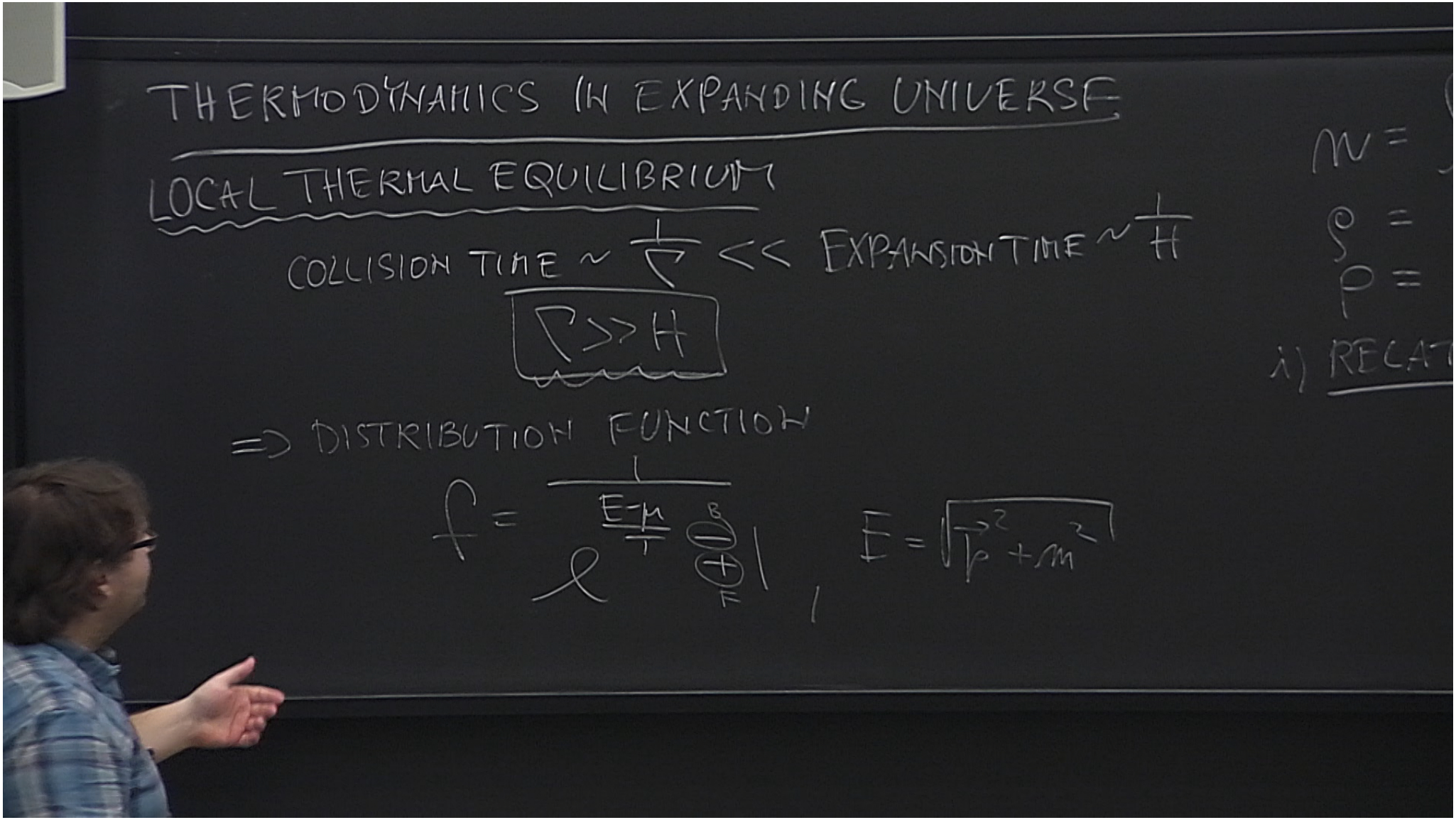
Title: PSI 17/18 - Cosmology - Lecture 5

Date: Feb 02, 2018 11:30 AM

URL: http://pirsa.org/18020014

Abstract:


THERMODYNAMICS IN EXPANDING UNIVERSE
LOCAL THERMAL EQUILIBRIUM
COLLISION TIME ~ 1/Γ << EXPANSION TIME ~ 1/H
Γ >> H
=> DISTRIBUTION FUNCTION
f = 1 / (e^((E-μ)/T) ± 1), E = √(p^2 + m^2)
n =
ρ =
p =
1) RECAT

$$n_i = \int \frac{d^3p}{(2\pi)^3} f = \frac{1}{(2\pi)^2} \int_m^\infty \frac{\sqrt{E^2 - m^2}\, E\, dE}{e^{\frac{E-\mu}{T}} \mp 1}$$

$\rho =$

$p =$

i) RELATIVISTIC LIMIT $T \gg m, \mu$

$$n \propto T^3, \quad \rho \propto T^4, \quad P \propto T^4$$

$$w = P/\rho = \frac{1}{3}$$

$$n_\nu = \int \frac{d^3p}{(2\pi)^3} f = \frac{1}{(2\pi)^2} \int_m^\infty \frac{\sqrt{E^2 - m^2}\, E\, dE}{e^{\frac{E-\mu}{T}} \mp 1}$$

$$s = \quad Ef$$

$$\rho =$$

a) RELATIVISTIC LIMIT $T \gg m, \mu$

$$n \propto T^3, \quad s \propto T^4, \quad P \propto T^4$$

$$w = P/\rho = \frac{1}{3}$$

$$n = \int \frac{d^3p}{(2\pi)^3} f = \frac{1}{(2\pi)^2} \int_m^\infty \frac{\sqrt{E^2 - m^2}\, E\, dE}{e^{\frac{E-\mu}{T}} \mp 1}$$

$$\rho = Ef$$

$$\rho =$$

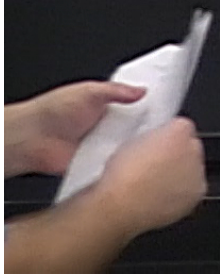
i) RELATIVISTIC LIMIT $T \gg m, \mu$

$$n \propto T^3, \quad \rho \propto T^4, \quad P \propto T^4$$

$$w = P/\rho = \frac{1}{3}$$

ii) NON-RELATIVISTIC $T \ll m, \mu < m$

$$n = \frac{1}{2\pi^2} \int_m^\infty \frac{\sqrt{E^2 - m^2}\, E\, dE}{e^{\frac{E-\mu}{T}}} = \left| \begin{array}{l} u = \frac{E-m}{T} \\ E = uT + m \end{array} \right|$$



$$n = \frac{1}{2\pi^2} \int_m^\infty \frac{\sqrt{E^2 - m^2}\, E\, dE}{e^{\frac{E-\mu}{T}}} = \left| \begin{array}{l} u = \frac{E-m}{T} \\ E = uT + m \end{array} \right|$$

$$= \frac{1}{2\pi^2} e^{\frac{\mu - m}{T}} \int_0^\infty \frac{\sqrt{u^2T^2 + m^2 + 2uTm - m^2}\,(uT+m)\,T\,du}{e^u}$$

$$n = \frac{1}{2\pi^2}\int_m^\infty \frac{\sqrt{E^2-m^2}\,E\,dE}{e^{\frac{E-\mu}{T}}} = \left| \begin{array}{l} u = \frac{E-m}{T} \\ E = uT+m \end{array} \right| \quad \text{KEEPING TERM TO LOWEST ORDER IN } \underline{u}.$$

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$$n = \frac{1}{2\pi^2}\int_m^\infty \frac{\sqrt{E^2-m^2}\,E\,dE}{e^{\frac{E-\mu}{T}}e^{\frac{\mu+m}{T}}} = \left| \begin{array}{l} u = \frac{E-m}{T} \\ E = uT+m \end{array} \right| \quad \text{KEEPING TERM TO LOWEST ORDER IN } u.$$

$$= \frac{1}{2\pi^2} e^{\frac{\mu-m}{T}} \int_0^\infty \sqrt{u^2T^2+m^2+2uTm-m^2}\,(uT+m)\,T\,du$$

$$= \frac{1}{2\pi^2} (mT)^{3/2} e^{\frac{\mu-m}{T}} \int_0^\infty \frac{e^u \sqrt{2u}\,du}{e^u}$$

$$n = \frac{1}{2\pi^2}\int_m^\infty \frac{\sqrt{E^2-m^2}\,E\,dE}{e^{\frac{E-\mu}{T}}} = \left| \begin{array}{l} u = \dfrac{E-m}{T} \\ E = uT+m \end{array} \right| \quad \text{KEEPING TERM TO LOWEST ORDER IN } u.$$

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$$= \frac{1}{2\pi^2} (mT)^{3/2} e^{\frac{\mu-m}{T}} \int_0^\infty \frac{e^u \sqrt{2u}\,du}{e^u} \approx \left(\frac{mT}{2\pi}\right)^{3/2} e^{\frac{\mu-m}{T}}$$

