Title: PSI 17/18 - Cosmology - Lecture 15

Date: Feb 16, 2018 09:00 AM

URL: http://pirsa.org/18020024

Abstract:

YESTERDAY: QUANTIZED TEST SCALAR FIELD

$$S = \int d\eta\, d^3x \, \frac{1}{2} \left(v'^2 + v \nabla^2 v + \frac{a''}{a} v \right)$$

ANSATZ: $$v = \int d^3k \left(a_k u_k + a_k^\dagger u_k^* \right)$$

$$u_k = \frac{1}{(2\pi)^{3/2}} f_k(\eta)\, e^{i\vec{k}\cdot\vec{x}}$$

B

$\Rightarrow$ POWERSPECTRUM

$$\boxed{P_\phi(k) = \frac{|f_k|^2}{a^2}}$$

WHERE f_k SOLVES MUKHANOV'S EQ:

$$\boxed{f_k'' + \left(k^2 + \frac{a''}{a}\right) f_k = 0}$$

BUNCH-DAVIES $\lim_{\eta \to -\infty} f_k = \frac{1}{\sqrt{2k}} e^{-ik\eta}$

USED dS APPROX.

$$a = -\frac{1}{\eta H}$$

AT LATE TIMES $(\eta = 0)$

$$|f_k|^2 = \frac{1}{2k^3 \eta^2}$$

$$\boxed{P_\phi(k) = \frac{H^2}{2k^3}}$$

FAMOUS
SCALE INVARIANT
SPECTRUM.

d) PRIMORDIAL FLUCTUATIONS

$$S = m_{pl}^2 \int d^4 \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right)$$

WANT TO STUDY COUPLED SCALAR & TENSOR PERTURBATIONS.

