Title: PSI 17/18 - Quantum Field Theory III - Lecture 5

Date: Feb 02, 2018 09:00 AM

URL: http://pirsa.org/18020026

Abstract:

$|x_1 - x_2|^{2\Delta_A}$ $\qquad$ $|x_1 - x_2|^{2\Delta}$

$I_{\mu\nu}(x^\rho) \equiv \delta_{\mu\nu} - 2x_\mu x_\nu$

Operators in CFT (Primary or descendant)

Primaries transform under conformal transformation $x^\mu \to \tilde{x}^\mu = (g x)^\mu$ s.t $ds^2 \to e^{2\omega(x)} ds^2$

$$\theta_A(x) \to \tilde{\theta}_A(x) = \left|\frac{\partial x}{\partial \tilde{x}}\right|^{\Delta/D} L_A^{\ B}(R)\, \theta_B(g^{-1}x) \qquad w/ \quad R^\mu_{\ \nu} = \frac{\partial \tilde{x}^\mu}{\partial x^\nu} e^{-\omega(x)}$$

1) Δ : scaling dimension

2) spin (under Lorentz)

Conformal symmetry greatly constraints correlators. Conformal invariance implies that:

$$\langle \theta_{A_1}(x_1)\, \theta_{A_2}(x_2) \cdots \theta_{A_n}(x_n) \rangle = \langle \tilde{\theta}_{A_1}(x_1)\, \tilde{\theta}_{A_2}(x_2) \cdots \theta_{A_n}(x_n) \rangle$$

$\Longrightarrow$ infinitessimal $\delta_\xi \langle \quad \rangle = 0$

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$\Rightarrow$ infinitessimal $\delta_\xi \langle \qquad \rangle = 0$

1. $\langle \mathcal{O}_A \rangle = 0$ unless $\mathcal{O}_A = \mathbb{1}$.

2. $$\langle \mathcal{O}_A(x_1)\, \mathcal{O}_B(x_2) \rangle = \frac{\delta_{\Delta_A, \Delta_B}}{|x_1 - x_2|^{2\Delta_A}}$$

; w/ spin $$\langle \mathcal{O}_\mu(x_1)\, \mathcal{O}_\nu(x_2) \rangle = \frac{I_{\mu\nu}(x_1 - x_2)}{|x_1 - x_2|^{2\Delta}}$$

$$I_{\mu\nu}(x^\rho) \equiv \delta_{\mu\nu} - \frac{2 x_\mu x_\nu}{x^2} = I_{\mu\nu}\left(\frac{x^\rho}{x^2}\right)$$

what is Δ for a conserved current $\partial^\mu J_\mu = 0$?

$\partial^\mu \langle \qquad \rangle = 0$ when ?

3. $$\langle \mathcal{O}_A(x_1)\, \mathcal{O}_B(x_2)\, \mathcal{O}_C(x_3) \rangle = \frac{C_{ABC}}{(x_{12}^2)^{\frac{1}{2}(\Delta_3 - \Delta_1 - \Delta_2)} (x_{13}^2)^{\frac{1}{2}(\Delta_2 - \Delta_1 - \Delta_3)} (x_{23}^2)^{\frac{1}{2}(\Delta_1 - \Delta_2 - \Delta_3)}}$$

where $x_{ij} \equiv x_i - x_j$

4 pt. functions

$$u = \frac{x_{12}^2 \, x_{34}^2}{x_{13}^2 \, x_{24}^2}$$

$$v = \frac{x_{14}^2 \, x_{23}^2}{x_{13}^2 \, x_{24}^2}$$

4 pt of identical primary.

$$\langle \phi(x_1)\, \phi(x_2)\, \phi(x_3)\, \phi(x_3) \rangle = \frac{g(u,v)}{x_{12}^{2\Delta} \, x_{34}^{2\Delta}}$$

$g(u,v)$ is completely fixed in terms of the CFT data

$$\{ \Delta_A, C_{ABC} \}.$$

$0 \qquad x_2'' \qquad (1 0 \cdots 0) \qquad \infty$

M_{2j}

$j > 2$

$(a, b, 0, 0)$

$x_1 \; \cdots\cdots \; x_n$

$4n-15$ is the number of cross-ratios

4 pt functions

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$g(u,v)$ is completely fixed in terms of the CFT data

