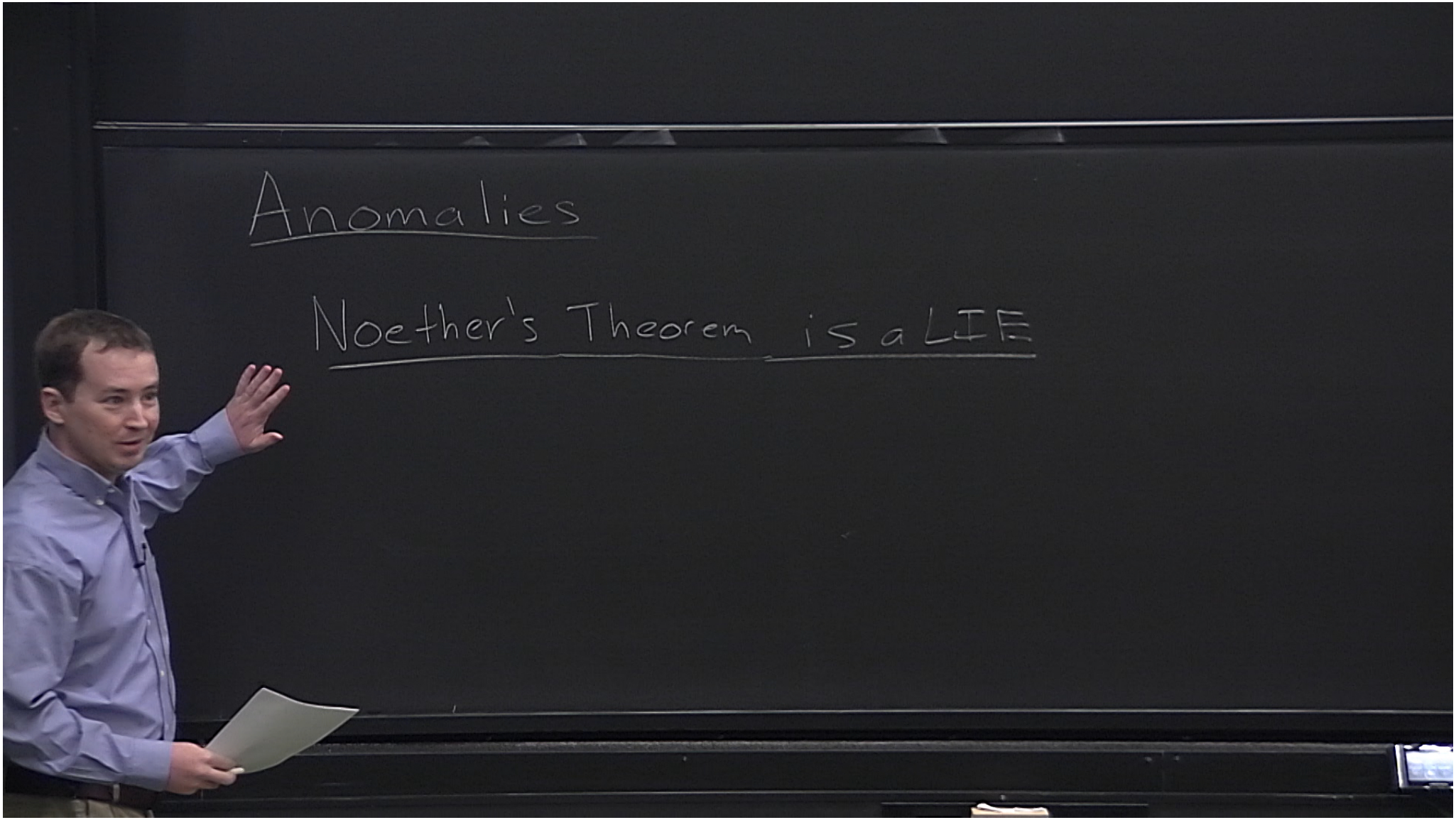
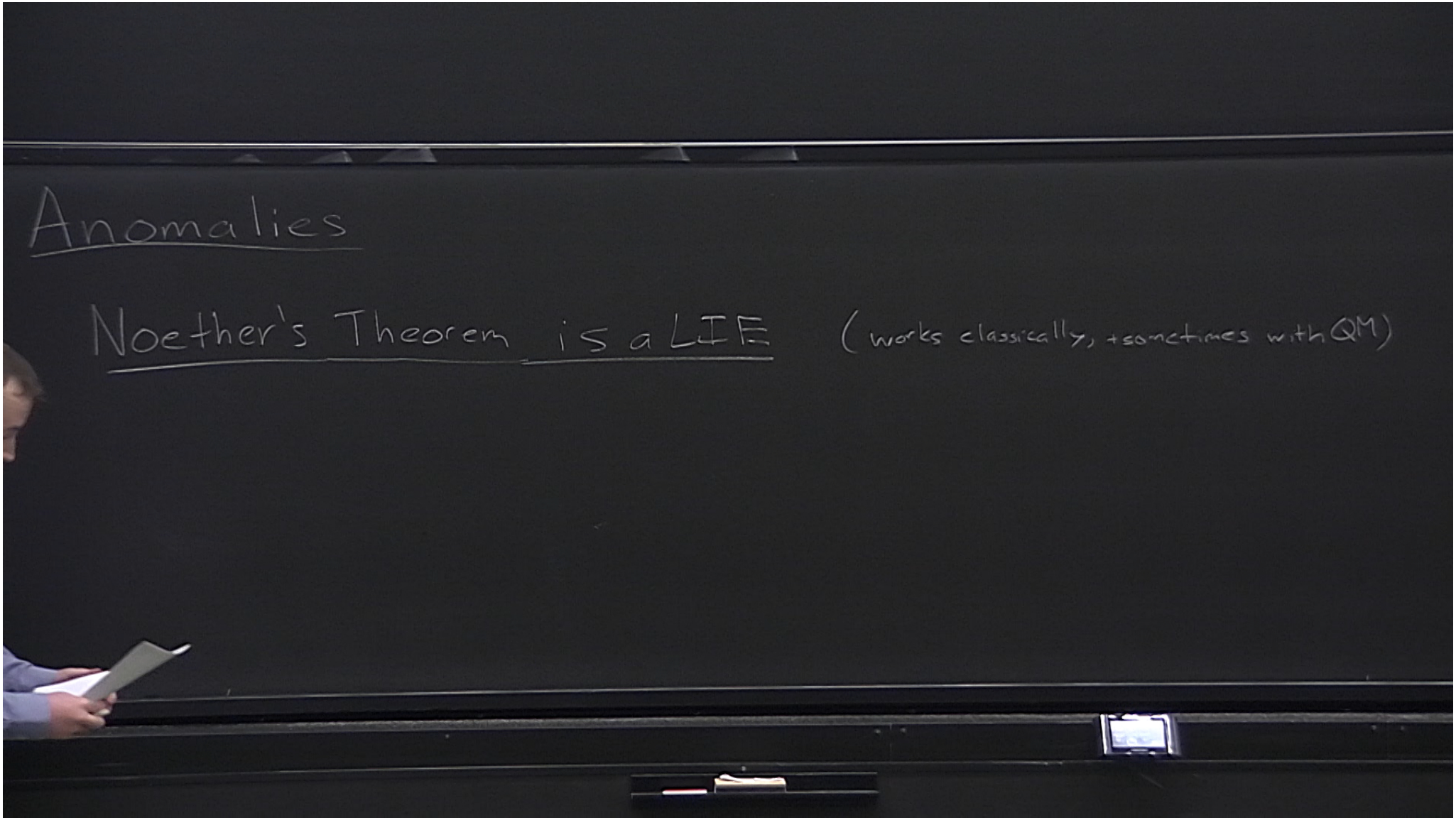
Title: PSI 17/18 - Quantum Field Theory III - Lecture 7

