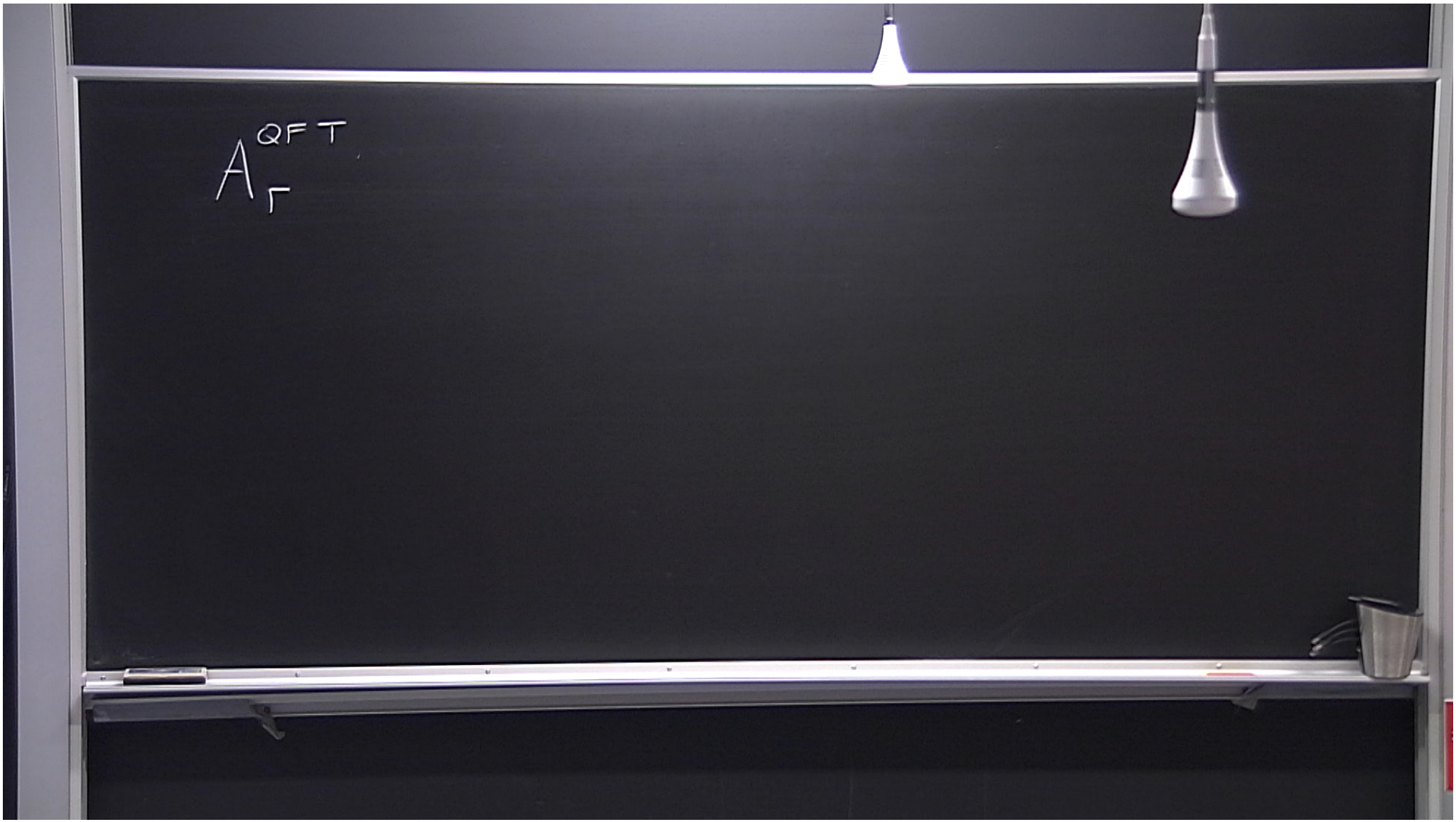
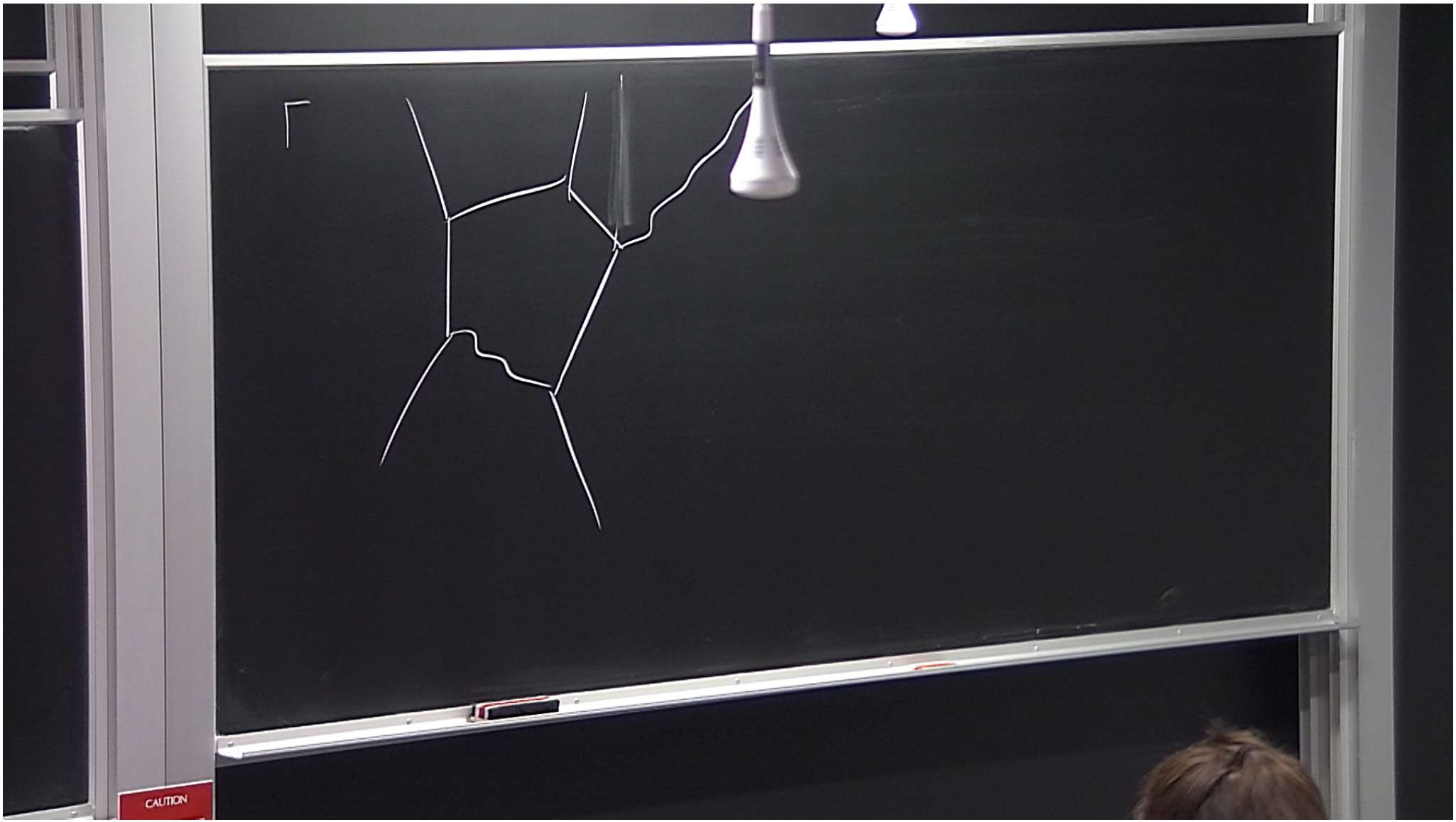
Title: PSI 17/18 - String Theory - Lecture 2

Date: Feb 21, 2018 10:15 AM

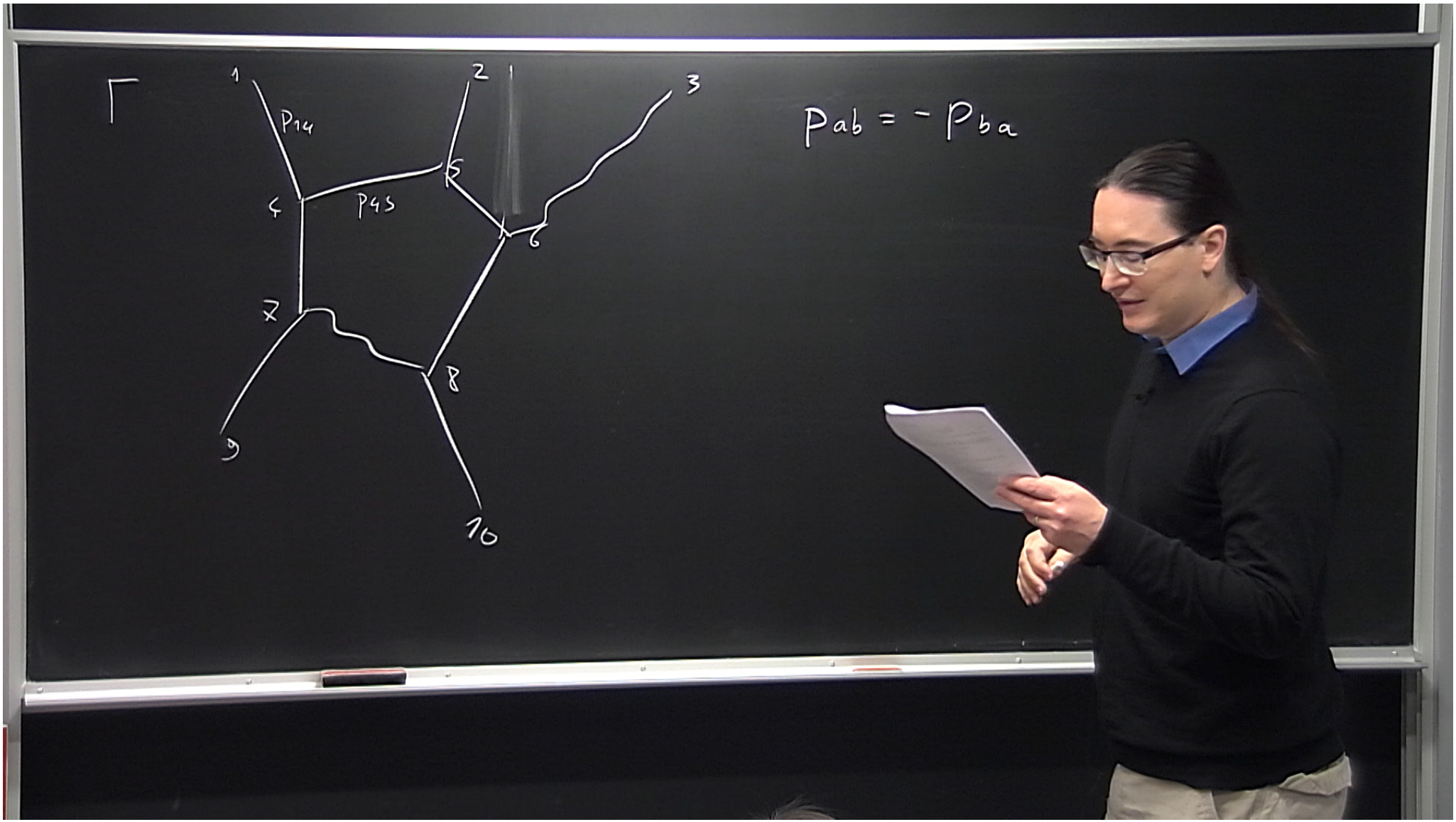
URL: http://pirsa.org/18020038

Abstract:


A_{Γ}^{QFT}


CAUTION

$$A_\Gamma^{QFT} = \int_{\mathcal{M}[\Gamma]} d\ell_{ab} \int_{x:\Gamma \to \mathbb{R}^{d,1}} e^{-S[x,\ell_{ab}]}$$