BACKGROUND: $g_{\mu\nu}$ .... FRW $(k=0)$

$\phi(t)$ ... INFLATON

$V(\phi)$

TENSOR

PERTURBATIONS

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

$$\phi \to \phi + \varphi$$

GRAVITY .. GAUGE THEORY

→ GAUGE FIX

SHOULD EXTRACT

2+3 = 3 DOF

THEORY

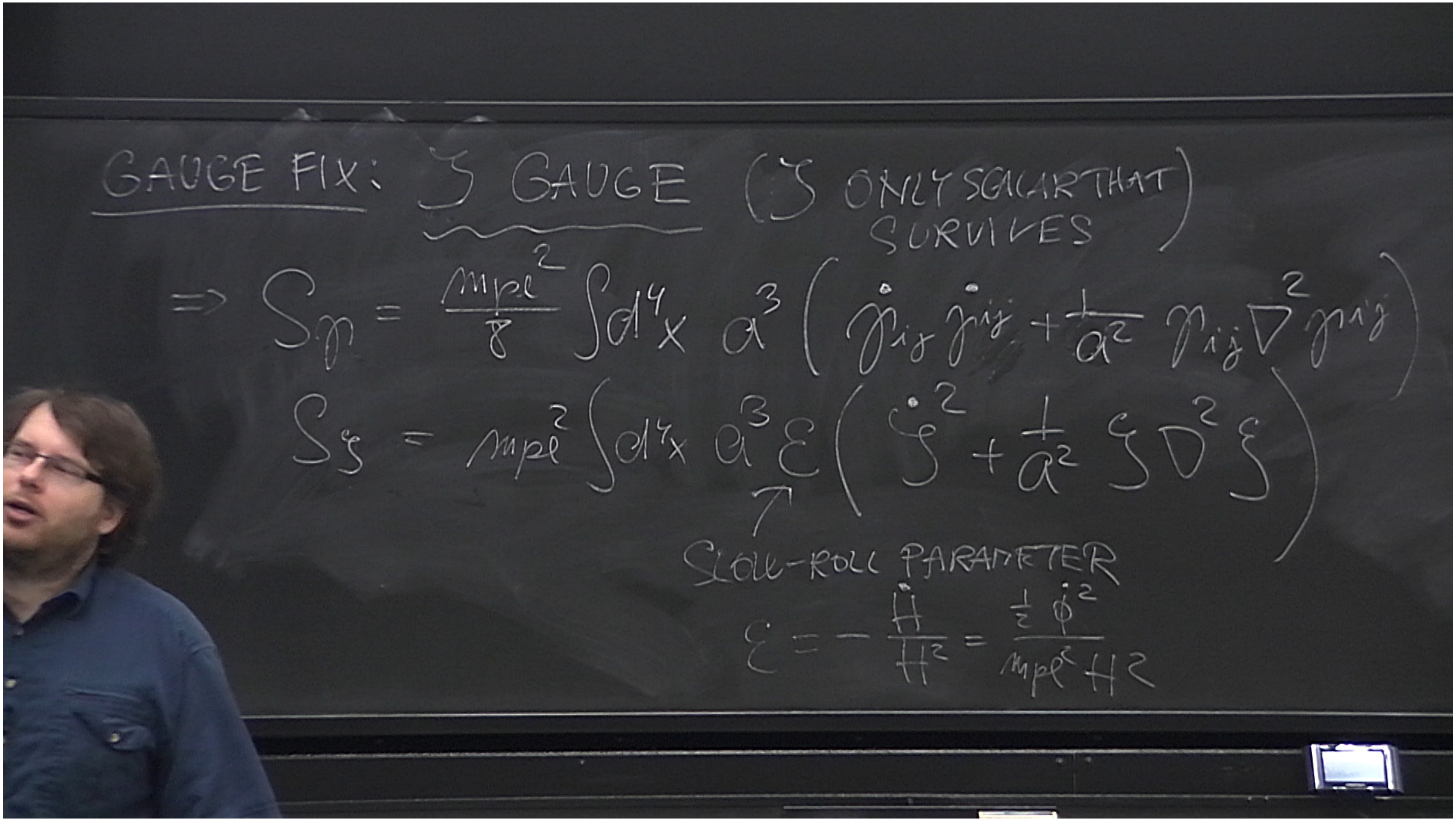
- SVT DECOMPOSITION

(COSMOLOGICAL PERT. THEORY)

$$h_{00} = \Phi \,, \quad h_{0i} = a(V_i + \partial_i B)$$

$$h_{ij} = a^2\left(\gamma_{ij} + 2\,\partial_{(i}W_{j)} + \partial_i\partial_j \chi + 2\zeta\,\delta_{ij}\right)$$

$$\partial^i V_i = 0 \,, \quad \gamma^i{}_i = 0 = \partial^i \gamma_{ij}$$


GAUGE FIX: ζ GAUGE (ζ ONLY SCALAR THAT SURVIVES)
$\Rightarrow S_\gamma = \frac{m_{pl}^2}{8}\int d^4x\, a^3\left(\dot\gamma_{ij}\dot\gamma^{ij} + \frac{1}{a^2}\gamma_{ij}\nabla^2\gamma^{ij}\right)$
$S_\zeta = m_{pl}^2\int d^4x\, a^3\varepsilon\left(\dot\zeta^2 + \frac{1}{a^2}\zeta\nabla^2\zeta\right)$
SLOW-ROLL PARAMETER
$\varepsilon = -\frac{\dot H}{H^2} = \frac{\frac{1}{2}\dot\phi^2}{m_{pl}^2 H^2}$

<u>SCALARS</u>: $v = z\zeta, \quad z^2 = 2a^2 m_{pl}^2 \epsilon$

$$S_\zeta = \int d^3x\, d\eta\, \frac{1}{2}\left(v'^2 + v\nabla^2 v + \frac{z''}{z} v^2\right)$$

$$P_\zeta(k) = \frac{|f_k|^2}{z^2}$$

<u>dS-APPROX.</u>

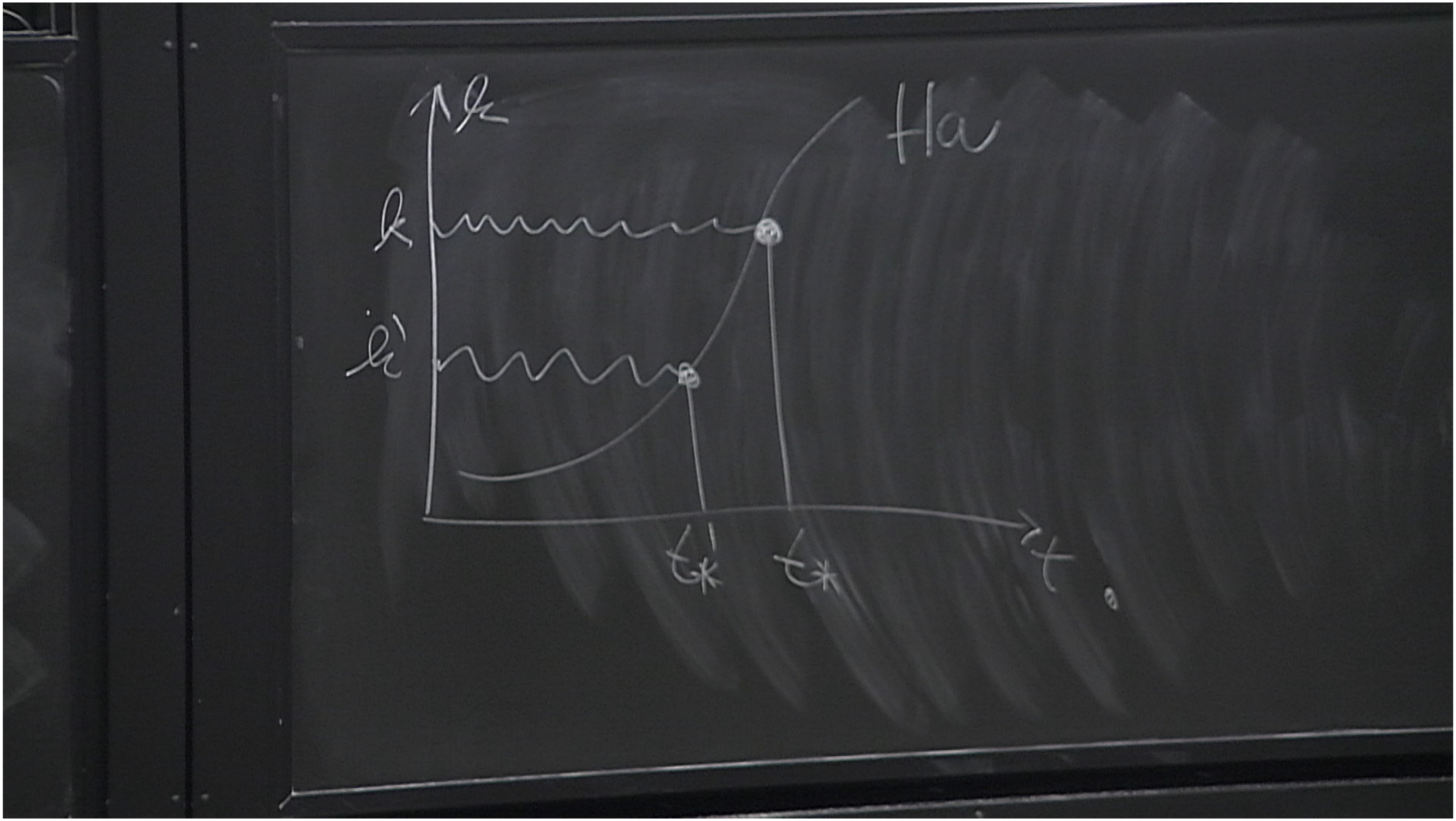
SCALARS: $v = z\zeta$, $z^2 = 2a^2 m_{pl}^2 \varepsilon$

