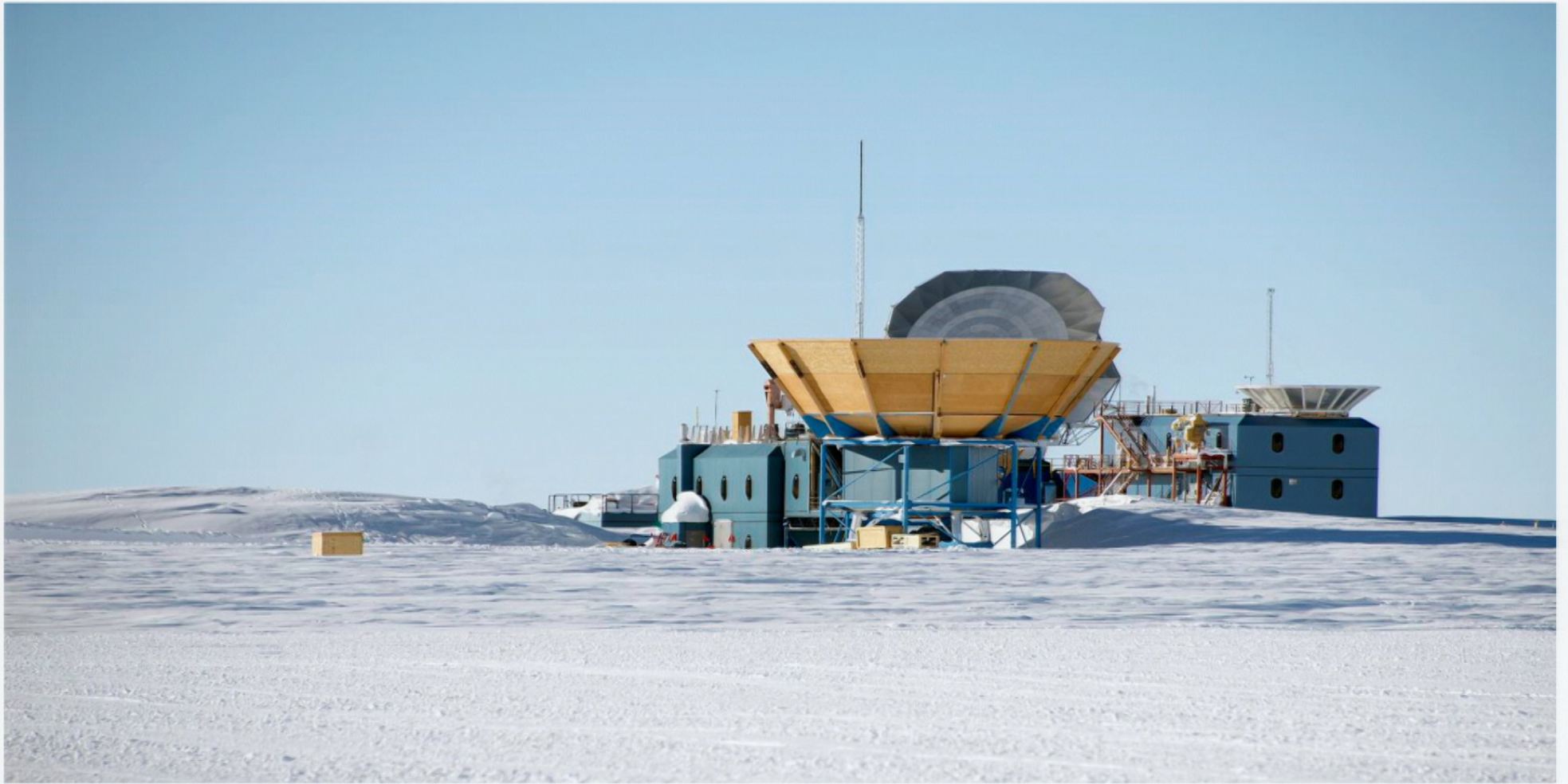


Title: PSI 17/18 - Cosmology - Lecture 12

Date: Feb 13, 2018 09:00 AM

URL: <http://pirsa.org/18020021>

Abstract:



BICEP 2

KECK:



YESTERDAY: CMB ENCODES INFO:

TEMPERATURE: $\delta T = \sum_{l,m} a_{lm}^T Y_{lm}(\hat{n})$

POLARIZATION: $f_{ij} = \sum_{l,m} a_{lm}^E Y_{ij,lm}^E + \sum_{l,m} a_{lm}^B Y_{ij,lm}^B$

POWER SPECTRA (2-PT CORR. FUNCTIONS)

