Title: PSI 17/18 - Cosmology - Lecture 4

Date: Feb 01, 2018 11:30 AM

URL: http://pirsa.org/18020013

Abstract:

DYNAMICS OF FRW:

PROVIDED

$$ds^2 = -dt^2 + a^2(t)\, g^{(3,0)k}_{ij}\, dx^i dx^j$$

$$T_{\mu\nu} = P g_{\mu\nu} + (\rho + P) u_\mu u_\nu$$

WANT TO SOLVE FOR $a(t)$, $\rho(t)$, $P(t)$

$$\dot{\rho} = -3H(\rho + P)$$

$$H^2 = \frac{8\pi G_N}{3}\rho - k/a^2$$

$$\rho = \rho(P)$$

FRIEDMANN EQS. + EQ OF STATE

PROVIDED $w = P/\rho = \text{CONST.}$ CAN SOLVE 1ST. EQ $\boxed{\rho \propto a^{-3(1+w)}}$

THE SECOND STEP IS TO SOLVE

2ND EQ FOR $a = a(t) = a(\eta)$

$\Rightarrow$ "DRIVEN HARMONIC OSCILLATOR"

$$\boxed{a(\eta) = \left[\frac{8\pi G}{3}\rho_0 a_0^{3(1+w)}\right]^{\frac{\alpha}{2}} S_K^{\alpha}(\eta/\alpha), \quad \alpha = \frac{2}{1+3w}}$$

SINGLE COMPONENT UNIVERSE $\quad t \sim \int a(\eta)\, d\eta$

$H^2 = \frac{8\pi G_N}{3}\rho - k/a^2$

$\rho = \rho(P)$

+ EQ OF STATE

SINGLE COMPONENT UNIV

K=0

COMPONENT	w	DENSITY	$a(\eta)$	$a(t)$
MATTER	0	$1/a^3$	η^2	$t^{2/3}$
RADIATION	$1/3$	$1/a^4$	η	$t^{1/2}$
Λ	-1	a^0	$-\frac{1}{\eta H}$	e^{Ht} CONST

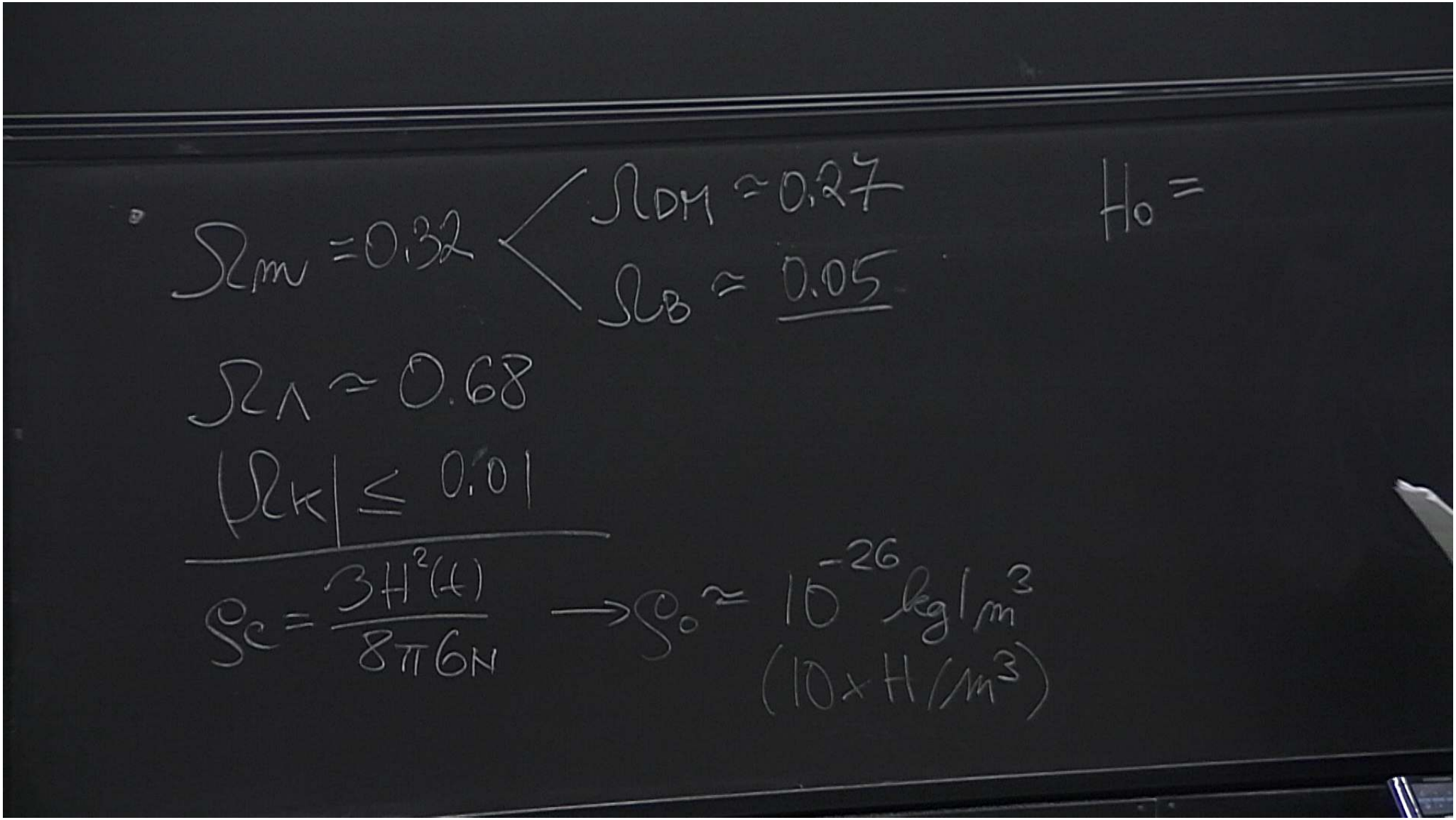
SINGLE COMPONENT UNIVERSE $t \sim \int a(\eta) d\eta$