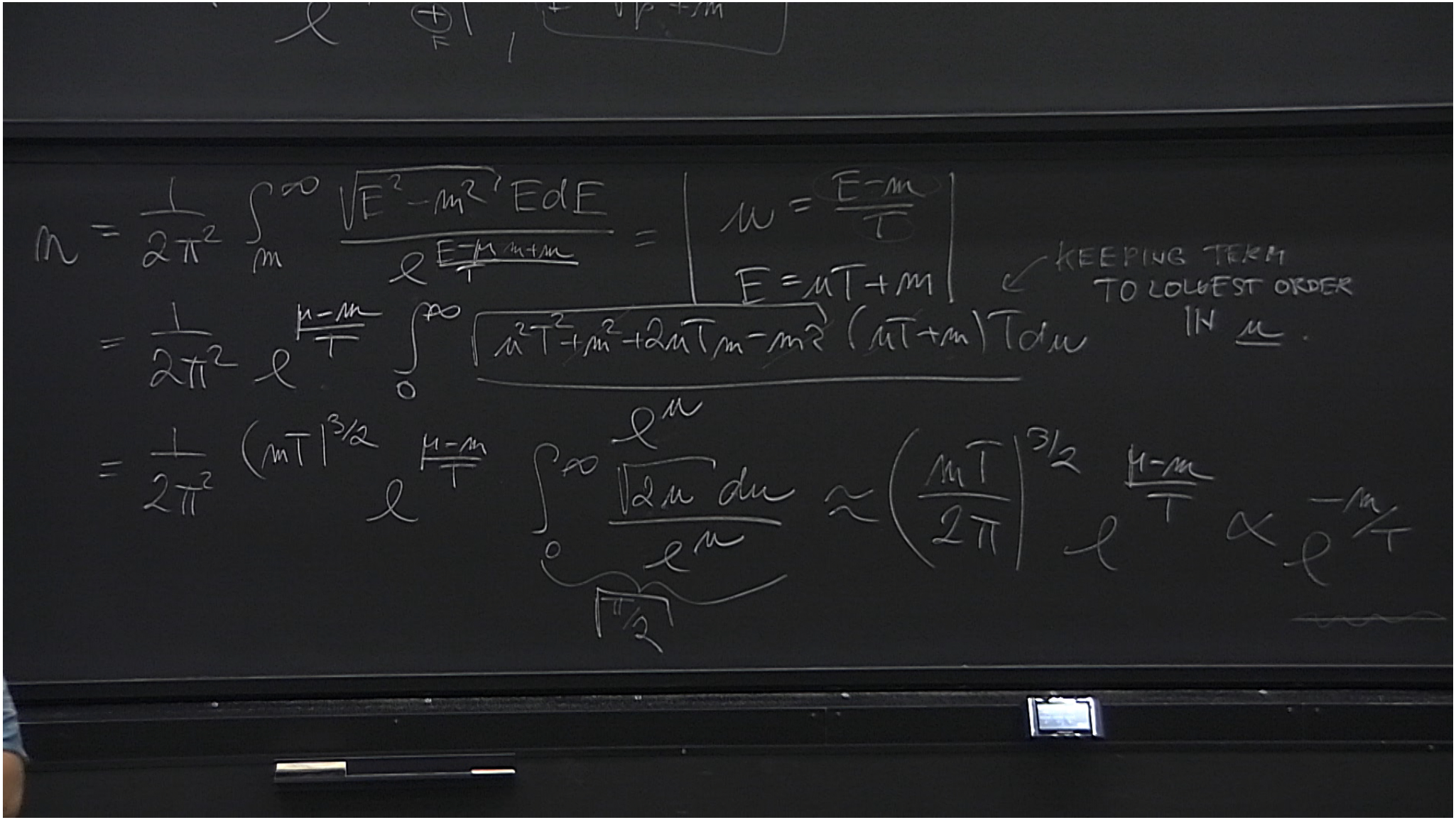
$$\Gamma(3/2)$$

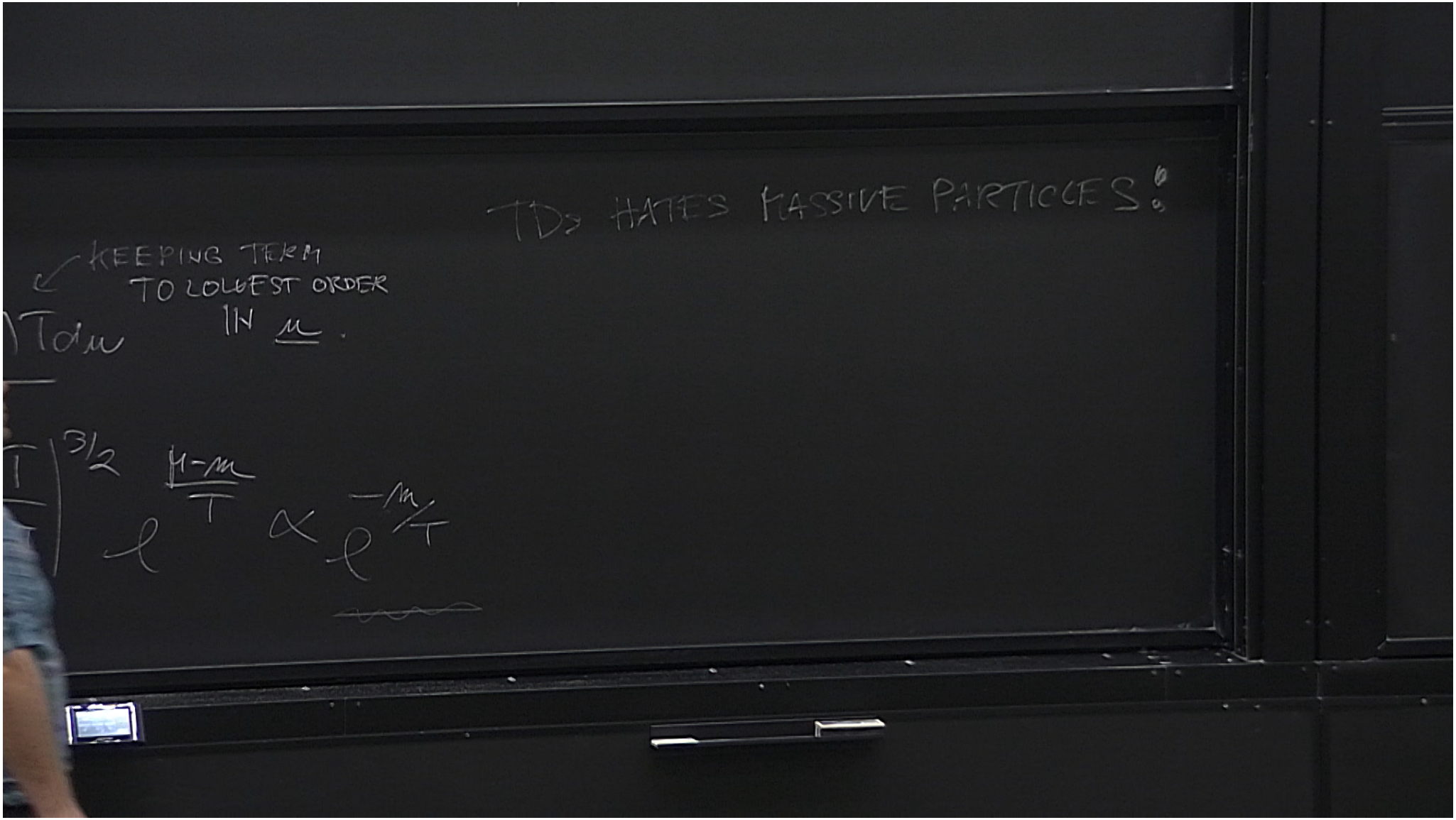
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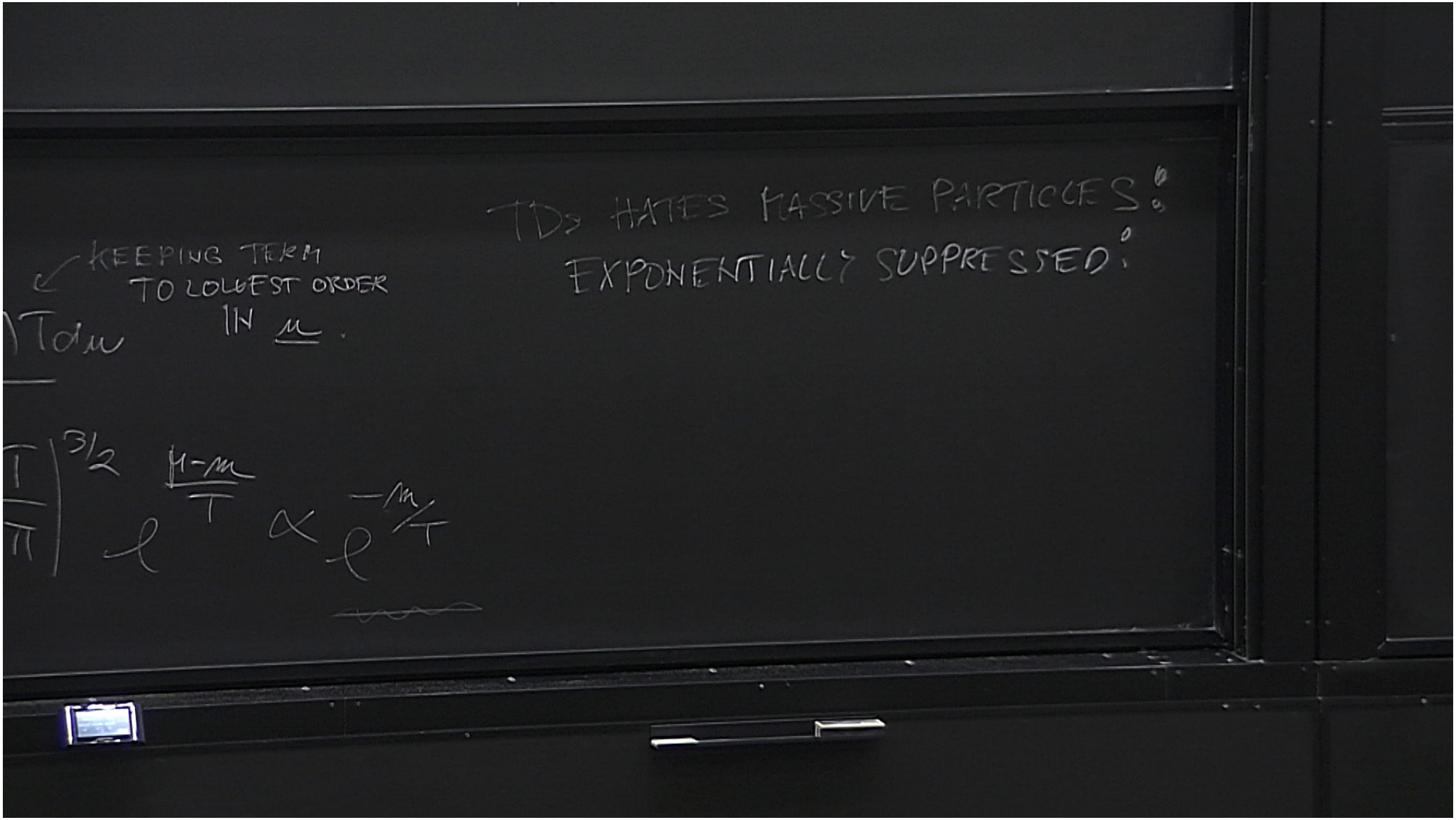
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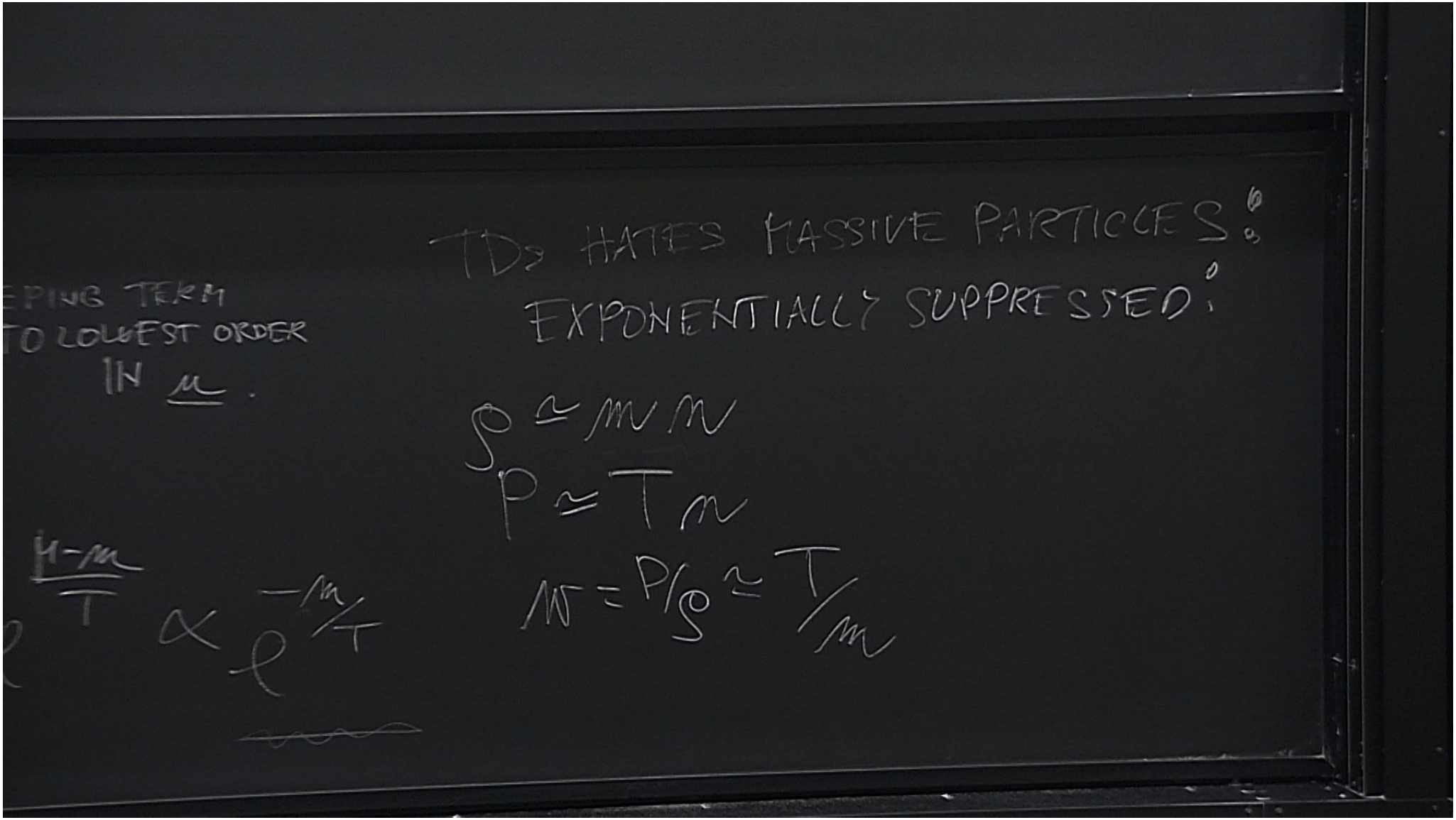
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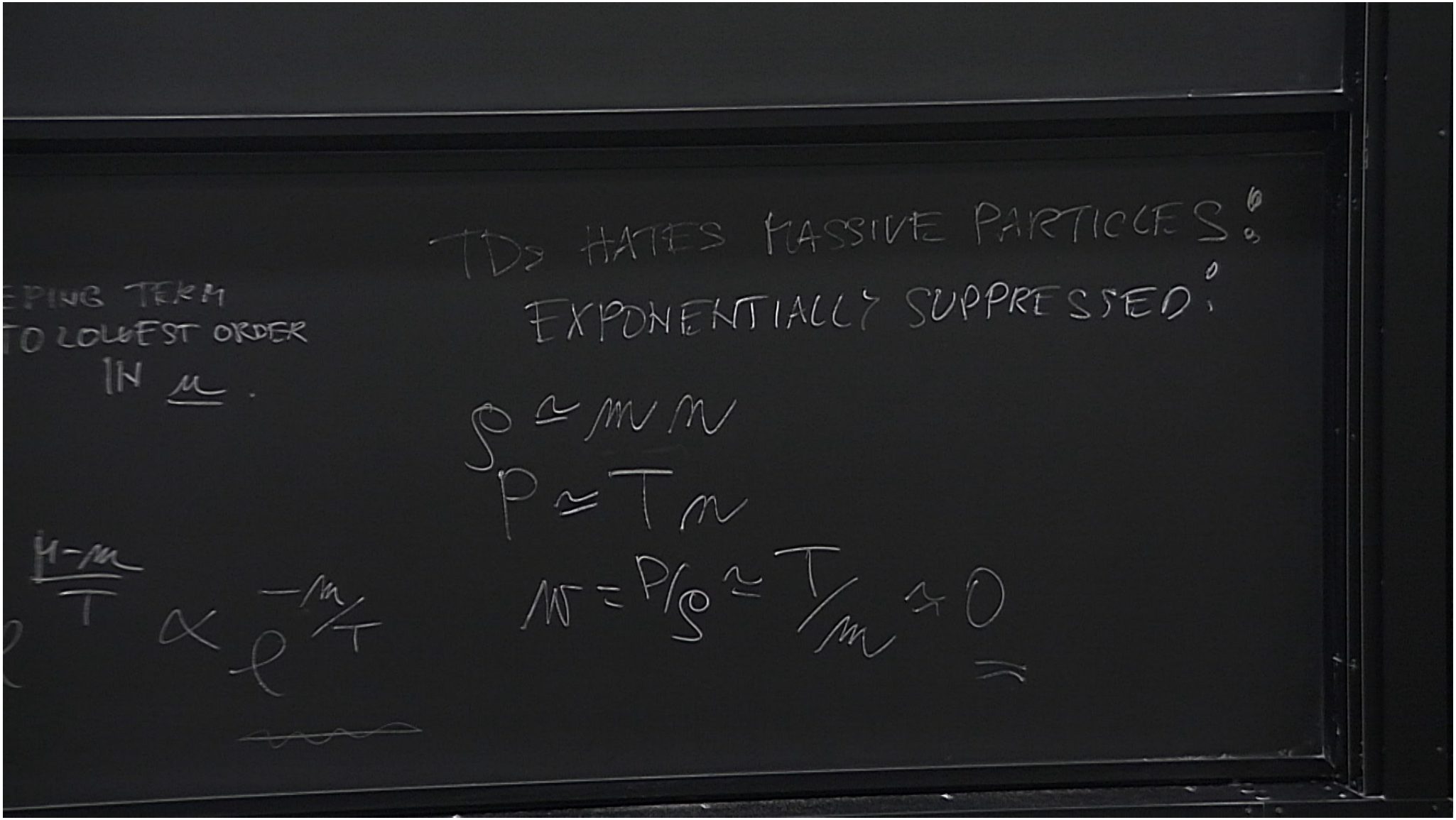
$\Gamma(3/2)$

