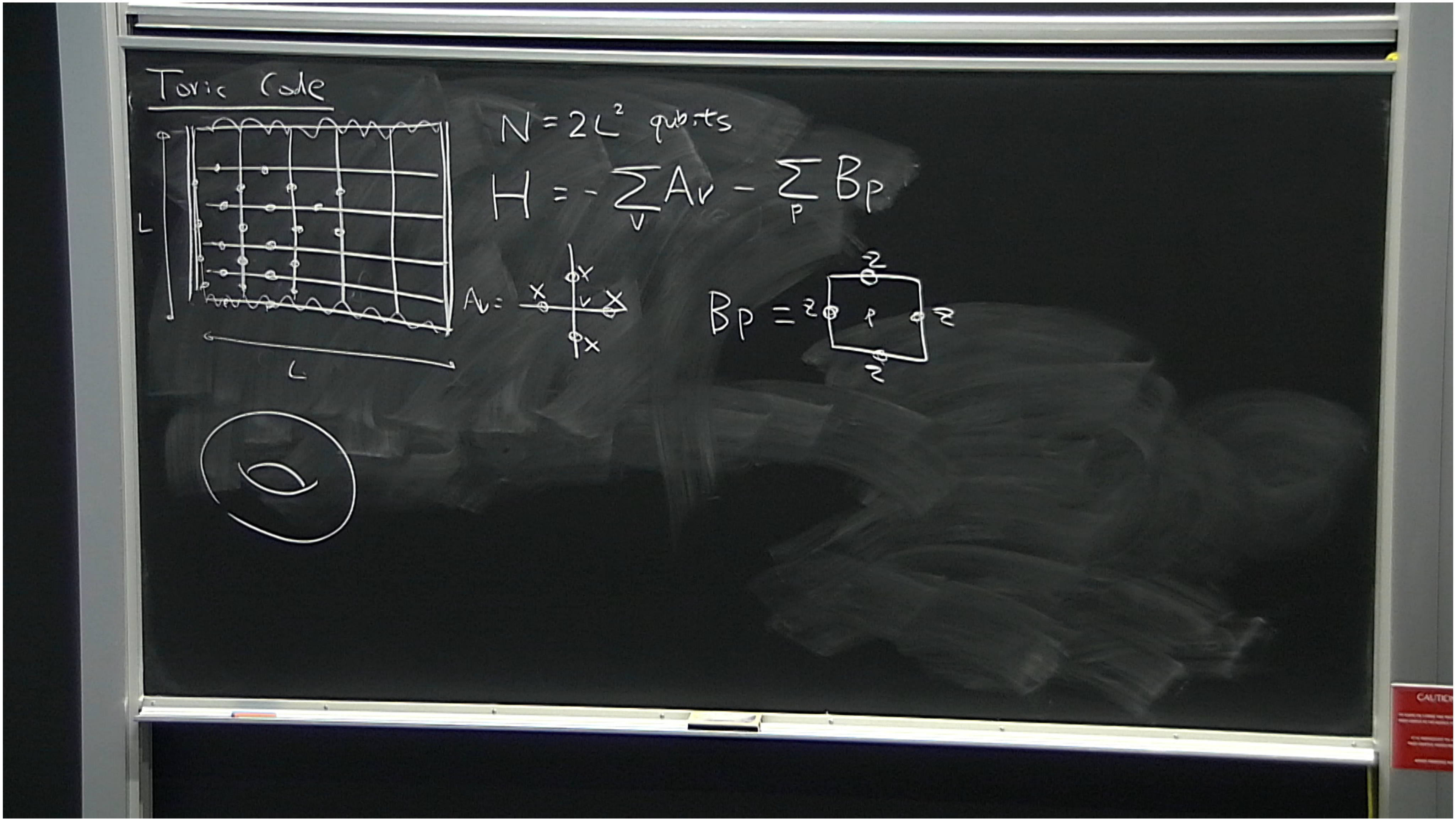
Title: IQC - Quantum Error Correction - Lecture 7

Date: Feb 15, 2018 02:00 PM

URL: http://pirsa.org/18020011

Abstract:


Toric Code
L
L
$N = 2L^2$ qubits
$H = -\sum_v A_v - \sum_p B_p$
$A_v =$
$B_p =$

...al code

- Repetition code

$$0 \longrightarrow 00\cdots\cdots00$$
$$1 \longrightarrow 11\cdots\cdots11$$

Hamming distance is n.

$$00\cdots\cdots00 \xrightarrow{\text{error}} 000100110 0\cdots0 \xrightarrow{\text{recovery}} 0\cdots\cdots00$$

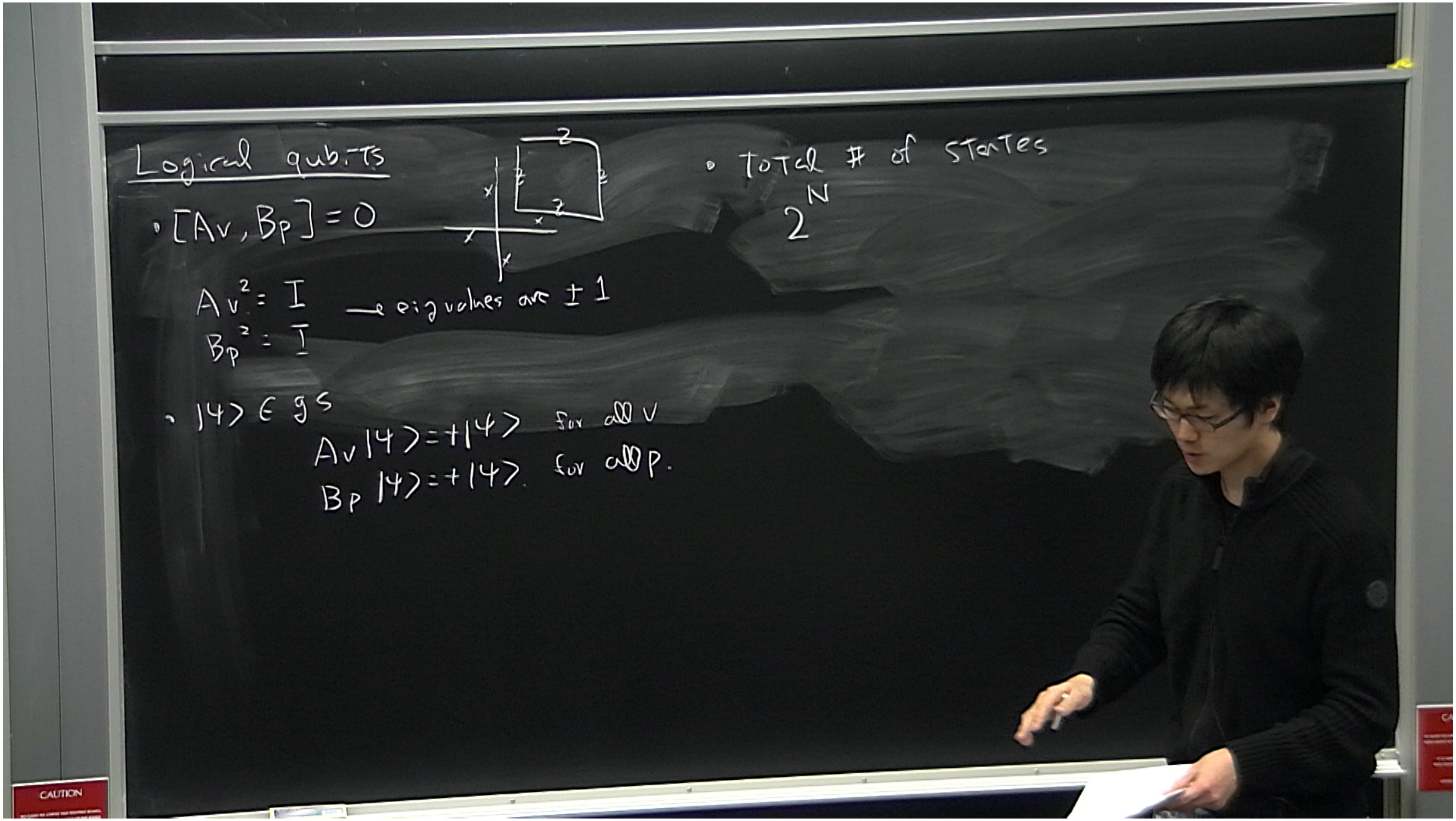
- Ferromagnet (HDD)

$$H = -\sum_{i,j} Z_i Z_j \qquad |0\cdots\cdots0\rangle,\ |11\cdots\cdots1\rangle$$

ground space = codeword space.

