

Title: PSI 17/18 - Cosmology - Lecture 10

Date: Feb 07, 2018 09:00 AM

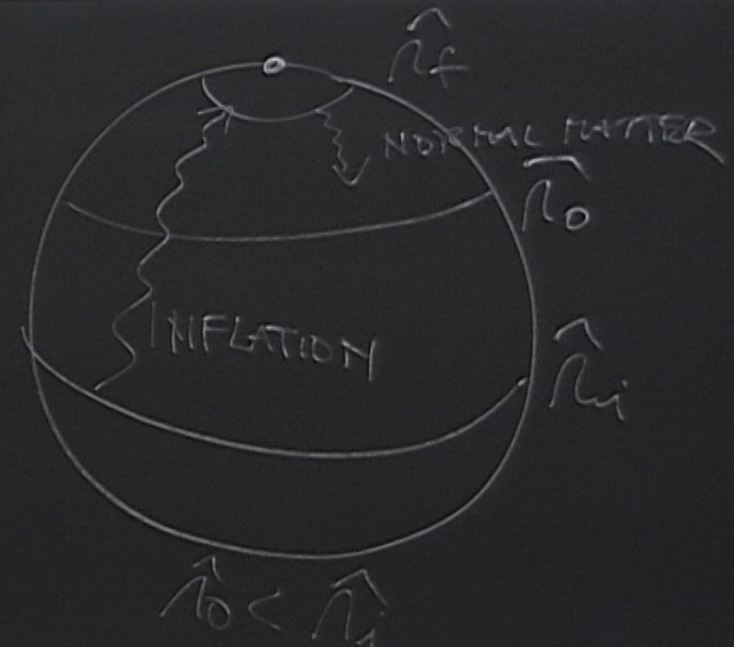
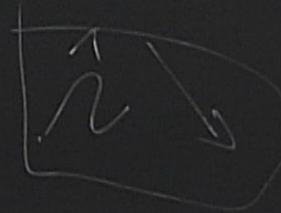
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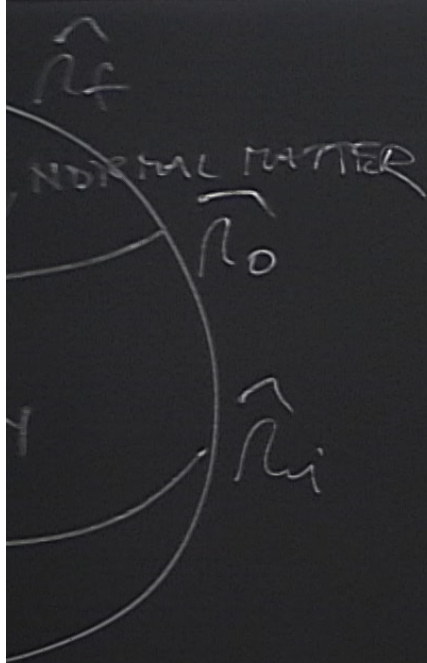
Abstract:

INFLATION $\equiv$ ERA OF RAPID EXPANSION
OF UNIVERSE THAT PRECEDED
RADIATION ERA

COMOVING HUBBLE RADIUS

$$\hat{r} = \frac{1}{aH}$$





$$N = \frac{\hat{\rho}_i}{\hat{\rho}_f}, \quad \boxed{N \approx 60}$$

• TWO KINEMATIC CONDITIONS

$$\boxed{\epsilon = -\frac{\dot{H}}{H^2} \ll 1}$$

(RAPIDLY SHRINKING $\hat{\lambda}$)

$$\boxed{|\eta| = \left| \frac{\ddot{\epsilon}}{H\epsilon} \right| \ll 1}$$

(INFLATION LASTS LONG ENOUGH)

MATTER DRIVING INFLATION

$$\rho + 3p < 0$$

$$w < -\frac{1}{3}$$

(VIOLATES STRONG E. COND.)