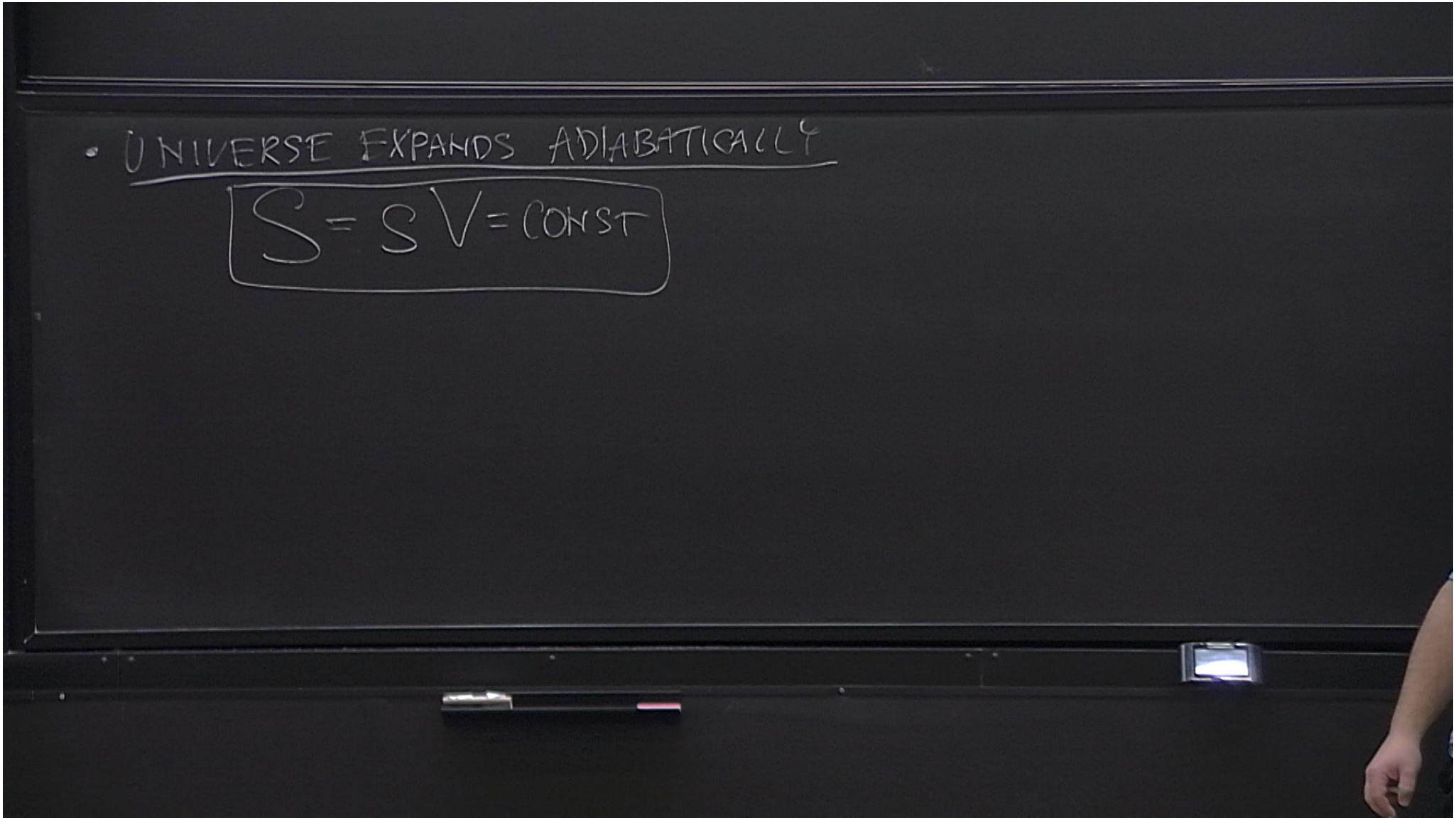

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KEEPING TERM TO LOWEST ORDER IN u.
$= \frac{1}{2\pi^2} e^{\frac{\mu-m}{T}} \int_0^\infty \sqrt{u^2T^2 + m^2 + 2uTm - m^2}\,(uT+m)\,T\,du$
$= \frac{1}{2\pi^2} (mT)^{3/2} e^{\frac{\mu-m}{T}} \int_0^\infty \frac{\sqrt{2u}\, du}{e^u} \approx \left(\frac{mT}{2\pi}\right)^{3/2} e^{\frac{\mu-m}{T}} \propto e^{-m/T}$
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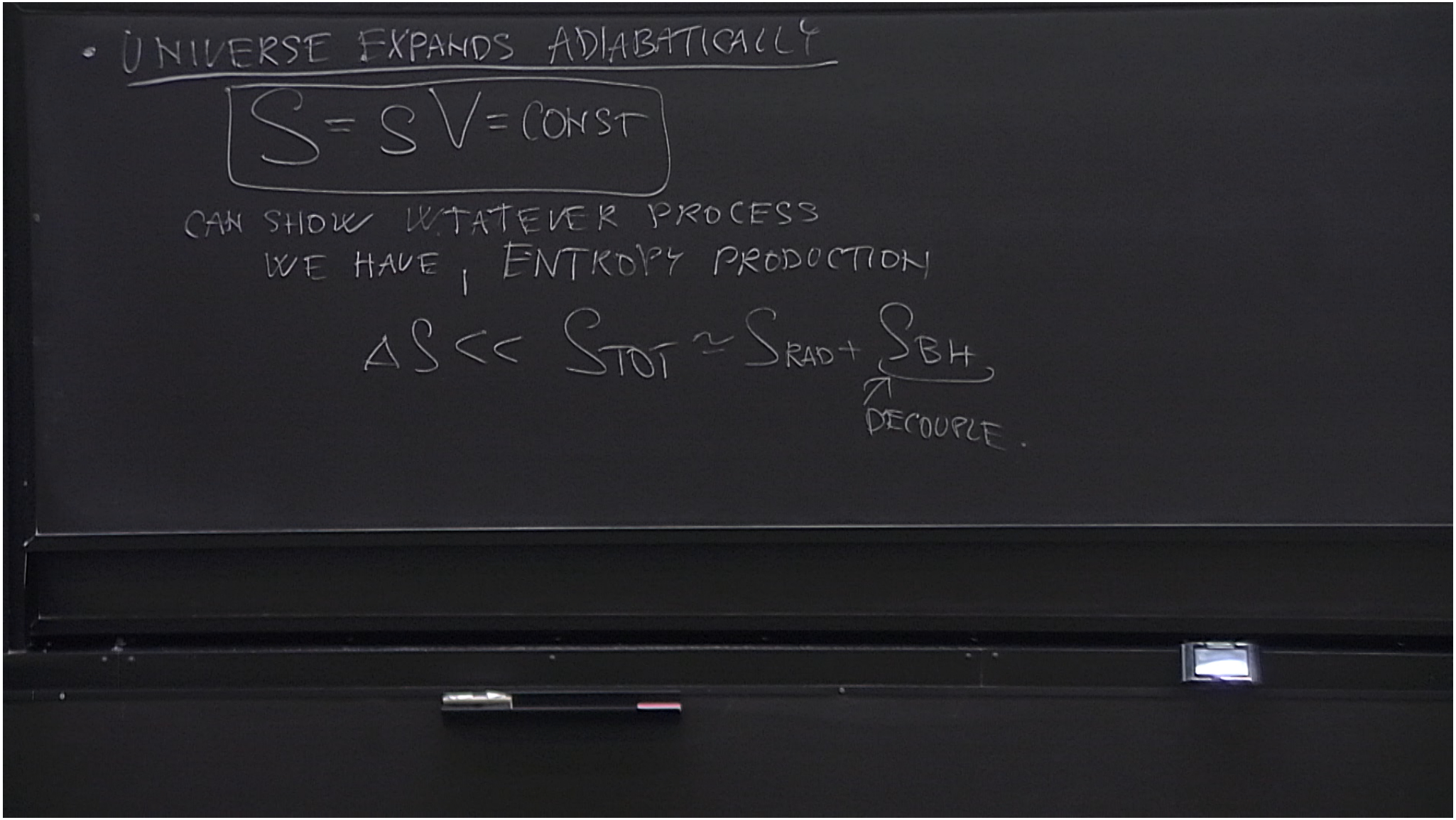

TD: HATES MASSIVE PARTICLES!
KEEPING TERM
TO LOWEST ORDER
IN m.


KEEPING TERM
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ING TERM
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ρ ≃ m n
P ≃ T n
n = P/ρ ≃ T/m
μ-m/T ∝ e^{-m/T}


EPING TERM
TO LOWEST ORDER
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TD: HATES MASSIVE PARTICLES!
EXPONENTIALLY SUPPRESSED!
ρ ≃ m n
P ≃ T n
w = P/ρ ≃ T/m ≪ 1
μ-m/T
∝ e^{-m/T}


• UNIVERSE EXPANDS ADIABATICALLY
S = S V = CONST


• UNIVERSE EXPANDS ADIABATICALLY
S = S V = CONST
CAN SHOW WHATEVER PROCESS WE HAVE, ENTROPY PRODUCTION
ΔS << S_TOT ≃ S_RAD + S_BH
DECOUPLE.

- UNIVERSE EXPANDS ADIABATICALLY

$$S = S\,V = \text{CONST}$$

CAN SHOW WHATEVER PROCESS WE HAVE, ENTROPY PRODUCTION

$$\Delta S \ll S_{TOT} \simeq S_{RAD} + S_{BH}$$

(S_{BH}: DECOUPLE.)

($10^9 \times$ MORE PHOTONS THAN BARYONS.)

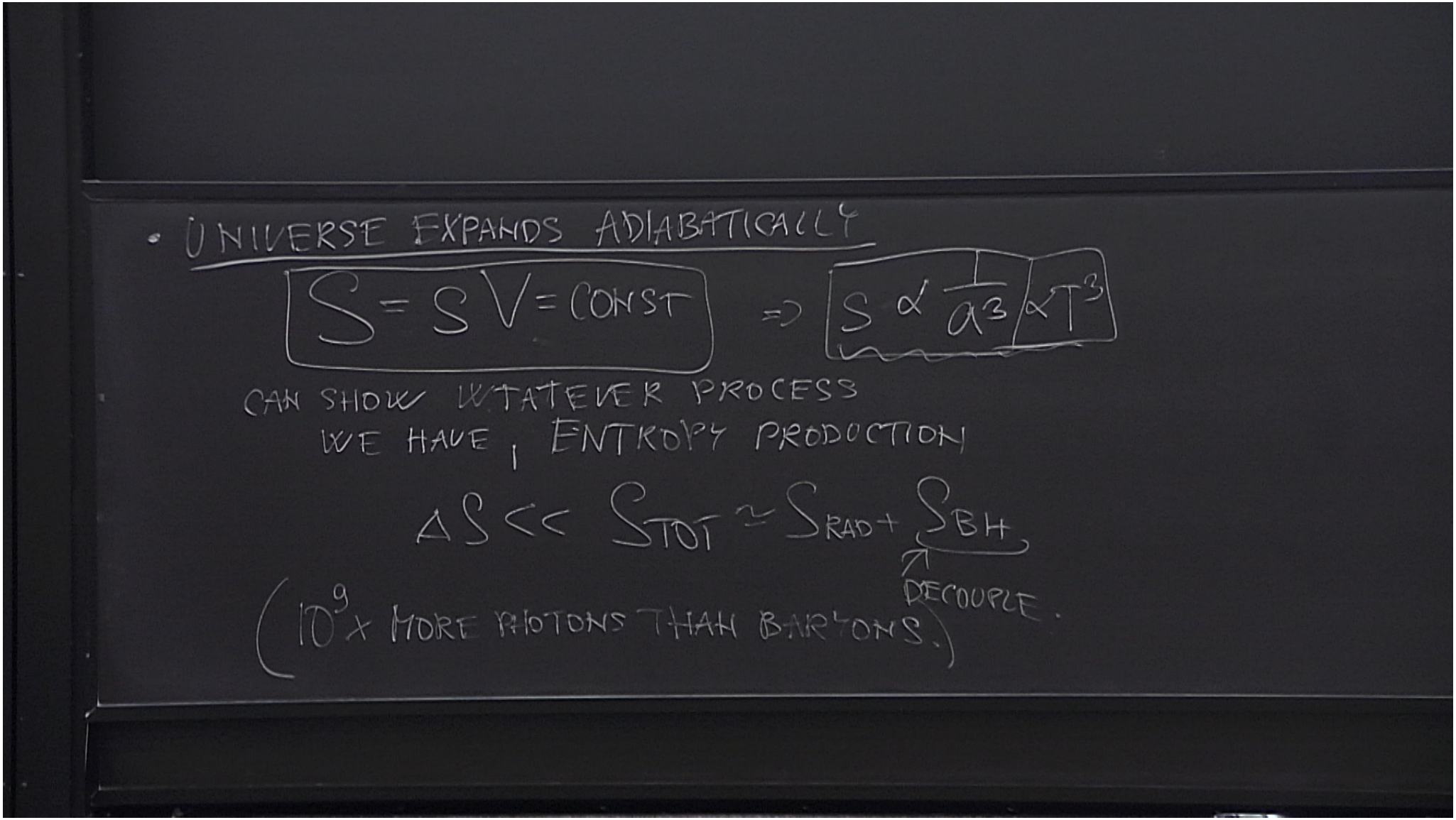
- UNIVERSE EXPANDS ADIABATICALLY

$$S = S\,V = \text{CONST} \Rightarrow S \propto \frac{1}{a^3}$$

CAN SHOW WHATEVER PROCESS WE HAVE, ENTROPY PRODUCTION

$$\Delta S \ll S_{TOT} \simeq S_{RAD} + S_{BH}$$

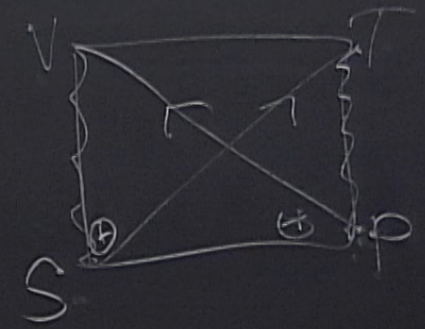
($10^9\times$ MORE PHOTONS THAN BARYONS.) ← DECOUPLE.


• UNIVERSE EXPANDS ADIABATICALLY
$S = sV = \text{CONST} \Rightarrow s \propto \frac{1}{a^3} \propto T^3$
CAN SHOW WHATEVER PROCESS WE HAVE, ENTROPY PRODUCTION
$\Delta S \ll S_{TOT} \simeq S_{RAD} + S_{BH}$
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($10^9 \times$ MORE PHOTONS THAN BARYONS.)

- BATH OF RELATIVISTIC PARTICLES

$$\rho = \rho(T), \quad P = P(T), \quad S = S(T)$$

H1: $$\frac{\partial P}{\partial T} = \frac{\rho + P}{T} = \left(\frac{\partial S}{\partial V}\right)_P = \left(\frac{\partial S}{\partial V}\right)_T$$



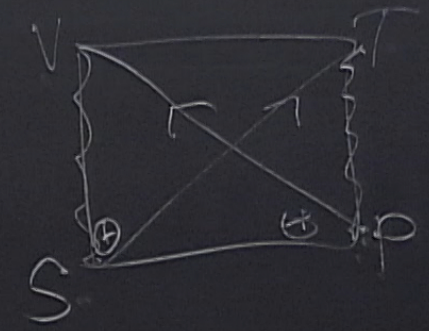
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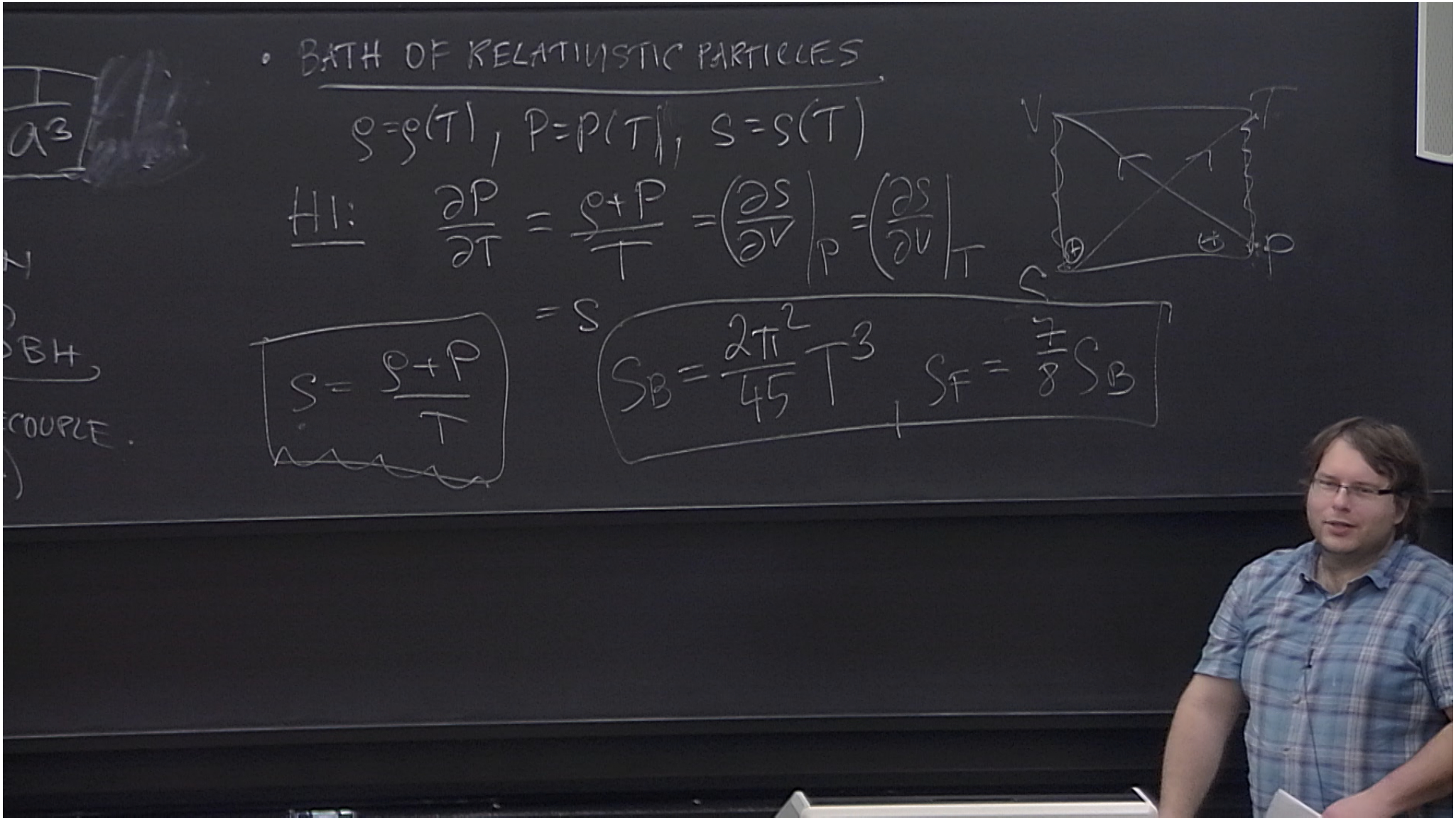
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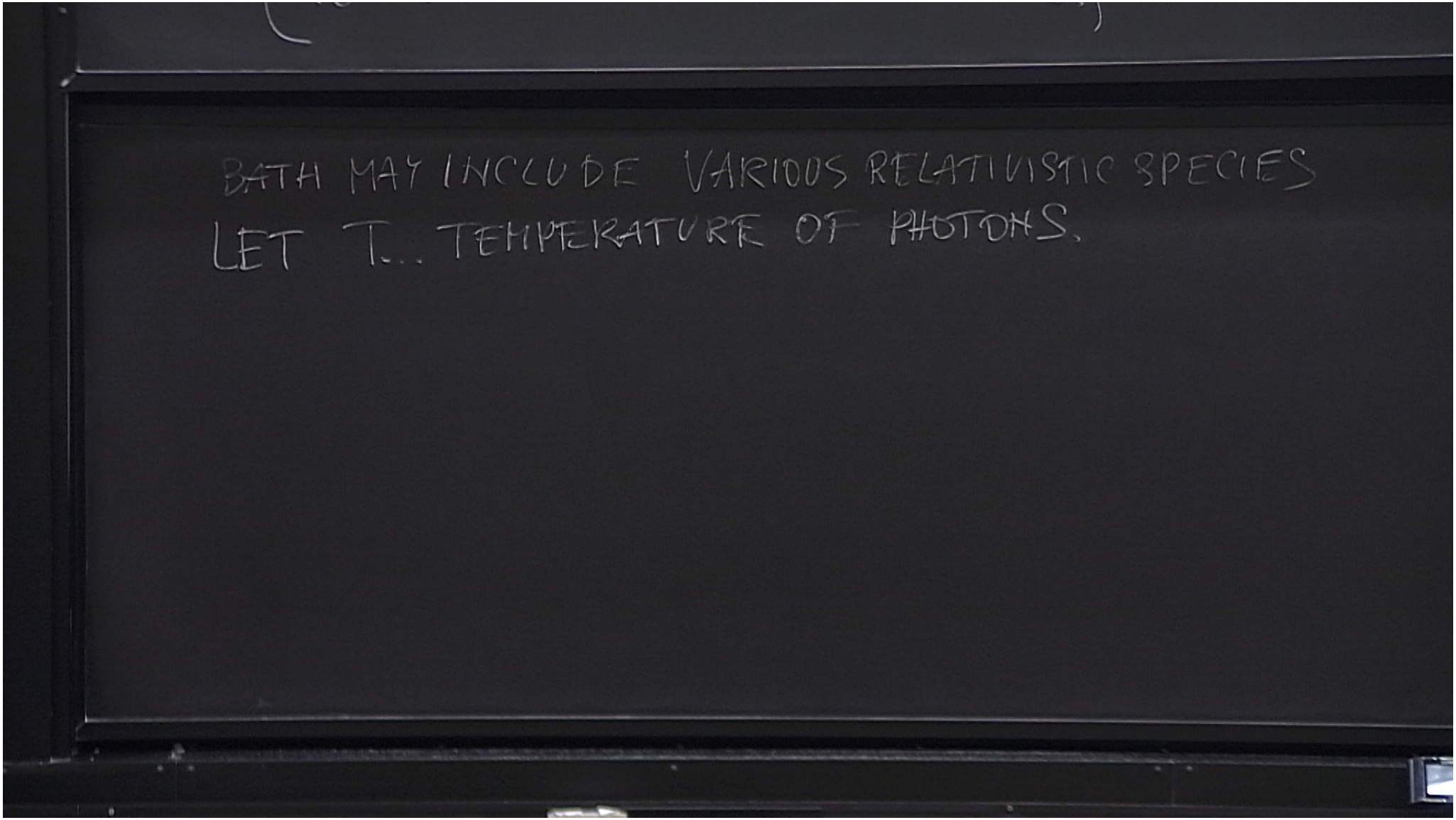
H1: $$\frac{\partial P}{\partial T} = \frac{\rho + P}{T} = \left(\frac{\partial S}{\partial V}\right)_P = \left(\frac{\partial S}{\partial V}\right)_T$$