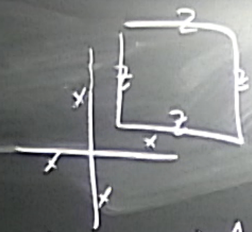
- Quantum repetition does not exist $\quad |\psi\rangle \longrightarrow \cancel{|\psi\rangle\otimes\cdots\otimes|\psi\rangle}$

due to no-cloning


Logical qubits
• [Av, Bp] = 0
Av² = I — eigvalues are ± 1
Bp² = I
• |ψ> ∈ gs
Av|ψ> = +|ψ> for all v
Bp |ψ> = +|ψ>. for all p.
• Total # of states
2^N
CAUTION

Logical qubits

- $[A_v, B_p] = 0$



$A_v^2 = I$ → eigvalues are ± 1
$B_p^2 = I$

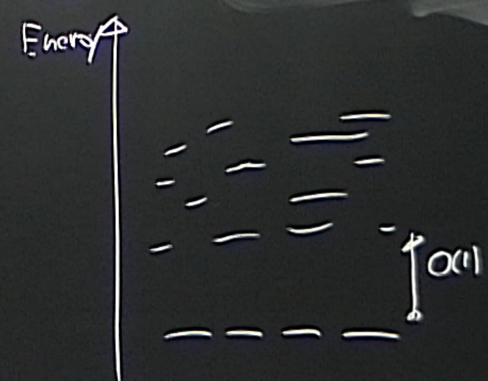
- $|\psi\rangle \in$ gs
  $A_v|\psi\rangle = +|\psi\rangle$ for all v (L^2 constraints)(only L^2-1 are indep)
  $B_p|\psi\rangle = +|\psi\rangle$ for all p.

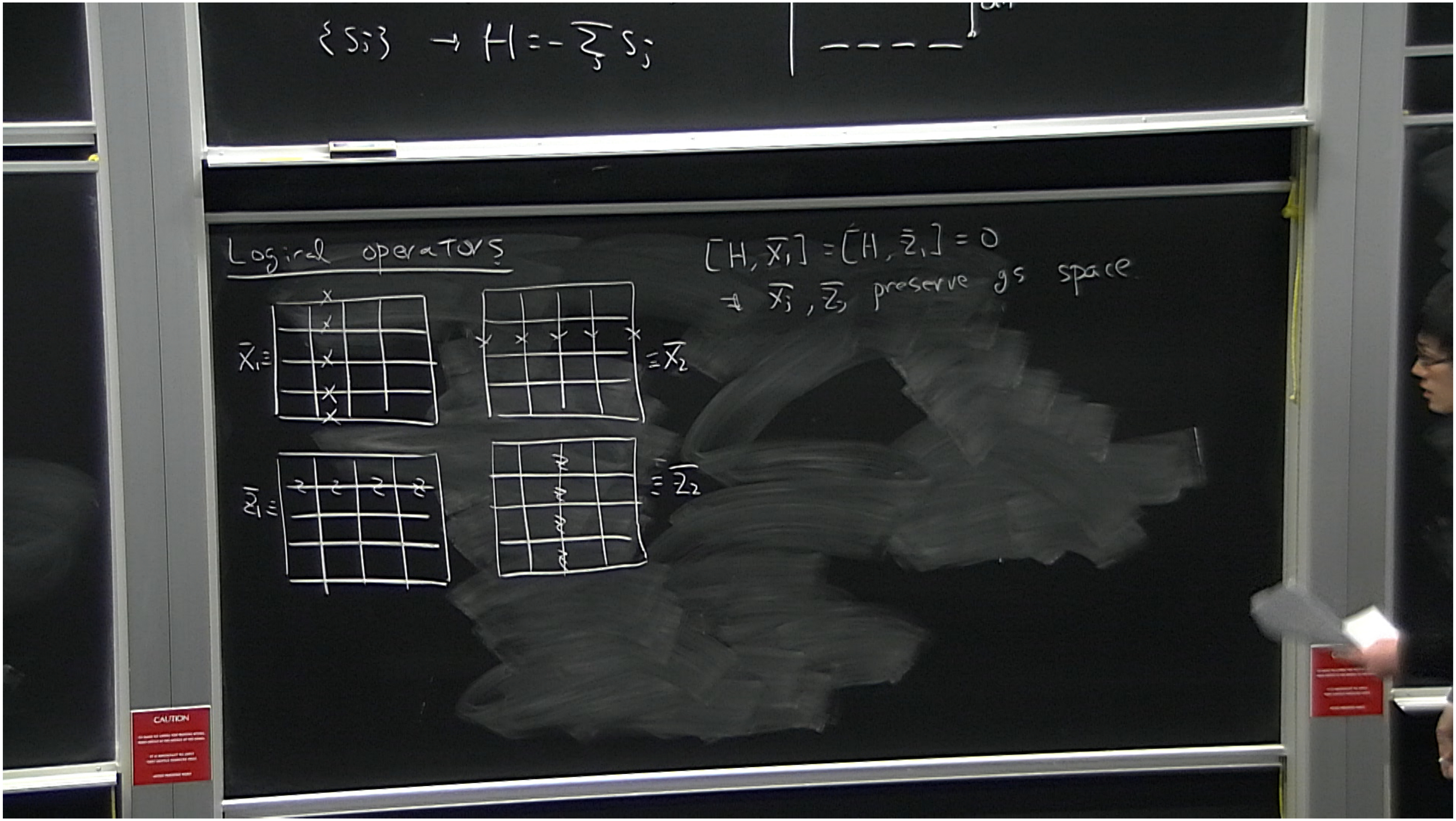
$\prod_v A_v = I \qquad \prod_p B_p = I$

