Title: PSI 17/18 - Quantum Field Theory III - Lecture 15

Date: Feb 15, 2018 11:30 AM

URL: http://pirsa.org/18020035

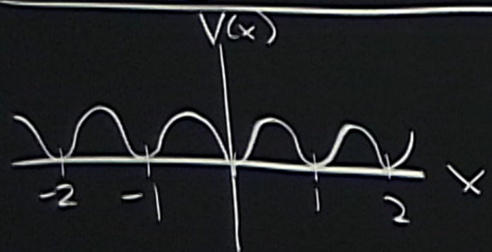
Abstract:

Today: Instantons in QM (continued)

Decay of False Vacuum in Scalar QFT

Resummation + Instantons

Periodic Potential



n instantons

$\bar{n}$ anti instantons

$$\langle x=\ell | e^{-HT/\hbar} | x=k \rangle = \overbrace{\left(\frac{\omega}{\pi\hbar}\right)^{1/2} e^{-\omega T/2}}^{C} \sum_{n=0}^{\infty} \sum_{\bar{n}=0}^{\infty} \frac{1}{n!}\frac{1}{\bar{n}!} \left(K T e^{-S_0/\hbar}\right)^{n+\bar{n}} \cdot \delta_{n-\bar{n},\, \ell-k}$$

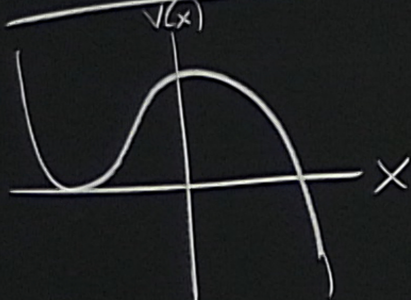
use $\delta_{a,b} = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{-i\theta(a-b)}$

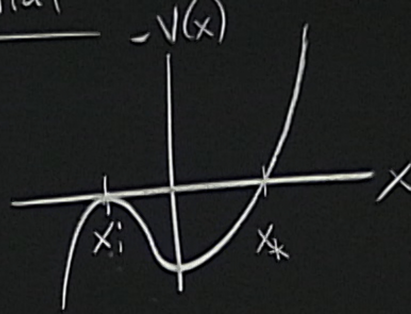
$$\overbrace{\langle x=\ell | e^{-HT/\hbar} | x=k \rangle}^{Z} = C \int_0^{2\pi} \frac{d\theta}{2\pi}\, e^{-i\theta(k-\ell)} \underbrace{\sum_{n,\bar{n}} \left(kT e^{-S_0/\hbar}\right)^{n+\bar{n}} e^{i\theta(n-\bar{n})}}_{=\exp\left[2kT\cos\theta\, e^{-S_0/\hbar}\right]}$$

$$E(\theta) = \frac{1}{2}\hbar\omega + 2\hbar k \cos\theta\, e^{-S_0/\hbar}$$

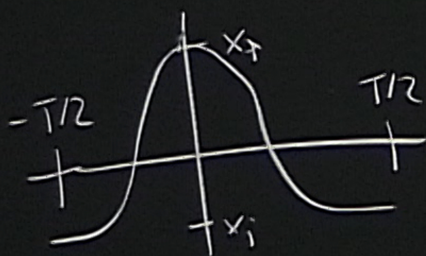
$$E_0 = -\lim_{T\to\infty} \frac{\hbar}{T} \ln Z$$

Metastable Potential





$\langle x_i | e^{-HT/\hbar} | x_i \rangle$

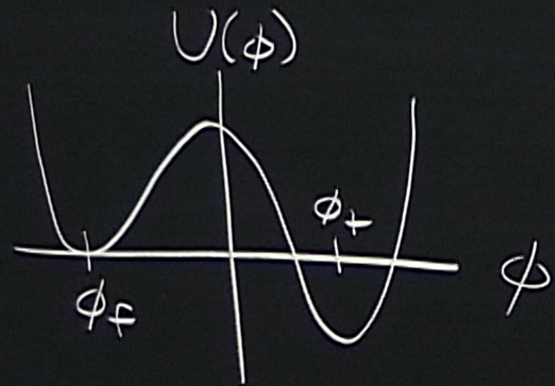


"bounce"

k will be purely imaginary (one negative eigenvalue of S'')