$$= S$$

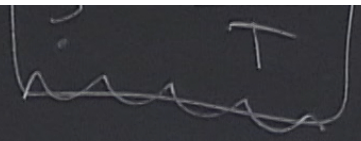
$$\boxed{S = \frac{\rho + P}{T}}$$




• BATH OF RELATIVISTIC PARTICLES
$\rho = \rho(T)$, $P = P(T)$, $S = S(T)$
H1: $\frac{\partial P}{\partial T} = \frac{\rho + P}{T} = \left(\frac{\partial S}{\partial V}\right)_P = \left(\frac{\partial S}{\partial V}\right)_T$
$= S$
$S = \frac{\rho + P}{T}$
$S_B = \frac{2\pi^2}{45} T^3$, $S_F = \frac{7}{8} S_B$
V
T
P
a³
BH
COUPLE.


BATH MAY INCLUDE VARIOUS RELATIVISTIC SPECIES
LET T.. TEMPERATURE OF PHOTONS.

($10^9 \times$ MORE PHOTONS THAN BARYONS.) DECOUPLE.



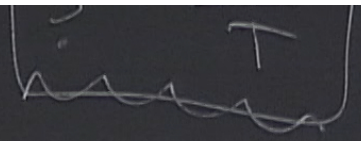
BATH MAY INCLUDE VARIOUS RELATIVISTIC SPECIES

LET T .. TEMPERATURE OF PHOTONS.

AND T_i ... TEMP OF OTHER SPECIES.

$$S = g_{*S} \frac{2\pi^2}{45} T^3 \quad , \quad g_{*S} = \sum_{\text{BOSONS}} g_i \left(\frac{T_i}{T}\right)^3 + \sum_{\text{FERMIONS}} \frac{7}{8} g_i \left(\frac{T_i}{T}\right)^3$$

(10^9 × MORE PHOTONS THAN BARYONS.) DECOUPLE.



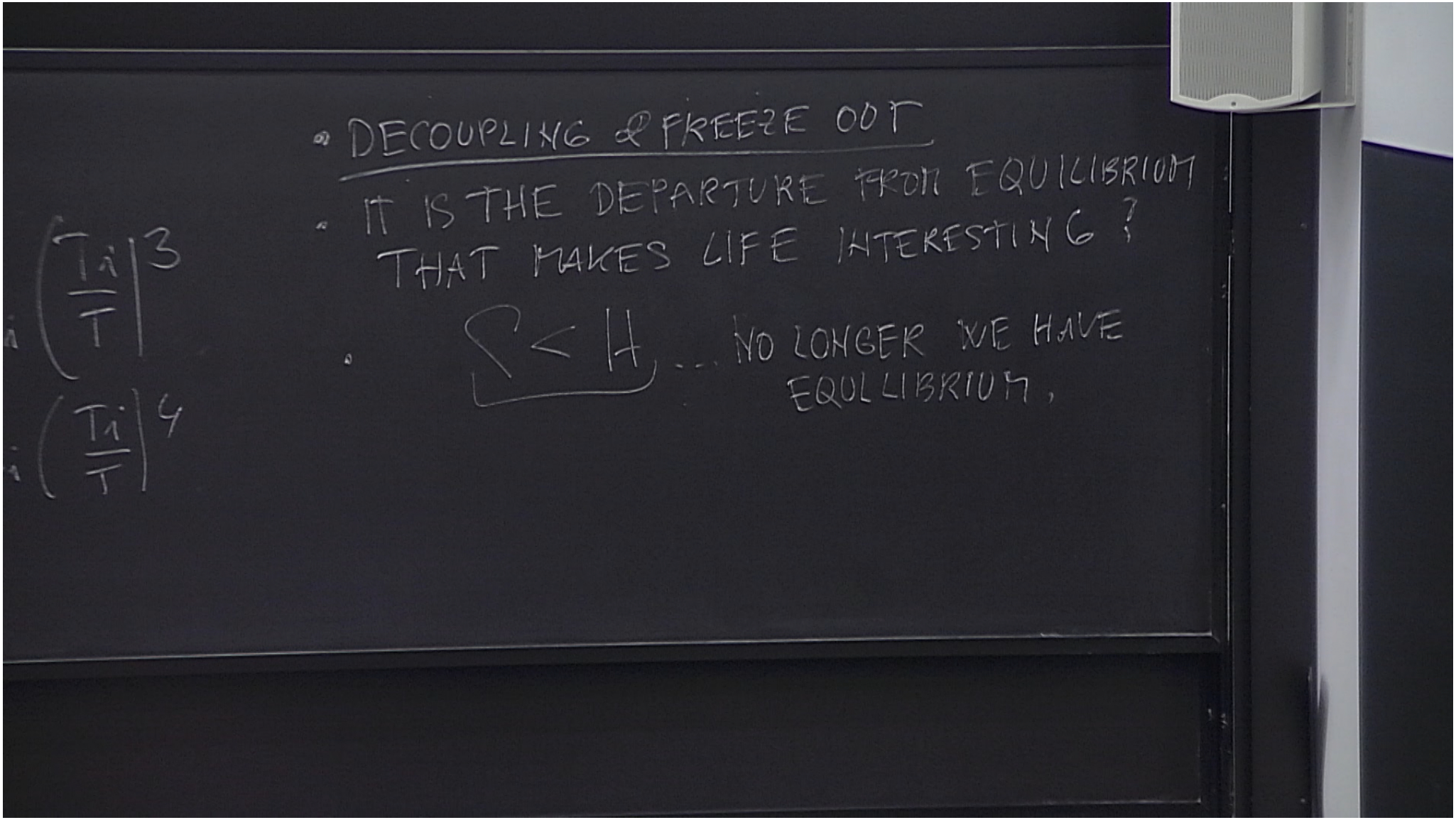
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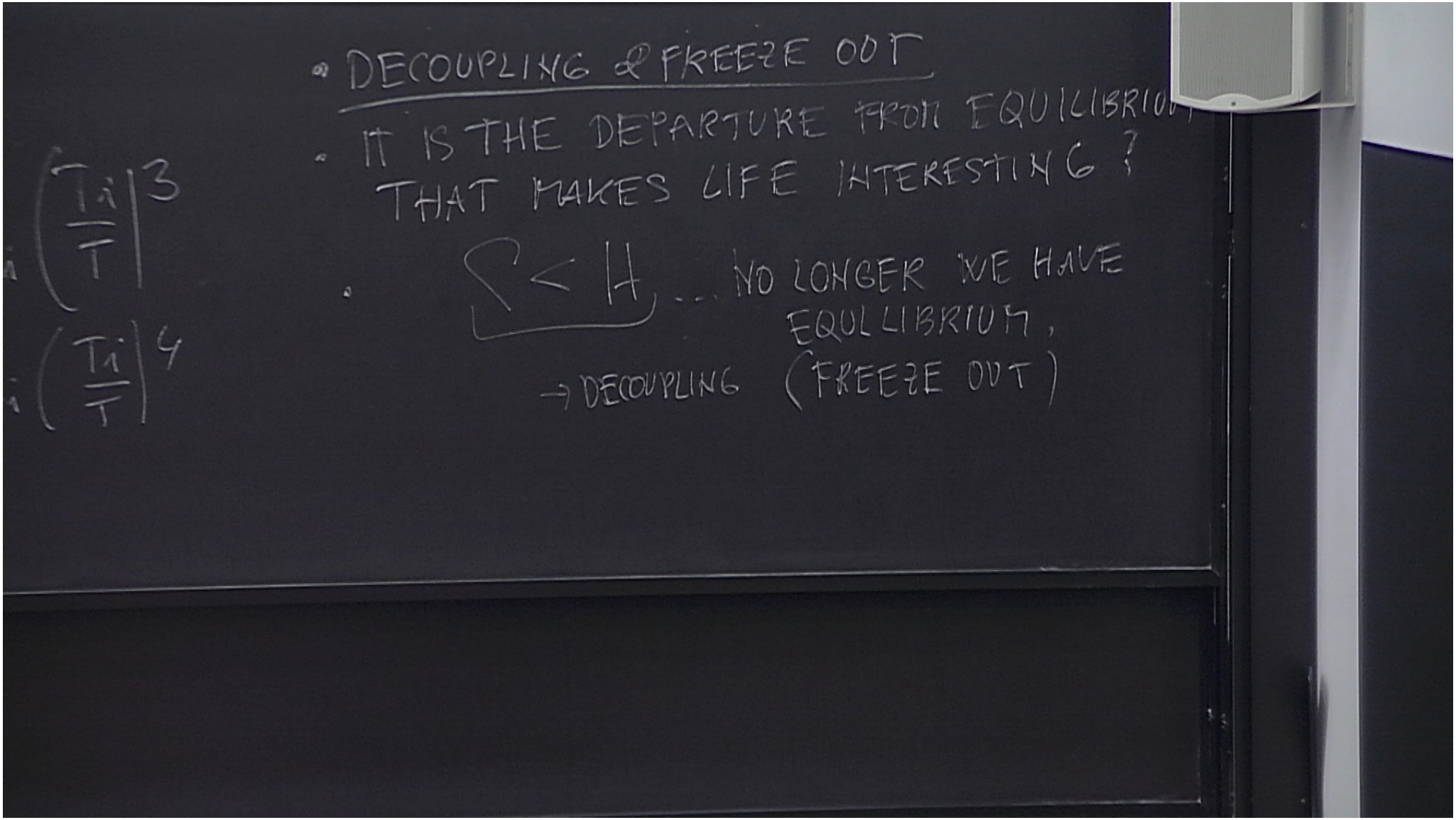
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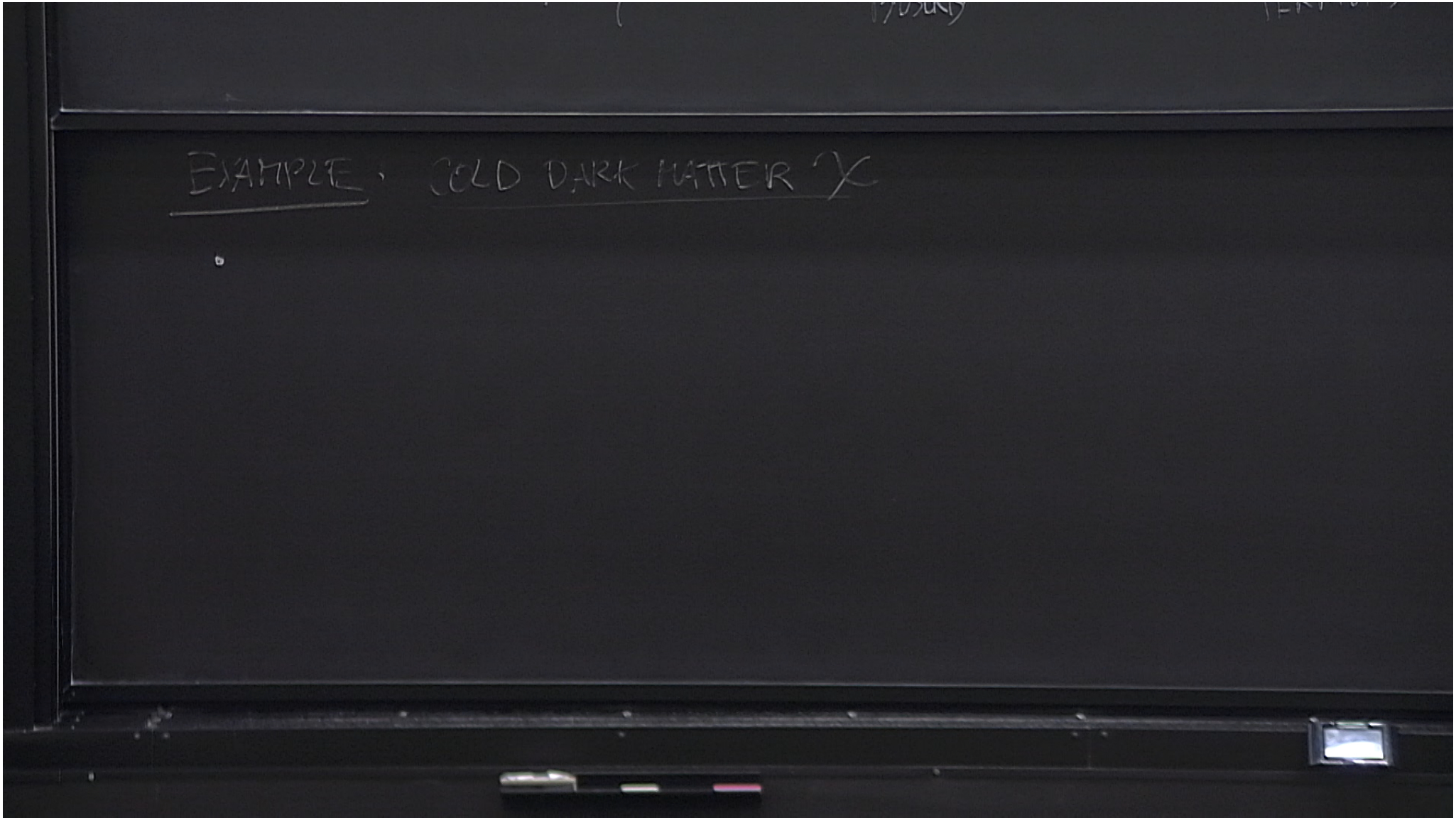
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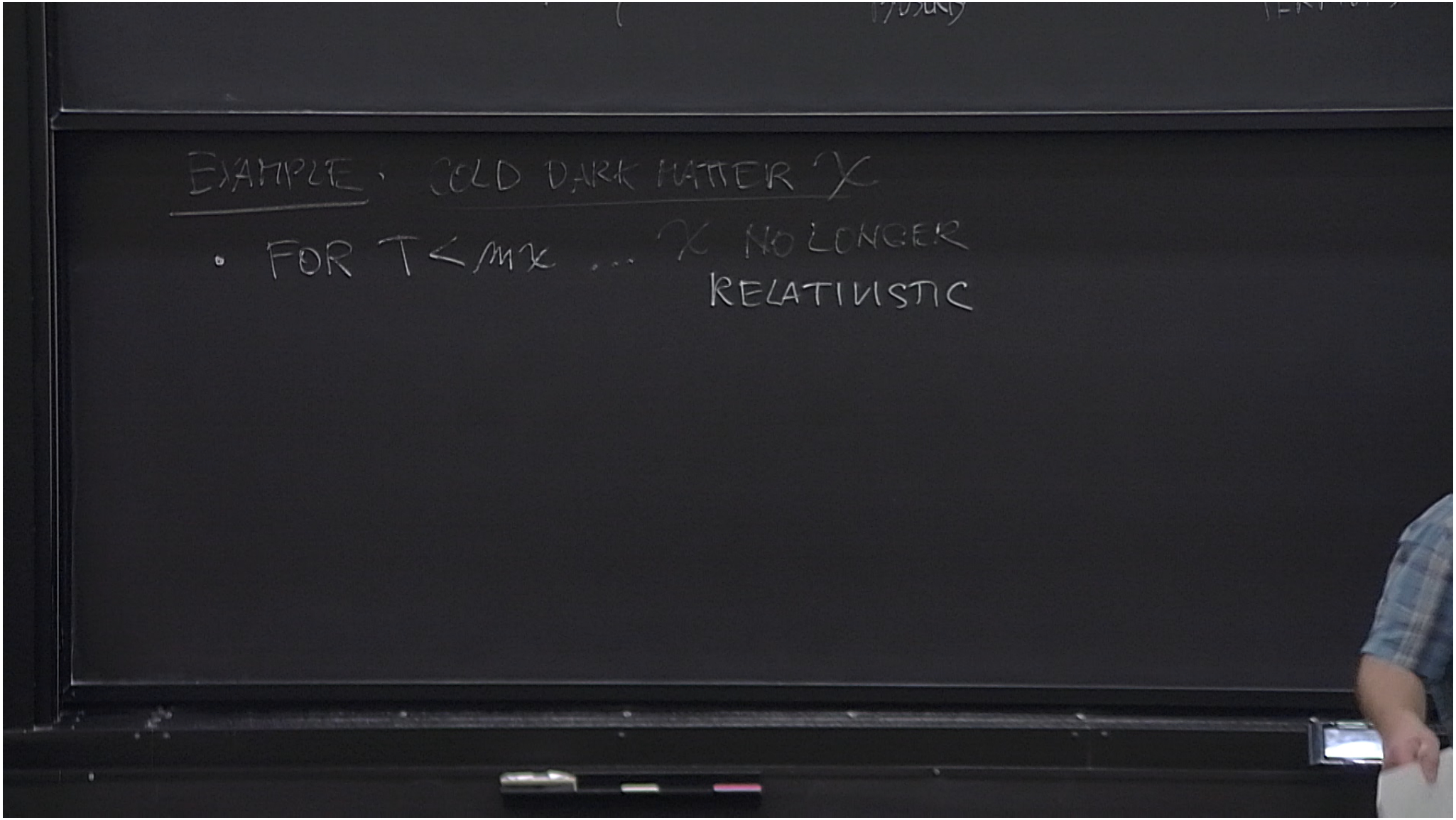
$$s = g_{*s}\frac{2\pi^2}{45}T^3, \quad g_{*s} = \sum_{\text{BOSONS}} g_i\left(\frac{T_i}{T}\right)^3 + \sum_{\text{FERMIONS}} \frac{7}{8} g_i\left(\frac{T_i}{T}\right)^3$$

