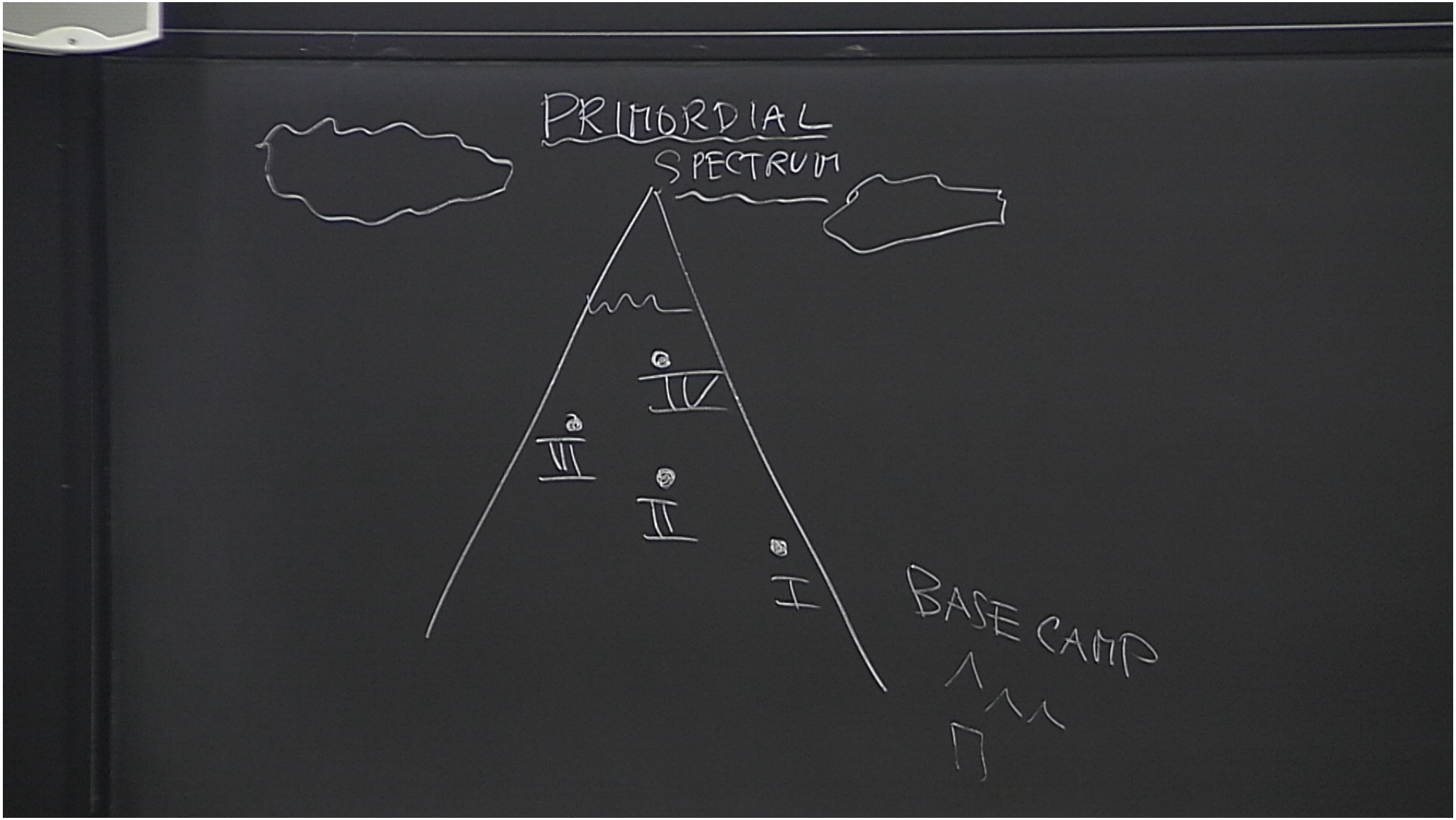
Title: PSI 17/18 - Cosmology - Lecture 13

Date: Feb 14, 2018 09:00 AM

URL: http://pirsa.org/18020022

Abstract:


PRIMORDIAL
SPECTRUM
IV
III
II
I
BASE CAMP

- TIME DEPENDENT HARMONIC OSCILLATOR

- $S = \int dt \; \frac{1}{2}\underbrace{(\dot{q}^2 - \omega^2(t) q^2)}_{L}$

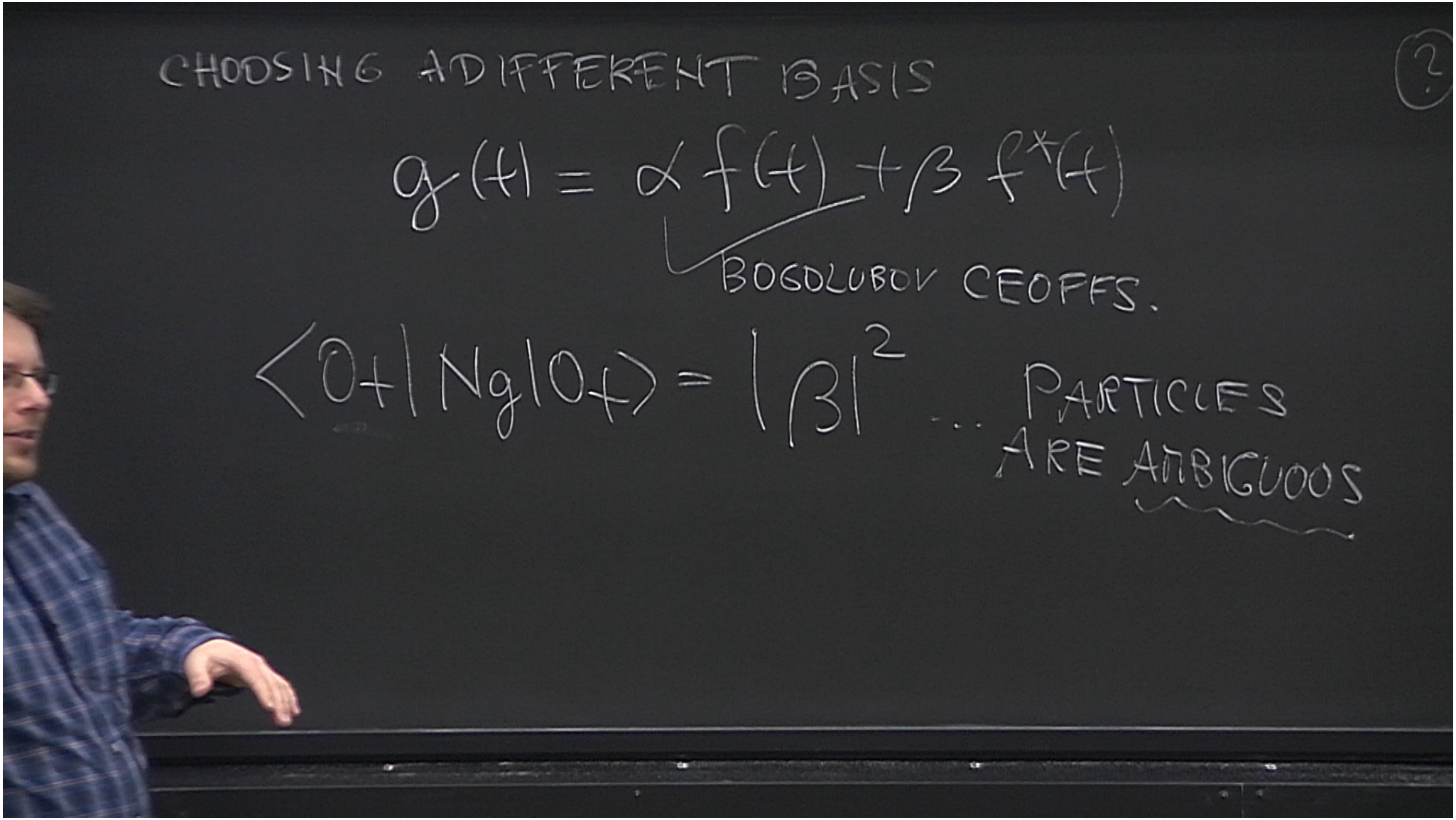
- EXPANDED CLASSICAL SOLUTION

$$q(t) = a_f f(t) + a_f^\dagger f^*(t) \qquad (f, f) = 1$$

W.R.T SYMPLECTIC PRODUCT $\quad (f_1, f_2) = \frac{1}{i}\left(f_1^* \frac{df_2}{dt} - \frac{df_1^*}{dt} f_2\right)\Big|_{t=0}$

- QUANTIZE → OPERATORS

$|0_f\rangle \quad |n_f\rangle \quad N_f = a_f^\dagger a_f$


CHOOSING A DIFFERENT BASIS
$g(t) = \alpha f(t) + \beta f^*(t)$
BOGOLUBOV COEFFS.
$\langle 0_+ | N_g | 0_+ \rangle = |\beta|^2$ ... PARTICLES ARE AMBIGUOUS

$|0_f\rangle$ $|n_f\rangle$ $N_f = a_f^\dagger a_f$

(2) IS THE VACUUM THE LOWEST ENERGY STATE?

$$H = p\dot{q} - L\Big|_{p=\dot{q}} = \frac{1}{2}p^2 + \frac{1}{2}q^2\omega^2(t) = H(t)$$

$$= \frac{1}{2}\left(a_f \dot{f} + a_f^\dagger \dot{f}^*\right)^2 + \frac{1}{2}\omega^2\left(a_f f + a_f^\dagger f^*\right)^2$$