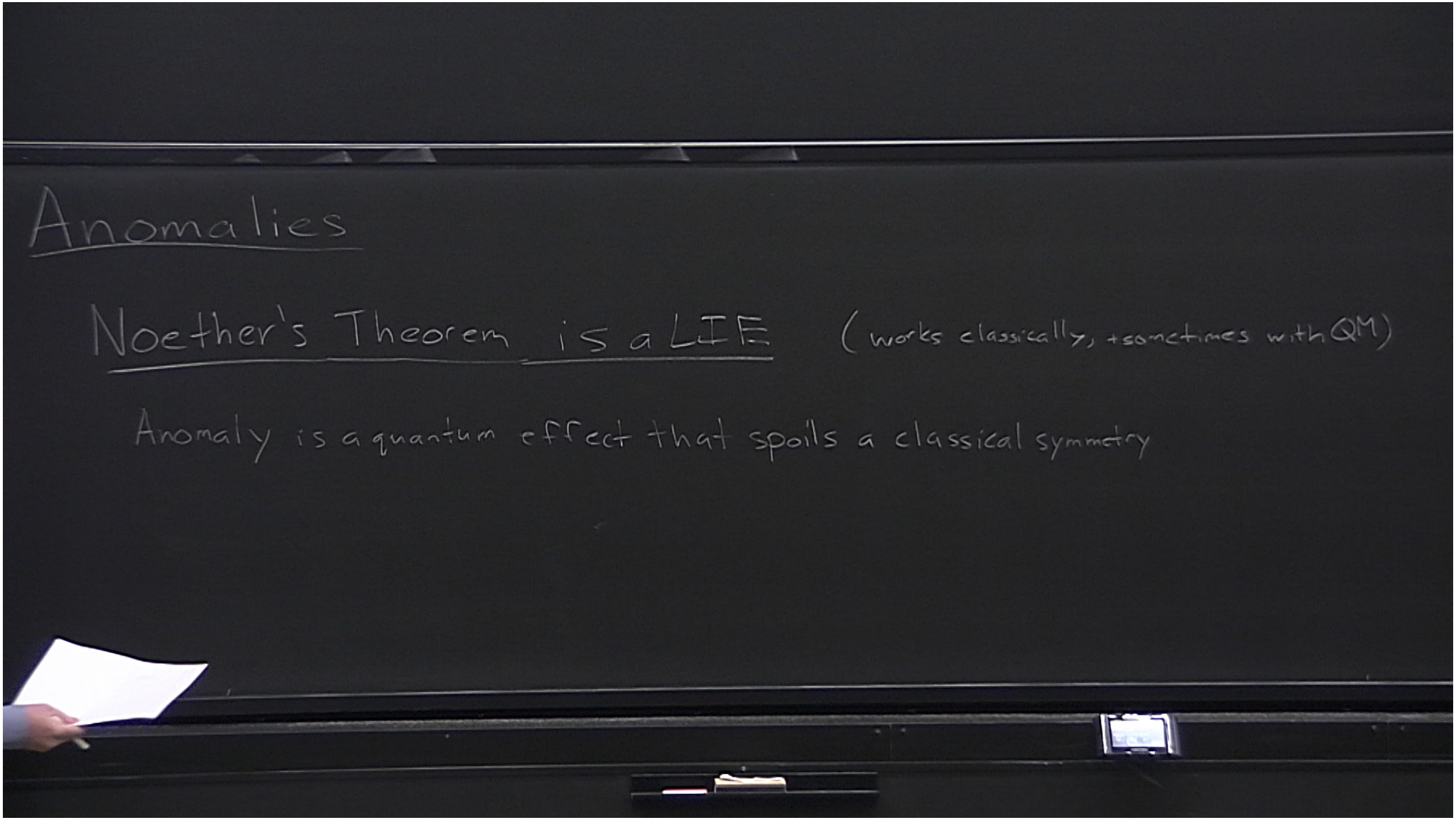
Date: Feb 07, 2018 11:30 AM

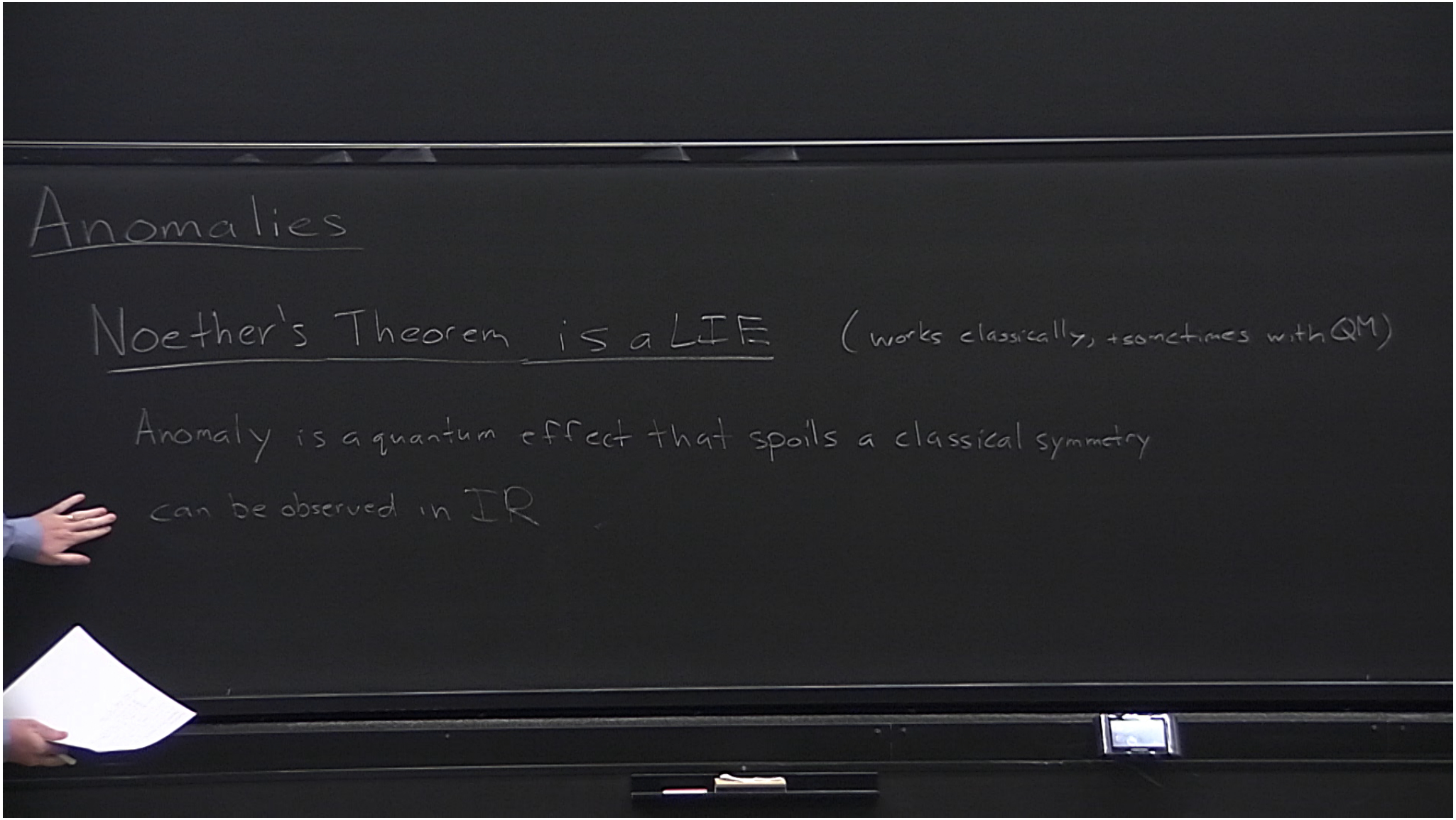
URL: http://pirsa.org/18020029

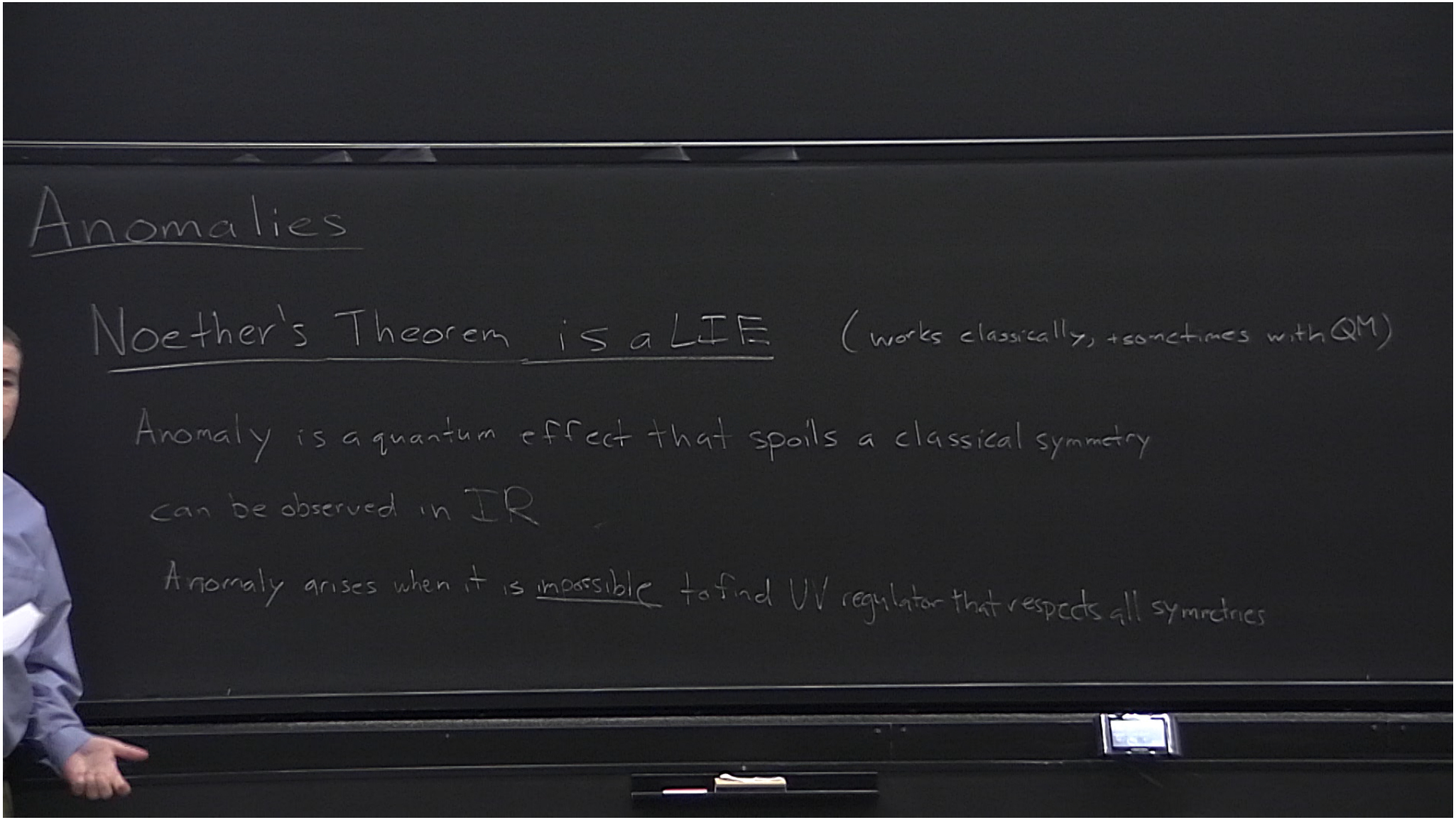
Abstract:

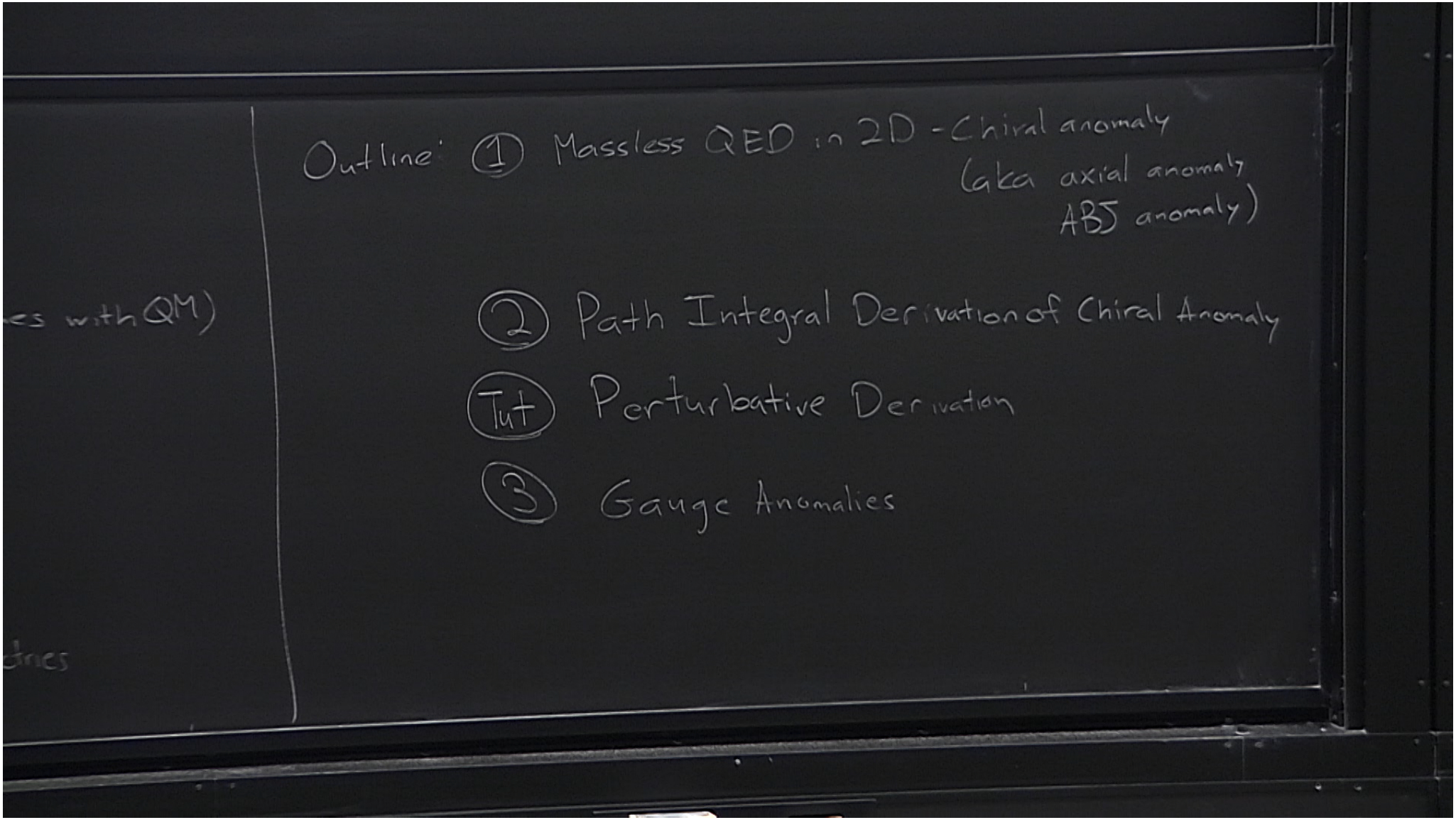

Anomalies
Noether's Theorem is a LIE


Anomalies
Noether's Theorem is a LIE (works classically, + sometimes with QM)


Anomalies
Noether's Theorem is a LIE (works classically, + sometimes with QM)
Anomaly is a quantum effect that spoils a classical symmetry


Anomalies
Noether's Theorem is a LIE (works classically, +sometimes with QM)
Anomaly is a quantum effect that spoils a classical symmetry
can be observed in IR


Anomalies
Noether's Theorem is a LIE (works classically, sometimes with QM)
Anomaly is a quantum effect that spoils a classical symmetry
can be observed in IR
Anomaly arises when it is impossible to find UV regulator that respects all symmetries


Outline:
(1) Massless QED in 2D - Chiral anomaly
(aka axial anomaly
ABJ anomaly)
(2) Path Integral Derivation of Chiral Anomaly
(Tut) Perturbative Derivation
(3) Gauge Anomalies

Outline: (1) Massless QED in 2D - Chiral anomaly
(aka axial anomaly
ABJ anomaly)

(2) Path Integral Derivation of Chiral Anomaly

(Tut) Perturbative Derivation

(3) Gauge Anomalies

Today: Classical Aspects of Schwinger Model (massless 2D QED)