TENSORIAL
HARMONICS

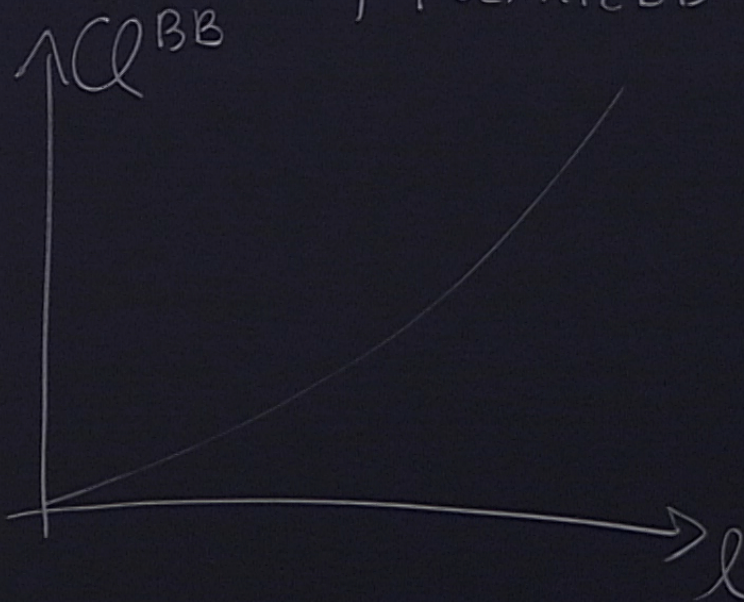
$$\langle a_{lm}^T a_{l'm'}^B \rangle = C_l^{TB} \delta_{ll'} \delta_{mm'} \stackrel{\text{PARITY}}{=} 0 = C_l^{EB}$$

B-MODES ONLY NONTRIVIAL Q^{BB}

3 SOURCES i) GRAVITY WAVES

ii) GRAVITATIONAL LENSING

iii) POLARIZED DUST



$B \vee B$
 $m \vee j, l, m$

TENSORIAL
HARMONICS

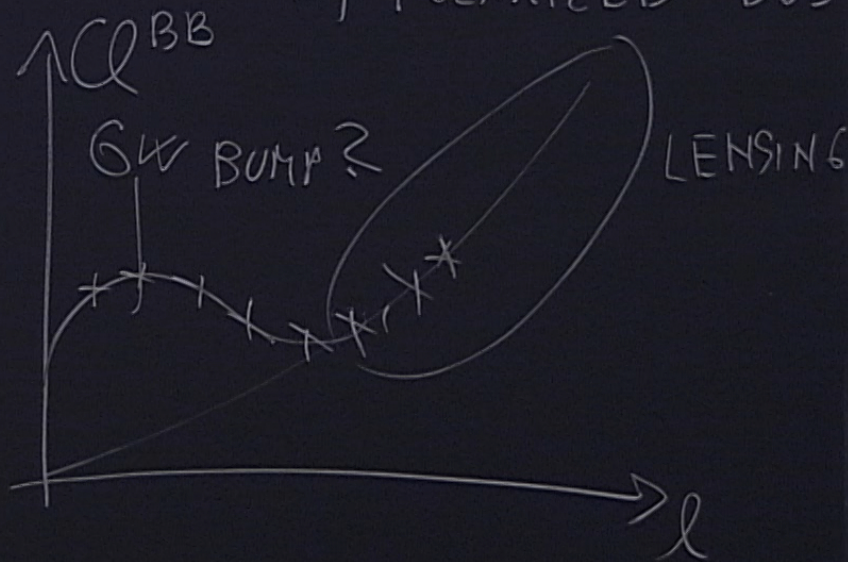
EB
 l

B-MODES ONLY NONTRIVIAL Q^{BB}

3 SOURCES i) GRAVITY WAVES

ii) GRAVITATIONAL LENSING

iii) POLARIZED DUST



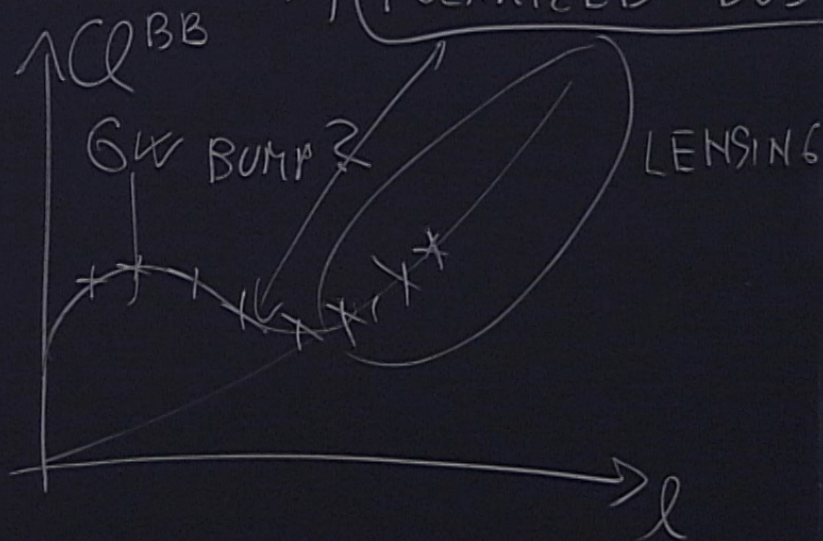
2014 - BICEP 2

B-MODES ONLY NONTRIVIAL Q^{BB}

3 SOURCES i) GRAVITY WAVES

ii) GRAVITATIONAL LENSING

iii) POLARIZED DUST



2014 - BICEP 2

DISCOVERY OF GWs?

CONTRADICTION
PLANCK!

