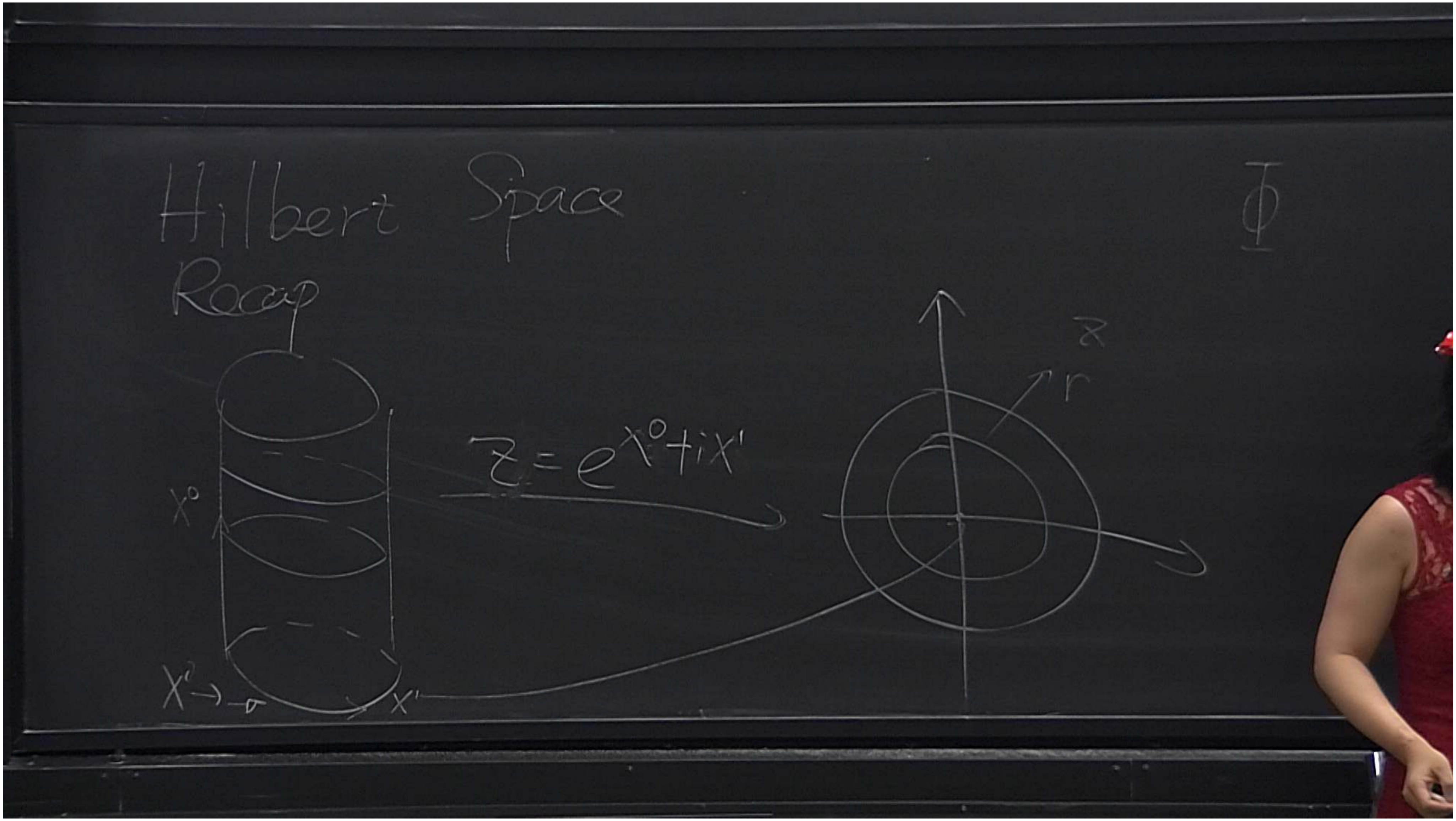
Title: PSI 17/18 - Quantum Field Theory III - Lecture 12

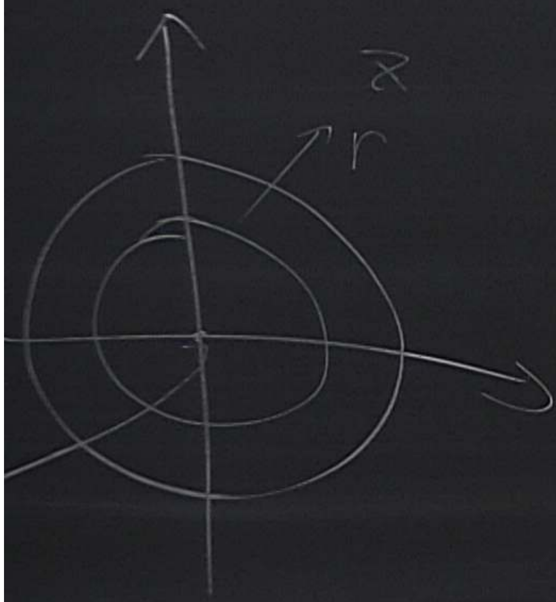
Date: Feb 12, 2018 11:30 AM

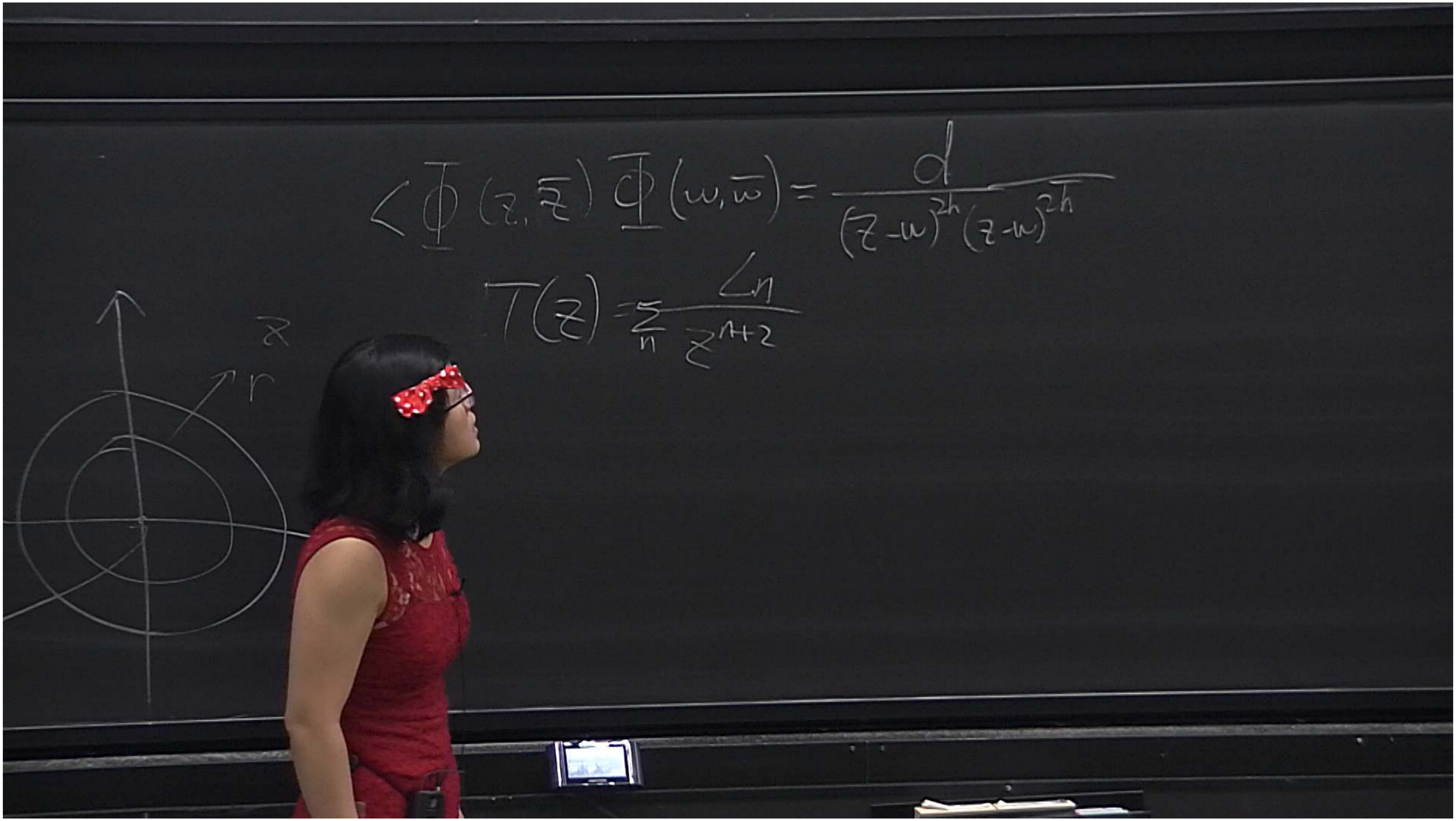
URL: http://pirsa.org/18020032

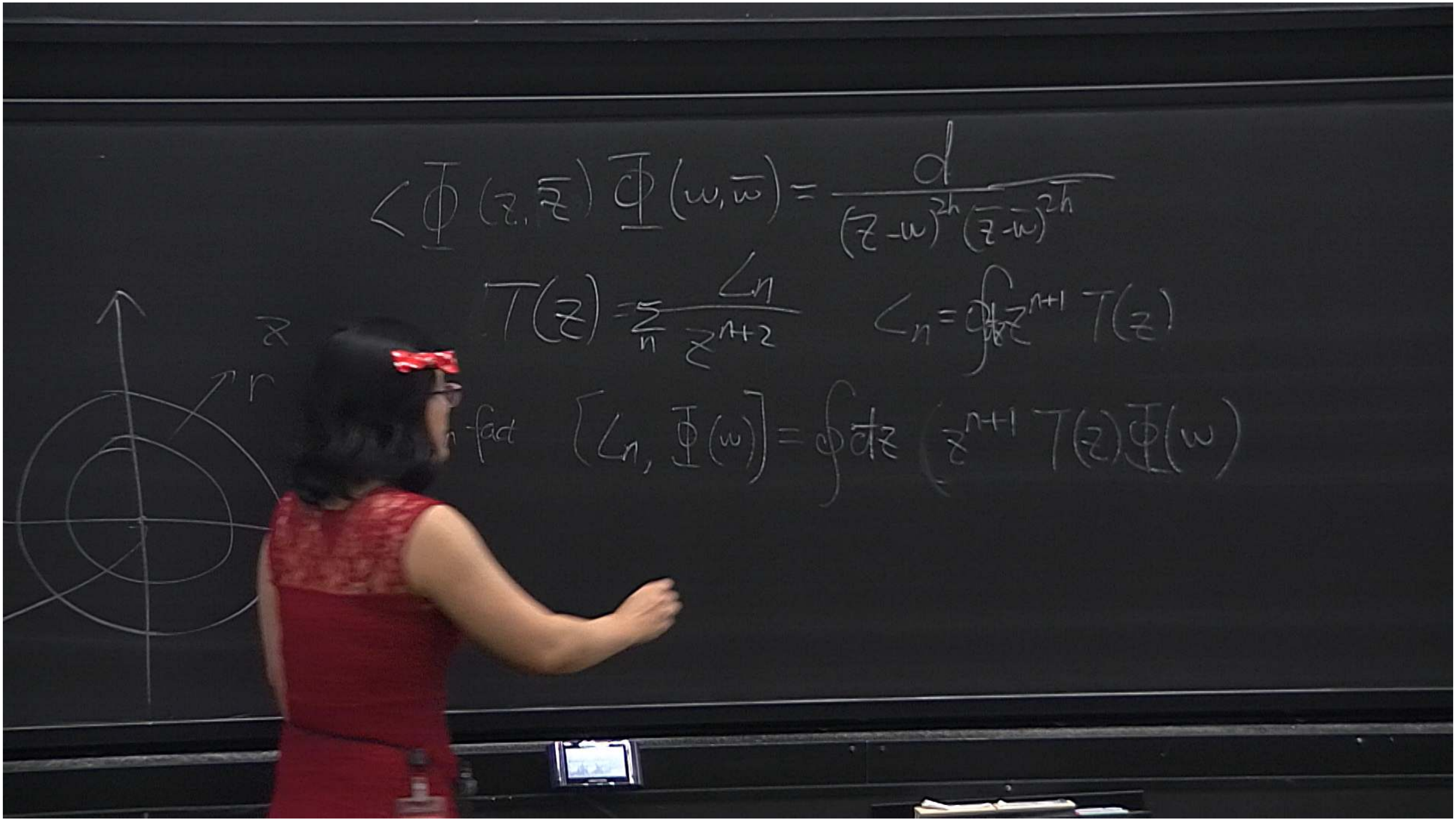
Abstract:


Hilbert Space
Recap
Φ
$z = e^{X^0 + iX^1}$
X^0
X^1
z
r

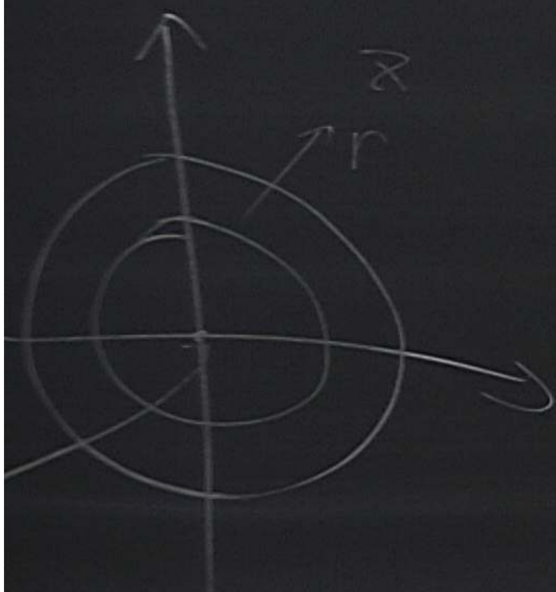
$$\langle \Phi(z,\bar{z})\, \Phi(w,\bar{w}) = \frac{d}{(z-w)^{2h}(\bar{z}-\bar{w})^{2\bar{h}}}$$




$\langle \Phi(z,\bar{z})\, \Phi(w,\bar{w}) = \frac{d}{(z-w)^{2h}(\bar{z}-\bar{w})^{2\bar{h}}}$
$T(z) = \sum_n \frac{L_n}{z^{n+2}}$
z
r



$\langle \Phi(z,\bar z)\,\Phi(w,\bar w) = \frac{d}{(z-w)^{2h}(\bar z-\bar w)^{2h}}$
$T(z) = \sum_n \frac{L_n}{z^{n+2}}$
$L_n = \oint dz\, z^{n+1}\, T(z)$
$[L_n, \Phi(w)] = \oint dz\, (z^{n+1}\, T(z)\,\Phi(w)$
z
r



$$\langle \Phi(z,\bar{z})\,\Phi(w,\bar{w})\rangle = \frac{d}{(z-w)^{2h}(\bar{z}-\bar{w})^{2h}}$$

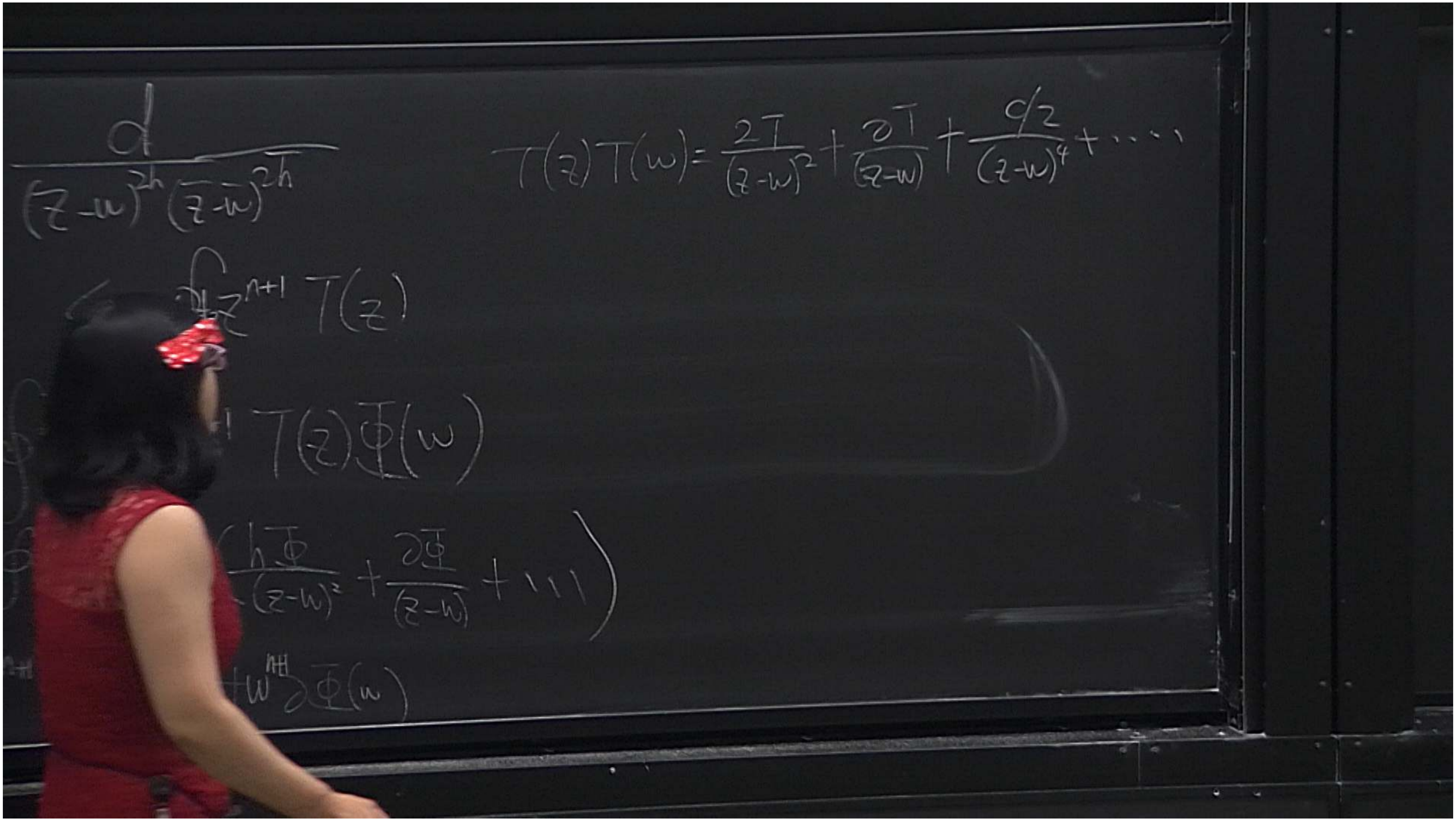
$$T(z) = \sum_n \frac{L_n}{z^{n+2}} \qquad L_n = \oint dz\, z^{n+1}\, T(z)$$

Fun-fact $[L_n, \Phi(w)] = \oint dz \left(z^{n+1}\, T(z)\Phi(w) \right)$

$$= \oint dz\, z^{n+1}\left(\frac{h\Phi}{(z-w)^2} + \frac{\partial\Phi}{(z-w)} + \dots\right)$$

$$= (n+1)\,w^n h\Phi(w) + w^{n+1}\partial\Phi(w)$$

$$\oint dz \frac{f(z)}{(z-z_0)^{n+1}} = \frac{f^{(n)}(z_0)}{n!}$$


d / (z-w)^{2h} (z̄-w̄)^{2h̄}
T(z)T(w) = 2T/(z-w)^2 + ∂T/(z-w) + c/2/(z-w)^4 + ...
z^{n+1} T(z)
T(z)Φ(w)
(hΦ/(z-w)^2 + ∂Φ/(z-w) + ...)
w^{n+1} ∂Φ(w)

$$\frac{d}{(z-w)^{2h}(\bar{z}-\bar{w})^{2h}}$$

$$T(z)T(w) = \frac{2T}{(z-w)^2} + \frac{\partial T}{(z-w)} + \frac{c/2}{(z-w)^4} + \cdots$$

$$L_n = \oint dz\, z^{n+1}\, T(z)$$

$$[L_n, L_m] = \frac{c}{12} n(n+1)(n-1)\,\delta_{n+m} + (n-m) L_{n+m}$$

$$\oint dz\, \left(z^{n+1}\, T(z)\, \Phi(w) \right)$$

$$\oint dz\, z^{n+1} \left(\frac{h\Phi}{(z-w)^2} + \frac{\partial\Phi}{(z-w)} + \cdots \right)$$

$$(n+1) w^n h\Phi(w) + w^{n+1} \partial\Phi(w)$$

Outline today

1) $|in\rangle \langle out|$

2) $|0\rangle$

3) highes weight method.

4) descendant states

5) Miracle: descendant operators/fields do not matter

State-local operator Correspondence.