Chiral Anomaly in Schwinger Model

Schwinger Model

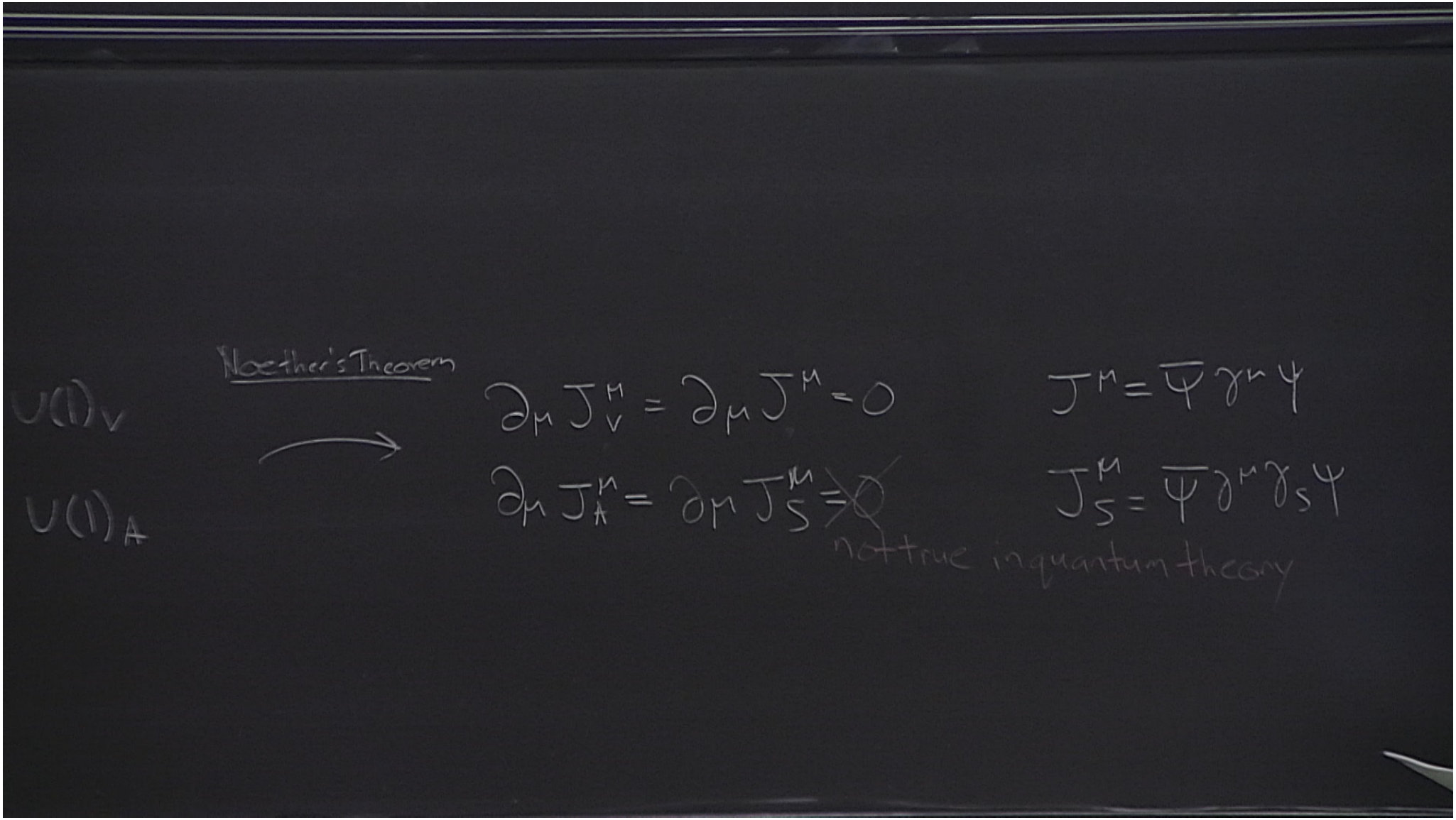
Recall: in 4D with $m=0$

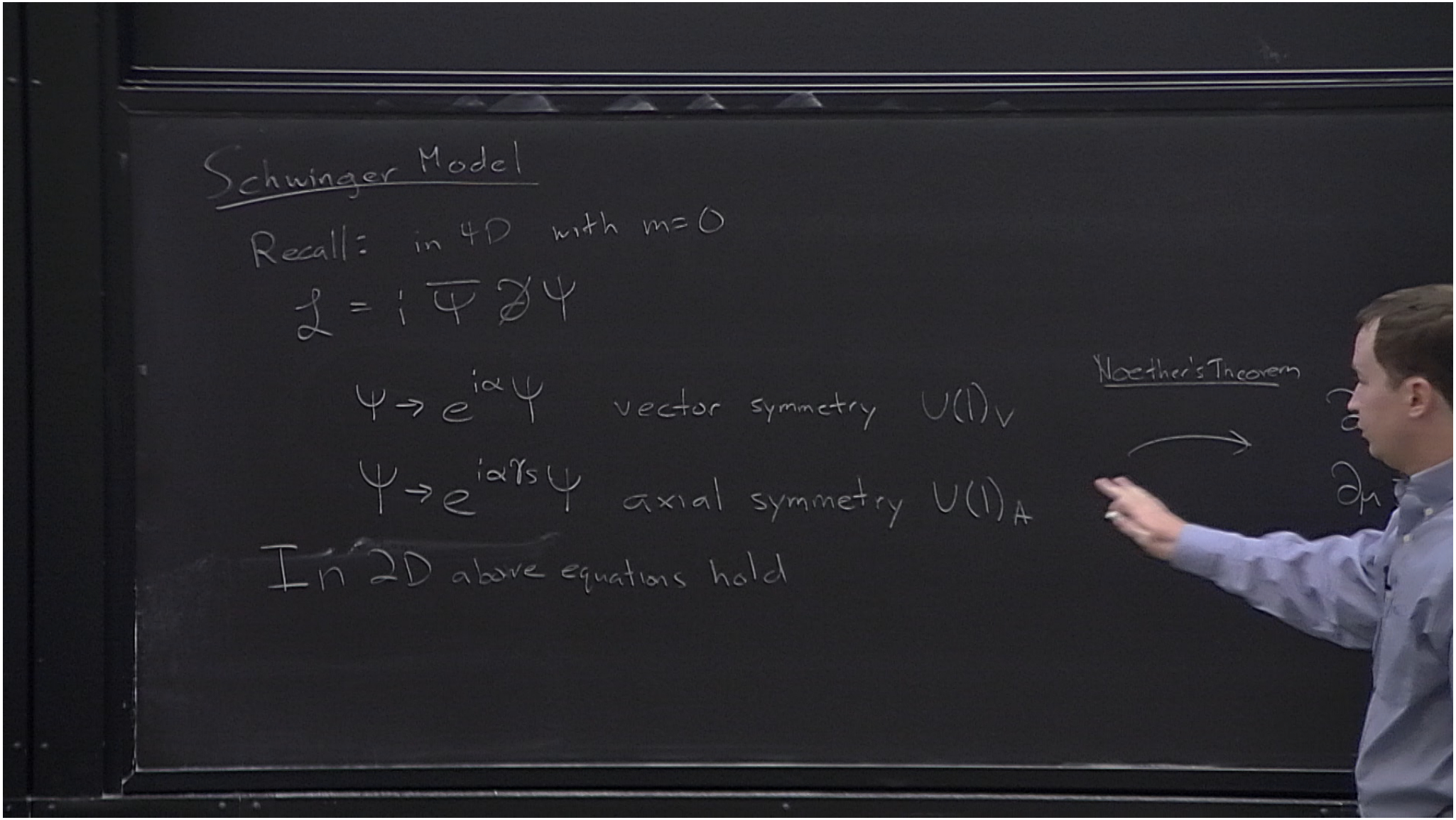
$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi$$

