Title: PSI 17/18 - Cosmology - Lecture 11

Date: Feb 12, 2018 09:00 AM

URL: http://pirsa.org/18020020

Abstract:

3) FLUCTUATIONS

a) CMB POWER SPECTRUM (AGAIN)

3 SOURCES:
- PRIMORDIAL FLUCTUATIONS
- PLASMA OSCILLATIONS

} "PRIMARY"

- SECONDARY ANISOTROPIES

3) FLUCTUATIONS

a) CMB POWER SPECTRUM (AGAIN)

3 SOURCES:
- PRIMORDIAL FLUCTUATIONS
- PLASMA OSCILLATIONS

} "PRIMARY"

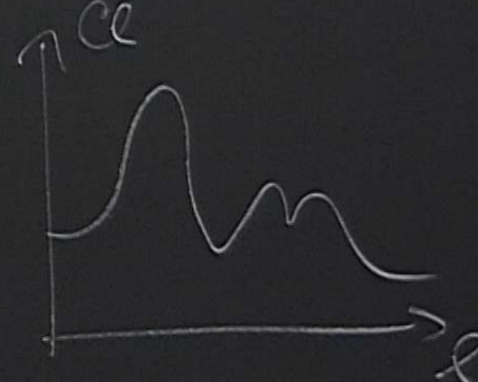
- SECONDARY ANISOTROPIES (AFTER CMB WAS RELEASED)
  (INTEGRATED) SACHS-WOLFE EFFECT
  SUNYAEV-ZELDOVICH EFFECT
  DOPPLER SHIFT..

- OBSERVED TEMPERATURE $T = T(\hat{n})$. HAS FLUCTUATIONS

$$\delta T = \frac{T - T_0}{T_0}, \quad T_0 = \frac{1}{4\pi}\int d\Omega\, T(\hat{n}) \simeq \underline{2.7K}.$$

EXPAND IN SPHERICAL HARMONICS.

$$\delta T = \sum_{\ell} \sum_{m=-\ell}^{\ell} \underline{a_{\ell m}}\, Y_{\ell m}(\hat{n})$$



OBSERVED POWER SPECTRUM.

$$C_\ell^{(OBS)} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$

SUNYAEV-ZELDOVICH EFFECT
DOPPLER SHIFT..

- WANT TO THEORETICALLY PREDICT C_ℓ.
  - CONSIDER AN ENSAMBLE OF POSSIBLE UNIVERSES
  - WE MEASURE RANDOMLY CHOSEN $\{a_{\ell m}\}$
    WITH PROB. DISTRIBUTION $P(\{a_{\ell m}\})$
    (GIVEN BY OUR THEORY)

$\Rightarrow$ CAN CALCULATE "CORRELATION FUNCTIONS"

$$\langle a_{\ell m} a_{\ell' m'} \ldots \rangle = \int \left(\prod d a_{\ell m}\right) P(\{a_{\ell m}\}) \left[a_{\ell m} a_{\ell' m'} \ldots\right]$$

• P IS ROTATIONALLY INVARIANT ✓

IS GAUSSIAN ?

... ALL n-POINT C.F GIVEN IN TERMS OF 2-POINT. C.F. VIA WICK THEOREM

$$\langle \underbrace{a \ldots a}_{\text{ODD}} \rangle = 0$$

$$\langle a_1 a_2 a_3 a_4 \rangle = \langle a_1 a_2 \rangle \langle a_3 a_4 \rangle + \langle a_1 a_3 \rangle \langle a_2 a_4 \rangle + \langle a_1 a_4 \rangle \langle a_2 a_3 \rangle$$

MALDACENA - ASTRO-PH/0210603

.. NON-GAUSSIANITIES.

$$\langle \delta T(\hat{n}) \rangle = 0$$

$$\langle \delta T(\hat{n})\, \delta T(\hat{n}') \rangle \underset{\text{INVAR.}}{\overset{\text{ROT}}{=}} f(\hat{n}\cdot\hat{n}') = \sum \frac{2\ell+1}{4\pi} C_\ell \underbrace{P_\ell(\hat{n}\cdot\hat{n}')}_{\frac{4\pi}{2\ell+1}\sum_m Y_{\ell m}(\hat{n})\, Y^*_{\ell m}(\hat{n}')}$$

$$\Rightarrow \boxed{\langle a_{\ell m}\, a^*_{\ell' m'} \rangle = \delta_{\ell\ell'}\,\delta_{mm'}\, C_\ell}$$

THEORETICAL

$P_\ell(\hat{n}\cdot\hat{n}')$

$\sum_m Y_{\ell m}(\hat{n}) Y^*_{\ell m}(\hat{n}')$

• WANT TO RELATE C_ℓ & $C_\ell^{(OBS)}$

AVERAGE:

$$\langle C_\ell^{OBS}\rangle = \left\langle \frac{1}{2\ell+1}\sum_m a_{\ell m}^* a_{\ell m}\right\rangle$$

$$= \frac{1}{2\ell+1}\sum_m \underbrace{\langle a_{\ell m}^* a_{\ell m}\rangle}_{\delta_{\ell\ell}\,\delta_{mm}}$$

- WANT TO RELATE C_ℓ & C_ℓ^{OBS}.

