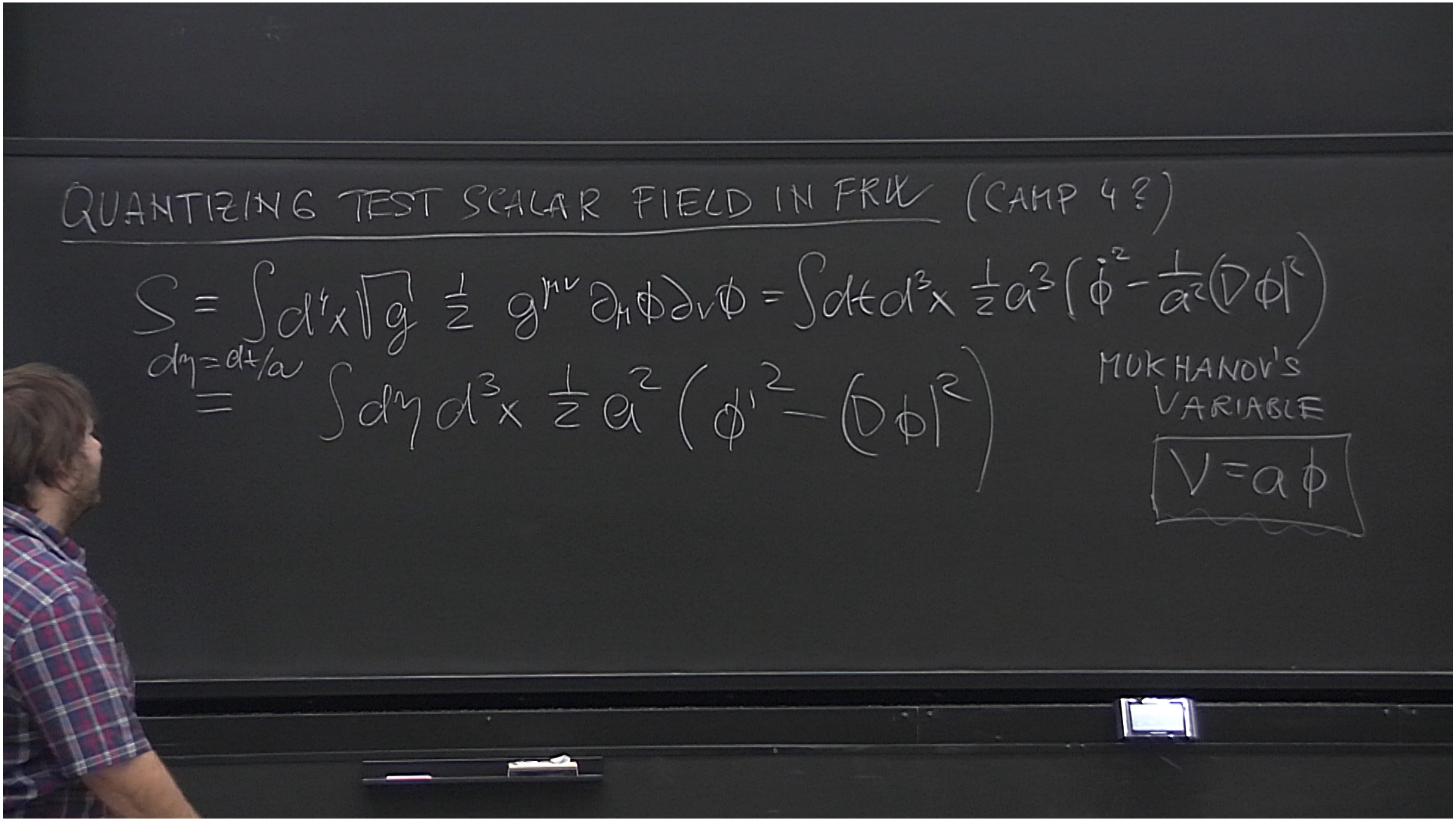
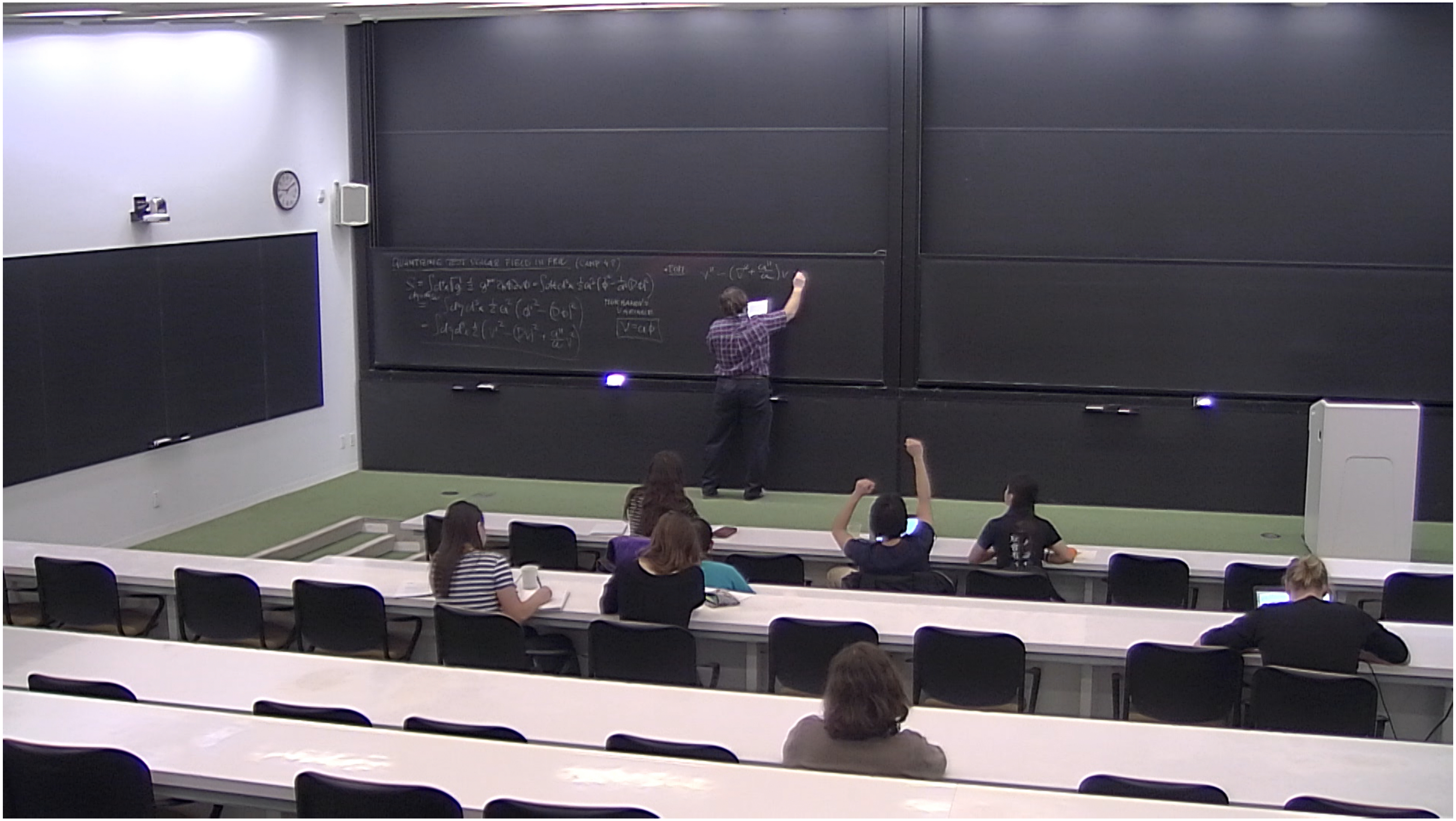
Title: PSI 17/18 - Cosmology - Lecture 14