FRAMEWORK

THE PHYSICS OF INFLATION (CONCRETE THEORIES)

- FALSE VACUUM INFLATION CONSTANT ENERGY DENSITY... Δ

→ EXPANDING DE-SITTER U.

$$\underline{K=0}: ds^2 = -dt^2 + e^{2Ht} dx_i^2, \quad H = \text{CONST.}$$

$$\mathcal{E} = 0 = \mathcal{P} \quad \checkmark$$

CLASSICALLY NEVER ENDS, QM... CAN TUNNEL

$$\vec{\Lambda}_0 < \vec{\Lambda}_1$$

(RAPIDLY SHRINKING $\vec{\Lambda}$)

(INFLATION LASTS LONG ENOUGH)

THE PHYSICS OF INFLATION (CONCRETE THEORIES)

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$$K=0: ds^2 = -dt^2 + e^{2Ht} dx_i^2, \quad H = \text{CONST.}$$

$$\mathcal{E} = 0 = \mathcal{M} \quad \checkmark$$

CLASSICALLY NEVER ENDS, QM... QM TUNNEL... DOES NOT CORRESPOND

WE KNOW IT
TO UNIVERSE AS

$$\lambda_0 < \lambda_i$$

(RAPIDLY SHRINKING λ)

(INFLATION LASTS LONG ENOUGH)

THE PHYSICS OF INFLATION (CONCRETE THEORIES)

• FALSE VACUUM INFLATION CONSTANT ENERGY DENSITY... Δ

→ EXPANDING DE-SITTER U.

$$K=0: ds^2 = -dt^2 + e^{2Ht} dx_i^2, \quad H = \text{CONST.}$$

$$E=0=\rho \quad \checkmark$$

CLASSICALLY NEVER ENDS, QM... QM TUNNEL... DOES NOT CORRESPOND

WE KNOW IT
TO UNIVERSE AS

A SINGLE SCALAR FIELD ... INFLATION,

• HYBRID INFLATION, DBI INFLATION,
GAUGA FIELD INFL.

SLOW ROLL INFLATION

$$S = \int d^4x \sqrt{g} \left(\frac{R}{16\pi G} - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right)$$

- $\nabla^2 \phi - V'(\phi) = 0$ EOM

- $T_{\mu\nu} = -\frac{2}{\sqrt{g}} \frac{\delta \mathcal{L}}{\delta g^{\mu\nu}} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} (\partial\phi)^2 - g_{\mu\nu} V(\phi)$
 $\Leftrightarrow T_{\mu\nu} = (\mathcal{S} + P) u_\mu u_\nu + P g_{\mu\nu}, \quad u^2 = -1$

$$P = -\frac{1}{2} (\partial\phi)^2 - V, \quad \mathcal{S} = -\frac{1}{2} (\partial\phi)^2 + V$$

$$u^\mu = -\frac{\partial^\mu \phi}{\sqrt{(\partial\phi)^2}}$$

$$w = P/\rho = \frac{-\frac{1}{2}(\partial\phi)^2 - V}{-\frac{1}{2}(\partial\phi)^2 + V}$$

$$\rho \propto a^{-3(1+w)}$$

• "WIGGLY FIELD" $(\partial\phi)^2 \gg V$

$w=1$, $\rho \propto \frac{1}{a^6}$. INHOT. UNIVERSE.

• "SMOOTH FIELD" $V \gg (\partial\phi)^2$

$w=-1 \rightarrow \rho = \text{CONST} \dots$ INFLATION.

STILL ASSUME FRW. (HOM & ISOTR.)

$$\Rightarrow \phi = \phi(t)$$

$$3m_{\text{pl}}^2 H^2 = \frac{1}{2} \dot{\phi}^2 + V$$

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

$$\rho_{\text{KIN}} \ll \rho_{\text{TOT}}$$

SLOW LOW INF

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{\frac{1}{2} \dot{\phi}^2}{m_{\text{pl}}^2 H^2} \approx \frac{\rho_{\text{KIN}}}{\rho_{\text{TOT}}} \ll 1$$

$$\eta = \frac{\dot{\epsilon}}{\epsilon H} = 2(\epsilon - \delta), \quad \delta = -\frac{\ddot{\phi}}{H\dot{\phi}}$$

$$|\delta| = \left| \frac{\ddot{\phi}}{H\dot{\phi}} \right| \ll 1$$

EOM:

$$3H\dot{\phi} + V' = 0$$

$$3m_p^2 H = V$$

DESIGNING THE RIGHT POTENTIAL:

$$\epsilon_V = \epsilon = \frac{m_p^2}{2} \left(\frac{V'}{V} \right)^2 \ll 1$$

$$\eta_V = \epsilon + \delta = m_p^2 \frac{V''}{V}$$

$$|\eta_V| \ll 1$$

$$\frac{(11)^2}{(21)^2} = \frac{g_H^2 \dot{\phi}^2}{g_{mp}^4 H^2} = \frac{V'^2}{V^2} = \frac{2\epsilon}{m_{pl}^2}$$

GIVEN V HOW MANY E-FOLDS CAN IT SUPPORT?