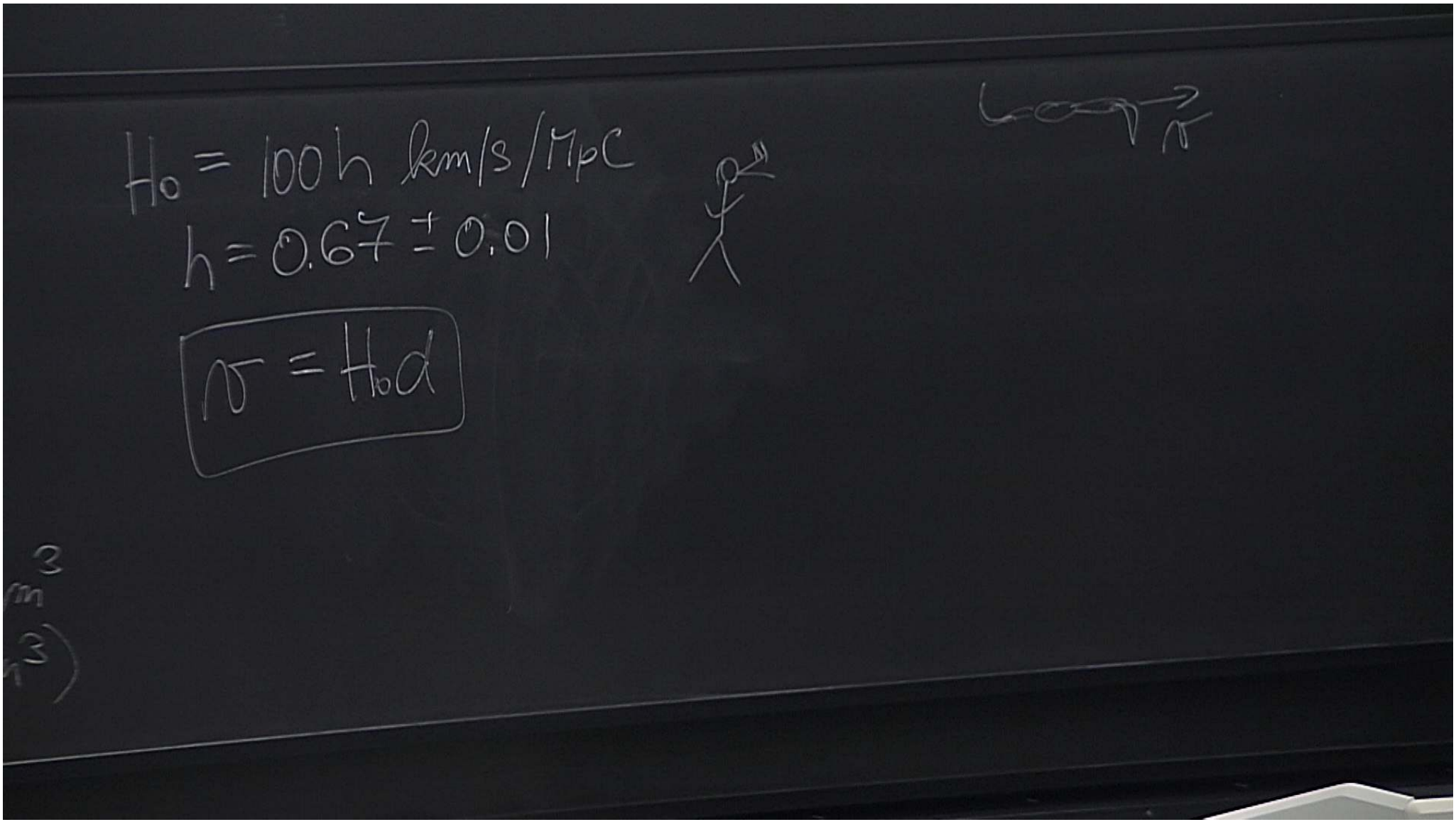
TY	$a(\eta)$	$a(t)$
3	η^2	$t^{2/3}$
4	η	$t^{1/2}$
0	$-\frac{1}{\eta H}$	e^{Ht} CONST