same produre → $\Gamma = |k| e^{-S_E[\bar{x}]/\hbar}$

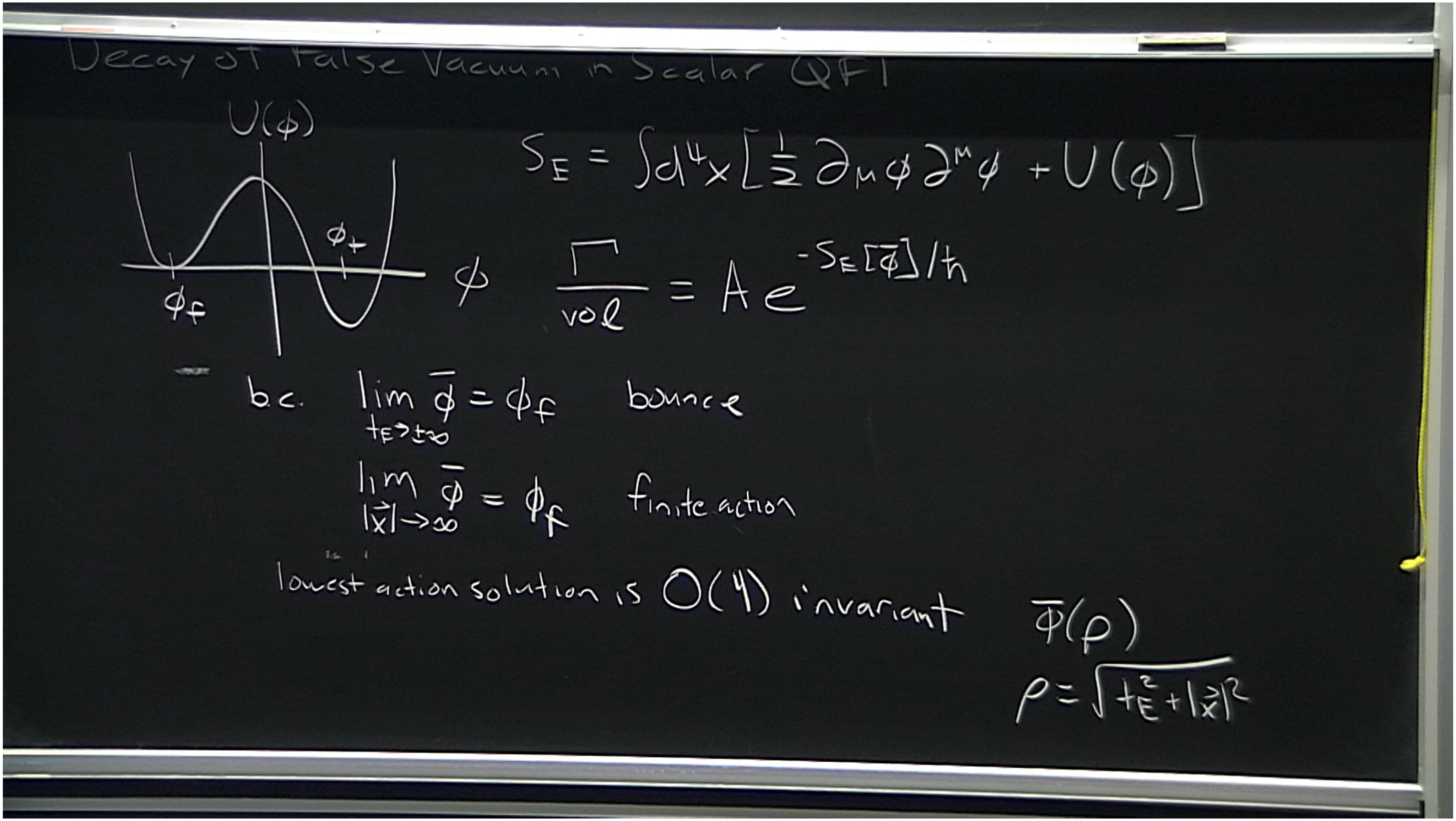
Decay of False Vacuum in Scalar QFT



$$S_E = \int d^4x \left[\frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + U(\phi) \right]$$