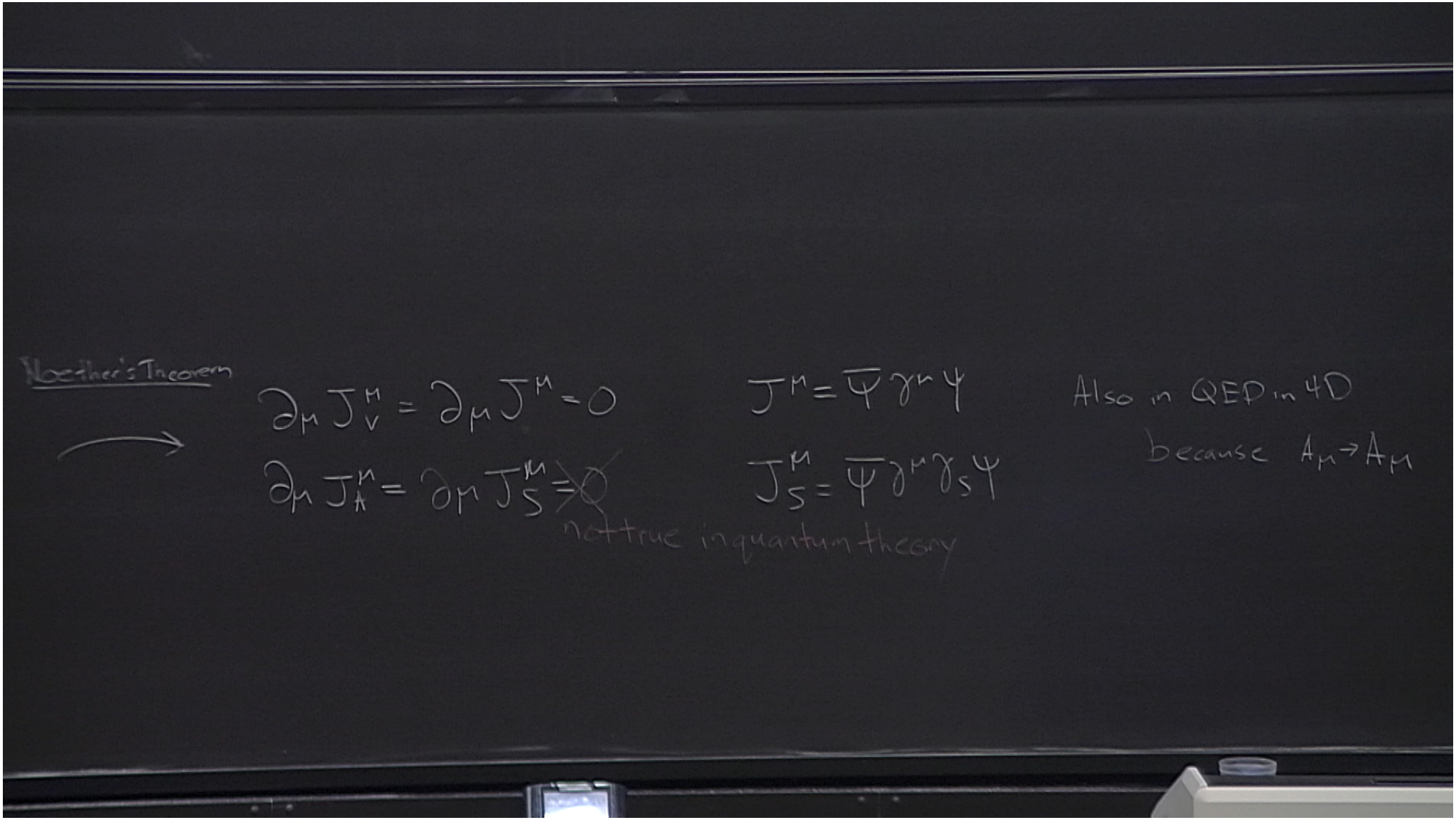
$\psi \to e^{i\alpha}\psi$ vector symmetry $U(1)_V$

Noether's Theorem

$\psi \to e^{i\alpha\gamma_5}\psi$ axial symmetry $U(1)_A$


Noether's Theorem
$U(1)_V$
$\partial_\mu J^\mu_V = \partial_\mu J^\mu = 0$
$J^\mu = \bar{\psi}\gamma^\mu\psi$
$U(1)_A$
$\partial_\mu J^\mu_A = \partial_\mu J^\mu_5 \neq 0$
$J^\mu_5 = \bar{\psi}\gamma^\mu\gamma_5\psi$
not true in quantum theory


Schwinger Model
Recall: in 4D with m=0
$\mathcal{L} = i\bar{\psi}\not{\partial}\psi$
$\psi \to e^{i\alpha}\psi$ vector symmetry $U(1)_V$
$\psi \to e^{i\alpha\gamma_5}\psi$ axial symmetry $U(1)_A$
In 2D above equations hold
Noether's Theorem
∂_μ


Noether's Theorem
$\partial_\mu J^\mu_V = \partial_\mu J^\mu = 0$
$J^\mu = \bar{\psi}\gamma^\mu\psi$
Also in QED in 4D
because $A_\mu \to A_\mu$
$\partial_\mu J^\mu_A = \partial_\mu J^\mu_5 = 0$
$J^\mu_5 = \bar{\psi}\gamma^\mu\gamma_5\psi$
not true in quantum theory

Recall: in 4D with $m=0$

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi$$

$\psi \to e^{i\alpha}\psi$ vector symmetry $U(1)_V$

$\psi \to e^{i\alpha\gamma_5}\psi$ axial symmetry $U(1)_A$

Noether's Theorem →

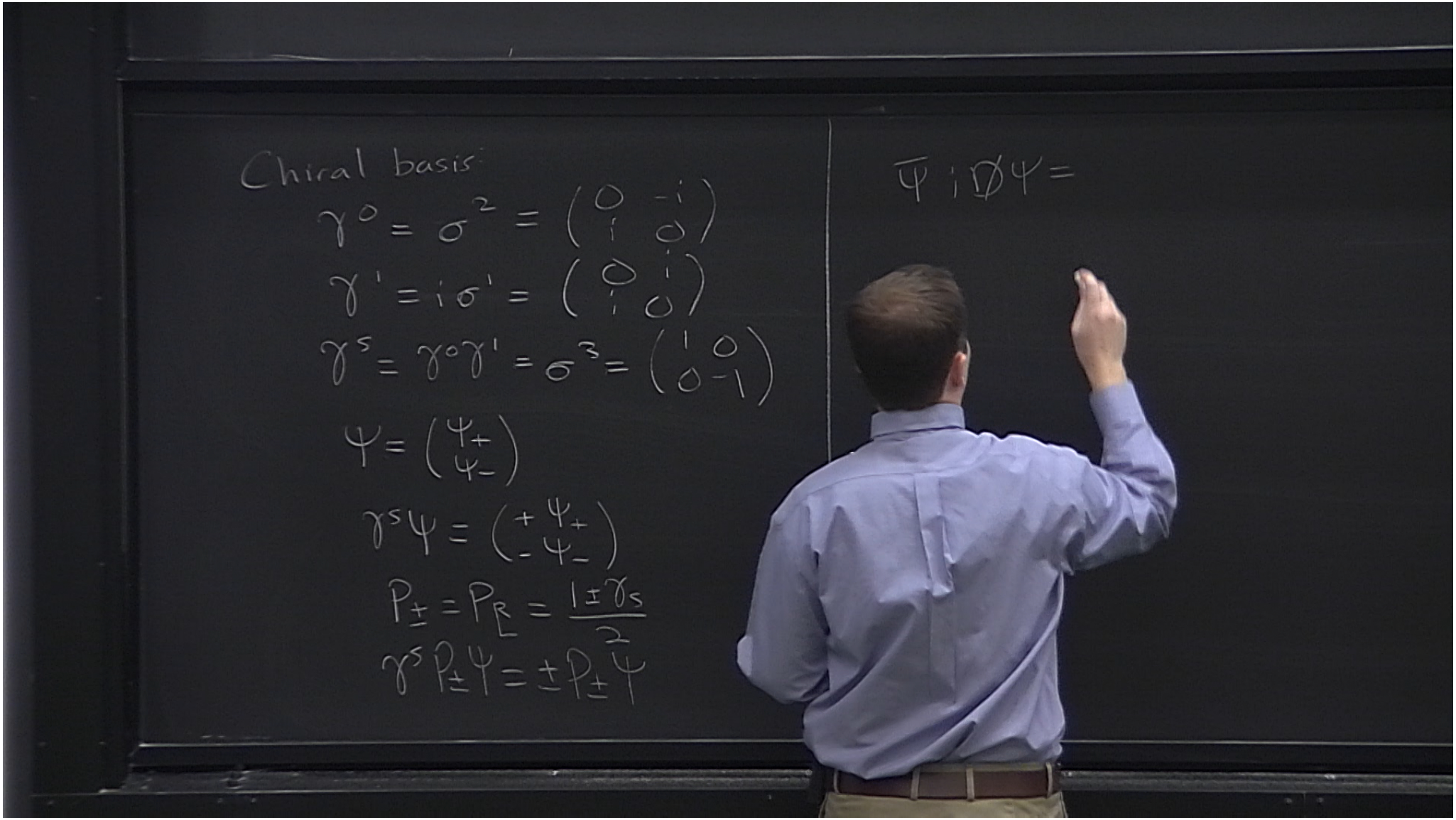
$\partial_\mu J^\mu_V = 0$

$\partial_\mu J^\mu_A = 0$

In 2D above equations hold

$$\mathcal{L} = \bar{\psi} i D\!\!\!/ \psi - \frac{1}{4e_0^2} F_{\mu\nu}^2$$

$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ $\quad g^{\mu\nu} = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$