$$\rho = g_*\frac{\pi^2}{30}T^4, \quad g_* = \sum_{\text{BOSONS}} g_i\left(\frac{T_i}{T}\right)^4 + \sum_{\text{FERMIONS}} \frac{7}{8} g_i\left(\frac{T_i}{T}\right)^4$$


DECOUPLING & FREEZE OUT
IT IS THE DEPARTURE FROM EQUILIBRIUM THAT MAKES LIFE INTERESTING ?
$\Gamma < H$ ... NO LONGER WE HAVE EQUILIBRIUM ,
$\left(\frac{T_i}{T}\right)^3$
$\left(\frac{T_i}{T}\right)^4$


DECOUPLING & FREEZE OUT
IT IS THE DEPARTURE FROM EQUILIBRIUM THAT MAKES LIFE INTERESTING ?
$\Gamma < H$ ... NO LONGER WE HAVE EQUILIBRIUM,
→ DECOUPLING (FREEZE OUT)
$\left(\frac{T_i}{T}\right)^3$
$\left(\frac{T_i}{T}\right)^4$


EXAMPLE: COLD DARK MATTER X


EXAMPLE: COLD DARK MATTER χ
• FOR T < mχ ... χ NO LONGER
RELATIVISTIC

EXAMPLE: COLD DARK MATTER χ

- FOR $T < m_\chi$ ... χ NO LONGER RELATIVISTIC
  - $\Rightarrow$ EXPONENTIALLY SUPPRESSED
- ASSUME THAT $\chi + \bar{\chi} \to 2\gamma$

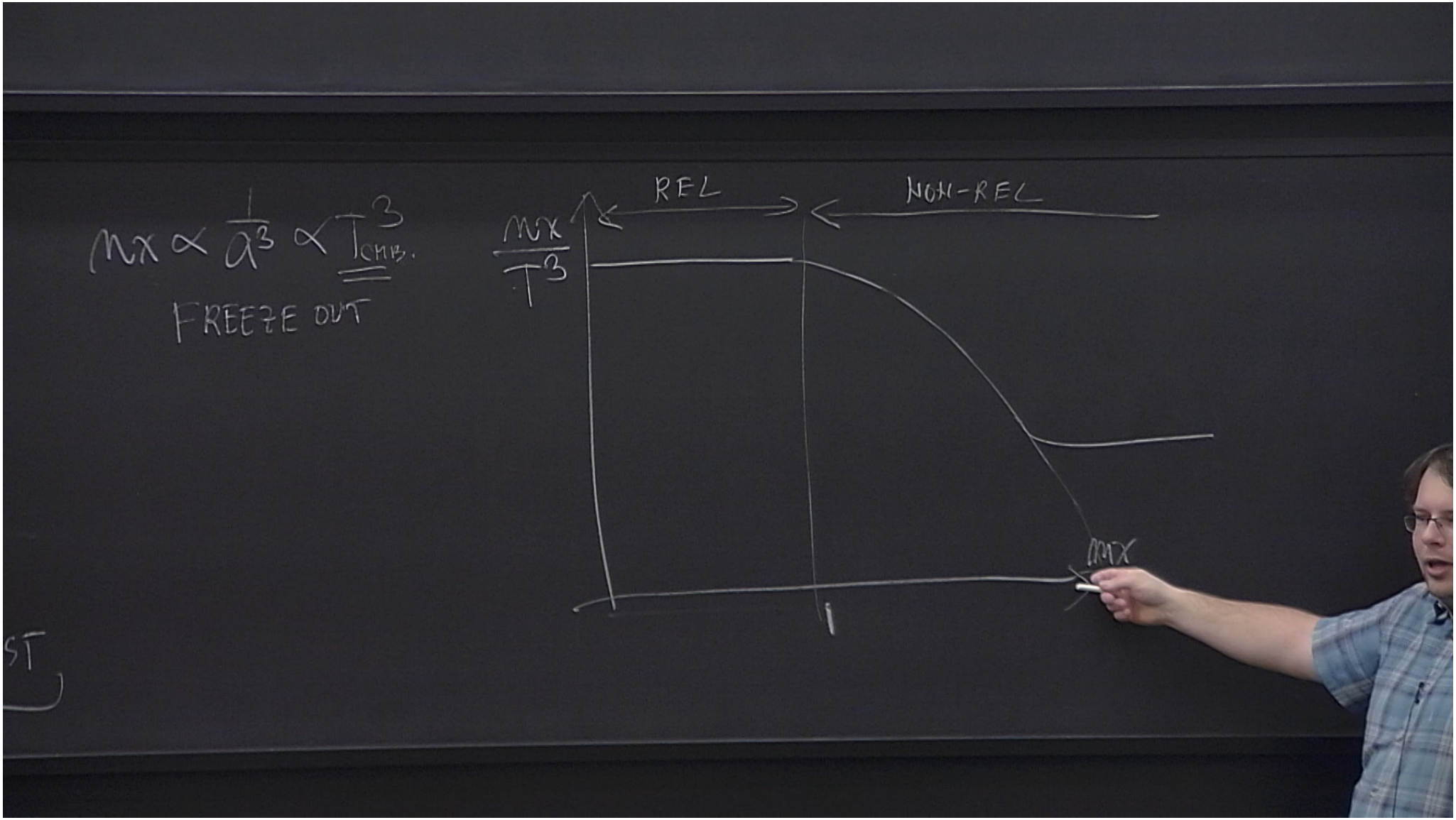
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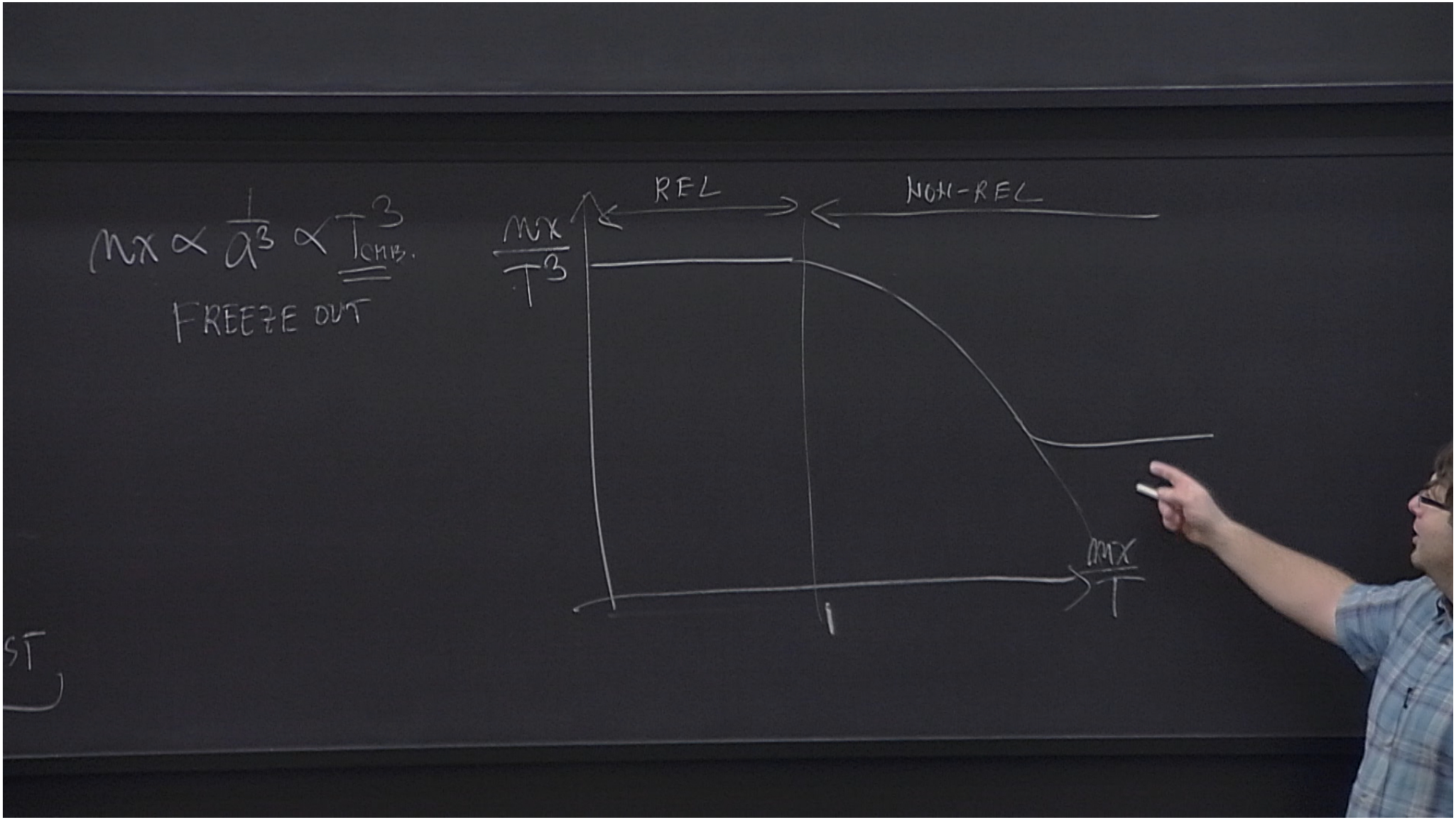
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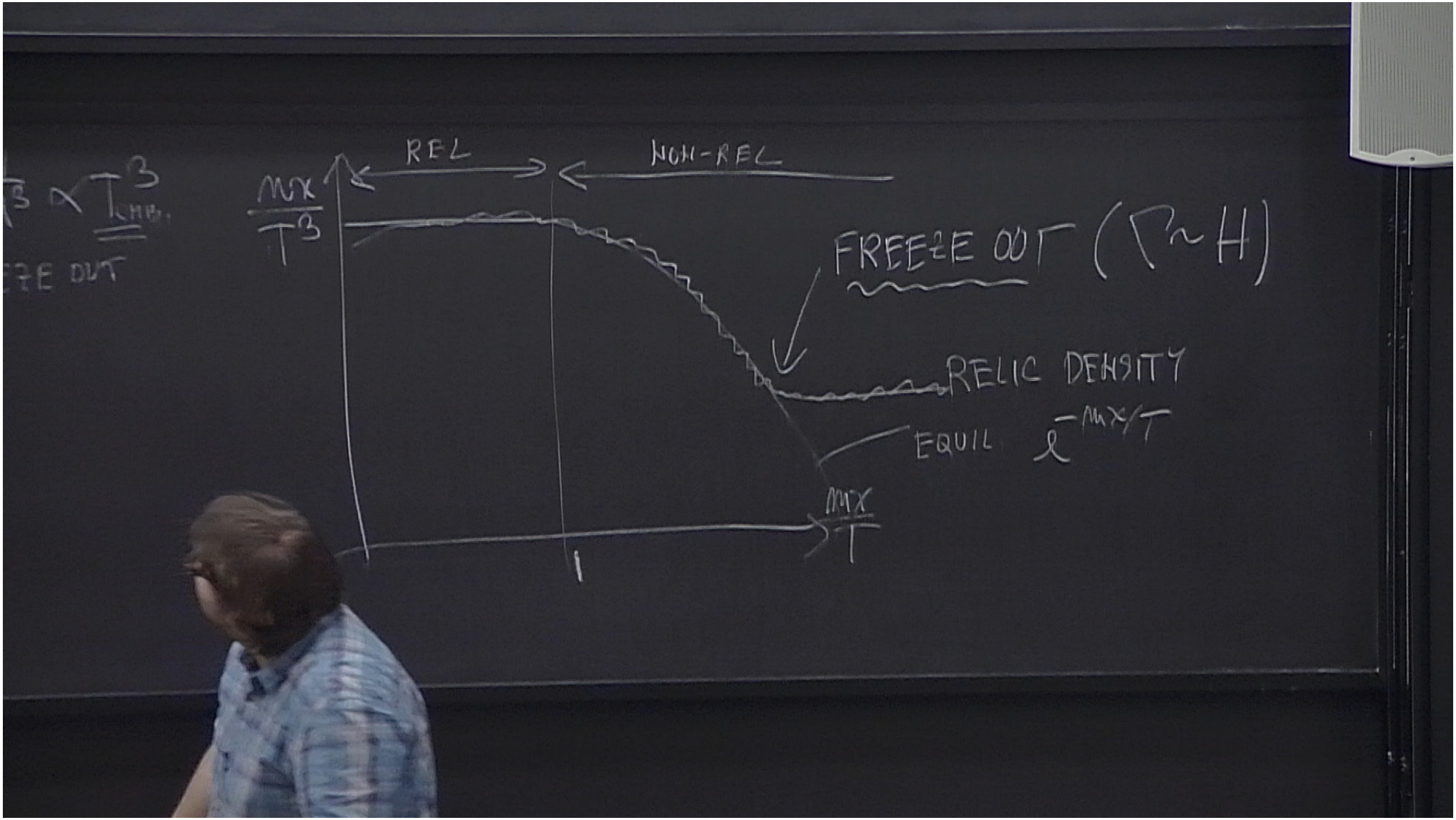
EXAMPLE: COLD DARK MATTER χ