QFT $\quad |k\rangle = a_k^\dagger |0\rangle$

$$P^i = \int d^3x \, \pi \cdot \nabla\phi$$

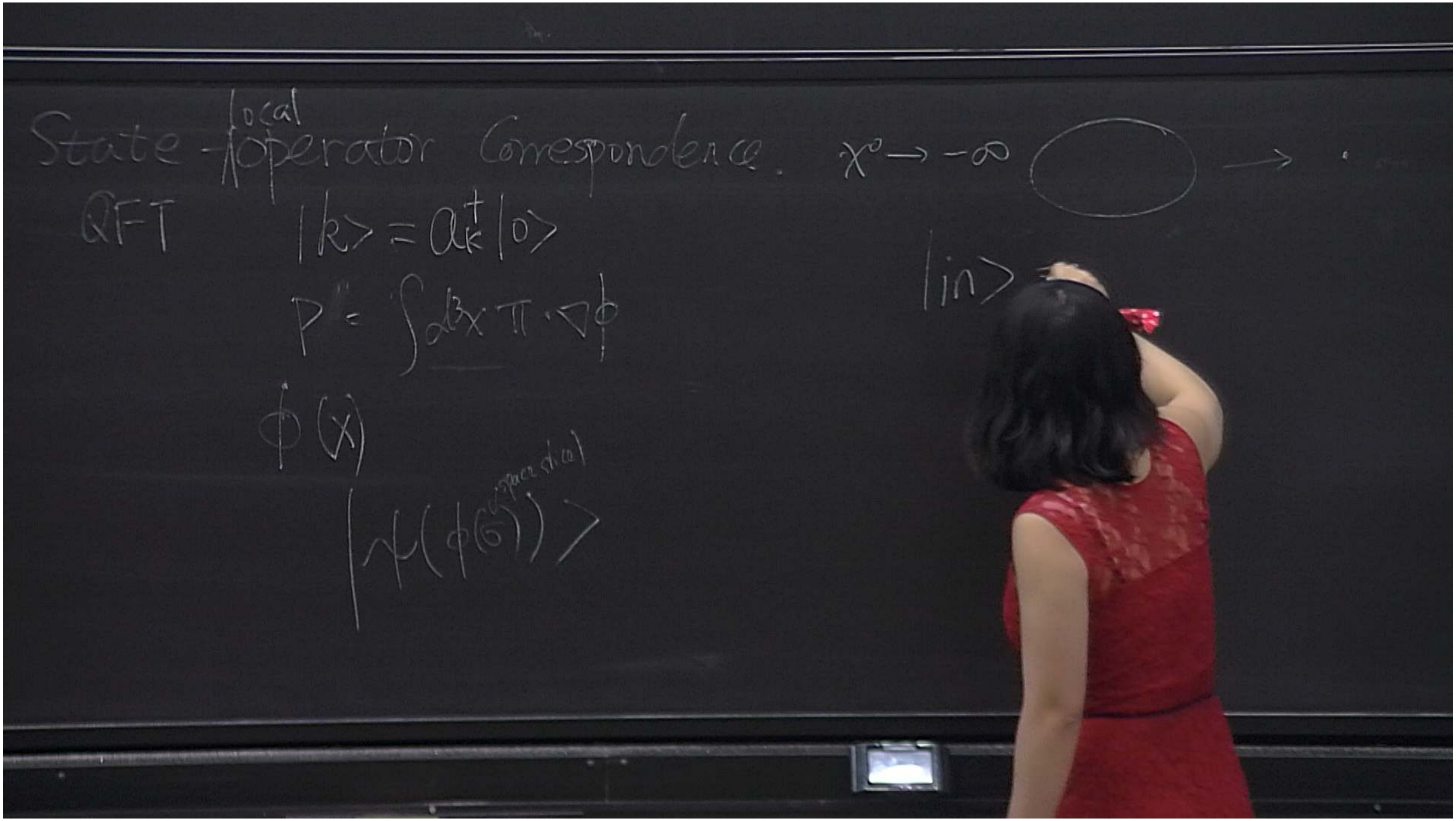
$$\phi(x)$$

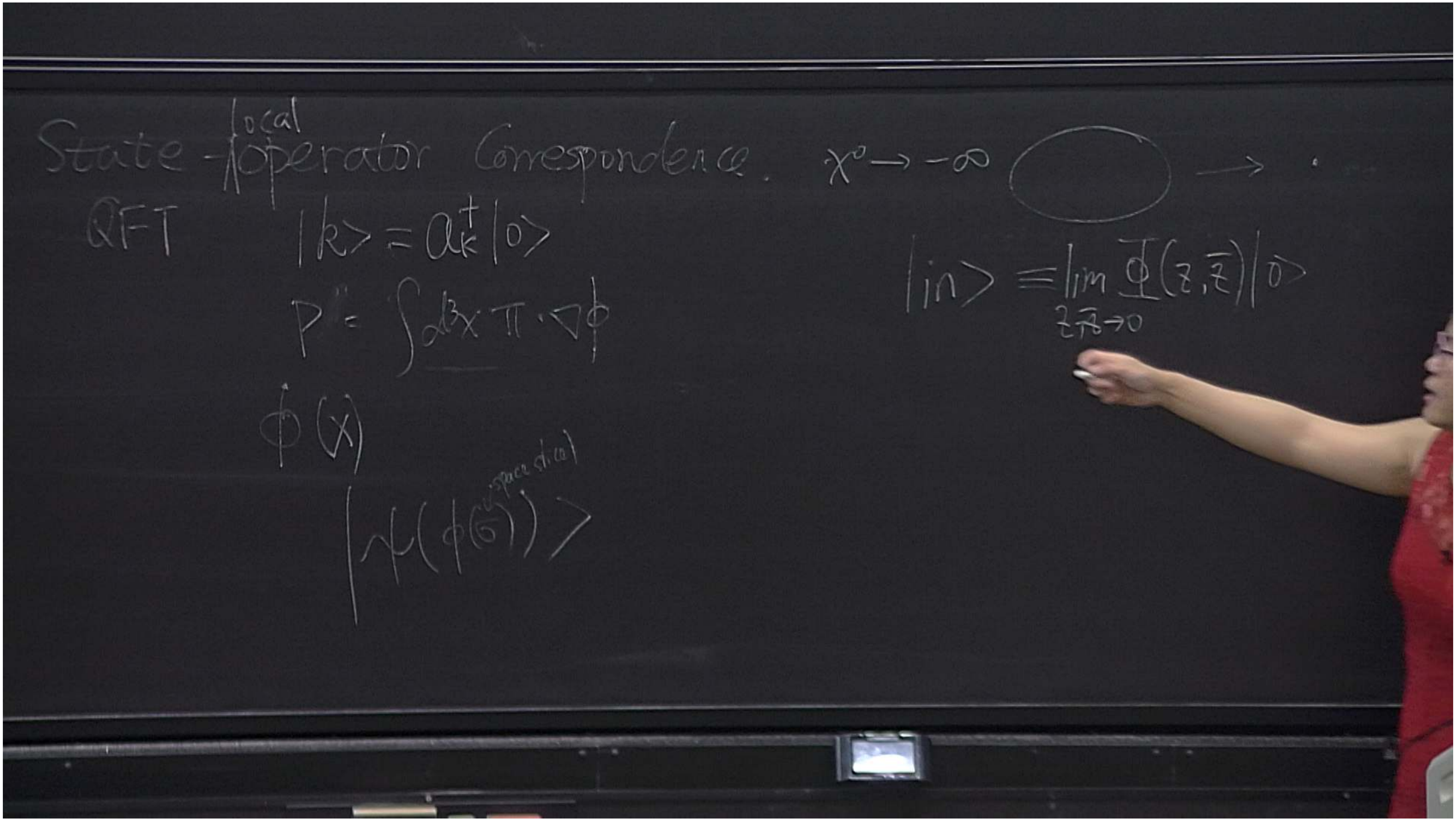
$$|\psi(\phi(\sigma)^{\text{space slice}})\rangle$$

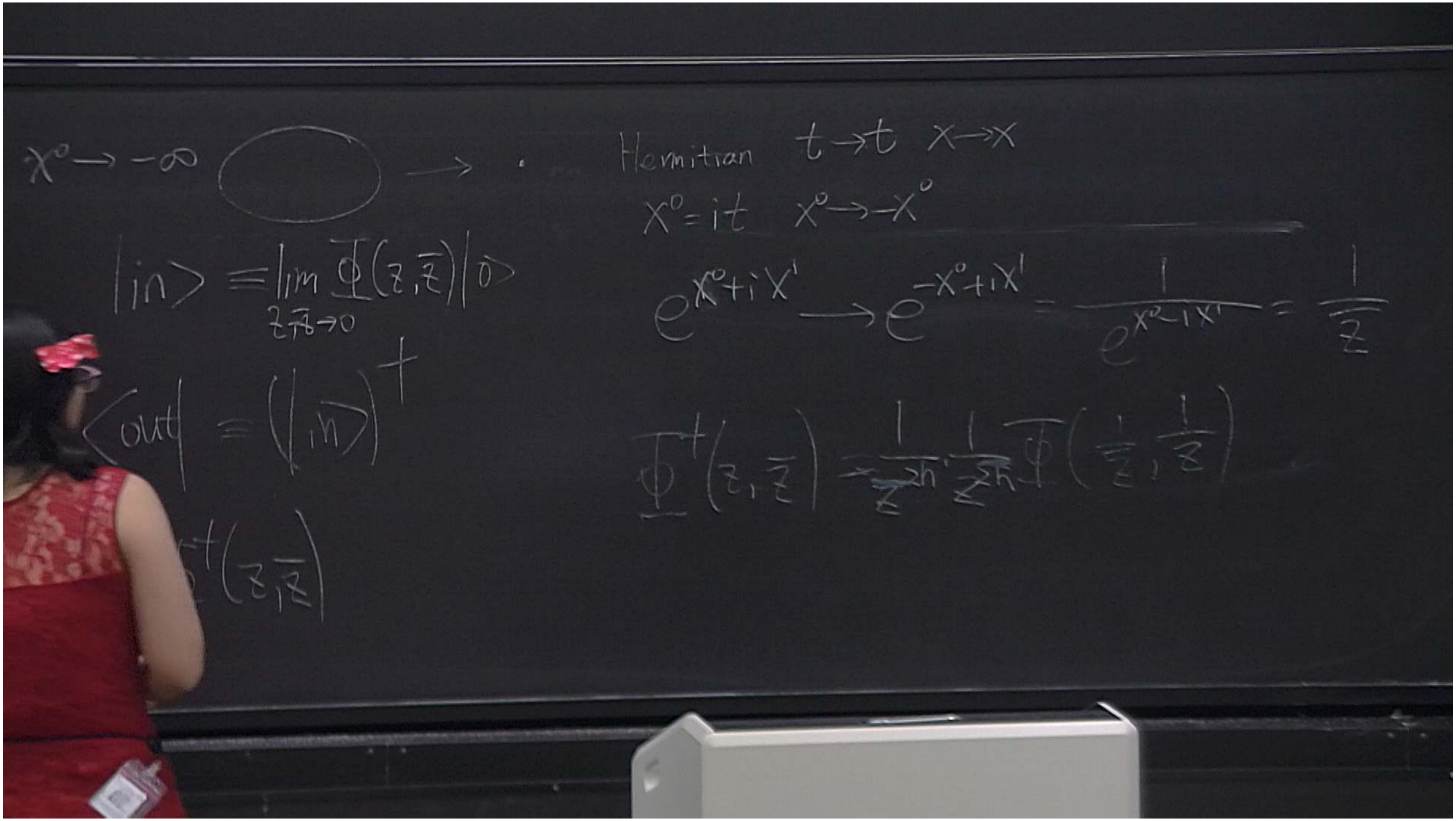
d.

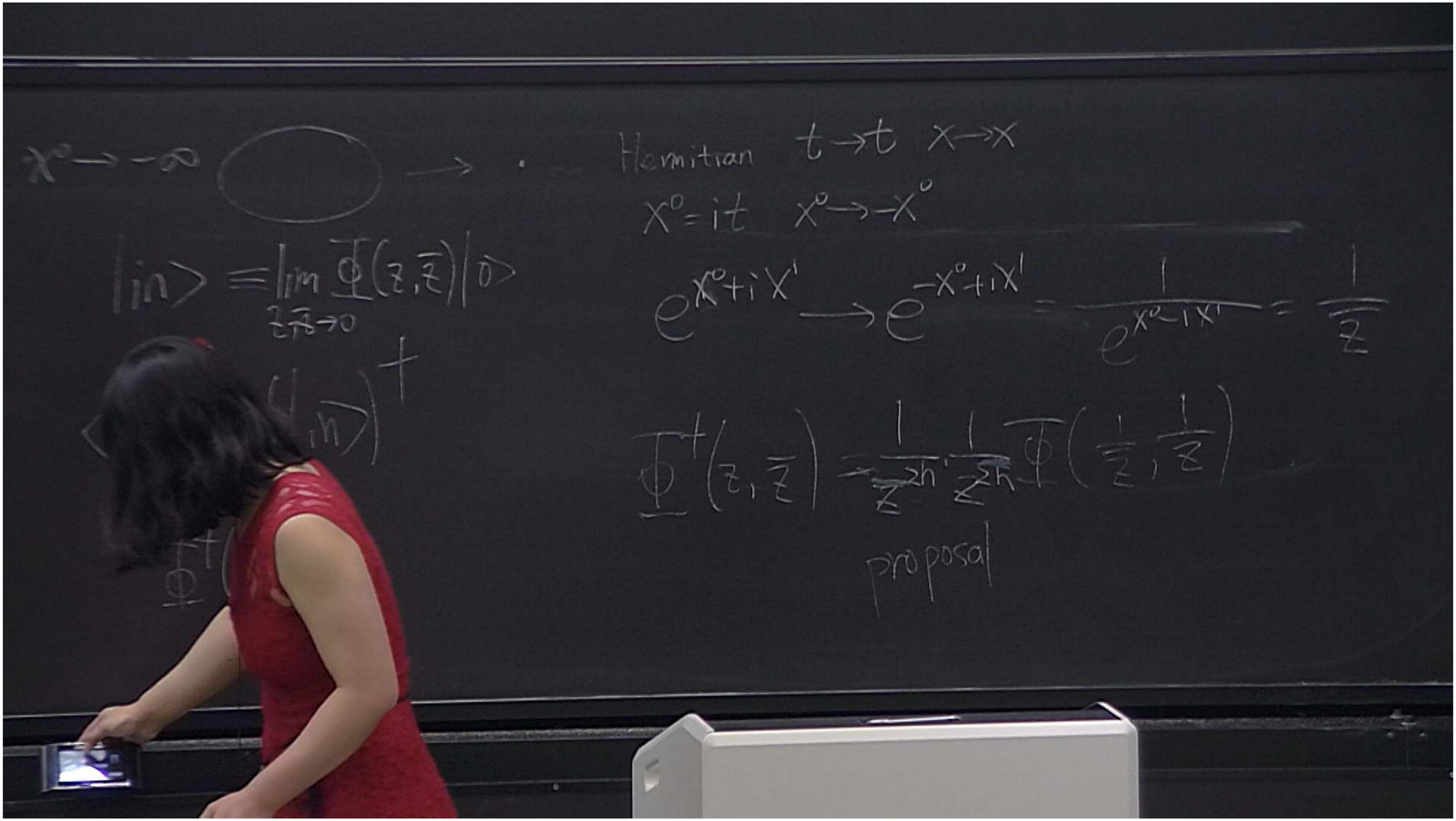
or/fields

tter


State-local operator Correspondence. $x^0 \to -\infty$
QFT
$|k\rangle = a_k^\dagger |0\rangle$
$P = \int d^3x\, \pi \cdot \nabla\phi$
$\phi(x)$
$|\psi(\phi(\sigma))\rangle$
$|in\rangle$


State-local operator Correspondence. x⁰ → −∞
QFT
|k> = a_k^† |0>
P^i = ∫d³x π · ∇φ
φ(x)
|ψ(φ(σ))>
space slice
|in> ≡ lim Φ(z, z̄) |0>
z, z̄ → 0


$x^0 \to -\infty$
Hermitian $t \to t$ $x \to x$
$x^0 = it$ $x^0 \to -x^0$
$|in\rangle \equiv \lim_{z,\bar{z}\to 0} \Phi(z,\bar{z})|0\rangle$
$e^{x^0+ix^1} \to e^{-x^0+ix^1} = \frac{1}{e^{x^0-ix^1}} = \frac{1}{\bar{z}}$
$\langle out| = (|in\rangle)^\dagger$
$\Phi^\dagger(z,\bar{z}) = \frac{1}{\bar{z}^{2h}} \frac{1}{z^{2\bar{h}}} \Phi\left(\frac{1}{\bar{z}}, \frac{1}{z}\right)$
$\Phi^\dagger(z,\bar{z})$


$x^0 \to -\infty$
Hermitian $t \to t$ $x \to x$
$x^0 = it$ $x^0 \to -x^0$
$|in\rangle = \lim_{z,\bar{z} \to 0} \Phi(z,\bar{z})|0\rangle$
$e^{x^0+ix^1} \to e^{-x^0+ix^1} = \frac{1}{e^{x^0-ix^1}} = \frac{1}{\bar{z}}$
$(|in\rangle)^\dagger$
$\Phi^\dagger(z,\bar{z}) = \frac{1}{\bar{z}^{2h}} \frac{1}{z^{2\bar{h}}} \Phi\left(\frac{1}{\bar{z}}, \frac{1}{z}\right)$
proposal