Chiral basis
$\gamma^0 = \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
$\gamma^1 = i\sigma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$
$\gamma^5 = \gamma^0\gamma^1 = \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$
$\gamma^5\psi = \begin{pmatrix} +\psi_+ \\ -\psi_- \end{pmatrix}$
$P_\pm = P_{R,L} = \frac{1 \pm \gamma_5}{2}$
$\gamma^5 P_\pm \psi = \pm P_\pm \psi$
$\bar{\psi} i \not{D} \psi =$

$$\bar{\Psi} i \not{D} \Psi = \Psi_+^\dagger \, i(D_0 + D_1)\Psi_+ + \Psi_-^\dagger i(D_0 - D_1)\Psi_-$$

$\Rightarrow$ number currents for $\Psi_\pm$ are separately conserved

$$(\partial_\mu \bar{\Psi}_\pm \gamma^\mu \Psi_\pm = 0)$$

$$\bar{\psi} i \not{D} \psi = \psi_+^\dagger i(D_0 + D_1)\psi_+ + \psi_-^\dagger i(D_0 - D_1)\psi_-$$

$\Rightarrow$ number currents for $\psi_\pm$ are separately conserved

$$(\partial_\mu \bar{\psi}_\pm \gamma^\mu \psi_\pm = 0)$$

free fermion EOM: $i(\partial_0 \pm \partial_1)\psi_\pm = 0$

Solutions are waves moving to left (−) + right (+) at speed 1

$\bar{\psi} i\not{D}\psi = \psi_+^\dagger i(D_0 + D_1)\psi_+ + \psi_-^\dagger i(D_0 - D_1)\psi_-$

$\Rightarrow$ number currents for $\psi_\pm$ are separately conserved

$(\partial_\mu \bar{\psi}_\pm \gamma^\mu \psi_\pm = 0)$

free fermion EOM: $i(\partial_0 \pm \partial_1)\psi_\pm = 0$

solutions are waves moving to left (-) + right (+) at speed 1

in 2D Coulomb potential grows linearly

$\bar{\psi} i \not{D} \psi = \psi_+^\dagger i(D_0 + D_1)\psi_+ + \psi_-^\dagger i(D_0 - D_1)\psi_-$

$\Rightarrow$ number currents for $\psi_\pm$ are separately conserved

$(\partial_\mu \bar{\psi}_\pm \gamma^\mu \psi_\pm = 0)$

free fermion EOM: $i(\partial_0 \pm \partial_1)\psi_\pm = 0$

solutions are waves moving to left (-) + right (+) at speed 1

in 2D Coulomb potential grows linearly $\rightarrow$ confinement in 2D Minkowski

compactify on a circle w/ circumference L with $e_0 L \ll 1$

$\bar{\psi} i \not{D} \psi = \psi_+^\dagger i(D_0 + D_1)\psi_+ + \psi_-^\dagger i(D_0 - D_1)\psi_-$

$\Rightarrow$ number currents for $\psi_\pm$ are separately conserved

$(\partial_\mu \bar{\psi}_\pm \gamma^\mu \psi_\pm = 0)$

free fermion EOM: $i(\partial_0 \pm \partial_1)\psi_\pm = 0$

solutions are waves moving to left (−) + right (+) at speed 1

in 2D Coulomb potential grows linearly $\rightarrow$ confinement in 2D Minkowski

compactify on a circle w/ circumference L with $e_0 L \ll 1$

$[\psi] = \frac{1}{2}$ $[A] = 1$ $[e_0] = 1$

spects all symmetries

Today: Classical Aspects of Schw

Chiral Anomaly in Schwinger Model

$\psi_{-}^{\dagger}(D_0 - D_1)\psi = \bar{\psi}\, i\gamma^{\mu}(\partial_{\mu} + iA_{\mu})\psi$

re separately conserved

$\pm = 0$

eft(−) + right(+)

$\mathcal{L} = \bar{\psi} i \not{D} \psi - \frac{1}{4e^2} F_{\mu\nu}^2$ $\quad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad g^{\mu\nu} = \begin{pmatrix} 1 & & \\ & -1 & \\ & & \ddots \end{pmatrix}$

$A_\mu = \sum a_\mu(t) \sin\left(\frac{2\pi n}{L} x\right)$

$A_\mu \to A_\mu + \partial_\mu \alpha$

$\alpha(t,x) = \sum_{n \neq 0} \frac{L}{2\pi n} a_1(t) \sin\left(\frac{2\pi n}{L} x\right)$

$\to$ set $A_1(t,x) = A_1(t)$

$\mathcal{L} = \bar{\psi} i \not{D} \psi - \frac{1}{4e^2} F_{\mu\nu}^2$ $\quad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$

$A_\mu = \sum a_\mu(t) \sin\left(\frac{2\pi n}{L} x\right)$

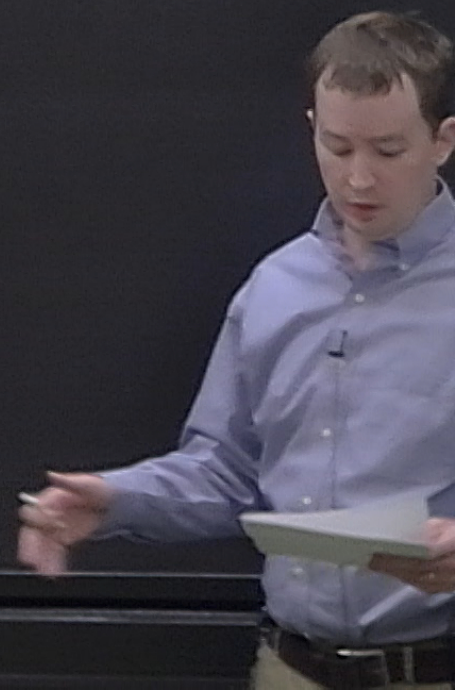
$A_\mu \to A_\mu + \partial_\mu \alpha$

$\alpha(t,x) = \sum_{n \neq 0} \frac{L}{2\pi n} a_1(t) \sin\left(\frac{2\pi n}{L} x\right)$

$\to$ set $A_1(t,x) = A_1(t)$

$\alpha = \frac{2\pi n}{L} x$

$A_1 + \frac{2\pi n}{L}$ are gauge equivalent $\to A_1$ lives on a circle of circumference $\frac{2\pi}{L}$