$x_1 \,\ldots\ldots\, x_n$

$n > 4$ $\quad \underbrace{4n-15}$ is the number of cross-ratios = # of generators of conf transf in $D=4$ $\quad SO(2,4)$

4 pt functions

$$u = \frac{x_{12}^2\, x_{34}^2}{x_{13}^2\, x_{24}^2}$$

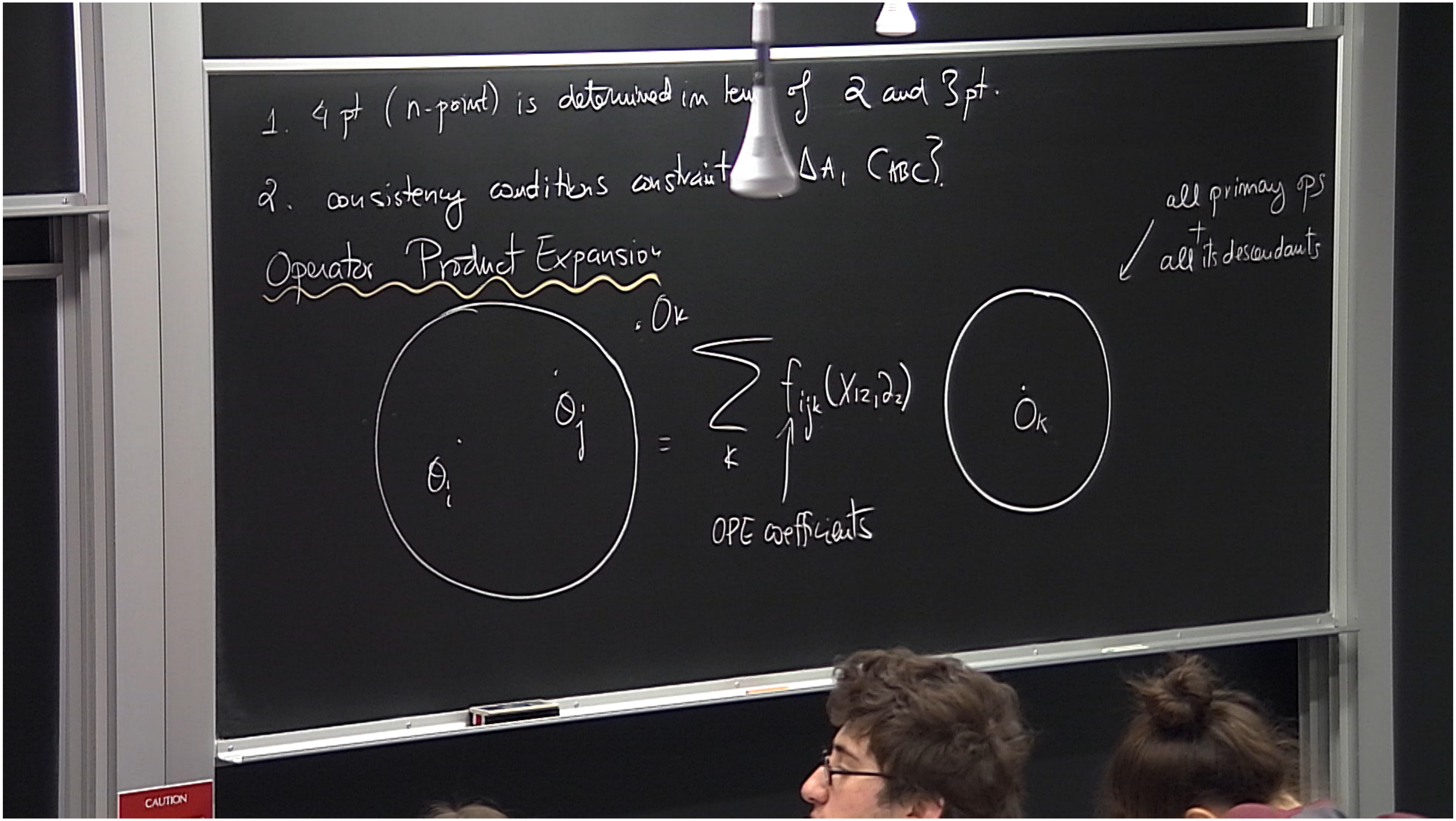
$$v = \frac{x_{14}^2\, x_{23}^2}{x_{13}^2\, x_{24}^2}$$

