

Title: PSI 17/18 - Quantum Field Theory III - Lecture 11

Date: Feb 09, 2018 11:30 AM

URL: <http://pirsa.org/18020031>

Abstract:

Gauge Anomalies

$$\mathcal{L} = \bar{\Psi}_L^i (\not{\partial} + i A^a (T_a)^i_j) \Psi_L^j - \frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu}_a$$

$$\Psi_L = P_L \Psi$$

gauge invariant: $\Psi_L \rightarrow e^{i\beta^a(x)T^a} \Psi_L = U \Psi_L$

$$A_\mu \rightarrow U A_\mu U^{-1} + U^{-1} \partial_\mu U$$

conserved current: $J^a_\mu = \bar{\Psi}_L \gamma_\mu T^a \Psi_L \rightarrow \text{not gauge invariant}$
 $\rightarrow \text{not observable}$

$$\Psi_L = PL\Psi$$

EOM for A_μ^a :
$$\partial^\nu \frac{\partial \mathcal{L}}{\partial (\partial^\nu A_\mu^a)} = \frac{\partial \mathcal{L}}{\partial A_\mu^a}$$

$$\frac{1}{g^2} (\partial^\nu F_{\mu\nu})^a = \bar{\Psi}_L \gamma_\mu T^a \Psi_L = J_{LM}^a$$

apply D^μ to EOM: $0 = \cancel{(D_\mu J_{LM}^a)^a} \rightarrow \text{covariantly conserved current}$
 wrong when including quantum effects

correct equation is:

$$\begin{aligned} (D^\mu \mathcal{J}_\mu)^a &= \frac{1}{24\pi^2} \epsilon^{\mu\nu\rho\sigma} \partial_\mu \text{tr} [T^a A_\nu \partial_\rho A_\sigma - \frac{i}{4} A_\nu [A_\rho, A_\sigma]] \\ &= \frac{1}{24\pi^2} \epsilon^{\mu\nu\rho\sigma} \partial_\mu [A_\nu^b \partial_\rho A_\sigma^c - \frac{i}{4} A_\nu^b [A_\rho, A_\sigma]^c] D^a_{bc} \end{aligned}$$

$$D^{abc} = \text{tr} [T^a T^b T^c]$$

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$$D^{abc} = \text{tr} [T^a T^b T^c]$$

EOM are wrong! $\rightarrow$ need extra terms in $\mathcal{L} \rightarrow$ not gauge invariant $\rightarrow$ not renormalizable or unitary

Functional Integral Derivation

extra factor T^a

Euclidean $Z[A] = \int \mathcal{D}\Psi_L \mathcal{D}\bar{\Psi}_L e^{-\int \bar{\Psi}_L i \not{D} \Psi_L} \propto \text{Det}(i \not{D} P_L)$

$$i \not{D} P_L = i P_R \not{D} \quad \text{maps } \Psi_L \rightarrow \Psi_R$$

$Z[A]$ existence unclear

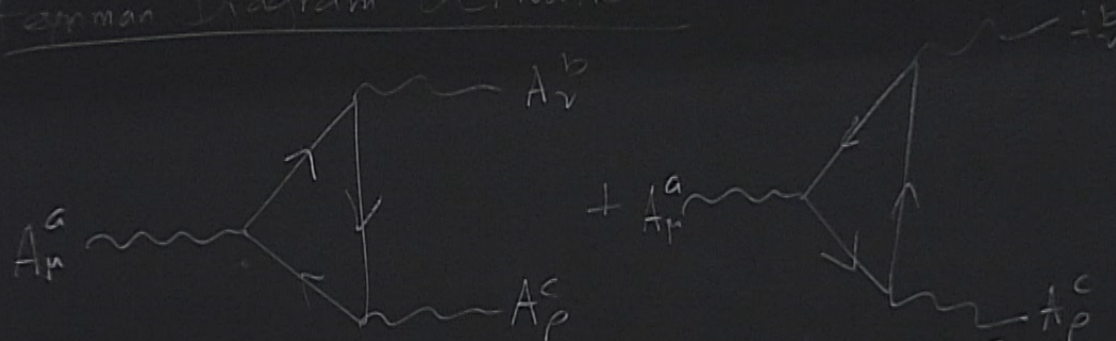
Consider $(i \not{D} P_L)^\dagger (i \not{D} P_L)$ maps $\Psi_L \rightarrow \Psi_L$

$$\text{Det}[(i\mathcal{D}P_L)^\dagger (i\mathcal{D}P_L)] = |\text{Det}(i\mathcal{D}P_L)|^2$$

