Title: PSI 17/18 - Cosmology - Lecture 4

Date: Feb 01, 2018 11:30 AM

URL: http://pirsa.org/18020013

Abstract:

PROVIDED

DYNAMICS OF FRW:

$$\boxed{\begin{aligned} ds^2 &= -dt^2 + a^2(t)\, g^{(3,0)k}_{ij}\, dx^i dx^j \\ T_{\mu\nu} &= P g_{\mu\nu} + (\rho + P) u_\mu u_\nu \end{aligned}}$$

WANT TO SOLVE FOR $a(t)$, $\rho(t)$, $P(t)$

$$\boxed{\begin{aligned} \dot{\rho} &= -3H(\rho + P) \\ H^2 &= \frac{8\pi G_N}{3}\rho - k/a^2 \\ \rho &= \rho(P) \end{aligned}}$$

FRIEDMANN EQS.

+

EQ OF STATE

PROVIDED $w = P/\rho = \text{CONST.}$ CAN SOLVE 1ST. EQ $\boxed{\rho \propto a^{-3(1+w)}}$

THE SECOND STEP IS TO SOLVE

2ND EQ FOR $a = a(t) = a(\underline{\eta})$

=> "DRIVEN HARMONIC OSCILLATOR"

$$\boxed{a(\eta) = \left[\frac{8\pi G}{3}\rho_0 a_0^{3(1+w)}\right]^{\frac{\alpha}{2}} S_k^{\alpha}(\eta/\alpha), \quad \alpha = \frac{2}{1+3w}}$$

SINGLE COMPONENT UNIVERSE

$t \sim \int a(\eta)\, d\eta$

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$$
$$\rho = \rho(P)$$

+ EQ OF STATE

SINGLE COMPONENT UNIV

$\boxed{K=0}$

COMPONENT	w	DENSITY	$a(\eta)$	$a(t)$
MATTER	0	$1/a^3$	η^2	$t^{2/3}$
RADIATION	$1/3$	$1/a^4$	η	$t^{1/2}$
Λ	-1	a^0	$-\frac{1}{\eta H}$	e^{Ht}, H CONST

SINGLE COMPONENT UNIVERSE $\qquad t \sim \int a(\eta)\, d\eta$

...TY	$a(\eta)$	$a(t)$
3	η^2	$t^{2/3}$
4	η	$t^{1/2}$
0	$-\frac{1}{\eta H}$	e^{Ht} (H: CONST)

CURRENT COSMOLOGICAL MODEL

- Λ CDM
  COLD DARK MATTER
- $\boxed{\Omega_I(t) = \frac{\rho_I(t)}{\rho_c(t)}}$

$I \in \{\text{MATTER}, \text{RAD}, \Lambda, K\}$

$$\Omega_m \simeq 0.32 \begin{cases} \Omega_{DM} \simeq 0.27 \\ \Omega_B \simeq \underline{0.05} \end{cases}$$