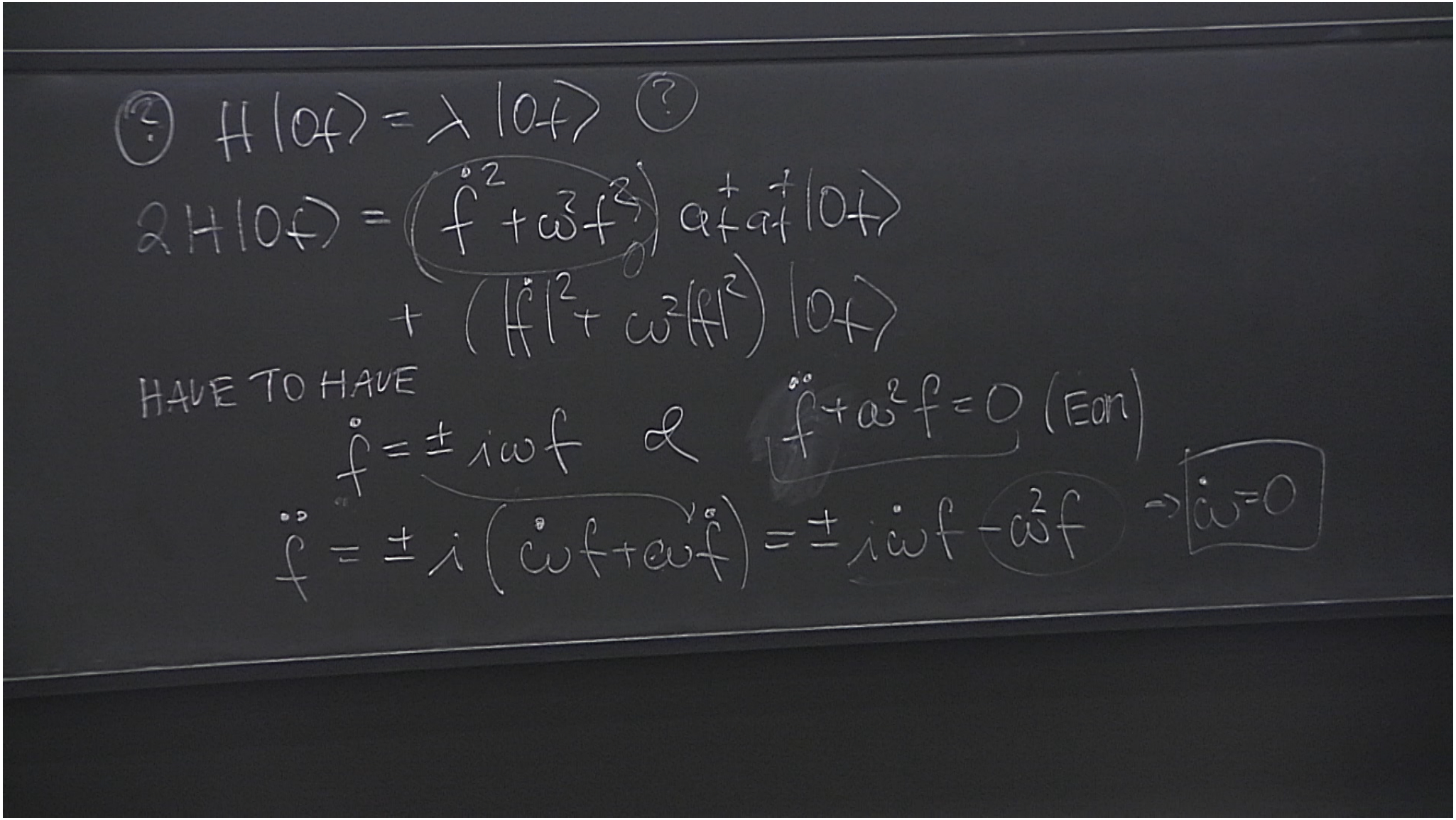
$$= \frac{1}{2}\Big[\left(\dot{f}^2 + \omega^2 f^2\right) a_f a_f + \left(\dot{f}^{*2} + \omega^2 f^{*2}\right) a_f^\dagger a_f^\dagger + \left(|\dot{f}|^2 + \omega^2 |f|^2\right)\left(a_f a_f^\dagger + a_f^\dagger a_f\right)\Big]$$

(f)

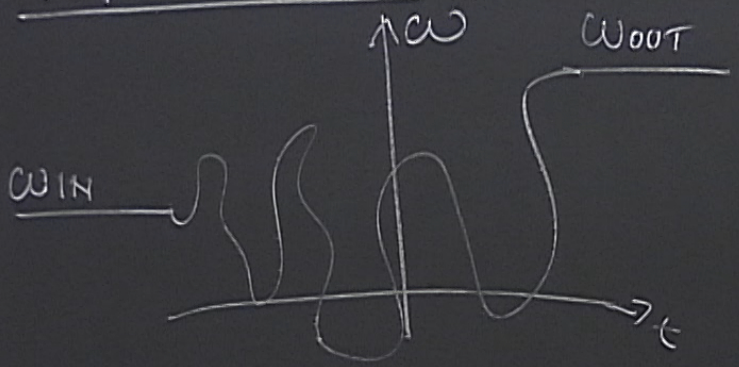
FFS.

PARTICLES

RE AMBIG


? H|0f⟩ = λ|0f⟩ ?
2H|0f⟩ = (ḟ² + ω³f³) a†a†|0f⟩
+ (|ḟ|² + ω²|f|²)|0f⟩
HAVE TO HAVE
ḟ = ± iωf & f̈ + ω²f = 0 (Eom)
f̈ = ± i(ω̇f + ωḟ) = ± iω̇f − ω²f → ω̇ = 0

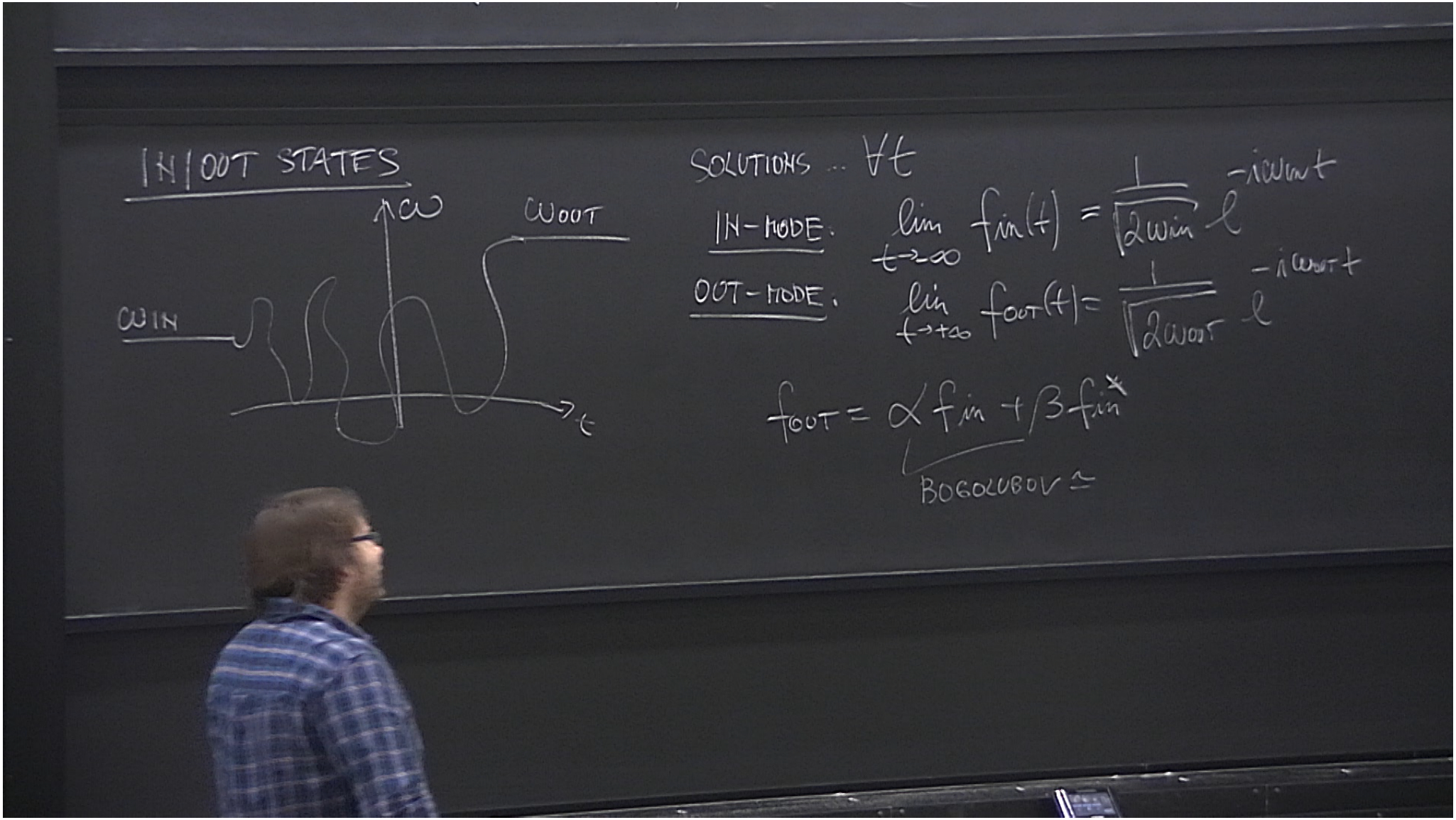
IN/OUT STATES



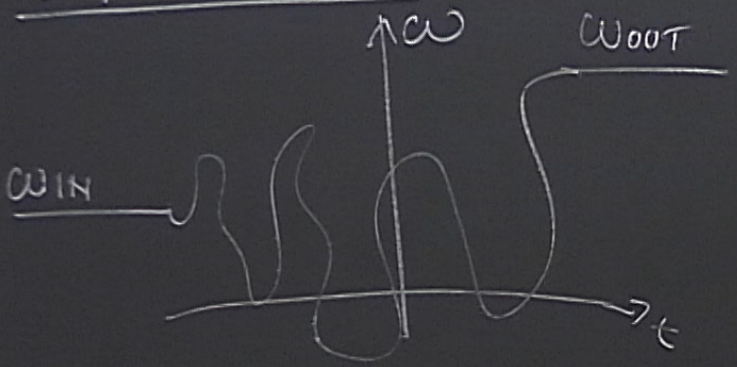
SOLUTIONS ... $\forall t$

IN-MODE: $\lim_{t \to -\infty} f_{in}(t) = \frac{1}{\sqrt{2\omega_{in}}} e^{-i\omega_{in} t}$

OUT-MODE: $\lim_{t \to +\infty} f_{OUT}(t) = \frac{1}{\sqrt{2\omega_{OUT}}} e^{-i\omega_{OUT} t}$


IN/OUT STATES
ω
ω_OUT
ω_IN
t
SOLUTIONS ... ∀t
IN-MODE: lim_{t→-∞} f_in(t) = 1/√(2ω_in) e^{-iω_in t}
OUT-MODE: lim_{t→+∞} f_OUT(t) = 1/√(2ω_OUT) e^{-iω_OUT t}
f_OUT = α f_in + β f_in*
BOGOLUBOV ≃