↑ use to define $|Z[A]|$

if phase of $|Z[A]|$ (or $\text{Im} \Gamma[A]$) cannot be
defined in gauge-invariant manner $\leftrightarrow$ anomaly

Feynman Diagram Derivation



$$\propto \text{tr}[T^a T^c T^b] + \text{tr}[T^a T^b T^c] \propto \text{tr}[T^a T^b T^c] \propto \text{tr}[T^a \{T^b, T^c\}]$$

Anomaly Cancellation

Need $A^{abc} = 0$

$$\begin{aligned} A^{abc} &\equiv \sum_{\text{fermions}} \text{tr} [T^a \{T^b, T^c\}] \\ &= \sum_L \text{tr} [T_L^a \{T_L^b, T_L^c\}] - \sum_R \text{tr} [T_R^a \{T_R^b, T_R^c\}] \end{aligned}$$

Cases with

marking

Cases with trivial cancellation or no contributions

- If $\psi_L + \psi_R$ live in same rep $\rightarrow$ cancel

Dirac $\psi = \psi_L + \psi_R$ in same rep

- If $\psi_L + \psi_R$ live in equivalent reps

$$T_L = U T_R U^{-1}$$

$$\text{tr} [T_R^a \{T_R^b, T_R^c\}]$$

- fermions in real or pseudoreal reps don't contribute
- pseudoreal: $T_L^T = -V T_L V^{-1}$ for V unitary
- real if $V = \mathbb{1}$

$$\begin{aligned}
 \text{tr} [T_L^a \{T_L^b, T_L^c\}] &= \text{tr} [\{T_L^b, T_L^c\} T_L^a] \\
 &= \text{tr} [T_L^a \{T_L^b, T_L^c\}] \\
 &= (-1)^3 \text{tr} [T_L^a \{T_L^b, T_L^c\}] \rightarrow \text{tr} [] = 0
 \end{aligned}$$

$$SU(2)$$

$$SO(2N+1) \quad N \geq 1$$

$$SO(4N) \quad N \geq 2$$

$$Sp(2N) \quad N \geq 3$$

$$G_2, F_4, E_6, E_7, E_8$$

+ direct products only involving these groups

only pseudoreal reps

$$\rightarrow \frac{1}{\text{Tr}} = 0$$

$SU(2)$

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G_2, F_4, E_7, E_8

+ direct products only involving these groups

Only pseudoreal reps

- $SO(4N+2) \quad N \geq 1$ and E_6 have reps that are not pseudoreal but none contribute to anomalies

$$\frac{1}{2} \text{tr}(\gamma_5) = 0$$

massive fermions do not contribute

$$m \psi_L^T \sigma^2 \psi_L + \text{h.c.}$$

$$\psi_L \rightarrow e^{i\beta T^a} \psi_L$$

$$\psi_L^T \rightarrow \psi_L^T e^{i\beta T^{aT}}$$

$$m \psi_L^T \sigma^2 \psi_L \rightarrow m \psi_L^T \sigma^2 e^{i\beta(T^a + T^{aT})} \psi_L$$

$$T^a = -T^{aT} \rightarrow \text{real rep}$$

also works for $M_{ij} \psi_{Li}^T \sigma^2 \psi_{Lj}$

only need to com

only need to compute for massless fermions
if gauge group is $SU(N)$ $N \geq 3$
 $U(1)$
or products involving these groups

$$SM = SU(3) \times SU(2) \times U(1)$$

Summary

Chiral Anomalies 😊

Gauge Anomalies ☹️