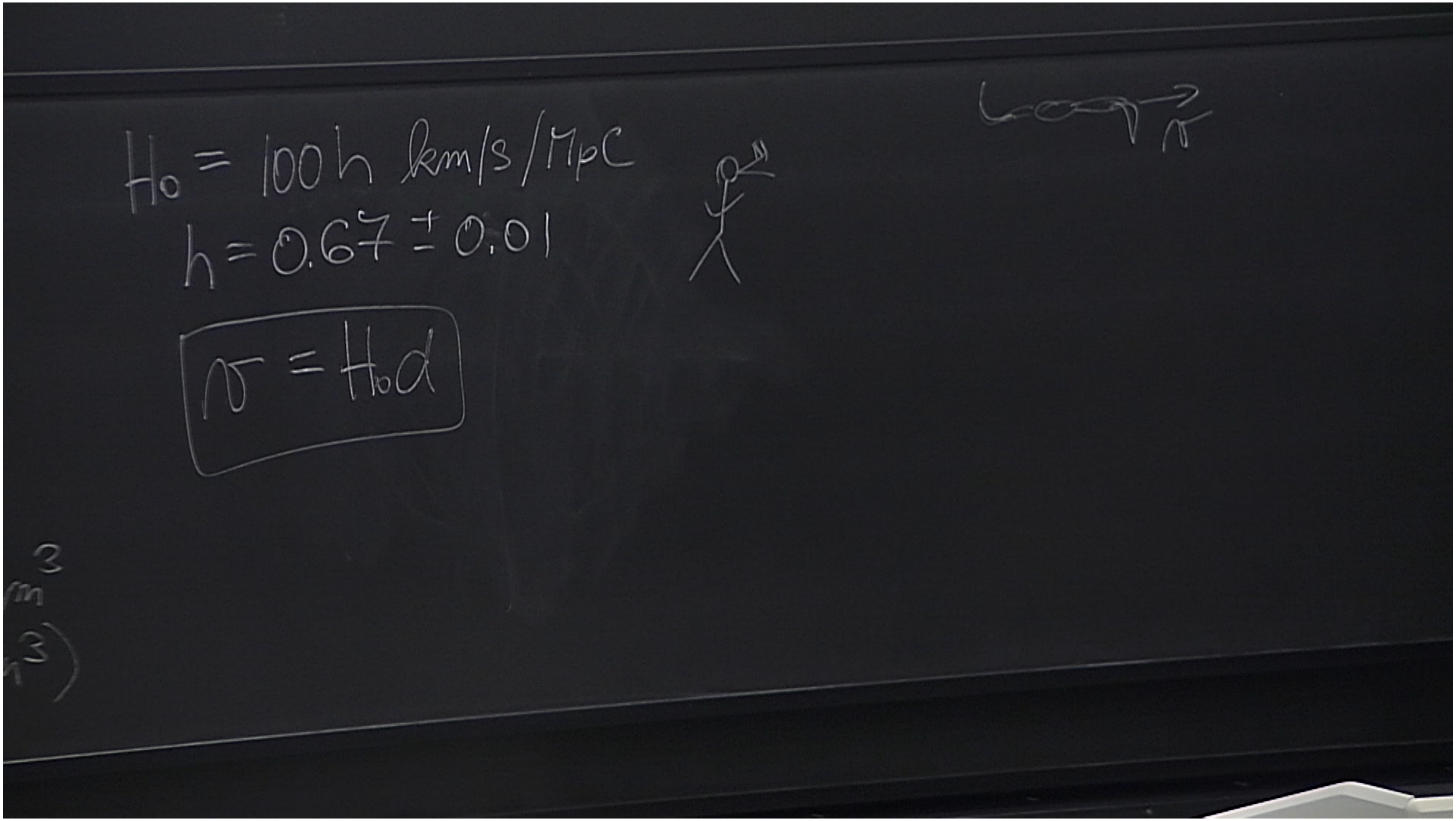
$$H_0 =$$

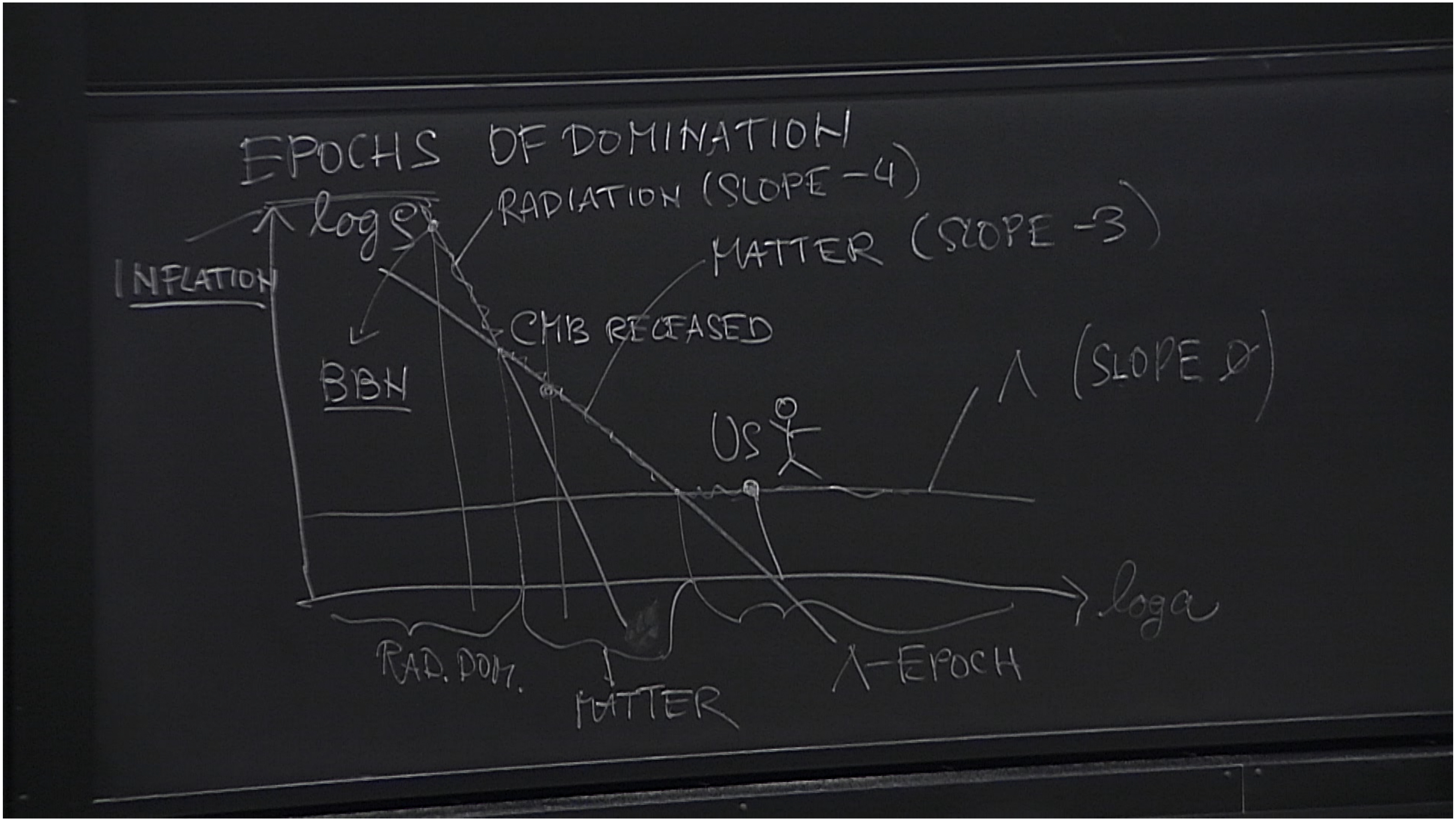
$$\Omega_\Lambda \simeq 0.68$$

$$|\Omega_K| \leq 0.01$$

$$\rho_c = \frac{3H^2(t)}{8\pi G_N} \longrightarrow \rho_0 \simeq 10^{-26}\ \mathrm{kg/m^3}$$

$$(10 \times \mathrm{H/m^3})$$


H0 = 100 h km/s/MpC
h = 0.67 ± 0.01
v = H0 d



EPOCHS OF DOMINATION
log ρ
RADIATION (SLOPE −4)
MATTER (SLOPE −3)
INFLATION
CMB RELEASED
BBN
Λ (SLOPE 0)
US
log a
RAD. DOM.
MATTER
Λ-EPOCH

"WE ARE HERE TO OBSERVE IT."