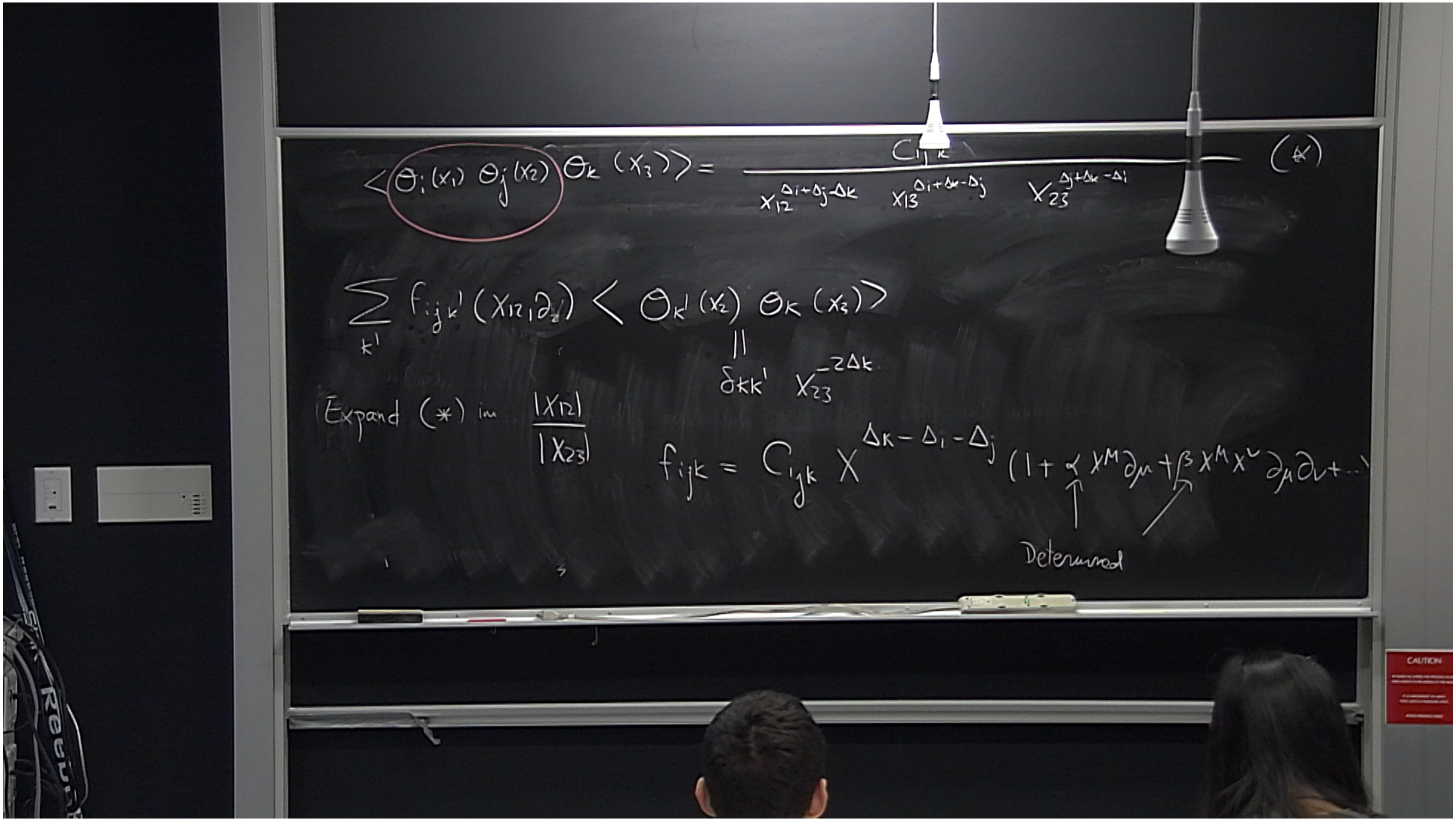
4 pt of identical primary.