$$S_\zeta = \int d^3x\, d\eta\, \frac{1}{2}\left(v'^2 + v\nabla^2 v + \frac{z''}{z} v^2\right)$$

$$P_\zeta(k) = \frac{|f_k|^2}{z^2}$$

dS-APPROX: $|f_k|^2 \sim \frac{1}{2k^3\eta^2}$, $a^2 = \frac{1}{\eta^2 H^2}$

$$\boxed{P_\zeta = \frac{1}{4k^3}\frac{H^2}{m_{pl}^2 \varepsilon}}$$


k
Ha
k
k'
t
t*
t*

SPECTRAL INDEX:

TECHNICAL PREP:

- SLOW ROLL-PARAMS:

$$\boxed{\varepsilon = -\frac{\dot{H}}{H^2} \qquad \eta = \frac{\dot{\varepsilon}}{H\varepsilon} = 2(\varepsilon - \delta)}$$

- EXIT TIME t^*: $k = Ha$ ... DIFF W.R.T. k

$$1 = \frac{dH}{dk} a + H \frac{da}{dk} = \dot{H} \frac{dt}{dk} a + H \frac{\dot{a}}{a} \frac{dt}{dk} a$$

$$= (\dot{H} a + H^2 a) \frac{dt}{dk}$$

k

$$k\frac{dt}{dk} = \frac{k}{\dot{H}a + H^2 a} = \frac{Ha}{H^2 a\left(1 + \frac{\dot{H}}{H^2}\right)} = \frac{1}{H(1-\varepsilon)}$$

SPECTRAL INDEX: $k\frac{dt}{dk}\frac{d}{dt}$

$$n_s - 1 = k\frac{d}{dk}\log\left(P(k)\,k^3\right) = \frac{1}{H(1-\varepsilon)}\frac{d}{dt}\log\left(\frac{H^2}{4\pi m_{pl}^2\varepsilon}\right)$$

$.T.\ k$

$H\frac{\dot{a}}{a}\frac{dt}{dk}a$

$$k\frac{dt}{dk} = \frac{k}{\dot{H}a + H^2 a} = \frac{Ha}{H^2 a\left(1 + \frac{\dot{H}}{H^2}\right)} = \frac{1}{H(1-\varepsilon)}$$

SPECTRAL INDEX: $k\frac{dt}{dk}\frac{d}{dt}$

$$n_s - 1 = k\frac{d}{dk}\log\left(P(k)\,k^3\right) = \frac{1}{H(1-\varepsilon)}\frac{d}{dt}\log\left(\frac{H^2}{4\pi m_{pl}^2\varepsilon}\right)$$

$$= \frac{1}{1-\varepsilon}\left(\frac{2\dot{H}}{H^2} - \frac{\dot{\varepsilon}}{H\varepsilon}\right) = -\frac{2\varepsilon + \eta_N}{1-\varepsilon} \simeq -2\varepsilon - \eta_N = \underline{-4\varepsilon + 2\delta}.$$

k

$\frac{dt}{dk}\,a$

$$\boxed{n_s = 1 - 4\varepsilon + 2\delta}$$ ... RED TILTED

- TENSOR MODES (H3)

$$\boxed{P_\gamma = \frac{2H^2}{k^3 m_{pl}^2}\Big|_{t=t_*}}$$ PER POLAR.

MISSING $\frac{1}{\varepsilon}$ ...