YESTERDAY: CMB ENCODES INFO:

TEMPERATURE: $\delta T = \sum_{l,m} a_{lm}^T Y_{lm}(\hat{n})$

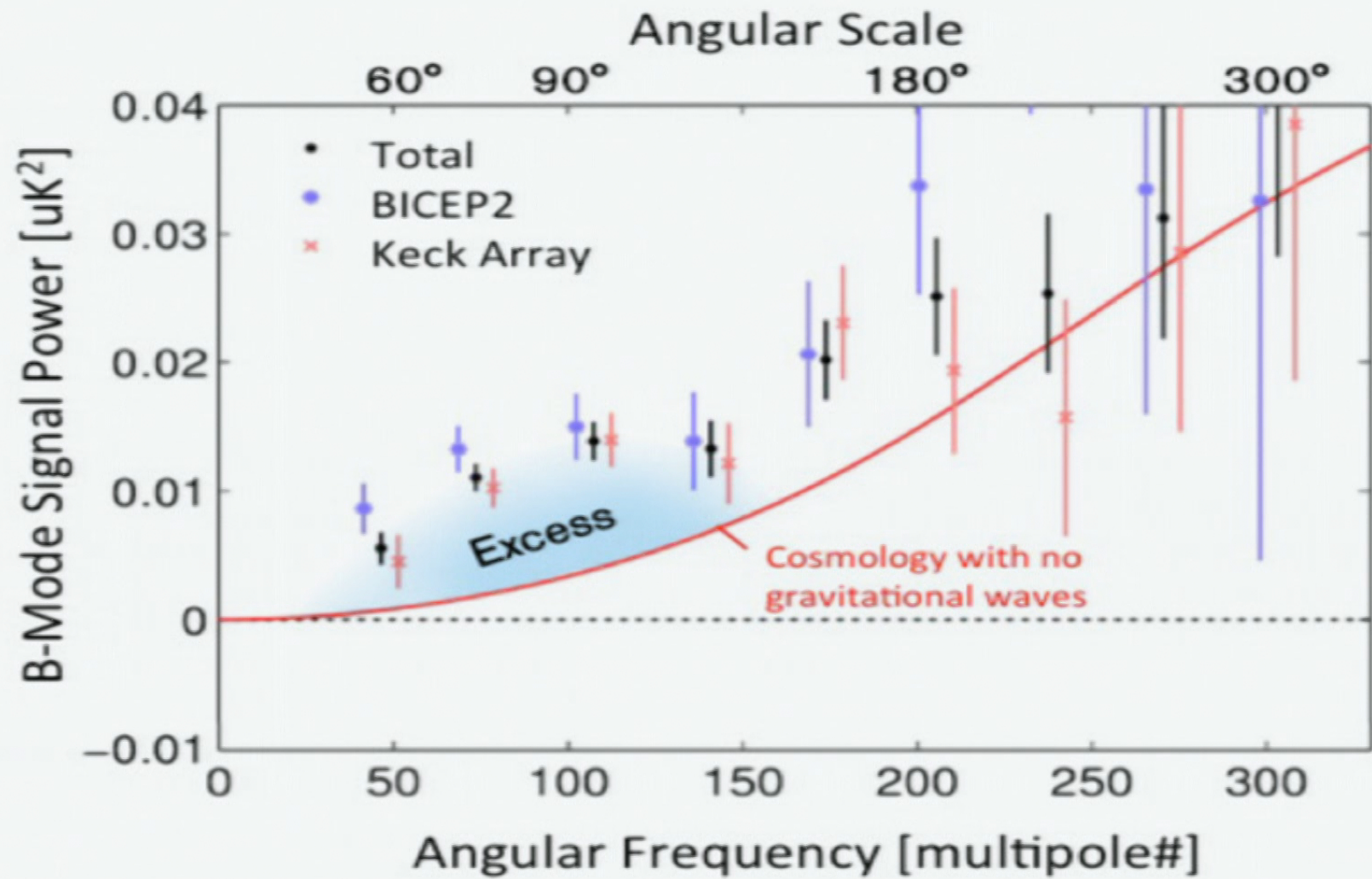
POLARIZATION: $f_{ij} = \sum_{l,m} a_{lm}^E Y_{ij,lm}^E + \sum_{l,m} a_{lm}^B Y_{ij,lm}^B$

POWER SPECTRA (2-PT CORR. FUNCTIONS)

TENSORIAL
HARMONICS

$$\langle a_{lm}^T a_{l'm'}^{*B} \rangle = C_l^{TB} \delta_{ll'} \delta_{mm'} \stackrel{\text{PARITY}}{=} 0 = C_l^{EB}$$

$$r = \frac{P_t}{P_s} = \underline{\underline{0.2 \pm 0.1}}$$



b) INFLATION: GENERATOR OF PERTURBATIONS

$\delta\phi$:

• TOY MODEL: TEST MASSLESS "INFLATON"
IN FRW WITH $K=0$.

$$ds^2 = -dt^2 + a^2 d\vec{x}^2$$

$$S = \int d^4x \sqrt{g} \left(-\frac{1}{2} (\partial\phi)^2 \right) = \int dt d^3x \frac{1}{2} a^3 \left(\dot{\phi}^2 - \frac{1}{a^2} |\nabla\phi|^2 \right)$$

$\delta\phi$:

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi = 0$$

$$\phi(\vec{k}, t) = \int d^3x \phi(x, t) e^{i\vec{k}\cdot\vec{x}}$$

$$\ddot{\phi} + 3H\dot{\phi} + \omega^2\phi = 0, \quad \omega = \frac{k}{a}$$

"FRICTION" "OSCILLAT."