Γ
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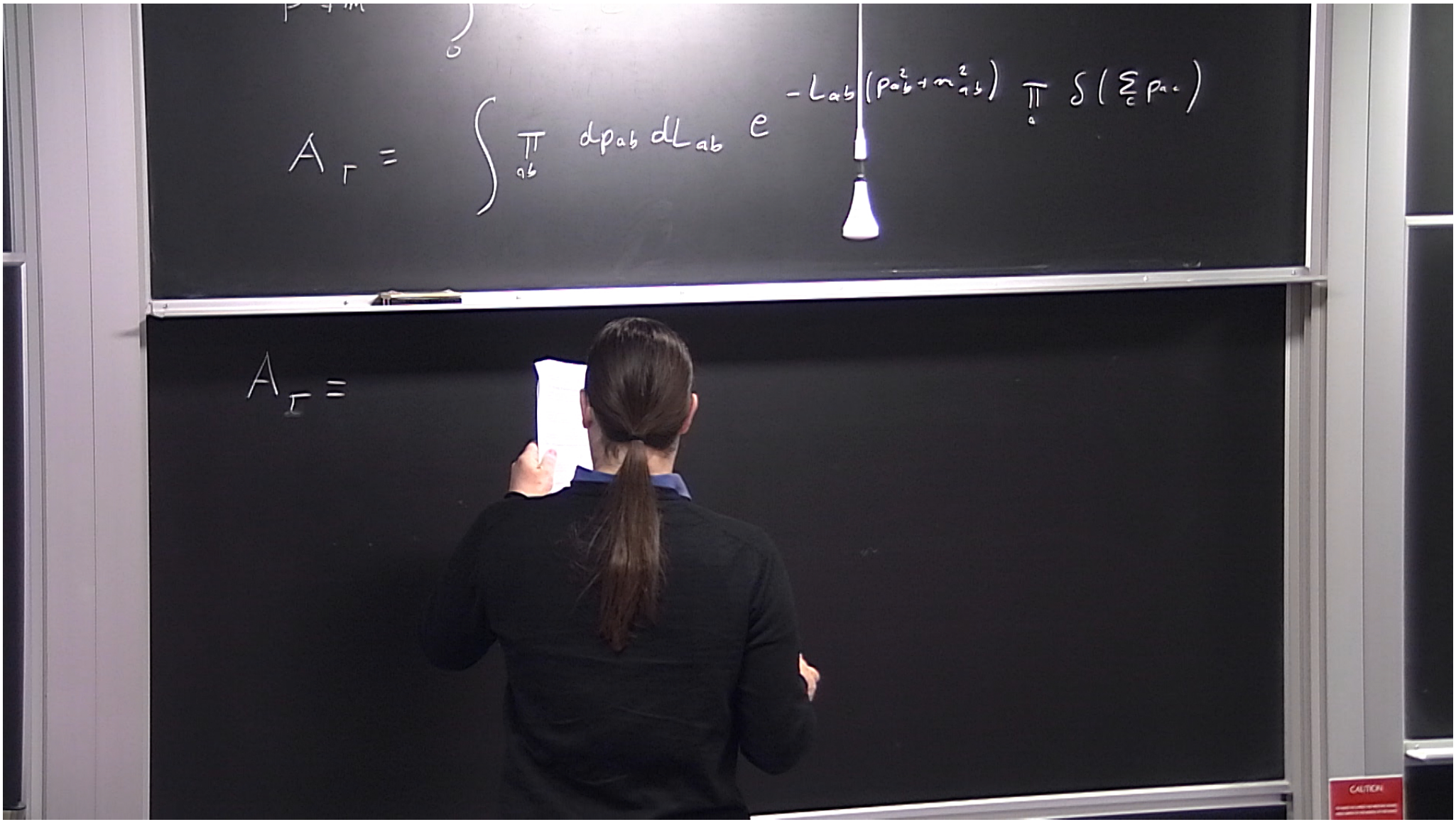
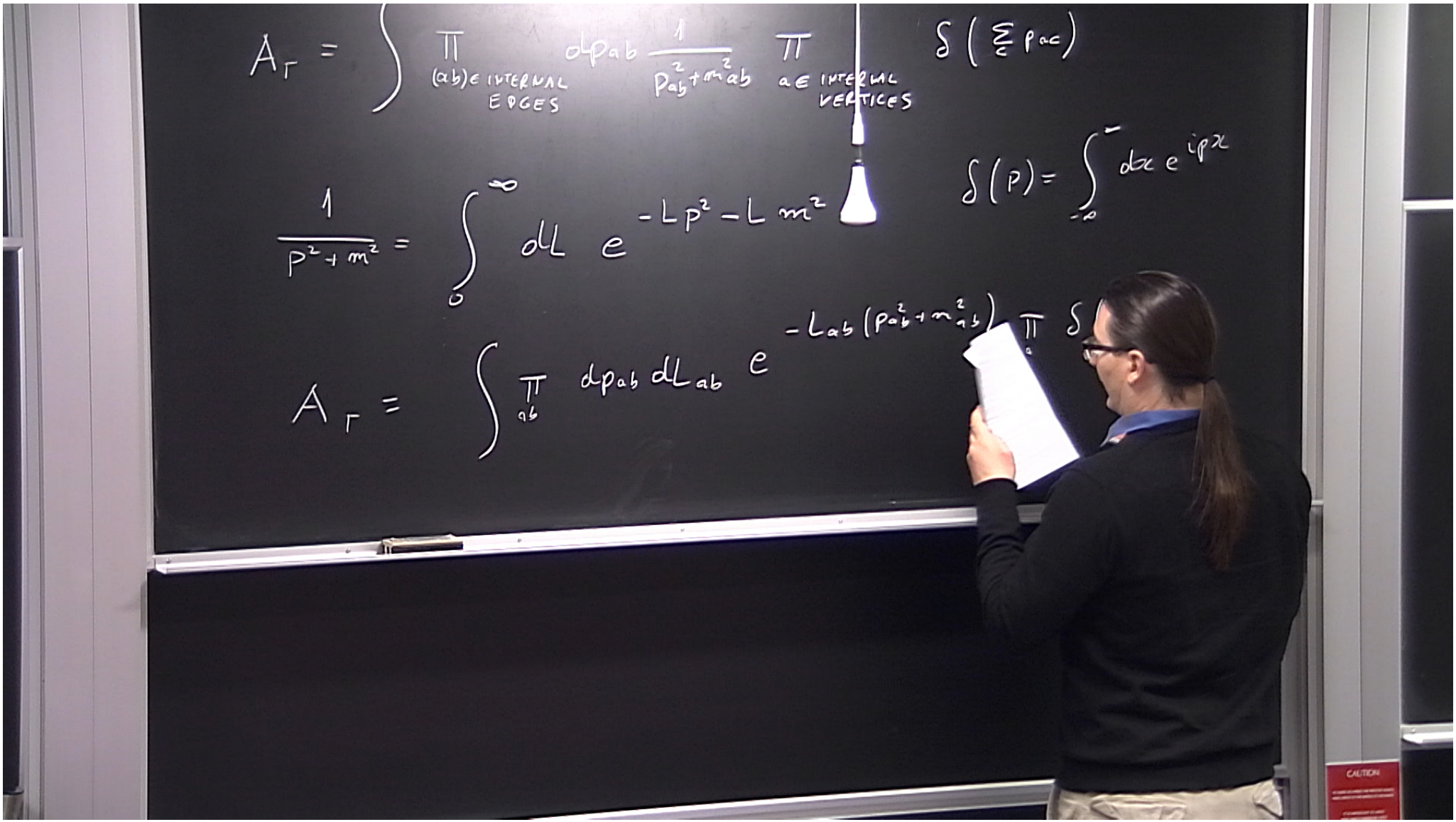
p14
p45
pab = − pba

$$A_\Gamma = \int \prod_{(ab)\in \text{INTERNAL EDGES}} dp_{ab} \frac{1}{p_{ab}^2 + m_{ab}^2} \prod_{a \in \text{INTERNAL VERTICES}} \delta\left(\sum_c p_{ac}\right)$$

$$A_\Gamma = \int \prod_{(ab)\in \text{INTERNAL EDGES}} dp_{ab} \frac{1}{p_{ab}^2 + m_{ab}^2} \prod_{a \in \text{INTERNAL VERTICES}} \delta\left(\sum_c p_{ac}\right)$$

$$\frac{1}{p^2 + m^2} = \int_0^\infty dL \, e^{-Lp^2 - Lm^2}$$


$A_\Gamma = \int \prod_{ab} dp_{ab}\, dL_{ab}\, e^{-L_{ab}(p_{ab}^2 + m_{ab}^2)} \prod_a \delta\left(\sum_c p_{ac}\right)$
$A_\Gamma =$


$A_\Gamma = \int \prod_{(ab)\in \text{INTERNAL EDGES}} dp_{ab} \frac{1}{p_{ab}^2 + m_{ab}^2} \prod_{a \in \text{INTERNAL VERTICES}} \delta\left(\sum_c p_{ac}\right)$
$\delta(p) = \int_{-\infty}^{\infty} dx\, e^{ipx}$
$\frac{1}{p^2+m^2} = \int_0^\infty dL\, e^{-Lp^2 - Lm^2}$
$A_\Gamma = \int \prod_{ab} dp_{ab}\, dL_{ab}\, e^{-L_{ab}(p_{ab}^2 + m_{ab}^2)} \prod_a \delta$

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$$\frac{1}{p^2+m^2} = \int_0^\infty dL \, e^{-Lp^2 - Lm^2}$$

$$\delta(p) = \int_{-\infty}^{\infty} dx \, e^{ipx}$$

$$A_\Gamma = \int \prod_{ab} dp_{ab} \, dL_{ab} \, e^{-L_{ab}(p_{ab}^2 + m_{ab}^2)} \prod_a dx_a \, e^{i\sum_c x_a p_{ac}}$$

$$A_\Gamma = \int \prod_{ab} dp_{ab}\, dL_{ab}\, e^{-L_{ab}(p_{ab}^2 + m_{ab}^2)} \prod_a dx_a\, e^{i\sum_c x_a p_{ac}}$$

$$\int dp\, e^{-Lp^2 + ixp - Lm^2} = e^{-\frac{x^2}{4L} - Lm^2}$$

$$A_\Gamma =$$

$$A_\Gamma = \int \prod_{ab} dp_{ab}\, dL_{ab}\, e^{-L_{ab}(p_{ab}^2 + m_{ab}^2)} \prod_a dx_a\, e^{i\sum_c x_a p_{ac}}$$

$$\int dp\, e^{-Lp^2 + ixp - Lm^2} = e^{-\frac{x^2}{4L} - Lm^2}$$

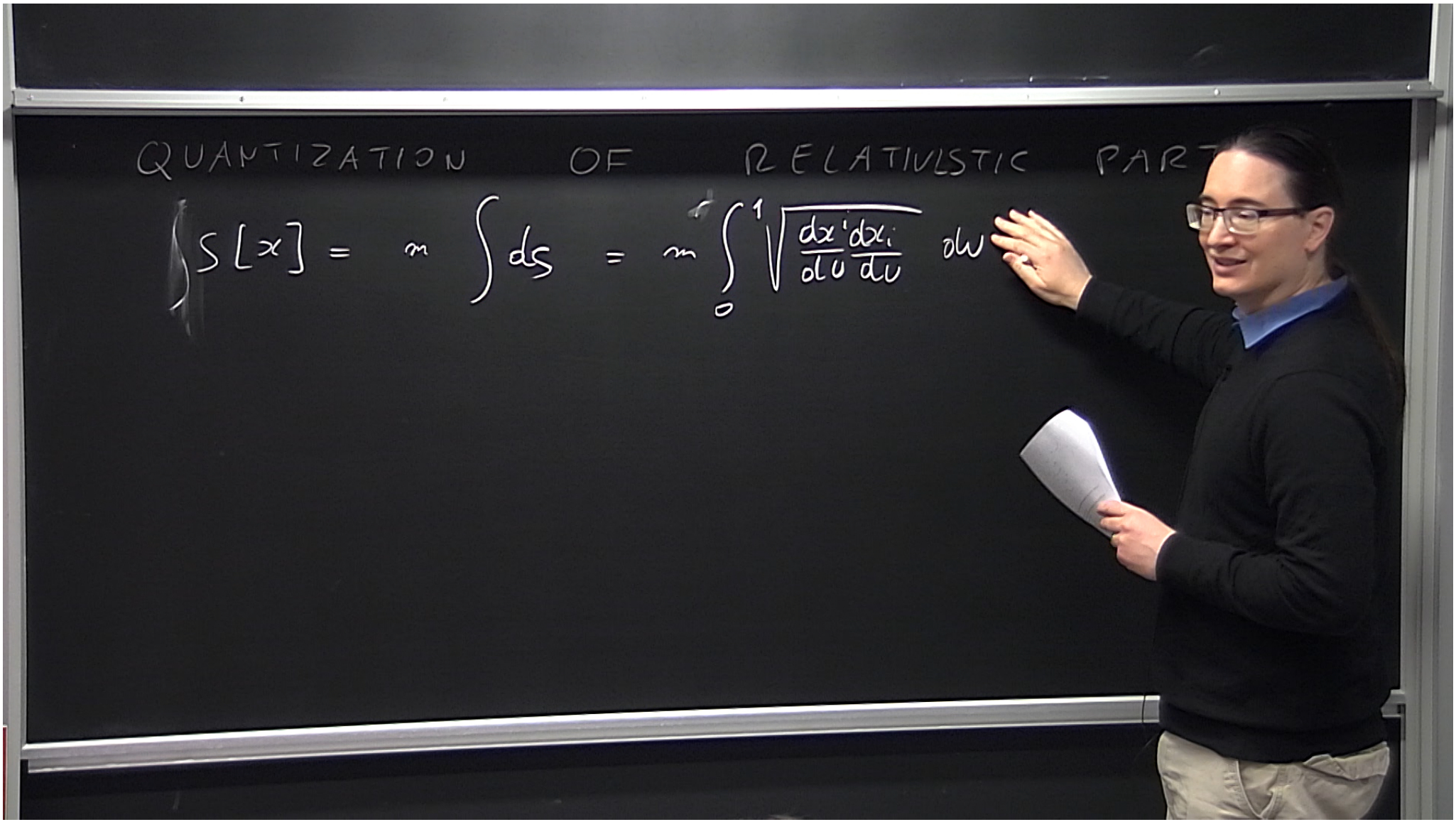
$$A_\Gamma = \prod_a \int dx_a\, e^{i x_a \sum_{b \in \text{EXTERNAL EDGES}} p_{ab}} \prod_{ab} \int_0^\infty dL_{ab}\, e^{-\frac{(x_a - x_b)^2}{4L_{ab}} - L_{ab} m_{ab}^2}$$