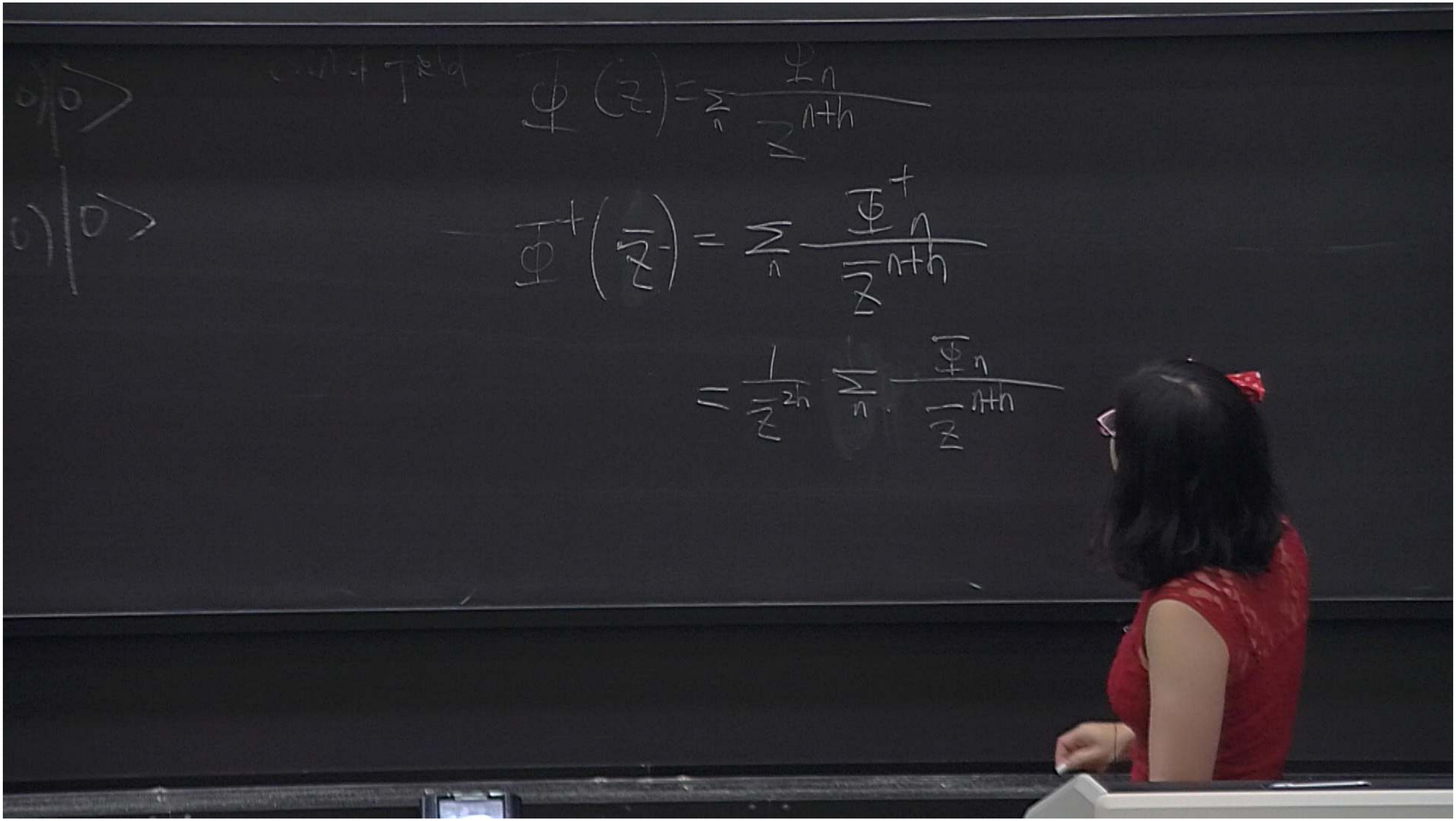
5) Miracle: [illegible] do not matter

$$\langle \mathrm{out} | \mathrm{in} \rangle = \lim_{z,\bar z \to 0} \langle 0 | \frac{1}{z^{2h}} \frac{1}{\bar z^{2\bar h}} \Phi\left(\frac{1}{z}, \frac{1}{\bar z}\right) \Phi(0,0) | 0 \rangle$$

$$\xrightarrow{\xi = \frac{1}{z}} \lim_{\bar\xi, \xi \to 0} \langle 0 | \xi^{2h} \bar\xi^{2\bar h} \Phi(\xi, \bar\xi) \Phi(0,0) | 0 \rangle$$

$$\frac{d}{\xi^{2h} \bar\xi^{2\bar h}}$$

$$= d$$

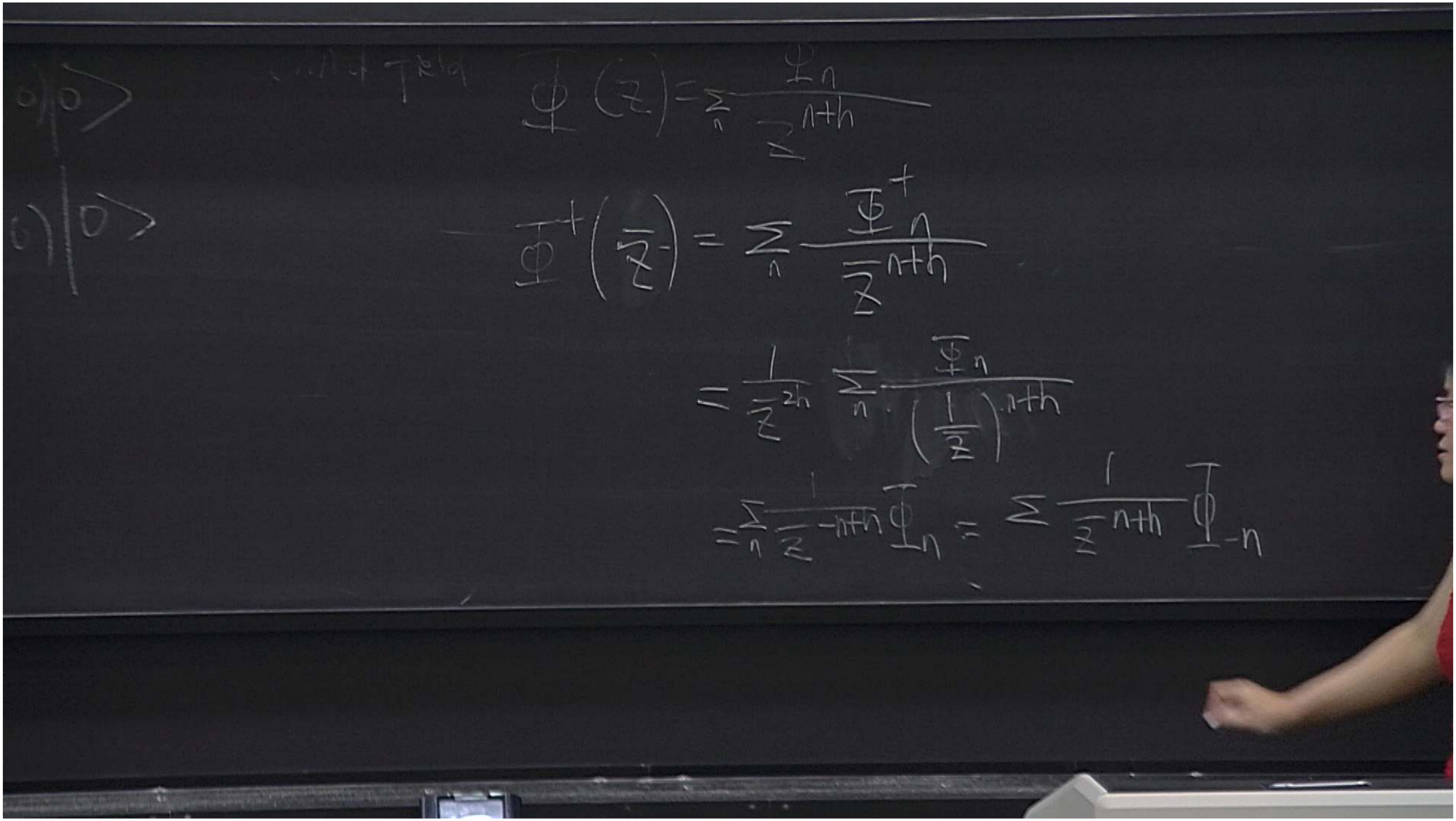

$\Phi(z)=\sum_n \frac{\Phi_n}{z^{n+h}}$
$\Phi^\dagger(\bar z)=\sum_n \frac{\Phi^\dagger_n}{\bar z^{n+h}}$
$=\frac{1}{\bar z^{2h}}\sum_n \frac{\Phi_n}{\bar z^{n+h}}$

$$\Phi(z) = \sum_n \frac{\Phi_n}{z^{n+h}}$$

$$\bar{\Phi}^\dagger(\bar{z}) = \sum_n \frac{\Phi^\dagger_n}{\bar{z}^{n+h}}$$

$$= \frac{1}{\bar{z}^{2h}} \sum_n \frac{\bar{\Phi}_n}{\left(\frac{1}{\bar{z}}\right)^{n+h}}$$

$$= \sum_n \frac{1}{\bar{z}^{-n+h}} \bar{\Phi}_n = \sum \frac{1}{\bar{z}^{n+h}}$$

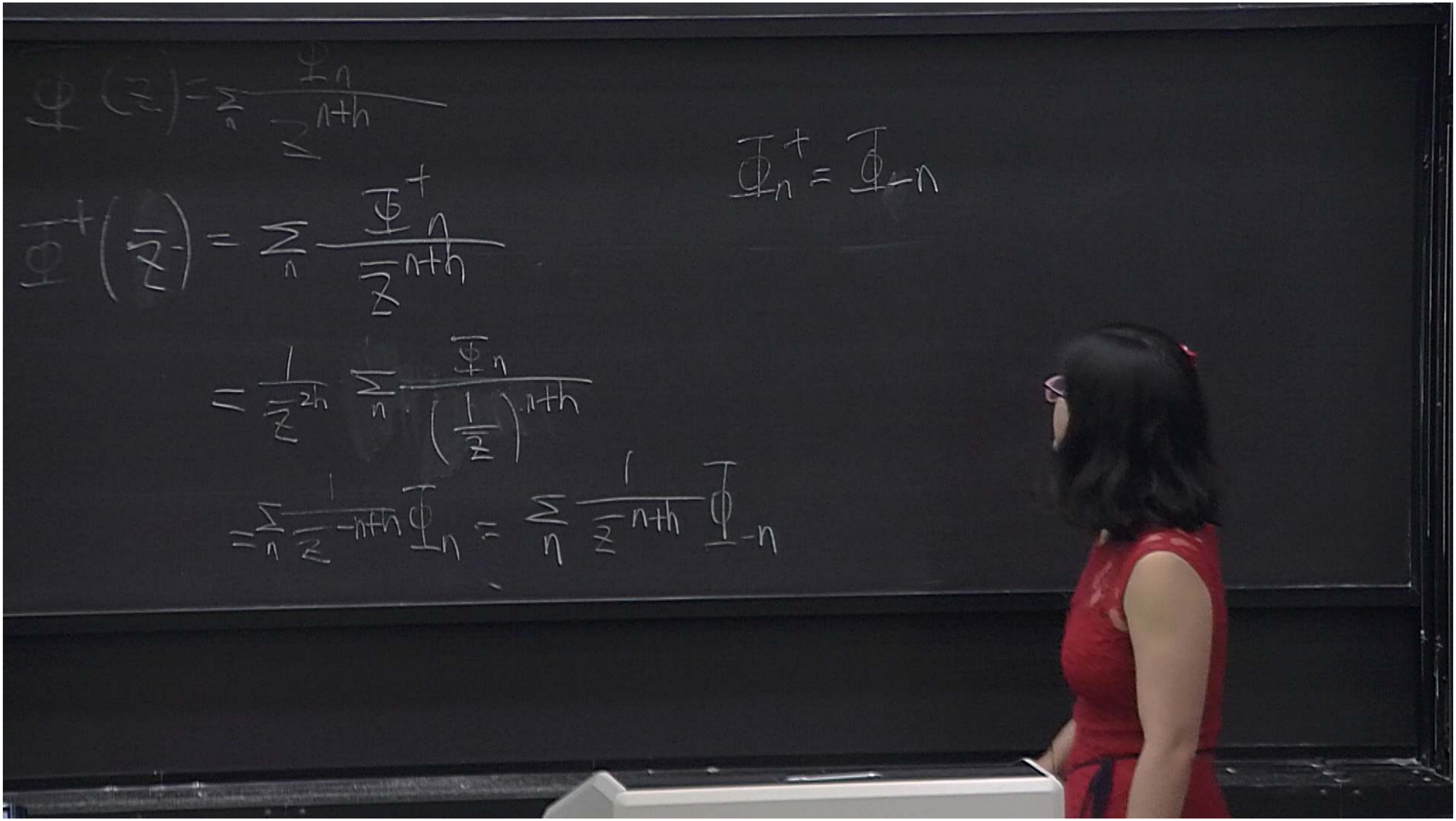

$\Phi(z) = \sum_n \frac{\Phi_n}{z^{n+h}}$
$\Phi^\dagger(\bar{z}) = \sum_n \frac{\Phi^\dagger_n}{\bar{z}^{n+h}}$
$= \frac{1}{\bar{z}^{2h}} \sum_n \frac{\Phi_n}{\left(\frac{1}{\bar{z}}\right)^{n+h}}$
$= \sum_n \frac{1}{\bar{z}^{-n+h}} \Phi_n = \sum \frac{1}{\bar{z}^{n+h}} \Phi_{-n}$