$$\left(\ddot{\phi} - \frac{1}{a^2}(\nabla\phi)^2 \right)$$

$$H a$$

... COMOVING HORIZON
(COMPARE TO $\hat{\lambda} = \frac{1}{H a}$)
 k -SPACE ANALOGUE

$$k \gg H a$$

... MODE IS INSIDE HORIZON
(MODE OSCILLATES)

$$k \ll H a$$

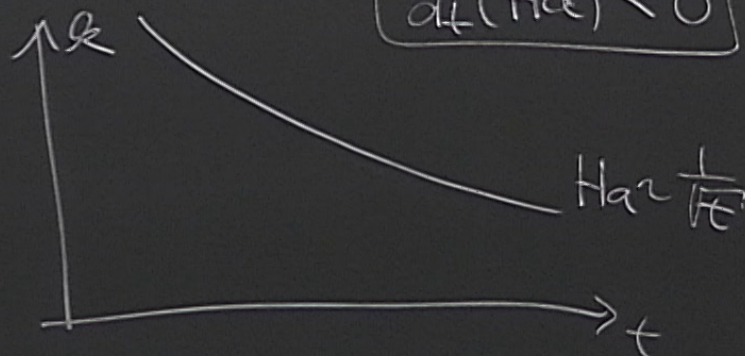
... OUTSIDE HORIZON (FROZEN
MODE)

① NON-INFLATIONARY PHASE FOR EXAMPLE RADIATION.

$$a \propto t^{1/2}, \quad H = \frac{\dot{a}}{a} \sim \frac{1}{t}, \quad \underline{Ha \sim t^{-1/2}}$$

MORE GENERALLY:

$$\boxed{\frac{d}{dt}(Ha) < 0}$$



NON

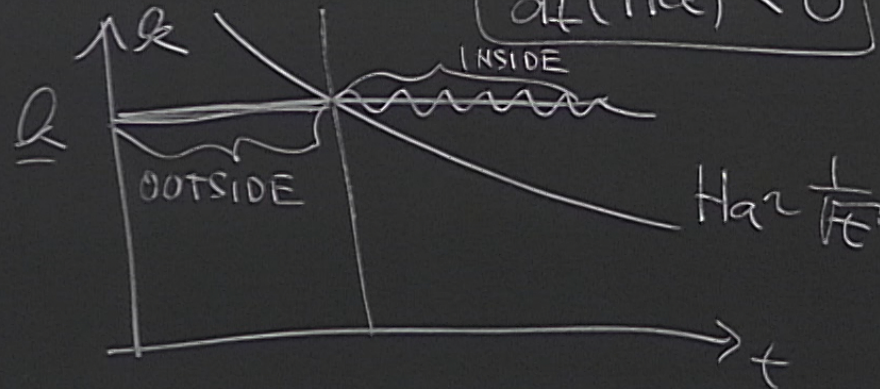
FROZEN
MODE

① NON-INFLATIONARY PHASE ... FOR EXAMPLE RADIATION

$$a \propto t^{1/2}, \quad H = \frac{\dot{a}}{a} \sim \frac{1}{t}, \quad \underline{Ha \sim t^{-1/2}}$$

MORE GENERALLY:

$$\boxed{\frac{d}{dt}(Ha) < 0}$$

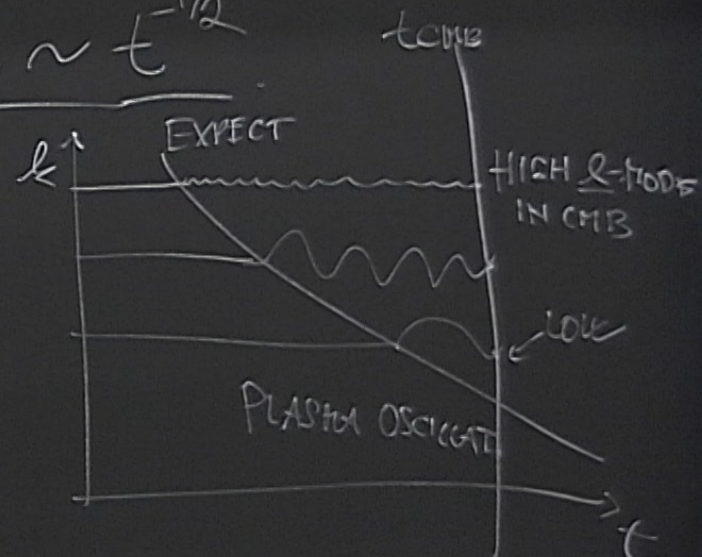
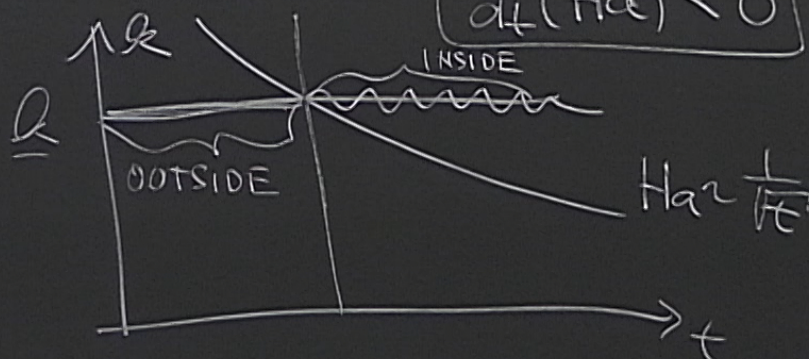