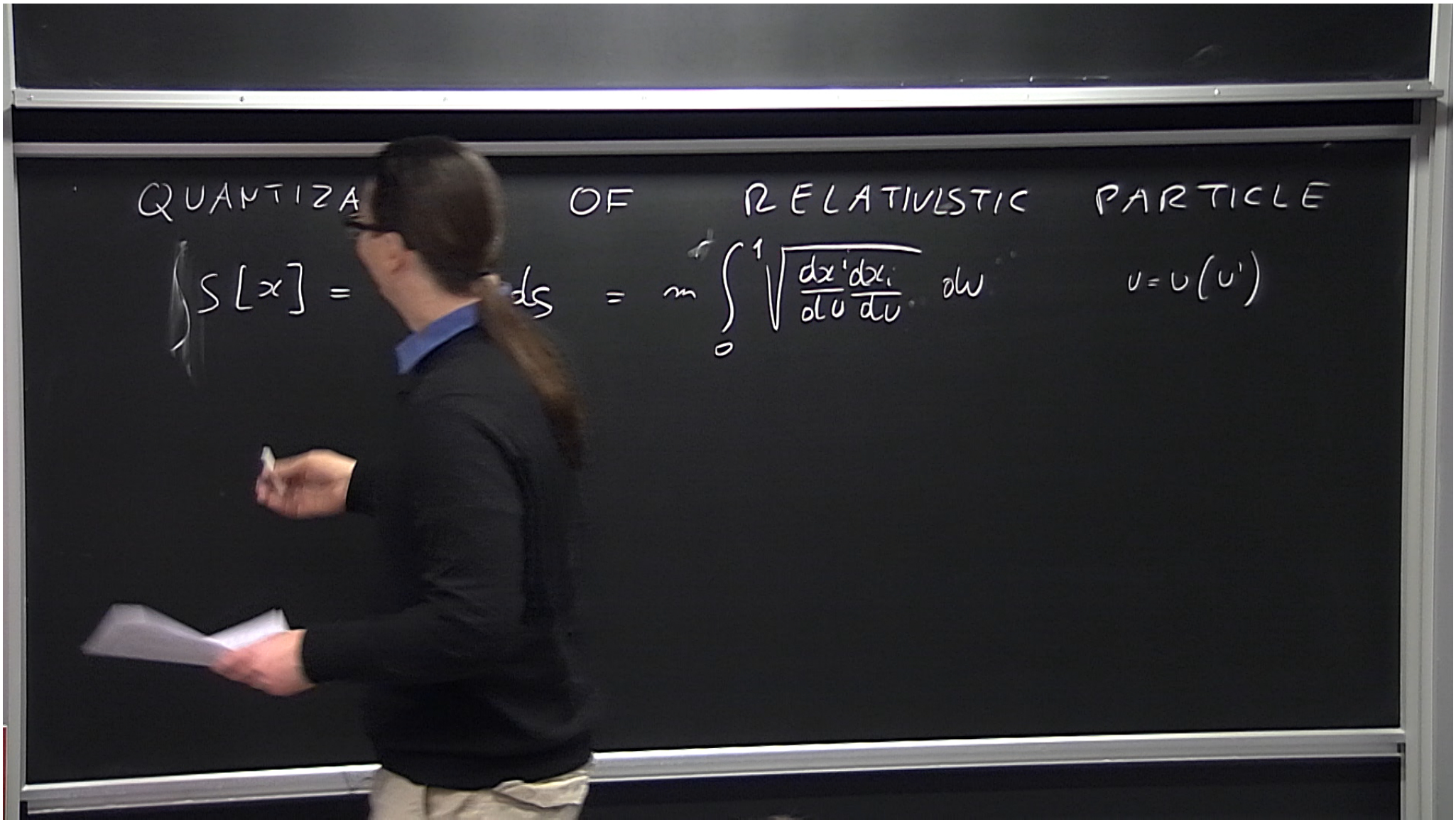
$$A_\Gamma = \int \prod_{ab} dp_{ab}\, dL_{ab}\, e^{-L_{ab}(p_{ab}^2 + m_{ab}^2)} \prod_a dx_a\, e^{i\sum_c x_a p_{ac}}$$

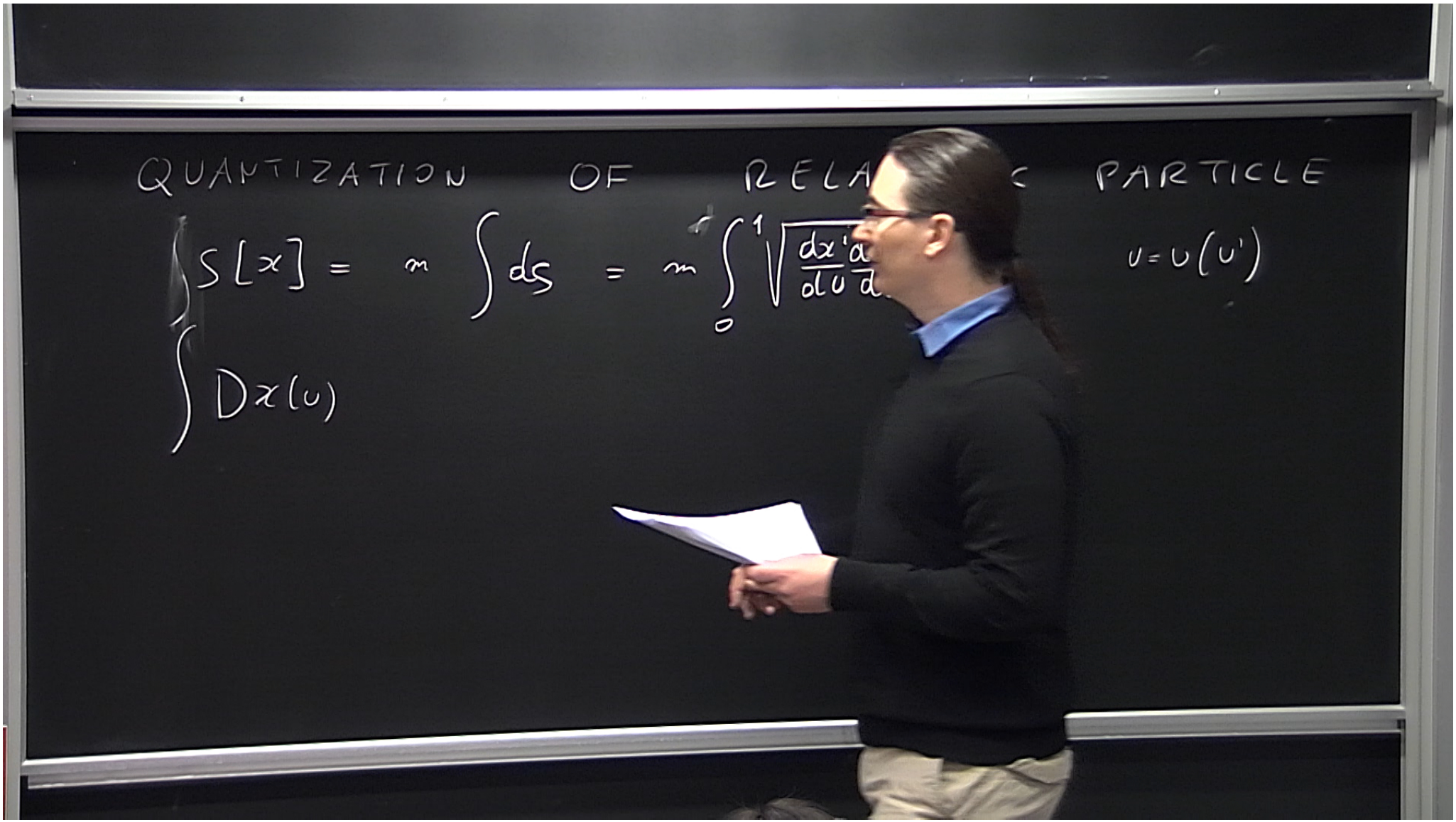
$$\int dp\, e^{-Lp^2 + ixp - Lm^2} = e^{-\frac{x^2}{4L} - Lm^2}$$

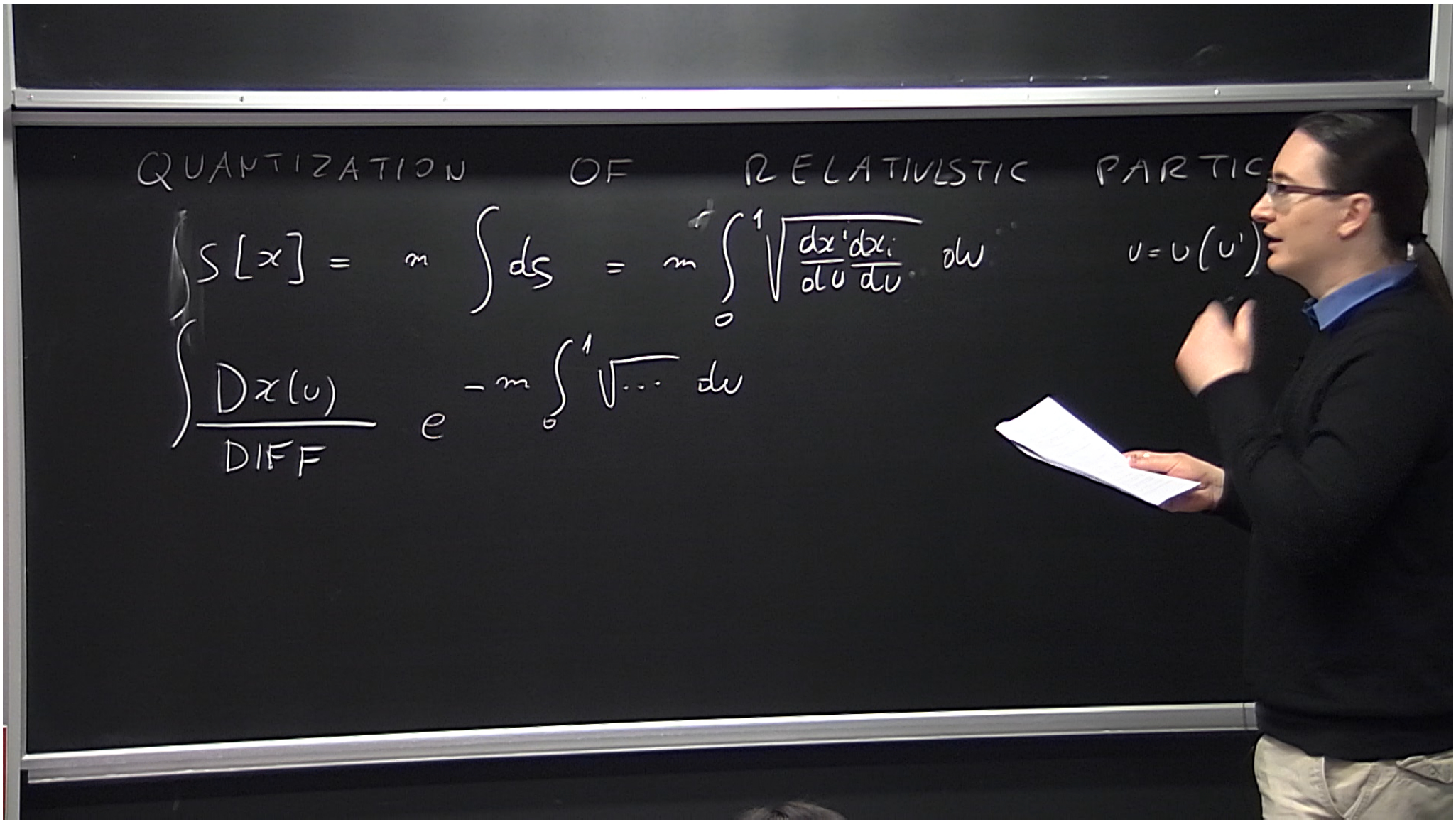
$$A_\Gamma = \prod_a \int dx_a\, e^{i x_a \sum_{b \in \text{EXTERNAL EDGES}} p_{ab}} \prod_{ab} \underbrace{\int_0^\infty dL_{ab}\, e^{-\frac{(x_a - x_b)^2}{4L_{ab}} - L_{ab} m_{ab}^2}}_{G(x_a, x_b)}$$