Quantum Mechanical Aspects

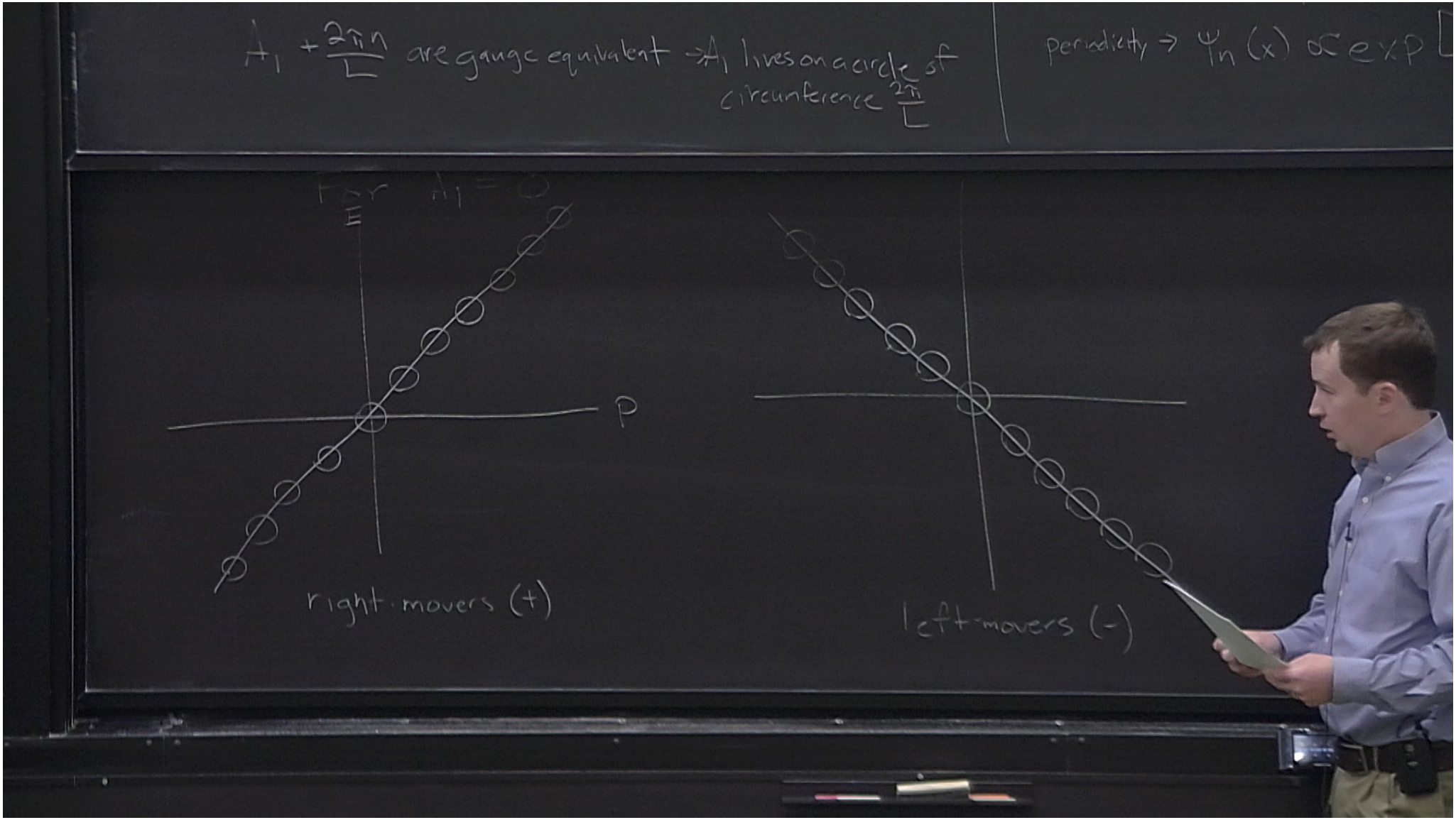
work in adiabatic approximation $A_1(t)$ is external + slowly varying

set $A_0 = 0$ ((Coulomb int is weak if $eaL \ll 1$)

$$i\not{D}\psi = 0 \rightarrow \partial_0 + \gamma^5(\partial_1 - iA_1)\psi = 0 \quad \text{(multiply by } -i\gamma^0\text{)}$$

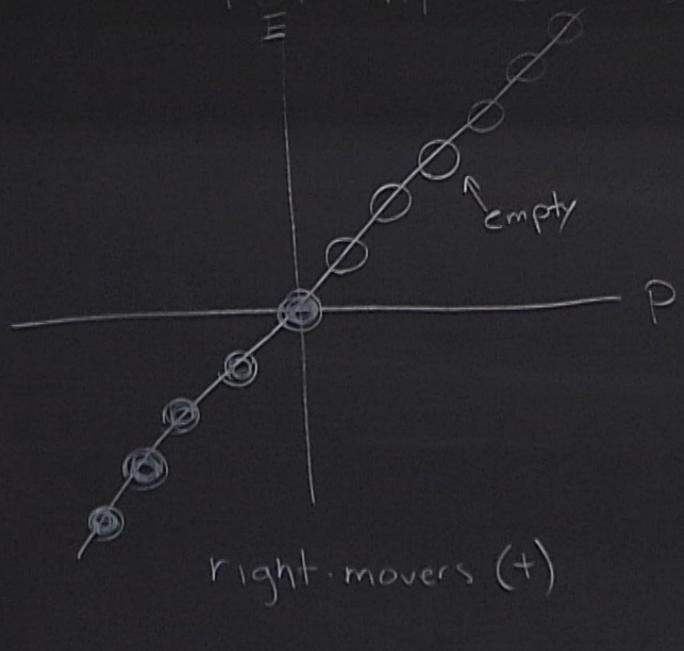
$$\left.\begin{array}{l} \psi \propto \exp[-iE_n t]\,\psi_n(x) \\ \text{periodicity} \Rightarrow \psi_n(x) \propto \exp\left[\frac{2\pi i n x}{L}\right] \end{array}\right\} \rightarrow E_n^{\mp} = \pm\frac{2\pi n}{L} \mp A_1$$

circle of
ce $\frac{2\pi}{L}$

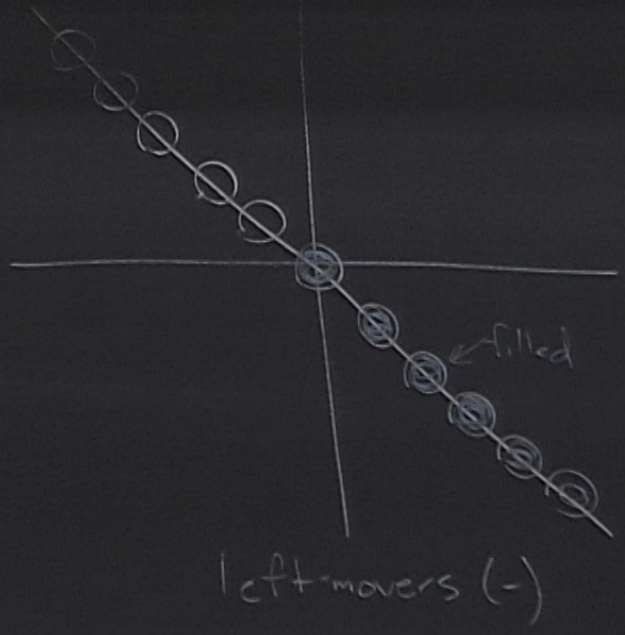

periodicity → $\psi_n(x) \propto \exp$
$A_1 + \frac{2\pi n}{L}$ are gauge equivalent → A_1 lives on a circle of circumference $\frac{2\pi}{L}$
for $A_1 = 0$
E
p
right-movers (+)
left-movers (-)

$A_1 + \frac{2\pi n}{L}$ are gauge equivalent $\Rightarrow A_1$ lives on a circle of circumference $\frac{2\pi}{L}$