① NON-INFLATIONARY PHASE ... FOR EXAMPLE RADIATION

$$a \propto t^{1/2}, \quad H = \frac{\dot{a}}{a} \sim \frac{1}{t}, \quad H a \sim t^{-1/2}$$

MORE GENERALLY:

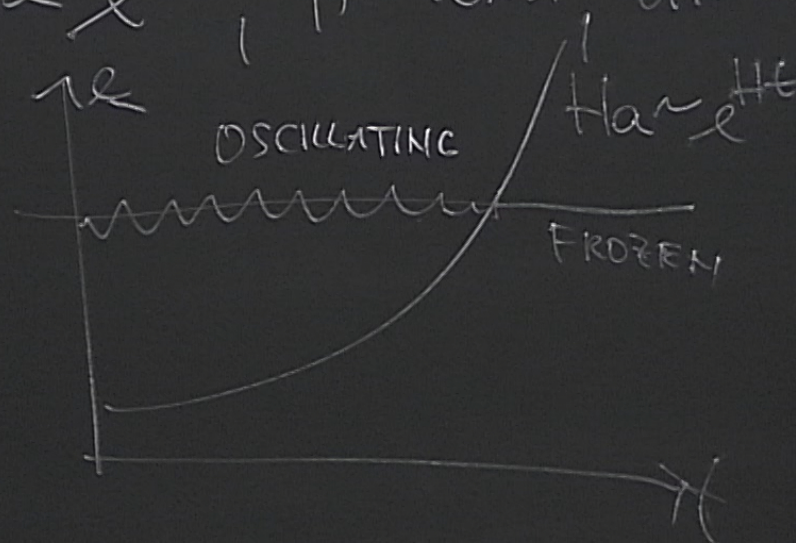