QUANTIZATION OF RELATIVISTIC PARTICLE


QUANTIZATION OF RELATIVISTIC PART
$S[x] = m \int ds = m \int_0^1 \sqrt{\frac{dx^i}{du}\frac{dx_i}{du}}\, du$


QUANTIZA OF RELATIVISTIC PARTICLE
S[x] = ds = m ∫₀¹ √(dx^i/du dx_i/du) du
u = u(u')


QUANTIZATION OF RELA C PARTICLE
S[x] = m ∫ ds = m ∫ 1 √ dx' d / du d
u = u(u')
∫ Dx(u)

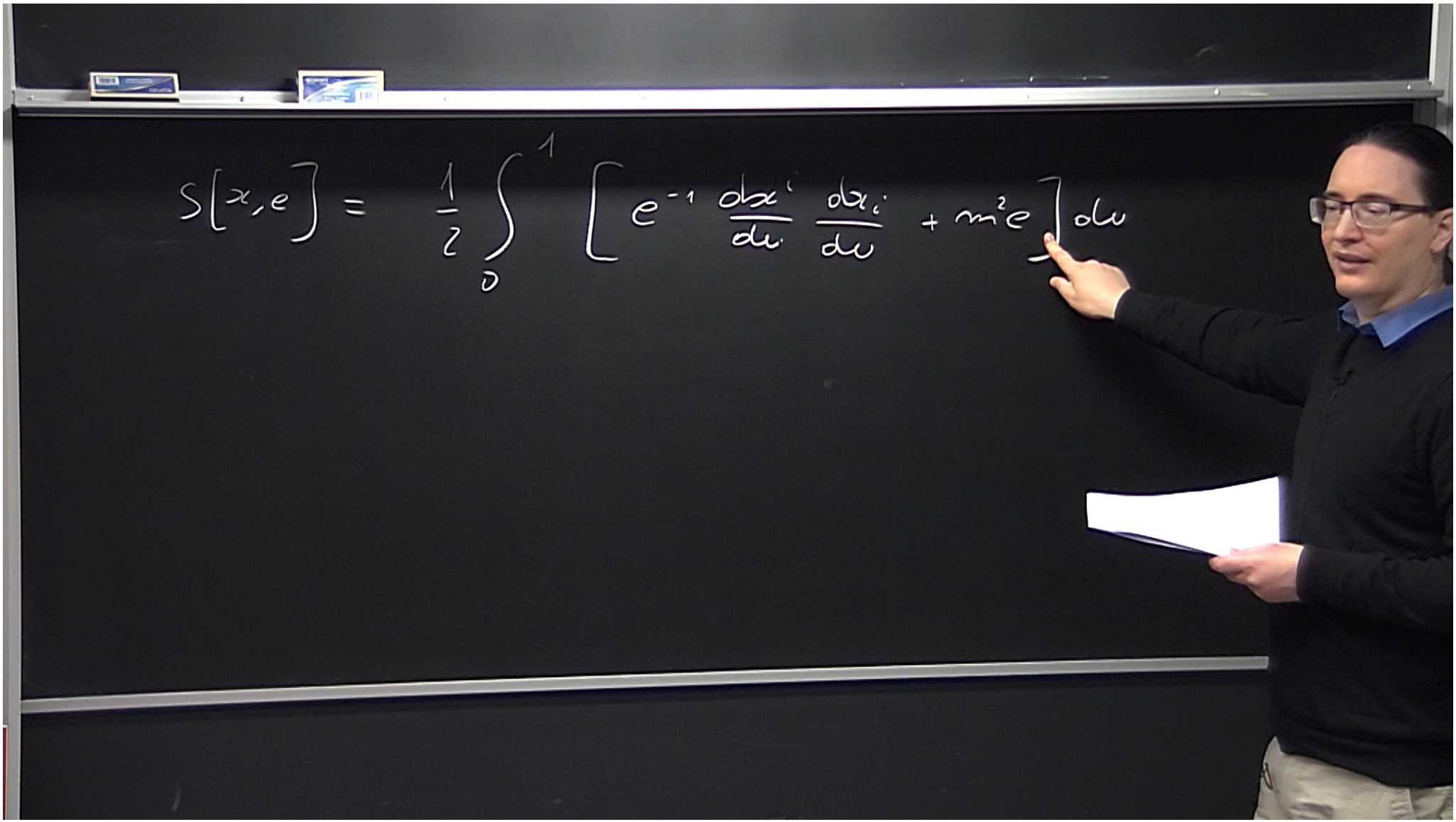

QUANTIZATION OF RELATIVISTIC PARTIC
$S[x] = m \int ds = m \int_0^1 \sqrt{\frac{dx^i}{du}\frac{dx_i}{du}}\, du$
$u = u(u')$
$\int \frac{Dx(u)}{DIFF} e^{-m \int_0^1 \sqrt{\cdots}\, du}$

QUANTIZATION OF RELATIVISTIC PARTICLE

$$S[x] = m \int ds = m \int_0^1 \sqrt{\frac{dx^i}{du}\frac{dx_i}{du}}\, du \qquad u = u(u')$$

$$\int \frac{Dx(u)}{DIFF}\, e^{-m\int_0^1 \sqrt{\cdots}\, du}$$

$$S[x,e] = \frac{1}{2}\int_0^1$$


$S[x,e] = \frac{1}{2}\int_0^1 \left[e^{-1} \frac{dx^i}{du} \frac{dx_i}{du} + m^2 e \right] du$

$$S[x,e] = \frac{1}{2}\int_0^1 \left[e^{-1} \frac{dx^i}{du}\frac{dx_i}{du} + m^2 e \right] du$$

$u = u(u')$

$e\,du = e'\,du'$

$$S[x,e] = \frac{1}{2}\int_0^1 \left[e^{-1} \frac{dx^i}{du} \frac{dx_i}{du} + m^2 e \right] du$$

$u = u(u')$

$e\,du = e'\,du'$

$e^2 \equiv$ 1d metric

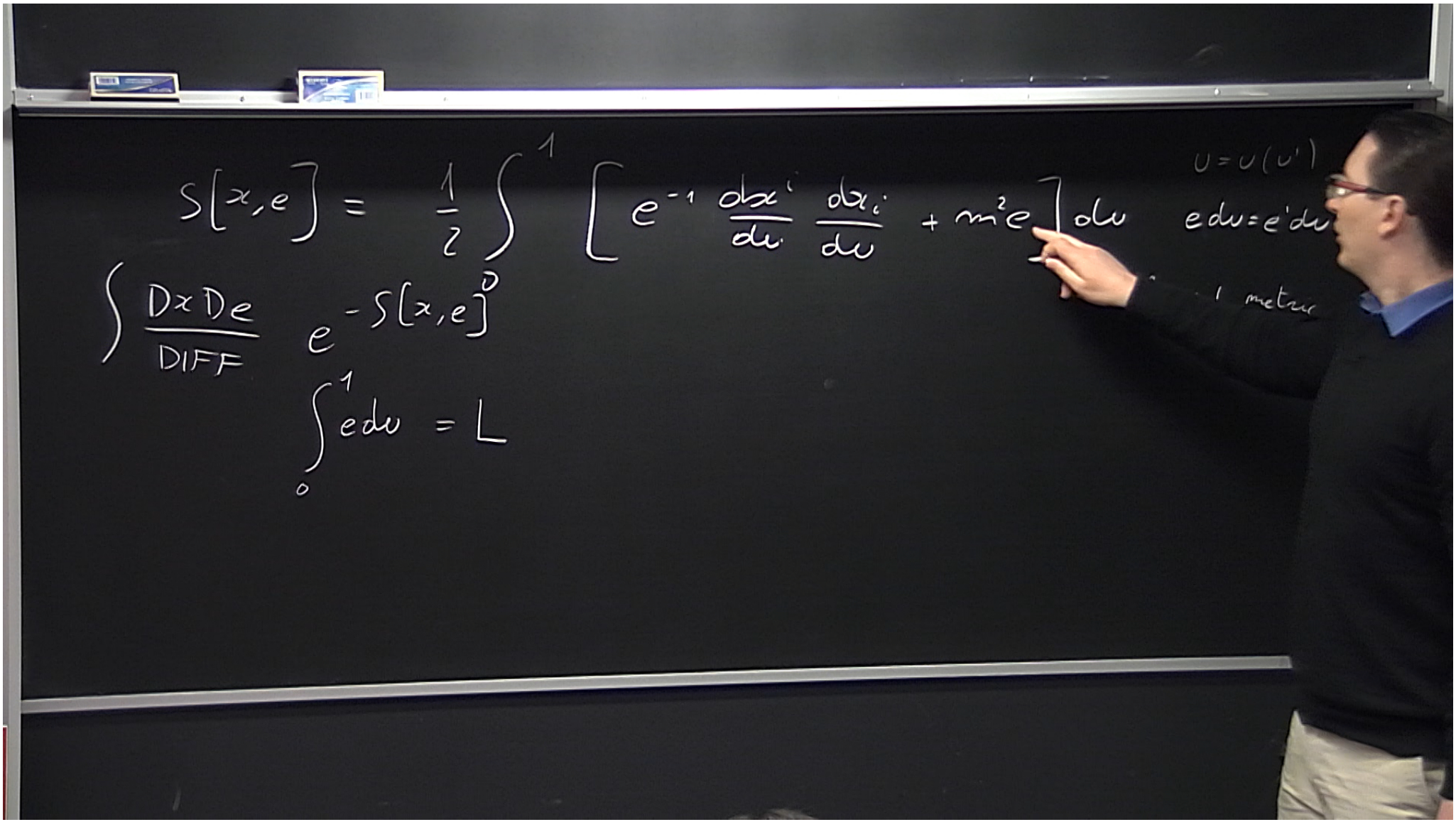
$$S[x,e] = \frac{1}{2}\int^{1} \left[e^{-1} \frac{dx^i}{du} \frac{dx_i}{du} + m^2 e \right] du$$

