Title: PSI 17/18 - Quantum Field Theory III - Lecture 13

Date: Feb 13, 2018 11:30 AM

URL: http://pirsa.org/18020033

Abstract:

Recap

$$T(z) = \sum \frac{L_n}{z^{n+2}}$$

$$[L_n, L_m] = (n-m) L_{n+m} + \frac{c}{12} n(n+1)(n-1)\, \delta_{n+m,0}$$

$$|h\rangle \equiv |h\rangle = \lim_{\substack{z \to 0 \\ \bar{z} \to 0}} \Phi(z, \bar{z}) |0\rangle$$

$$L_0 |h\rangle = h |h\rangle$$

$$n \geq 1 \quad L_n |h\rangle = 0 \qquad L_{-n} = L_n^\dagger$$

$$L_0 L_{-n} |h\rangle = (h+n) L_{-n} |h\rangle$$

$$T(z)\Phi(w) = \sum_{n\geq 0} (z-w)^{2-n} L_{-n}\Phi$$

$$L_0\Phi = h\Phi$$

$$L_{-1}\Phi = \partial\Phi$$

$$\langle \Phi_1 \Phi_2 \cdots L_{-k}\Phi_n \rangle = \mathcal{L}_{-k}\langle \Phi_1 \Phi_2 \cdots \Phi_n \rangle$$

$+m,0$

$L_{-n} = L_n^\dagger$

$L_{-n}|h\rangle$

How to build your own unitary solvable $D=2$ CFT.

① unitarity: $\| |\psi\rangle \|^2 \geq 0$ $\quad \langle h|h\rangle \geq 0$

$L_{-1}|h\rangle$ $\quad L_{-1} = L_1^\dagger$

$$| L_{-1}|h\rangle |^2$$

$$= \langle h| L_1 L_{-1} |h\rangle$$

$$= \langle h| \left([L_1, L_{-1}] + L_{-1} L_1 \right) |h\rangle$$

$$= \langle h| 2L_0 |h\rangle \qquad h \geq 0$$

$$= 2h \langle h|h\rangle \geq 0$$

…uable $D=2$ CFT.

$\rangle \geq 0$

$L_{-1} = L_1^\dagger$

$$|L_{-1}|h\rangle|^2$$
$$= \langle h|L_1 L_{-1}|h\rangle$$
$$= \langle h|\left([L_1, L_{-1}] + L_{-1}L_1\right)|h\rangle$$
$$= \langle h|2L_0|h\rangle \qquad h \geq 0$$
$$= 2h\langle h|h\rangle \geq 0$$

$$|L_{-2}|h\rangle|^2$$
$$= \langle h|L_2 L_{-2}|h\rangle \qquad c \geq 0$$
$$= \langle h|[L_2, L_{-2}]|h\rangle$$
$$= \langle h|4L_0 + \frac{c}{2}|h\rangle$$
$$= \left(4h + \frac{c}{2}\right)\langle h|h\rangle \geq 0$$

$$\left|L_{-2}|h\rangle\right|^2$$

$$= \langle h|L_2 L_{-2}|h\rangle \qquad c \geq 0$$

$$= \langle h|[L_2, L_{-2}]|h\rangle$$

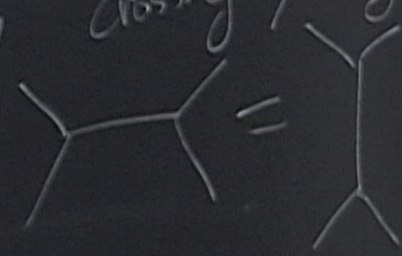
$$= \langle h|4L_0 + \frac{c}{2}|h\rangle$$

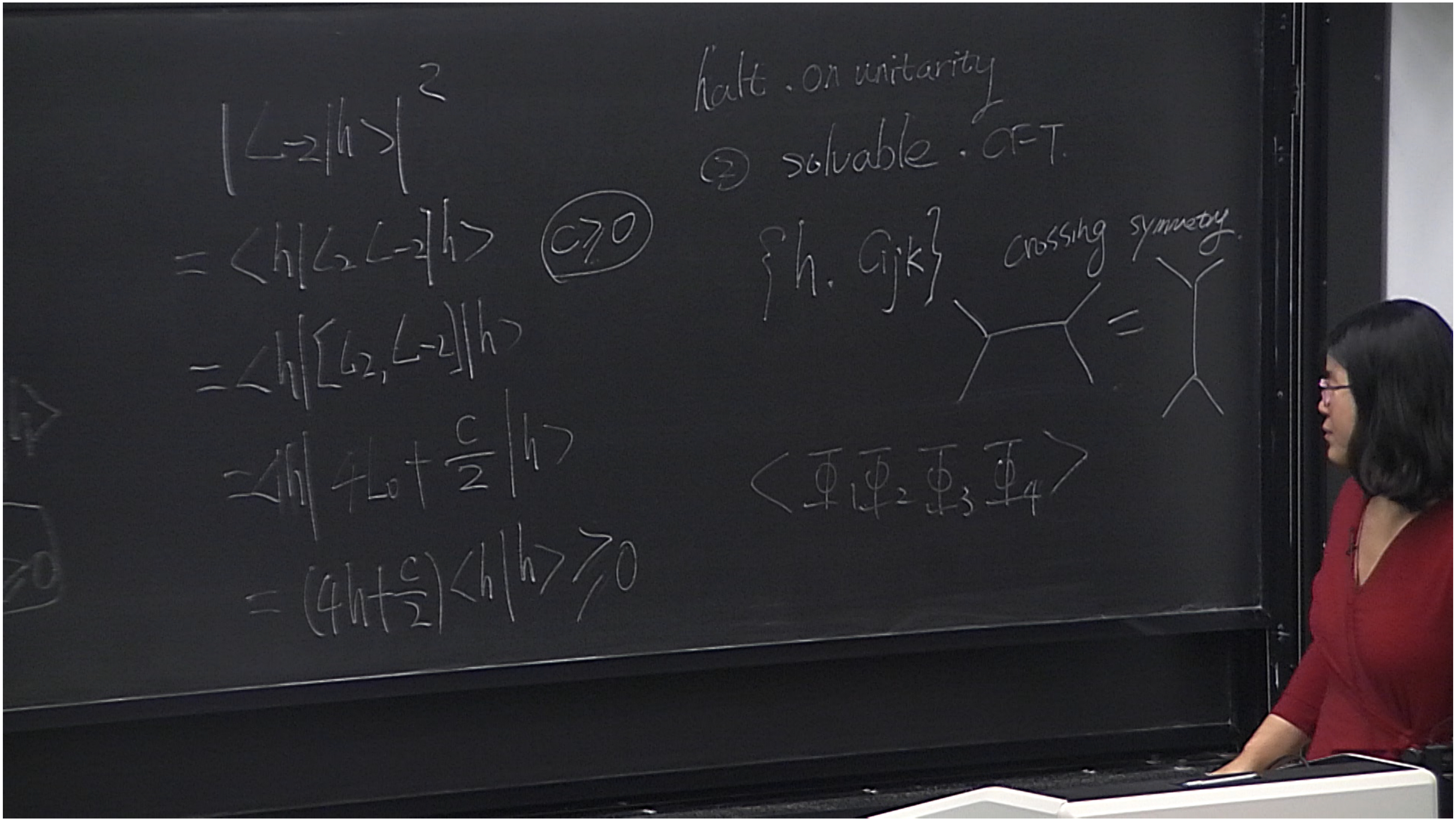
$$= \left(4h + \frac{c}{2}\right)\langle h|h\rangle \geq 0$$

halt . on unitarity

② solvable . CFT.