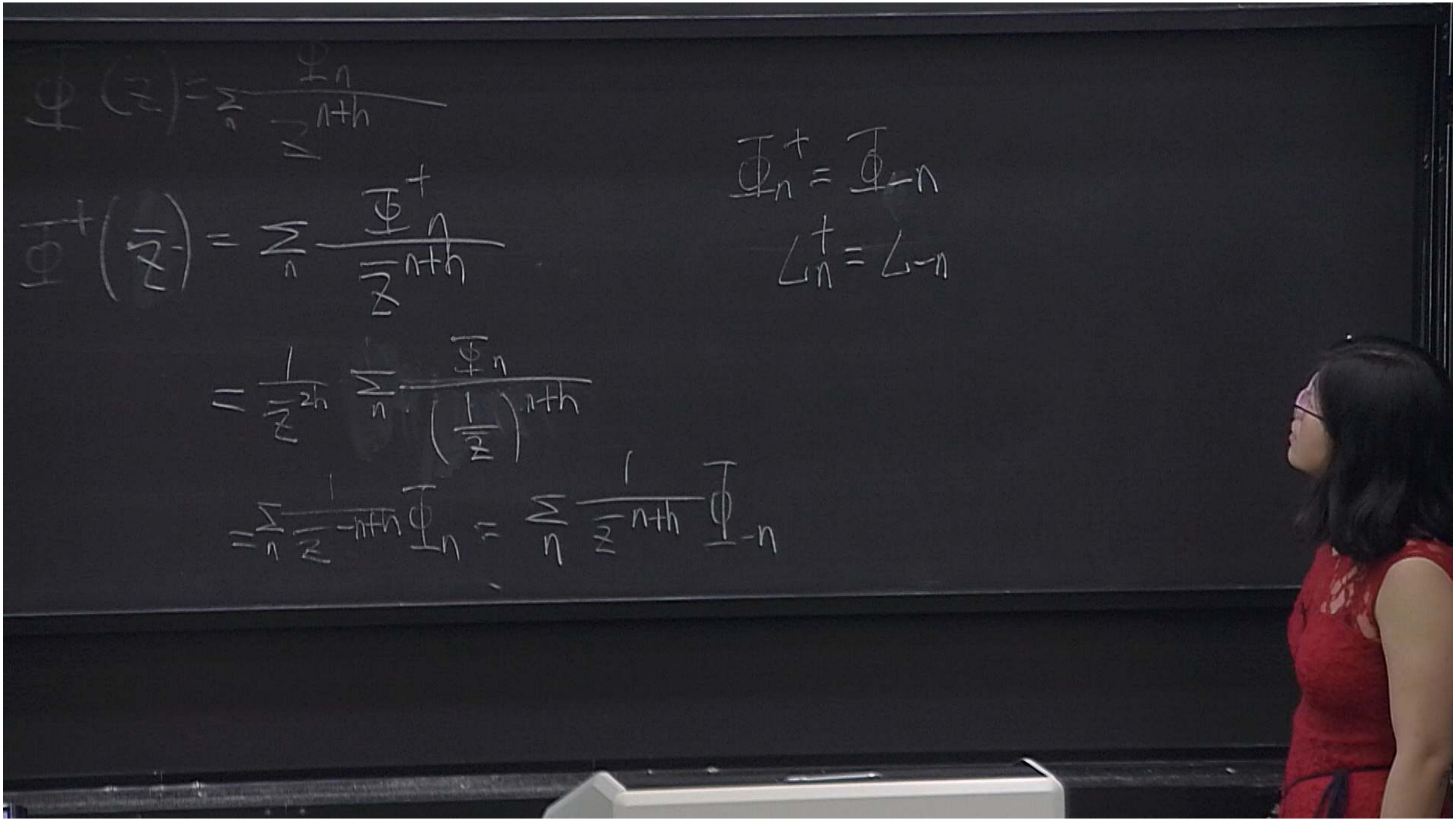
$$\Phi(z) = \sum_n \frac{\Phi_n}{z^{n+h}}$$

$$\Phi^\dagger(\bar{z}) = \sum_n \frac{\Phi^\dagger_n}{\bar{z}^{n+h}}$$

$$= \frac{1}{\bar{z}^{2h}} \sum_n \frac{\Phi_n}{\left(\frac{1}{\bar{z}}\right)^{n+h}}$$

$$= \sum_n \frac{1}{\bar{z}^{-n+h}} \Phi_n = \sum_n \frac{1}{\bar{z}^{n+h}} \Phi_{-n}$$


$\Phi(z) = \sum_n \frac{\Phi_n}{z^{n+h}}$
$\Phi^+_n = \Phi_{-n}$
$\Phi^+(\bar{z}) = \sum_n \frac{\Phi^+_n}{\bar{z}^{n+h}}$
$= \frac{1}{\bar{z}^{2h}} \sum_n \frac{\Phi_n}{\left(\frac{1}{\bar{z}}\right)^{n+h}}$
$= \sum_n \frac{1}{\bar{z}^{-n+h}} \Phi_n = \sum_n \frac{1}{\bar{z}^{n+h}} \Phi_{-n}$



$\Phi(z) = \sum_n \frac{\Phi_n}{z^{n+h}}$
$\Phi^\dagger(\bar{z}) = \sum_n \frac{\Phi^\dagger_n}{\bar{z}^{n+h}}$
$= \frac{1}{\bar{z}^{2h}} \sum_n \frac{\Phi_n}{\left(\frac{1}{\bar{z}}\right)^{n+h}}$
$= \sum_n \frac{1}{\bar{z}^{-n+h}} \Phi_n = \sum_n \frac{1}{\bar{z}^{n+h}} \Phi_{-n}$
$\Phi^\dagger_n = \Phi_{-n}$
$L^\dagger_n = L_{-n}$

$$\Phi(z) = \sum_n \frac{\Phi_n}{z^{n+h}}$$

$$\Phi^+(\bar z) = \sum_n \frac{\Phi^+_n}{\bar z^{n+h}}$$

$$= \frac{1}{\bar z^{2h}} \sum_n \frac{\Phi_n}{\left(\frac{1}{\bar z}\right)^{n+h}}$$

$$= \sum_n \frac{1}{\bar z^{-n+h}} \Phi_n = \sum_n \frac{1}{\bar z^{n+h}} \Phi_{-n}$$

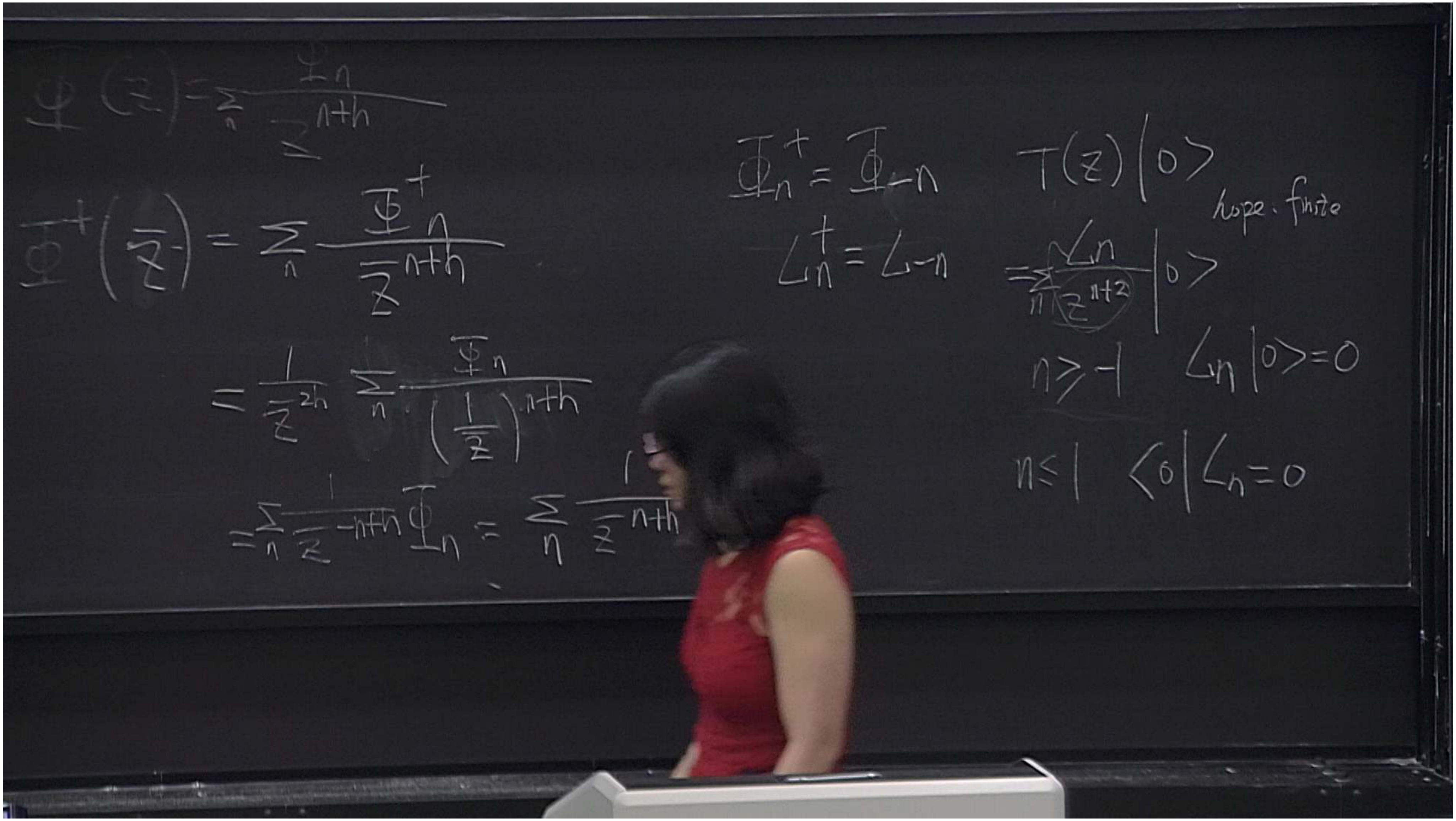
$$\Phi_n^+ = \Phi_{-n}$$

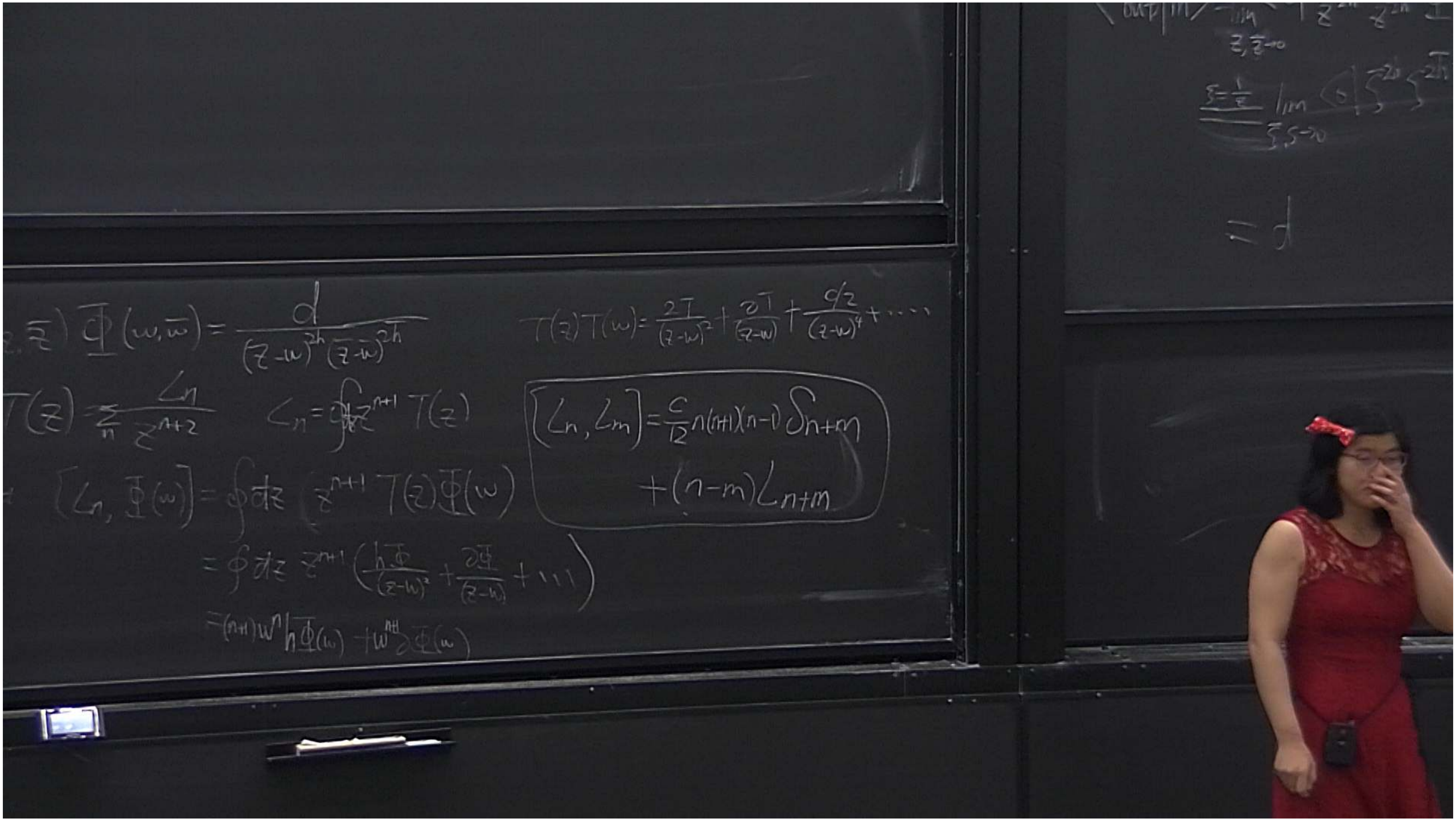
$$L_n^+ = L_{-n}$$

$T(z)|0\rangle$ hope: finite

$$= \sum_n \frac{L_n}{z^{n+2}} |0\rangle$$

$n \geq -1 \quad L_n|0\rangle = 0$


$\Phi(z) = \sum_n \frac{\Phi_n}{z^{n+h}}$
$\Phi^\dagger(\bar{z}) = \sum_n \frac{\Phi^\dagger_n}{\bar{z}^{n+h}}$
$= \frac{1}{\bar{z}^{2h}} \sum_n \frac{\Phi_n}{\left(\frac{1}{\bar{z}}\right)^{n+h}}$
$= \sum_n \frac{1}{\bar{z}^{-n+h}} \Phi_n = \sum_n \frac{1}{\bar{z}^{n+h}}$
$\Phi_n^\dagger = \Phi_{-n}$
$L_n^\dagger = L_{-n}$
$T(z)|0\rangle$
hope. finite
$= \sum_n \frac{L_n}{z^{n+2}} |0\rangle$
$n \geq -1 \quad L_n|0\rangle = 0$
$n \leq 1 \quad \langle 0| L_n = 0$



$\Phi(z,\bar z)\,\Phi(w,\bar w) = \frac{d}{(z-w)^{2h}(\bar z-\bar w)^{2h}}$
$T(z)T(w) = \frac{2T}{(z-w)^2} + \frac{\partial T}{(z-w)} + \frac{c/2}{(z-w)^4} + \cdots$
$T(z) = \sum_n \frac{L_n}{z^{n+2}}$
$L_n = \oint dz\, z^{n+1} T(z)$
$[L_n, L_m] = \frac{c}{12} n(n+1)(n-1)\delta_{n+m} + (n-m)L_{n+m}$
$[L_n, \Phi(w)] = \oint dz\, z^{n+1} T(z)\Phi(w)$
$= \oint dz\, z^{n+1}\left(\frac{h\Phi}{(z-w)^2} + \frac{\partial\Phi}{(z-w)} + \cdots\right)$
$= (n+1)w^n h\Phi(w) + w^{n+1}\partial\Phi(w)$
$\lim_{\xi,\bar\xi\to 0}$
$= d$

$= d \qquad \dfrac{d}{\xi^{-2h}\,\xi^{-2\bar{h}}}$

OB(1) is our only hope.