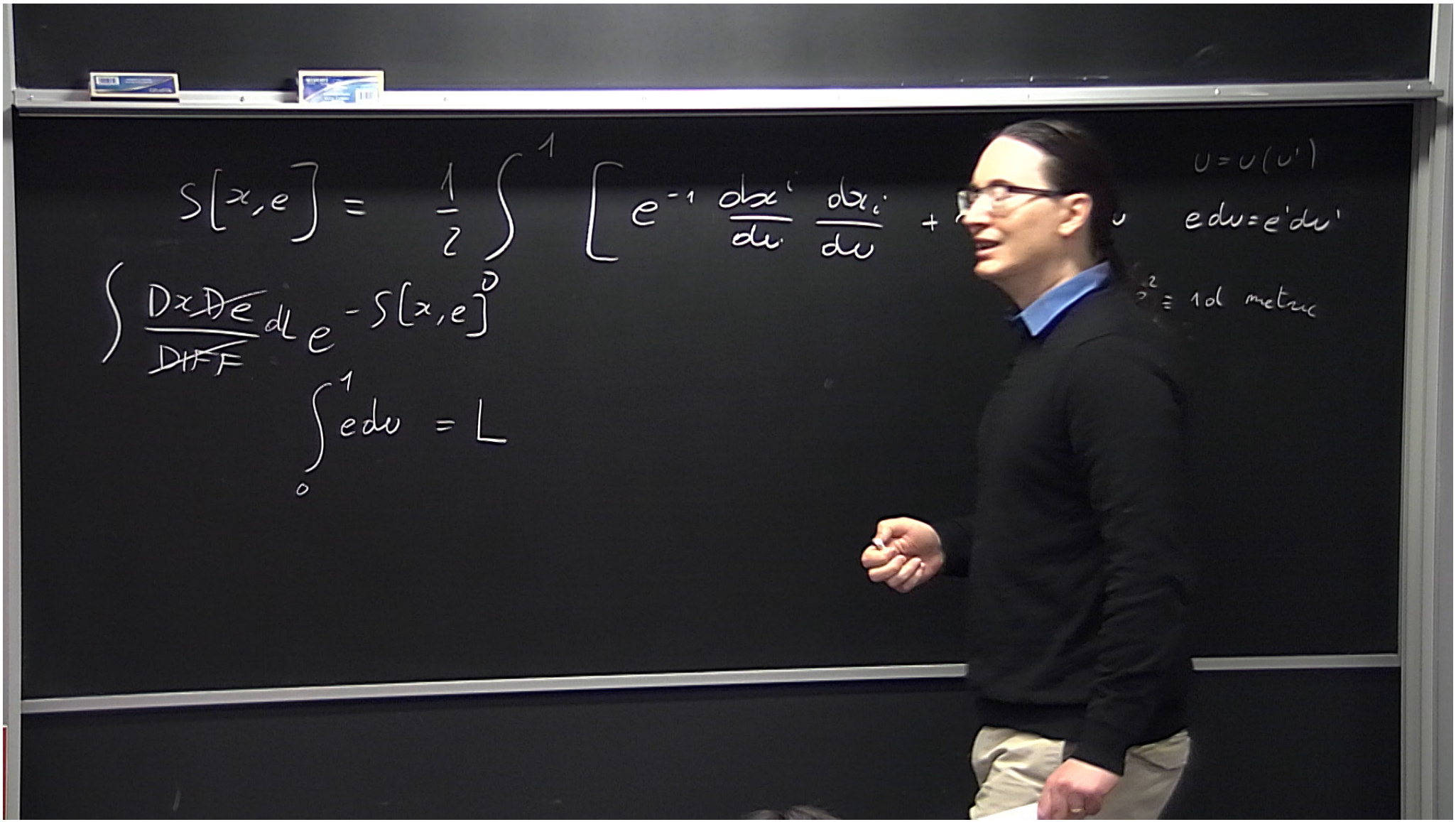
$u = u(u')$

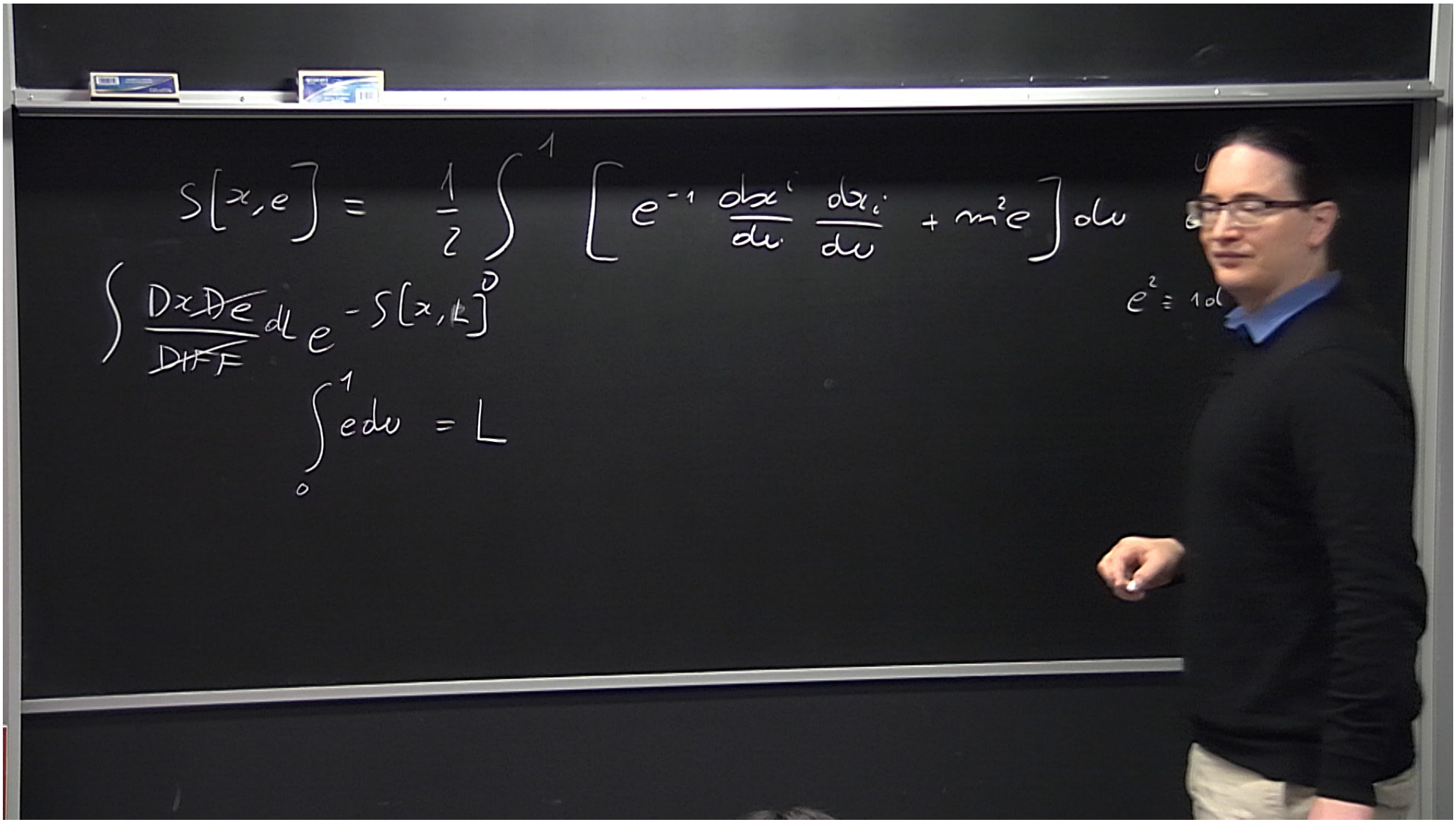
$e\,du = e'\,du'$

$$\int \frac{Dx\,De}{DIFF}\, e^{-S[x,e]}$$

$e^2 \equiv$ 1d metric


$S[x,e] = \frac{1}{2}\int^1 \left[e^{-1} \frac{dx^i}{du} \frac{dx_i}{du} + m^2 e \right] du$
$u = u(u')$
$e\,du = e'\,du'$
1 metric
$\int \frac{Dx\,De}{DIFF} e^{-S[x,e]}$
$\int_0^1 e\,du = L$


$S[x,e] = \frac{1}{2}\int_0^1 \left[e^{-1} \frac{dx^i}{du} \frac{dx_i}{du} + \right.$
$u = u(u')$
$e\,du = e'\,du'$
$\int \frac{Dx\,\cancel{De}}{\cancel{DIFF}}\, dL\, e^{-S[x,e]}$
$\int_0^1 e\,du = L$
$\equiv$ 1d metric


$S[x,e] = \frac{1}{2}\int^1 \left[e^{-1} \frac{dx^i}{du}\frac{dx_i}{du} + m^2 e \right] du$
$\int \frac{Dx\,De}{DIFF} dL\, e^{-S[x,L]}$
$\int_0^1 e\,du = L$
$e^2 = 1$

$$S[x,e] = \frac{1}{2}\int_0^1 \left[e^{-1}\frac{dx^i}{du}\frac{dx_i}{du} + m^2 e \right] du$$

$u = u(u')$

$e\,du = e'\,du'$

$$\int \frac{Dx\,De}{DIFF}\, dL\, e^{-S[x,L]}$$

$e^2 \equiv$ 1d metric

$$S[x,L] = \frac{1}{2}\int_0^L \left[\frac{dx^i}{du}\frac{dx_i}{du} + m^2 \right] du$$

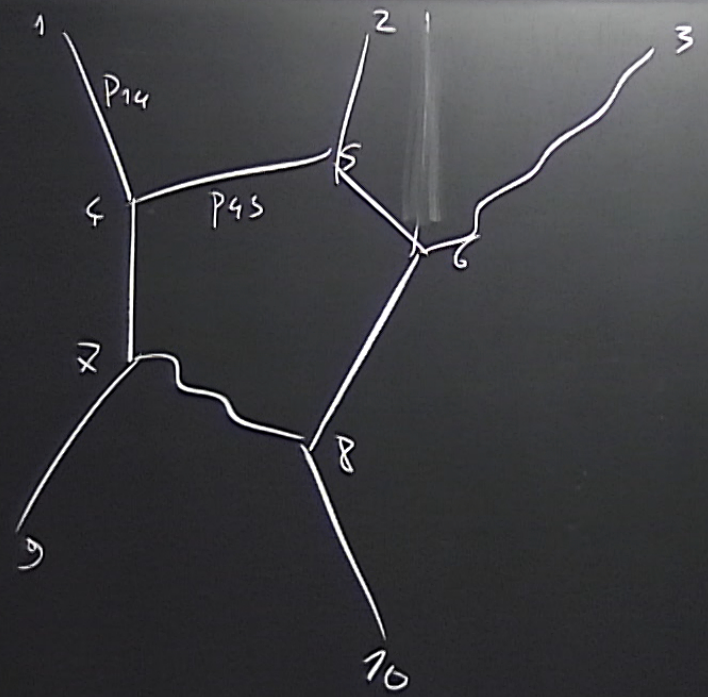
$$\int_0^1 e\,du = L$$

$$A_{\Gamma}^{QFT} = \int_{\mathcal{M}[\Gamma]} d\ell_{ab} \int_{x:\Gamma \to \mathbb{R}^{d,1}} e^{-S[x,\ell_{ab}]}$$

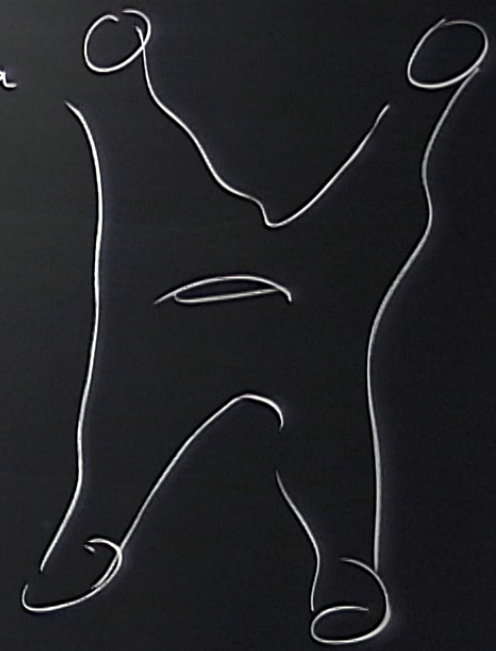
$$A_{\Sigma}^{STRING} = \int_{\mathcal{M}[\Sigma]} d\mu_i \int_{x:\Sigma \to \mathbb{R}^{d,1}} e^{-S[x,\mu_i]}$$

$$S[x,e] = \frac{1}{2}$$

Γ



$$p_{ab} = -p_{ba}$$



$u = u(u')$

QUANTIZATION OF RELATIVISTIC PARTICLE

$$\boxed{S[x] = m \int ds} = m \int_0^1 \sqrt{\frac{dx^i}{du}\frac{dx_i}{du}}\, du \qquad u = u(u')$$

$$\int \frac{Dx(u)}{\text{DIFF}}\, e^{-m \int_0^1 \sqrt{\cdots}\, du}$$

$$S[x,e] = \frac{1}{2} \int^1$$

$$S(X) = T \int da = T \int \sqrt{\det \frac{dX^i}{du^a} \frac{dX_i}{du^b}}\, du^1 du^2$$

$$S(X) = T \int da = T \int \sqrt{\det \frac{\partial X^i}{\partial u^a} \frac{\partial X_i}{\partial u^b}}\, du^1 du^2$$

$$g_{ab} = \frac{\partial X^i}{\partial u^a} \frac{\partial X^j}{\partial u^b} g_{ij}$$

$$S(X) = T \int da = T \int \sqrt{\det \frac{\partial X^i}{\partial u^a} \frac{\partial X_i}{\partial u^b}}\, du^1 du^2$$

$$g_{ab}^{(\text{INDUCED})} = \frac{\partial X^i}{\partial u^a} \frac{\partial X^j}{\partial u^b} g_{ij}$$

$$u^1 \to u'(u^1)$$

$$\frac{\partial X^i}{\partial u^a} = \frac{\partial X^i}{\partial u'^b} \frac{\partial u'^b}{\partial u^a}$$

POLYAKOV

$h = \det h_{ab}$ $\quad h^{ab} h_{bc} = \delta^a{}_c$

$$S[X, h_{ab}] = \frac{T}{2} \int \sqrt{h}\, h^{ab} \frac{\partial X^i}{\partial u^a} \frac{\partial X_i}{\partial u^b}\, du^1 du^2$$