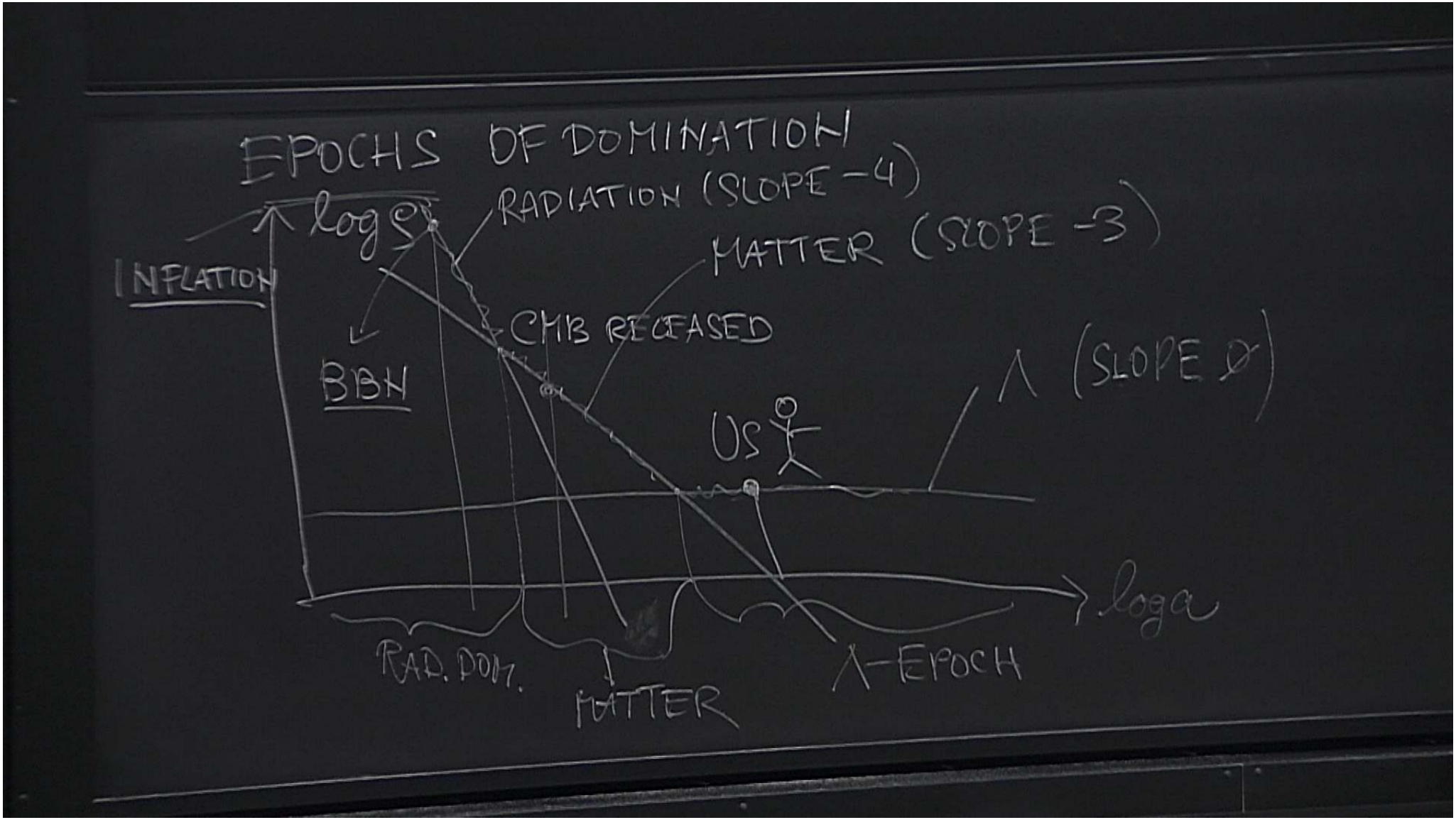
CURRENT COSMOLOGICAL MODEL

- ΛCDM
  - Λ: COLD DARK MATTER
- $\Omega_I(t) = \frac{\rho_I(t)}{\rho_c(t)}$

$I \in \{\text{MATTER, RAD, } \Lambda, K\}$


$\Omega_m = 0.32$
$\Omega_{DM} \simeq 0.27$
$\Omega_B \simeq 0.05$
$H_0 =$
$\Omega_\Lambda \simeq 0.68$
$|\Omega_K| \leq 0.01$
$\rho_c = \frac{3H^2(t)}{8\pi G_N} \rightarrow \rho_0 \simeq 10^{-26}\ kg/m^3$
$(10 \times H/m^3)$


$H_0 = 100h$ km/s/MpC
$h = 0.67 \pm 0.01$
$v = H_0 d$


EPOCHS OF DOMINATION
log ρ
RADIATION (SLOPE -4)
MATTER (SLOPE -3)
INFLATION
CMB RELEASED
BBN
Λ (SLOPE 0)
US
log a
RAD. DOM.
MATTER
Λ-EPOCH

WE ARE HERE TO OBSERVE IT!

II) MATTER

a) THERMODYNAMICS IN EXPANDING UNIVERSE

- DISTRIBUTION FUNCTION

$$\#\text{PARTICLES}_i = dN_i = f_i(\vec{x}, \vec{p}, t)\,\frac{d^3x\,d^3p}{(2\pi\hbar)^3}$$

f_i: DISTR. F.