Date: Feb 15, 2018 09:00 AM

URL: http://pirsa.org/18020023

Abstract:


QUANTIZING TEST SCALAR FIELD IN FRW (CAMP 4?)
$S = \int d^4x \sqrt{g}\, \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = \int dt\, d^3x\, \frac{1}{2} a^3 \left(\dot{\phi}^2 - \frac{1}{a^2} (\nabla \phi)^2 \right)$
$d\eta = dt/a$
$= \int d\eta\, d^3x\, \frac{1}{2} a^2 \left(\phi'^2 - (\nabla \phi)^2 \right)$
MUKHANOV'S VARIABLE
$\boxed{v = a\phi}$



QUANTIZING TEST SCALAR FIELD IN FRW (CAMP 4?)

$$S = \int d^4x \sqrt{g}\, \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = \int dt\, d^3x\, \frac{1}{2} a^3 \left(\dot{\phi}^2 - \frac{1}{a^2} (\nabla \phi)^2 \right)$$

$d\eta = dt/a$

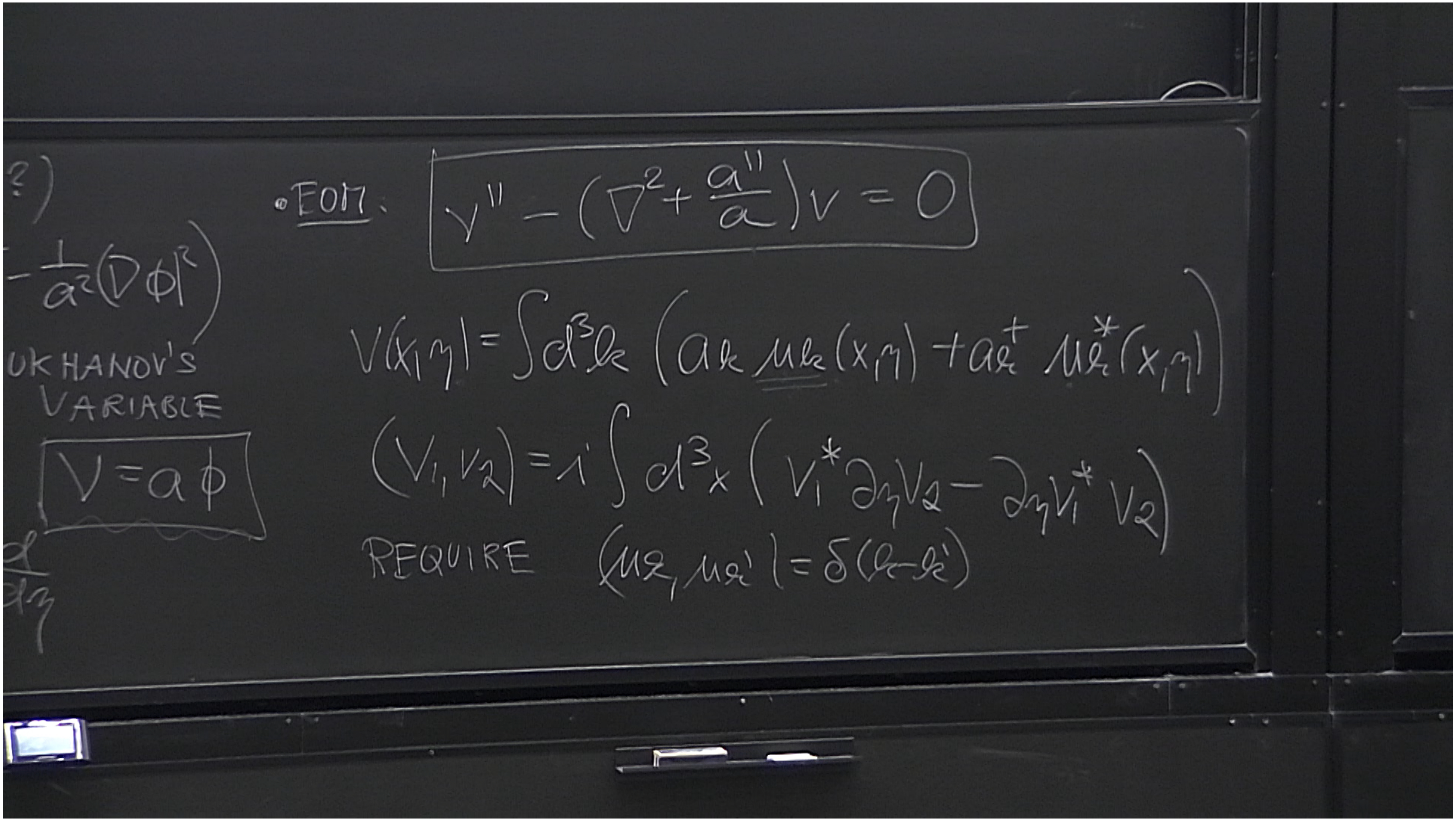
$$= \int d\eta\, d^3x\, \frac{1}{2} a^2 \left(\phi'^2 - (\nabla \phi)^2 \right)$$

$$= \int d\eta\, d^3x\, \frac{1}{2} \left(v'^2 - (\nabla v)^2 + \frac{a''}{a} v^2 \right)$$

MUKHANOV'S VARIABLE

$$\boxed{v = a\phi}$$

• EO


• EOM: $\boxed{v'' - \left(\nabla^2 + \frac{a''}{a}\right)v = 0}$
$-\frac{1}{a^2}(\nabla\phi)^2$
UKHANOV'S VARIABLE
$\boxed{v = a\phi}$
$v(x,\eta) = \int d^3k \left(a_k u_k(x,\eta) + a_k^\dagger u_k^*(x,\eta)\right)$
$(v_1, v_2) = i\int d^3x \left(v_1^* \partial_\eta v_2 - \partial_\eta v_1^* v_2\right)$
REQUIRE $(u_k, u_{k'}) = \delta(k-k')$