POLYAKOV

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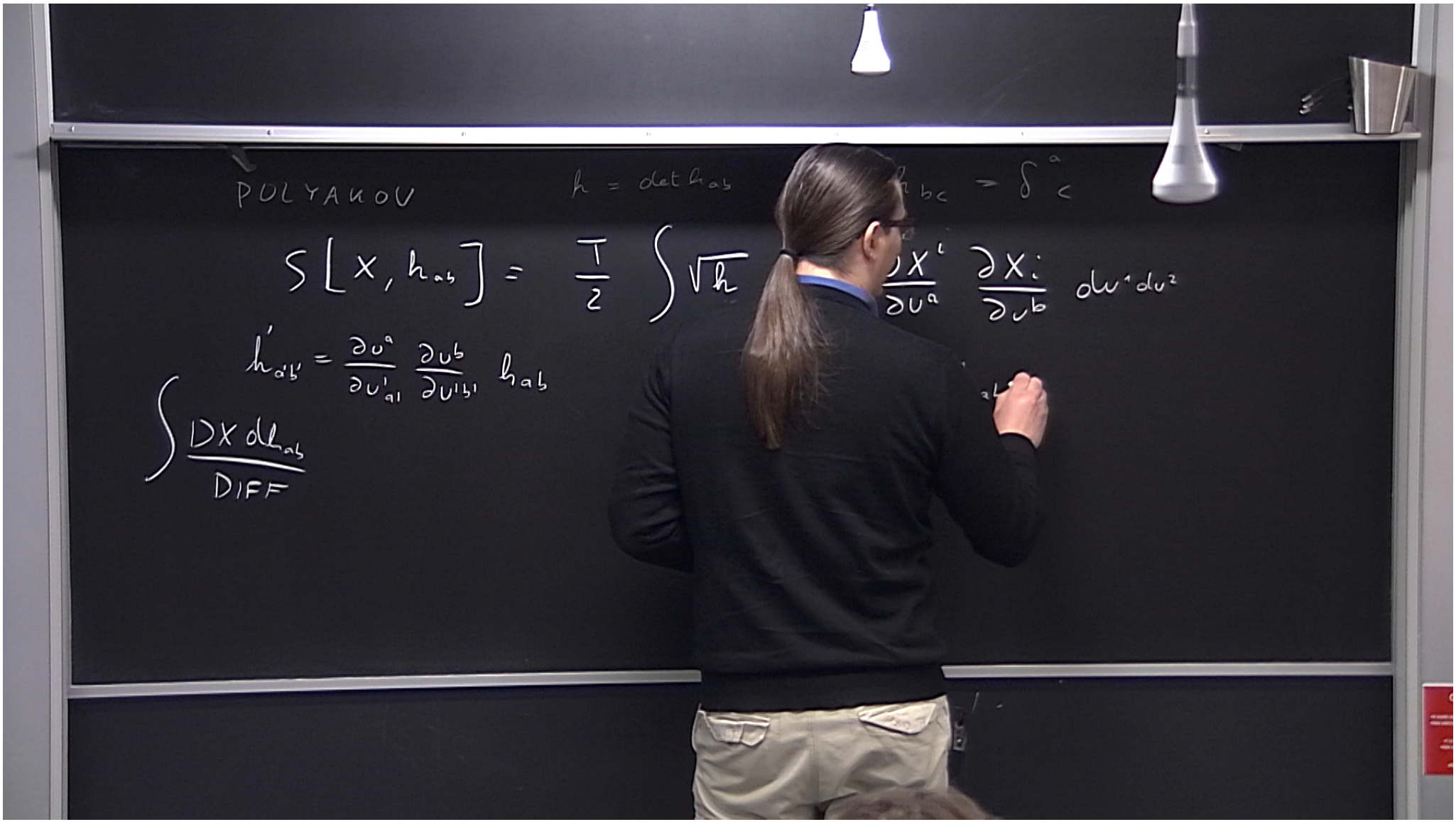
$$h'_{a'b'} = \frac{\partial u^a}{\partial u'_{a'}} \frac{\partial u^b}{\partial u'_{b'}} h_{ab}$$


POLYAKOV
h = det h_ab
S[X, h_ab] = T/2 ∫ √h
h'_a'b' = ∂u^a/∂u'_a' ∂u^b/∂u'_b' h_ab
∫ DX dh_ab / DIFF

POLYAKOV

$h = \det h_{ab}$ $\quad h^{ab} h_{bc} = \delta^a{}_c$

$$S[X, h_{ab}] = \frac{T}{2} \int \sqrt{h}\, h^{ab} \frac{\partial X^i}{\partial u^a} \frac{\partial X_i}{\partial u^b}\, du^1 du^2$$

$$h'_{a'b'} = \frac{\partial u^a}{\partial u'_{a'}} \frac{\partial u^b}{\partial u'_{b'}} h_{ab}$$

LOCALLY $\quad h_{ab} = e^{\phi} \delta_{ab}$

$$\int \frac{DX\, dh_{ab}}{\text{DIFF}}$$

POLYAKOV

$h = \det h_{ab}$ $\quad h^{ab} h_{bc} = \delta^a{}_c$

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$$\frac{T}{2} \int \frac{\partial X^i}{\partial u^a} \frac{\partial X_i}{\partial u_a} du^1 du^2$$

POLYAKOV $\quad h = \det h_{ab} \quad h^{ab} h_{bc} = \delta^a{}_c$

$$S[X, h_{ab}] = \frac{T}{2} \int \sqrt{h}\, h^{ab} \frac{\partial X^i}{\partial u^a} \frac{\partial X_i}{\partial u^b} du^1 du^2$$

DIFF : $h'_{a'b'} = \frac{\partial u^a}{\partial u'_{a'}} \frac{\partial u^b}{\partial u'_{b'}} h_{ab}$

$$\int \frac{DX\, dh_{ab}}{\text{DIFF}}$$

WEYL $\quad h_{ab} \to e^{\phi} h_{ab}$

LOCALLY $\quad h_{ab} = e^{\phi} \delta_{ab}$

$$\frac{T}{2} \int \frac{\partial X^i}{\partial u^a} \frac{\partial X_i}{\partial u_a} du^1 du^2$$

POLYAKOV $\quad h = \det h_{ab} \quad h^{ab} h_{bc} = \delta^a{}_c$

$$S[X, h_{ab}] = \frac{T}{2} \int \sqrt{h}\, h^{ab} \frac{\partial X^i}{\partial u^a} \frac{\partial X_i}{\partial u^b} du^1 du^2$$

DIFF : $h'_{a'b'} = \frac{\partial u^a}{\partial u'_{a'}} \frac{\partial u^b}{\partial u'_{b'}} h_{ab}$

$$\int \frac{DX\, dh_{ab}}{\text{DIFF} \times \text{WEYL}}$$

WEYL $\quad h_{ab} \to e^{\phi} h_{ab}$

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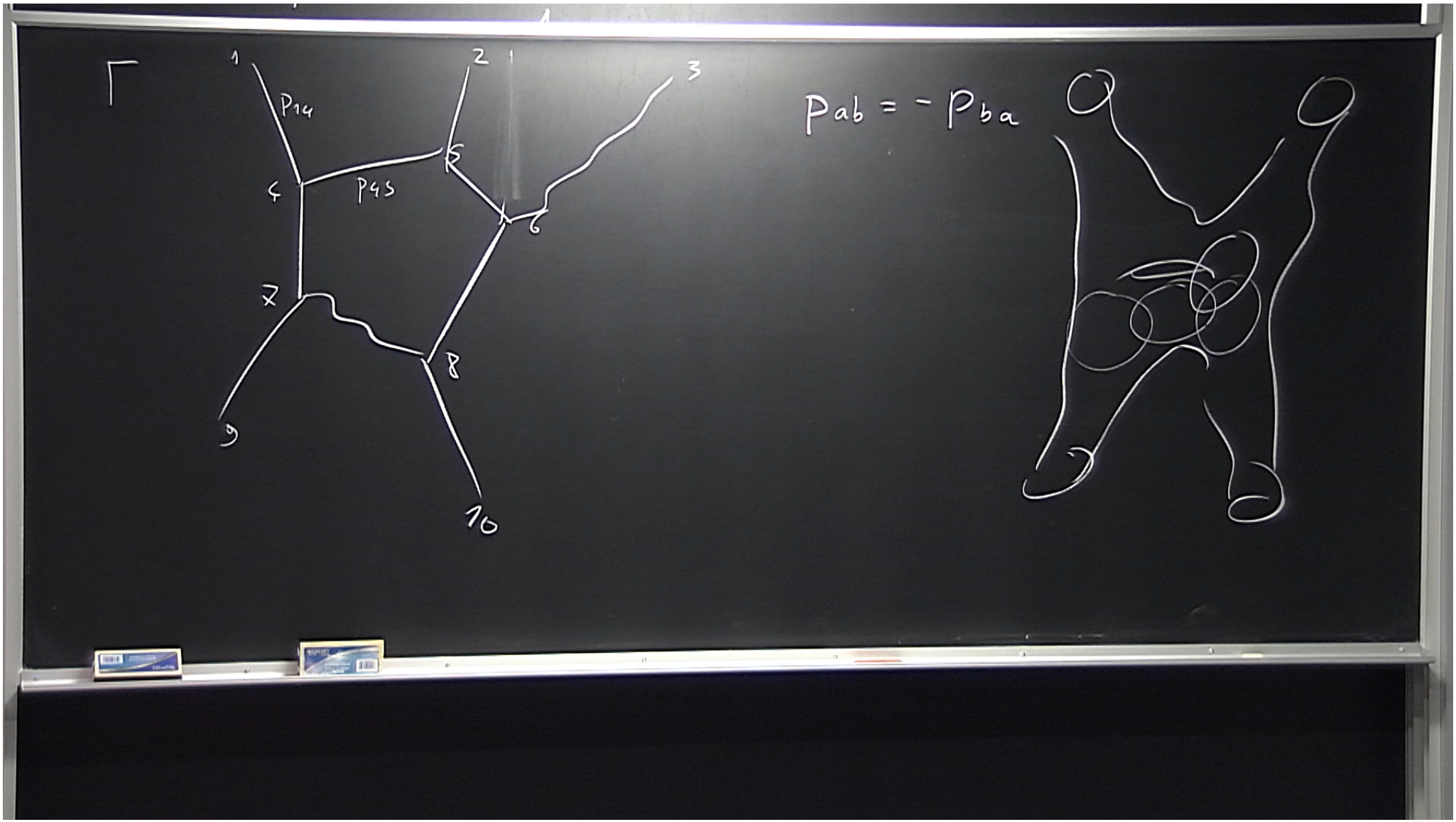
$$\frac{T}{2} \int \frac{\partial X^i}{\partial u^a} \frac{\partial X_i}{\partial u_a} du^1 du^2$$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POLYAKOV

$h = \det h_{ab}$ $\quad h^{ab} h_{bc} = \delta^a{}_c$

$$S[X, h_{ab}] = \frac{T}{2} \int \sqrt{h}\, h^{ab} \frac{\partial X^i}{\partial u^a} \frac{\partial X_i}{\partial u^b} du^1 du^2$$

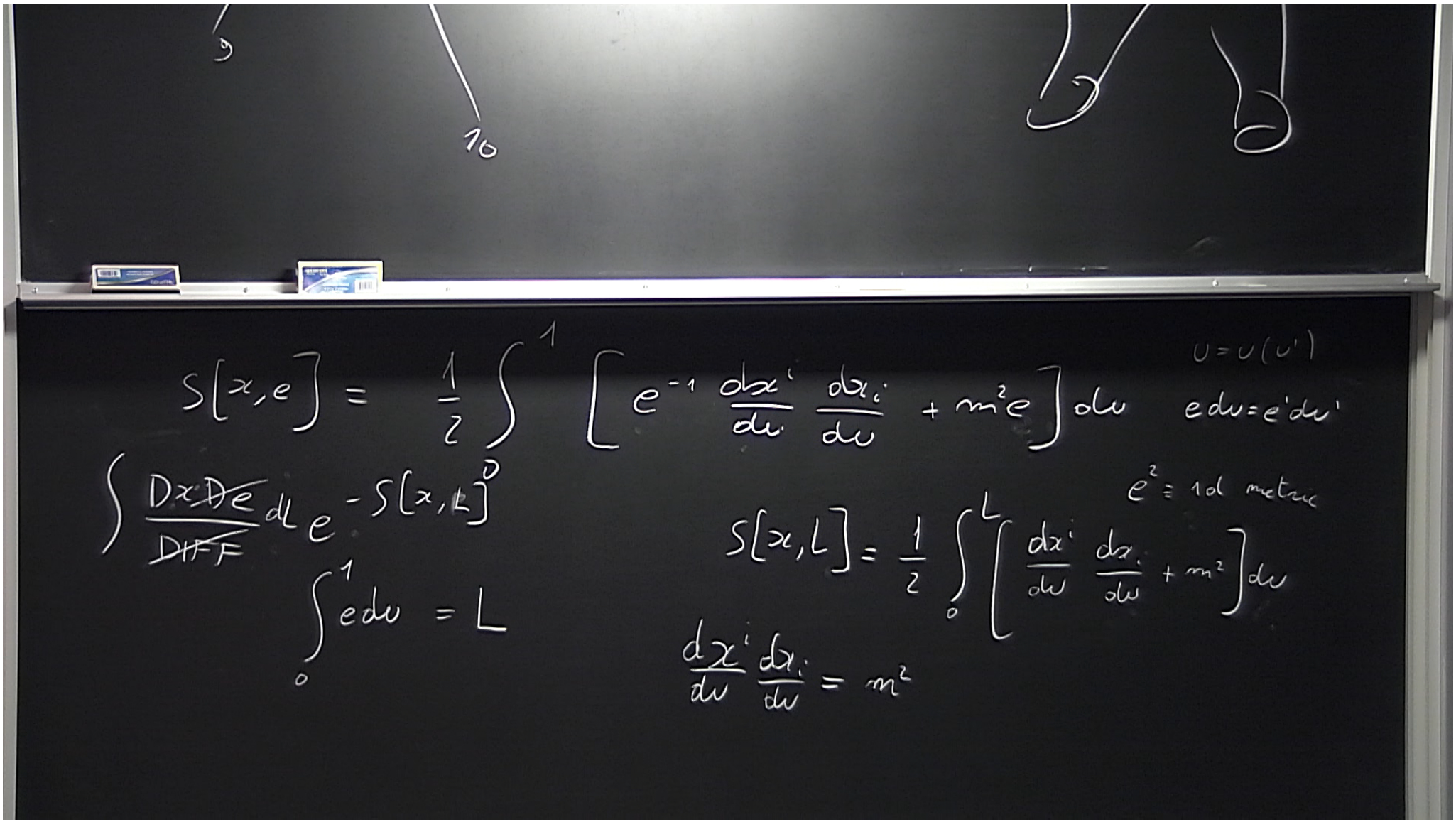
DIFF : $h'_{a'b'} = \frac{\partial u^a}{\partial u'_{a'}} \frac{\partial u^b}{\partial u'_{b'}} h_{ab}$

