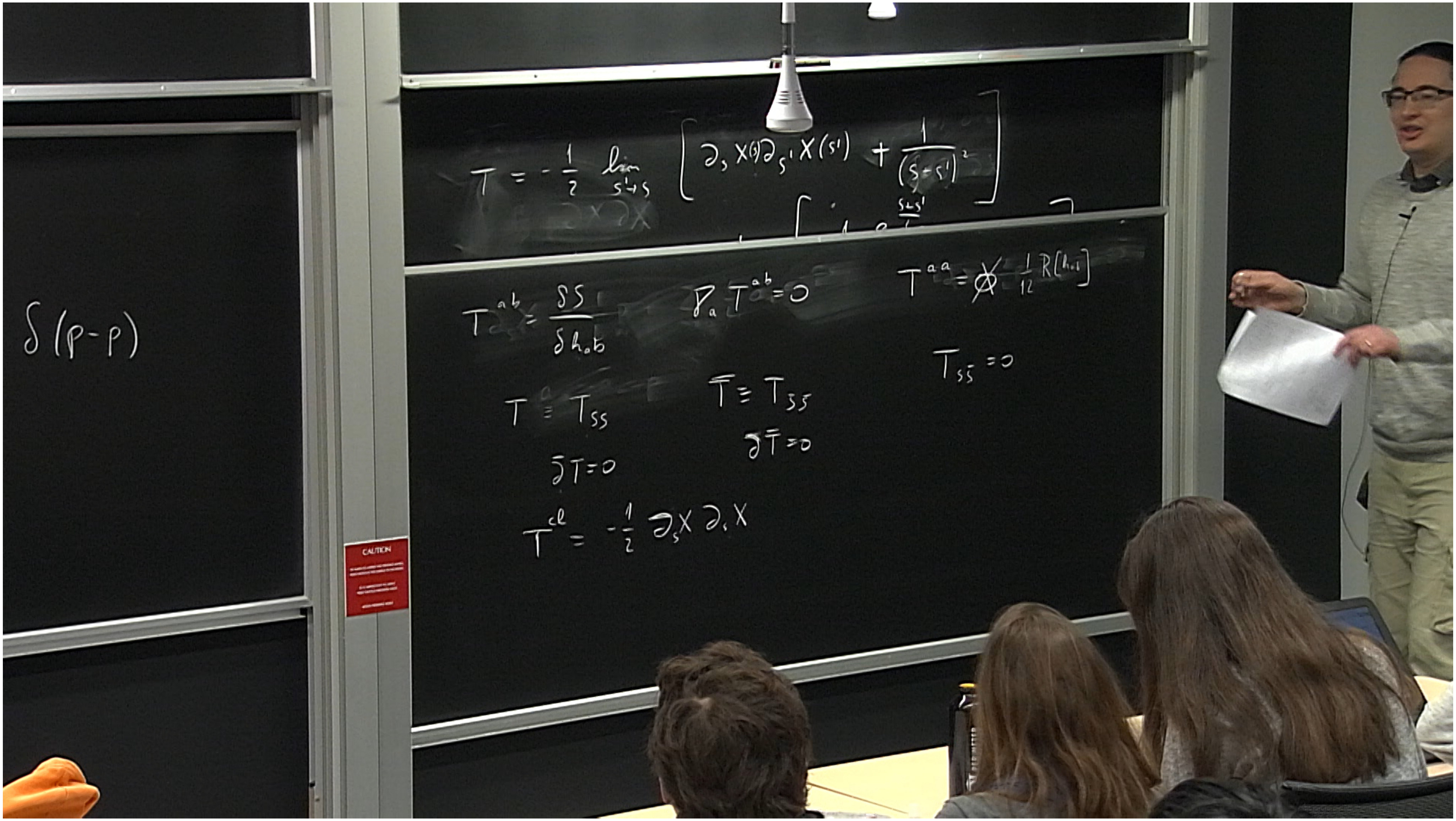
$$\bar{T}(\bar{s}) \;\cdots\cdots \quad \bar{L}_n$$