- ANSATZ (DUE TO HOM & ISOTROPY ... $\vec{x}$ ... KILLING)

$$u_k(x,\eta) = \frac{1}{(2\pi)^{3/2}} f_k(\eta)\, e^{i\vec{k}\cdot\vec{x}}$$

PLUG TO EOM. $\Rightarrow$

$$\boxed{f_k'' + \left(k^2 - \frac{a''}{a}\right) f_k = 0}$$

MUKHANOV'S EQUATION

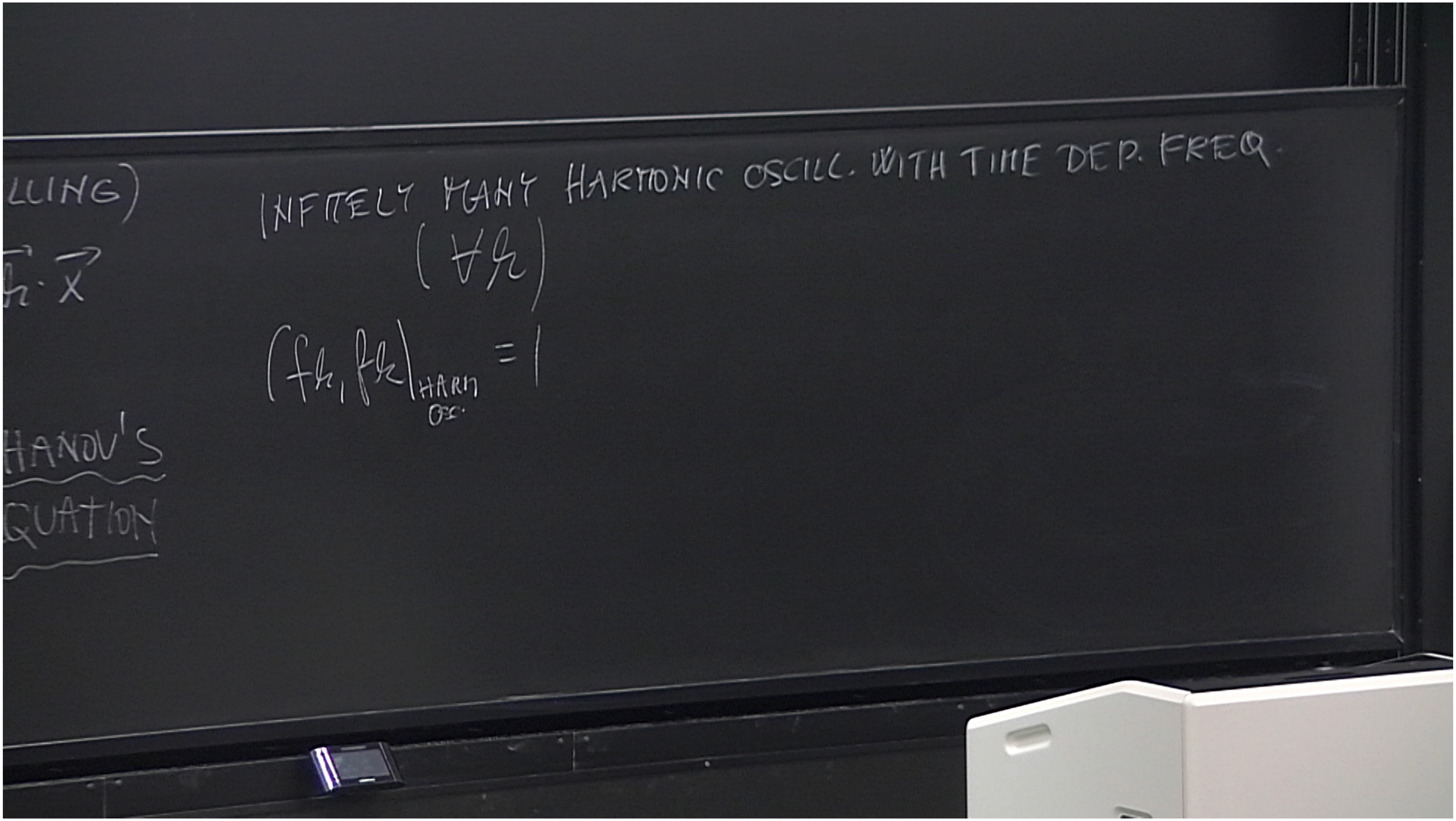
• ANSATZ (DUE TO HOM & ISOTROPY ... $\vec{x}$ .. KILLING)

$$u_k(x,\eta) = \frac{1}{(2\pi)^{3/2}} f_k(\eta)\, e^{i\vec{k}\cdot\vec{x}}$$

PLUG TO EOM. $\Rightarrow$

$$\boxed{f_k'' + \underbrace{\left(k^2 - \frac{a''}{a}\right)}_{\omega_k^2(\eta)} f_k = 0}$$

MUKHANOV'S EQUATION


LLING)
k · x
HANOV'S
QUATION
INFITELY MANY HARMONIC OSCILL. WITH TIME DEP. FREQ.
(∀k)
(fk, fk) HARM OSC. = 1

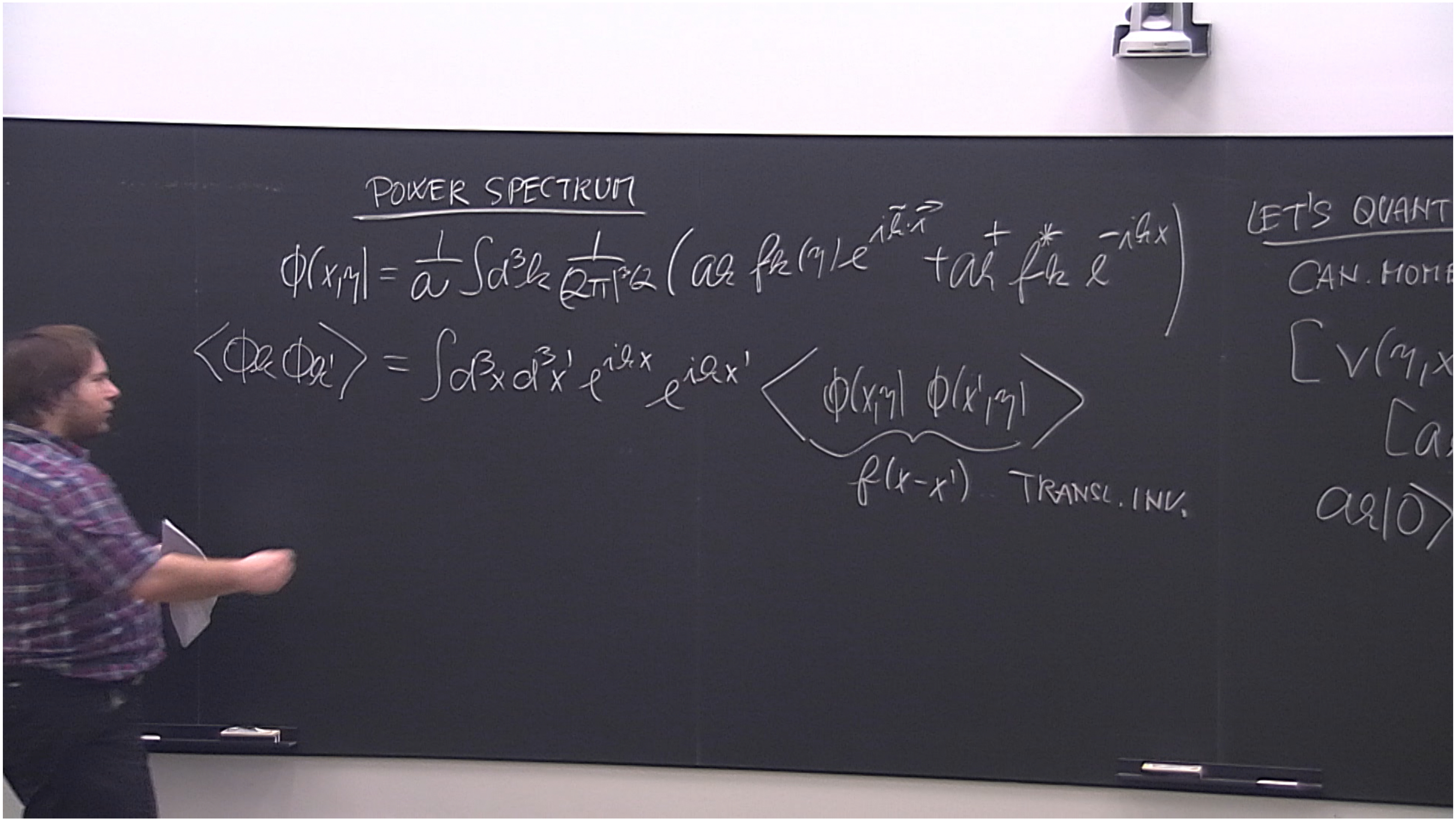
LET'S QUANTIZE:

CAN. MOMENTUM $\pi = v'$

$$[v(\eta, x), \pi(\eta, x')] = i\delta^3(x - x')$$

$$[a_k, a_{k'}^\dagger] = \delta^3(k - k')$$

$$a_k|0\rangle = 0 \quad \forall k$$


POWER SPECTRUM
$\phi(x,\eta) = \frac{1}{a}\int d^3k \frac{1}{(2\pi)^{3/2}}\left(a_k f_k(\eta) e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger f_k^* e^{-ik x}\right)$
$\langle \phi_k \phi_{k'} \rangle = \int d^3x\, d^3x'\, e^{ikx} e^{ikx'} \langle \underbrace{\phi(x,\eta)\, \phi(x',\eta)}_{f(x-x')} \rangle$
TRANSL. INV.

POWER SPECTRUM

$$\phi(x,\eta) = \frac{1}{a}\int d^3k \, \frac{1}{(2\pi)^{3/2}} \left(a_k f_k(\eta) e^{i\vec{k}\cdot\vec{x}} + a_k^{+} f_k^{*} e^{-ikx} \right)$$

$$\langle \phi_k \phi_{k'} \rangle = \int d^3x \, d^3x' \, e^{ikx} e^{ik'x'} \langle \underbrace{\phi(x,\eta)\,\phi(x',\eta)}_{f(x-x')} \rangle$$

TRANSL. INV.

$$= (2\pi)^3 \delta^3(k+k') \, \underline{P_\phi(|\vec{k}|,\eta)}$$

POWER SPECTRUM

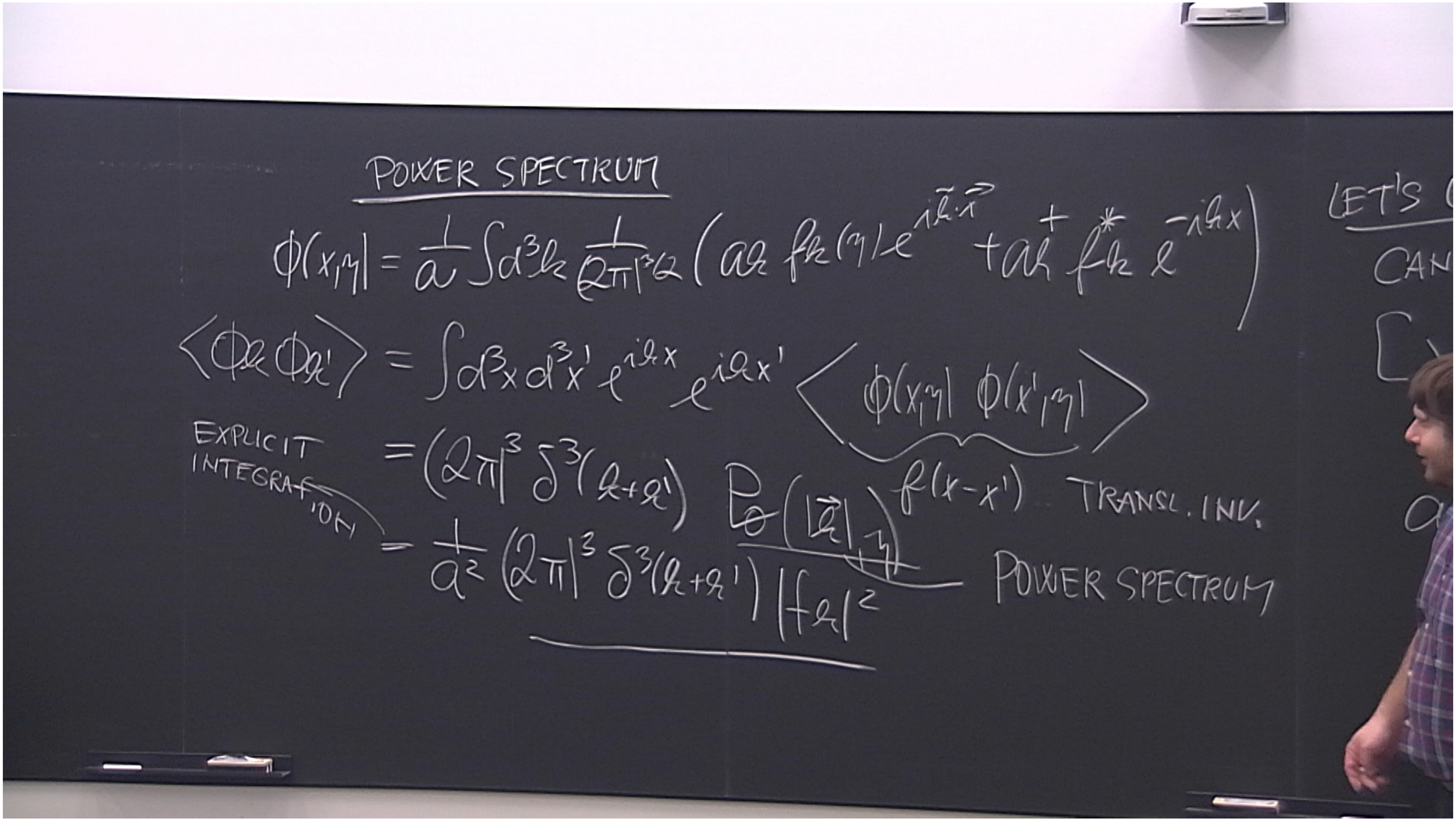
LET'S QUANT

CAN. MOME

$[v(\eta,x$

$[a$

$a_k|0\rangle$


POWER SPECTRUM
$\phi(x,\eta) = \frac{1}{a}\int d^3k \frac{1}{(2\pi)^{3/2}} \left(a_k f_k(\eta) e^{i\vec{k}\cdot\vec{x}} + a_k^{+} f_k^{*} e^{-ikx} \right)$
$\langle \phi_k \phi_{k'} \rangle = \int d^3x\, d^3x'\, e^{ikx} e^{ik'x'} \langle \underbrace{\phi(x,\eta)\, \phi(x',\eta)}_{f(x-x')} \rangle$
TRANSL. INV.
EXPLICIT INTEGRATION
$= (2\pi)^3 \delta^3(k+k') P_\phi(|\vec{k}|,\eta)$
POWER SPECTRUM
$= \frac{1}{a^2} (2\pi)^3 \delta^3(k+k') |f_k|^2$
LET'S

$$\boxed{P_\phi(k) = \frac{|f_k|^2}{a^2}}$$

f_k ... CLASSICAL SOL OF MUKHANOV'S EQ.