$\hbar = 1$

MATTER

ONCE WE FIND f_i, WE CAN CALCULATE

PARTICLE DENSITY $n_i = \int \frac{d^3p}{(2\pi)^3} f_i(t, \vec{x}, \vec{p})$

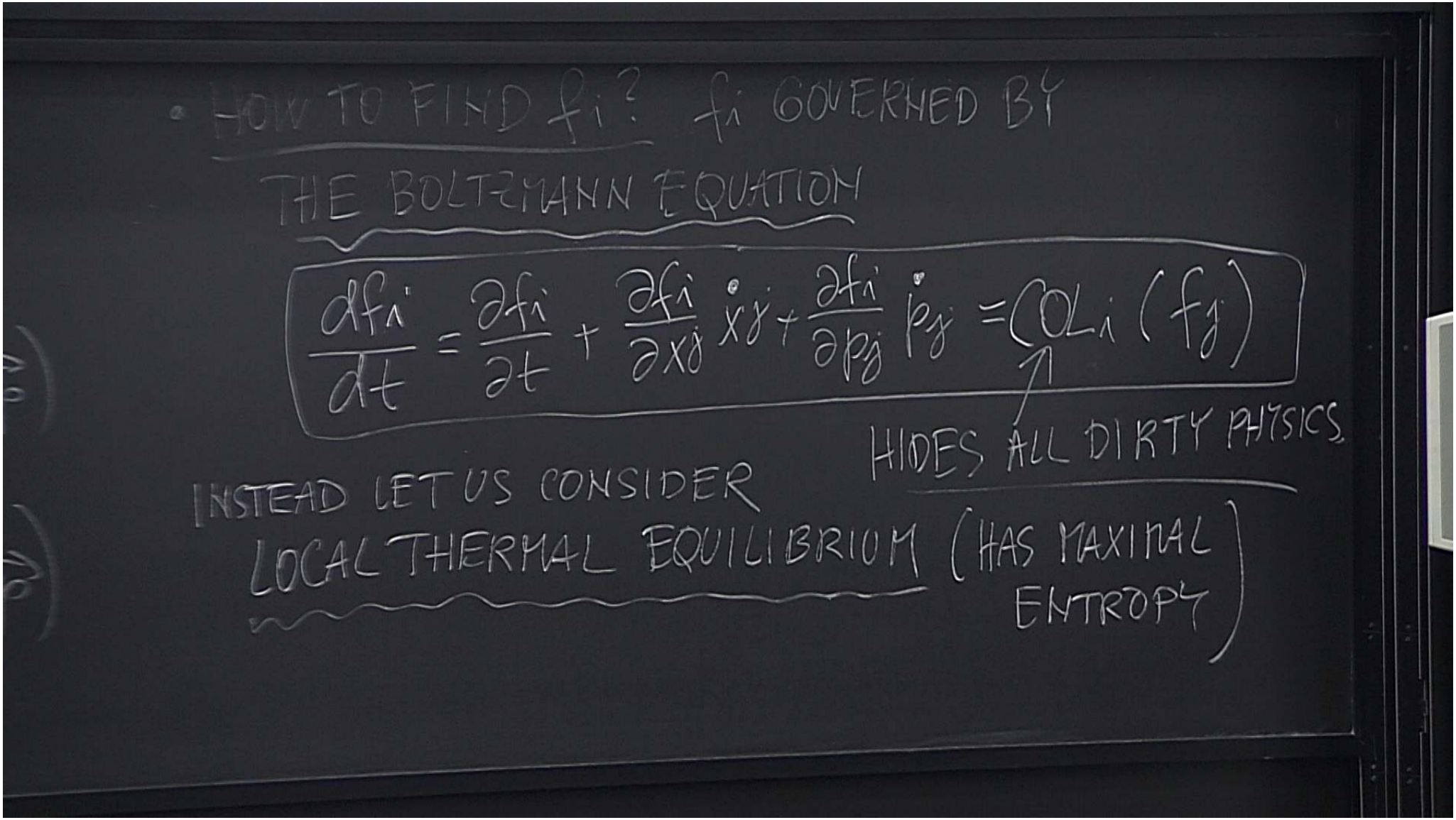
MATTER

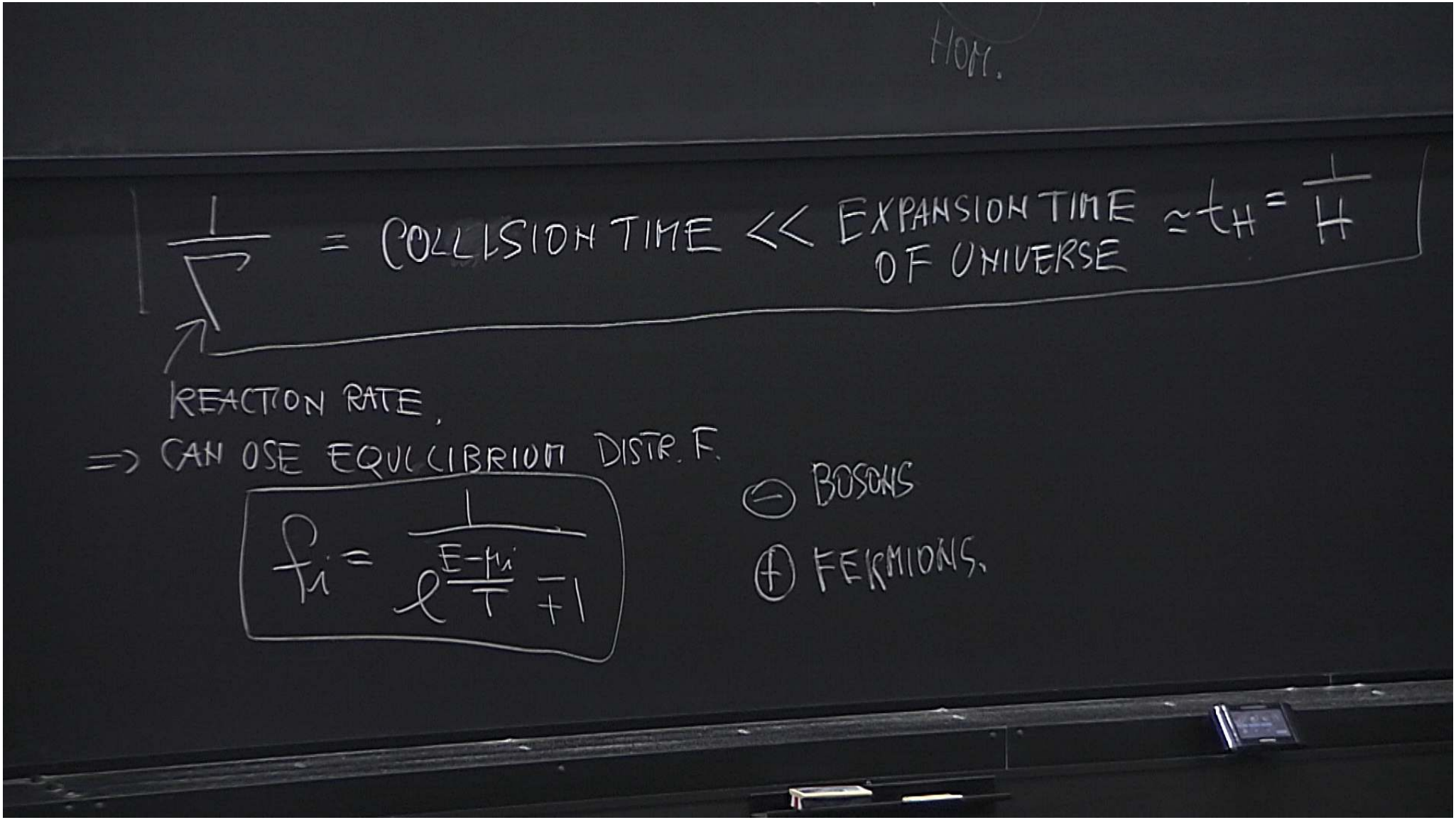
ONCE WE FIND f_i, WE CAN CALCULATE

PARTICLE DENSITY $n_i = \int \frac{d^3p}{(2\pi)^3} f_i(t, \vec{x}, \vec{p})$

ENERGY DENSITY $\rho_i = \int \frac{d^3p}{(2\pi)^3} E(\vec{x}, \vec{p}) f_i(t, \vec{x}, \vec{p})$

PRESSURE $P_i = \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^2}{3E} f_i(t, \vec{x}, \vec{p})$


HOW TO FIND f_i? f_i GOVERNED BY
THE BOLTZMANN EQUATION
$\frac{df_i}{dt} = \frac{\partial f_i}{\partial t} + \frac{\partial f_i}{\partial x_j}\dot{x}_j + \frac{\partial f_i}{\partial p_j}\dot{p}_j = C[L_i(f_j)]$
HIDES ALL DIRTY PHYSICS
INSTEAD LET US CONSIDER
LOCAL THERMAL EQUILIBRIUM (HAS MAXIMAL ENTROPY)


$\frac{1}{\Gamma}$ = COLLISION TIME $\ll$ EXPANSION TIME OF UNIVERSE $\simeq t_H = \frac{1}{H}$
REACTION RATE.
=> CAN USE EQUILIBRIUM DISTR. F.
$f_i = \frac{1}{e^{\frac{E-\mu_i}{T}} \mp 1}$
(-) BOSONS
(+) FERMIONS.