$R_2 \times D_2$ is . . . . . .

su(2) is our only hope ✓

$$= \sum_n \frac{1}{z^{-n+h}} \Phi_n = \sum_n \frac{1}{z^{n+h}}$$

SU(2) algebra

$$[J_+, J_-] = 2J_3$$

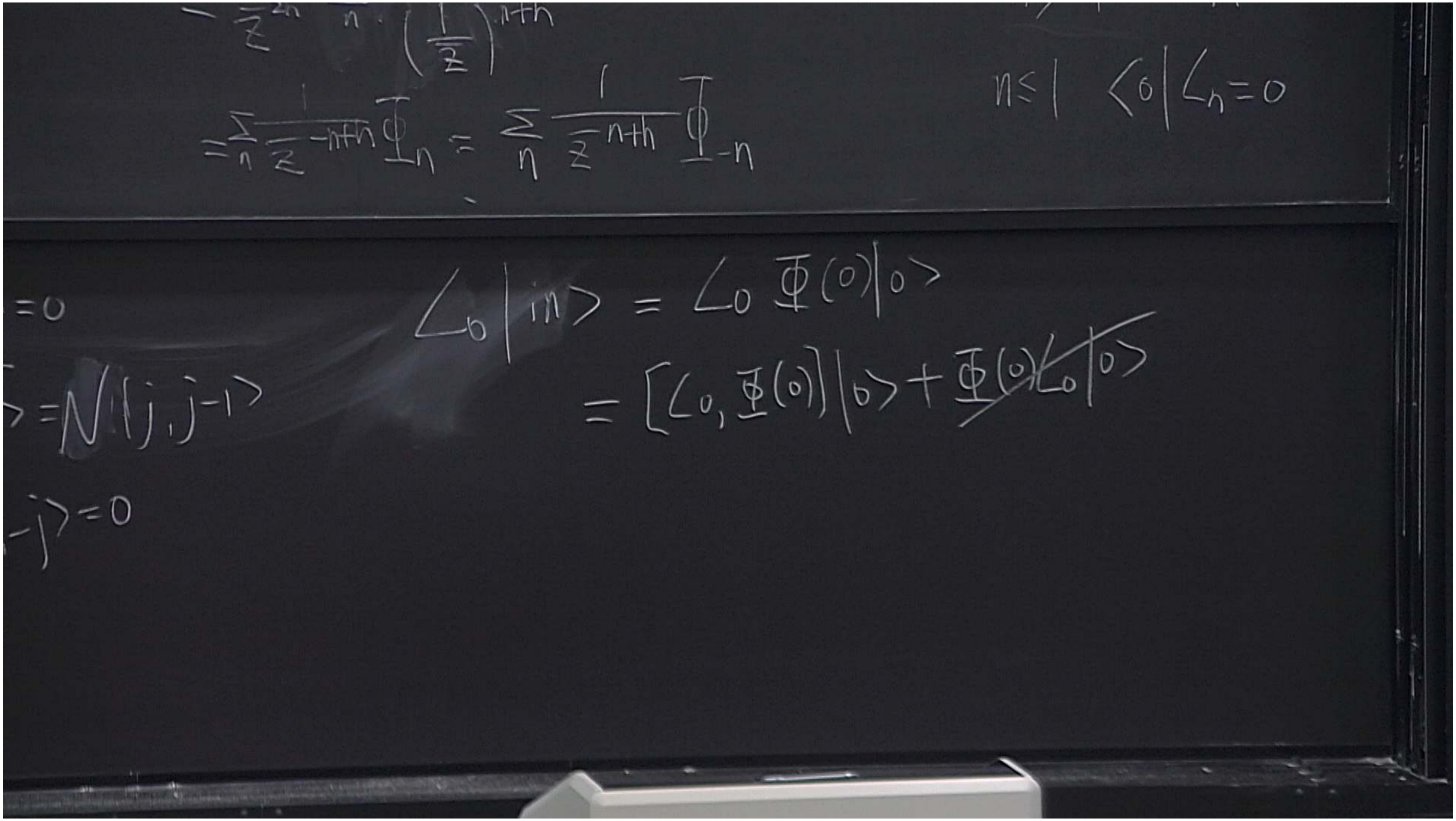
$$[J_\pm, J_3] = \pm J_3$$

$$J_3 |j, j\rangle = j |j, j\rangle$$

$$J_3 |j, m\rangle = m |j, m\rangle$$

$$J_+ |j, j\rangle = 0$$

$$J_- |j, j\rangle = N |j, j-1\rangle$$


= Σ_n 1/z^{-n+h} Φ_n = Σ_n 1/z^{-n+h} Φ_{-n}
n ≤ 1 ⟨0| L_n = 0
= 0
= N |j, j-1⟩
-j⟩ = 0
L_0 |in⟩ = L_0 Φ(0)|0⟩
= [L_0, Φ(0)]|0⟩ + Φ(0) L_0 |0⟩

$$= \sum_n \frac{1}{z^{-n+h}} \Phi_n = \sum_n \frac{1}{z^{-n+h}} \Phi_{-n}$$

$n \leq 1 \quad \langle 0 | L_n = 0$

$= 0$

$\rangle = N |j, j-1\rangle$

$-j\rangle = 0$

$$L_0 | in \rangle = L_0 \Phi(0) | 0 \rangle$$

$$= [L_0, \Phi(0)] | 0 \rangle + \cancel{\Phi(0) L_0 | 0 \rangle}$$

$$= h \Phi(0) | 0 \rangle$$

$$= \sum_n z^{-n+h} \Phi_n = \sum_n z^{n+h} \Phi_{-n}$$

$= 0$

$\rangle = N |j, j-1\rangle$

$-j\rangle = 0$

$$|in\rangle \equiv |h\rangle$$

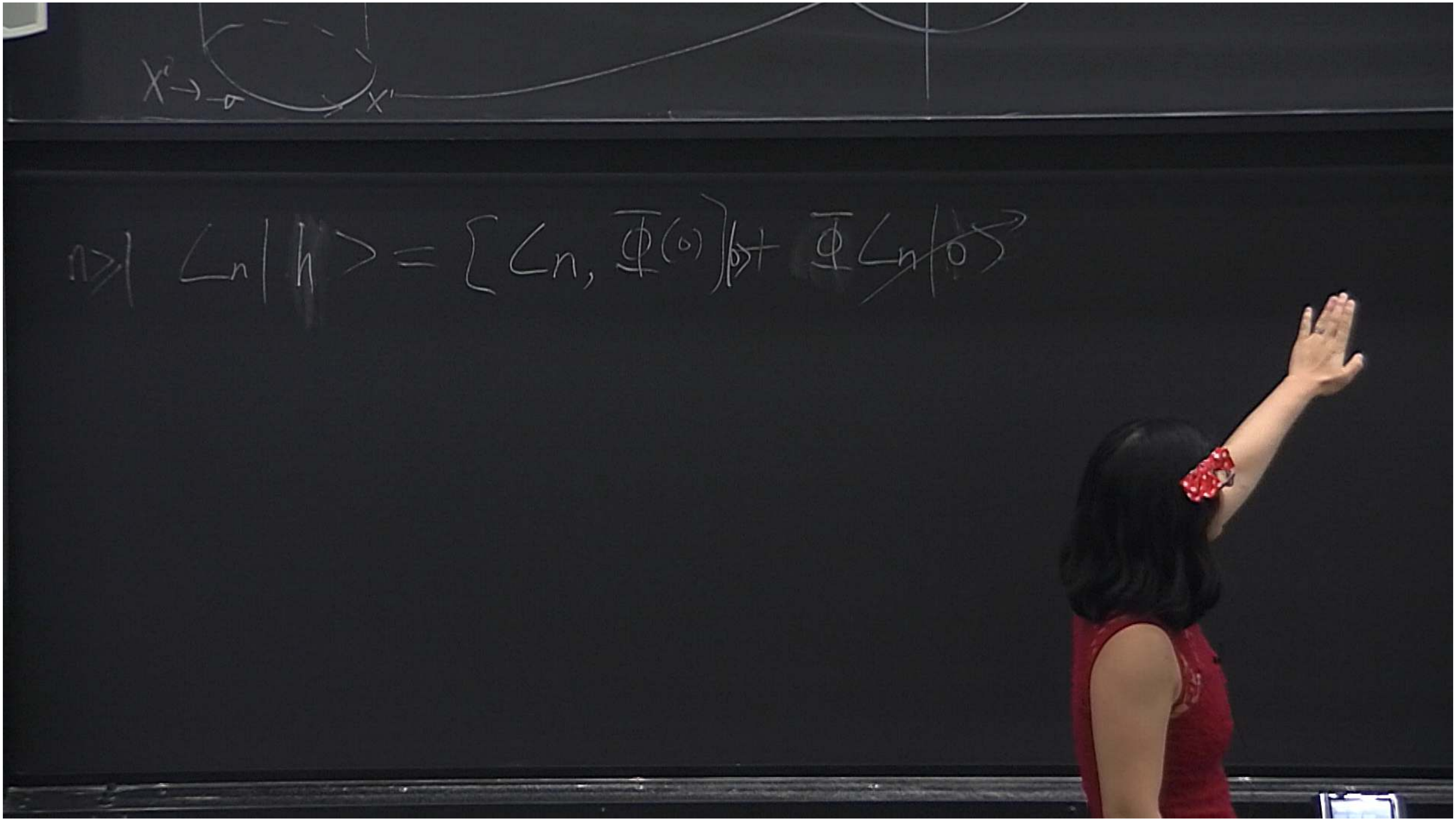
$$L_0 |h\rangle = h|h\rangle$$

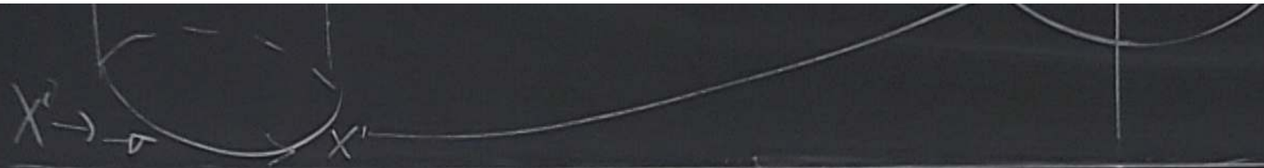
$$L_0 |in\rangle = L_0 \Phi(0)|0\rangle$$

$$= [L_0, \Phi(0)]|0\rangle + \cancel{\Phi(0) L_0 |0\rangle}$$

$$= h\,\Phi(0)|0\rangle$$

$$= h|in\rangle$$


$n>1 \quad \langle L_n | h \rangle = [L_n, \Phi(0)]|0\rangle + \Phi \cancel{L_n|0\rangle}$



$n>1 \quad L_n|h\rangle = [L_n, \Phi(0)]|0\rangle + \cancel{\Phi L_n|0\rangle}$

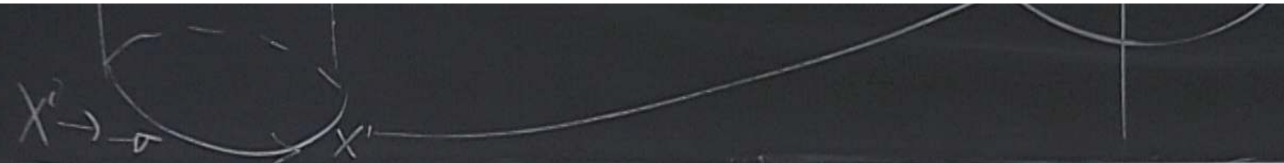
$L_n|h\rangle = 0$

$n \geq 1 \quad L_n|h\rangle = [L_n, \Phi(0)]|0\rangle + \cancel{\Phi L_n|0\rangle}$