$\{h, C_{ijk}\}$ crossing symmetry




$|L_{-2}|h\rangle|^2$
$= \langle h|L_2 L_{-2}|h\rangle$
$c \geq 0$
$= \langle h|[L_2, L_{-2}]|h\rangle$
$= \langle h|4L_0 + \frac{c}{2}|h\rangle$
$= (4h + \frac{c}{2})\langle h|h\rangle \geq 0$
halt . on unitarity
② solvable . CFT.
$\{h, C_{ijk}\}$ crossing symmetry.
$\langle \Phi_1 \Phi_2 \Phi_3 \Phi_4 \rangle$

$$|L_{-2}|h\rangle|^2$$

$$= \langle h|L_2 L_{-2}|h\rangle \qquad c \geq 0$$

$$= \langle h|[L_2, L_{-2}]|h\rangle$$

$$= \langle h|4L_0 + \frac{c}{2}|h\rangle$$

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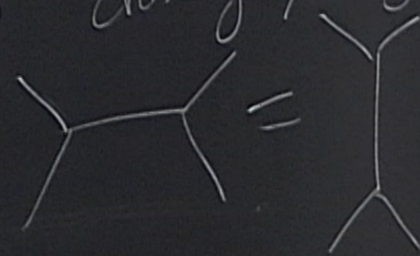
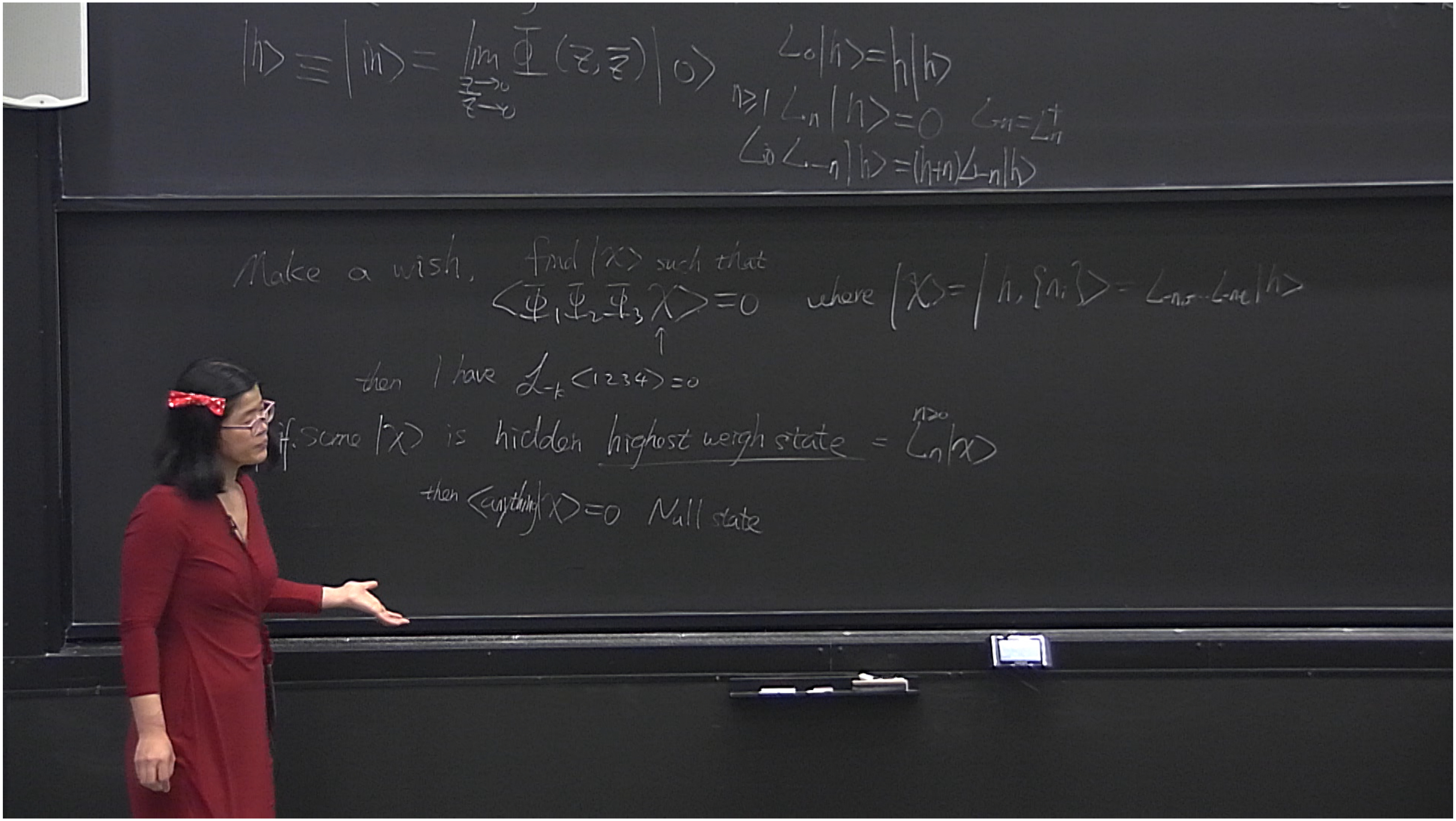
halt . on unitarity

② solvable . CFT.