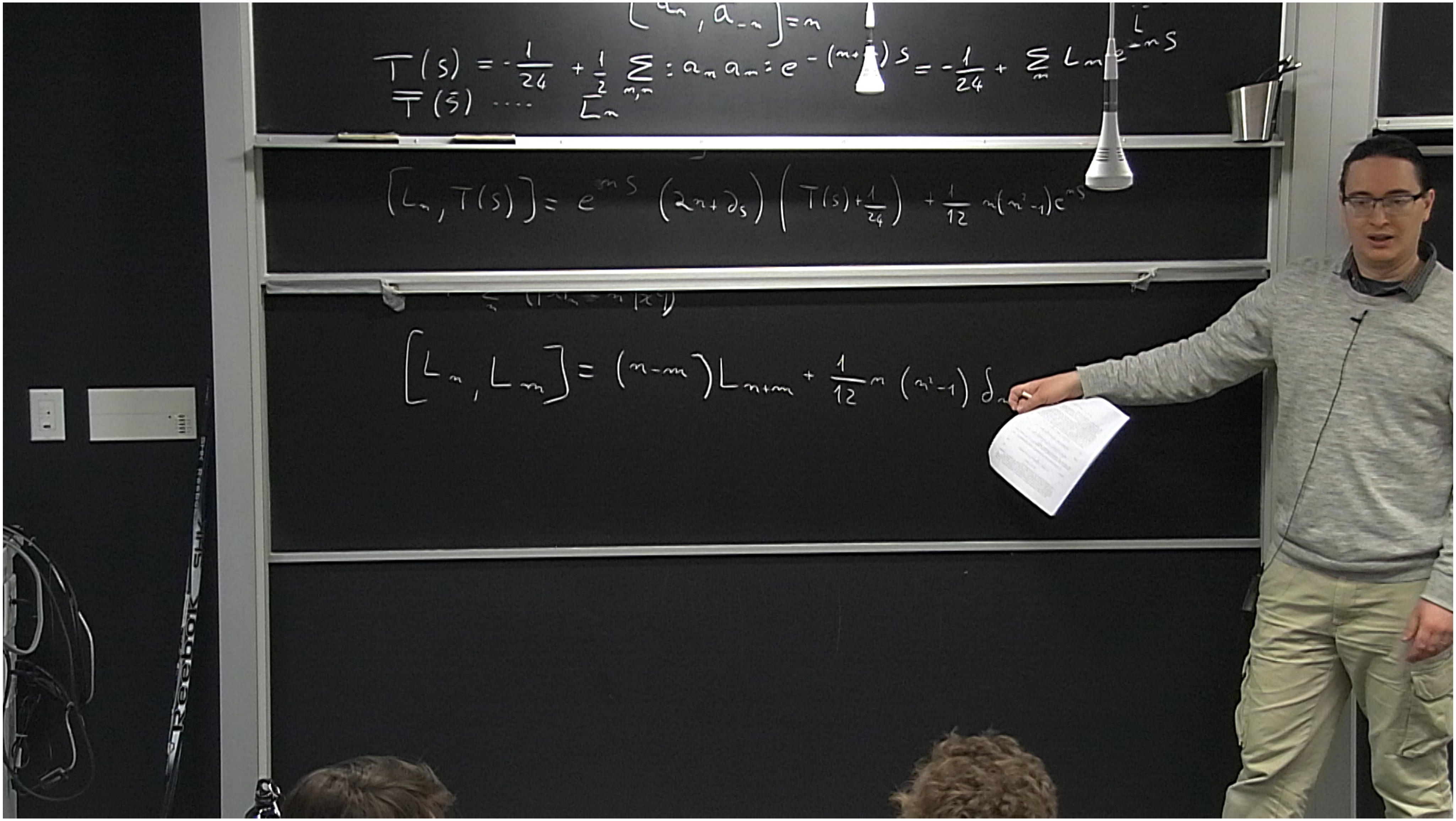
$$L_n = \frac{1}{2}\sum_m :a_{n-m} a_m: \qquad L_n = e^{ns}\partial_s$$