$$\frac{\Gamma}{\mathrm{vol}} = A \, e^{-S_E[\bar{\phi}]/\hbar}$$

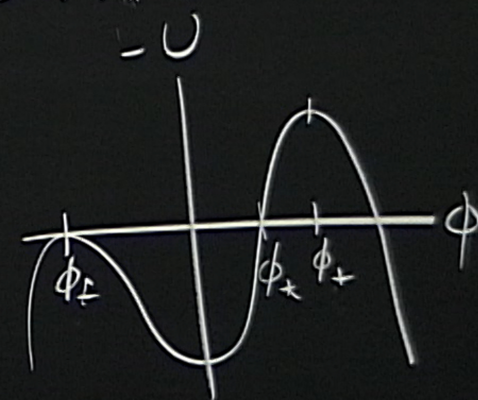
b.c. $\lim_{t_E \to \pm\infty} \bar{\phi} = \phi_f$


Decay of False Vacuum in Scalar QFT
$U(\phi)$
ϕ_+
ϕ_f
ϕ
$S_E = \int d^4x \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + U(\phi) \right]$
$\frac{\Gamma}{vol} = A e^{-S_E[\bar{\phi}]/\hbar}$
b.c. $\lim_{t_E \to \pm\infty} \bar{\phi} = \phi_f$ bounce
$\lim_{|\vec{x}| \to \infty} \bar{\phi} = \phi_f$ finite action
lowest action solution is $O(4)$ invariant
$\bar{\Phi}(\rho)$
$\rho = \sqrt{t_E^2 + |\vec{x}|^2}$

$$\frac{d^2\Phi}{d\rho^2} + \frac{3}{\rho}\frac{d\Phi}{d\rho} = U'(\Phi) \qquad \text{Euclidean EOM}$$

"time" dep damping

interpret ρ as a "time"



b.cs. $\lim_{\rho\to\infty} \Phi(\rho) = \phi_f$

$$\left.\frac{d\Phi}{d\rho}\right|_{\rho=0} = 0 \qquad \text{non-singular}$$

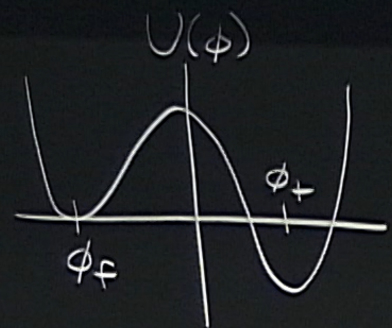
solution always exists

undershoot - start to left of Φ_*

overshoot - start near ϕ_f

Continuity $\rightarrow$ solution exists

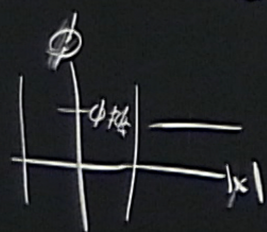
Decay of False Vacuum in Scalar QFT



$$S_E = \int d^4x \left[\frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi + U(\phi) \right]$$

$$\frac{\Gamma}{vol} = A e^{-S_E[\bar{\phi}]/\hbar}$$

b.c. $\lim_{t_E \to \pm\infty} \bar{\phi} = \phi_f$ bounce



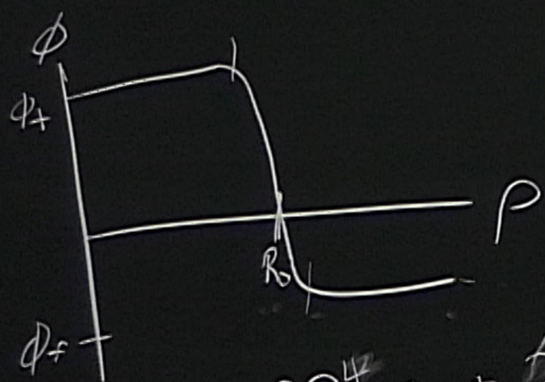
$\lim_{|\vec{x}| \to \infty} \bar{\phi} = \phi_f$ finite action