AVERAGE:

$$\langle C_\ell^{OBS} \rangle = \left\langle \frac{1}{2\ell+1} \sum_m a_{\ell m}^* a_{\ell m} \right\rangle$$

$$= \frac{1}{2\ell+1} \sum_m \underbrace{\langle a_{\ell m}^* a_{\ell m} \rangle}_{\delta_{\ell\ell}\,\delta_{mm}\,C_\ell} = \underline{\underline{C_\ell}}.$$

$$\frac{P_\ell(\hat{n}\cdot\hat{n}')}{\sum_m Y_{\ell m}(\hat{n})\, Y^*_{\ell m}(\hat{n}')}$$

E:

$$\rangle = \left\langle 1 - \frac{2C_\ell^{OBS}}{C_\ell} + \left(\frac{C_\ell^{OBS}}{C_\ell}\right)^2 \right\rangle = 1 - 2 + \frac{1}{C_\ell^2}\left\langle C_\ell^{OBS\,2} \right\rangle = -1 + \ldots$$

$$\times \sum_{m,m'} \underbrace{\left\langle a_{\ell m}\, a^*_{\ell m}\; a_{\ell m'}\, a^*_{\ell m'} \right\rangle}_{\langle a_{\ell m} a^*_{\ell m}\rangle \langle a_{\ell m'} a^*_{\ell m'}\rangle + \langle a_{\ell m} a_{\ell m'}\rangle \langle a^*_{\ell m} a^*_{\ell m'}\rangle + \langle a_{\ell m} a^*_{\ell m'}\rangle \langle a \ldots}$$

$$\ldots\frac{C_\ell^{OBS}}{C_\ell} + \left(\frac{C_\ell^{OBS}}{C_\ell}\right)^2 \Big\rangle = 1 - 2 + \frac{1}{C_\ell^2}\left\langle C_\ell^{OBS\,2}\right\rangle = -1 + \frac{1}{C_\ell^2(2\ell+1)^2} \times$$

$$\ldots a^*_{\ell m}\, a_{\ell m'}\, a^*_{\ell m'} \Big\rangle$$

$$\ldots \langle a_{\ell m'}\, a^*_{\ell m'} \rangle + \langle a_{\ell m}\, a_{\ell m'} \rangle \langle a^*_{\ell m}\, a^*_{\ell m'} \rangle + \langle a_{\ell m}\, a^*_{\ell m'} \rangle \langle a^*_{\ell m}\, a_{\ell m'} \rangle$$

$$- \frac{2C_\ell^{OBS}}{C_\ell} + \left(\frac{C_\ell^{OBS}}{C_\ell}\right)^2 \rangle = 1 - 2 + \frac{1}{C_\ell^2}\langle C_\ell^{OBS\,2}\rangle = -1 + \frac{1}{C_\ell^2(2\ell+1)^2} \times$$

$$\underbrace{\langle a_{\ell m} a^*_{\ell m} \; a_{\ell m'} a^*_{\ell m'} \rangle}$$

$$\underbrace{\langle a_{\ell m} a^*_{\ell m}\rangle\langle a_{\ell m'} a^*_{\ell m'}\rangle}_{\delta_{mm}\delta_{\ell\ell} C_\ell \; \delta_{\ell\ell}\delta_{m'm'} C_\ell \;=\; C_\ell^2} + \langle a_{\ell m} a_{\ell m'}\rangle\langle a^*_{\ell m} a^*_{\ell m'}\rangle + \underbrace{\langle a_{\ell m} a^*_{\ell m'}\rangle\langle a^*_{\ell m} a_{\ell m'}\rangle}_{\delta_{mm'} C_\ell^2}$$

$$\boxed{a^*_{\ell m} = (-1)^m a_{\ell -m}}$$

$$(2\ell+1)\, C_\ell^2 \qquad (2\ell+1)\, C_\ell^2$$

$$(2\ell+1)^2 C_\ell^2 \qquad\qquad (2\ell+1)\, C_\ell^2$$

$$= -1 + \frac{1}{C_\ell^2 (2\ell+1)^2}\Big((2\ell+1)^2 C_\ell^2 + (2\ell+1) C_\ell^2 + (2\ell+1) C_\ell^2 \Big) = \underline{\frac{2}{2\ell+1}}$$

$$\sqrt{\left\langle \left(\frac{C_\ell - C_\ell^{\mathrm{OBS}}}{C_\ell} \right)^2 \right\rangle} = \sqrt{\frac{2}{2\ell+1}}$$

• CMB IS POLARIZED ... PRODUCED BY THOMSON SCATTERING.