PROVIDED RIGH INITIAL CONDS.

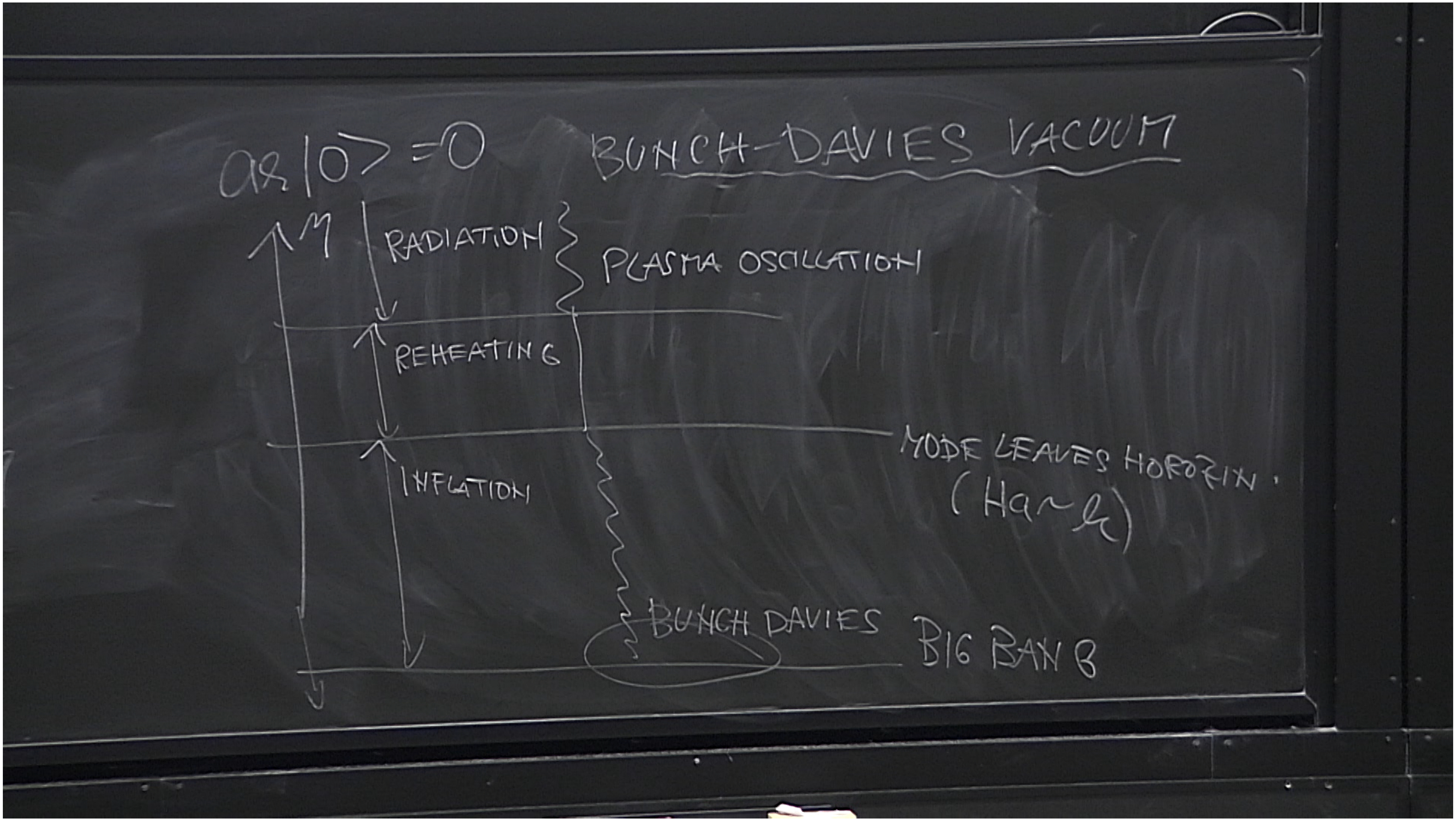
TYPICAL CHOISE

AT EARLY TIMES $\boxed{\frac{a''}{a} \to 0}$

$\omega_k(\eta) = \text{CONST} = \underline{k}$.

$$f_k(\eta) \to \frac{1}{\sqrt{2k}} e^{-ik\eta}$$

$\longrightarrow$ SOLUTIONS GIVE VACUUM.



$a_k|0\rangle = 0$
BUNCH-DAVIES VACUUM
η
RADIATION
PLASMA OSCILLATION
REHEATING
INFLATION
MODE LEAVES HORIZON.
($Ha \sim k$)
BUNCH DAVIES
BIG BANG

$-i k x$

L. INV.

ECTRUM

FOR SIMPLICITY CONSIDER INFLATION ~ DESITTER EXP.

$$a = e^{Ht} \quad , \; H = \text{CONST.}$$

$$\eta = \int^t \frac{dt'}{a(t')} = -\frac{1}{H} e^{-Ht} = -\frac{1}{Ha}$$

$$\boxed{a = -\frac{1}{H\eta}}$$

NOTE THAT AS $\eta \to -\infty$ $\omega_k \to k$ ... BUNCH-DAVIES

SOLUTION: $f_k = \frac{1}{\sqrt{2k}} e^{-ik\eta} ($

$$\boxed{f_k'' + \underbrace{\left(k^2 - \frac{a''}{a}\right)}_{\omega_k^2(\eta)} f_k = 0 = f_k'' + \left(k^2 - \frac{2}{\eta^2}\right) f_k}$$

NOTE THAT AS $\eta \to -\infty$ $\underline{\omega_k \to k}$, ∴ BUNCH-DAVIES

SOLUTION:

$$\boxed{f_k = \frac{1}{\sqrt{2k}}\, e^{-ik\eta}\left(1 - \frac{i}{k\eta}\right)}$$

$$\boxed{f_k'' + \underbrace{\left(k^2 - \frac{a''}{a}\right)}_{\omega_k^2(\eta)} f_k = 0 = f_k'' + \left(k^2 - \frac{2}{\eta^2}\right) f_k}$$