$$\boxed{\langle \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_3)\rangle = \frac{g(u,v)}{x_{12}^{2\Delta}\; x_{34}^{2\Delta}}}$$

$g(u,v)$ is completely fixed in terms of the $\underline{\text{CFT data}}$

$$\{\Delta_A,\ C_{ABC}\}.$$

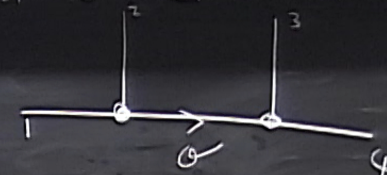

1. 4 pt (n-point) is determined in te of 2 and 3pt.
2. consistency conditions constraint ΔA, CABC}
Operator Product Expansion
O_k
O_j
O_i
= Σ_k f_ijk(x12, ∂2)
O_k
OPE coefficients
all primary ops
+
all its descendants
CAUTION


$\langle O_i(x_1)\, O_j(x_2)\, O_k(x_3)\rangle = \frac{C_{ijk}}{x_{12}^{\Delta_i+\Delta_j-\Delta_k}\; x_{13}^{\Delta_i+\Delta_k-\Delta_j}\; x_{23}^{\Delta_j+\Delta_k-\Delta_i}}$ (*)
$\sum_{k'} f_{ijk'}(x_{12}, \partial_2) \langle O_{k'}(x_2)\, O_k(x_3)\rangle$
$\delta_{kk'}\, x_{23}^{-2\Delta_k}$
Expand (*) in $\frac{|x_{12}|}{|x_{23}|}$
$f_{ijk} = C_{ijk}\, x^{\Delta_k-\Delta_i-\Delta_j}\,(1 + \alpha\, x^\mu \partial_\mu + \beta\, x^\mu x^\nu \partial_\mu \partial_\nu + \dots)$
Determined

Determined

Strategy: Apply sequentially to reduce correlator to ...