$\{h, C_{ijk}\}$ crossing symmetry



$$\langle \Phi_1 \Phi_2 \Phi_3 \Phi_4 \rangle$$


$|h\rangle \equiv |in\rangle = \lim_{z\to 0, \bar z \to 0} \Phi(z,\bar z)|0\rangle$
$L_0|h\rangle = h|h\rangle$
$n\geq 1 \; L_n|h\rangle = 0 \quad L_{-n} = L_n^\dagger$
$L_0 L_{-n}|h\rangle = (h+n)L_{-n}|h\rangle$
Make a wish, find $|\chi\rangle$ such that
$\langle \Phi_1 \Phi_2 \Phi_3 \chi\rangle = 0$ where $|\chi\rangle = |h, \{n_i\}\rangle = L_{-n_1} \cdots L_{-n_k}|h\rangle$
then I have $\mathcal{L}_{-k}\langle 1234\rangle = 0$
if some $|\chi\rangle$ is hidden highest weigh state $= L_n|\chi\rangle$ $n>0$
then $\langle \text{anything}|\chi\rangle = 0$ Null state

$$|h\rangle \equiv |in\rangle = \lim_{\substack{z\to 0\\ \bar z\to 0}} \Phi(z,\bar z)\,|0\rangle \qquad L_0|h\rangle = h|h\rangle$$

$$n\geq 1 \quad L_n|h\rangle = 0 \qquad L_{-n} = L_n^\dagger$$

$$L_0 L_{-n}|h\rangle = (h+n)L_{-n}|h\rangle$$

Make a wish, find $|\chi\rangle$ such that

$\langle \Phi_1\Phi_2\Phi_3\chi\rangle = 0$ where $|\chi\rangle = |h,\{n_i\}\rangle = L_{-n_1}\cdots L_{-n_k}|h\rangle$ $\langle$

then I have $\mathcal{L}_{-k}\langle 1234\rangle = 0$

claim if some $|\chi\rangle$ is hidden <u>highest weigh state</u> $= \overset{n>0}{L_n}|\chi\rangle$

then $\langle \text{anything}|\chi\rangle = 0$ Null state

$= \lim_{\substack{z \to 0 \\ \bar{z} \to 0}} \Phi(z, \bar{z}) |0\rangle$

$L_0 |h\rangle = h |h\rangle$

$n \geq 1 \quad L_n |h\rangle = 0 \qquad L_{-n} = L_n^\dagger$

$L_0 L_{-n} |h\rangle = (h+n) L_{-n} |h\rangle$

ish, find $|\chi\rangle$ such that

$\langle \Phi_1 \Phi_2 \Phi_3 \chi \rangle = 0$ where $|\chi\rangle = |h, \{n_i\}\rangle = L_{-n_5} \cdots L_{-n_1} |h\rangle$ $\quad \langle \chi' | \chi \rangle = \delta_{hh'} \delta_{\Sigma n_i \Sigma n_i'}$

I have $\mathcal{L}_{-k} \langle 1234 \rangle = 0$

same energy same level

$\langle \chi' | \chi \rangle = \langle h | L_{-n_i} \cdots L_{-n_1} | \chi \rangle$

$L_{-n}^\dagger = L_n$

$= \langle h | L_{n_1} \cdots L_{n_1} | \chi \rangle$

$= 0$

$\rangle$ is hidden highest weigh state $= \overset{n>0}{L_n} |\chi\rangle$

then $\langle \text{anything} | \chi \rangle = 0$ Null state

$= 2h\langle h|h\rangle \geq 0$

Go hunting: for a Null state $L_n|\chi\rangle$ for all $n \geq 1$

At level 1. $L_n L_{-1}|h\rangle = 0$

$[L_n, L_{-1}]|h\rangle$ $\partial 1 = 0$

$= (n+1) L_{n-1}|h\rangle$

$$= \begin{cases} n=1 & 2h|h\rangle \\ n\geq 2 & 0 \end{cases}$$

At level 2

$|\chi\rangle = L_{-2}|h\rangle + \alpha L_{-1}^2|h\rangle$

1) demand $L_1|\chi\rangle = 0$

$L_1 L_{-2}|h\rangle + \alpha L_1 L_{-1}^2|h\rangle$

$= 3L_{-1}|h\rangle +$

for all $n > 1$

$\cdot f_1 = 0$

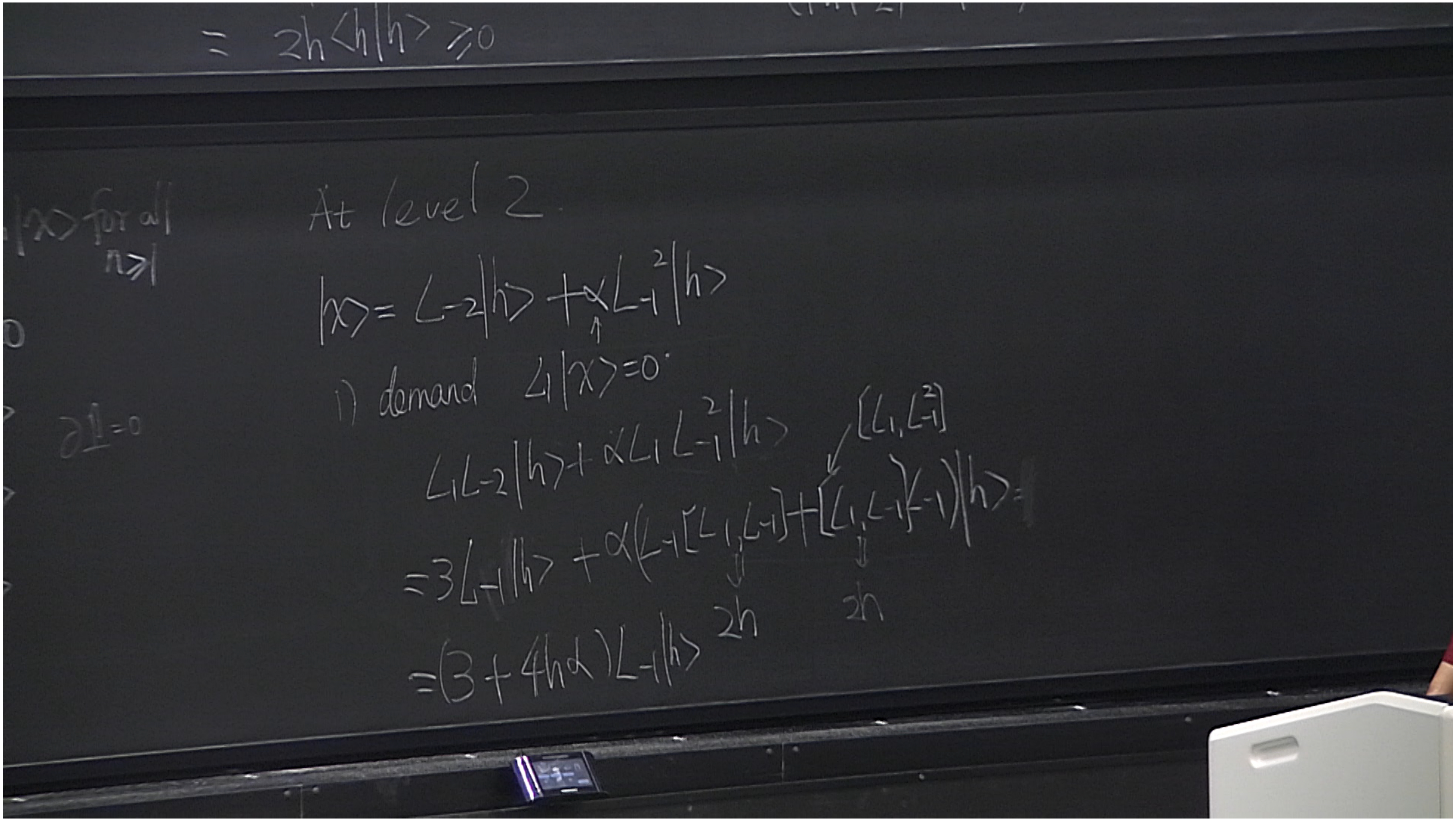
At level 2.