SCALAR MODES ENHANCED
W.R.T. TENSORS

TED

$$P_t = 2P_{\gamma\gamma}$$

$$n_T = k \frac{d}{dk} \log\left(k^3 P_k\right) = \underline{-2\varepsilon}$$

OLAP.

$$\boxed{r = \frac{P_t}{P_\zeta} = 16\varepsilon = -8\, n_T}$$

$$= 2P_{\gamma}$$

$$= k\frac{d}{dk}\log\left(k^3 P_{\gamma}\right) = -2\varepsilon$$

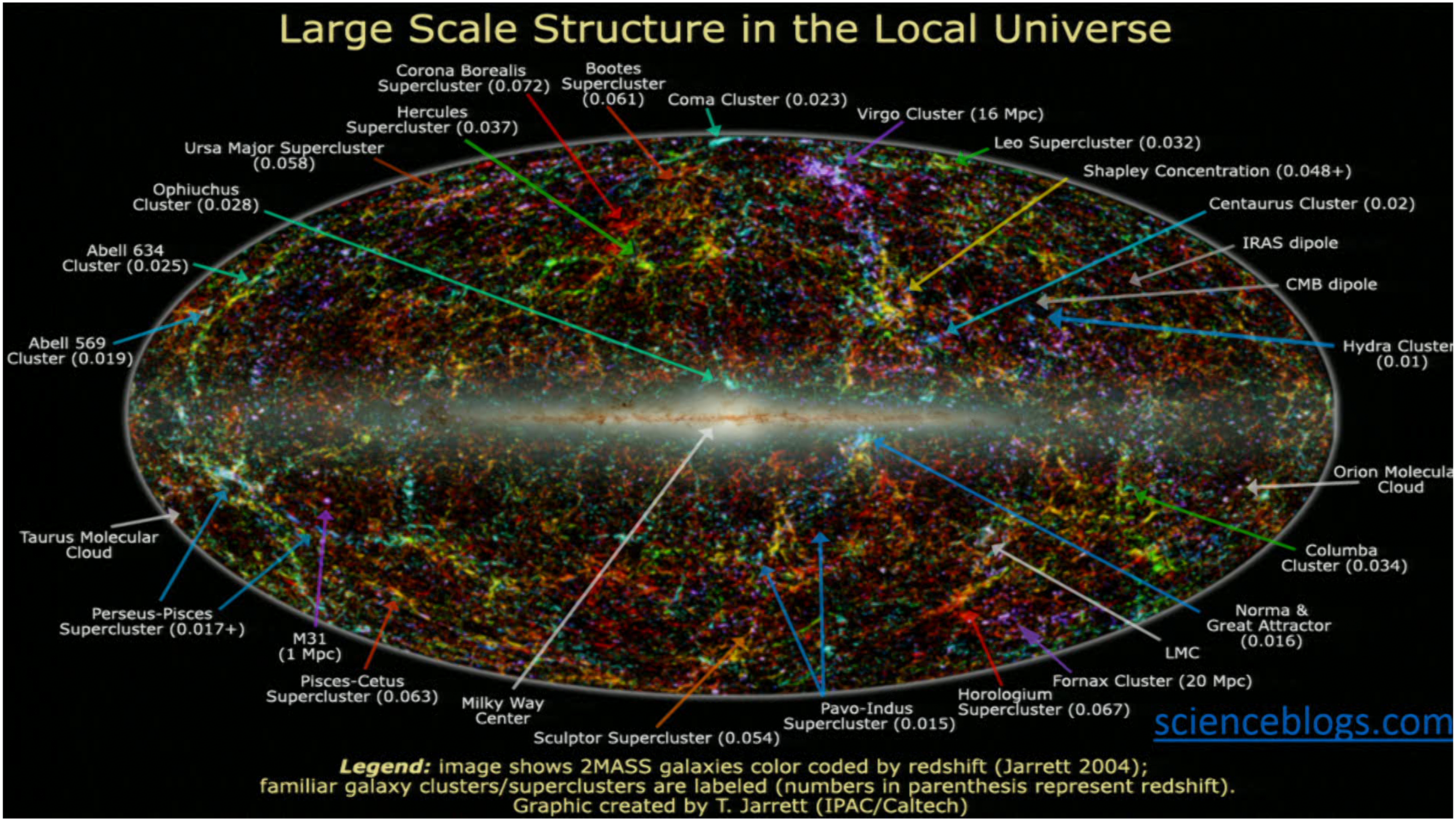
$$\boxed{\frac{P_t}{P_{\zeta}} = 16\varepsilon = -8n_T}$$