- Stabilizer code $\quad S_j|\psi\rangle = +|\psi\rangle$

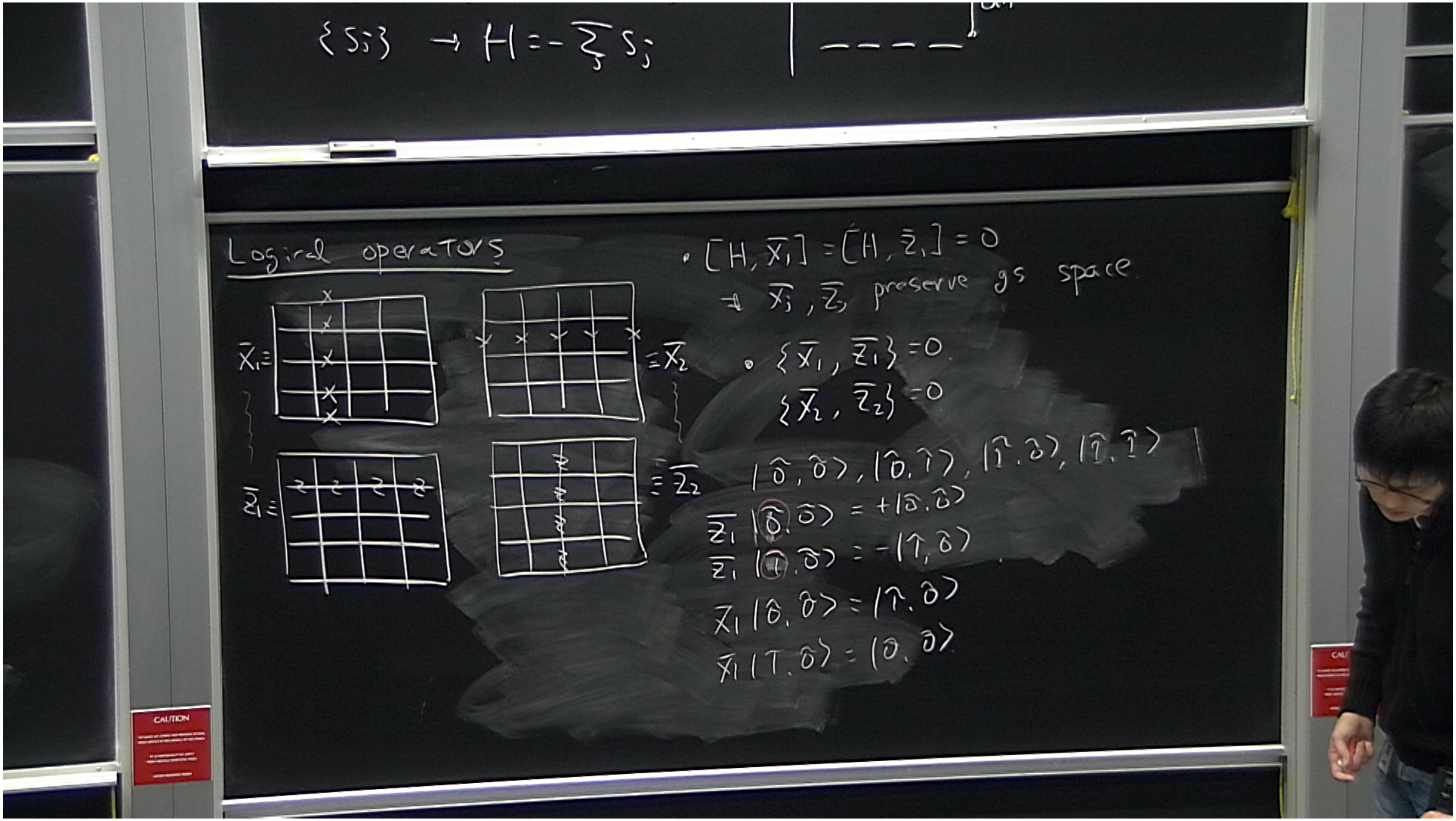
$\{S_j\} \rightarrow H = -\sum_j S_j$

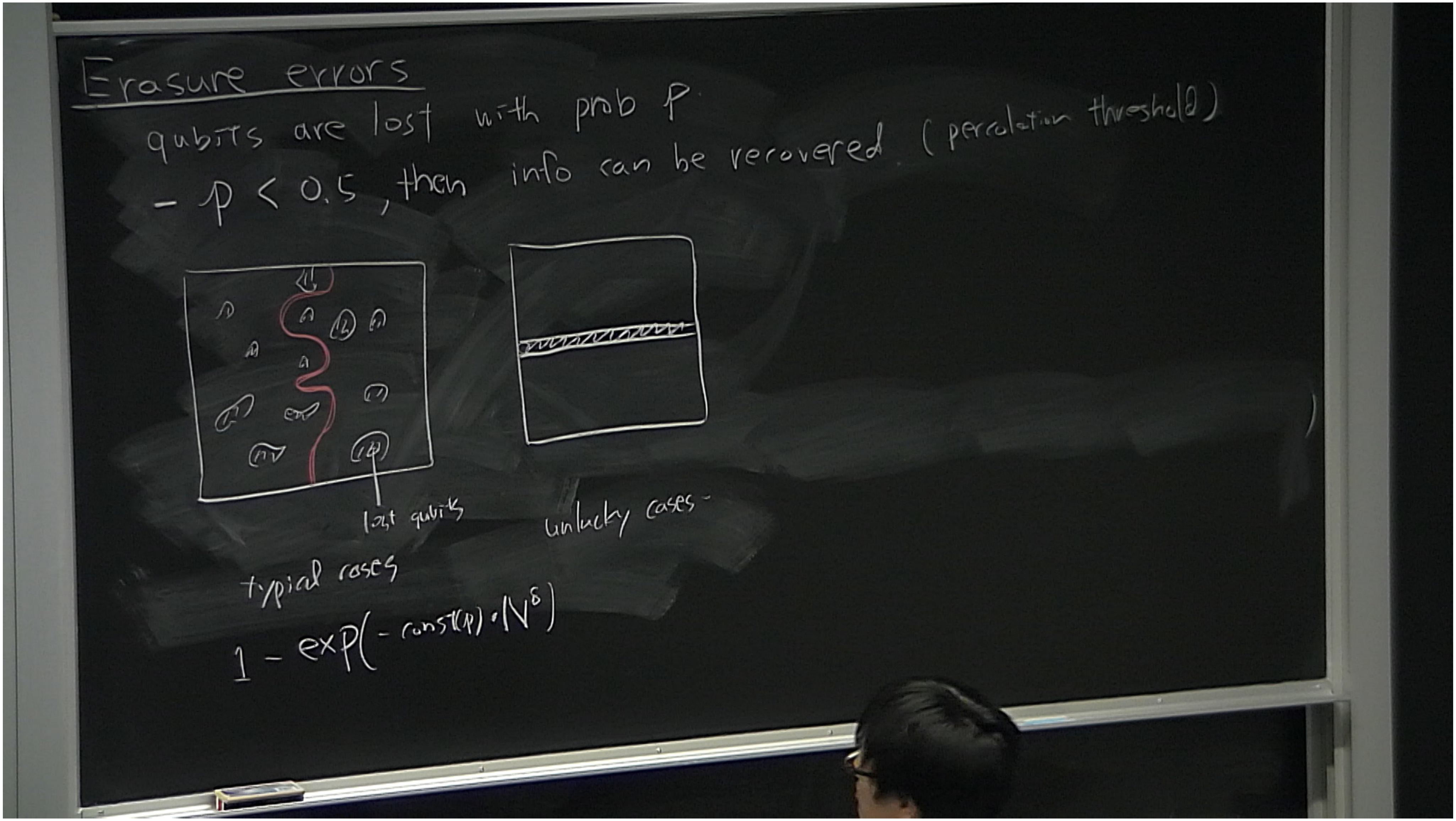
- Total # of states
  2^N
- $2^N / 2^{L^2-1} \cdot 2^{L^2-1} = 4$ (# of gs)
  (2 logical qubits)