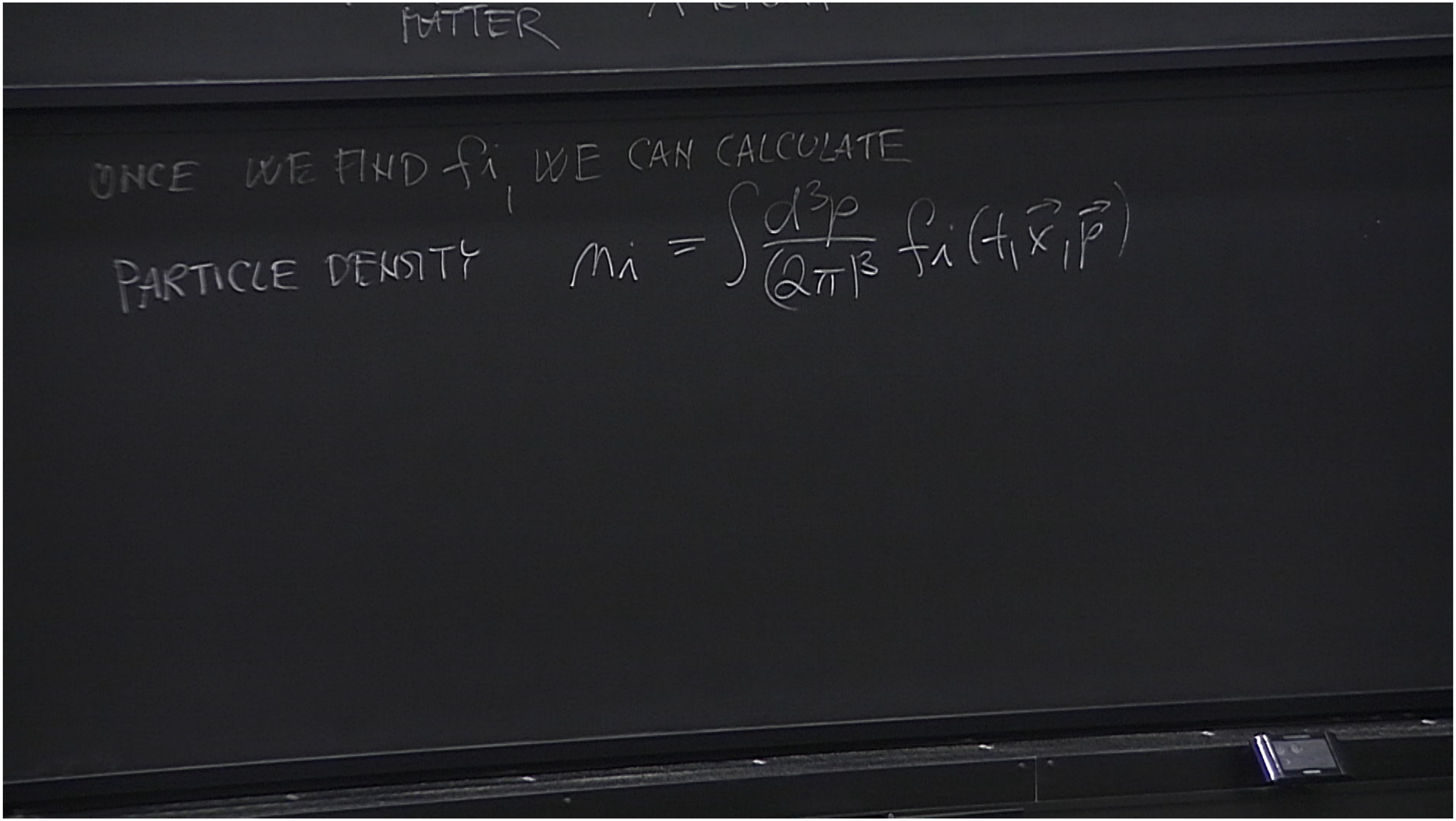
$\boxed{\text{II) MATTER}}$

a) THERMODYNAMICS IN EXPANDING UNIVERSE

- DISTRIBUTION FUNCTION

$$\#\text{PARTICLES}_i = dN_i = \underbrace{f_i}_{\text{DISTR. F.}}(\vec{x}, \vec{p}, t)\,\frac{d^3x\,d^3p}{(2\pi\hbar)^3}$$

$\hbar = 1$


MATTER
ONCE WE FIND f_i, WE CAN CALCULATE
PARTICLE DENSITY $n_i = \int \frac{d^3p}{(2\pi)^3} f_i(t, \vec{x}, \vec{p})$