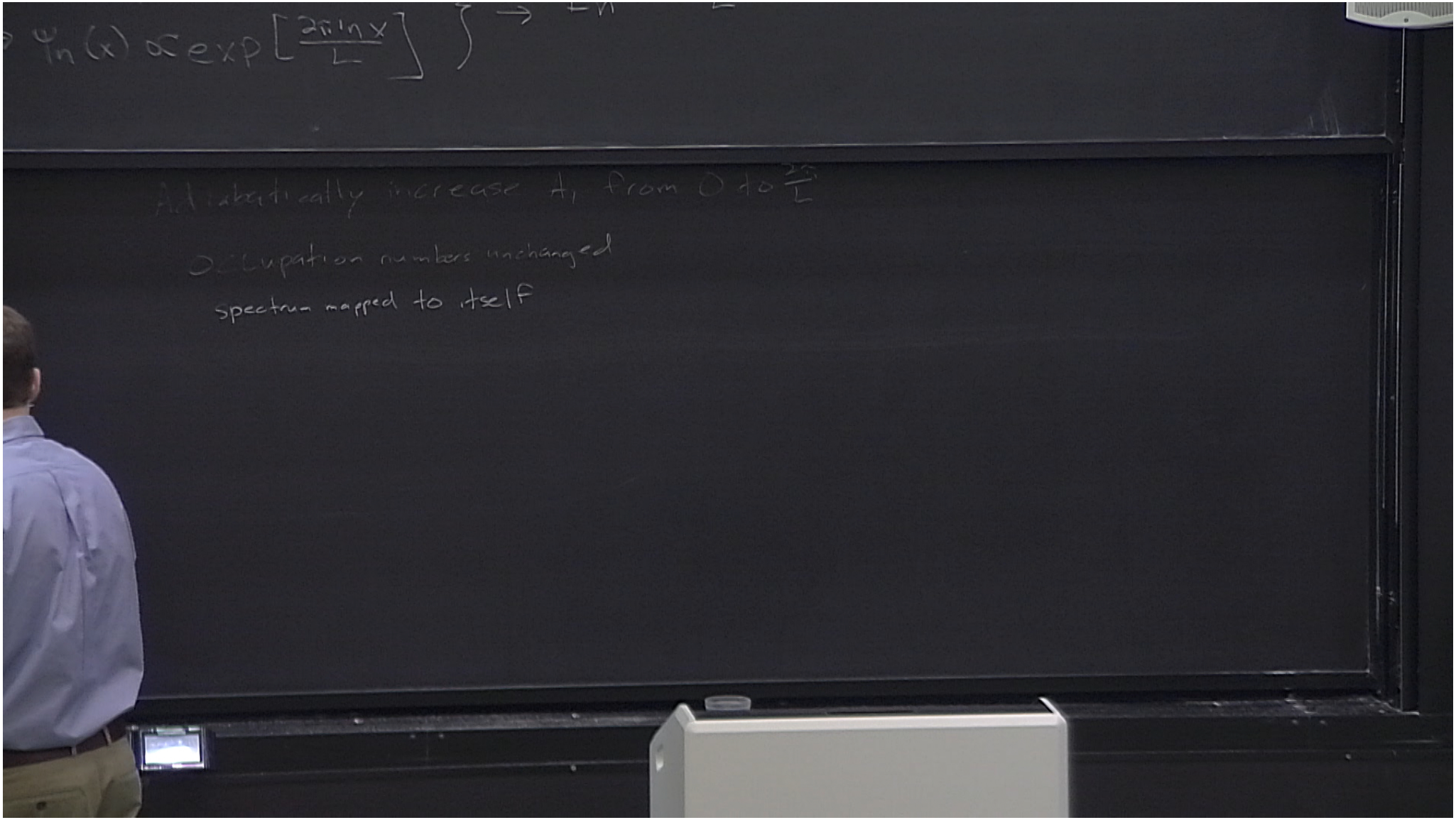
periodicity $\Rightarrow \psi_n(x) \propto \exp\left[\frac{2\pi}{L}\right]$

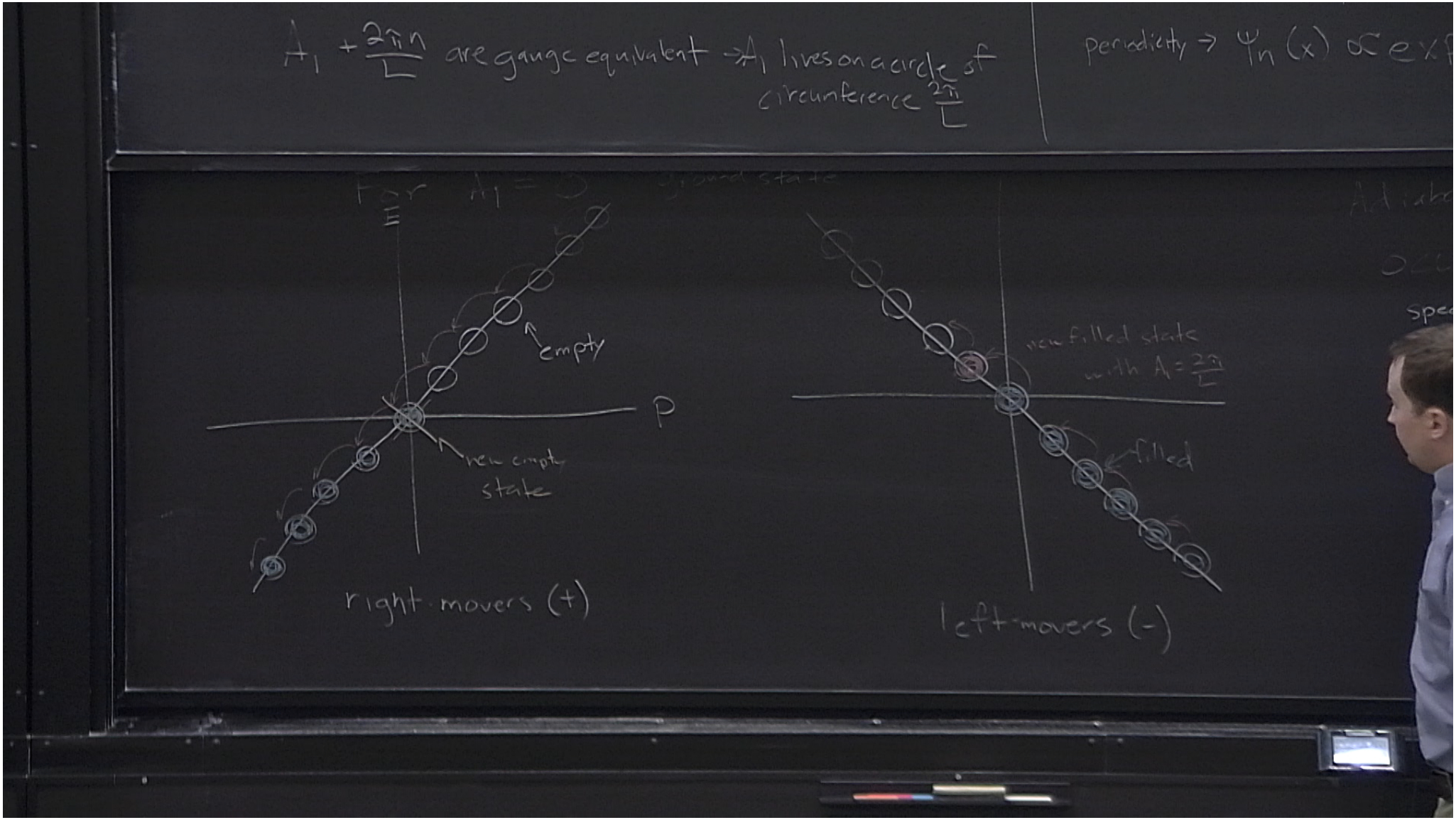


right-movers (+)



left-movers (-)


$\psi_n(x) \propto \exp\left[\frac{2\pi i n x}{L}\right]$
Adiabatically increase t_1 from 0 to $\frac{2\pi}{L}$
Occupation numbers unchanged
spectrum mapped to itself


$A_1 + \frac{2\pi n}{L}$ are gauge equivalent $\Rightarrow A_1$ lives on a circle of circumference $\frac{2\pi}{L}$
periodicity $\Rightarrow \psi_n(x) \propto$
For $A_1 = 0$
ground state
E
p
empty
new empty state
right-movers (+)
new filled state with $A = \frac{2\pi}{L}$
filled
left-movers (-)

periodicity → $\psi_n(x) \propto \exp\left[\frac{2\pi i n x}{L}\right]$ → L_n

Adiabatically increase A_i from 0 to $\frac{2\pi}{L}$

occupation numbers unchanged

spectrum mapped to itself

1 (−) particle

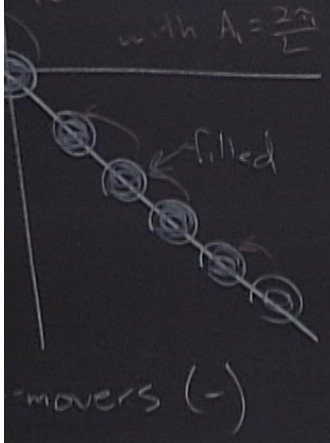
1 (+) anti-particle

Q is unchanged

assign $Q_S = 1$ (+) particles

$Q_S = -1$ (−) particles

$\Delta Q_S = -2$



Anomaly arises when it is impossible to find UV regulator that respects all sy

$$\frac{\Delta Q_5}{\Delta t} = -\frac{L}{\tilde{\pi}} \frac{\Delta A_1}{\Delta t}$$

$$\rightarrow \dot{Q}_5 = -\frac{L}{\tilde{\pi}} \dot{A}_1$$

$$Q_5 = \int dx\, J_5^0$$

$$L A_1 = \int \epsilon^{0M} A_M\, dx$$

$$\boxed{\partial_M J_5^M = \frac{1}{\tilde{\pi}} \epsilon^{M\nu} F_{M\nu}}$$

→ anomalous non-conservation equation