lowest action solution is $O(4)$ invariant $\bar{\Phi}(\rho)$

$$\rho = \sqrt{t_E^2 + |\vec{x}|^2}$$

Thin-wall limit

If $U(\phi_+) \simeq U(\phi_f)$



$$S_E = -\frac{1}{2}\pi^2 R_0^4 \epsilon + 2\pi^2 R_0^3 \sigma$$

tension

$U(\phi_f) - U(\phi_+)$

$$\frac{dS_E}{dR_0} = 0 \rightarrow R_0 = \frac{3\sigma}{\epsilon}$$

$$S_E = \frac{27\pi^2 \sigma^4}{2\epsilon^3}$$

prefactor often unimportant

$$\frac{27\pi^2}{2} \simeq 100$$

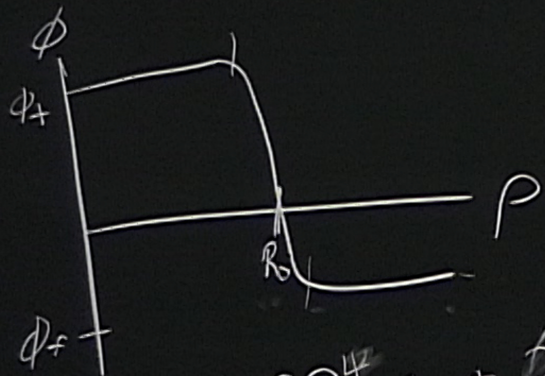
$S_E = O(10^3)\,\hbar$ is not atypical

$$\left(\frac{1}{H_0 l_p}\right)^4 \simeq e^{560}$$

$$e^{560} e^{-1000}$$

Thin-wall limit

If $U(\phi_+) \simeq U(\phi_f)$



tension

$$S_E = -\frac{1}{2}\pi^2 R_0^4 \epsilon + 2\pi^2 R_0^3 \sigma$$

$U(\phi_f)-U(\phi_+)$

$$\sigma = \int_{\phi_f}^{\phi_+} d\phi \sqrt{2U}$$

$$\frac{dS_E}{dR_0} = 0 \rightarrow R_0 = \frac{3\sigma}{\epsilon}$$

$$S_E = \frac{27\pi^2\sigma^4}{2\epsilon^3}$$

prefactor often unimportant

$$\frac{27\pi^2}{2} \simeq 100$$

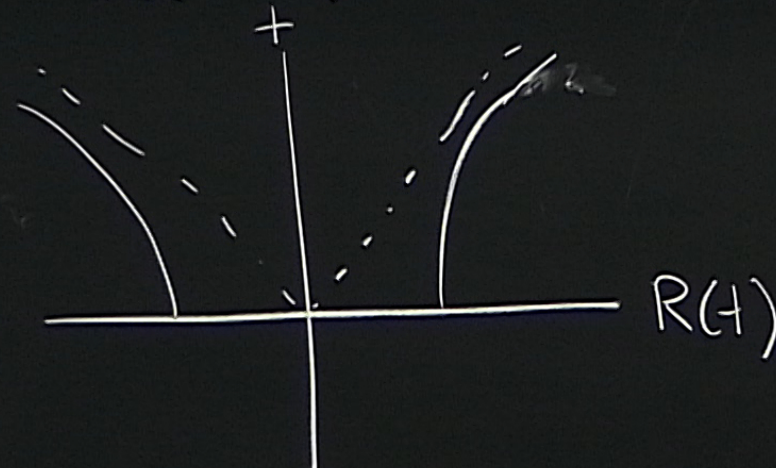
$S_E = O(10^3)\,\hbar$ is not atypical

$$\left(\frac{1}{H_0 l_P}\right)^4 \simeq e^{560}$$

$$e^{560}\, e^{-1000}$$

Real time evolution $t_E \to it$ $O(4) \to O(3,1)$

$$R(t) = \sqrt{t^2 + R_0^2} \qquad t \gtrsim R_0 \qquad \frac{dR}{dt} \simeq 1$$



Perturbation Theory + Instantons

$$f(g) \sim \sum_{n \geq 0} a_n g^n \equiv \varphi(g)$$

exact quantity

$$f(g) \underset{\substack{\text{Borel resummation (sometimes)} \\ \text{+ instantons}}}{\overset{\text{pert theory}}{\rightleftarrows}} \varphi(g)$$

Dyson's argument:

QM particle w/ potential $V(q) = \frac{q^2}{2} + \frac{g}{4} q^4$

$f(g) = E_0(g)$

If $\varphi(g)$ has a finite radius of convergence

$g > 0$ stable

$\mathrm{Im} E_0 = 0$

$g < 0$ unstable

$\mathrm{Im} E_0 \neq 0$

connection between pert + non pert physics