$n_\chi \propto \frac{1}{a^3} \propto T^3$

- FOR $T < m_\chi$ ... χ NO LONGER RELATIVISTIC
  - $\Rightarrow$ EXPONENTIALLY SUPPRESSED
- ASSUME THAT $\chi + \bar{\chi} \to 2n$
- WHEN $\Gamma < H$ χ... CANNOT FIND $\bar{\chi}$ TO ANNIHILATE WITH $\Rightarrow$ $N_\chi = \text{CONST}$


$n_X \propto \frac{1}{a^3} \propto T_{CMB}^3$
FREEZE OUT
$\frac{n_X}{T^3}$
REL
NON-REL
m_X


$n_X \propto \frac{1}{a^3} \propto T_{CMB}^3$
FREEZE OUT
REL
NON-REL
$\frac{n_X}{T^3}$
$\frac{m_X}{T}$
ST


REL
NON-REL
m_X/T_3
FREEZE OUT ($\Gamma \sim H$)
RELIC DENSITY
EQUIL $e^{-m_X/T}$
m_X/T

- 3 REMARKS ON CHEM. POTENTIALS

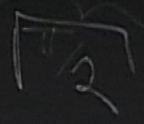
- IN QFT PARTICLES CAN POP OUT OF VACUUM

  UNLESS THERE IS A CONSERVATION LAW PROTECTING PARTICLE #: $\boxed{M = 0}$ ∴ PHOTONS

- EQUIL. PROCESS $(dE = TdS + \mu dN)$

$$A + B \rightleftarrows C + D$$

$$\boxed{\mu_A + \mu_B = \mu_C + \mu_D}$$



- 3 REMARKS ON CHEM. POTENTIALS

- IN QFT PARTICLES CAN POP OUT OF VACUUM

  UNLESS THERE IS A CONSERVATION LAW

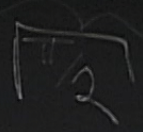
  PROTECTING PARTICLE #: $\mu = 0$ ∴ PHOTONS

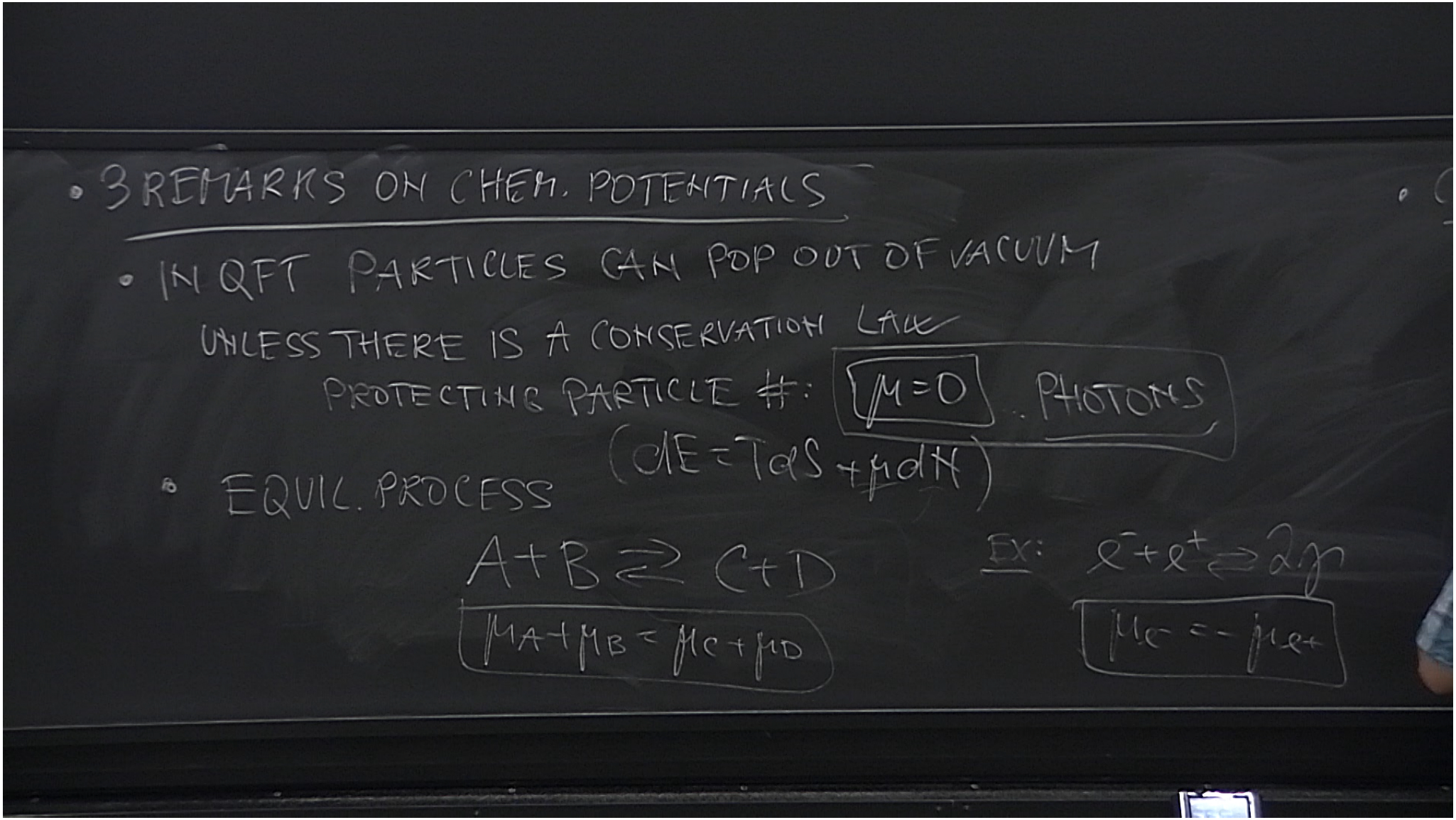
- EQUIL. PROCESS $(dE = TdS + \mu dN)$

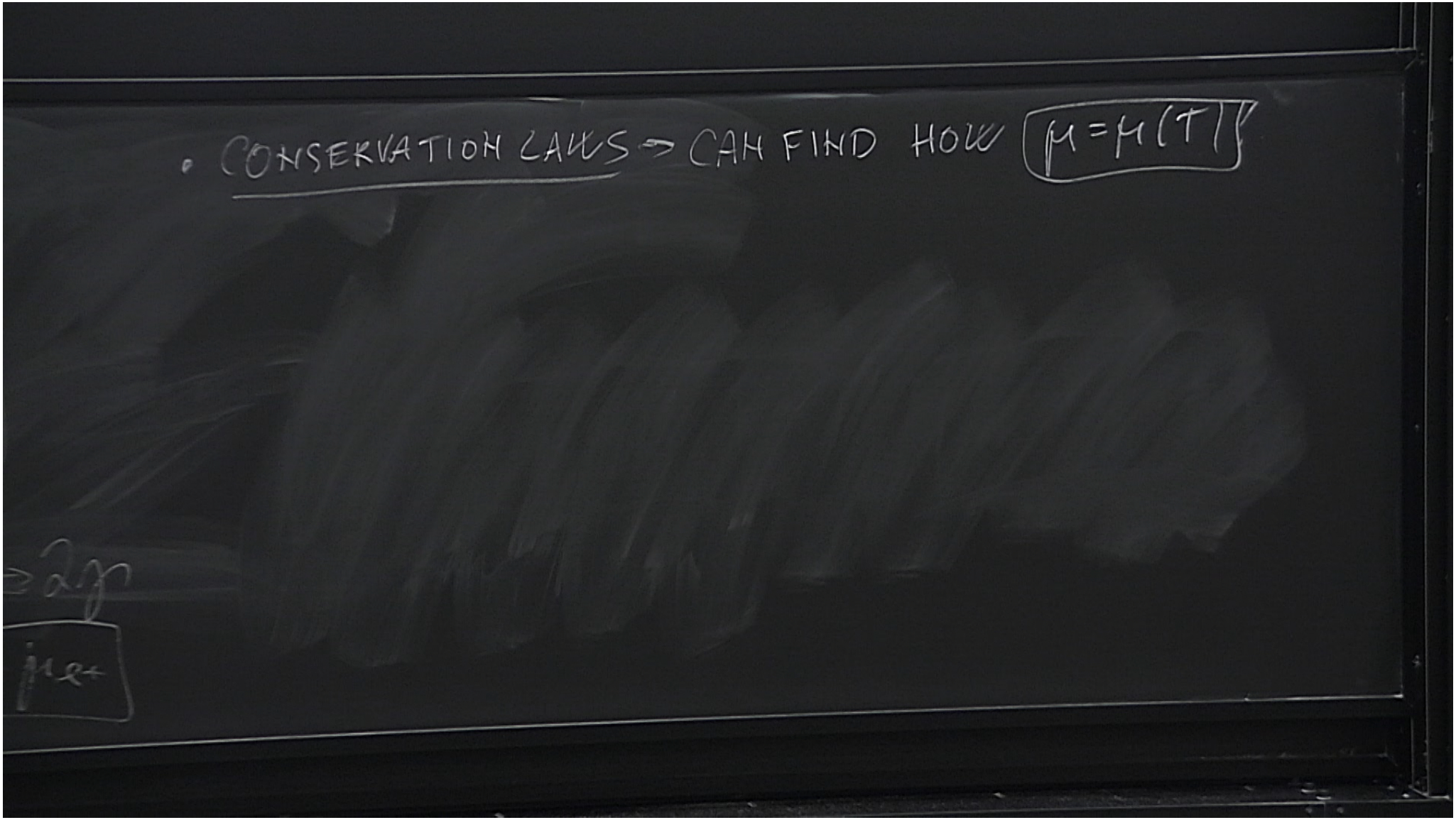
$$A + B \rightleftarrows C + D$$

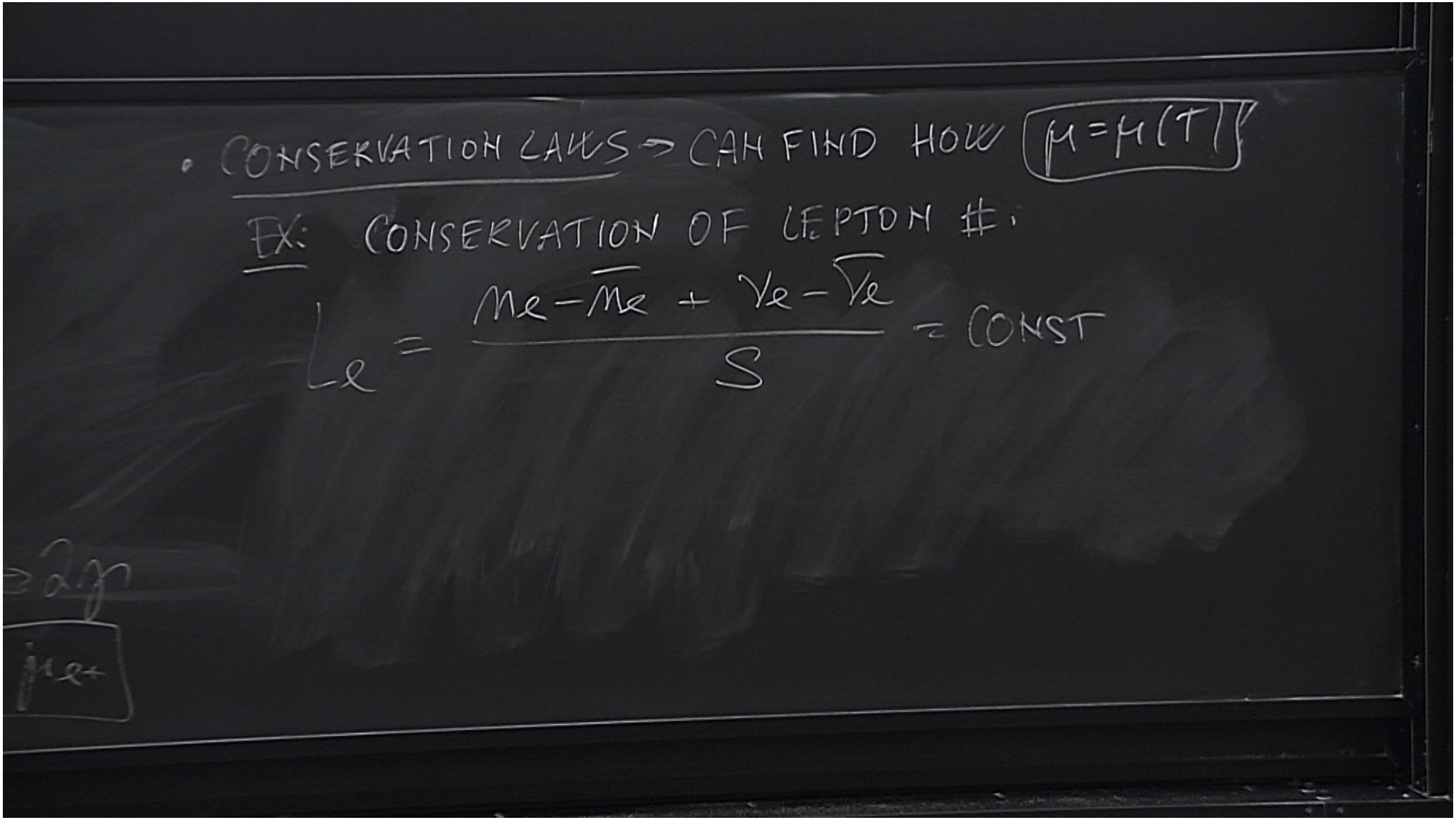
$$\mu_A + \mu_B = \mu_C + \mu_D$$

Ex: $e^- + e^+ \rightleftarrows 2\gamma$




• 3 REMARKS ON CHEM. POTENTIALS
• IN QFT PARTICLES CAN POP OUT OF VACUUM
UNLESS THERE IS A CONSERVATION LAW
PROTECTING PARTICLE #: μ=0 ∴ PHOTONS
(dE = TdS + μdN)
• EQUIL. PROCESS
A + B ⇄ C + D
μA + μB = μC + μD
Ex: e⁻ + e⁺ ⇄ 2γ
μe⁻ = −μe⁺