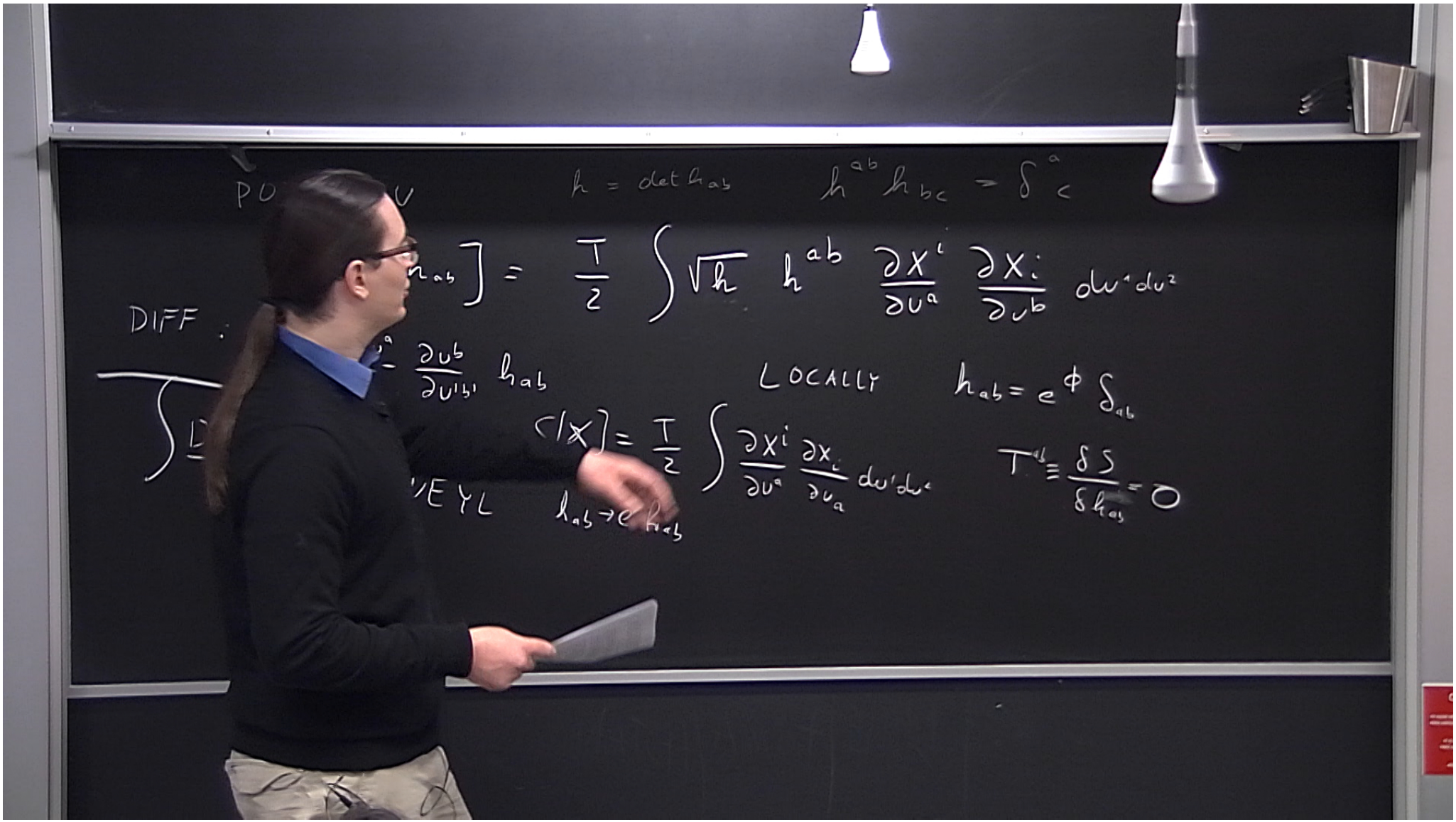
$$\boxed{\int \frac{DX\, dh_{ab}}{\text{DIFF} \times \text{WEYL}}}$$

WEYL $\quad h_{ab} \to e^{\phi} h_{ab}$

LOCALLY $\quad h_{ab} = e^{\phi} \delta_{ab}$

$$S[X] = \frac{T}{2} \int \frac{\partial X^i}{\partial u^a} \frac{\partial X_i}{\partial u_a} du^1 du^2$$


$S[x,e] = \frac{1}{2}\int_0^1 \left[e^{-1} \frac{dx^i}{du}\frac{dx_i}{du} + m^2 e \right] du$
$u = u(u')$
$e\,du = e'\,du'$
$\int \frac{Dx\,De}{DIFF}\, dL\, e^{-S[x,L]}$
$e^2 \equiv$ 1d metric
$S[x,L] = \frac{1}{2}\int_0^L \left[\frac{dx^i}{du}\frac{dx_i}{du} + m^2 \right] du$
$\int_0^1 e\,du = L$
$\frac{dx^i}{du}\frac{dx_i}{du} = m^2$


h = det h_ab
h^ab h_bc = δ^a_c
DIFF
LOCALLY
h_ab = e^Φ δ_ab
T^ab ≡ δS / δh_ab = 0
h_ab → e h_ab

POLYAKOV $\quad h = \det h_{ab} \quad h^{ab} h_{bc} = \delta^a{}_c$

$$S[X, h_{ab}] = \frac{T}{2} \int \sqrt{h}\, h^{ab} \frac{\partial X^i}{\partial u^a} \frac{\partial X_i}{\partial u^b} du^1 du^2$$

DIFF: $h'_{a'b'} = \frac{\partial u^a}{\partial u'_{a'}} \frac{\partial u^b}{\partial u'_{b'}} h_{ab}$

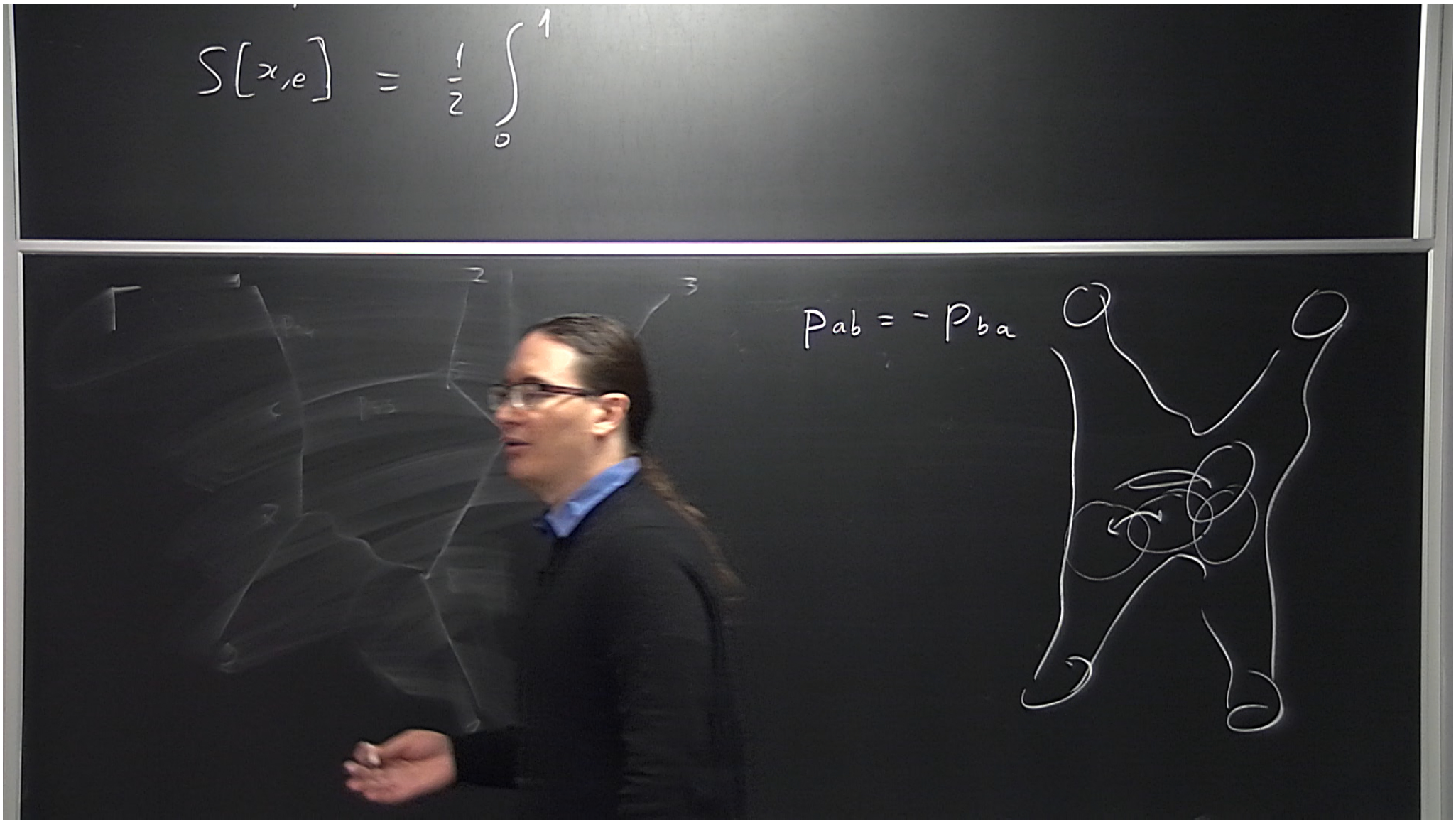
$$\int \frac{DX\, dh_{ab}}{\text{DIFF} \times \text{WEYL}}$$