$$|\chi\rangle = L_{-2}|h\rangle + \alpha L_{-1}^2|h\rangle$$

1) demand $L_1|\chi\rangle = 0$.

$$L_1 L_{-2}|h\rangle + \alpha L_1 L_{-1}^2|h\rangle \qquad [L_1, L_{-1}^2]$$

$$= 3L_{-1}|h\rangle + \alpha\left(L_{-1}[L_1, L_{-1}] + [L_1, L_{-1}]L_{-1}\right)|h\rangle$$



$= 2h\langle h|h\rangle \geq 0$
$|\chi\rangle$ for all $n \geq 1$
$\partial \mathbb{1} = 0$
At level 2
$|\chi\rangle = L_{-2}|h\rangle + \alpha L_{-1}^2|h\rangle$
1) demand $L_1|\chi\rangle = 0$
$L_1 L_{-2}|h\rangle + \alpha L_1 L_{-1}^2|h\rangle$
$[L_1, L_{-1}^2]$
$= 3L_{-1}|h\rangle + \alpha(L_{-1}[L_1, L_{-1}] + [L_1, L_{-1}]L_{-1})|h\rangle$
$2h$
$2h$
$= (3 + 4h\alpha)L_{-1}|h\rangle$

$$= 2h\langle h|h\rangle \geq 0$$

$|\chi\rangle$ for all $n \geq 1$

0

$\partial \mathbb{1} = 0$

At level 2.

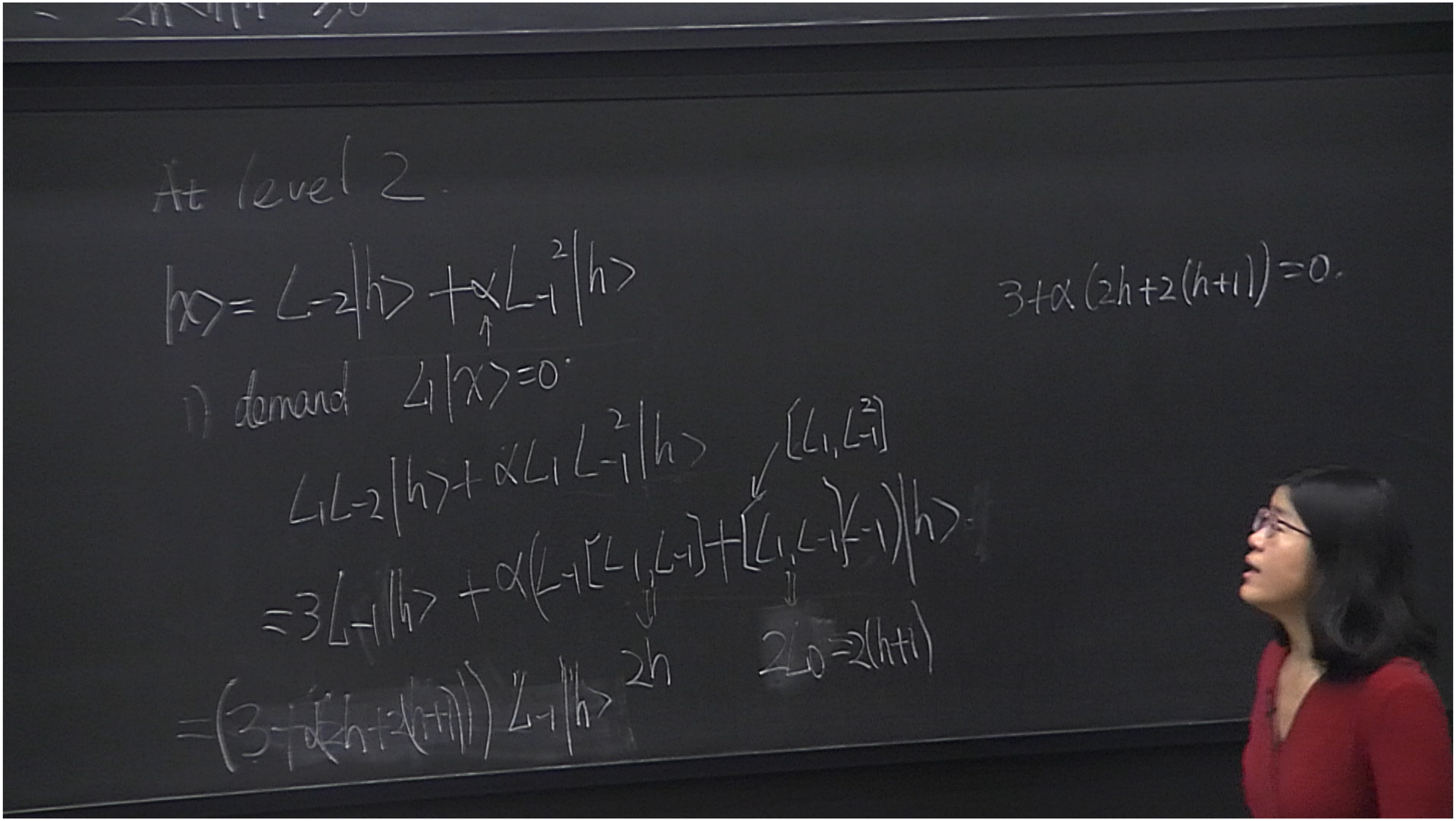
$$|\chi\rangle = L_{-2}|h\rangle + \alpha L_{-1}^2|h\rangle$$