IN/OUT STATES



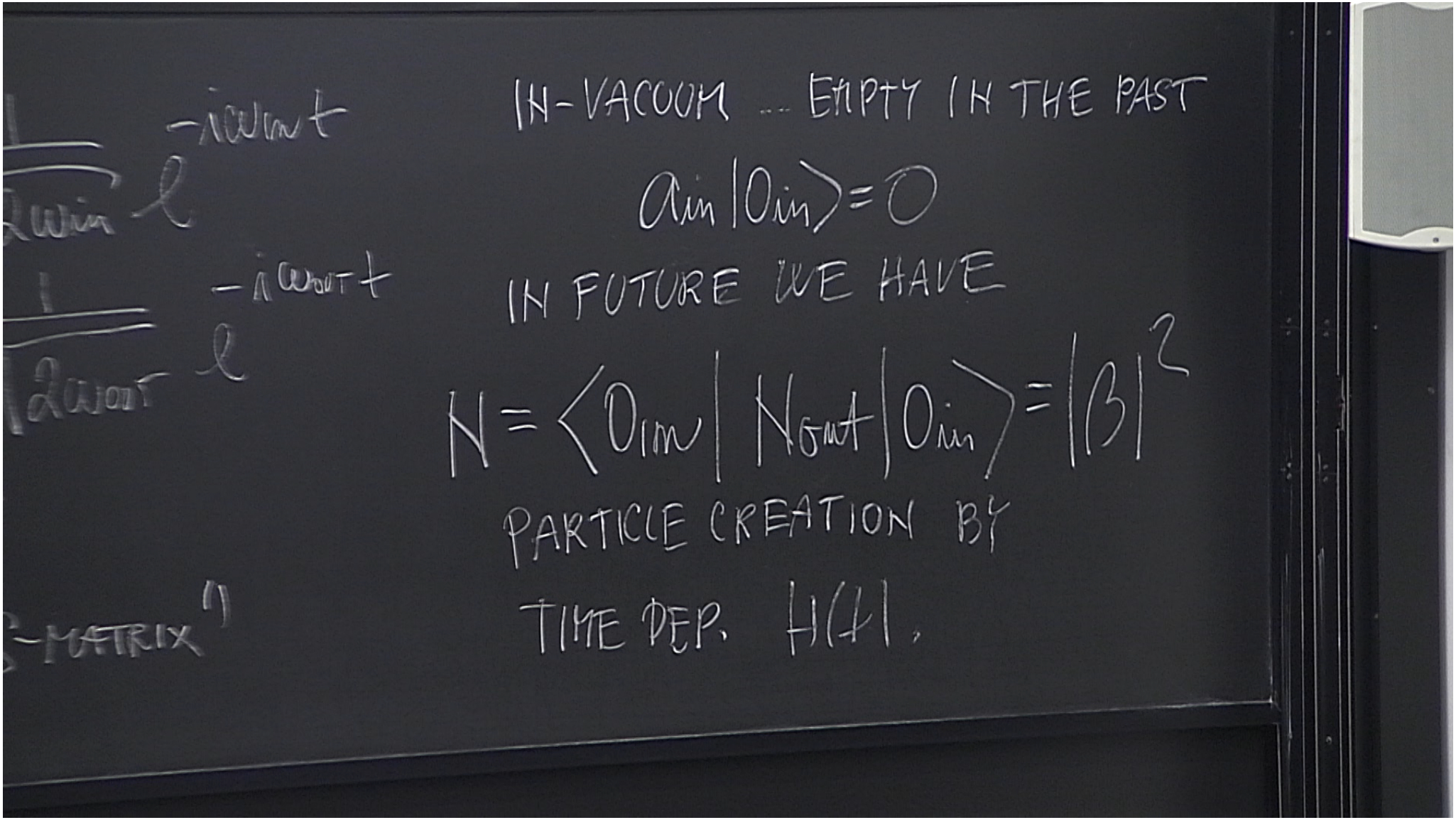
SOLUTIONS ... $\forall t$

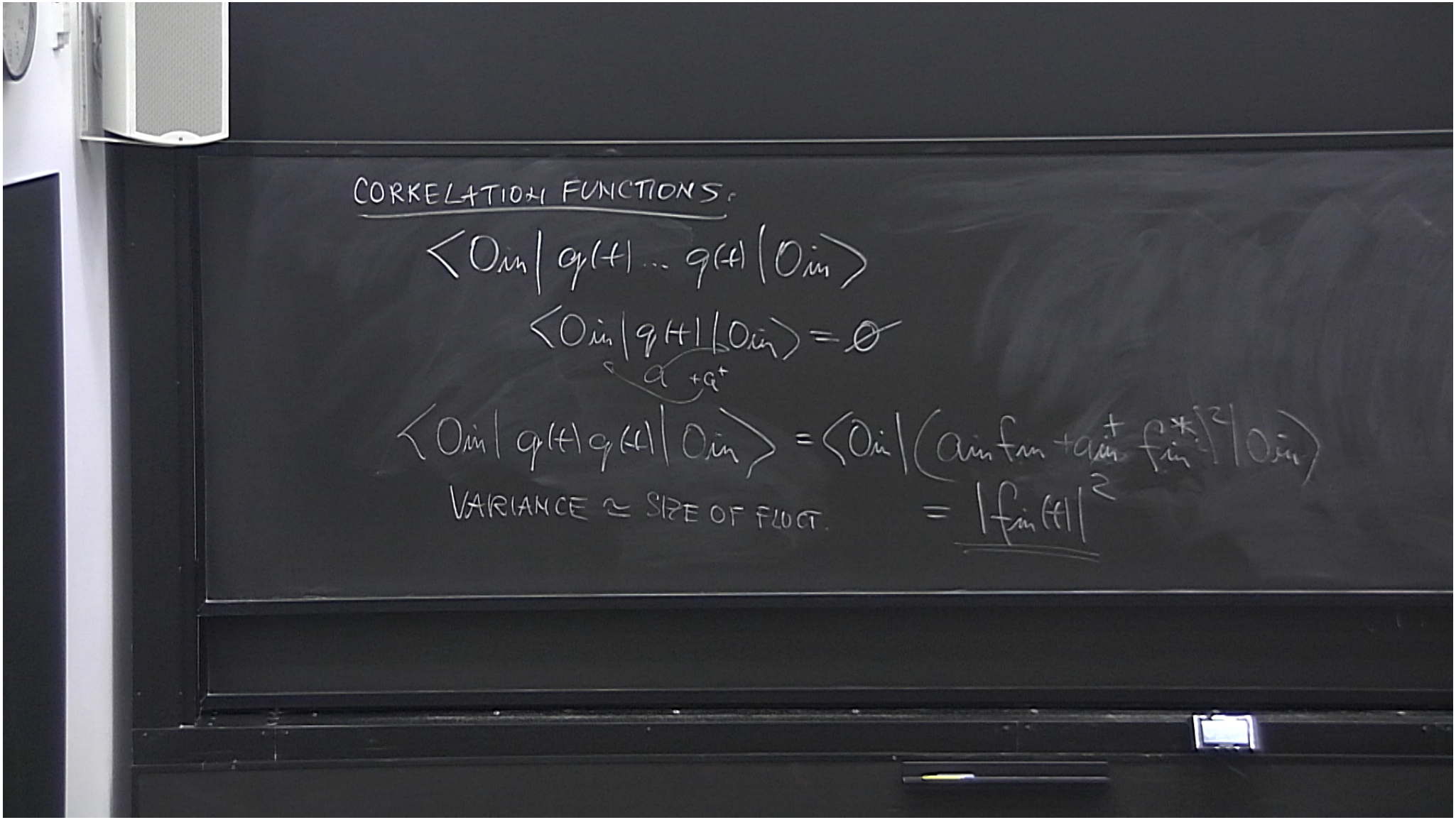
IN-MODE: $\lim_{t\to-\infty} f_{in}(t) = \frac{1}{\sqrt{2\omega_{in}}} e^{-i\omega_{in}t}$

OUT-MODE: $\lim_{t\to+\infty} f_{OUT}(t) = \frac{1}{\sqrt{2\omega_{OUT}}} e^{-i\omega_{OUT}t}$

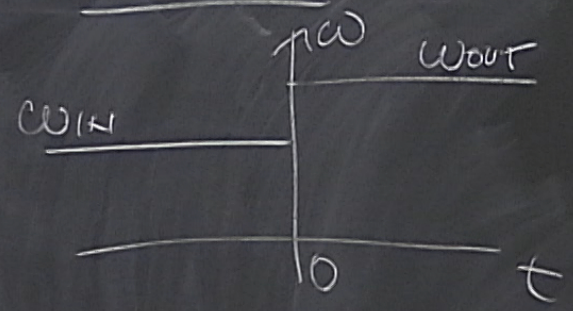
$$f_{OUT} = \alpha f_{in} + \beta f_{in}^*$$

BOGOLUBOV ≃ "S-MATRIX"


IN-VACUUM ... EMPTY IN THE PAST
$a_{in}|0_{in}\rangle = 0$
IN FUTURE WE HAVE
$N = \langle 0_{in}| N_{out} |0_{in}\rangle = |\beta|^2$
PARTICLE CREATION BY
TIME DEP. $H(t)$.
$e^{-i\omega_{in}t}$
$e^{-i\omega_{out}t}$
S-MATRIX


CORRELATION FUNCTIONS:
$\langle 0_{in}|\, q(t)\ldots q(t)\,|0_{in}\rangle$
$\langle 0_{in}|\, q(t)\,|0_{in}\rangle = 0$
$a + a^\dagger$
$\langle 0_{in}|\, q(t)q(t)\,|0_{in}\rangle = \langle 0_{in}|\,(a_{in} f_{in} + a_{in}^\dagger f_{in}^*)^2\,|0_{in}\rangle$
VARIANCE $\simeq$ SIZE OF FLUCT.
$= |f_{in}(t)|^2$