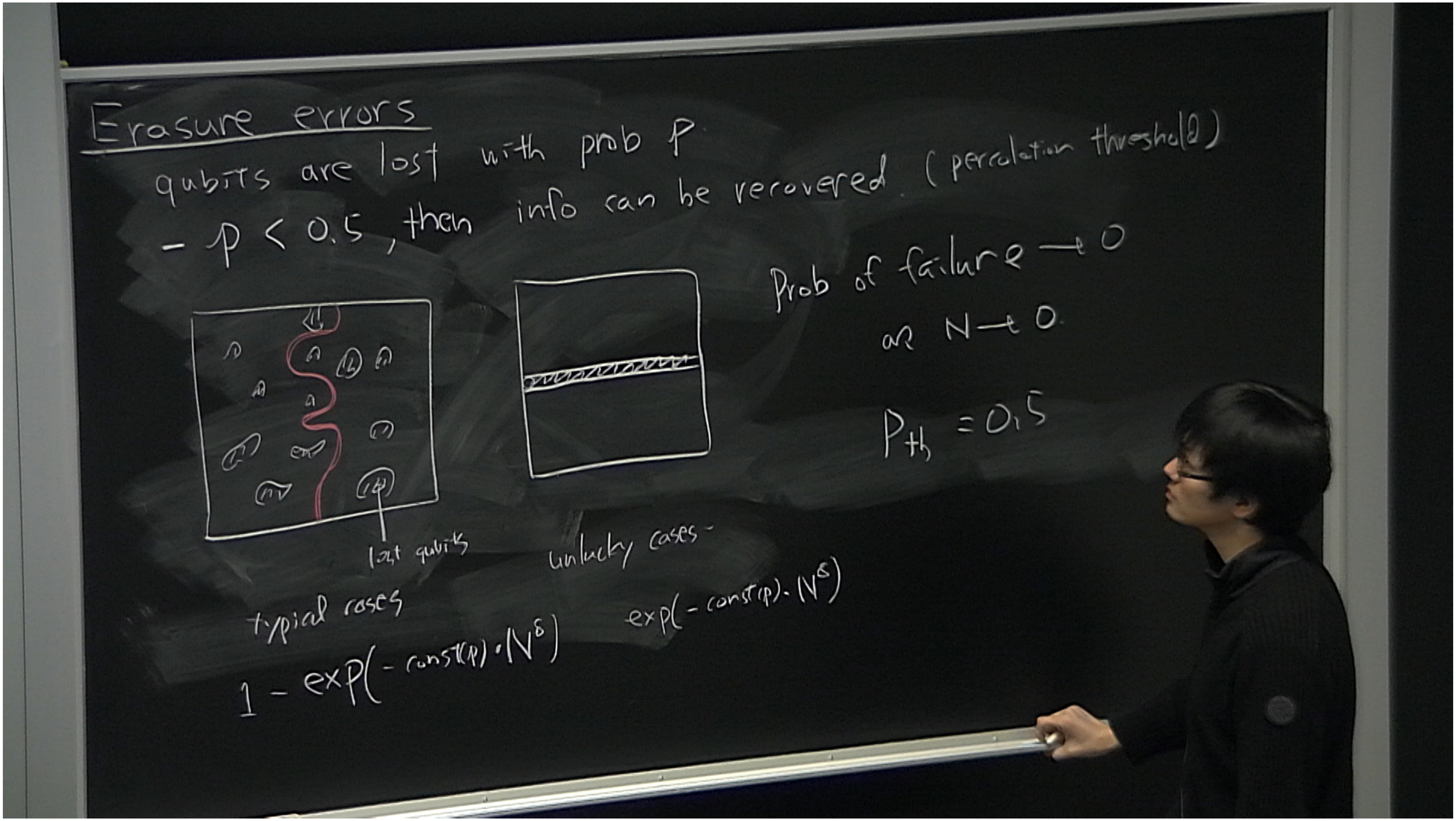


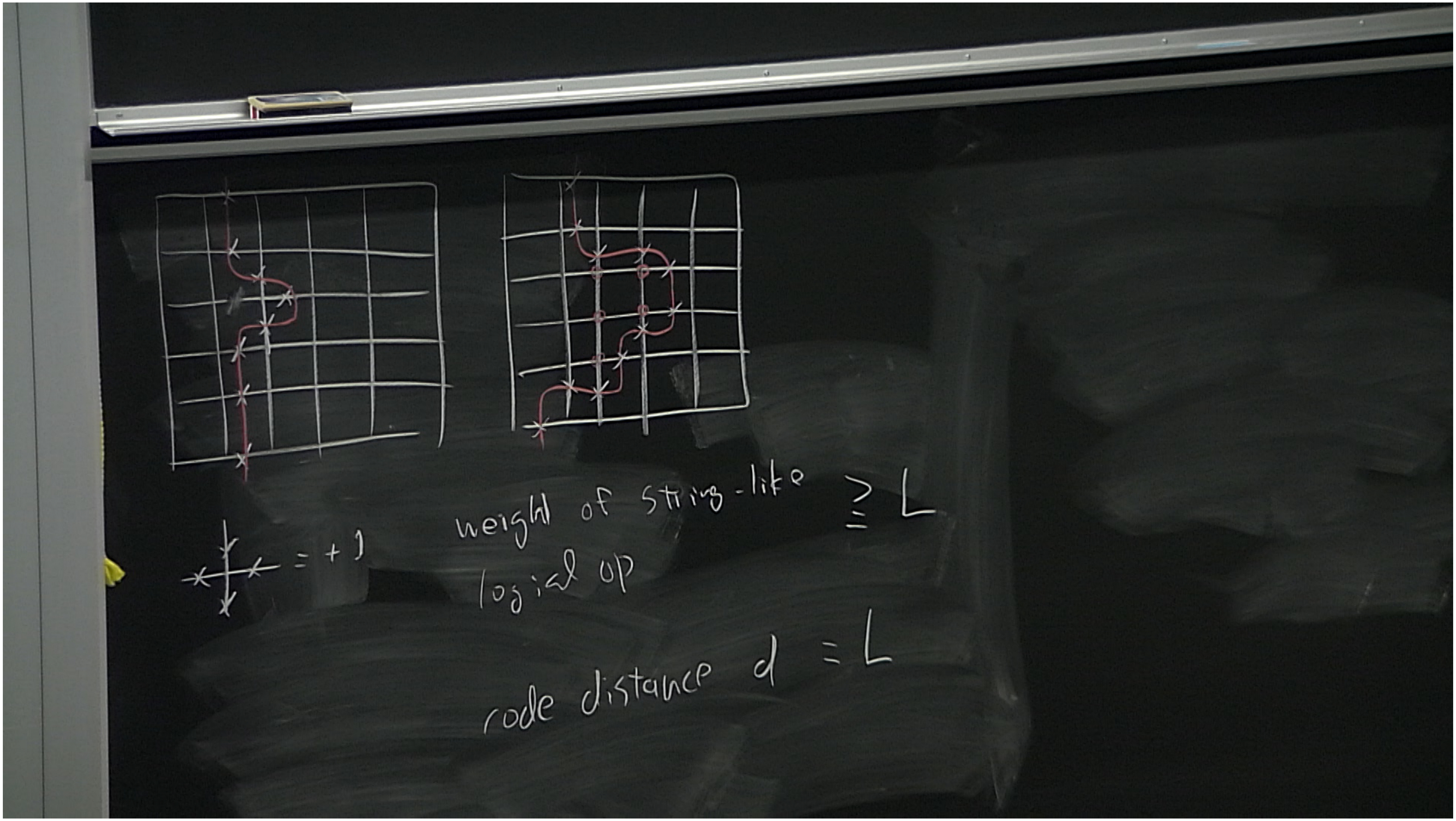


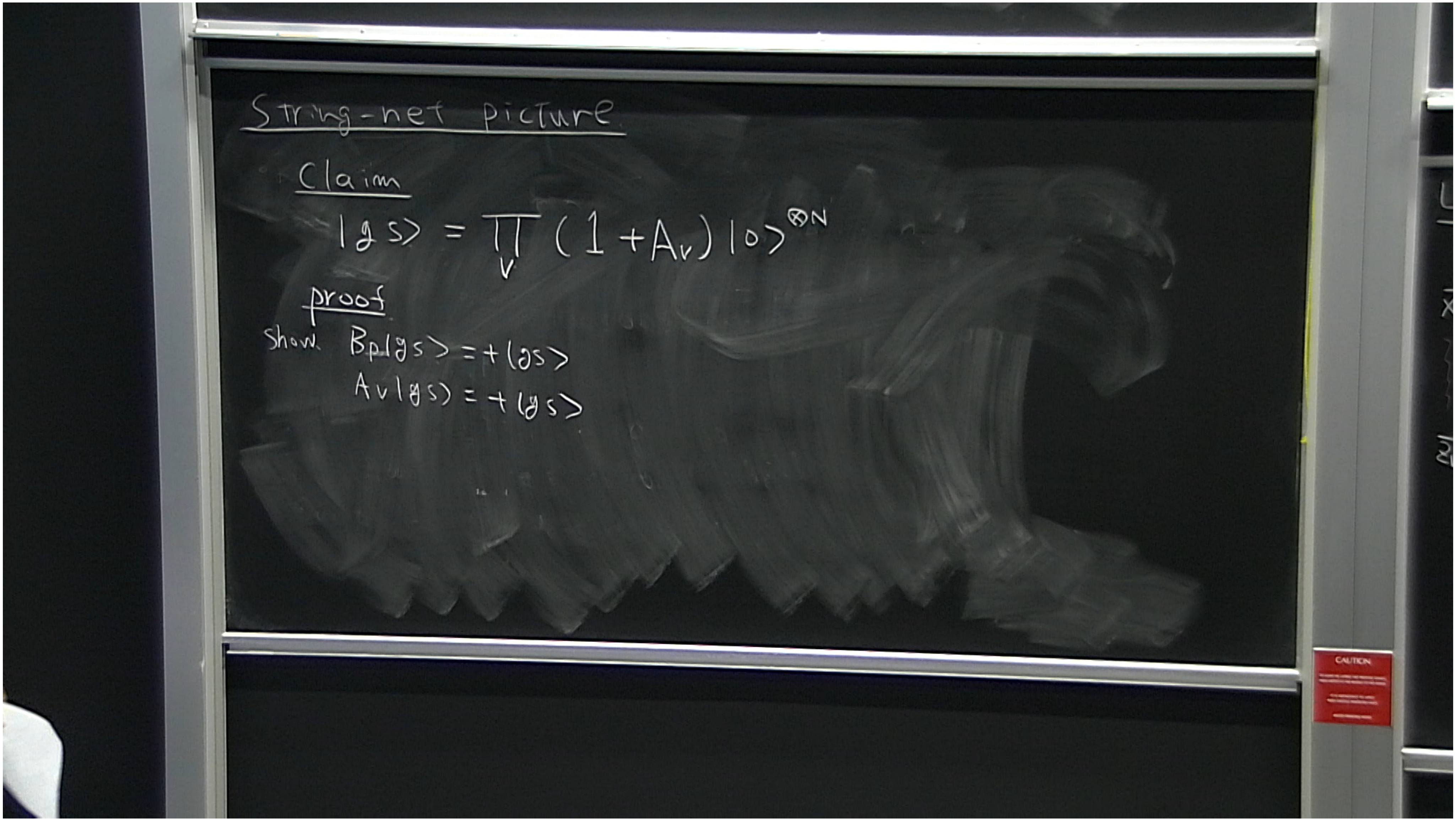
$\{S_j\} \rightarrow H = -\sum_j S_j$
Logical operators
$\bar{X}_1 =$
$= \bar{X}_2$
$\bar{Z}_1 =$
$= \bar{Z}_2$
$[H, \bar{X}_1] = [H, \bar{Z}_1] = 0$
$\Rightarrow \bar{X}_j, \bar{Z}_j$ preserve gs space