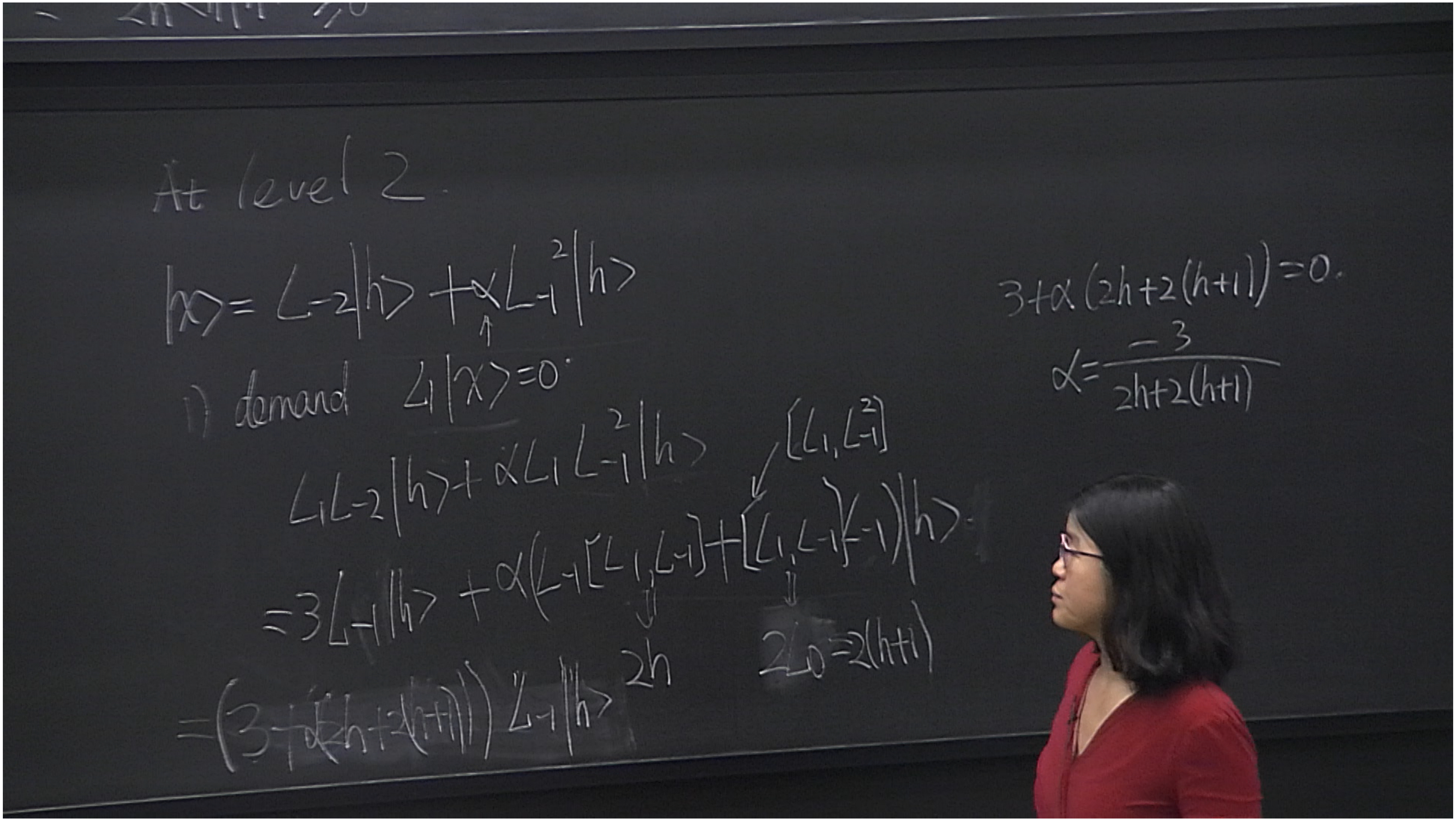
1) demand $L_1|\chi\rangle = 0$

$$L_1 L_{-2}|h\rangle + \alpha L_1 L_{-1}^2|h\rangle \qquad [L_1, L_{-1}^2]$$

$$= 3L_{-1}|h\rangle + \alpha\big(L_{-1}\underbrace{[L_1, L_{-1}]}_{2h} + \underbrace{[L_1, L_{-1}]}_{2L_0 = 2(h+1)}L_{-1}\big)|h\rangle =$$

$$= (3 + 4h\alpha)L_{-1}|h\rangle$$


At level 2.
$|\chi\rangle = L_{-2}|h\rangle + \alpha L_{-1}^2|h\rangle$
1) demand $L_1|\chi\rangle = 0$.
$L_1 L_{-2}|h\rangle + \alpha L_1 L_{-1}^2|h\rangle$
$[L_1, L_{-1}^2]$
$= 3L_{-1}|h\rangle + \alpha(L_{-1}[L_1, L_{-1}] + [L_1, L_{-1}]L_{-1})|h\rangle$.
$2h$
$2L_0 = 2(h+1)$
$= (3 + \alpha(2h + 2(h+1)))L_{-1}|h\rangle$
$3 + \alpha(2h + 2(h+1)) = 0$.


At level 2.
$|\chi\rangle = L_{-2}|h\rangle + \alpha L_{-1}^{2}|h\rangle$
1) demand $L_1|\chi\rangle = 0$.
$L_1 L_{-2}|h\rangle + \alpha L_1 L_{-1}^{2}|h\rangle$
$[L_1, L_{-1}^{2}]$
$= 3L_{-1}|h\rangle + \alpha(L_{-1}[L_1, L_{-1}] + [L_1, L_{-1}]L_{-1})|h\rangle$.
$2h$
$2L_0 = 2(h+1)$
$= (3 + \alpha(2h + 2(h+1)))L_{-1}|h\rangle$
$3 + \alpha(2h + 2(h+1)) = 0$.
$\alpha = \frac{-3}{2h+2(h+1)}$