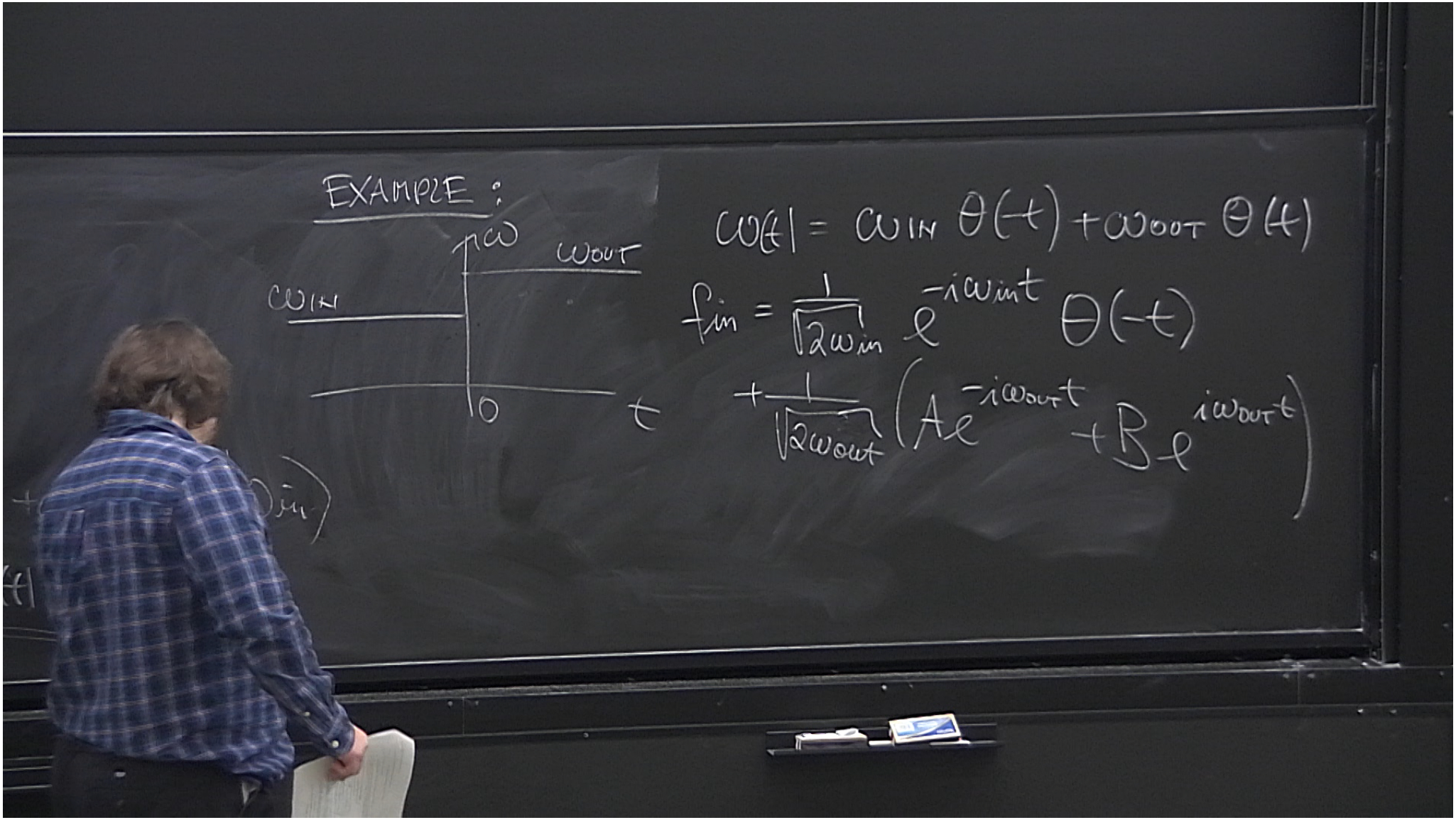
EXAMPLE :



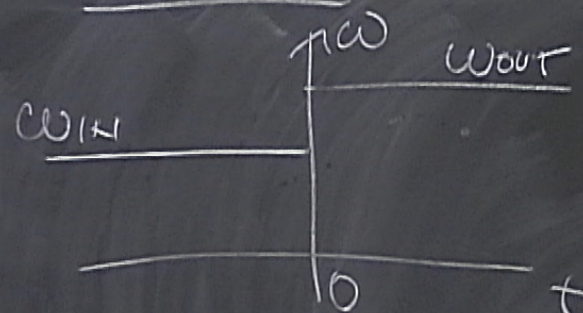
$$\omega(t) = \omega_{IN}\,\theta(-t) + \omega_{OUT}\,\theta(t)$$

$\ldots + a_{in}^{\dagger} f_{in}^{*}|^2 / O_{in})$

$|\ldots_{in}(t)|^2$


EXAMPLE :
ω
ω_OUT
ω_IN
0
t
ω(t) = ω_IN θ(−t) + ω_OUT θ(t)
f_in = 1/√(2ω_in) e^{−iω_in t} θ(−t)
+ 1/√(2ω_out) (A e^{−iω_out t} + B e^{iω_out t})

EXAMPLE:



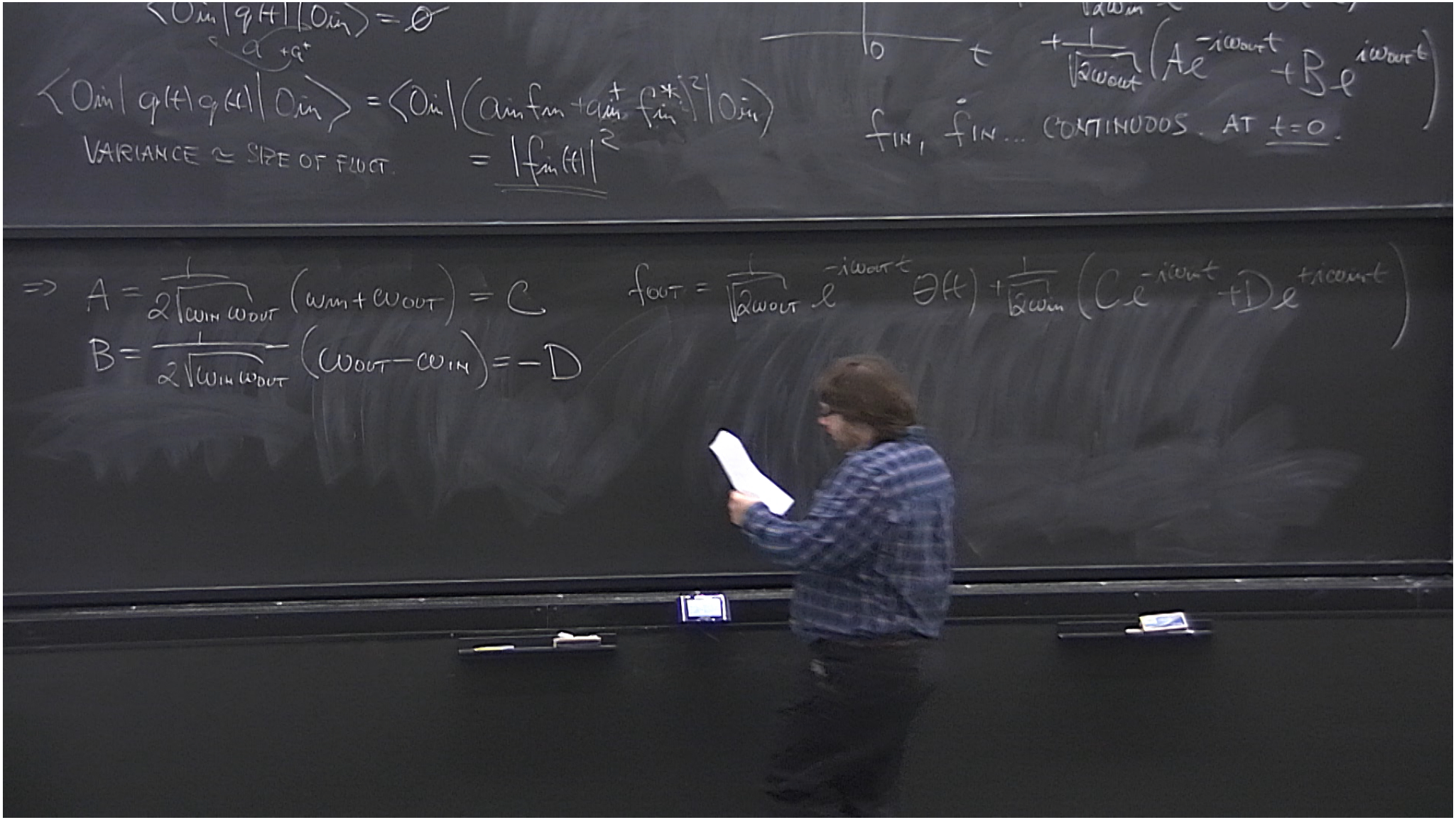
$$\omega(t) = \omega_{IN}\,\theta(-t) + \omega_{OUT}\,\theta(t)$$

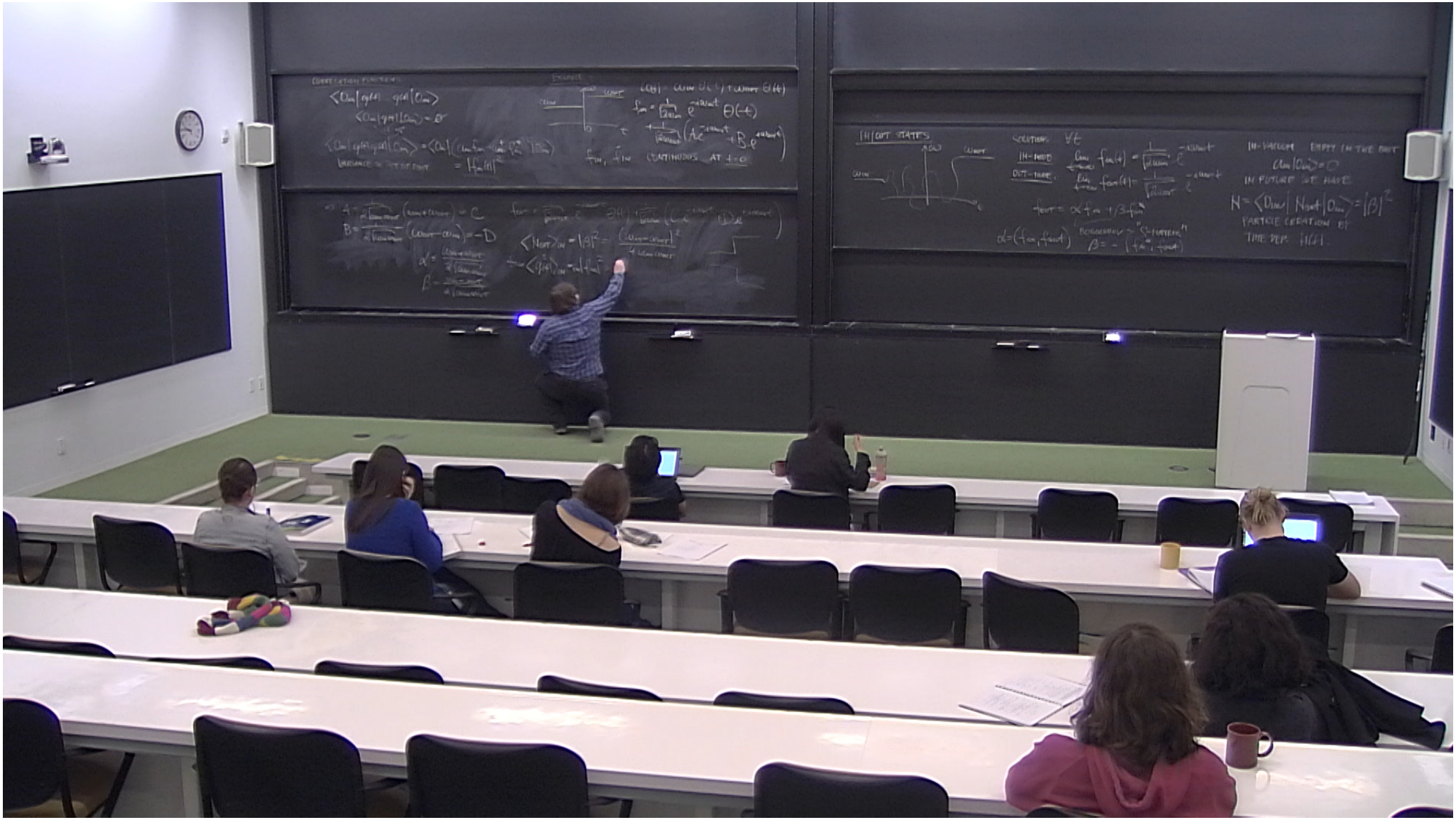
$$f_{in} = \frac{1}{\sqrt{2\omega_{in}}} e^{-i\omega_{in}t}\,\theta(-t) + \frac{1}{\sqrt{2\omega_{out}}}\left(A e^{-i\omega_{out}t} + B e^{i\omega_{out}t}\right)$$