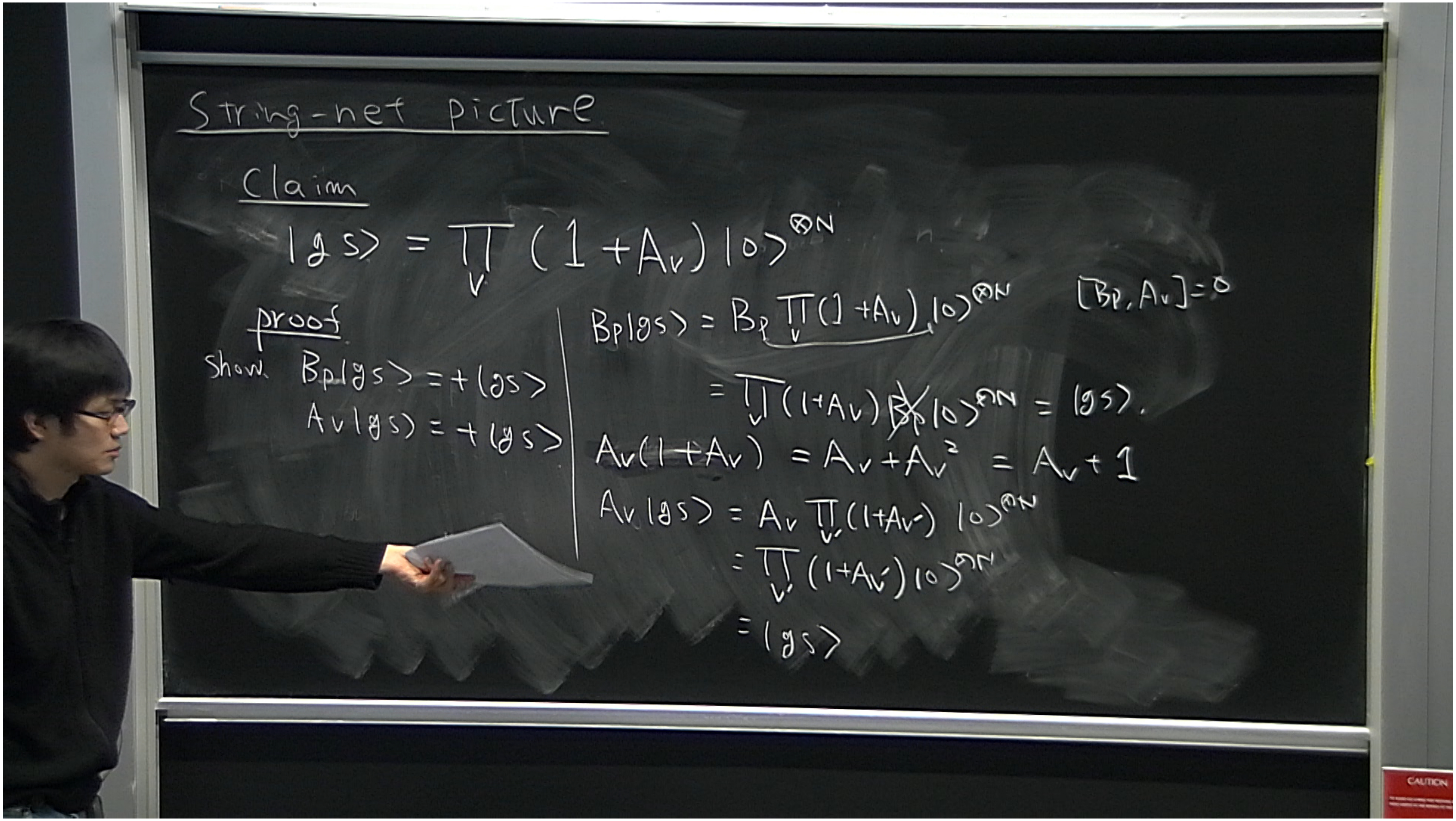

{S_j} → H = - Σ_j S_j
Logical operators
X̄₁ =
Z̄₁ =
= X̄₂
= Z̄₂
• [H, X̄₁] = [H, Z̄₁] = 0
⇒ X̄_j, Z̄_j preserve gs space
• {X̄₁, Z̄₁} = 0
{X̄₂, Z̄₂} = 0
|0̂, 0̂>, |0̂, 1̂>, |1̂, 0̂>, |1̂, 1̂>
Z̄₁ |0̂, 0̂> = +|0̂, 0̂>
Z̄₁ |1̂, 0̂> = -|1̂, 0̂>
X̄₁ |0̂, 0̂> = |1̂, 0̂>
X̄₁ |1̂, 0̂> = |0̂, 0̂>

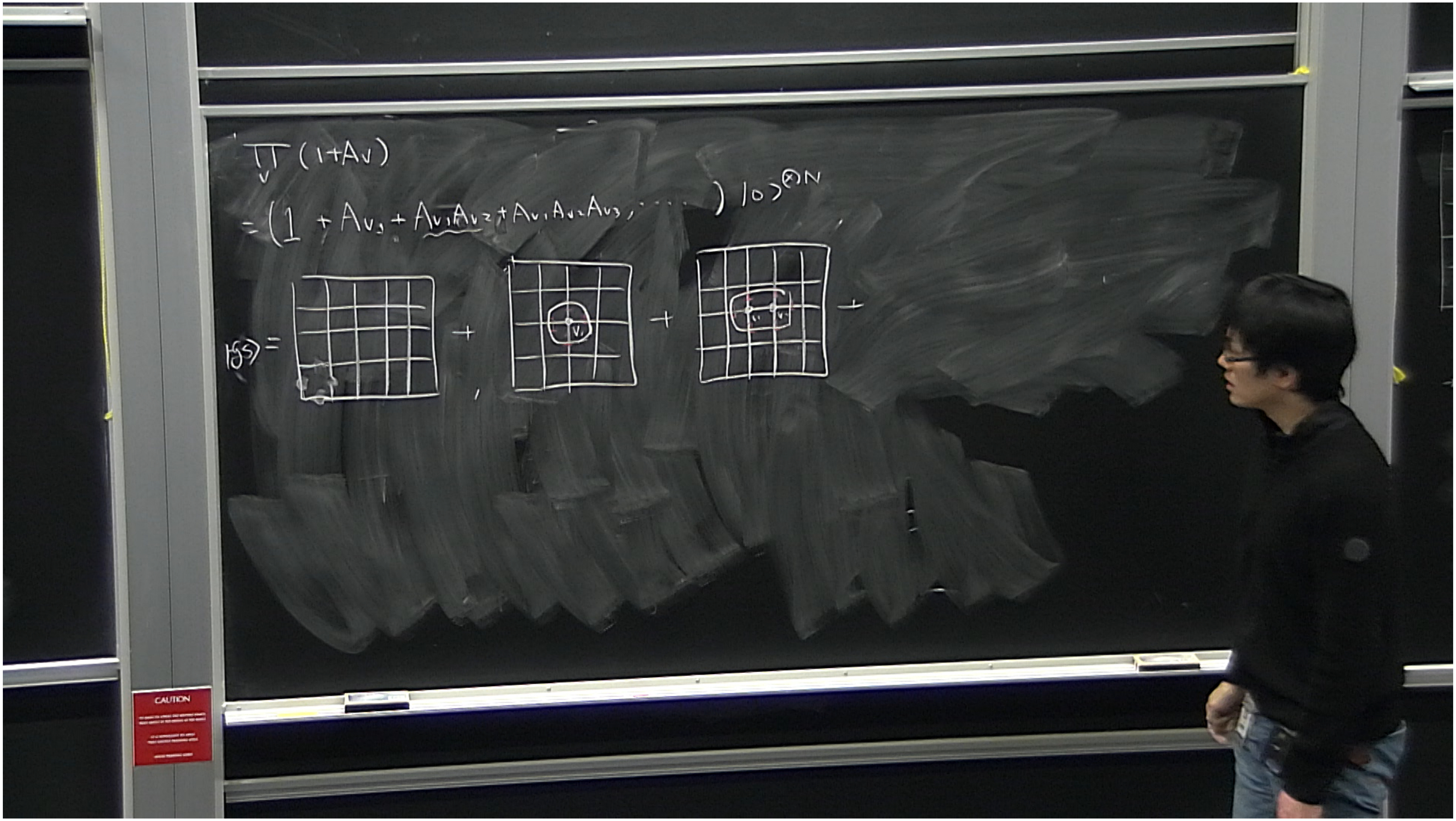

Erasure errors
qubits are lost with prob p
- p < 0.5, then info can be recovered (percolation threshold)
lost qubits
unlucky cases
typical cases
1 - exp(- const(p) · N^δ)

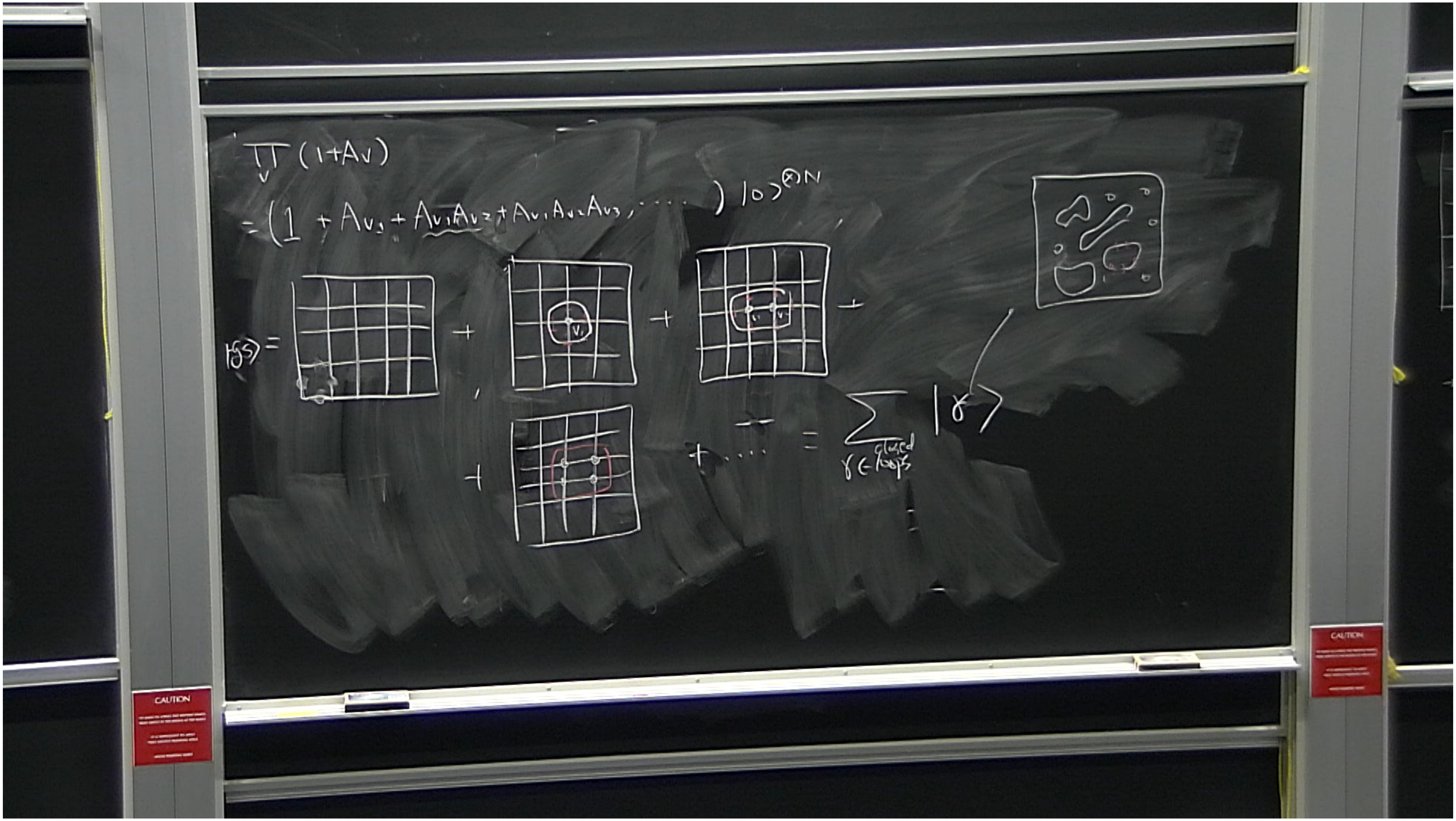

Erasure errors
qubits are lost with prob p
- p < 0.5, then info can be recovered (percolation threshold)
Prob of failure → 0
as N → 0.
P_th = 0.5
lost qubits
unlucky cases -
typical cases
1 - exp(- const(p) · N^δ)
exp(- const(p) · N^δ)