$$\boxed{\frac{d}{dt}(H a) < 0}$$

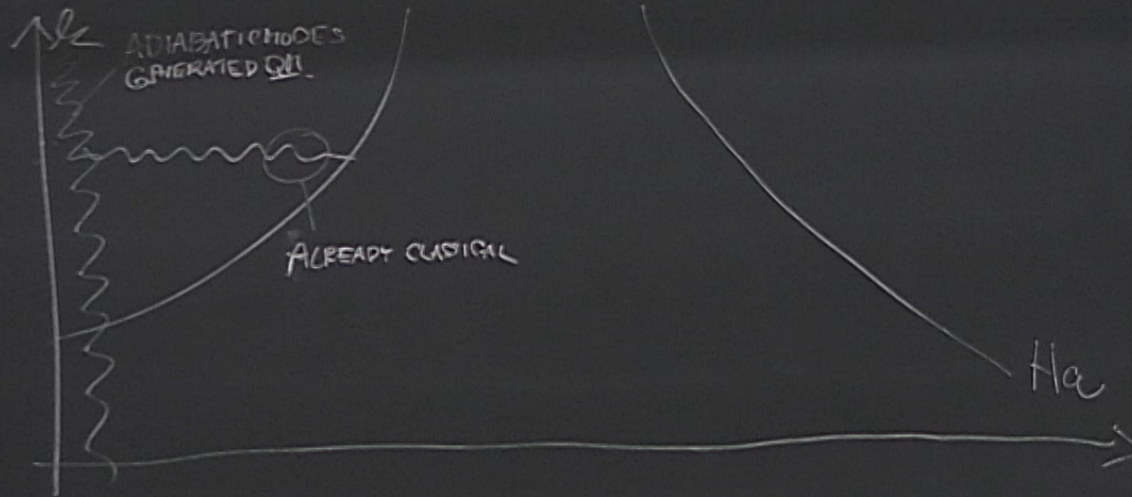


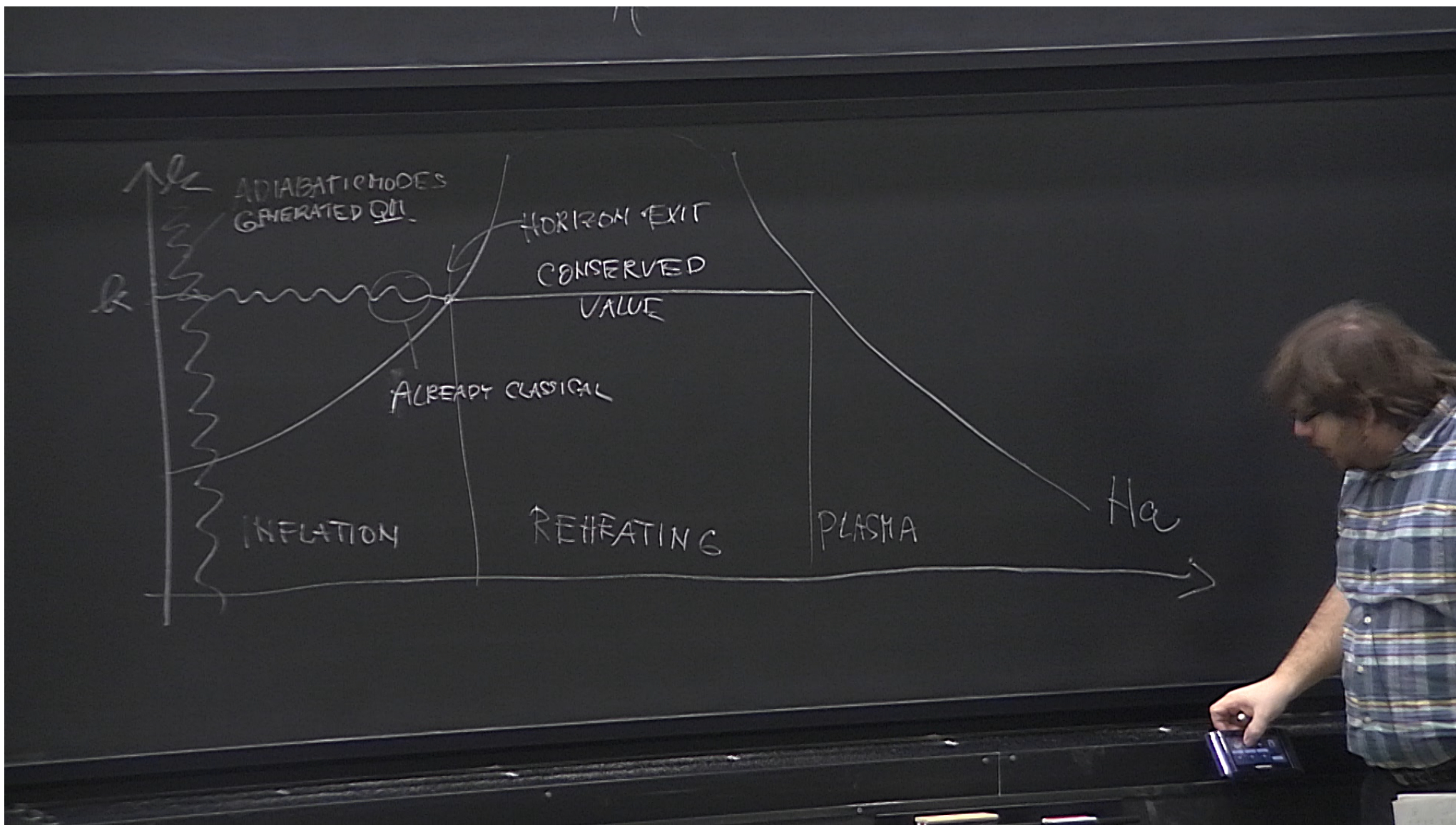
② INFLATIONARY PHASE

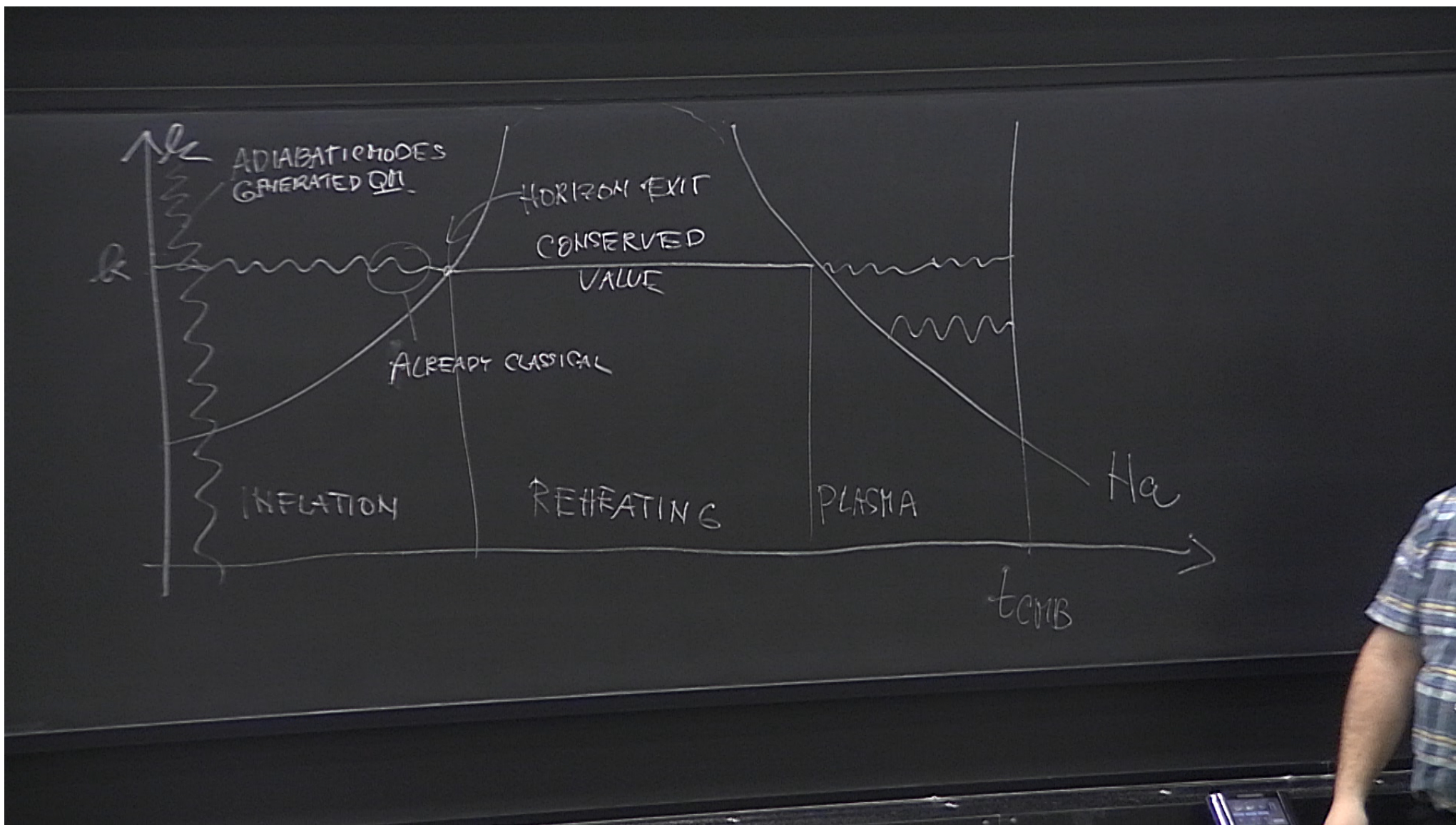
"SLOW ROLL"