• CONSERVATION LAWS ⇒ CAN FIND HOW μ=μ(T)


• CONSERVATION LAWS → CAN FIND HOW μ = μ(T)
EX: CONSERVATION OF LEPTON #:
$L_e = \frac{n_e - \bar{n}_e + \nu_e - \bar{\nu}_e}{S} = \text{CONST}$
≥ 2p
μe+

• CONSERVATION LAWS ⇒ CAN FIND HOW $\mu = \mu(T)$

EX: CONSERVATION OF LEPTON #:

$$L_e = \frac{n_e - \bar{n}_e + \nu_e - \bar{\nu}_e}{S} = \text{CONST}$$

$$n_e - \bar{n}_e = T^3 \frac{\mu_e}{T}$$

$\gtrsim 2p$

μ_{e^+}

• CONSERVATION LAWS → CAN FIND HOW $\mu = \mu(T)$

EX: CONSERVATION OF LEPTON #:

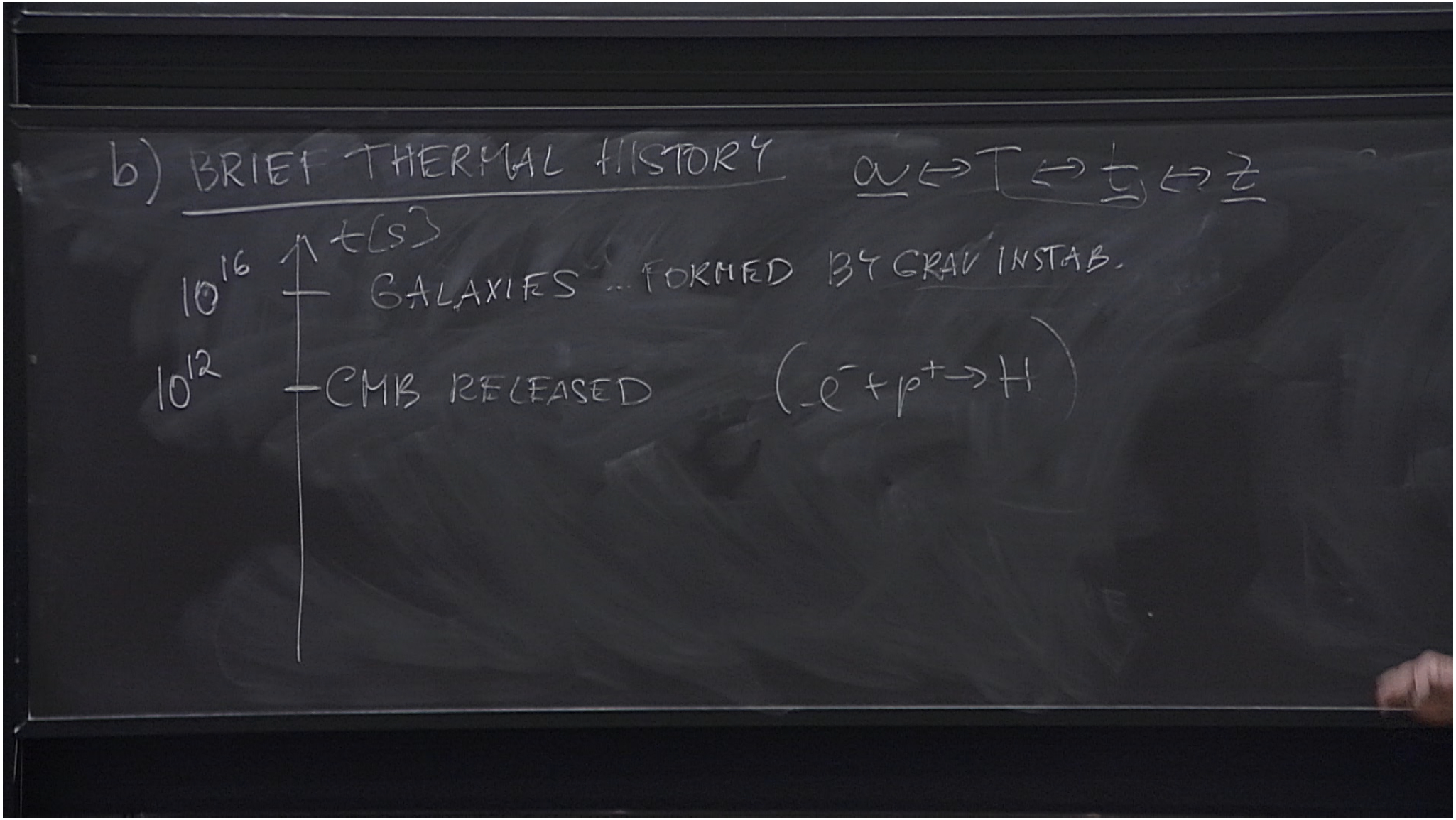
$$L_e = \frac{n_e - \bar{n}_e + \nu_e - \bar{\nu}_e}{S} = \text{CONST}$$

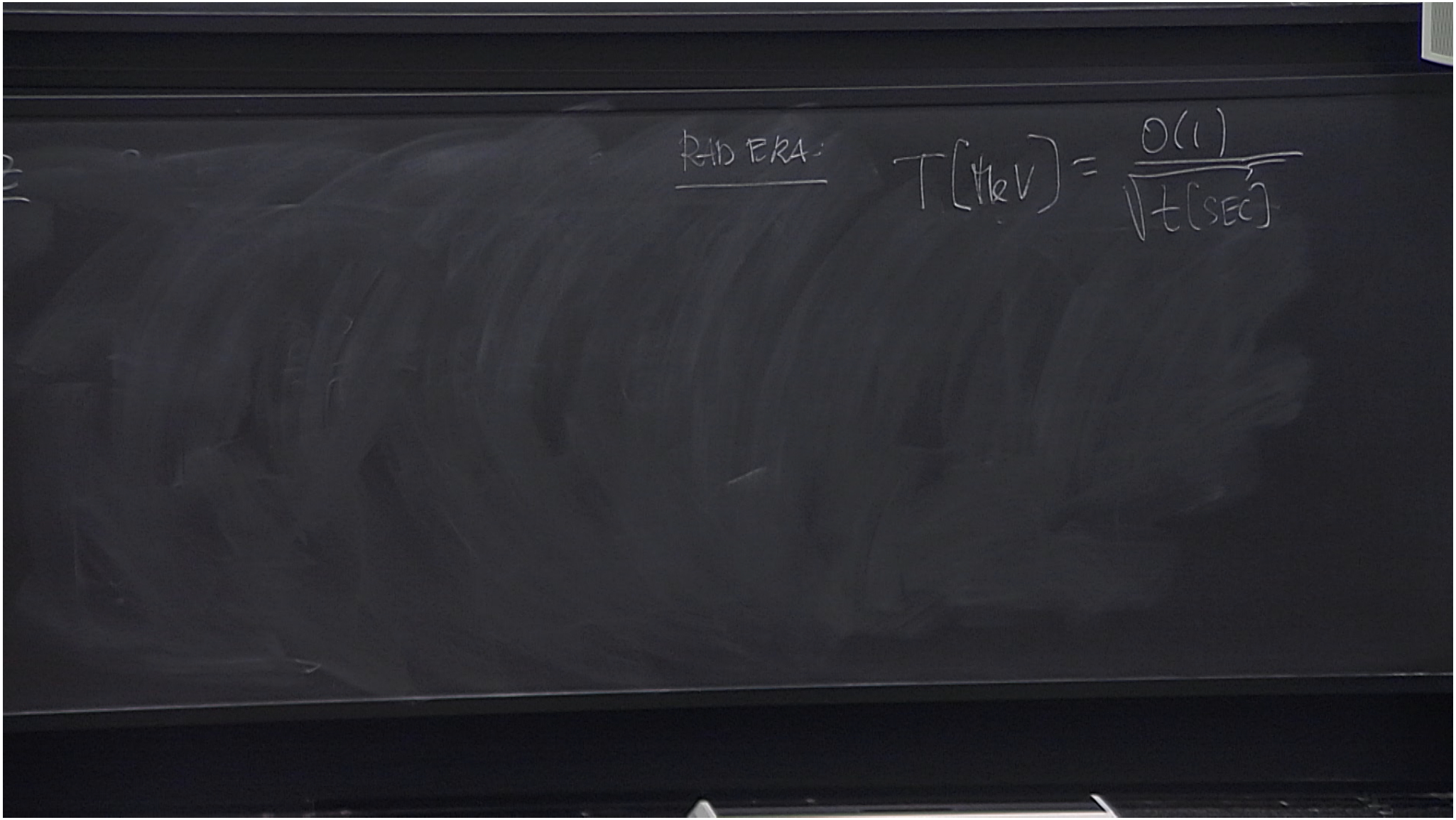
$$n_e - \bar{n}_e = T^3 \frac{\mu_e}{T}$$

$$\frac{\mu_e + \mu_{\nu_e}}{T} = \text{CONST}$$

$\Rightarrow 2\mu$

μ_{e^+}


b) BRIEF THERMAL HISTORY
a ↔ T ↔ t ↔ z
t[s]
10^{16}
GALAXIES .. FORMED BY GRAV INSTAB.
10^{12}
CMB RELEASED
($e^- + p^+ \rightarrow H$)


RAD ERA:
T[MeV] = O(1) / √t[SEC]

b) BRIEF THERMAL HISTORY

$a \leftrightarrow T \leftrightarrow t \leftrightarrow z$

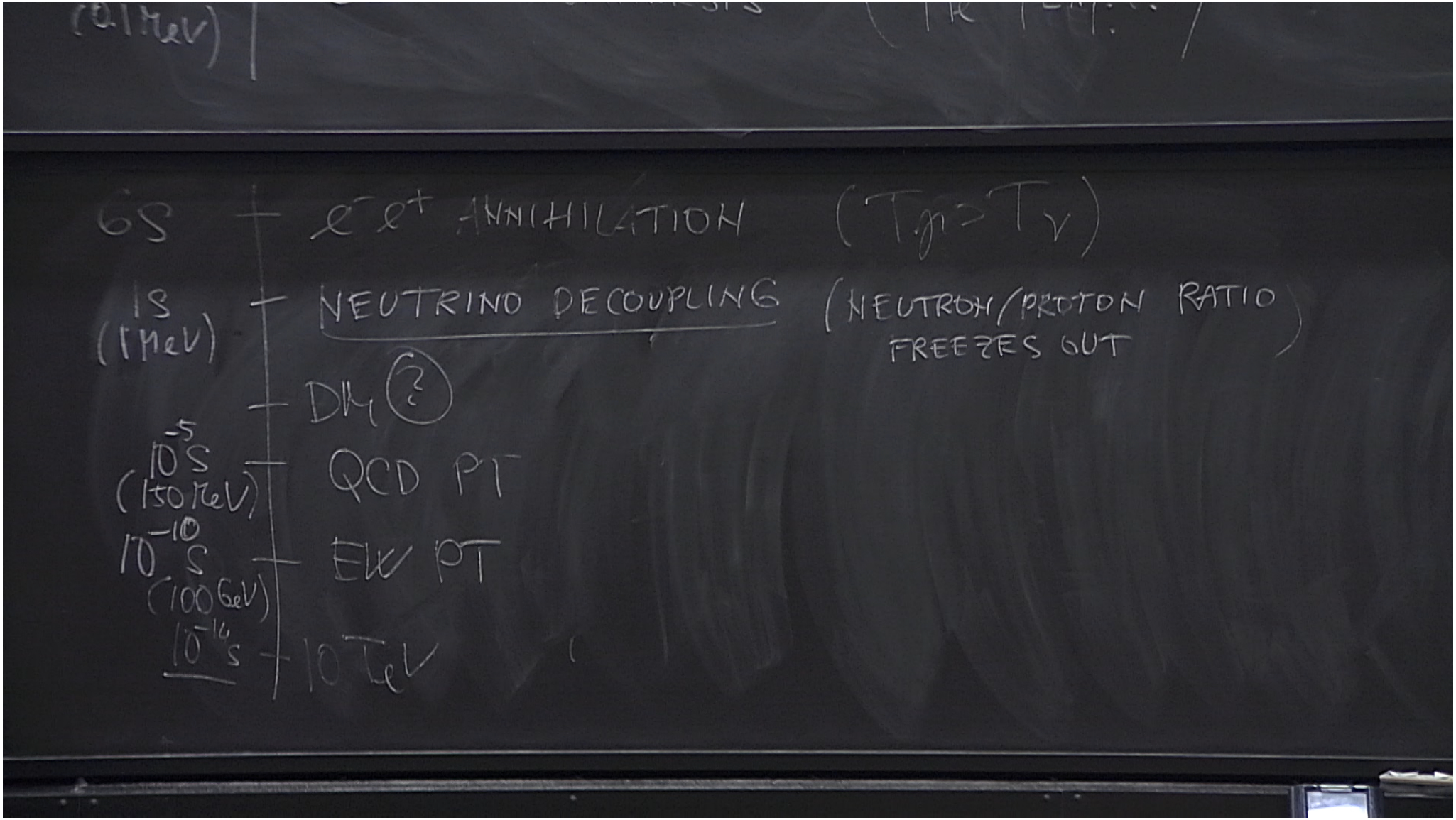
t [s]

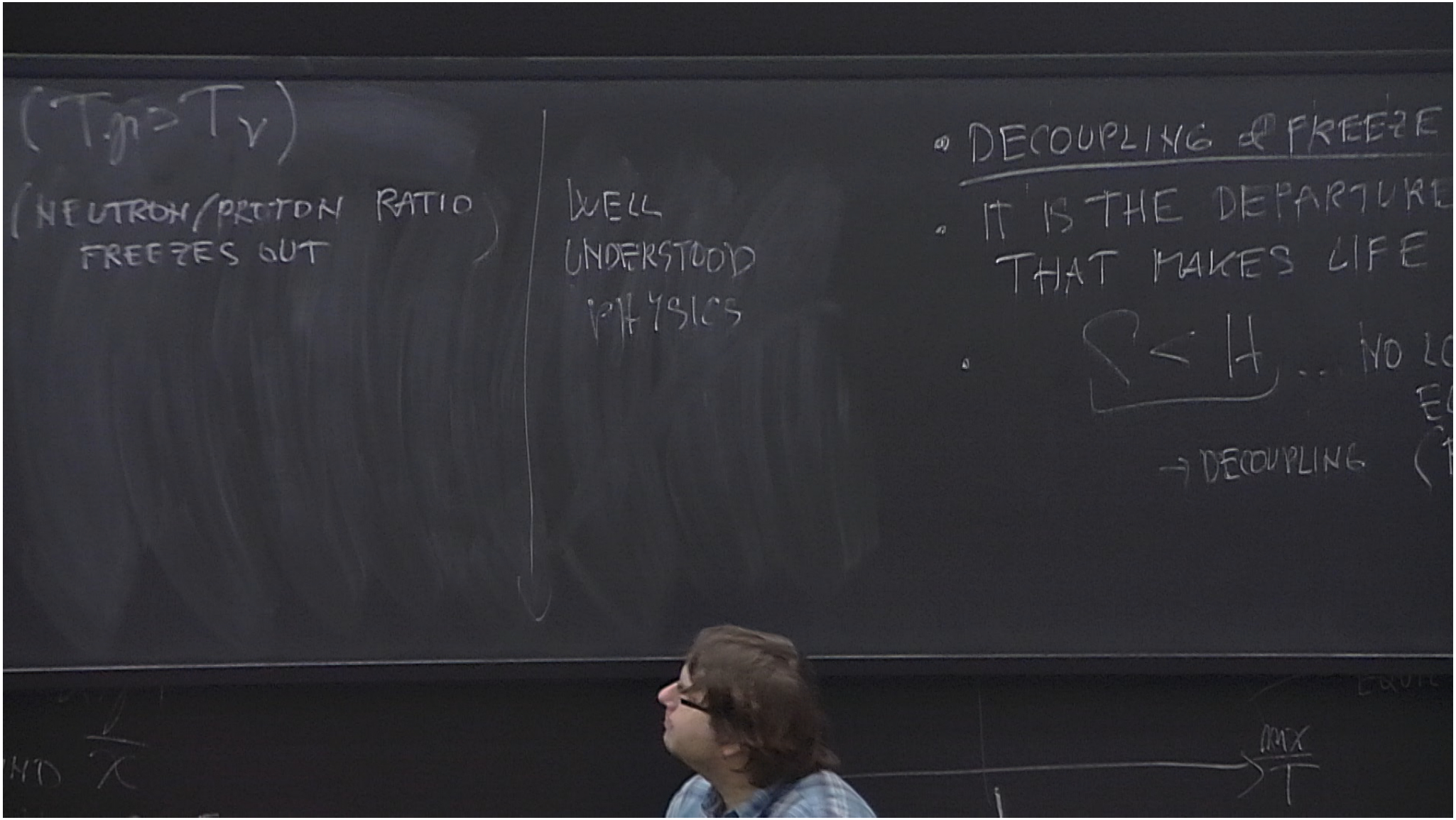
10^{16} — GALAXIES ... FORMED BY GRAV INSTAB.