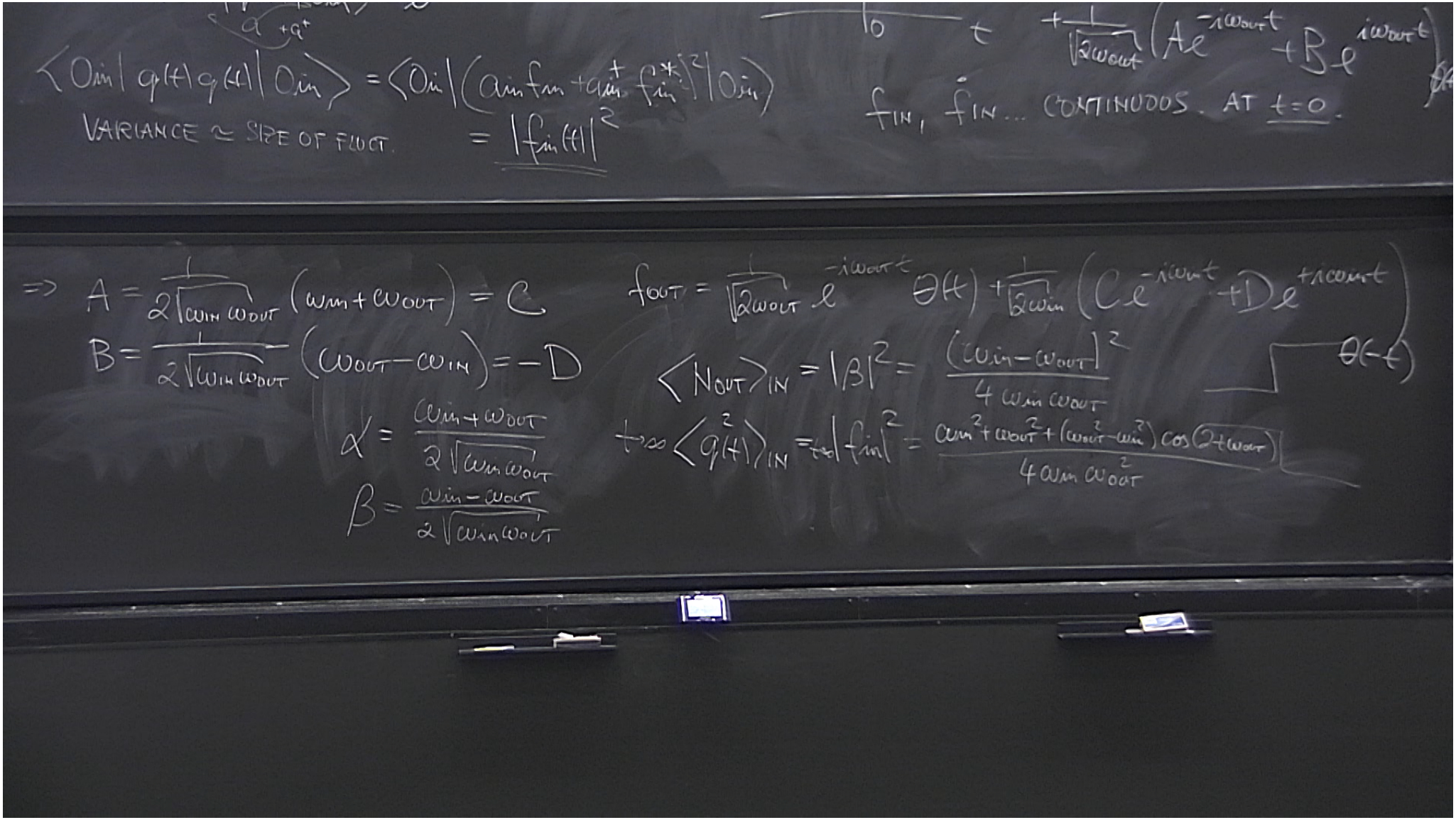
f_{IN}, $\dot{f}_{IN}$ ... CONTINUOUS AT $t=0$.

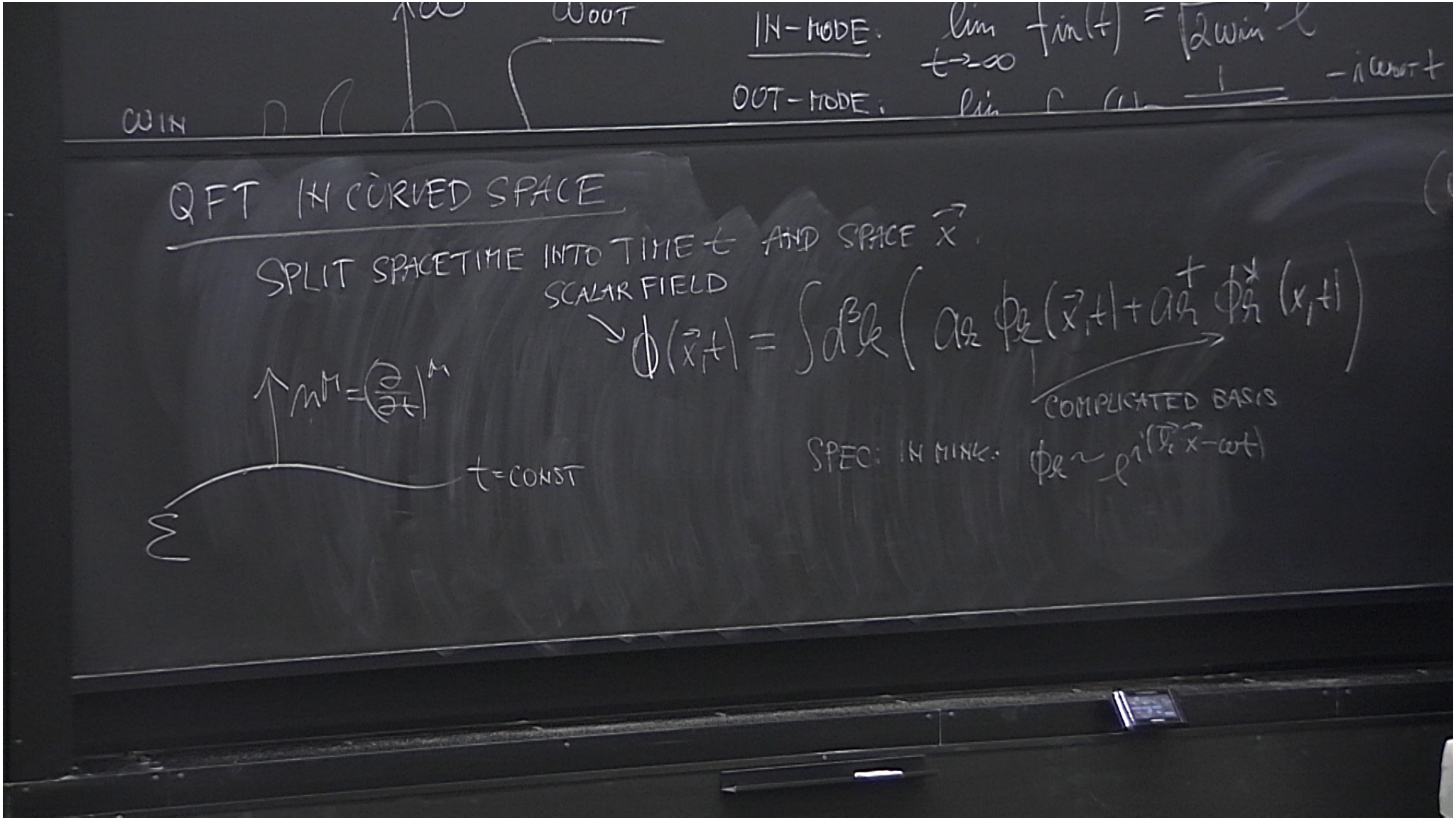
$$\frac{\dot{f}_{in}^{*}}{2} f_{in}^{*}|^2 / \Omega_{in})$$