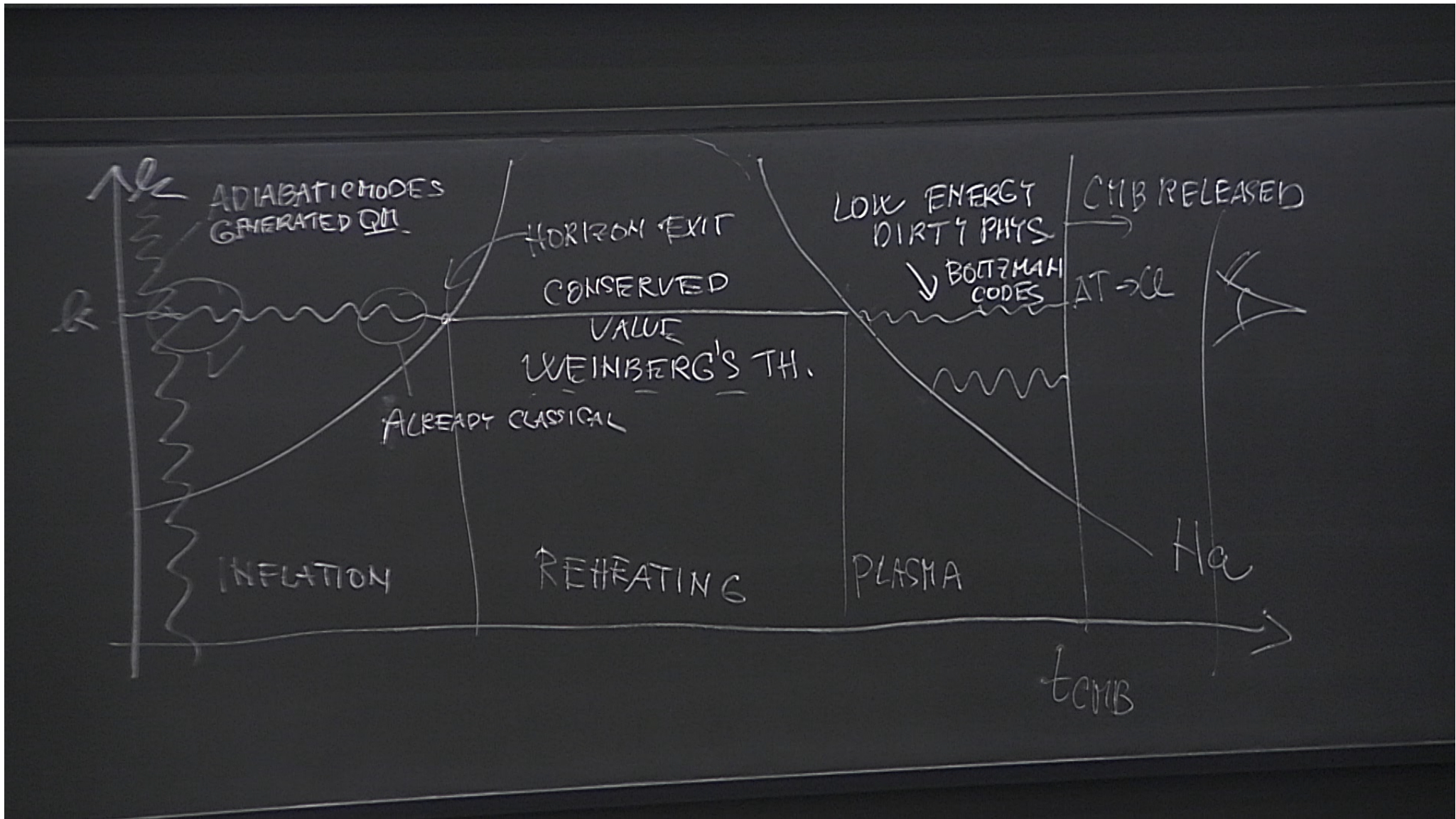
$$a \propto e^{Ht}, \quad H = \text{CONST}, \quad aH \sim e^{Ht}$$











C) BASICS OF QFT IN CURVED SPACE

• TIME DEPENDENT HARMONIC OSCILLATOR

$$L = \frac{1}{2} \dot{q}^2 - \frac{1}{2} \omega^2(t) q^2$$

$$\text{EOM: } \ddot{q} + \omega^2 q = 0$$

FOR 2 SOLUTIONS f_1 & f_2 :

$$(f_1, f_2) = f_1^* \frac{df_2}{dt} - \frac{df_1^*}{dt} f_2 \Big|_{t=0}$$

• GIVEN A SOLUTION f TO EOM: $(f, f) = 1$

$\Rightarrow f^*$ ALSO A SOLUTION: $(f^*, f^*) = -1$, $(f, f^*) = 0$

GENERAL SOLUTION

$$q(t) = a_f f(t) + a_{f^*} f^*(t)$$

$$p = \frac{\partial L}{\partial \dot{q}}$$

TO QUANTIZE, IMPOSE COMM. RELS.