POLARIZATION ℓ_i: $\delta_{ij}\ell^i\ell^j = 1$, $\ell^i p^j \delta_{ij} = 0$

OF PARTICLES $= \int f_{ij}\,\ell^i\ell^j \,\frac{d^3p\, d^3q}{(2\pi)^3}$

$f_{ij}(q,p,t)$ ... SYMMETRIC TENSOR.

• **SVT DECOMPOSITION** ANY SYMMETRIC TENSOR IN d-DIMENSION CAN BE DECOMPOSED AS:

$$T_{ij} = h^{TT}_{ij} + \nabla_{(i}V_{j)} + \nabla_i \nabla_j \phi - \frac{1}{2} g_{ij} \nabla^2 \phi$$

$$\frac{d(d+1)}{2} \qquad + \frac{1}{d} g_{ij} \psi$$

· SVT DECOMPOSITION ANY SYMMETRIC TENSOR IN d-DIMENSION CAN BE DECOMPOSED AS:

$$T_{ij} = h^{TT}_{ij} + \nabla_{(i} V^T_{j)} + \nabla_i \nabla_j \phi - \frac{1}{2} g_{ij} \nabla^2 \phi + \frac{1}{d} g_{ij} \psi$$

$\frac{d(d+1)}{2}$

$\nabla^i h^{TT}_{ij} = 0$ ... TRANSVERSE

$\nabla^i V^T_i = 0$

$g^{ij} h^{TT}_{ij} = 0$ ... TRACELESS

$\langle a_1 a_2 a_3 a_4 \rangle = \langle a_1 a_2 \rangle \langle a_3 a_4 \rangle$

$+ \langle a_1 a_4 \rangle \langle a_2 a_3 \rangle$

SPEC: 1H $d=2$:

$$h_{ij}^{TT}=0\,,\quad V_i^{T}=\varepsilon_{ij}\,\partial_j\chi$$

ONLY 3 SCALARS:

$$\underbrace{\chi \quad \phi}_{\text{POLAR}}\,,\quad \psi \longrightarrow \underline{\underline{\text{Temp}}}$$

• TENSORIAL HARMONICS:

$$T_{ij} = \underbrace{\epsilon_{(i}{}^{k} D_{j)} D_k}_{\text{"B"}} \kappa + \underbrace{\left(D_i D_j - \frac{1}{2} g_{ij} D^2\right)}_{\text{"E"}} \phi$$

$$Y^{E}_{ij,\ell m}(\hat{n}) = \sqrt{\frac{2(\ell-2)!}{(\ell+2)!}} \left(D_i D_j - \frac{1}{2} g_{ij} D^2\right) Y_{\ell m}(\hat{n})$$

• TENSORIAL HARMONICS.

$$T_{ij} = \underbrace{\varepsilon_{(i}{}^{k} D_{j)} D_k}_{\text{"B"}} \kappa + \big(\underbrace{D_i D_j - \tfrac{1}{2} g_{ij} D^2}_{\text{"E"}}\big)\phi$$

$$Y^{E}_{ij,\,lm}(\hat{n}) = \sqrt{\frac{2(l-2)!}{(l+2)!}} \left(D_i D_j - \frac{1}{2} g_{ij} D^2\right) Y_{lm}(\hat{n})$$

$$Y^{B}_{ij,\,lm}(\hat{n}) = \frac{1}{2}\ \text{—''—}\ \left(\varepsilon_i{}^{k} D_j D_k + \varepsilon_j{}^{k} D_i D_k\right) Y_{lm}(\hat{n})$$

$(2l+1)\Gamma^2 C_l^2$ $(2l+1)C_l^2$

... FORM COMPLETE BASIS.

=> CAN DECOMPOSE

$$f_{ij} = \sum_{lm} a^E_{lm} Y^E_{lm\,ij} + \sum_{lm} a^B_{lm} Y^B_{ij,lm}$$

STANDARTLY $a_{lm} \equiv a^T_{lm}$ ... TEMPERATURE.

CAN CALCULATE CORRELATIONS

$$\langle a^E_{lm} a^B_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l^{EB}$$