$L_n|h\rangle = 0 \longleftrightarrow J_+$

$n \leq -1 \quad L_{-n}|h\rangle$

$L_0 L_{-n}|h\rangle$



Virasor

$n \geq 1 \quad L_n|h\rangle = [L_n, \Phi(0)]|0\rangle + \cancel{\Phi L_n|0\rangle}$

$$L_n|h\rangle = 0 \longleftrightarrow J_+$$

$n \geq 1 \quad L_{-n}|h\rangle$

$$L_0 L_{-n}|h\rangle = [L_0, L_{-n}]|h\rangle + L_{-n}L_0|h\rangle \longleftrightarrow J_-$$

$$= nL_{-n}|h\rangle + L_{-n}h|h\rangle$$

energy is bounded below $\leftarrow = (h+n)L_{-n}|h\rangle$

$$= (n+1) w^n h \bar{\Phi}(w) + w^{n+1} \partial \bar{\Phi}(w)$$

Virasoro algebra		SU(2) algebra.
$n \geq 1$ L_0	Cartan generator	J_3
L_n	raising operator	J_+
L_{-n}	lowering operator	J_-
$\vert h\rangle$	highest weight state	$\vert j, j\rangle$
$\vert h+n\rangle = L_{-n_1} \cdots L_{-n_i} \cdots L_{-n_p} \vert h\rangle$	generic state	$\vert j, m\rangle = J_- \cdots J_- \vert j, j\rangle$

$= d$ $\qquad \xi^{2h} \quad \bar{\xi}^{2\bar{h}}$

Hilbert Space

Level 0 $\quad L_0|h\rangle = h|h\rangle$

Level 1 $\quad L_{-1}|h\rangle \sim |h+1\rangle$

Level 2 $\quad L_{-2},\ (L_{-1})^2\ |h\rangle \sim |h+2\rangle$

Level 3 $\quad L_{-3},\ L_{-2}L_{-1},\ (L_{-1})^3\ |h\rangle \sim |h+3\rangle$

$\vdots$

Level n $\quad p(n) \quad L_{-n_1} L_{-n_2} \cdots L_{-n_j}\ |h\rangle \sim |h+n\rangle$

$\quad \sum_i n_i = n$

$$= \sum_n \frac{1}{z^{-n+h}} \Phi_n = \sum_n \frac{1}{z^{n+h}} \Phi_{-n}$$

$n \leq$

descendant operator.

$T(z)$

$$= \sum_n \frac{1}{z^{-n+h}} \Phi_n = \sum_n \frac{1}{z^{n+h}} \Phi_{-n}$$

$n \leq$

descendant operator.

$$T(z)\,\Phi(w) = \frac{h\Phi}{(z-w)^2} + \frac{\partial\Phi}{(z-w)} + \cdots$$

$$= \sum_n \frac{1}{(z-w)^{2-n}} \hat{L}_{-n} \Phi$$

$$= \sum_n \frac{1}{z^{-n+h}} \Phi_n = \sum_n \frac{1}{z^{n+h}} \Phi_{-n}$$

$n \leq$

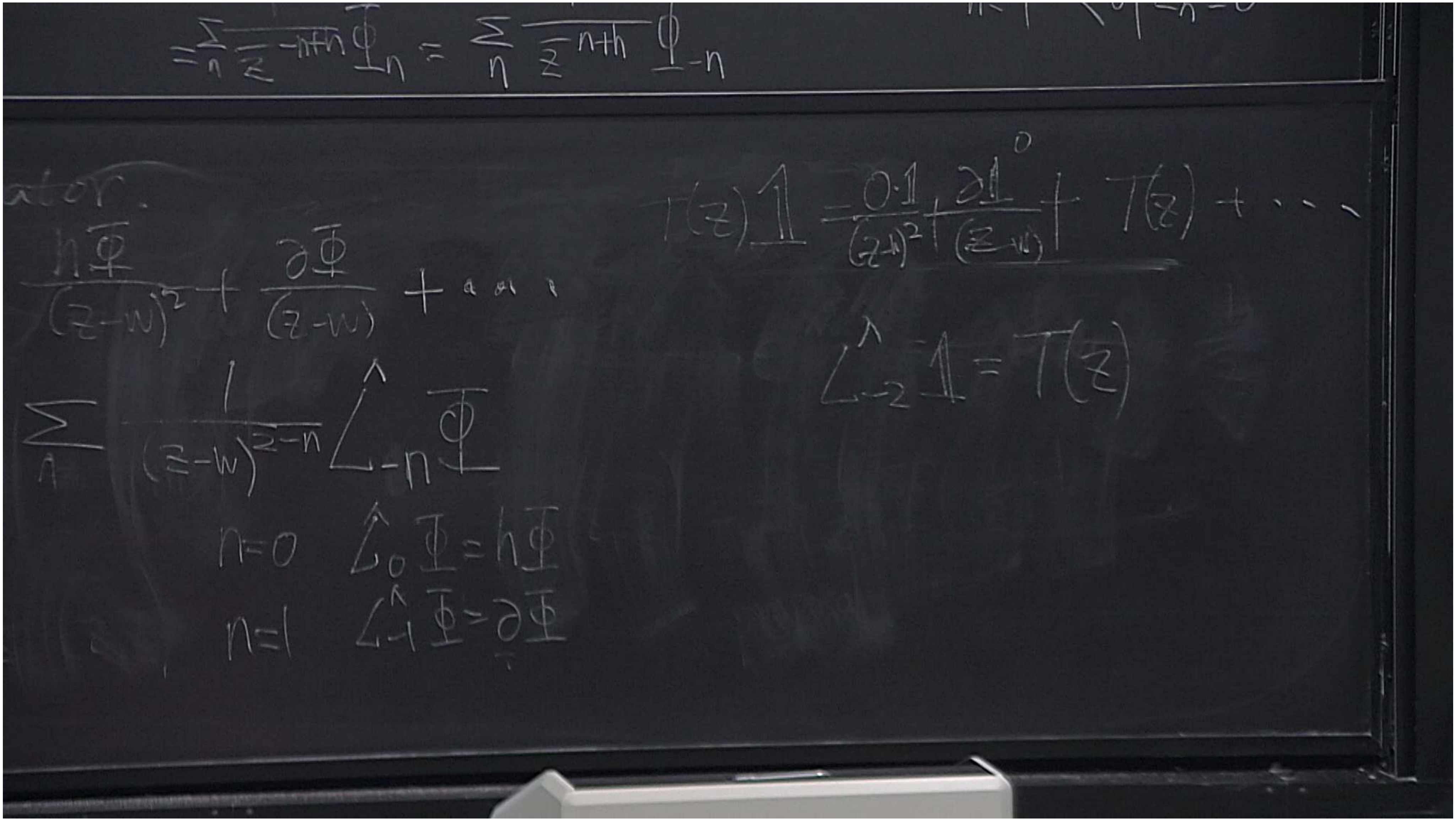
descendant operator.

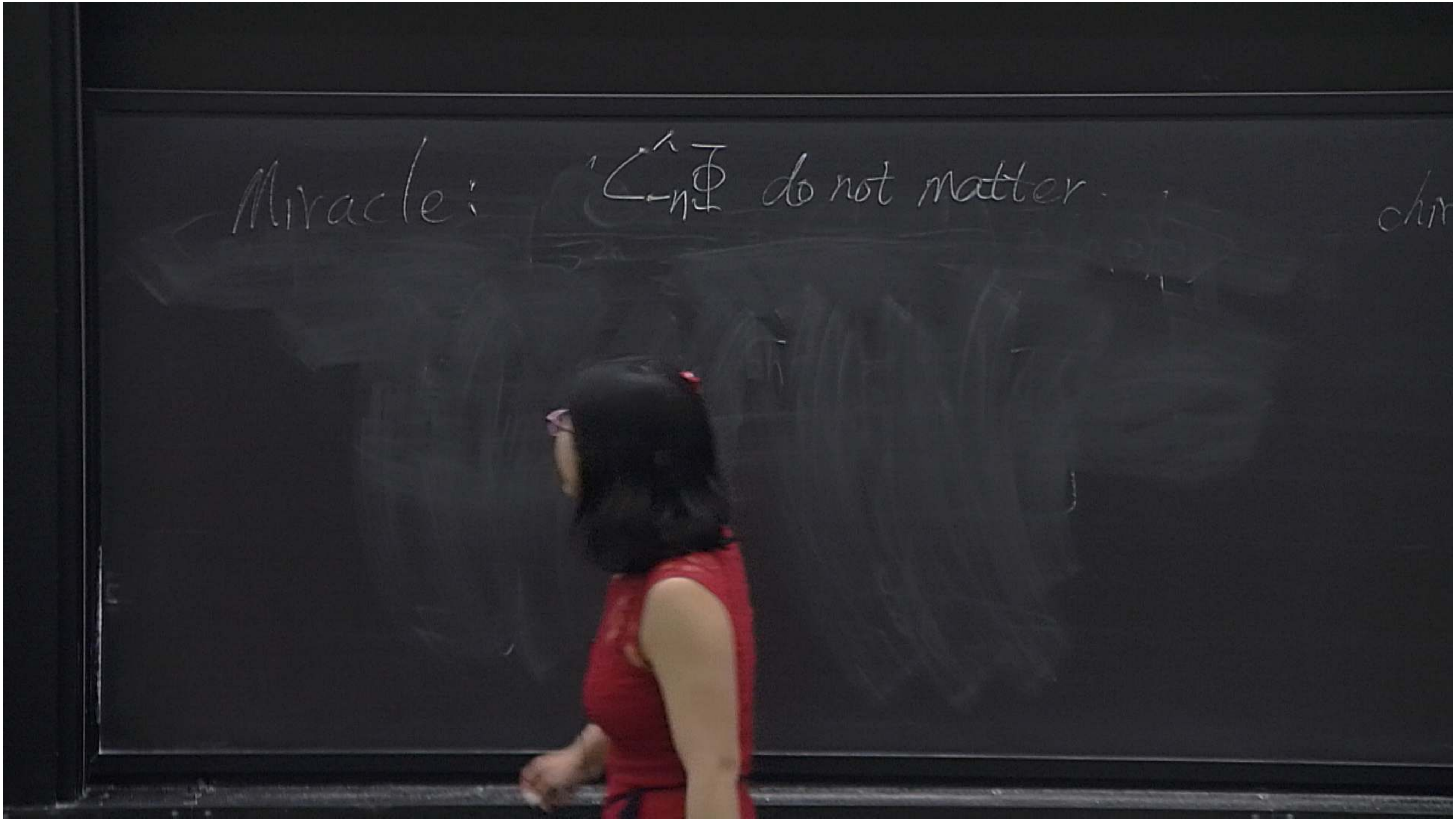
$$T(z)\,\Phi(w) = \frac{h\Phi}{(z-w)^2} + \frac{\partial\Phi}{(z-w)} + \ldots$$

$$= \sum_n \frac{1}{(z-w)^{2-n}} \hat{L}_{-n} \Phi$$

$n=0 \quad \hat{L}_0 \Phi = h\Phi$

$n=1 \quad \hat{L}_{-1} \Phi = \partial\Phi$


$= \sum_n z^{-n+h} \Phi_n = \sum_n z^{n+h} \Phi_{-n}$
ator.
$\frac{h\Phi}{(z-w)^2} + \frac{\partial\Phi}{(z-w)} + \ldots$
$\sum_n \frac{1}{(z-w)^{2-n}} \hat{L}_{-n} \Phi$
$n=0 \quad \hat{L}_0 \Phi = h\Phi$
$n=1 \quad \hat{L}_{-1} \Phi = \partial\Phi$
$T(z)\mathbb{1} = \frac{0 \cdot \mathbb{1}}{(z-w)^2} + \frac{\overset{0}{\partial \mathbb{1}}}{(z-w)} + T(z) + \ldots$
$\hat{L}_{-2}\mathbb{1} = T(z)$


Miracle: '$\hat{L}_{-n}\Phi$' do not matter.