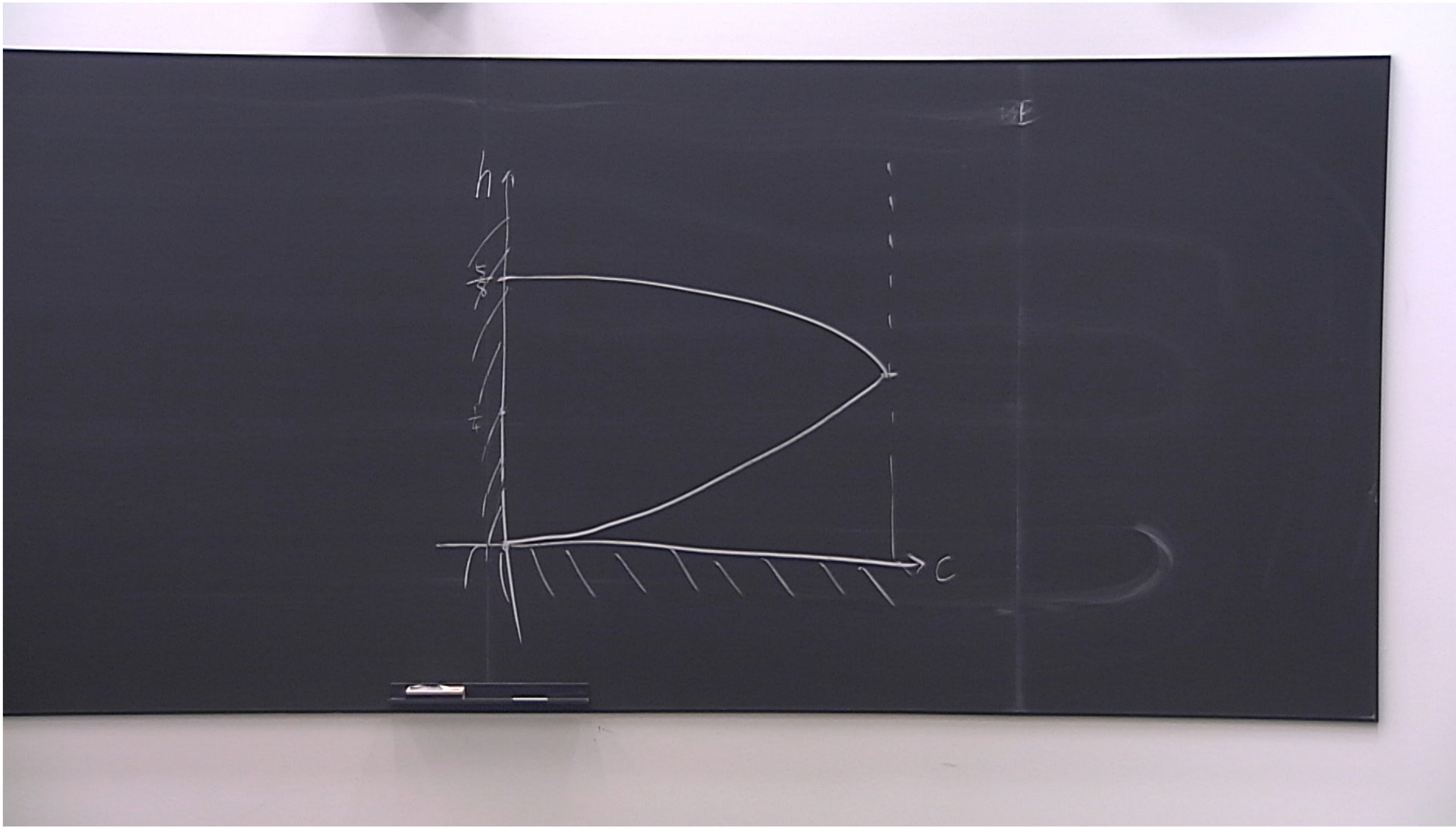
now: $L_2|\chi\rangle$

$$L_2\left(L_{-2}|h\rangle + \alpha L_{-1}^2|h\rangle\right)$$

$$[L_2, L_{-2}]|h\rangle = \left(4h + \frac{c}{2}\right)|h\rangle$$

$$[L_2, L_{-1}^2]|h\rangle = \left(L_{-1}[L_2, L_{-1}] + [L_2, L_{-1}]L_{-1}\right)|h\rangle$$

$$= \left(L_{-1} 3L_1 + 3L_1 L_{-1}\right)|h\rangle = 6h|h\rangle$$


h
5/8
1/4
c

claim if some $|\chi\rangle$ is hidden highest weigh state $= L_n|\chi\rangle$

then $\langle \text{anything}|\chi\rangle = 0$ Null state

$= \langle h| L_{n_1} \cdots L_{n_i}|\chi\rangle$
$= 0$

now $\cdot$ $L_2|\chi\rangle$

$L_2 \left(L_{-2}|h\rangle + \alpha L_{-1}^2|h\rangle \right)$

$= \left(\ldots \right) + 6h\alpha \Big) |h\rangle$

$\frac{c}{2} + 6h\alpha = 0$

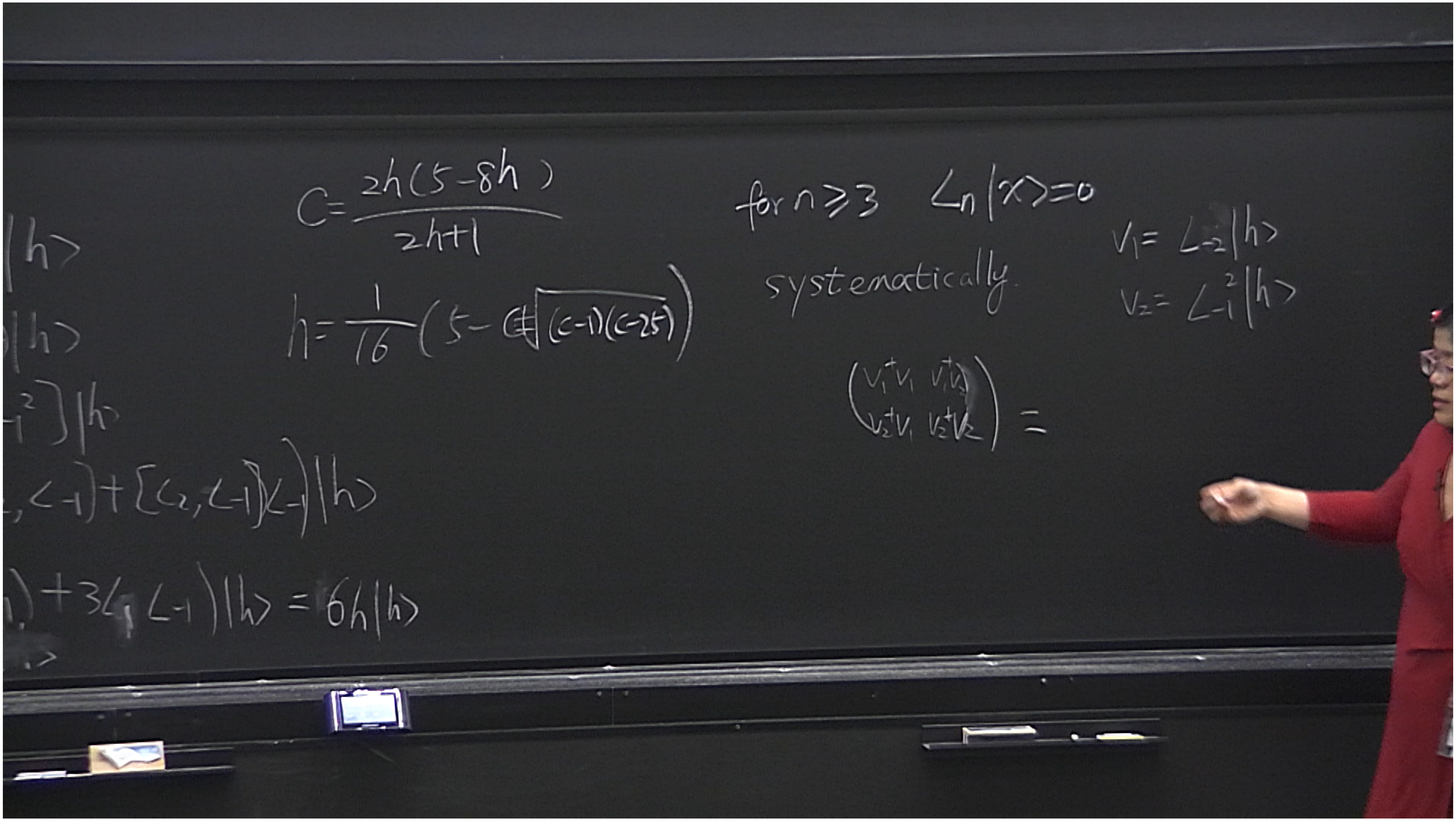
$[L_2, L_{-2}]|h\rangle$
$= (4h + \frac{c}{2})|h\rangle$