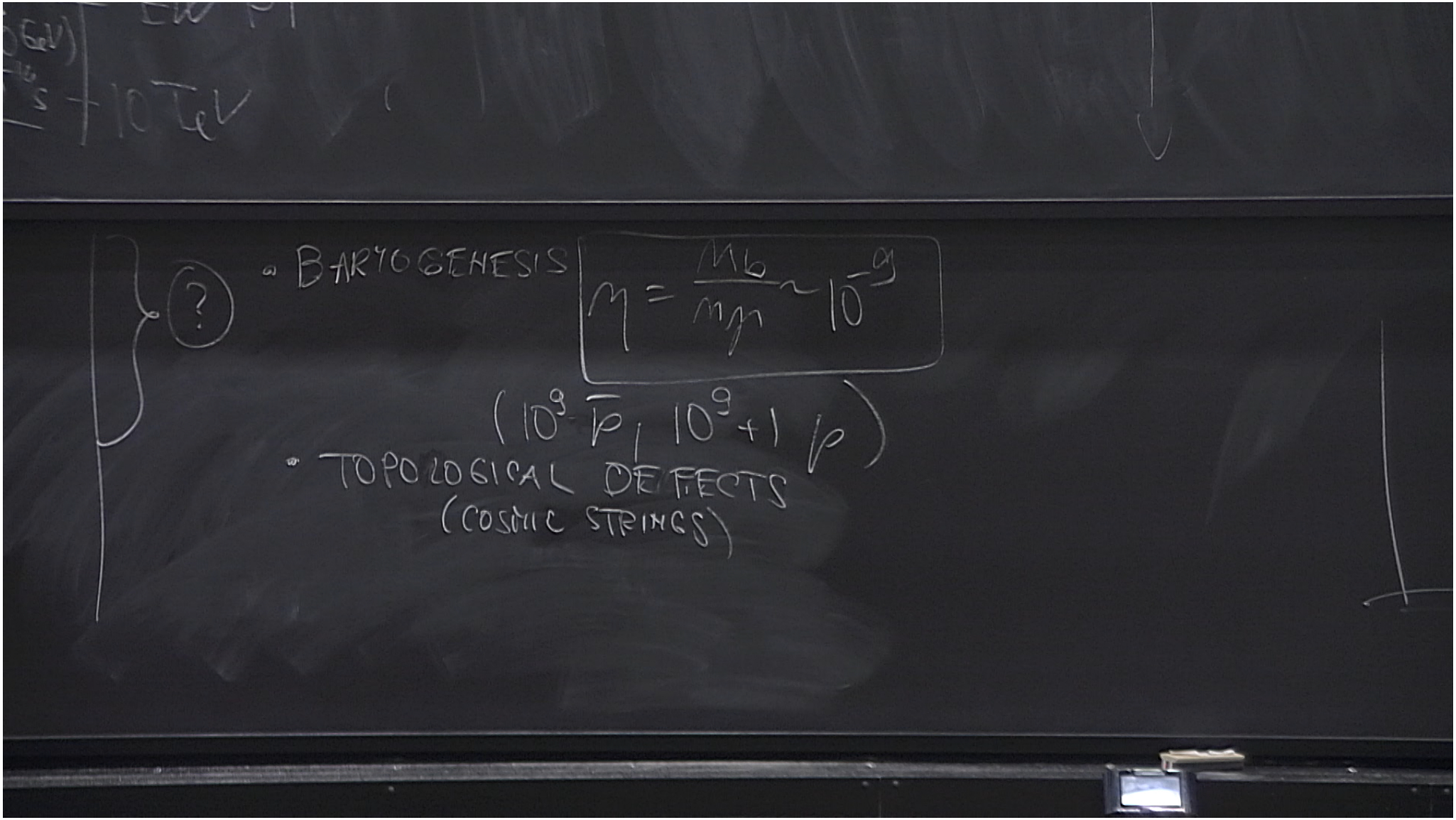
10^{12} — CMB RELEASED ($e^- + p^+ \rightarrow H$)

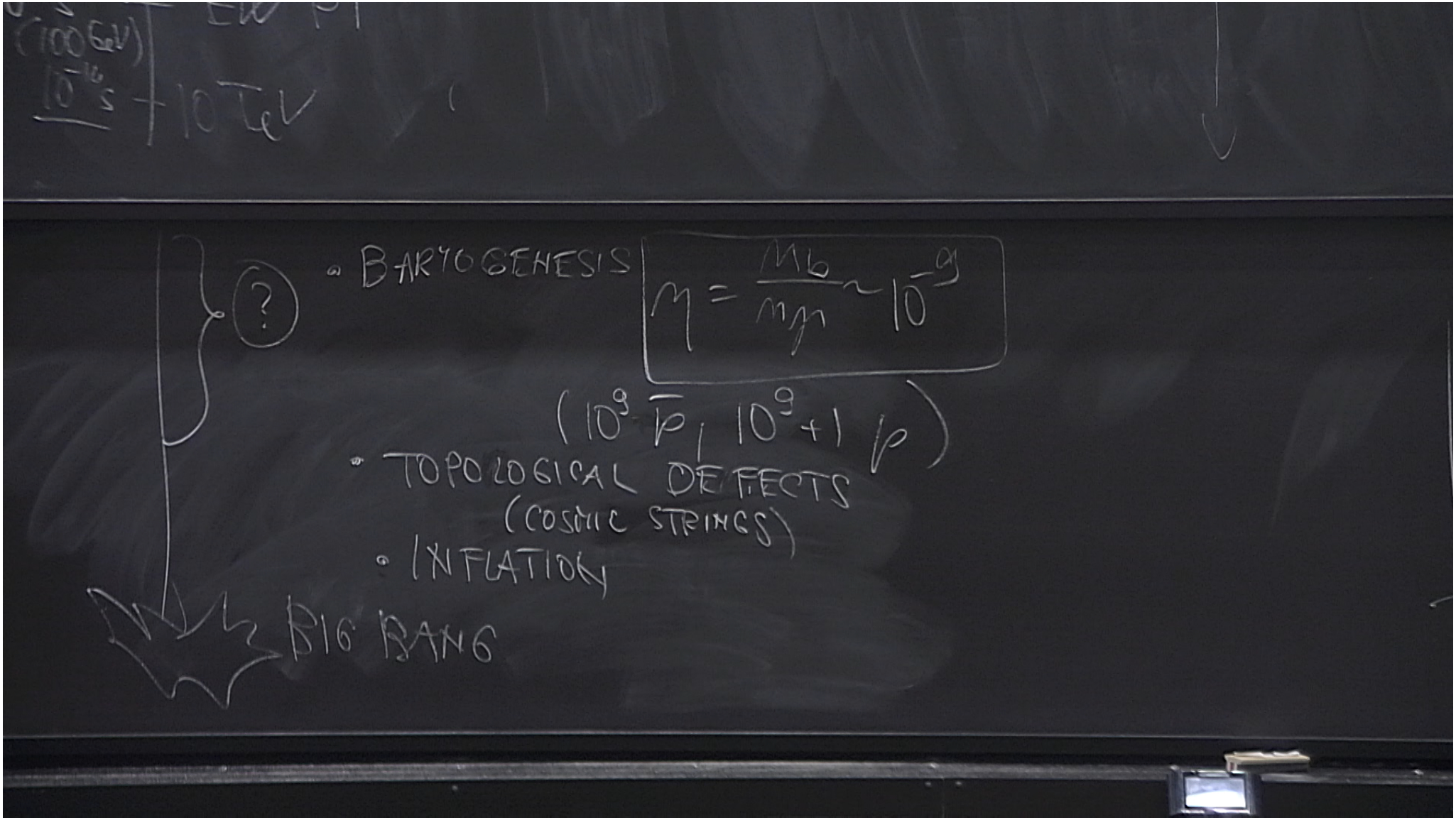
10^{11} — RAD-MATTAR EQ.

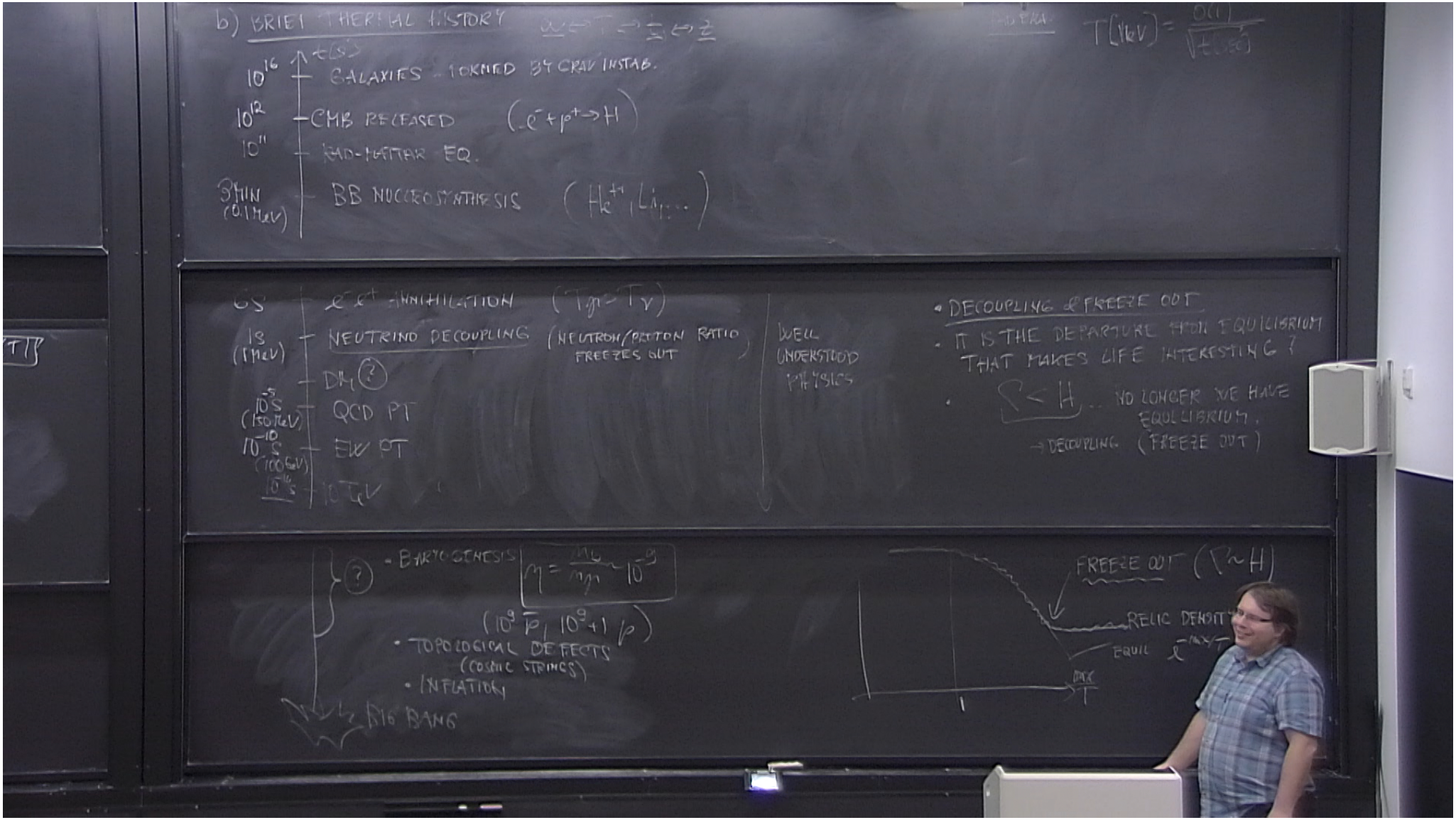
3 MIN (0.1 MeV) — BB NUCLEOSYNTHESIS (He^{+1}, Li, ...)


(0.1 MeV)
6S
$e^- e^+$ ANNIHILATION ($T_\gamma = T_\nu$)
1S
(1 MeV)
NEUTRINO DECOUPLING (NEUTRON/PROTON RATIO FREEZES OUT)
DM (?)
10^{-5} S
(150 MeV)
QCD PT
10^{-10} S
(100 GeV)
EW PT
10^{-14} S
10 TeV


$(T_{\gamma n} = T_\nu)$
(NEUTRON/PROTON RATIO FREEZES OUT)
WELL UNDERSTOOD PHYSICS
• DECOUPLING & FREEZE
• IT IS THE DEPARTURE THAT MAKES LIFE
$S < H$ ... NO LO
→ DECOUPLING
$\frac{1}{T}$
$\frac{mx}{T}$


+10 TeV
?
• BARYOGENESIS
$\eta = \frac{n_b}{n_\gamma} \sim 10^{-9}$
$(10^9 \bar{p}, 10^9 + 1\ p)$
• TOPOLOGICAL DEFECTS
(COSMIC STRINGS)


(100 GeV)
10^{-16} s
+10 TeV
?
• BARYOGENESIS
$\eta = \frac{n_b}{n_\gamma} \sim 10^{-9}$
$(10^9\,\bar{p},\ 10^9+1\,p)$
• TOPOLOGICAL DEFECTS
(COSMIC STRINGS)
• INFLATION
BIG BANG


b) BRIEF THERMAL HISTORY
GALAXIES - FORMED BY GRAV INSTAB.
CMB RELEASED
RAD-MATTER EQ.
BB NUCLEOSYNTHESIS
e⁻e⁺ ANNIHILATION
NEUTRINO DECOUPLING
QCD PT
EW PT
WELL UNDERSTOOD PHYSICS
• DECOUPLING & FREEZE OUT
• IT IS THE DEPARTURE FROM EQUILIBRIUM THAT MAKES LIFE INTERESTING?
NO LONGER WE HAVE EQUILIBRIUM.
→ DECOUPLING (FREEZE OUT)
• BARYOGENESIS
• TOPOLOGICAL DEFECTS (COSMIC STRINGS)
• INFLATION
BIG BANG
FREEZE OUT
RELIC DENSITY
EQUIL