$$E_i = \sqrt{\vec{p}^2 + m_i^2}$$

WE HAVE

$$d^3p = d\Omega\, |p|^2 d|p| =$$

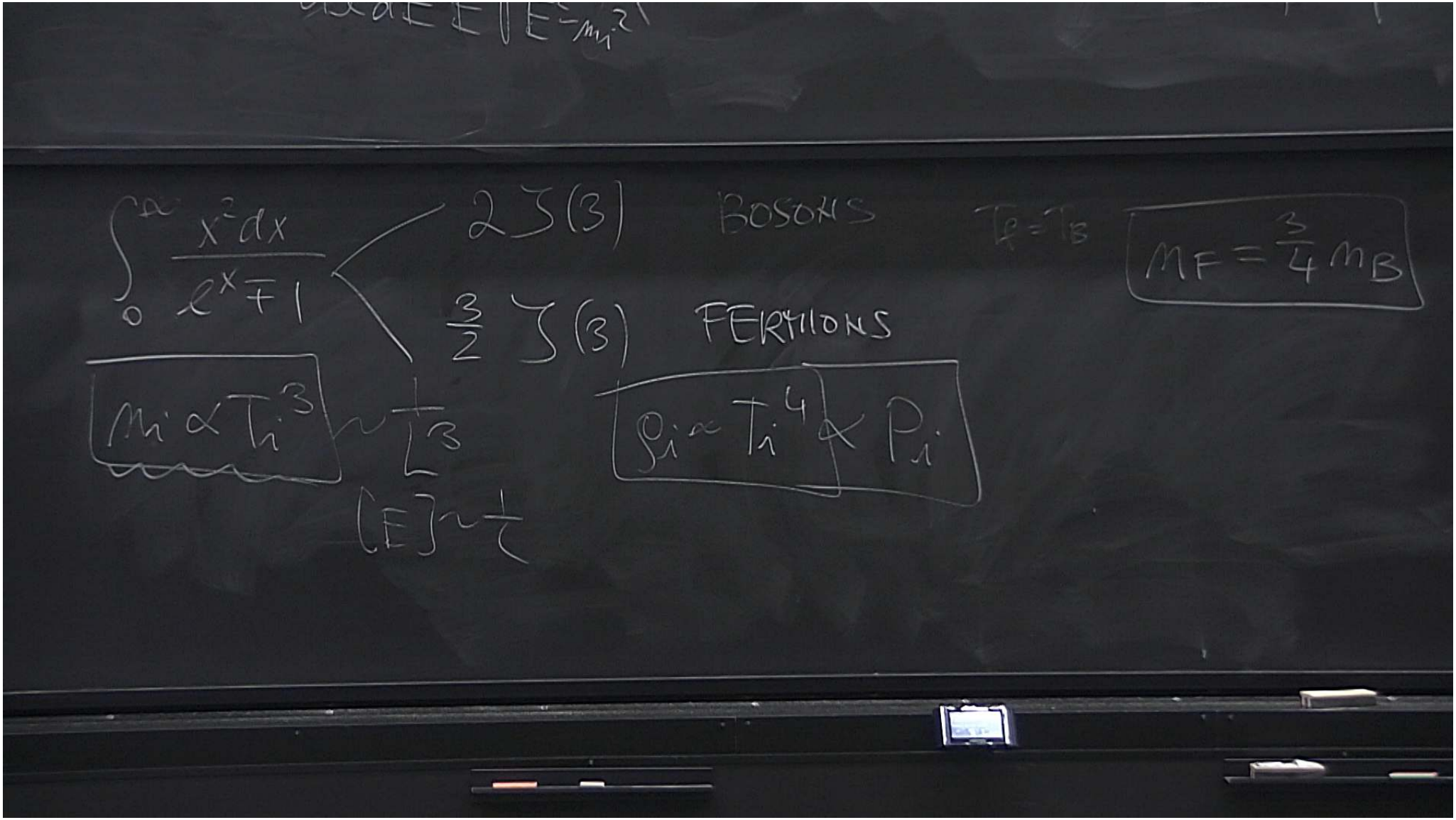
$$= d\Omega \frac{(E^2 - m_i^2)}{\sqrt{E^2 - m_i^2}} E\, dE$$