$$\langle \phi(x_1)\,\phi(x_2)\,\phi(x_3)\,\phi(x_4)\rangle =$$

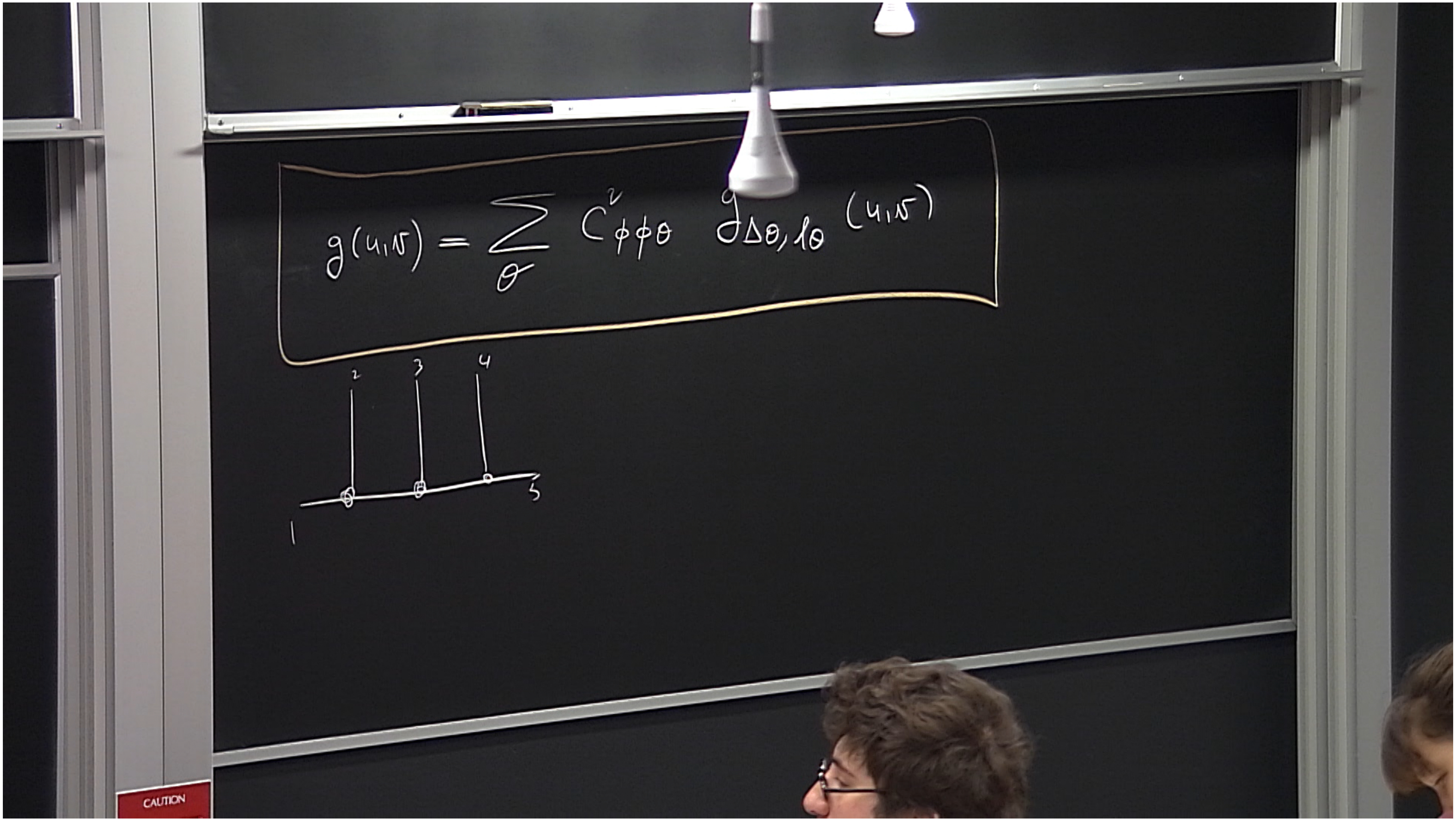


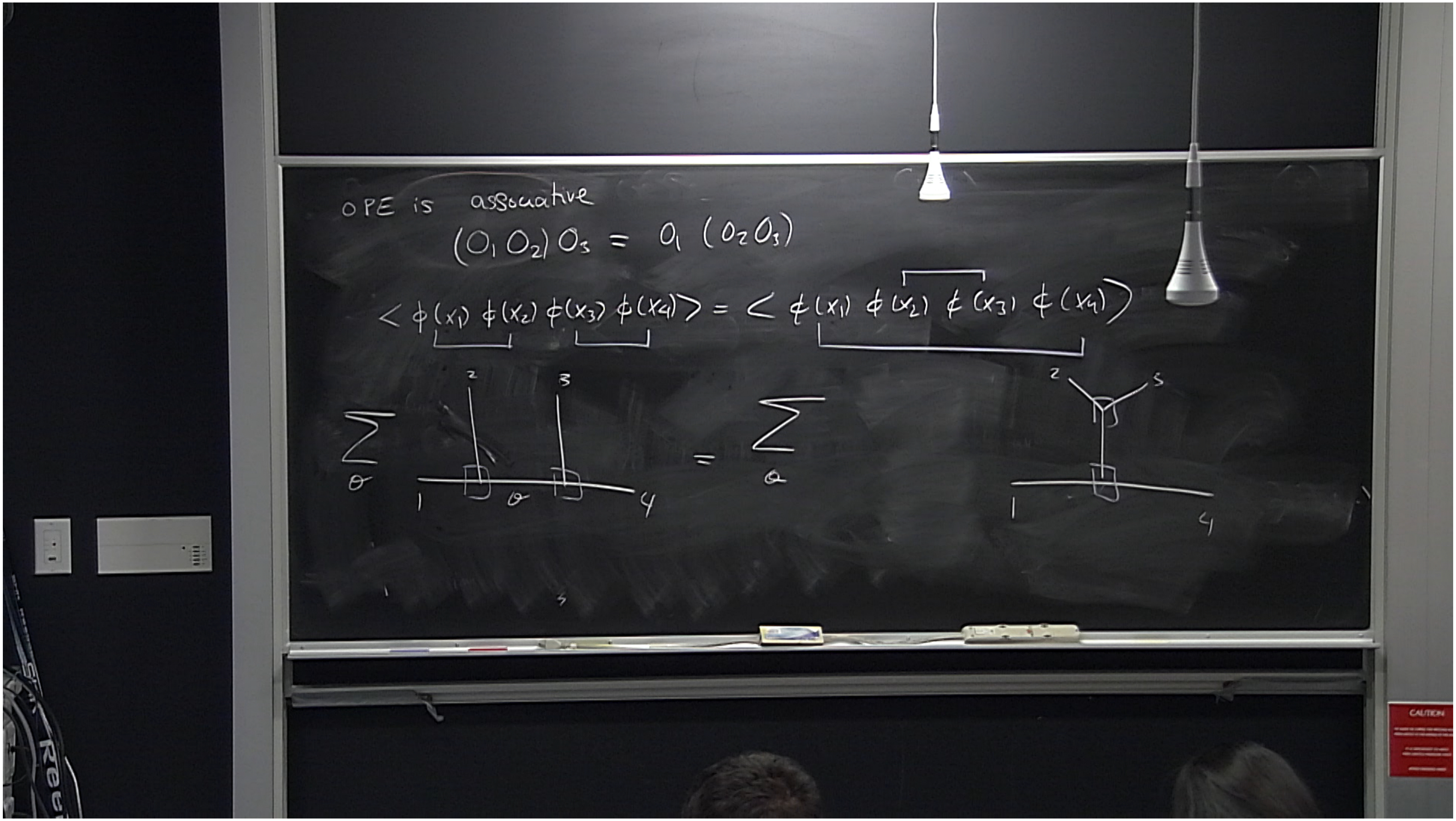
$$\sum_{\mathcal{O},\mathcal{O}'} C_{\phi\phi\mathcal{O}}\, C_{\phi\phi\mathcal{O}'}\, C_a(x_{12},\partial_2)\, C_b(x_{34},\partial_4)\, \langle \mathcal{O}^a(x_2)\, \mathcal{O}'^b(x_4)\rangle$$