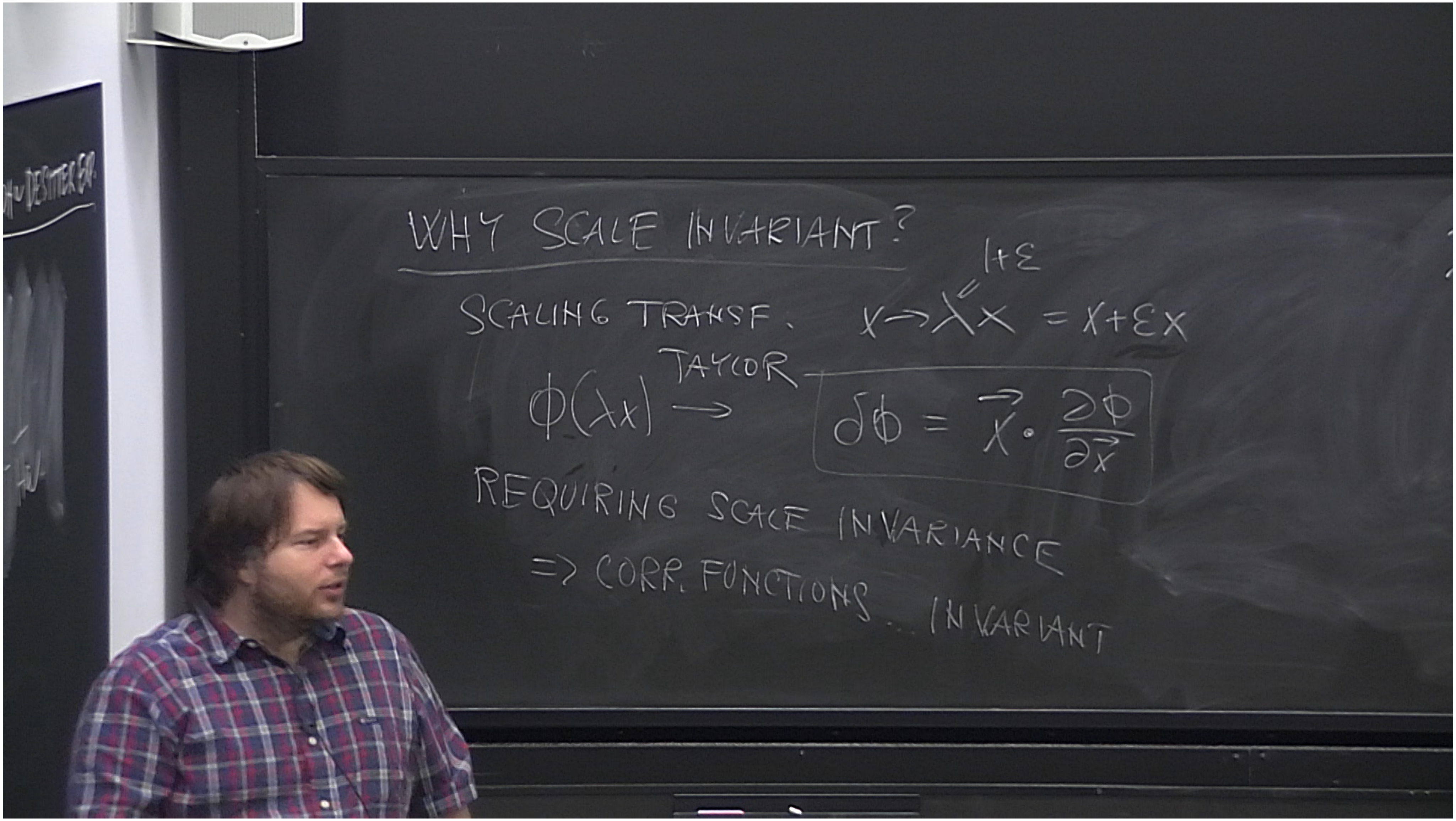
POWER SPECTRUM

$$P_\phi(k) = \frac{|f_k|^2}{a^2} = H^2 \eta^2 \frac{1}{2k}\left(1 + \frac{1}{k^2\eta^2}\right)$$

AT THE END OF INFLATION $(\eta \to 0)$

$$\boxed{P_\phi(k,\eta) = \frac{H^2}{2k^3}}$$

SCALE INVARIANT SPECTRUM


WHY SCALE INVARIANT?
SCALING TRANSF. $x \to \lambda x = x + \varepsilon x$
$\lambda = 1+\varepsilon$
TAYLOR
$\phi(\lambda x) \longrightarrow$ $\boxed{\delta\phi = \vec{x}\cdot\frac{\partial\phi}{\partial\vec{x}}}$
REQUIRING SCALE INVARIANCE
=> CORR. FUNCTIONS ... INVARIANT

$x+\varepsilon x$

$$\boxed{\frac{\partial\phi}{\partial\vec{x}}}$$

$$O = \vec{x}\cdot\frac{\partial}{\partial\vec{x}}\langle\phi(x)\,\phi(0)\rangle = \vec{x}\cdot\frac{\partial}{\partial\vec{x}}\int\frac{d^3k\,d^3k'}{(2\pi)^3(2\pi)^3}\,e^{-ik\cdot x}\underbrace{\langle\phi_k\,\phi_{k'}\rangle}_{(2\pi)^3\,\delta(k+k')\,P_\phi(k)}$$

$$= \vec{x}\cdot\frac{\partial}{\partial\vec{x}}\int\frac{d^3k}{(2\pi)^3}\,e^{-i\vec{k}\cdot\vec{x}}\,P(k)$$

NT

$x + \epsilon x$

$\frac{\partial \phi}{\partial \vec{x}}$

$$0 = \vec{x}\cdot\frac{\partial}{\partial \vec{x}}\langle \phi(x)\,\phi(0)\rangle = \vec{x}\cdot\frac{\partial}{\partial\vec{x}}\int\frac{d^3k\,d^3k'}{(2\pi)^3(2\pi)^3}\,e^{-ik\cdot x}\underbrace{\langle \phi_k\,\phi_{k'}\rangle}_{(2\pi)^3\,\underline{\delta(k+k')}\,P_\phi(k)}$$

$$= \vec{x}\cdot\frac{\partial}{\partial\vec{x}}\int\frac{d^3k}{(2\pi)^3}\,e^{-i\vec{k}\cdot\vec{x}}\,P(k)$$

$$= \int\frac{d^3k}{(2\pi)^3}\left(-\frac{\partial}{\partial\vec{k}}\left(e^{-i\vec{k}\cdot\vec{x}}\right)\cdot\vec{k}\;P(k)\right)$$

$$= \int\frac{d^3k}{(2\pi)^3}\,e^{-i\vec{k}\cdot\vec{x}}\left(3+\vec{k}\cdot\frac{\partial}{\partial\vec{k}}\right)P(k)$$