$$\boxed{r \simeq 0.01}$$

SUMMARY OF CURRENT COSMOLOGICAL MODEL

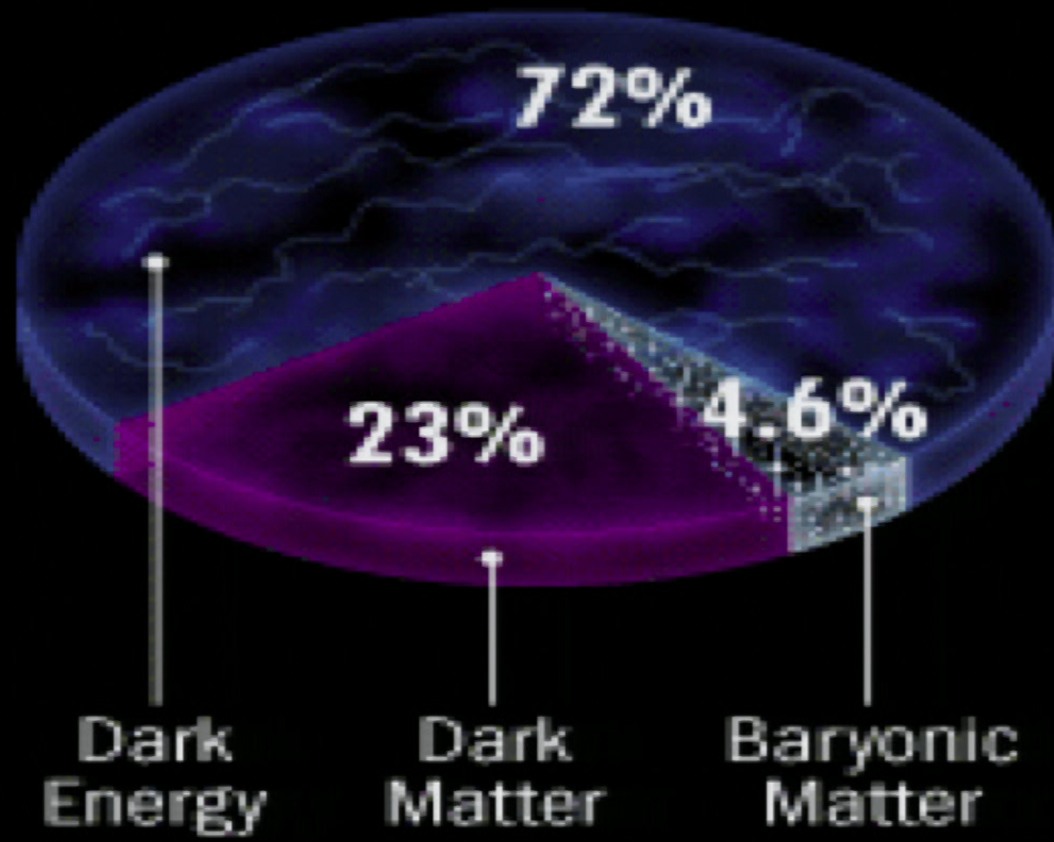
ΛCDM MODEL

- $\Omega_b, \Omega_{CDM}, \Omega_\Lambda$ ... "HOMOGENEOUS U."
- n_s, A_s ..... PRIMORDIAL FLUCT.
- τ ... RE-IONIZATION OPTICAL DEPTH