$[L_2, L_{-1}^2]|h\rangle$
$= \left(L_{-1}[L_2, L_{-1}] + [L_2, L_{-1}]L_{-1} \right)|h\rangle$
$= \left(L_{-1} 3L_1 + 3L_1 L_{-1} \right)|h\rangle = 6h|h\rangle$

$C = \frac{2h(5-8h)}{2h+1}$

$h = \frac{1}{16}\left(5 - c \pm \sqrt{(c-1)(c-25)}\right)$

for $n \geq 3$ $L_n|\chi\rangle = 0$


C = 2h(5-8h) / (2h+1)
h = 1/16 (5 - c ± √((c-1)(c-25)))
for n ≥ 3 L_n |χ> = 0
systematically.
V_1 = L_{-2}|h>
V_2 = L_{-1}^2|h>
= 6h|h>

$$C = \frac{2h(5-8h)}{2h+1}$$

$$h = \frac{1}{16}\left(5 - c \pm \sqrt{(c-1)(c-25)}\right)$$

$|h\rangle$

$|h\rangle$

$L_{-1}^2]|h\rangle$

$, L_{-1}) + [L_2, L_{-1}]L_{-1})|h\rangle$

$) + 3(L_1 L_{-1})|h\rangle = 6h|h\rangle$

for $n \geq 3$ $L_n|\chi\rangle = 0$

systematically

$v_1 = L_{-2}|h\rangle$

$v_2 = L_{-1}^2|h\rangle$

$$\det\begin{pmatrix} v_1^\dagger v_1 & v_1^\dagger v_2 \\ v_2^\dagger v_1 & v_2^\dagger v_2 \end{pmatrix} = 0$$

$|h\rangle$

$|h\rangle$

$L_{-1}^2]|h\rangle$

$,L_{-1}]+[L_2,L_{-1}]L_{-1})|h\rangle$

$)+3L_1L_{-1})|h\rangle = 6h|h\rangle$

$$C=\frac{2h(5-8h)}{2h+1}$$

$$h=\frac{1}{16}\left(5-c\pm\sqrt{(c-1)(c-25)}\right)$$

for $n\geq 3$ $L_n|\chi\rangle=0$

$V_1 = L_{-2}|h\rangle$

$V_2 = L_{-1}^2|h\rangle$

systematically

$$\det\begin{pmatrix} V_1^\dagger V_1 & V_1^\dagger V_2 \\ V_2^\dagger V_1 & V_2^\dagger V_2 \end{pmatrix} = 0$$

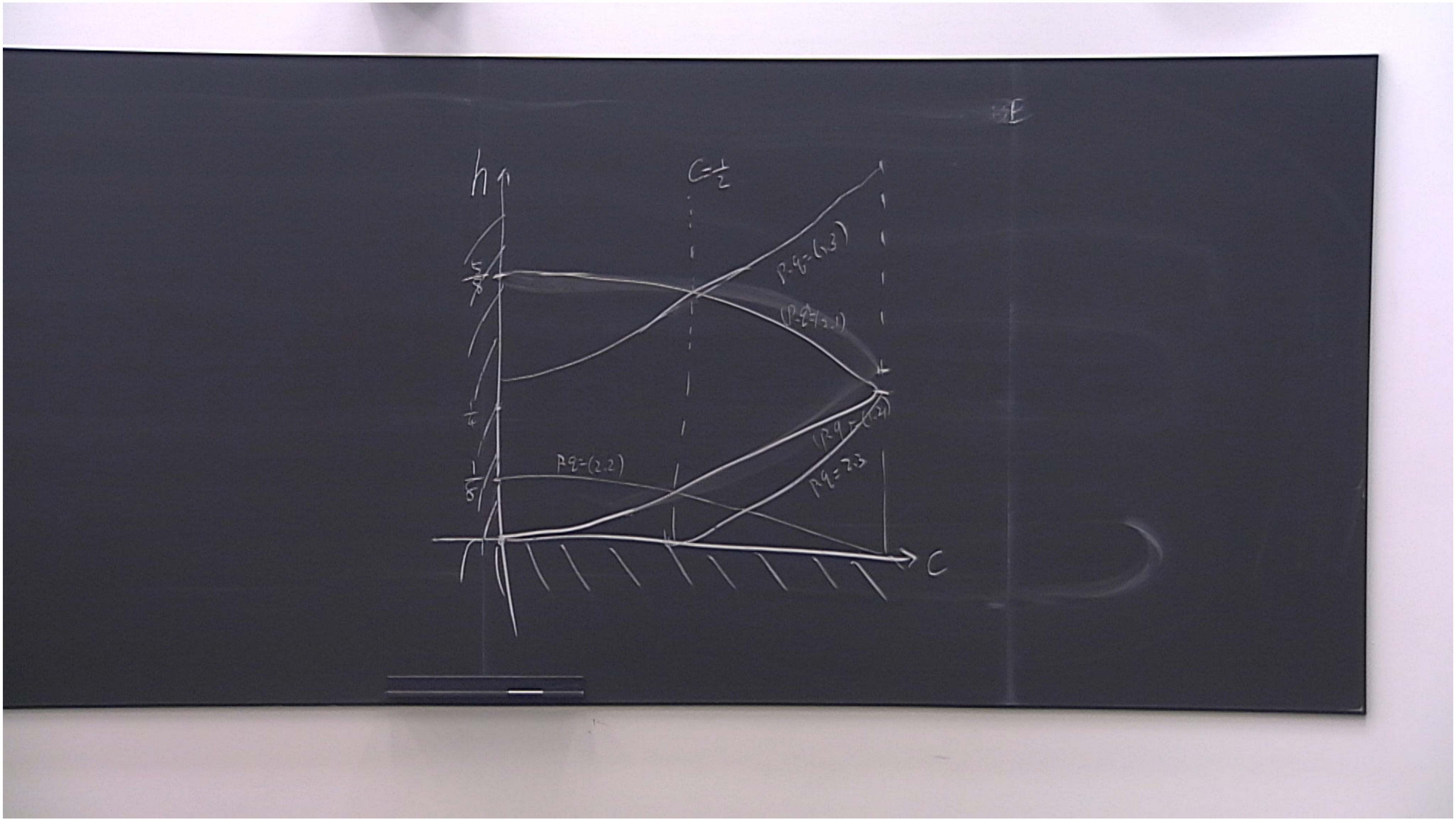
$$\det\begin{pmatrix} 8h^2+4h & 6h \\ 6h & 4h+\frac{c}{2} \end{pmatrix}=0$$