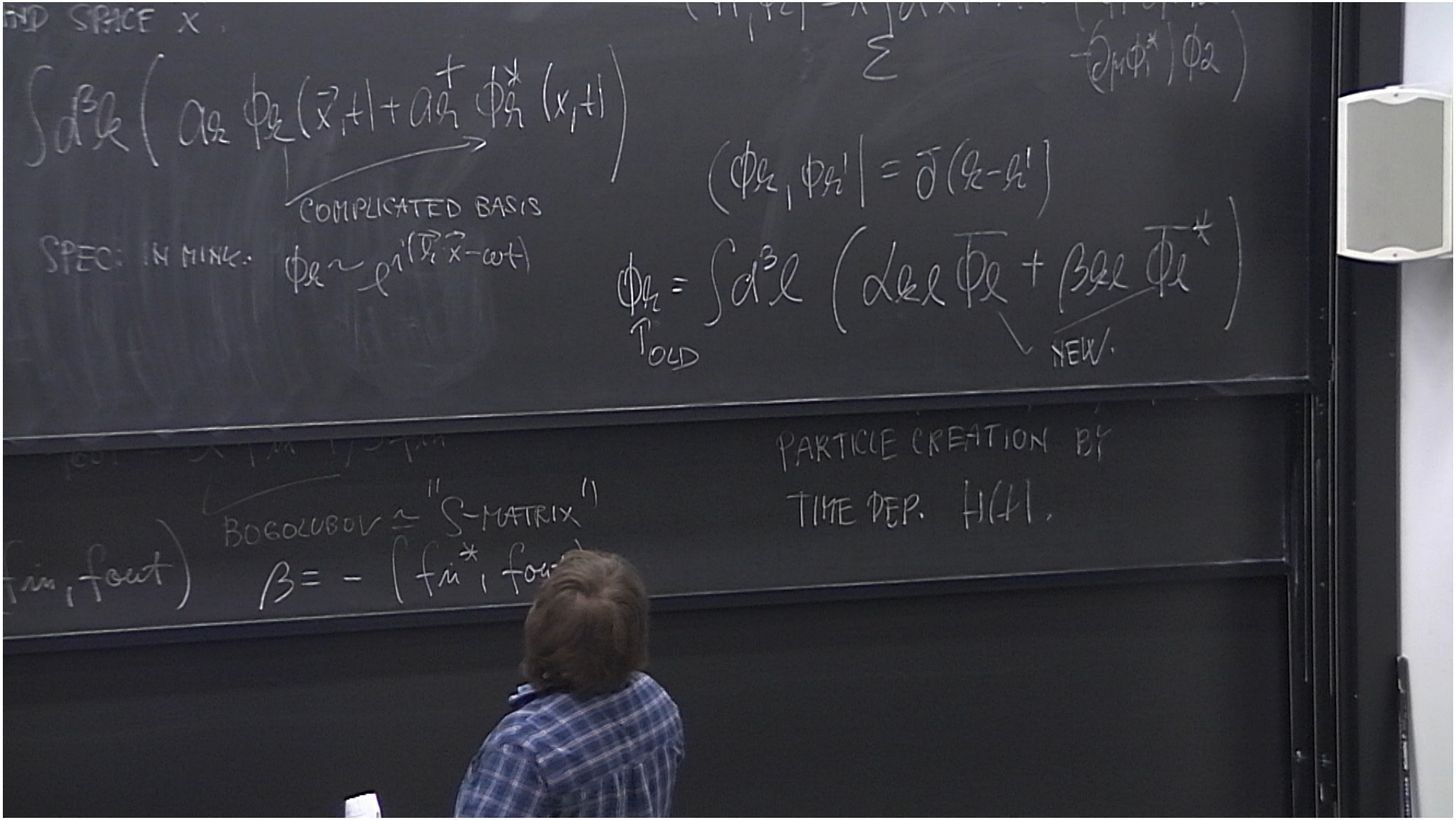
$$-2\Big[\left(\dot{f}^2+\omega^2 f^2\right) a_f a_f + \left(\dot{f}^{*2}+\omega^2 f^{*2}\right) a_f^{+} a_f^{+} + \left(|\dot{f}|^2+\omega^2|f|^2\right)\left(a_f a_f^{+} + a_f^{+} a_f\right)$$

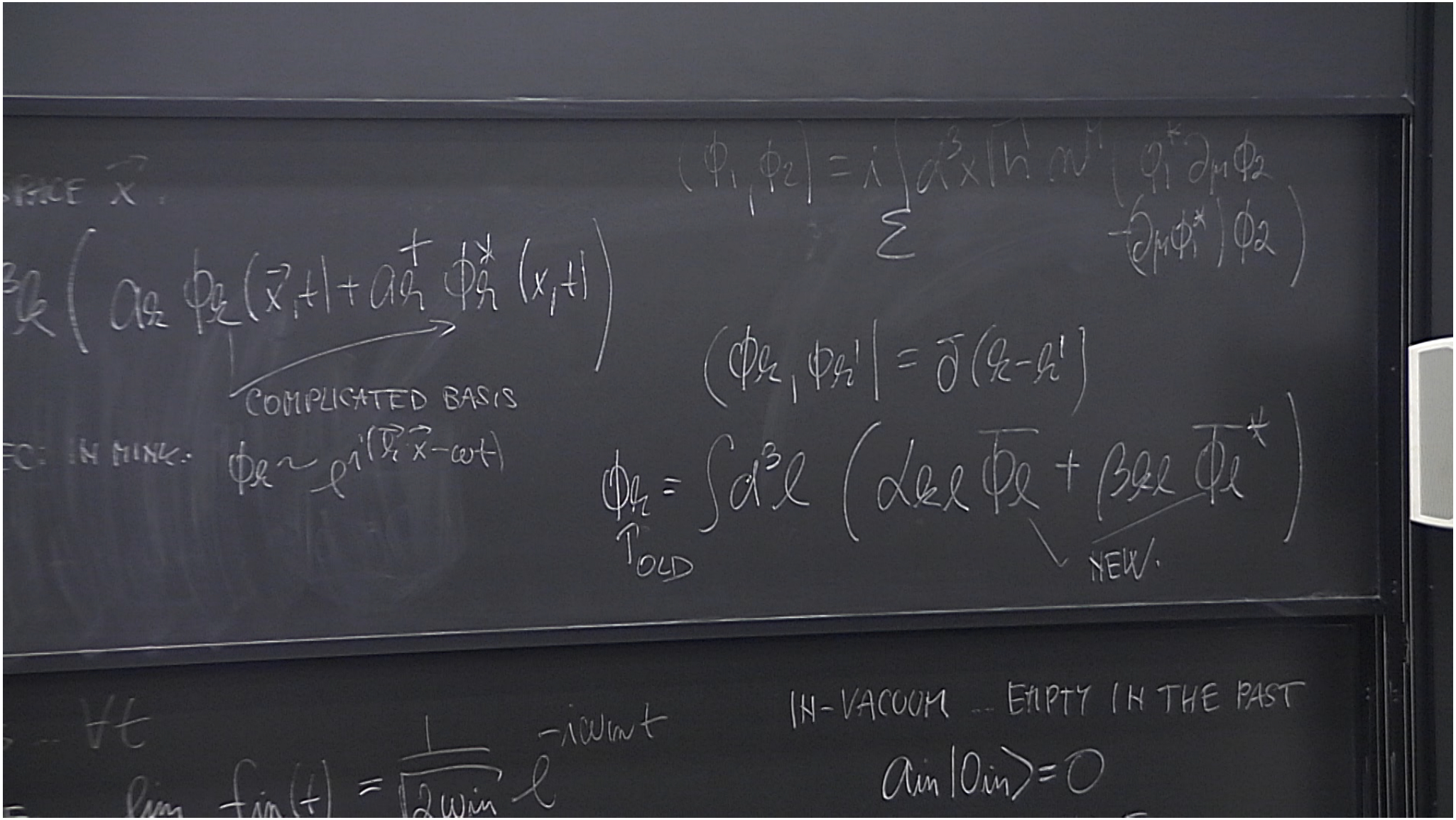

$\langle 0_{in}|q(t)|0_{in}\rangle = 0$
$a + a^\dagger$
$\langle 0_{in}|q(t)q(t)|0_{in}\rangle = \langle 0_{in}|(a_{in}f_{in} + a_{in}^\dagger f_{in}^*)^2|0_{in}\rangle$
VARIANCE ≃ SIZE OF FLUCT. $= |f_{in}(t)|^2$
f_{IN}, $\dot{f}_{IN}$ ... CONTINUOUS AT $t=0$.
$\Rightarrow A = \frac{1}{2\sqrt{\omega_{IN}\omega_{OUT}}}(\omega_{IN}+\omega_{OUT}) = C$
$B = \frac{1}{2\sqrt{\omega_{IN}\omega_{OUT}}}(\omega_{OUT}-\omega_{IN}) = -D$
$f_{OUT} = \frac{1}{\sqrt{2\omega_{OUT}}}e^{-i\omega_{OUT}t}\theta(t) + \frac{1}{\sqrt{2\omega_{in}}}(Ce^{-i\omega_{in}t} + De^{+i\omega_{in}t})$