= +1
weight of string-like logical op ≥ L
code distance d = L

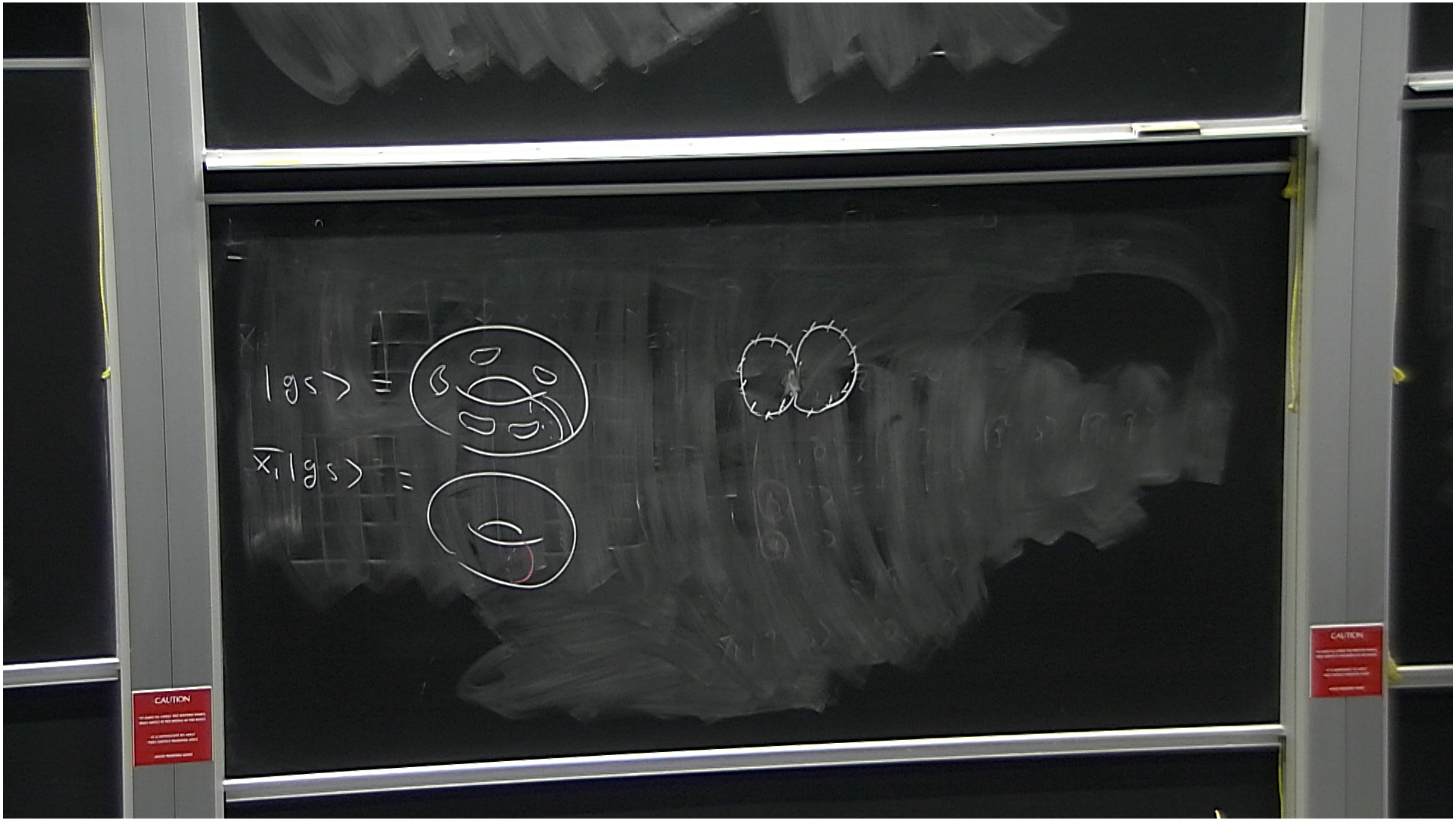

String-net picture
Claim
$|gs\rangle = \prod_v (1 + A_v) |0\rangle^{\otimes N}$
proof
Show $B_p |gs\rangle = + |gs\rangle$
$A_v |gs\rangle = + |gs\rangle$


String-net picture
Claim
$|gs\rangle = \prod_v (1 + A_v)|0\rangle^{\otimes N}$
proof
Show $B_p|gs\rangle = +|gs\rangle$
$A_v|gs\rangle = +|gs\rangle$
$B_p|gs\rangle = B_p \prod_v (1 + A_v)|0\rangle^{\otimes N}$
$[B_p, A_v] = 0$
$= \prod_v (1 + A_v) \cancel{B_p} |0\rangle^{\otimes N} = |gs\rangle.$
$A_v(1 + A_v) = A_v + A_v^2 = A_v + 1$
$A_v|gs\rangle = A_v \prod_{v'} (1 + A_{v'})|0\rangle^{\otimes N}$
$= \prod_{v'} (1 + A_{v'})|0\rangle^{\otimes N}$
$= |gs\rangle$


∏ (1+Av)
= (1 + Av1 + Av1Av2 + Av1Av2Av3, ...) |0>⊗N
|gs> =
CAUTION



∏_v (1+A_v)
= (1 + A_v1 + A_v1 A_v2 + A_v1 A_v2 A_v3, ...) |0⟩^⊗N
|gs⟩ =
+ + +
+ .... = Σ_{γ ∈ closed loops} |γ⟩
CAUTION
CAUTION

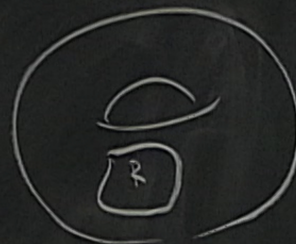


|gs> =
X1 |gs> =
CAUTION

Local indistinguishability

$|\psi\rangle \in gs$

$\rho^{|\psi\rangle} = |\psi\rangle\langle\psi|$



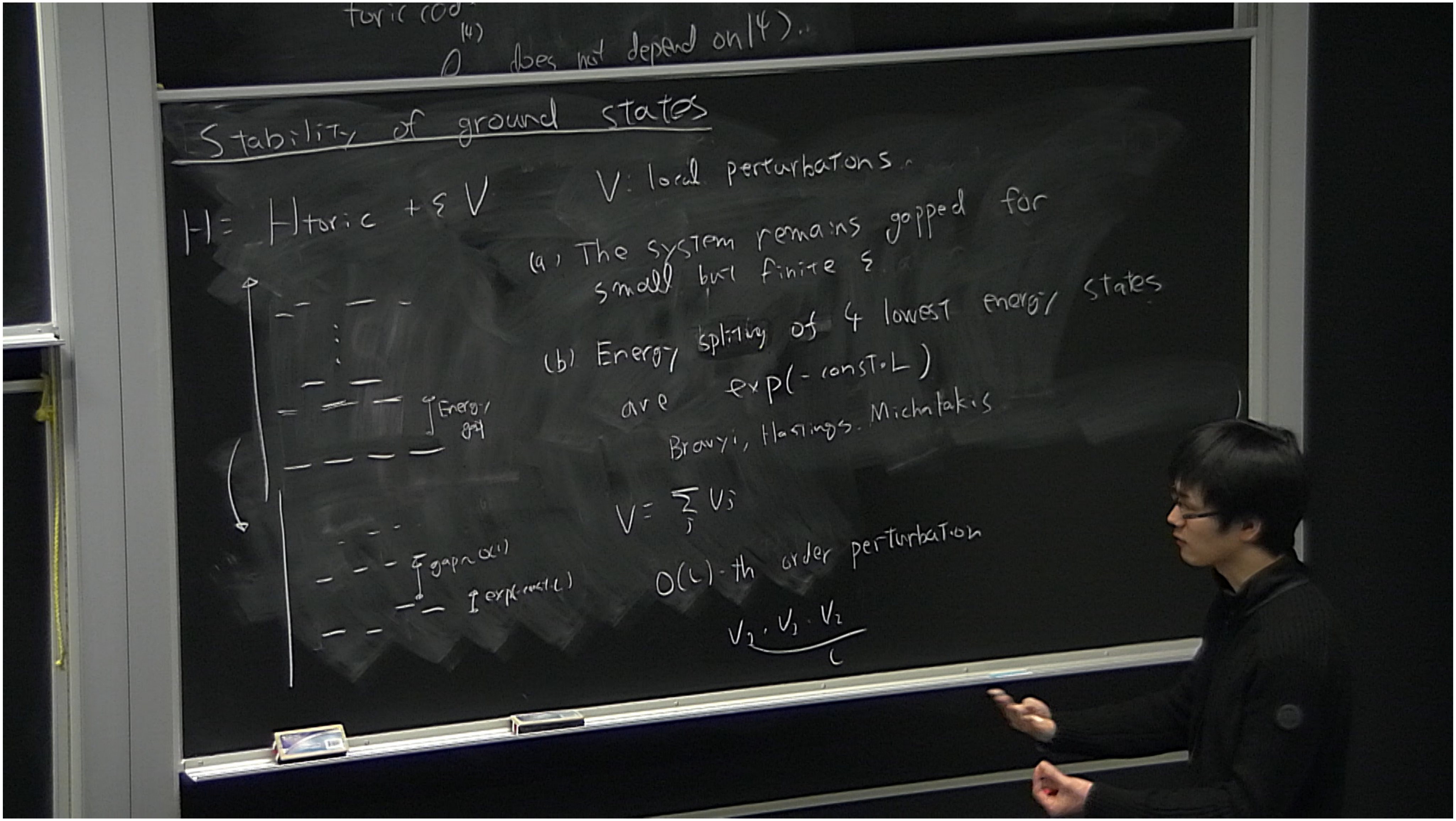
$$\rho_R^{|\psi\rangle} \equiv \mathrm{Tr}_{\bar{R}}\left(\rho^{|\psi\rangle}\right)$$