BY PARTS

$=$

NT

$x + \varepsilon x$

$\boxed{\frac{\partial \phi}{\partial \vec{x}}}$

$$0 = \vec{x}\cdot\frac{\partial}{\partial \vec{x}}\langle \phi(x)\,\phi(0)\rangle = \vec{x}\cdot\frac{\partial}{\partial \vec{x}}\int \frac{d^3k\, d^3k'}{(2\pi)^3(2\pi)^3}\, e^{-ik\cdot x}\underbrace{\langle \phi_k\, \phi_{k'}\rangle}_{(2\pi)^3\,\delta(k+k')\,P_\delta(k)}$$

$$= \vec{x}\cdot\frac{\partial}{\partial \vec{x}}\int \frac{d^3k}{(2\pi)^3}\, e^{-i\vec{k}\cdot\vec{x}}\, P(k)$$

$$= \int \frac{d^3k}{(2\pi)^3}\left(-\frac{\partial}{\partial \vec{k}}\left(e^{-i\vec{k}\cdot\vec{x}}\right) i\vec{k}\, P(k)\right)$$

BY PARTS $=$

$$= \int \frac{d^3k}{(2\pi)^3}\, e^{-i\vec{k}\cdot\vec{x}}\underbrace{\left(3 + \vec{k}\cdot\frac{\partial}{\partial \vec{k}}\right)P(k)}_{0}$$

$$\boxed{P(k) \sim \frac{1}{k^3}}$$

NT

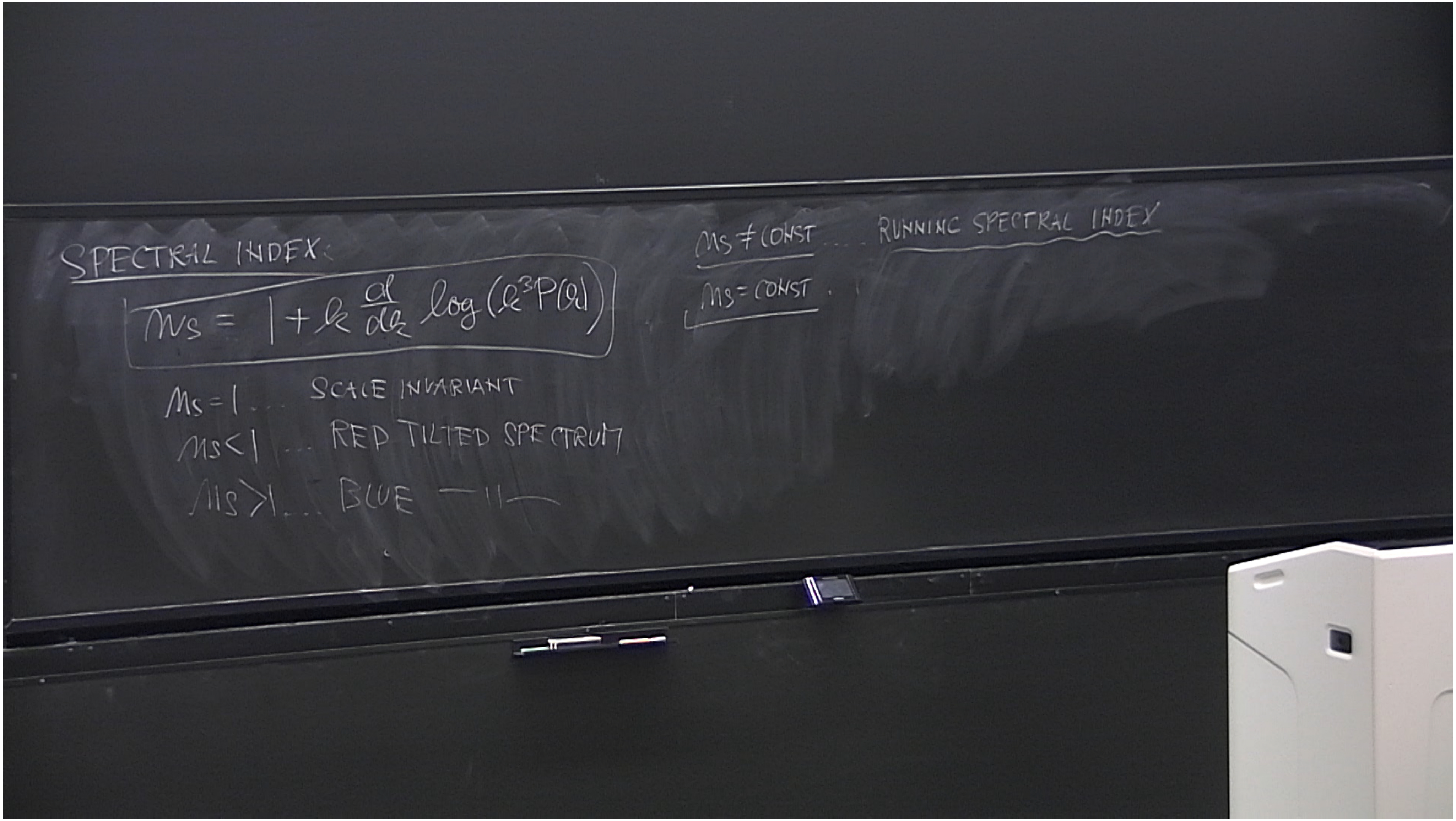
SPECTRAL INDEX:

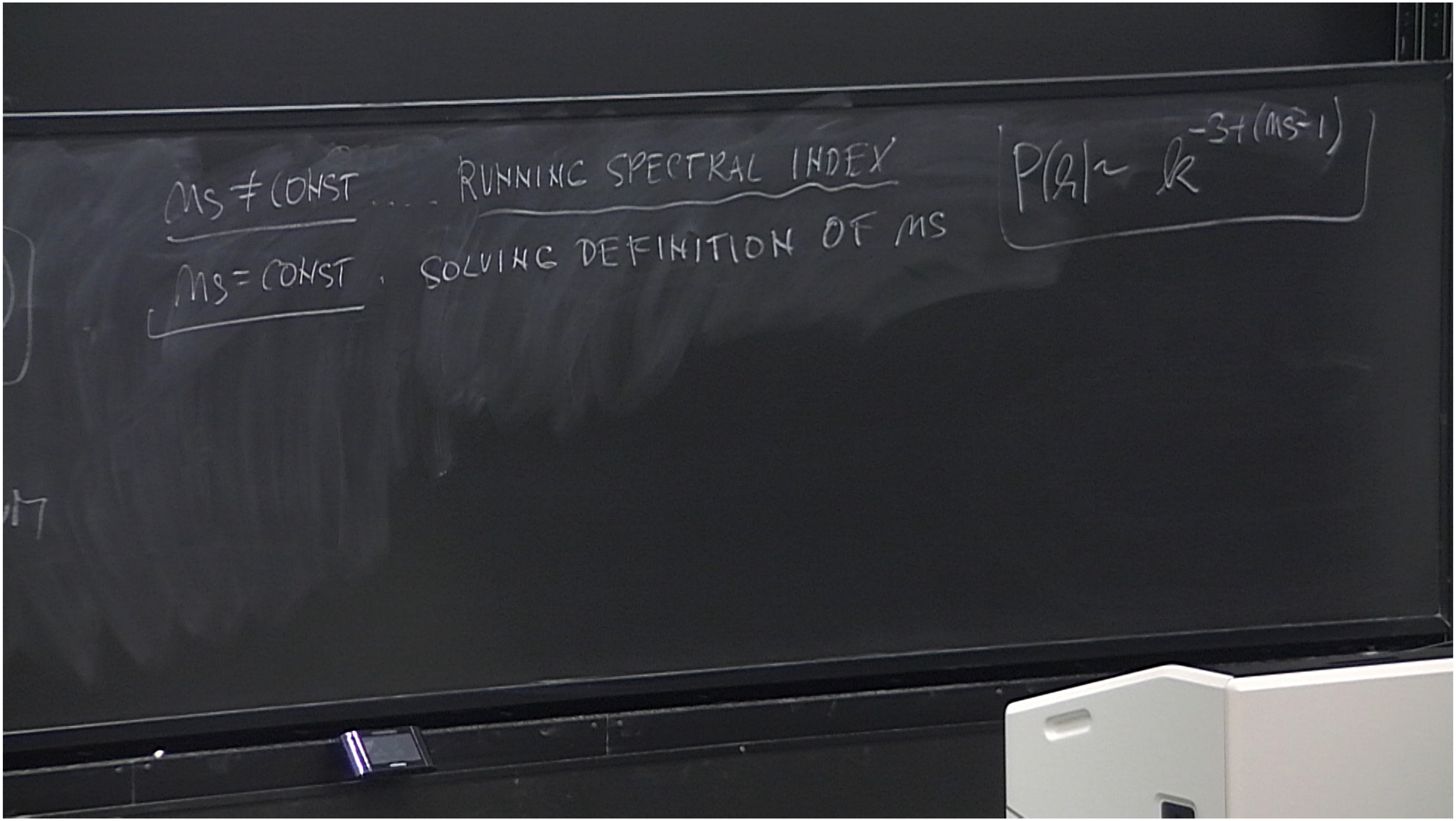
$$\boxed{n_s = 1 + k \frac{d}{dk} \log\left(k^3 P(k)\right)}$$