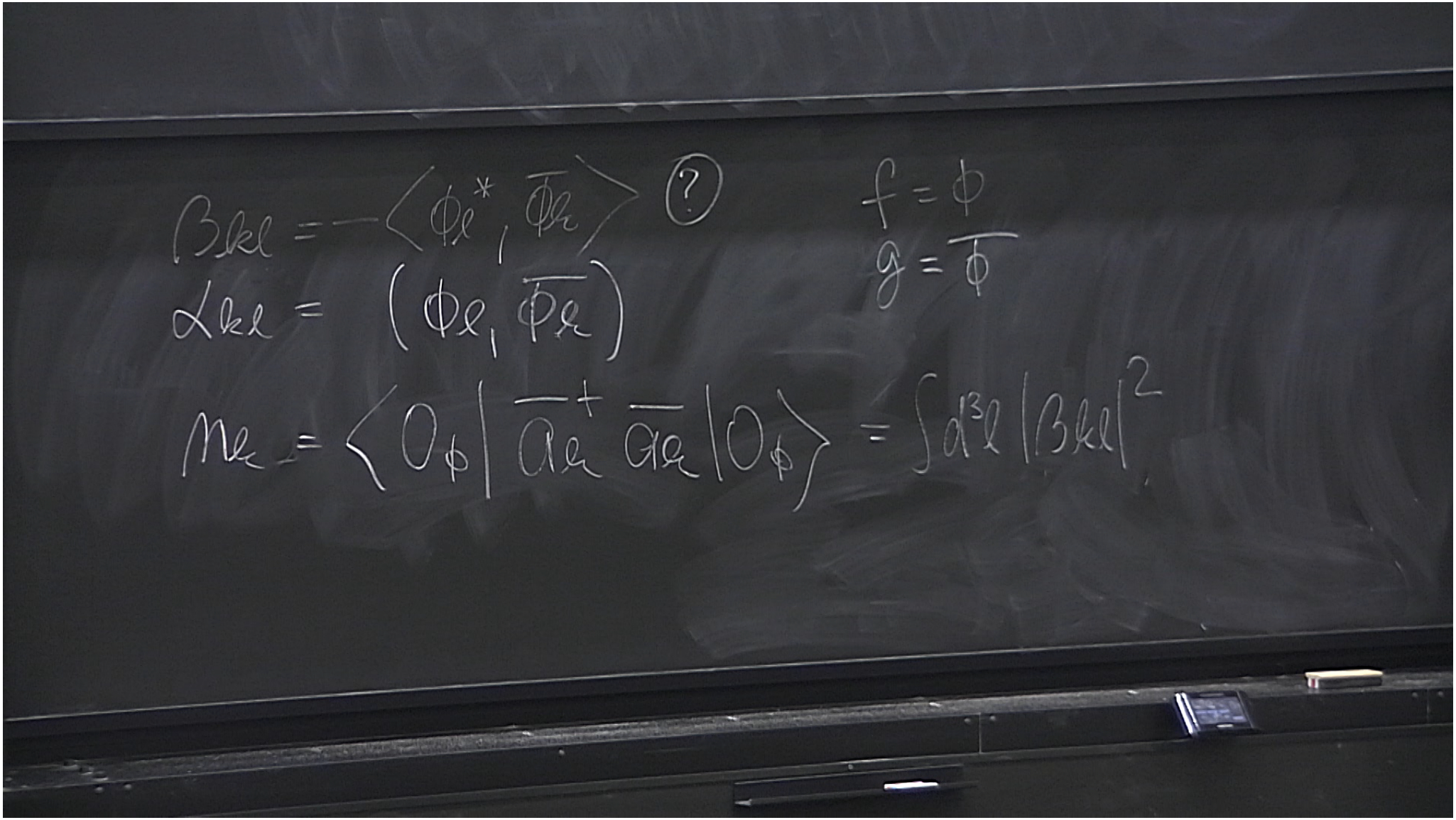



$\langle 0_{in}|\, q(t)q(t)\,|0_{in}\rangle = \langle 0_{in}|\left(a_{in} f_{in} + a_{in}^{\dagger} f_{in}^{*}\right)^2|0_{in}\rangle$
VARIANCE ≃ SIZE OF FLUCT.
$= |f_{in}(t)|^2$
f_{IN}, $\dot{f}_{IN}$ ... CONTINUOUS AT $t=0$.
$\Rightarrow A = \frac{1}{2\sqrt{\omega_{IN}\omega_{OUT}}}(\omega_{IN}+\omega_{OUT}) = C$
$B = \frac{1}{2\sqrt{\omega_{IN}\omega_{OUT}}}(\omega_{OUT}-\omega_{IN}) = -D$
$\alpha = \frac{\omega_{in}+\omega_{OUT}}{2\sqrt{\omega_{in}\omega_{OUT}}}$
$\beta = \frac{\omega_{in}-\omega_{OUT}}{2\sqrt{\omega_{in}\omega_{OUT}}}$
$\langle N_{OUT}\rangle_{IN} = |\beta|^2 = \frac{(\omega_{in}-\omega_{OUT})^2}{4\omega_{in}\omega_{OUT}}$
$t\to\infty \quad \langle q^2(t)\rangle_{IN} = |f_{in}|^2 = \frac{\omega_{in}^2+\omega_{OUT}^2+(\omega_{OUT}^2-\omega_{in}^2)\cos(2t\omega_{OUT})}{4\omega_{in}\omega_{OUT}^2}$

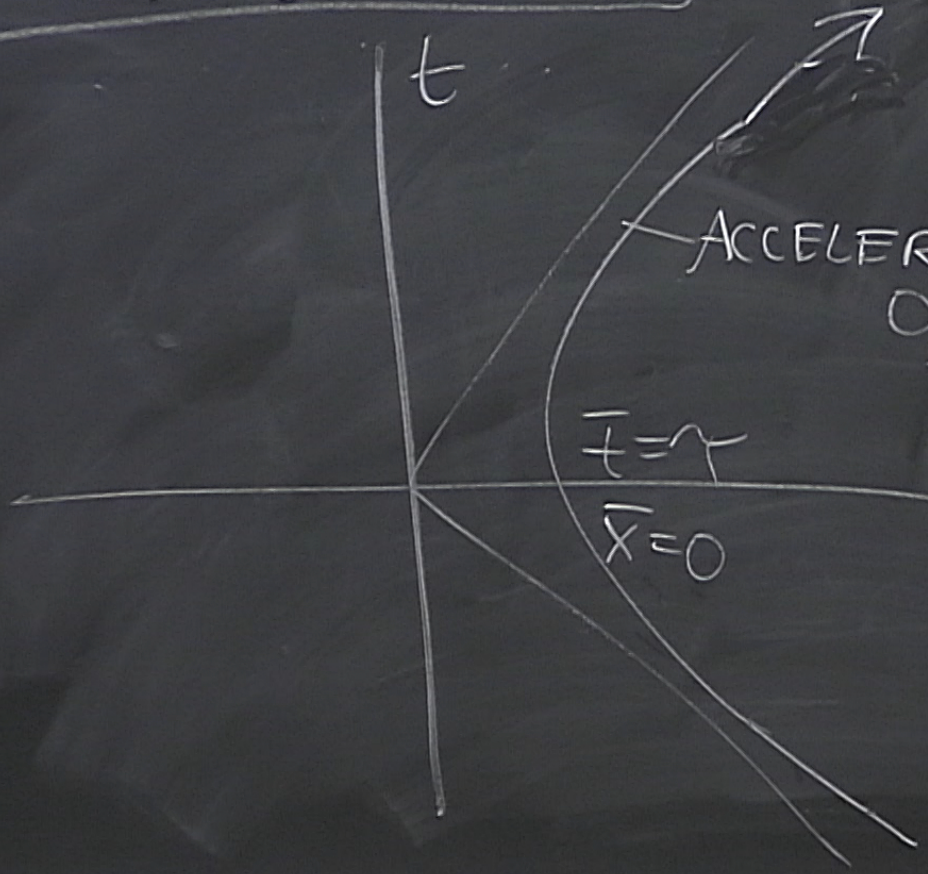

ω_{IN}
ω_{OUT}
IN-MODE: $\lim_{t\to-\infty} f_{in}(t) = \frac{1}{\sqrt{2\omega_{in}}} e^{-i\omega_{in}t}$
OUT-MODE: $\lim$
QFT IN CURVED SPACE
SPLIT SPACETIME INTO TIME t AND SPACE $\vec{x}$.
SCALAR FIELD
$\hat{\phi}(\vec{x},t) = \int d^3k \left(a_k \phi_k(\vec{x},t) + a_k^\dagger \phi_k^*(x,t) \right)$
COMPLICATED BASIS
$n^\mu = \left(\frac{\partial}{\partial t}\right)^\mu$
t=CONST
SPEC: IN MINK.: $\phi_k \sim e^{i(\vec{k}\cdot\vec{x}-\omega t)}$


ND SPACE X .
$\int d^3k \left(a_k \phi_k(\vec{x},t) + a_k^\dagger \phi_k^*(x,t) \right)$
COMPLICATED BASIS
SPEC: IN MINK.: $\phi_k \sim e^{i(\vec{k}\cdot\vec{x}-\omega t)}$
$(\phi_k, \phi_{k'}) = \delta(k-k')$
$\phi_k = \int d^3\ell \left(\alpha_{k\ell} \phi_\ell + \beta_{k\ell} \phi_\ell^* \right)$
OLD
NEW.
PARTICLE CREATION BY
TIME DEP. $H(t)$.
BOGOLUBOV $\simeq$ "S-MATRIX"
$\beta = -$


$(\phi_1, \phi_2) = i \int_\Sigma d^3x \sqrt{h}\, n^\mu \left(\phi_1^* \partial_\mu \phi_2 - (\partial_\mu \phi_1^*)\, \phi_2\right)$
$(\phi_{k_2}, \phi_{k_1}) = \delta(k_2 - k_1)$
$\phi_k = \int d^3l \left(\alpha_{kl}\, \phi_l + \beta_{kl}\, \phi_l^*\right)$
OLD
NEW.
COMPLICATED BASIS
$\phi_k \sim e^{i(\vec{k}\cdot\vec{x} - \omega t)}$
IN-VACUUM ... EMPTY IN THE PAST
$a_{in}|0_{in}\rangle = 0$
$f_{in}(t) = \frac{1}{\sqrt{2\omega_{in}}} e^{-i\omega_{in} t}$


$B_{kl} = -\langle \phi_l^*, \bar{\phi}_k \rangle$?
$\alpha_{kl} = (\phi_l, \bar{\phi}_k)$
$f = \phi$
$g = \bar{\phi}$
$N_k = \langle 0_\phi | \bar{a}_k^+ \bar{a}_k | 0_\phi \rangle = \int d^3l \, |B_{kl}|^2$