Kac

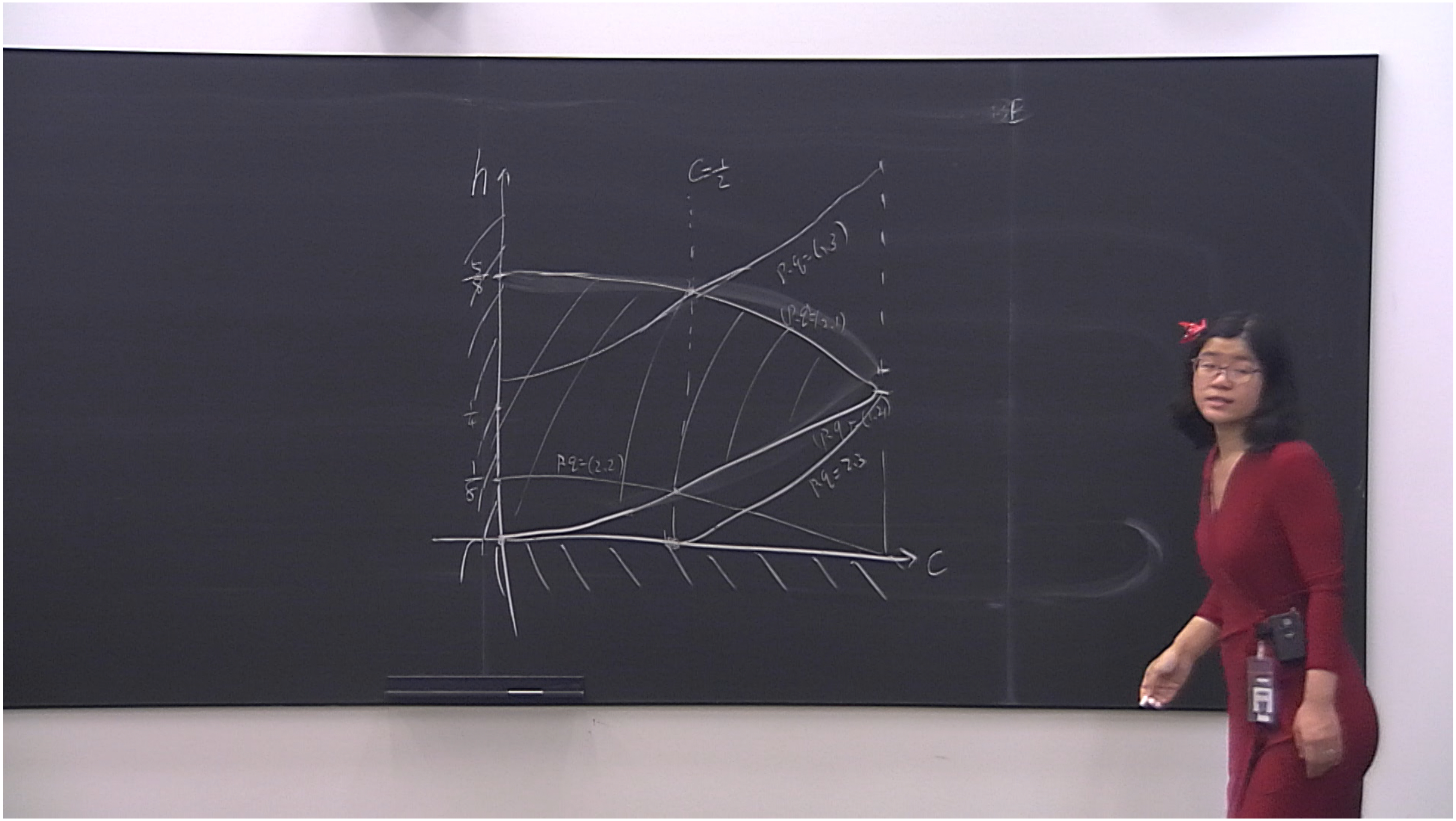
$$\det M_N \propto \prod_{pq \le N} \left(h - h_{p,q}(c)\right)^{P(N-pq)}$$

$$h(p,q)(c) = \frac{1-c}{96}\left(\left((p+q) \pm (p-q)\sqrt{\frac{25-c}{1-c}}\right)^2 - 4\right)$$

$p=1 \quad q=2$



h
c=1/2
p,q=(1,3)
(p,q)=(2,1)
(p,q)=(1,2)
p,q=2,3
p,q=(2,2)
5/8
1/4
1/8
c


h
c
$c=\frac{1}{2}$
$\frac{5}{8}$
$\frac{1}{4}$
$\frac{1}{8}$
p-q=(2,3)
(p-q)=(2,1)
p-q=(2,2)
(p-q)=(1,2)
p-q=2.3

$-4)$

$\det M \geq 0$ Unitary Minimal Model

$$c = 1 - \frac{6}{m(m+1)} \quad m = 3, 4, 5 \cdots$$

$$h_{pq}(m) = \frac{((m+1)p - mq)^2 - 1}{4m(m+1)} \qquad \begin{array}{l} p \to m - p \\ q \to m + 1 - q \end{array}$$

$\det M \geq 0$ Unitary Minimal Model

$$c = 1 - \frac{6}{m(m+1)} \qquad m = 3, 4, 5 \ldots$$

$$h_{pq}(m) = \frac{((m+1)p - mq)^2 - 1}{4m(m+1)} \qquad \begin{array}{l} p \to m-p \\ q \to m+1-q \end{array}$$

$m=3$ $\quad (2,1) \leftrightarrow (1,3)$

intersect

$(2,2) \leftrightarrow (1,2)$

$(1,1) \leftrightarrow (2,3)$