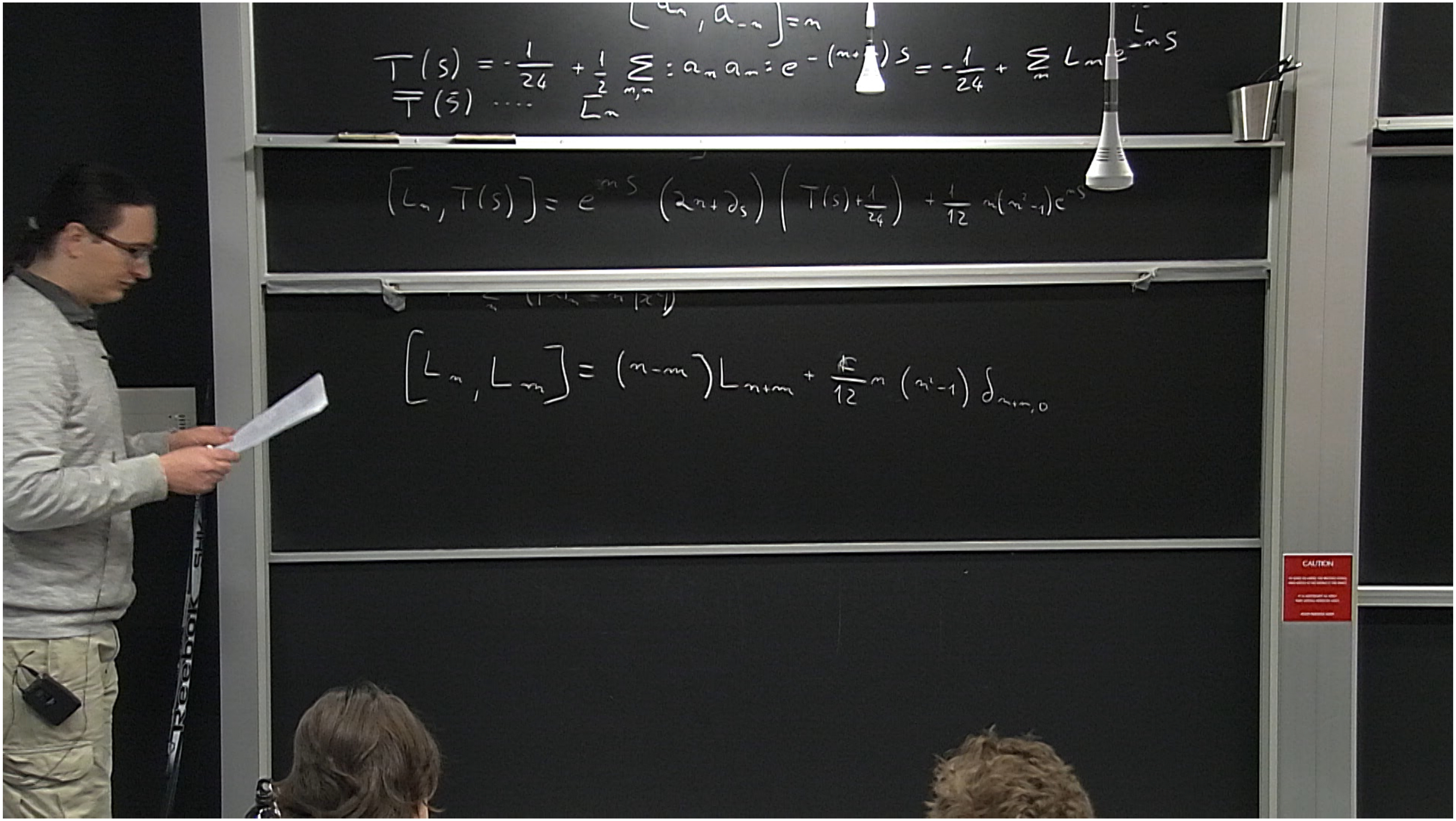
$$[L_n, a_m] = m\, a_{n+m} \qquad [L_n, \partial_s X(s)] = \partial_s\left(e^{ns}\partial_s X\right)$$