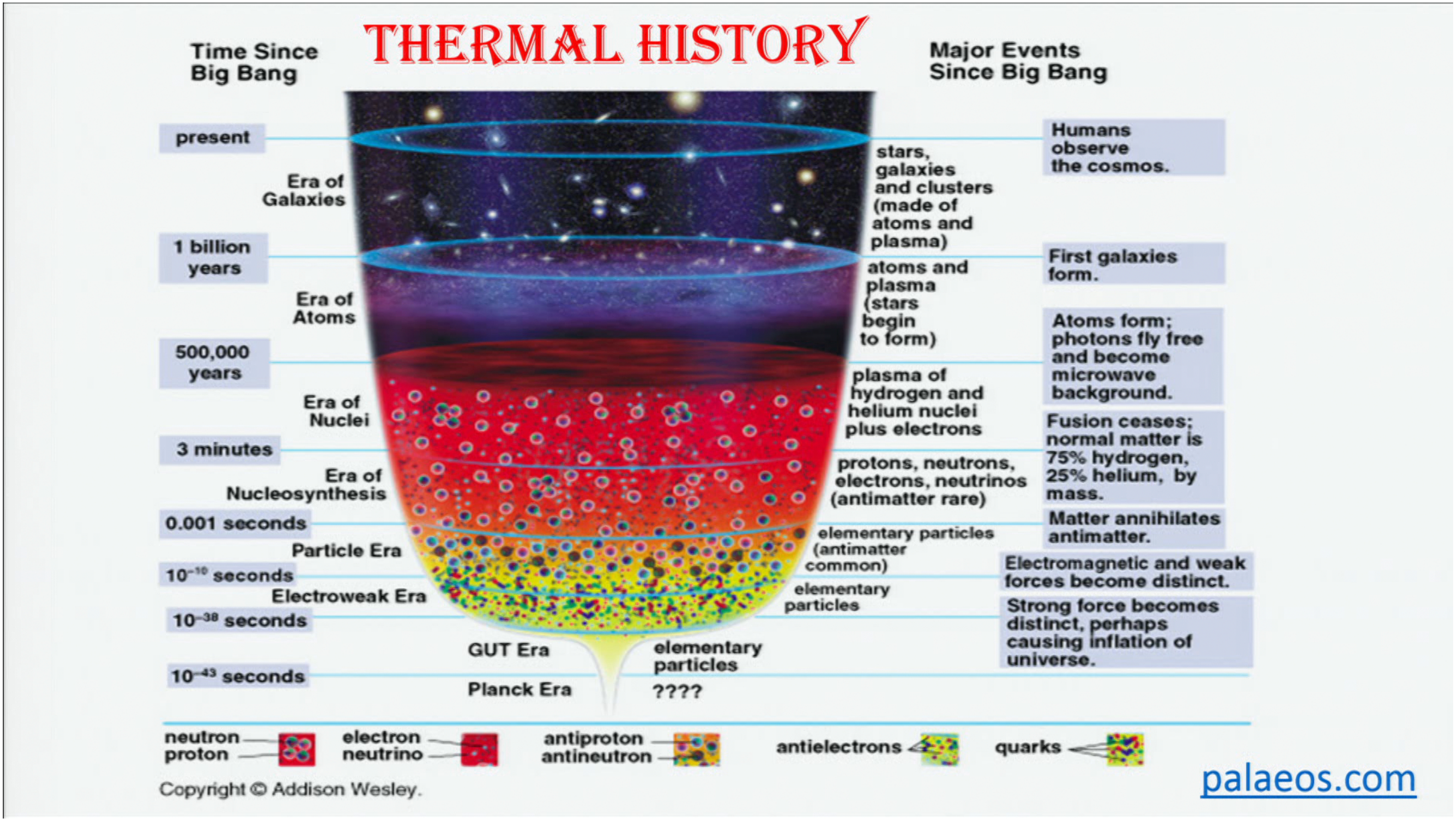


Large Scale Structure in the Local Universe
Corona Borealis Supercluster (0.072)
Bootes Supercluster (0.061)
Coma Cluster (0.023)
Virgo Cluster (16 Mpc)
Hercules Supercluster (0.037)
Leo Supercluster (0.032)
Ursa Major Supercluster (0.058)
Shapley Concentration (0.048+)
Ophiuchus Cluster (0.028)
Centaurus Cluster (0.02)
IRAS dipole
Abell 634 Cluster (0.025)
CMB dipole
Abell 569 Cluster (0.019)
Hydra Cluster (0.01)
Orion Molecular Cloud
Taurus Molecular Cloud
Columba Cluster (0.034)
Perseus-Pisces Supercluster (0.017+)
M31 (1 Mpc)
Norma & Great Attractor (0.016)
LMC
Pisces-Cetus Supercluster (0.063)
Fornax Cluster (20 Mpc)
Milky Way Center
Horologium Supercluster (0.067)
Pavo-Indus Supercluster (0.015)
Sculptor Supercluster (0.054)
scienceblogs.com
Legend: image shows 2MASS galaxies color coded by redshift (Jarrett 2004); familiar galaxy clusters/superclusters are labeled (numbers in parenthesis represent redshift). Graphic created by T. Jarrett (IPAC/Caltech)

WITH THE FOLLOWING MATTER CONTENT:



science.howstuffworks.com


THERMAL HISTORY
Time Since Big Bang
Major Events Since Big Bang
present
Era of Galaxies
1 billion years
Era of Atoms
500,000 years
Era of Nuclei
3 minutes
Era of Nucleosynthesis
0.001 seconds
Particle Era
10⁻¹⁰ seconds
Electroweak Era
10⁻³⁸ seconds
GUT Era
10⁻⁴³ seconds
Planck Era
stars, galaxies and clusters (made of atoms and plasma)
atoms and plasma (stars begin to form)
plasma of hydrogen and helium nuclei plus electrons
protons, neutrons, electrons, neutrinos (antimatter rare)
elementary particles (antimatter common)
elementary particles
elementary particles
????
Humans observe the cosmos.
First galaxies form.
Atoms form; photons fly free and become microwave background.
Fusion ceases; normal matter is 75% hydrogen, 25% helium, by mass.
Matter annihilates antimatter.
Electromagnetic and weak forces become distinct.
Strong force becomes distinct, perhaps causing inflation of universe.
neutron
proton
electron
neutrino
antiproton
antineutron
antielectrons
quarks
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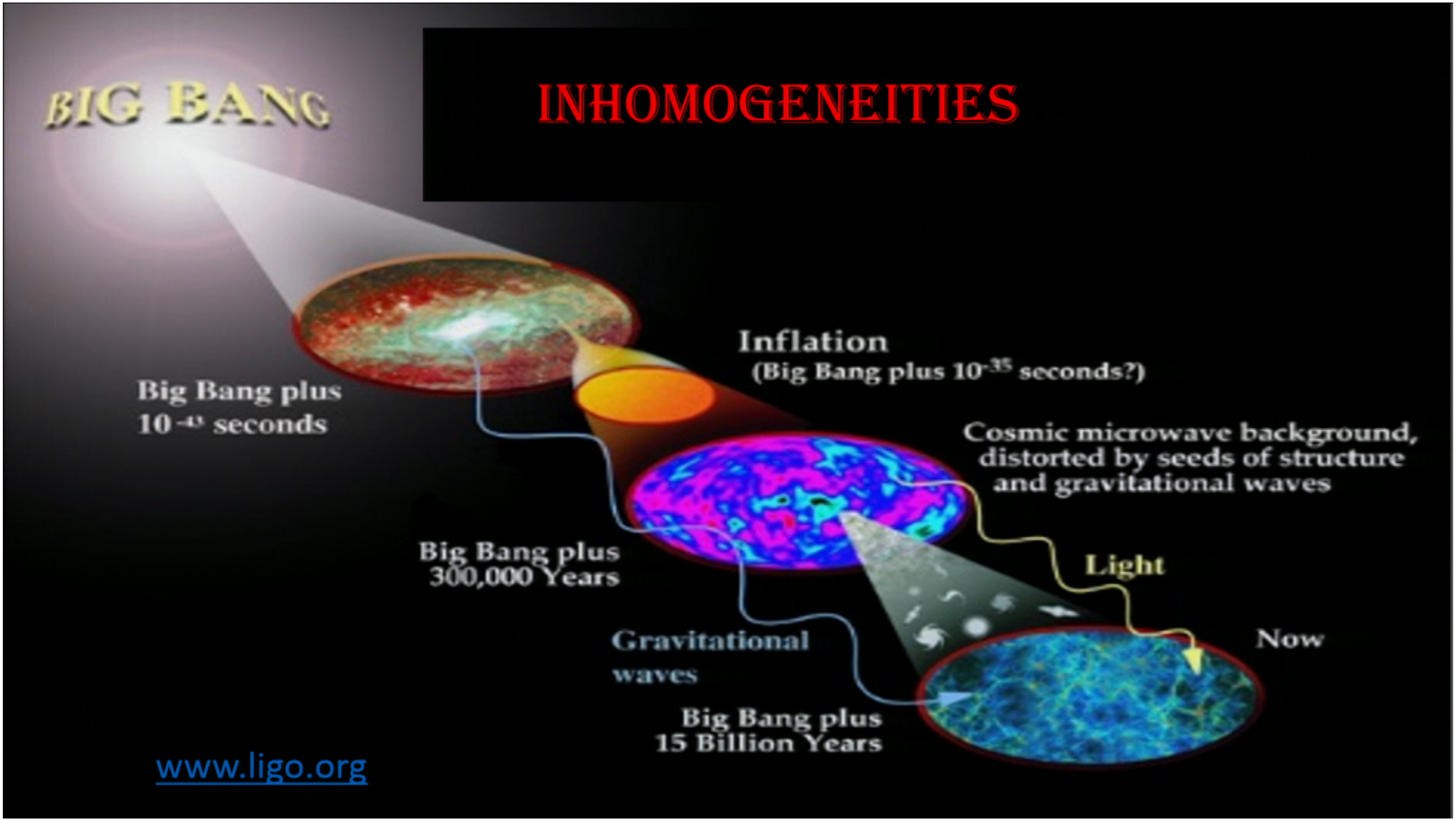
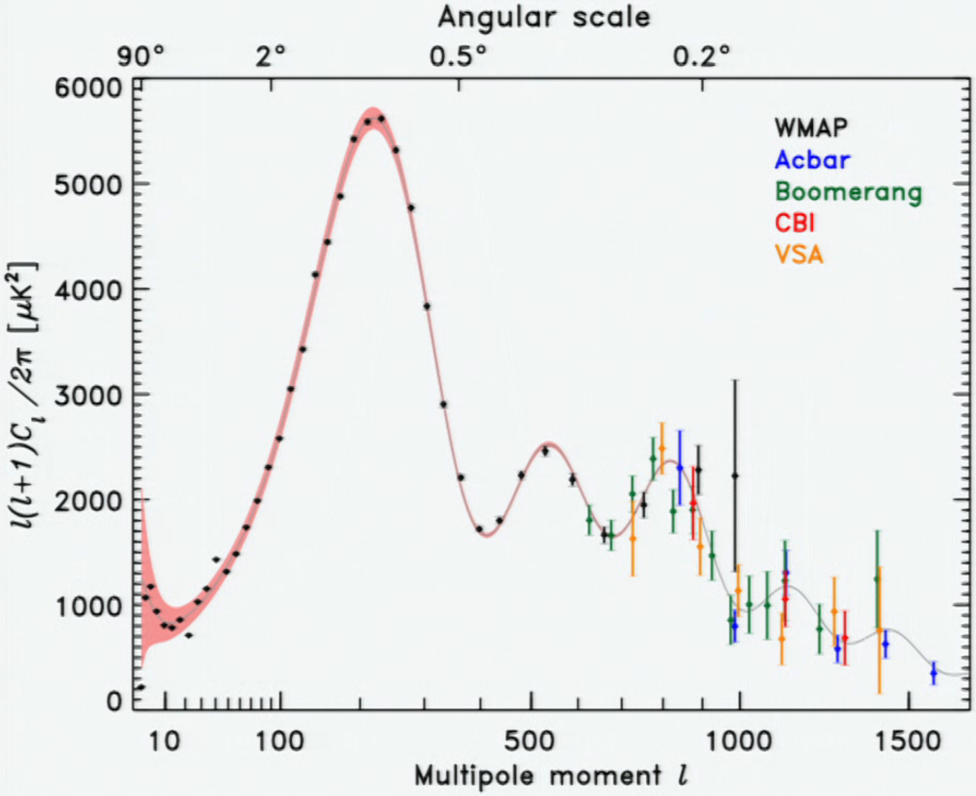