Miracle: $\hat{L}_{-n}\Phi$ do not matter.

chir

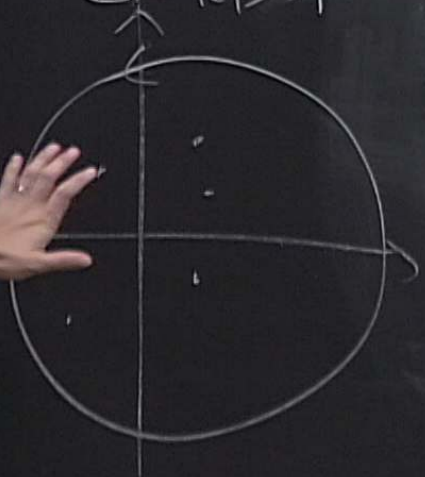
$$\langle \varepsilon(z)\, T(z)\, \Phi(w_1) \cdots \Phi(w_i) \cdots \Phi(w_n) \rangle$$

Miracle: $\hat{L}_{-n}\Phi$ do not matter.

chir

$$\oint_C dz \langle \varepsilon(z)\, T(z)\, \Phi(w_1) \cdots \Phi(w_i) \cdots \Phi(w_n) \rangle$$

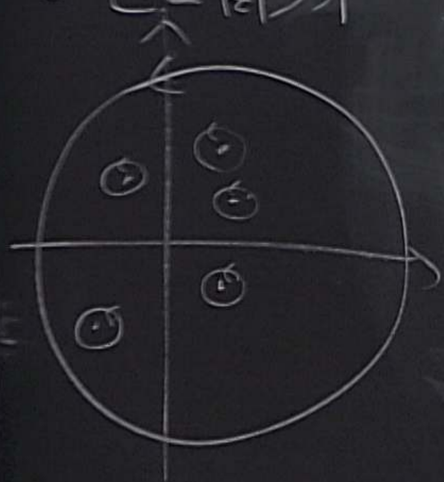
$C = |z| >> 1$



Miracle: $\hat{L}_{-n}\Phi$ do not matter.

$$\oint dz \langle \varepsilon(z)\, T(z)\, \Phi(w_1)\cdots\Phi(w_j)\cdots\Phi(w_n)\rangle$$

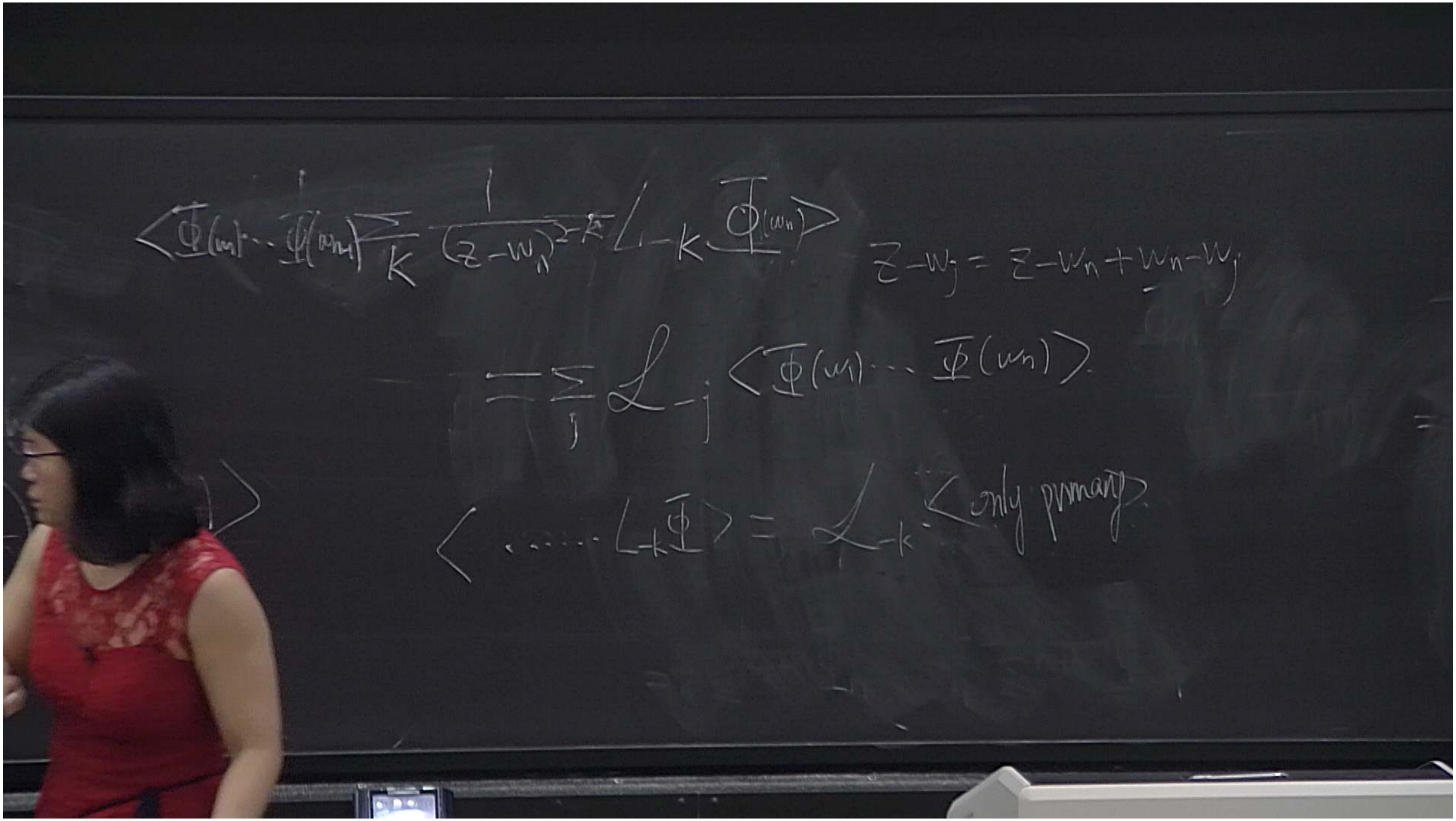
$C = |z| >> 1$



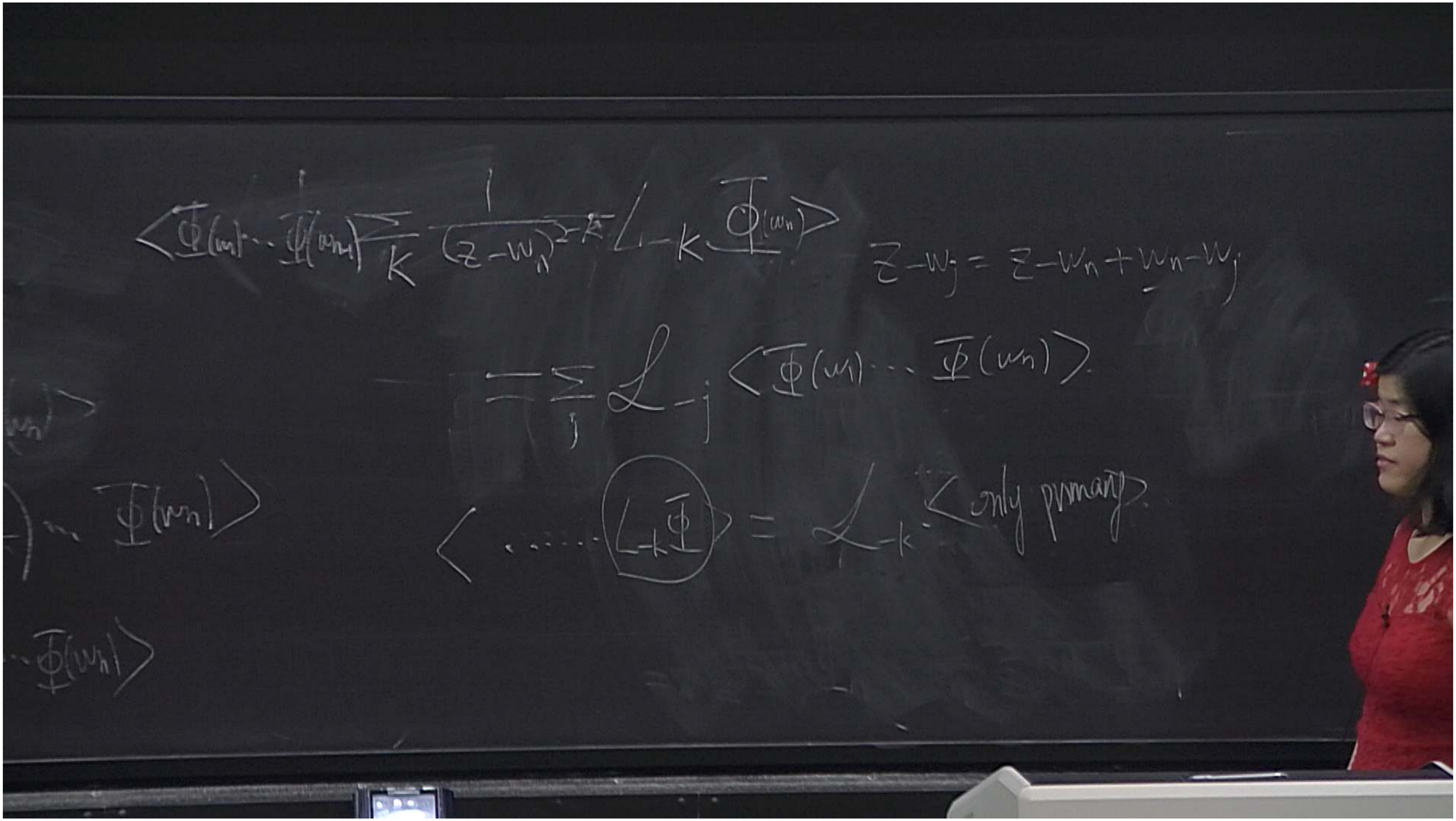
$$= \sum_j \oint_{C_j} dz \langle \varepsilon(z)\, \Phi(w_1)\cdots \left(T(z)\,\Phi(w_j)\right)\cdots\Phi(w_n)\rangle$$

$$= \sum_j \oint_{C_j} dz \langle \varepsilon(z)\, \Phi(w_1)\cdots\left(\frac{h\,\Phi(w_j)}{(z-w_j)^2}+\frac{\partial\Phi(w_j)}{(z-w_j)}\right)\cdots\Phi(w_n)\rangle$$

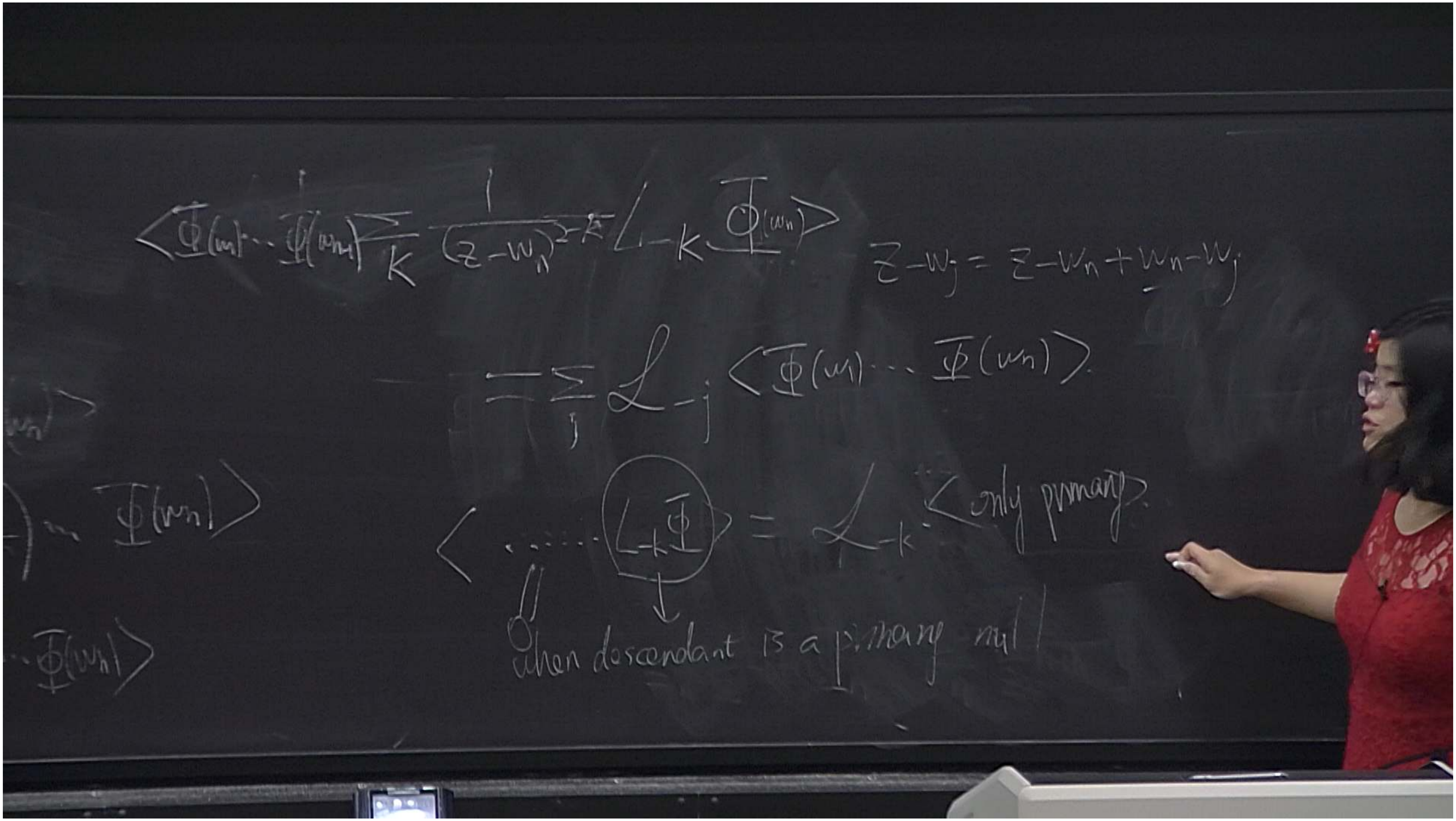
$$= \sum_j \oint_{C_j} dz \left(\frac{h}{(z-w_j)^2}+\frac{\partial}{(z-w_j)}\right)\langle\varepsilon(z)\,\Phi(w_1)\cdots\Phi(w_n)\rangle$$



$\langle \Phi(w_1)\cdots\Phi(w_{n-1}) \sum_k \frac{1}{(z-w_n)^{2-k}} L_{-k}\Phi(w_n)\rangle$
$z-w_j = z-w_n+w_n-w_j$
$= \sum_j \mathcal{L}_{-j} \langle \Phi(w_1)\cdots\Phi(w_n)\rangle$
$\langle \cdots\cdots L_{-k}\Phi\rangle = \mathcal{L}_{-k} \langle$only primary$\rangle$



$z - w_j = z - w_n + w_n - w_j$
only primary


only primary
when descendant is a primary null