There is no logical op on R

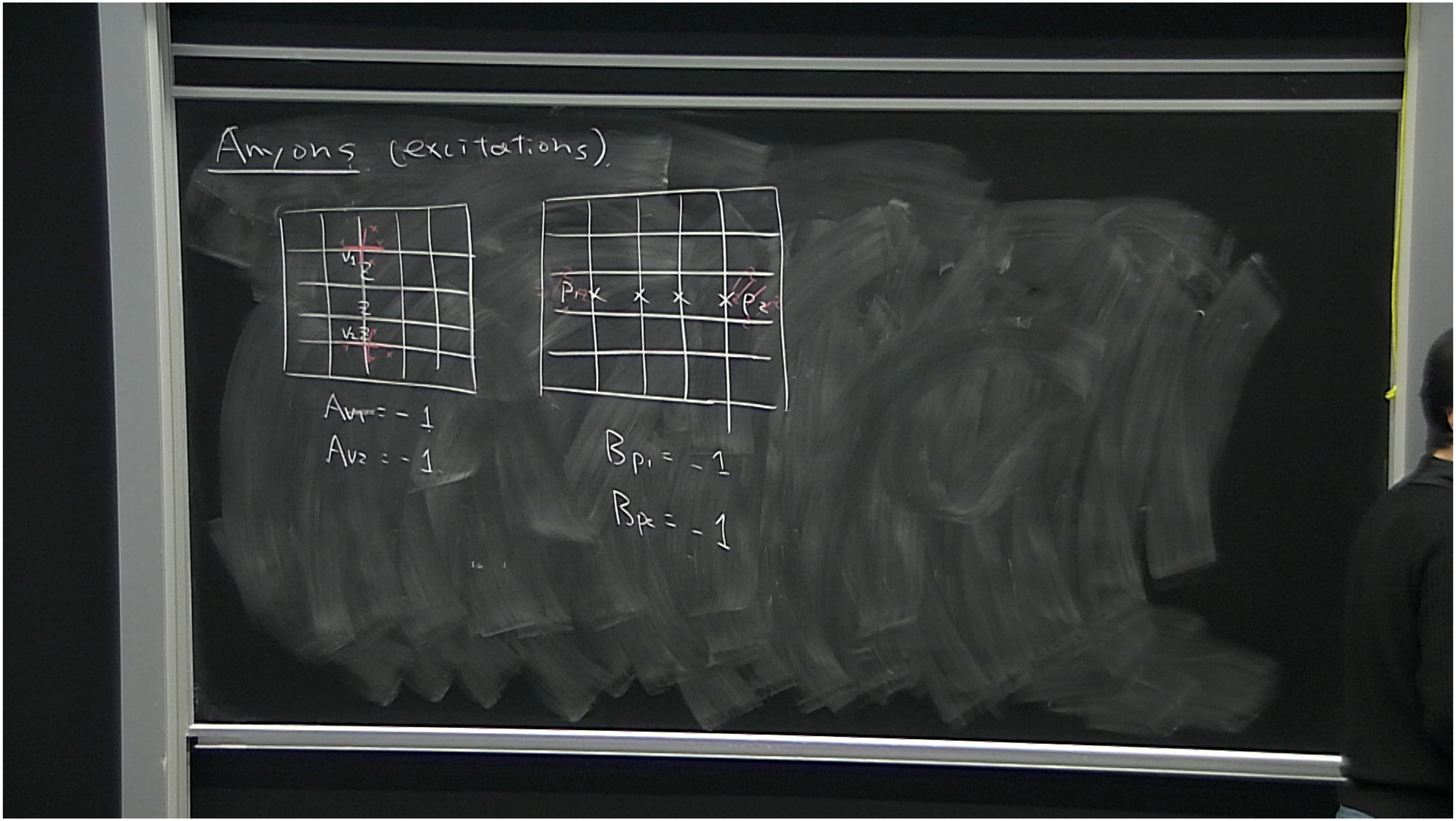
$\Downarrow$

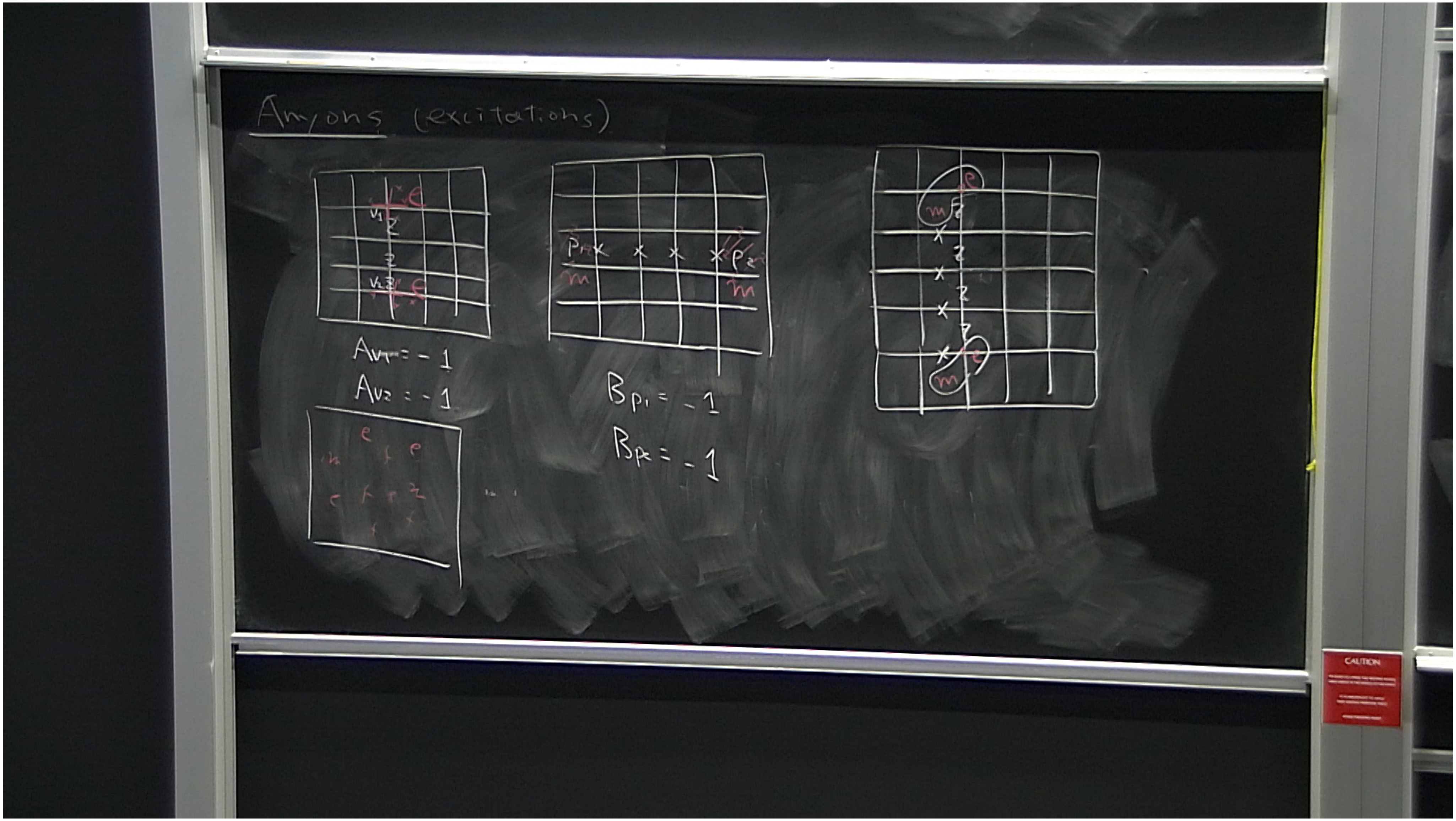
toric code

$\rho_R^{|\psi\rangle}$ does not depend on $|\psi\rangle$..



does not depend on |ψ⟩.
Stability of ground states
H = Htoric + ε V
V: local perturbations.
(a) The system remains gapped for small but finite ε.
(b) Energy splitting of 4 lowest energy states are exp(- const·L)
Bravyi, Hastings, Michalakis
Energy gap
gap ~ O(1)
exp(-const·L)
V = Σi Vi
O(L)-th order perturbation


Anyons (excitations)
v1
v2
Av1 = -1
Av2 = -1
p1
p2
Bp1 = -1
Bp2 = -1


Anyons (excitations)
Av1 = -1
Av2 = -1
Bp1 = -1
Bp2 = -1

Excitations can be viewed as a pair of quasi-particles.