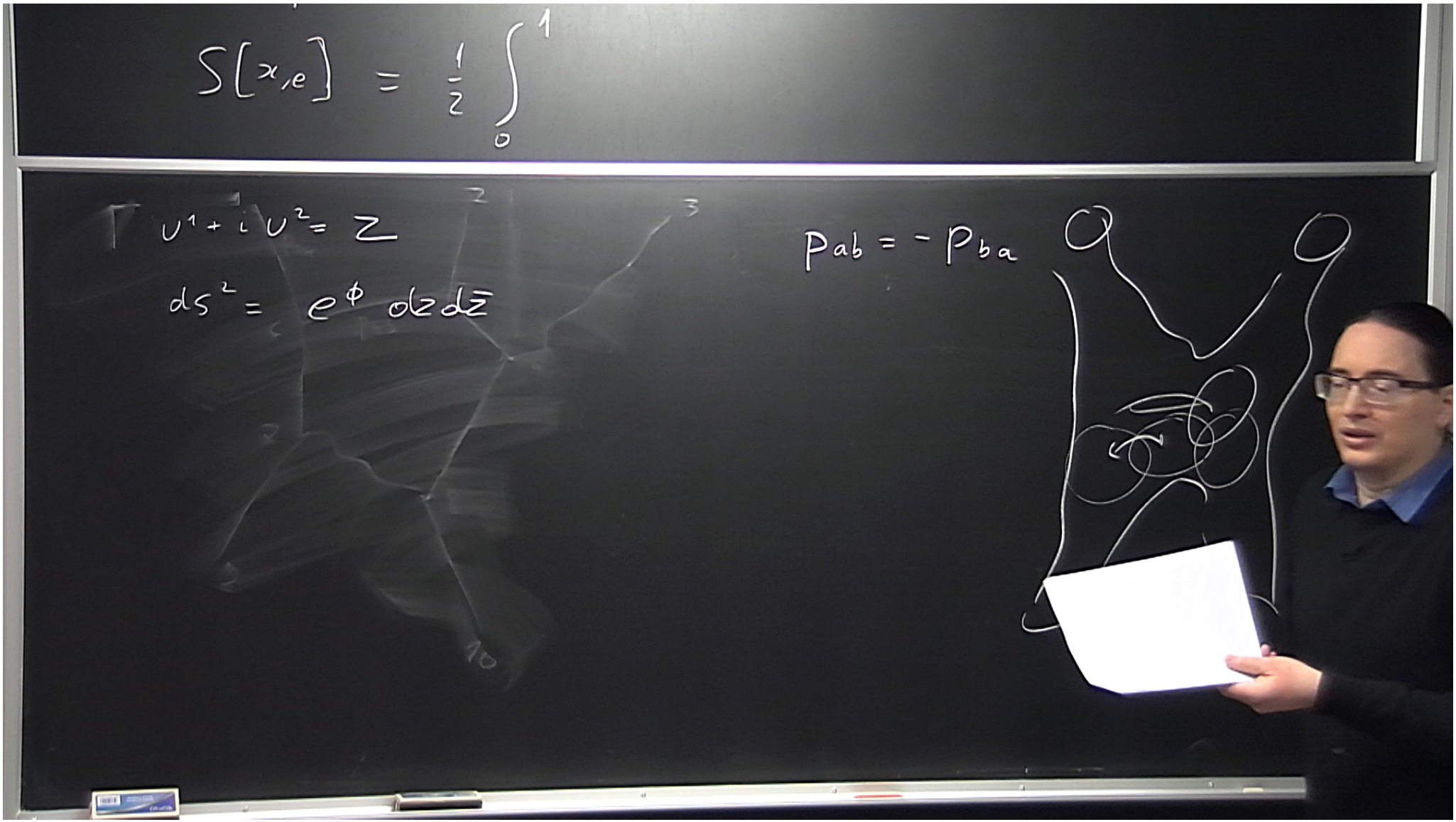
WEYL $\quad h_{ab} \to e^{\phi} h_{ab}$

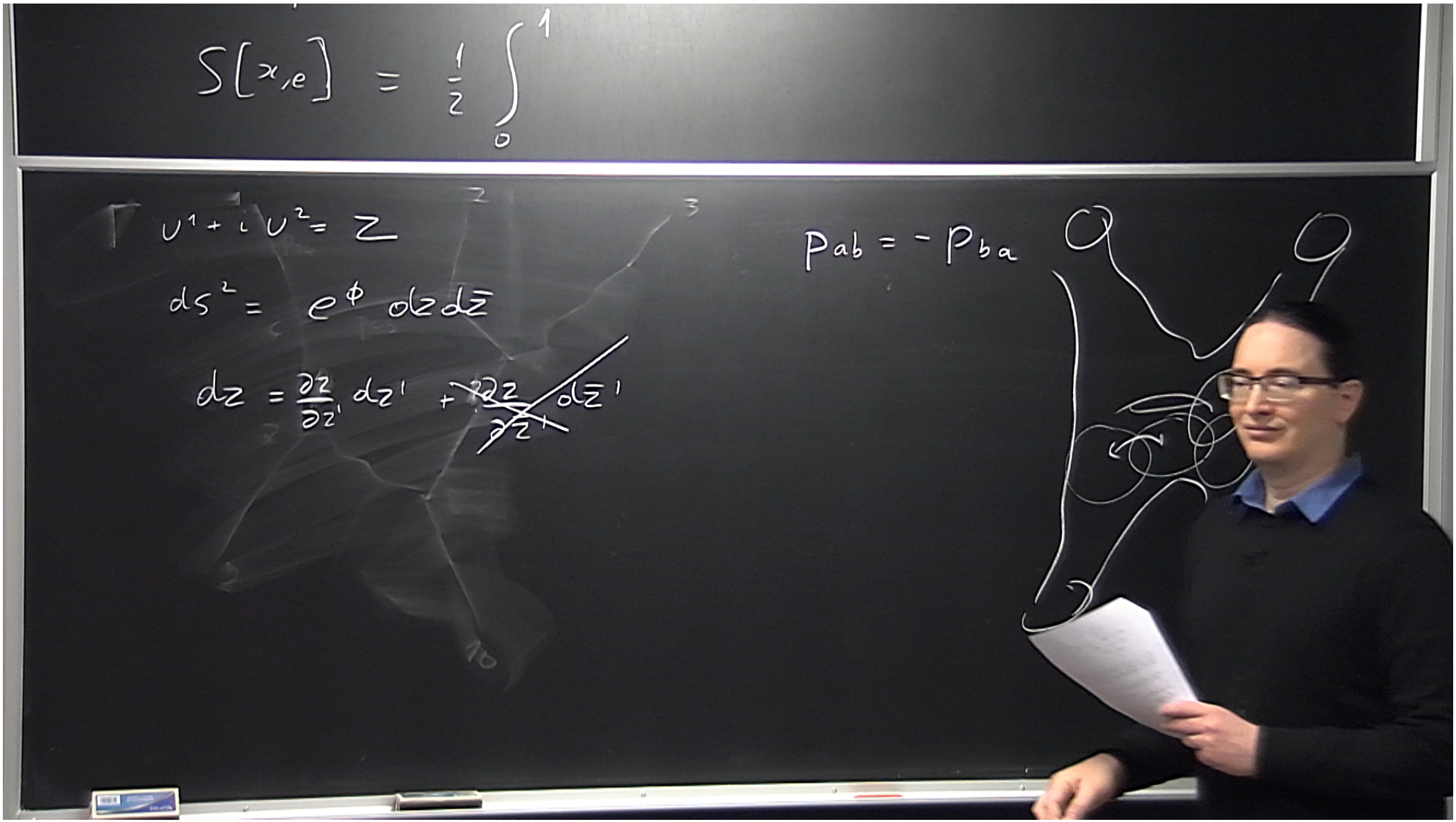
LOCALLY $\quad h_{ab} = e^{\phi} \delta_{ab}$

$$S[X] = \frac{T}{2} \int \frac{\partial X^i}{\partial u^a} \frac{\partial X_i}{\partial u_a} du^1 du^2$$

$$T^{ab} \equiv \frac{\delta S}{\delta h_{ab}} = 0$$


S[x,e] = 1/2 ∫_0^1
Pab = - Pba


$S[x,e] = \frac{1}{2}\int_0^1$
$u^1 + i u^2 = z$
$ds^2 = e^{\phi} dz d\bar{z}$
$p_{ab} = -p_{ba}$


$S[x,e] = \frac{1}{2}\int_0^1$
$u^1 + i u^2 = z$
$ds^2 = e^{\phi} dz d\bar{z}$
$dz = \frac{\partial z}{\partial z'} dz' + \cancel{\frac{\partial z}{\partial \bar{z}'} d\bar{z}'}$
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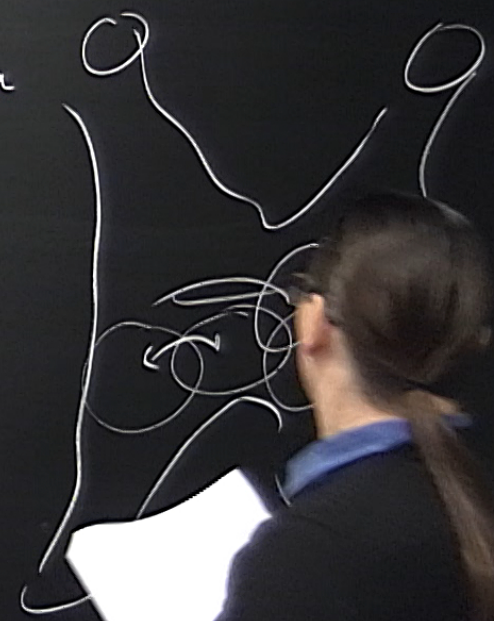

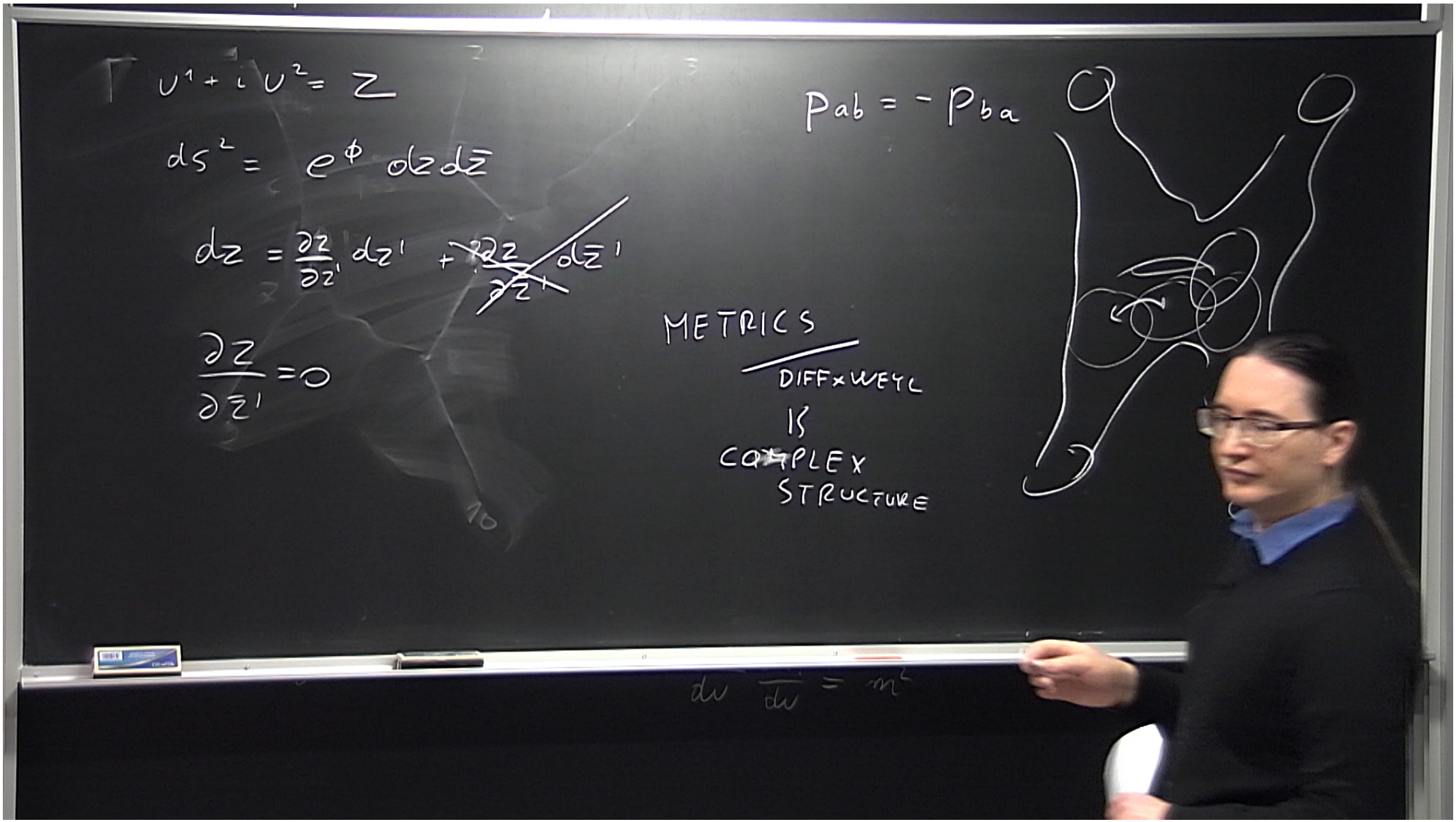
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METRICS / DIFF × WEYL
IS
COMPLEX STRUCTURE
$dw \, d\bar{w} = m^2$

$$A_{\Gamma}^{QFT} = \int_{\mathcal{M}[\Gamma]} d\ell_{ab} \int_{x:\Gamma \to R^{d,1}} e^{-S[x,\ell_{ab}]}$$

$$\boxed{A_{\Sigma}^{STRING} = \int_{\mathcal{M}[\Sigma]} d\mu_i \int_{x:\Sigma \to R^{d,1}} e^{-S[x,\mu_i]}}$$