UNRUH EFFECT



$t =$

$x =$

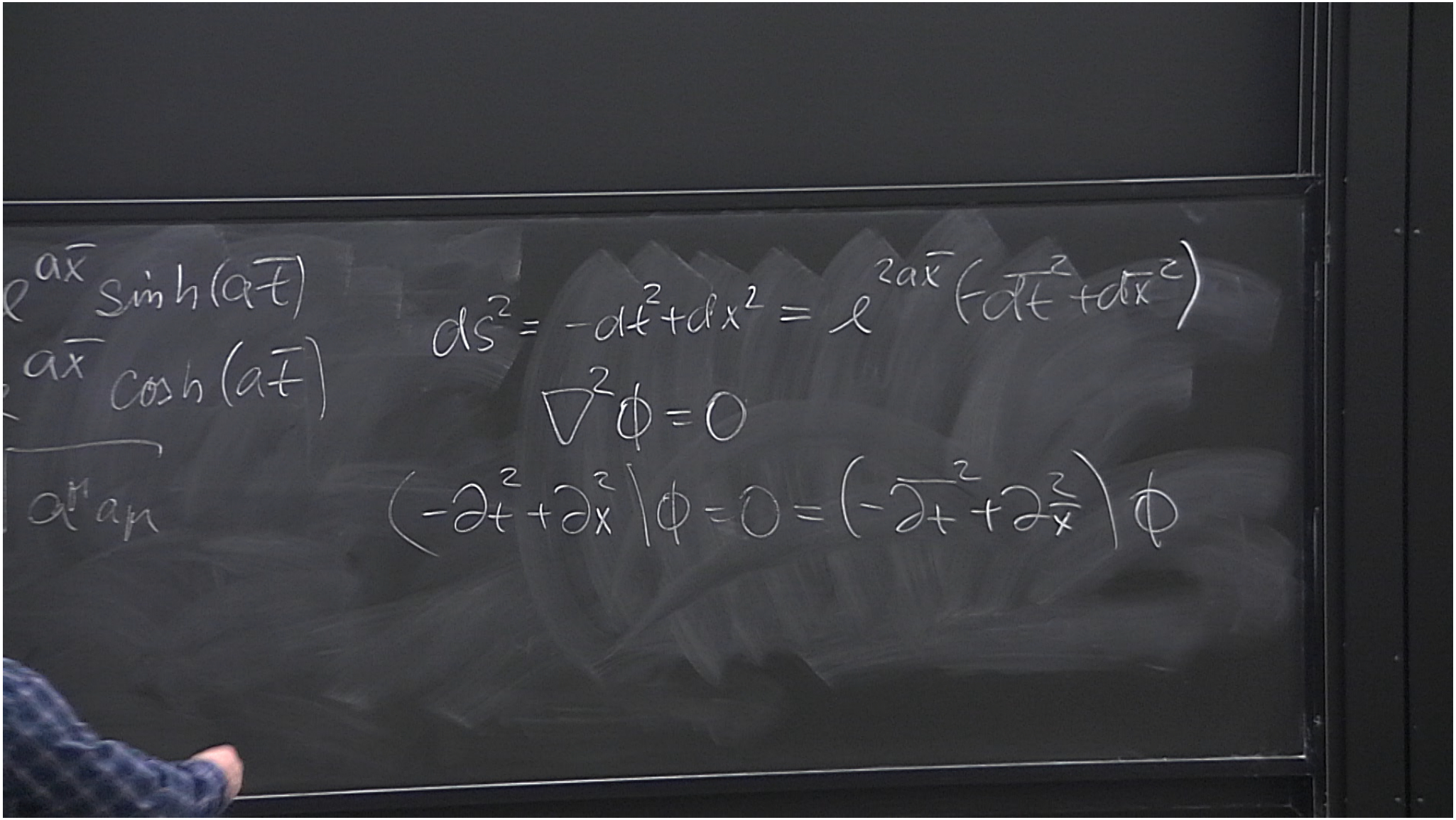
$\bar{t}, \bar{x}$

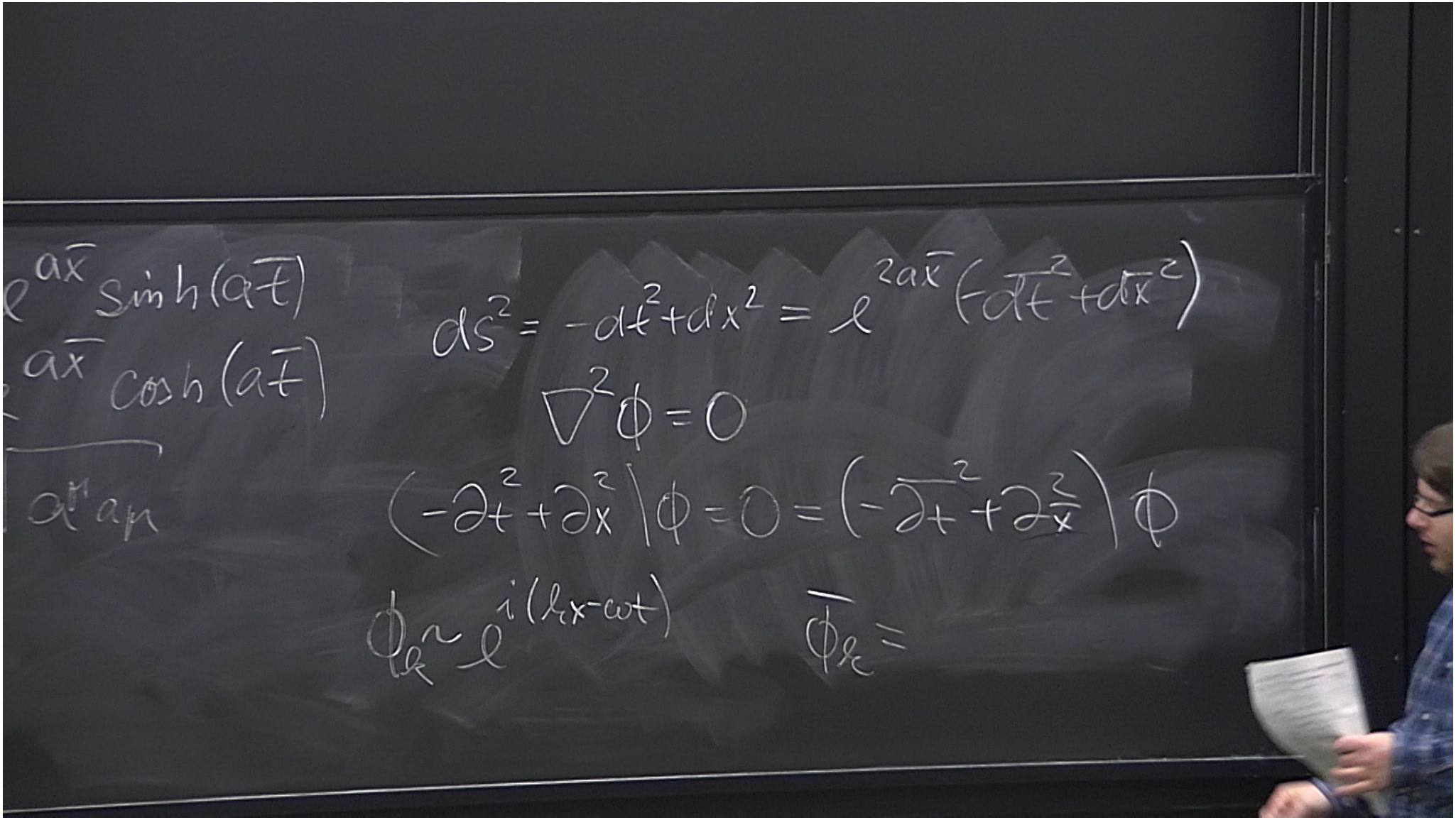
$$t = \frac{1}{a} e^{a\bar{x}} \sinh(a\bar{\tau})$$

$$x = \frac{1}{a} e^{a\bar{x}} \cosh(a\bar{\tau})$$

$$a = \sqrt{a^\mu a_\mu}$$

$\xi^\mu \bar{x}$


$e^{a\bar{x}}\sinh(a\bar{t})$
$e^{a\bar{x}}\cosh(a\bar{t})$
$a^m a_n$
$ds^2 = -dt^2 + dx^2 = e^{2a\bar{x}}(-d\bar{t}^2 + d\bar{x}^2)$
$\nabla^2\phi = 0$
$(-\partial_t^2 + \partial_x^2)\phi = 0 = (-\partial_{\bar{t}}^2 + \partial_{\bar{x}}^2)\phi$


$e^{a\bar{x}}\sinh(a\bar{t})$
$e^{a\bar{x}}\cosh(a\bar{t})$
$a^\mu a_\mu$
$ds^2 = -d\bar{t}^2 + d\bar{x}^2 = e^{2a\bar{x}}(-d\bar{t}^2 + d\bar{x}^2)$
$\nabla^2\phi = 0$
$(-\partial_{\bar{t}}^2 + \partial_{\bar{x}}^2)\phi = 0 = (-\partial_{\bar{t}}^2 + \partial_{\bar{x}}^2)\phi$
$\phi_k \sim e^{i(kx-\omega t)}$
$\bar{\phi}_k =$

$e^{a\bar{x}}\sinh(a\bar{t})$

$e^{a\bar{x}}\cosh(a\bar{t})$

$a^n a_n$

$$ds^2 = -dt^2 + dx^2 = e^{2a\bar{x}}(-d\bar{t}^2 + d\bar{x}^2)$$

$$\nabla^2\phi = 0$$

$$(-\partial_t^2 + \partial_x^2)\phi = 0 = (-\partial_{\bar{t}}^2 + \partial_{\bar{x}}^2)\phi$$

$$\phi_k \sim e^{i(kx-\omega t)} \qquad \bar{\phi}_k = e^{i(k\bar{x}-\omega\bar{t})}$$

$e^{a\bar{x}} \sinh(a\bar{t})$

$e^{a\bar{x}} \cosh(a\bar{t})$

$ds^2 = -dt^2 + dx^2 = e^{2a\bar{x}}(-d\bar{t}^2 + d\bar{x}^2)$

$\nabla^2 \phi = 0$

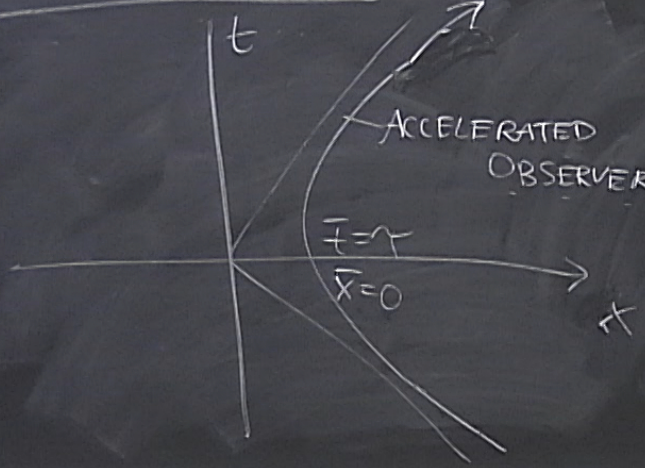
$(-\partial_t^2 + \partial_x^2)\phi = 0 = (-\partial_{\bar{t}}^2 + \partial_{\bar{x}}^2)\phi$

$\phi_k \sim e^{i(kx - \omega t)}$ $\quad \bar{\phi}_k = e^{i(k\bar{x} - \omega \bar{t})}$

$= \int_0^\infty dk \, |B_k|^2 \phi_k$ $\quad \frac{1}{e^{\frac{2\pi k}{a}} - 1}$ $\quad$ BOSE-EINSTEIN DIST.

$\bar{\phi}$ COMPLICATED

UNRUH EFFECT



$\bar{t}, \bar{x}$

$$t = \frac{1}{a} e^{a\bar{x}} \sinh(a\bar{t})$$

$$x = \frac{1}{a} e^{a\bar{x}} \cosh(a\bar{t})$$

$$a = \sqrt{a^\mu a_\mu}$$

$$T = \frac{a}{2\pi}$$

$$ds^2 = -dt^2 + dx^2 =$$

$$\nabla^2 \phi = 0$$

$$(-\partial_t^2 + \partial_x^2)\phi = 0 =$$

$$n_k = \langle 0_M | \bar{a}_k^\dagger \bar{a}_k | 0_M \rangle = \int_0^\infty d\omega \, |\beta_{\omega k}|^2$$

$$\phi_k \sim e^{i(kx - \omega t)}$$

COMPLICATED

$$\frac{1}{e^{\frac{2\pi\omega}{a}} - 1}$$

$$\beta = \frac{2\sqrt{\omega_{in}\omega_{out}}}{\omega_{in} - \omega_{out}}$$

$$\langle \psi(t) | N \quad \quad 4\omega_{in}\omega_{out}^2$$