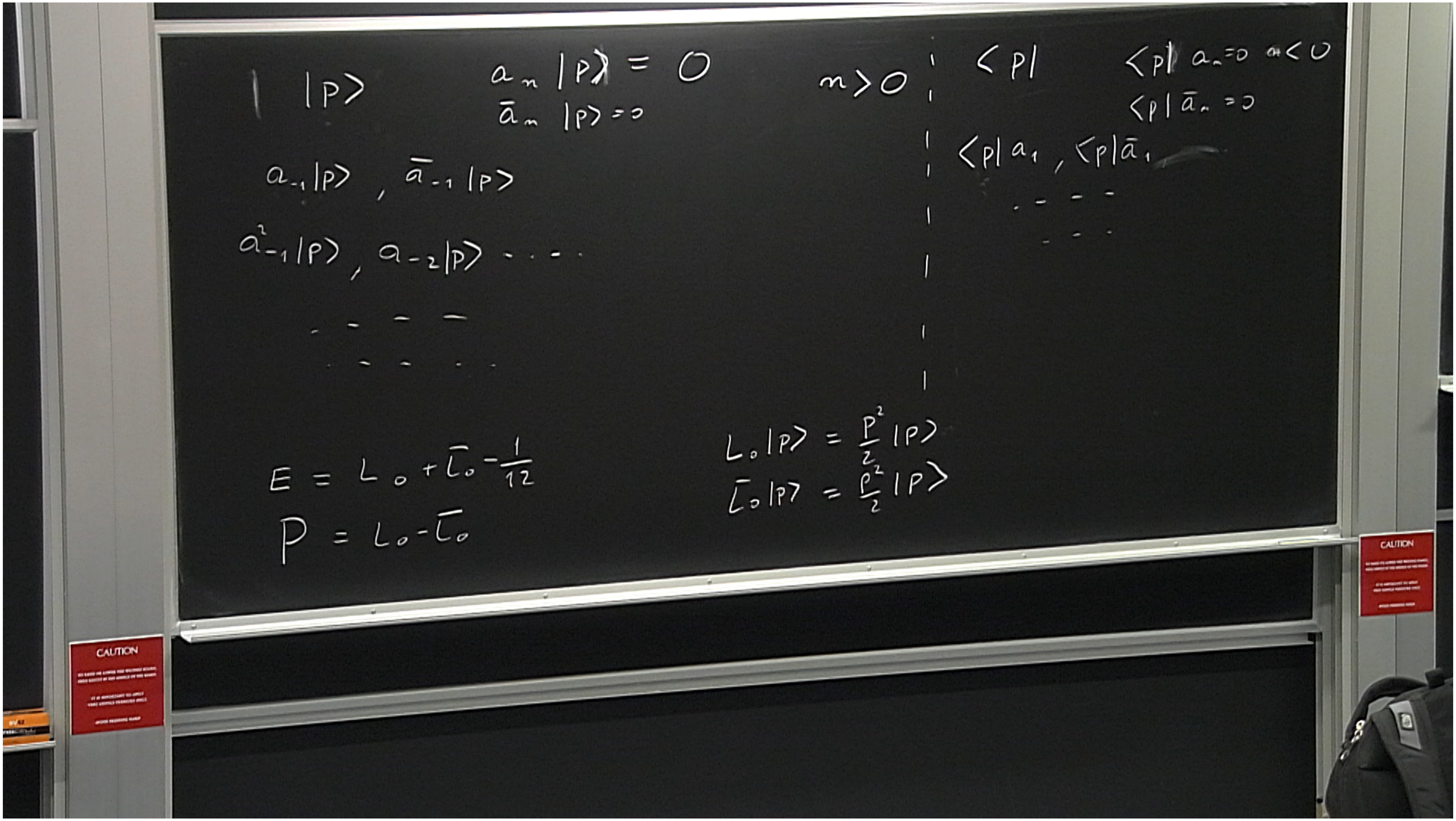
$$[L_n, \partial_s X(s)\, \partial_{s'} X(s')] = \cdots$$

$$[L_n, T(s)] = e^{ns}\left(2n + \partial_s\right)\left(T(s) + \frac{1}{24}\right) + \frac{1}{12} n(n^2-1) e^{ns}$$


CAUTION


$T(s) = -\frac{1}{24} + \frac{1}{2}\sum_{n,m} :a_n a_m: e^{-(n+m)s} = -\frac{1}{24} + \sum_n L_n e^{-ns}$
$T(\bar{s}) \cdots \bar{L}_n$
$[L_n, T(s)] = e^{ns}(2n+\partial_s)\left(T(s)+\frac{1}{24}\right) + \frac{1}{12}n(n^2-1)e^{ns}$
$[L_n, L_m] = (n-m)L_{n+m} + \frac{1}{12}n(n^2-1)\delta_{n+m}$


[L_n, T(s)] = e^{ns} (2n + ∂_s) (T(s) + 1/24) + 1/12 n(n²-1) e^{ns}
[L_n, L_m] = (n-m) L_{n+m} + c/12 n (n²-1) δ_{n+m,0}
CAUTION
Reebok