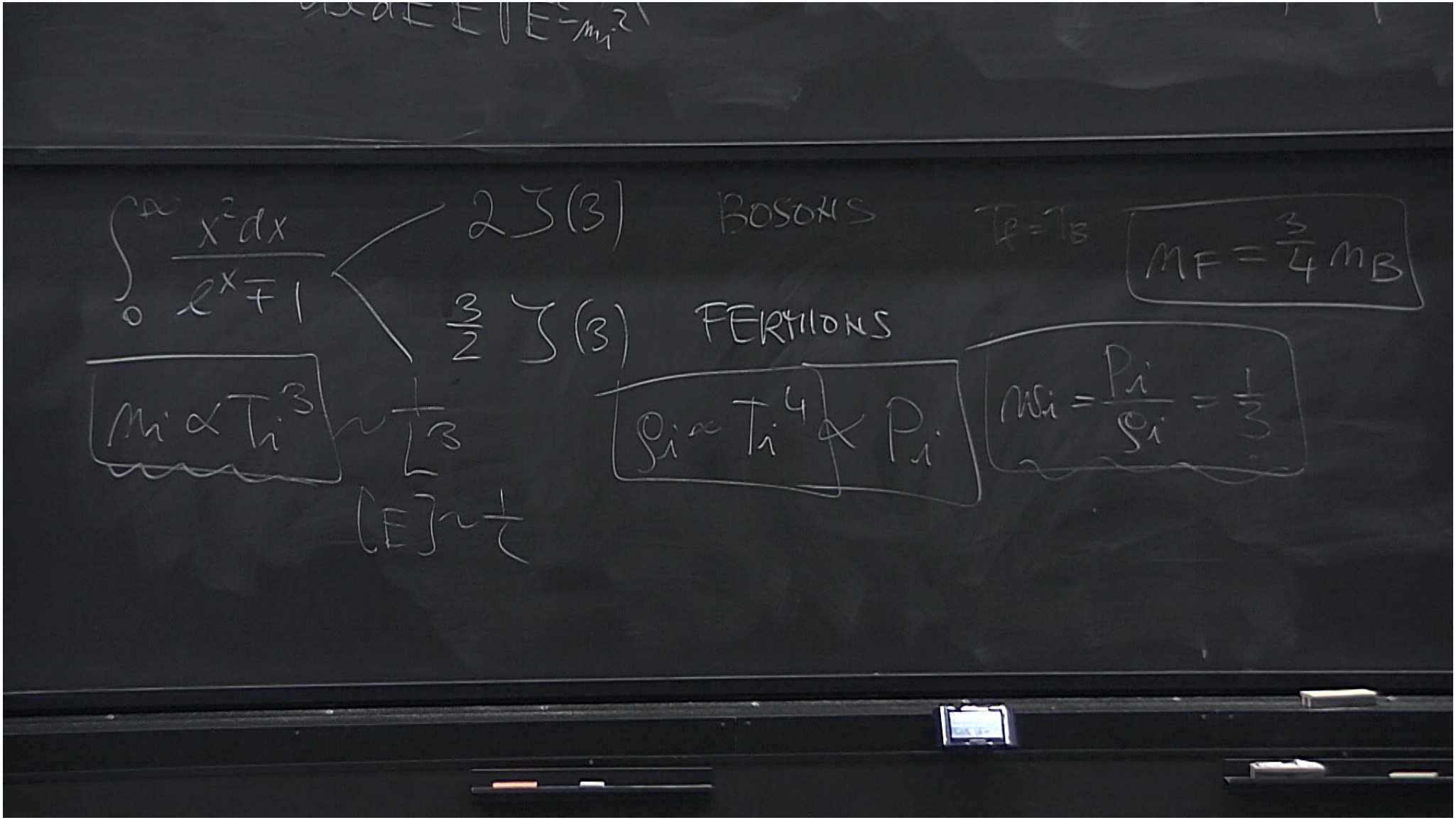
$$= d\Omega\, dE\, E \sqrt{E^2 - m_i^2}$$

$$n_i = \frac{1}{2\pi^2} \int_{m_i}^{\infty} \frac{\sqrt{E^2 - m_i^2}\, E\, dE}{e^{\frac{E-\mu_i}{T}} \mp 1}$$

i) RELATIVISTIC LIMIT $T \gg m_i, \mu_i$

$$n_i = \frac{1}{2\pi^2} \int_0^{\infty} \frac{E^2 dE}{e^{E/T} \mp 1} = \left| \begin{array}{l} E/T = x \\ dE = T\,dx \end{array} \right| = \frac{T^3}{2\pi^2} \int_0^{\infty} \frac{x^2 dx}{e^x \mp 1}$$


$\int_0^\infty \frac{x^2 dx}{e^x \mp 1}$
$2\,\zeta(3)$ BOSONS
$\frac{3}{2}\,\zeta(3)$ FERMIONS
$T_F = T_B$
$M_F = \frac{3}{4} M_B$
$M_i \propto T_i^3$
$\sim \frac{1}{L^3}$
$[E] \sim \frac{1}{t}$
$\rho_i \propto T_i^4 \propto P_i$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$T_F = T_B$
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$M_i \propto T_i^3$
$\sim \frac{1}{L^3}$
$[E] \sim t$
$\rho_i \propto T_i^4 \propto P_i$
$w_i = \frac{P_i}{\rho_i} = \frac{1}{3}$