BIG BANG
INHOMOGENEITIES
Inflation
(Big Bang plus 10-35 seconds?)
Big Bang plus
10 -43 seconds
Cosmic microwave background,
distorted by seeds of structure
and gravitational waves
Big Bang plus
300,000 Years
Light
Gravitational
waves
Now
Big Bang plus
15 Billion Years
www.ligo.org

Λ-CDM MODEL



physics.stackexchange.com

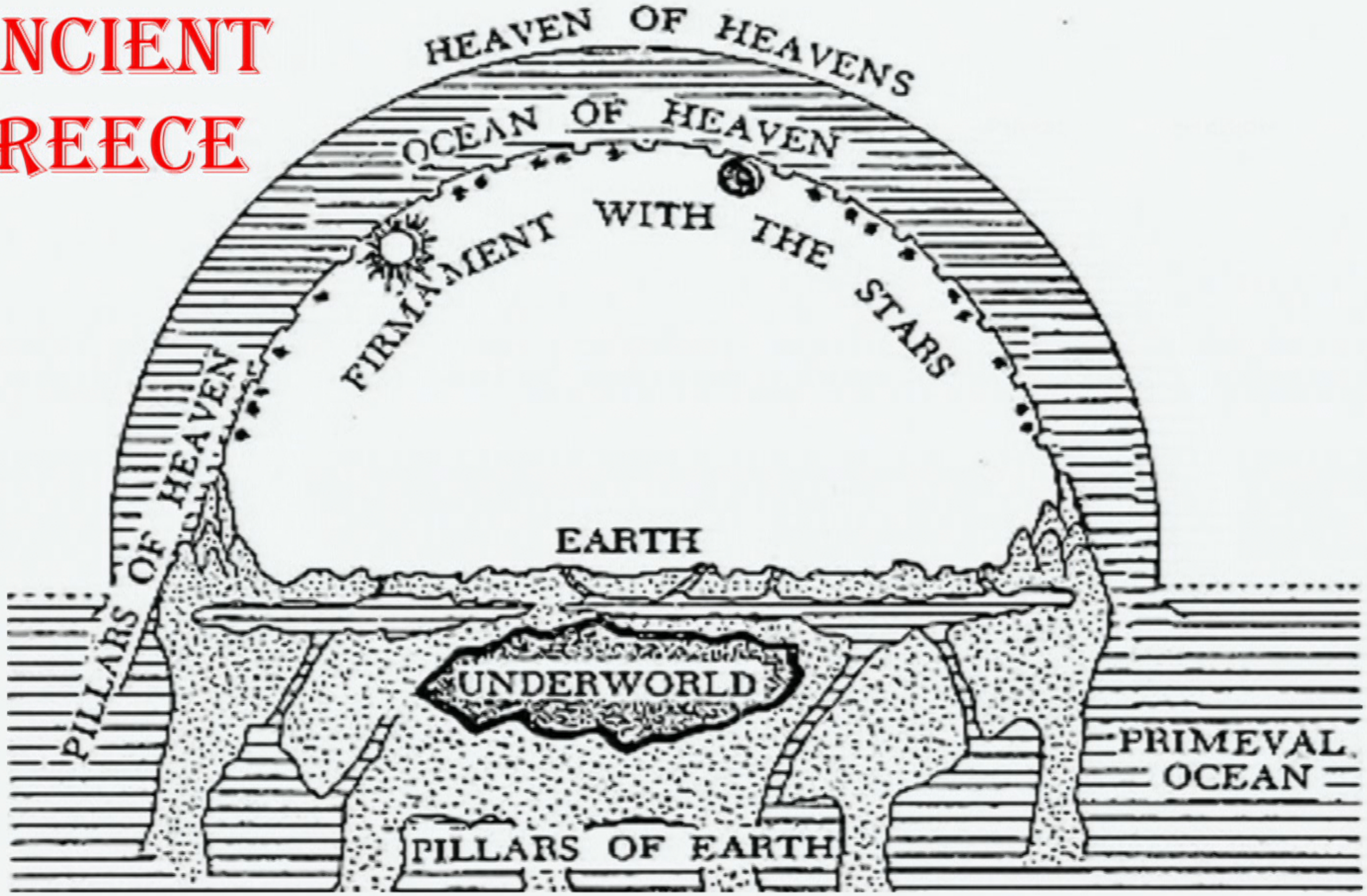
6 parameters:

Baryon density	Ω_b
Cold Dark Mater density	Ω_{CDM}
Dark energy density	Ω_Λ
Scalar spectral index	n_s
Primordial power spectrum amplitude	A_0
Re-ionization optical depth	τ


www.windows2universe.org


HAIDA
CULTURE
www.virtualmuseum.ca

ANCIENT GREECE



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