$|p\rangle$
$a_n |p\rangle = 0$
$\bar{a}_n |p\rangle = 0$
$n > 0$
$\langle p|$
$\langle p| a_n = 0 \quad n < 0$
$\langle p| \bar{a}_n = 0$
$a_{-1}|p\rangle, \bar{a}_{-1}|p\rangle$
$a_{-1}^2|p\rangle, a_{-2}|p\rangle \ldots$
$\langle p| a_1, \langle p| \bar{a}_1$
$E = L_0 + \bar{L}_0 - \frac{1}{12}$
$P = L_0 - \bar{L}_0$
$L_0|p\rangle = \frac{p^2}{2}|p\rangle$
$\bar{L}_0|p\rangle = \frac{p^2}{2}|p\rangle$
CAUTION
CAUTION

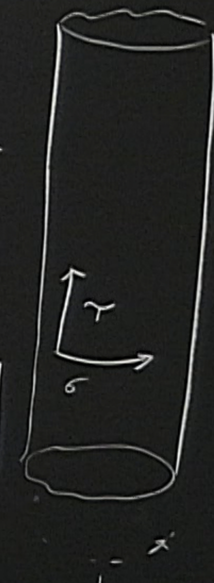
$$[L_n, \partial_s X(s)\, \partial_{s'} X(s')] = \dots$$

$$S[X, \delta_{ab}] = \frac{1}{2}\int \frac{dX}{d\sigma^a}\frac{dX}{d\sigma_a}\, d\sigma^1 d\sigma^2$$

$s = \tau - i\sigma$

$\tau = it$

$\sigma \equiv \sigma + 2\pi L$



$$X(\sigma,\tau) = x - \frac{2i}{L} p\tau + \sum_{n\neq 0}\left[\frac{i}{n} a_n e^{i\frac{n}{L}(\sigma+i\tau)} - \frac{i}{n}\bar{a}_n e^{-i\frac{n}{L}(\sigma-i\tau)}\right]$$

$$[x, p] = i \qquad [a_n, a_{-n}] = n$$

$$[\bar{a}_n, \bar{a}_{-n}] = n$$

$$T(s) = -\frac{1}{24} + \frac{1}{2}\sum_{m,n} :a_n a_m: e^{-(n+m)s} = -\frac{1}{24} + \sum_n L_n e^{-ns}$$

$$\bar{T}(\bar{s}) \quad \dots \quad \bar{L}_n$$

$$T(s) = -\frac{1}{24} + \frac{1}{2}\sum_{n,m} :a_n a_m: e^{-(n+m)s} = -\frac{1}{24} + \sum_m L_m e$$

$$\bar{T}(\bar{s}) \; \cdots \quad \bar{L}_n$$

$$\langle 0 | \partial_s X(s)\, \partial_{s'} X(s') | 0 \rangle = -\frac{1}{\ell^2} \sum_{n>0} n\, e^{\frac{n}{\ell}(s'-s)} = -\frac{1}{\ell^2} \frac{e^{\frac{1}{\ell}(s+s')}}{\left(e^{\frac{s}{\ell}} - e^{\frac{s'}{\ell}}\right)^2}$$

$$L_n = \frac{1}{2} \sum_m :a_{n-m} a_m: \qquad L_n := e^{ns} \partial_s$$

$$[L_n, a_m] = m\, a_{n+m} \qquad [L_n, \partial_s X(s)] = \partial_s \left(e^{ns} \partial_s X\right)$$

$$[L_n, \partial_s X(s)\, \partial_{s'} X(s')] = \cdots$$

$$[L_n, T(s)] = e^{ns} (2n + \partial_s)\left(T(s) + \frac{1}{24}\right) + \frac{1}{12} n(n^2-1) e^{ns}$$

$$T(s) = -\frac{1}{24} + \frac{1}{2}\sum_{n,m} :a_n a_m: e^{-(n+m)s} = -\frac{1}{24} + \sum_m L_m e$$

$$\bar{T}(\bar{s}) \cdots \quad \bar{L}_n$$

$$X(\sigma, 0) = x + \sum_n x_n e^{i\frac{n}{L}\sigma}$$

$$\dot{x}^2 + \sum_n \left(|\dot{x}|_n^2 + n^2|x^2|\right)$$

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12} n (n^2-1)\, \delta_{n+m,0}$$