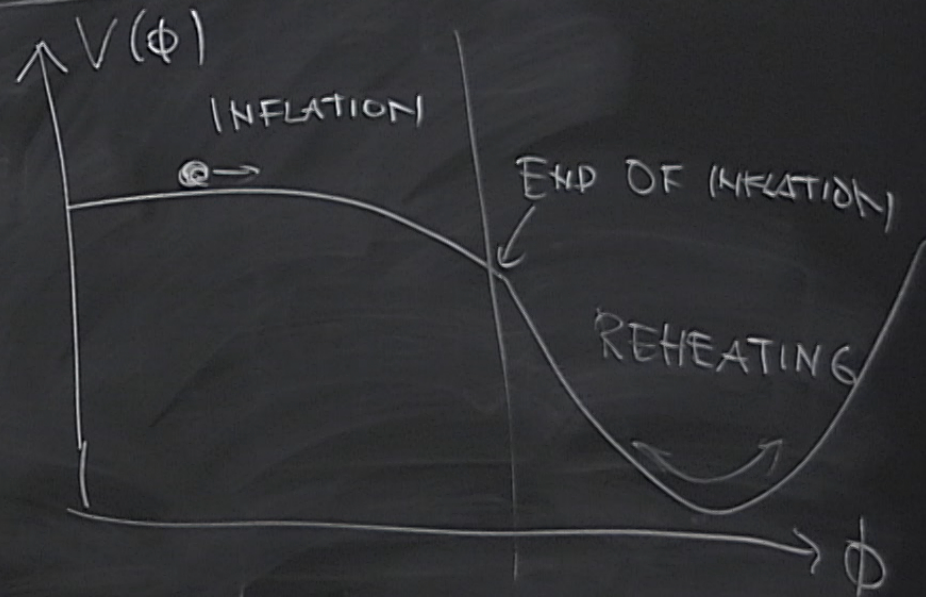
$$N = \log \frac{\hat{a}_f}{\hat{a}_i} = \log \frac{a_f H_f}{a_i H_i} \approx \log \frac{a_f}{a_i} - \int_{a_i}^{a_f} d \log a$$

$$= \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi \stackrel{(1)}{=} -3 \int_{\phi_i}^{\phi_f} \frac{H^2}{V'} d\phi \stackrel{(2)}{=} \frac{1}{M_{\text{Pl}}^2} \int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi$$

$\log a$

$$\frac{1}{V'} d\phi = \frac{1}{m_p^2} \int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi$$

TYPICAL POTENTIAL



HOLY GRAIL:

$$V = \frac{1}{2} m^2 \phi^2$$



$$\epsilon_V = m_V = \frac{2m^2}{\phi^2} \ll 1$$

$$\phi \gg m_{pl}$$

LARGE FIELD
INFLATION MODEL

TO TRUST THE THEORY

$$m^2 \phi^2 \ll \overset{4}{m_{pl}^4} = \text{PLANCK DENSITY}$$

$$m_{pl} \ll \phi \ll \frac{m_{pl}^2}{m}$$

$$m_{pl} \gg m$$

$$N = \frac{1}{4 m_{pl}^2} (\phi_i^2 - \phi_f^2) \approx 60$$

FOR EXAMPLE:

$$\phi_i = 10 m_p, \quad \dot{\phi}_+ = \frac{1}{10} \frac{m_p^2}{m}$$

$$N=60 \rightarrow \boxed{\frac{m_p^2}{m} = 60}$$

OBSERVATIONALLY NOW
DISFAVORED.

CONCLUDE:

- HOW INFLATION STARTED (BRANDERBERGER - 1601.0918)
- HOW IT ENDED? REHEATING