MATTER

ONCE WE FIND f_i, WE CAN CALCULATE

PARTICLE DENSITY $\quad n_i = \int \frac{d^3p}{(2\pi)^3} f_i(t,\vec{x},\vec{p})$

ENERGY DENSITY $\quad \rho_i = \int \frac{d^3p}{(2\pi)^3} E(\vec{x},\vec{p}) f_i(t,\vec{x},\vec{p})$

PRESSURE $\quad P_i = \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^{\,2}}{3E} f_i(t,\vec{x},\vec{p})$

• HOW TO FIND f_i? f_i GOVERNED BY

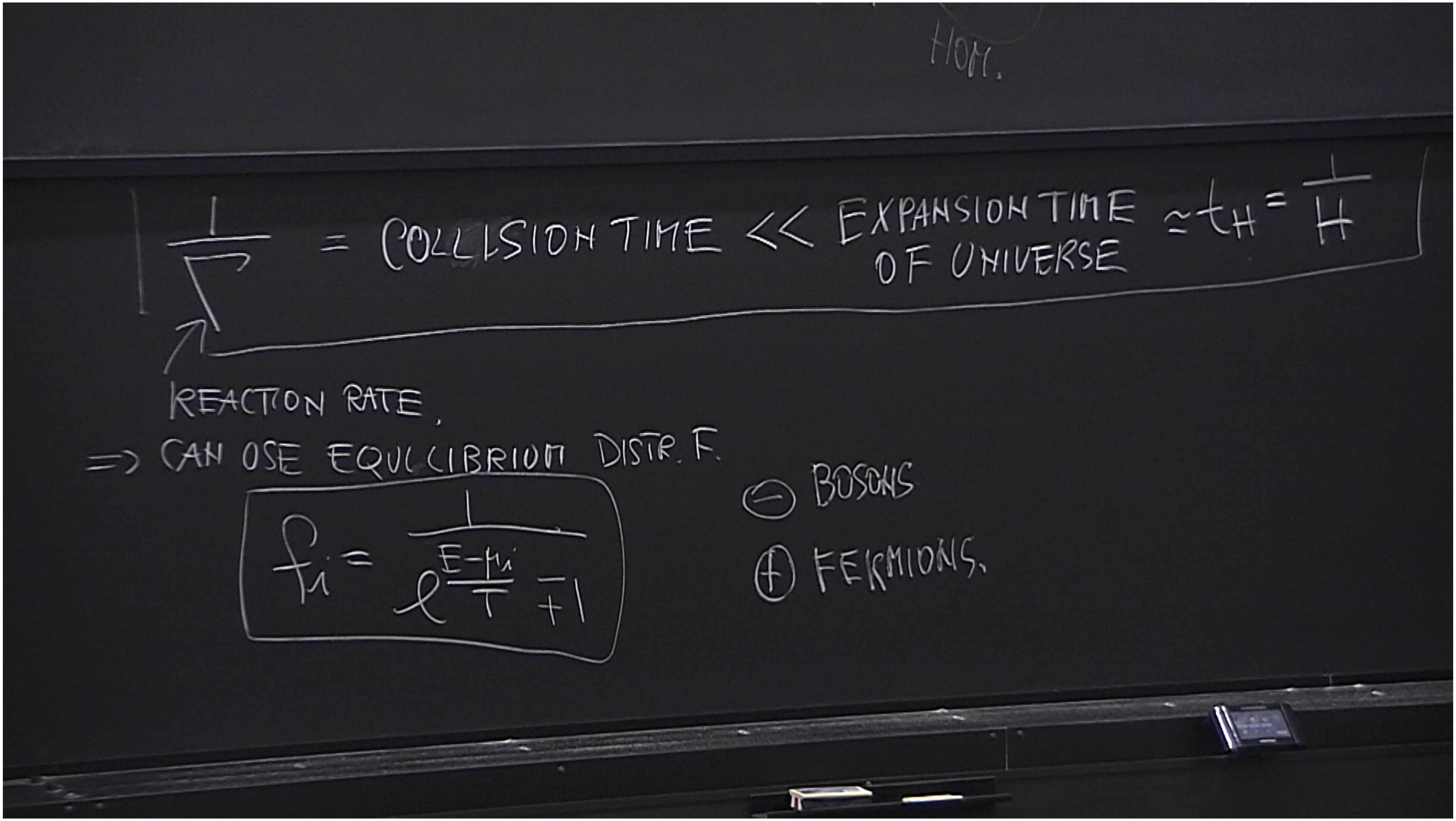
THE BOLTZMANN EQUATION

$$\boxed{\frac{df_i}{dt} = \frac{\partial f_i}{\partial t} + \frac{\partial f_i}{\partial x_j}\dot{x}_j + \frac{\partial f_i}{\partial p_j}\dot{p}_j = COL_i(f_j)}$$

HIDES ALL DIRTY PHYSICS

INSTEAD LET US CONSIDER

LOCAL THERMAL EQUILIBRIUM (HAS MAXIMAL ENTROPY)


HOM.
$\frac{1}{\Gamma}$ = COLLISION TIME << EXPANSION TIME OF UNIVERSE $\simeq t_H = \frac{1}{H}$
REACTION RATE.
=> CAN USE EQUILIBRIUM DISTR. F.
$f_i = \frac{1}{e^{\frac{E-\mu_i}{T}} \mp 1}$
(−) BOSONS
(+) FERMIONS.

$$E_1 = \sqrt{\vec{p}^{\,2} + m_1^2}$$

WE HAVE

$$d^3p = d\Omega\, |p|^2\, d|p| =$$

$$= d\Omega \frac{(E^2 - m_1^2)}{\sqrt{E^2 - m_1^2}} E\, dE$$

$$= d\Omega\, dE\, E \sqrt{E^2 - m_1^2}$$