$n_s = 1$ ... SCALE INVARIANT

$n_s < 1$ ... RED TILTED SPECTRUM

$n_s > 1$ ... BLUE — " —

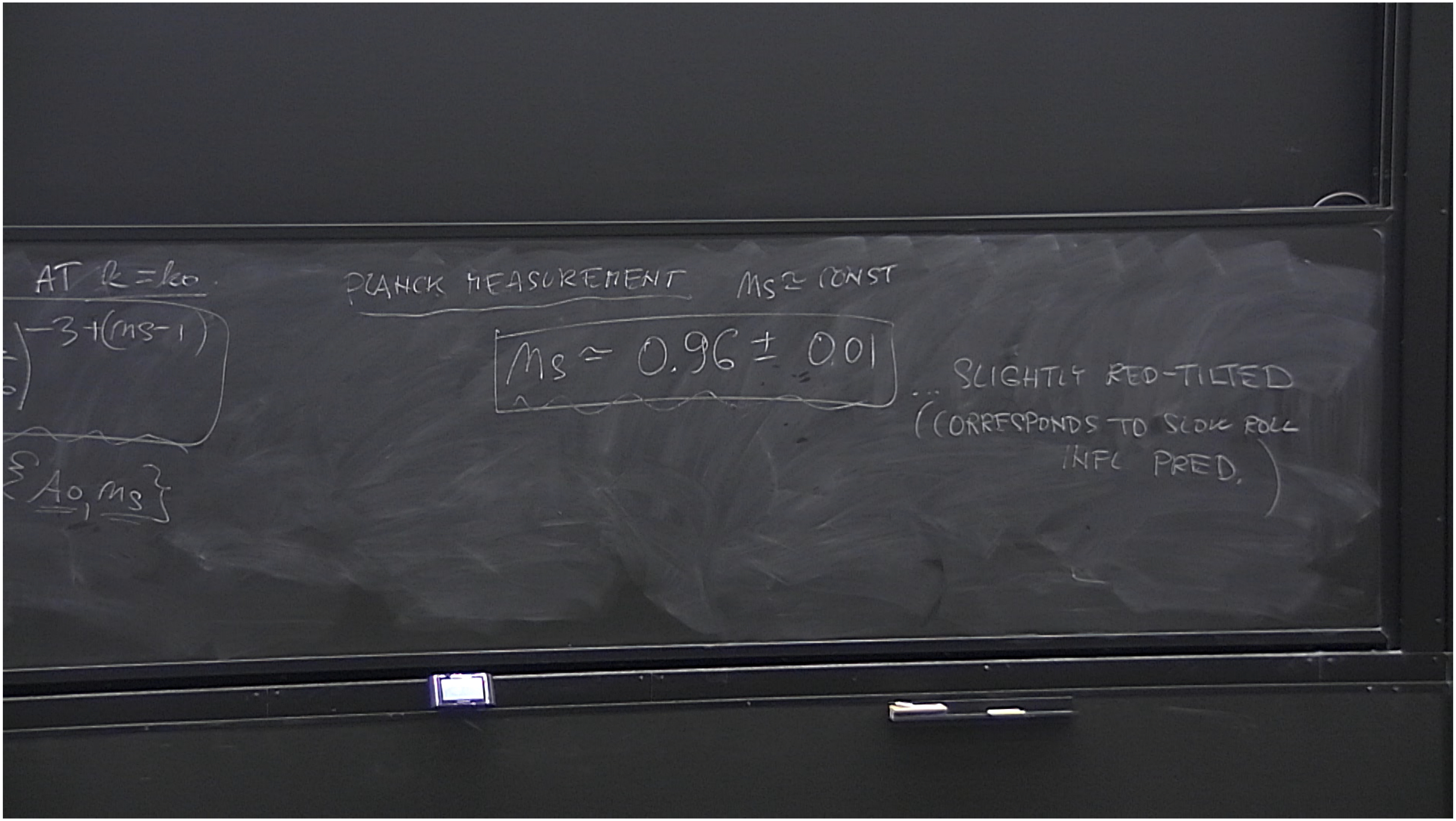

SPECTRAL INDEX:
$n_s = 1 + k \frac{d}{dk} \log(k^3 P(k))$
$n_s = 1$ ... SCALE INVARIANT
$n_s < 1$ ... RED TILTED SPECTRUM
$n_s > 1$ ... BLUE —"—
$n_s \neq$ CONST ... RUNNING SPECTRAL INDEX
$n_s =$ CONST .


ns ≠ CONST .... RUNNING SPECTRAL INDEX
$P(k) \sim k^{-3+(n_s-1)}$
ns = CONST , SOLVING DEFINITION OF ns

LET A_0 .. BY AMPLITUDE AT $\underline{k = k_0}$.

$$\boxed{P(k) = A_0 \left(\frac{k}{k_0}\right)^{-3+(n_s-1)}}$$

2 PARAMS ... $\{\underline{A_0}, \underline{n_s}\}$


AT k = k0
-3+(ns-1)
{A0, ns}
PLANCK MEASUREMENT
ns ≃ CONST
ns ≃ 0.96 ± 0.01
... SLIGHTLY RED-TILTED
(CORRESPONDS TO SLOW ROLL
INFL PRED.)