$$\langle \mathcal{O}^a(x_2)\, \mathcal{O}'^b(x_4)\rangle = \delta_{\mathcal{O},\mathcal{O}'}\, \frac{I^{ab}(x_2-x_4)}{x_{24}^{2\Delta_{\mathcal{O}}}}$$

$$\equiv \frac{1}{x_{12}^{2\Delta_\phi}\; x_{34}} \sum_{\mathcal{O}} C^2_{\phi\phi\mathcal{O}}\, \underline{g_{\Delta_{\mathcal{O}},\ell_{\mathcal{O}}}(x_i)}$$

$\uparrow$ conformal blocks


$g(u,v) = \sum_{\mathcal{O}} C^2_{\phi\phi\mathcal{O}}\, g_{\Delta_{\mathcal{O}}, l_{\mathcal{O}}}(u,v)$
2
3
4
1
5
CAUTION


OPE is associative
$(O_1 O_2) O_3 = O_1 (O_2 O_3)$
$\langle \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) \rangle = \langle \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) \rangle$
$\sum_{\mathcal{O}}$
$= \sum_{\mathcal{O}}$

$g(u,v)$ is completely fixed in terms of the ...

$1 \leftrightarrow 3$ (or $2 \leftrightarrow 4$)

$u \leftrightarrow v$

1) $\dfrac{g(u,v)}{X_{12}^{2\Delta} X_{34}^{2\Delta}}$

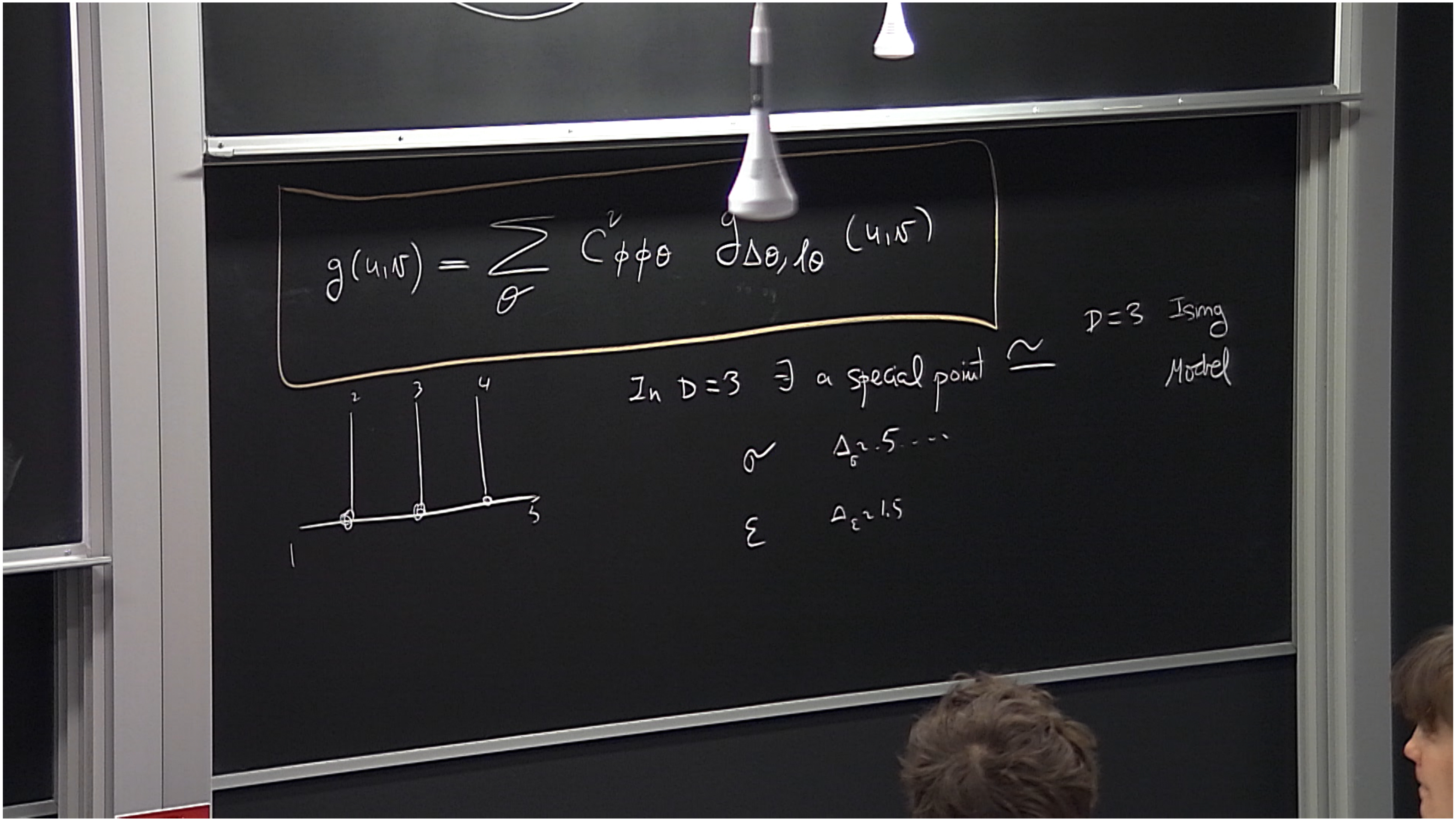
2) $\dfrac{g(v,u)}{X_{23}^{2\Delta} X_{14}^{2\Delta}}$

$$\boxed{\sum_{\mathcal{O}} C^2_{\phi\phi\mathcal{O}} \left[v^{\Delta}\, g_{\Delta_{\mathcal{O}},\ell_{\mathcal{O}}}(u,v) - u^{\Delta}\, g_{\Delta_{\mathcal{O}},\ell_{\mathcal{O}}}(v,u) \right] = 0}$$

- Non-linear equation that constraints Δ's and C's.

conformal bootstrap

- In $D=2$, solve analytically this constraints
- In $D \geq 3$ not known?


$g(u,v) = \sum_{\mathcal{O}} C^2_{\phi\phi\mathcal{O}}\, g_{\Delta_{\mathcal{O}},\ell_{\mathcal{O}}}(u,v)$
In $D=3$ $\exists$ a special point $\simeq$ $D=3$ Ising Model
σ $\Delta_\sigma \approx .5 \ldots$
ε $\Delta_\varepsilon \approx 1.5$
1
2
3
4
5