$$n_i = \frac{1}{2\pi^2}\int_{m_i}^{\infty} \frac{\sqrt{E^2-m_i^2}\,E\,dE}{e^{\frac{E-\mu_i}{T}} \mp 1}$$

i) RELATIVISTIC LIMIT $T \gg m_i, \mu_i$

$$n_i = \frac{1}{2\pi^2}\int_0^{\infty} \frac{E^2\,dE}{e^{\underbrace{E/T}_{x}} \mp 1} = \left| \begin{matrix} E/T = x \\ dE = T\,dx \end{matrix} \right| = \frac{T^3}{2\pi^2}\int_0^{\infty} \frac{x^2\,dx}{e^x \mp 1}$$

$\ldots dE\, E\sqrt{E^2 - m_i^2}$

$$\int_0^\infty \frac{x^2\,dx}{e^x \mp 1} \quad \begin{cases} 2\zeta(3) & \text{BOSONS} \\ \frac{3}{2}\zeta(3) & \text{FERMIONS} \end{cases}$$

$T_F = T_B$

$$\boxed{m_F = \frac{3}{4} m_B}$$

$$\boxed{n_i \propto T_i^3} \sim \frac{1}{L^3}$$

$$[E] \sim \frac{1}{L}$$

$$\boxed{\rho_i \propto T_i^4 \propto P_i}$$

$\ldots dE\, E \sqrt{E^2 - m_i^2}$

$$\int_0^\infty \frac{x^2\,dx}{e^x \mp 1} \quad \begin{cases} 2\zeta(3) & \text{BOSONS} \\ \frac{3}{2}\zeta(3) & \text{FERMIONS} \end{cases}$$

$T_F = T_B$ $\quad \boxed{n_F = \frac{3}{4} n_B}$

$$\boxed{n_i \propto T_i^3} \sim \frac{1}{L^3}$$

$$[E] \sim \frac{1}{L}$$

$$\boxed{\rho_i \propto T_i^4 \propto P_i}$$

$$\boxed{w_i = \frac{P_i}{\rho_i} = \frac{1}{3}}$$