$$[q(t), p(t)] = i \Leftrightarrow [a_f, a_f^\dagger] = i$$

f-VACUUM $|0_f\rangle$: $a_f|0_f\rangle = 0$

n-EXCITED STATE $|n_f\rangle$: $|n_f\rangle = \frac{1}{\sqrt{n!}} (a_f^\dagger)^n |0_f\rangle$

f-NUMBER OP $N_f = a_f^\dagger a_f$

$$p = \frac{\partial L}{\partial \dot{q}}$$

TO QUANTIZE, IMPOSE COMM. RELS.

$$[q(t), p(t)] = i \Leftrightarrow [a_f, a_f^\dagger] = i$$

f-VACUUM $|0_f\rangle$: $a_f|0_f\rangle = 0$

n-EXCITED STATE $|n_f\rangle$: $|n_f\rangle = \frac{1}{\sqrt{n!}} (a_f^\dagger)^n |0_f\rangle$

f-NUMBER OP $N_f = a_f^\dagger a_f$: $N_f |n_f\rangle = n |n_f\rangle$

WE CAN AS WELL CHOOSE g (INSTEAD OF f)

$$q(t) = a_g g(t) + a_g^+ g^*(t)$$

$$[a_g, a_g^+] = i$$

$$|0_g\rangle$$

$$|n_g\rangle$$

$$N_g$$

ADIABATIC MODES

• WE CAN WRITE

$$g(t) = \alpha f(t) + \beta f^*(t)$$

BOGOLUBOV COEFFS

$$\alpha = (f, g)$$

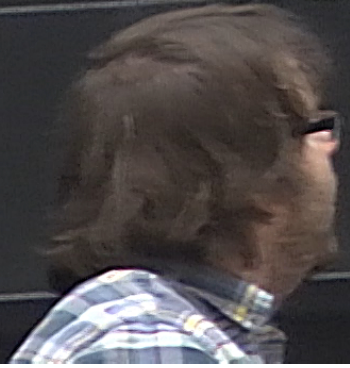
$$\beta = -(f^*, g)$$

$$|\alpha|^2 - |\beta|^2 = 1$$

$$q(t) = a_f f + a_f^\dagger f^* = a_g g + a_g^\dagger g^*$$

RELATION BETWEEN $a_f, a_g, \dots$

$$\begin{aligned} a_g &= \alpha a_f - \beta^* a_f^\dagger \\ a_g^\dagger &= \alpha^* a_f^\dagger - \beta a_f \end{aligned}$$



• WE CAN WRITE

$$g(t) = \alpha f(t) + \beta f^*(t)$$

BOGOLUBOV COEFFS.

$$\alpha = (f, g)$$

$$|\alpha|^2 - |\beta|^2 = 1$$

$$\beta = -(f^*, g)$$

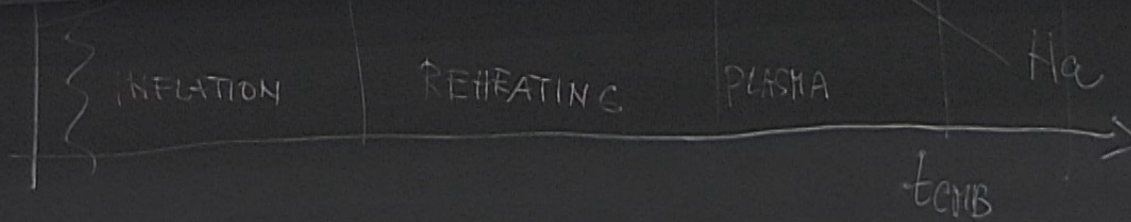
$$g(t) = a_f f + a_f^\dagger f^* = a_g g + a_g^\dagger g^*$$

RELATION BETWEEN $a_f, a_g, \dots$

$$\begin{aligned} a_g &= \alpha a_f - \beta^* a_f^\dagger \\ a_g^\dagger &= \alpha^* a_f^\dagger - \beta a_f \end{aligned}$$

HOW MANY g -PARTICLES ARE IN f -VACUUM.

$$\langle 0_f | N_g | 0_f \rangle = \langle 0_f | a_g^\dagger a_g | 0_f \rangle = \langle 0_f | (\alpha^* a_f^\dagger - \beta a_f) (\alpha a_f - \beta^* a_f^\dagger) | 0_f \rangle$$



$$q|H| = a_f^\dagger + a_f^\dagger{}^\dagger = a_g y + a_g y$$

RELATION BETWEEN $a_f, a_g, \dots$

$$\begin{aligned} a_g &= \alpha a_f - \beta^* a_f^\dagger \\ a_g^\dagger &= \alpha^* a_f^\dagger - \beta a_f \end{aligned}$$

$$\langle 0_f | a_g^\dagger a_g | 0_f \rangle = \langle 0_f | (\alpha^* a_f^\dagger - \beta a_f) (\alpha a_f - \beta^* a_f^\dagger) | 0_f \rangle = \underline{\underline{|\beta|^2}}$$

UNROH
HAWKING
PARTICLE PROD
IN EARLY U.