- "$A_v = -1$" electric charge e
- "$B_p = -1$" magnetic flux m.
- "$A_v = -1$ & $B_p = -1$" fermion (e & m).

Key properties:

- Excitations appear as pairs

$$\prod_v A_v = I \qquad a_v = \pm 1$$

$$\prod_p B_p = I \qquad b_p = \pm 1$$

$a_v = -1 \Rightarrow e$ at v

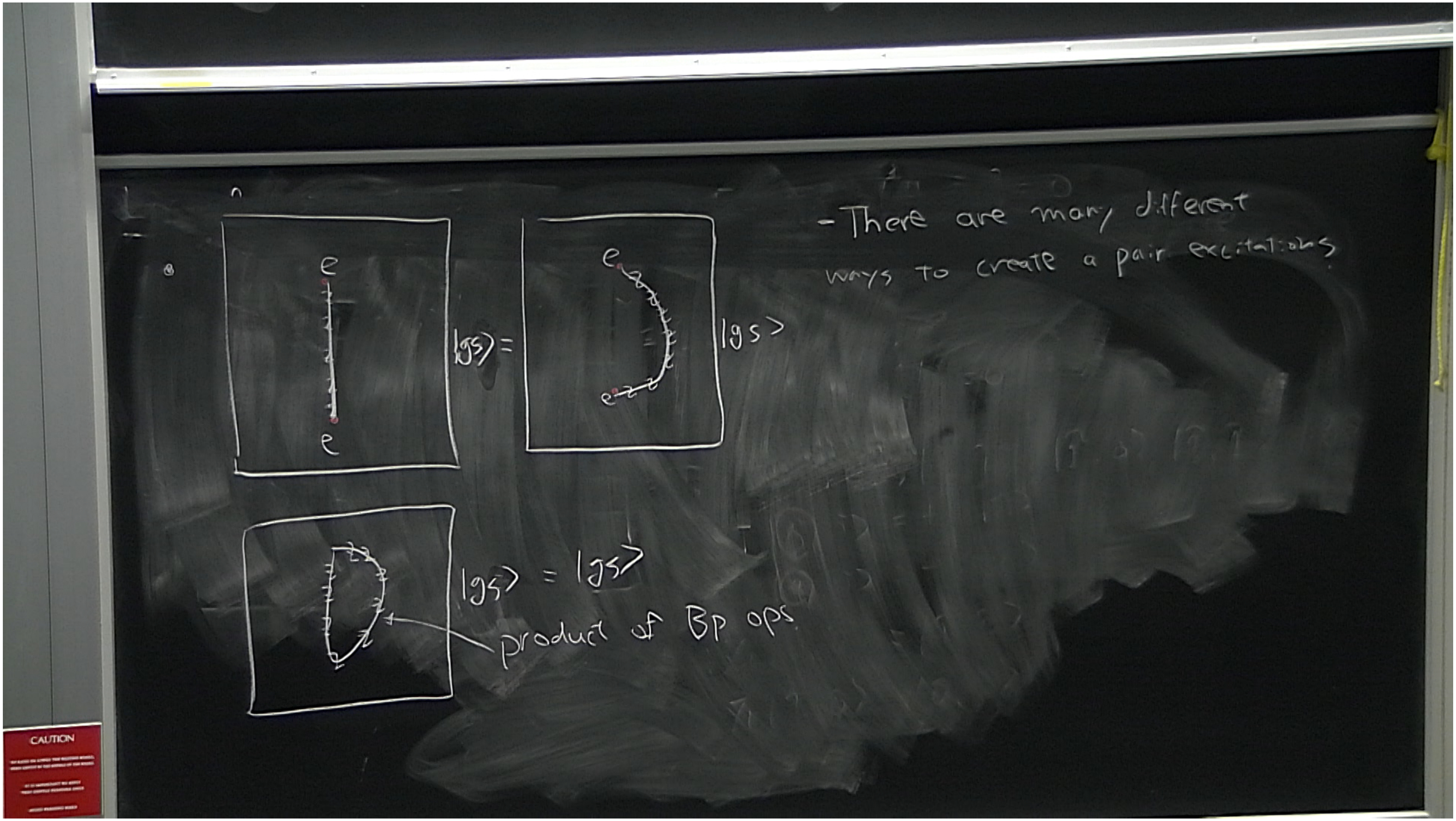
$b_p = -1 \Rightarrow m$ at p

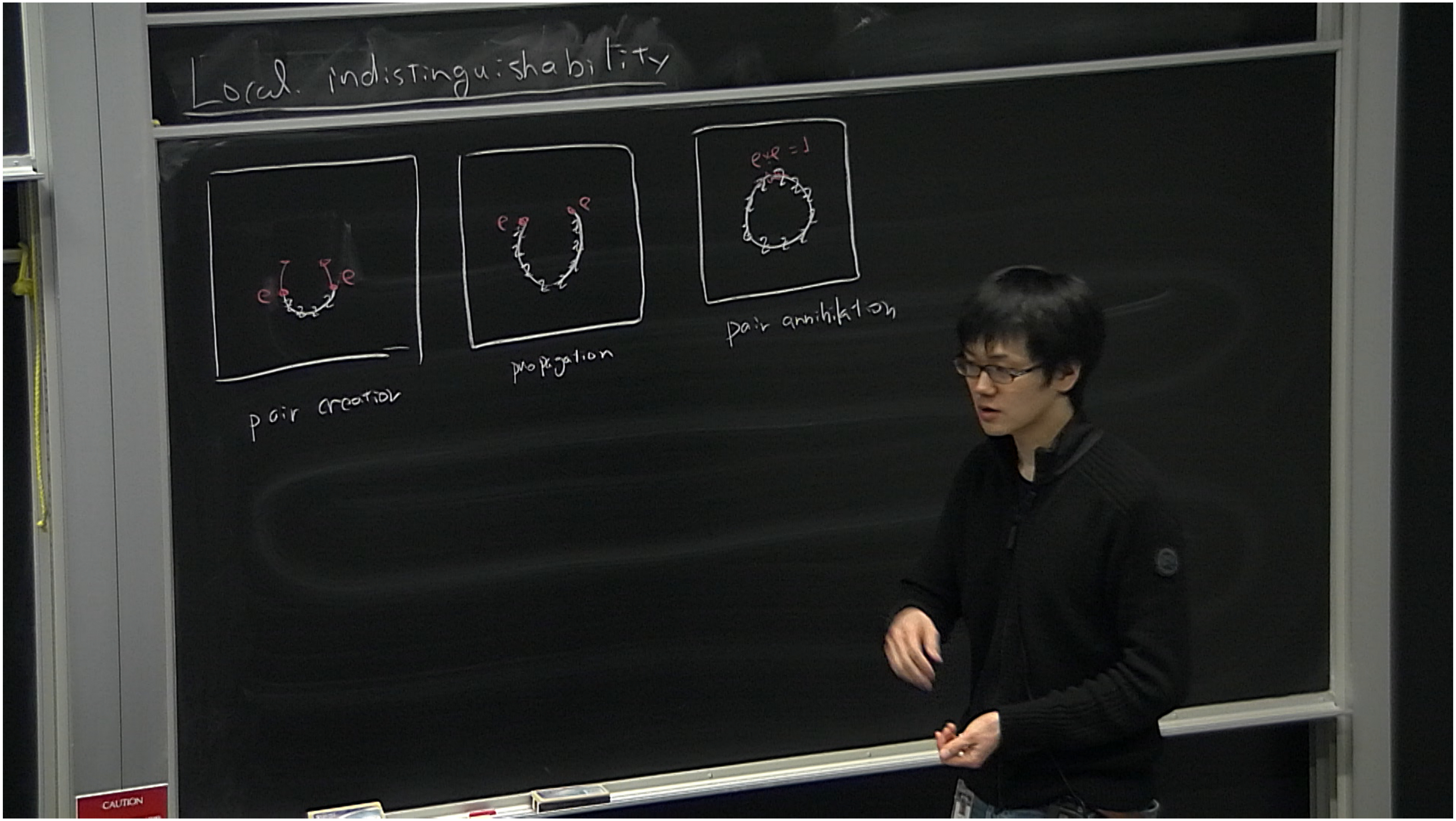
$$\prod_v a_v = 1$$

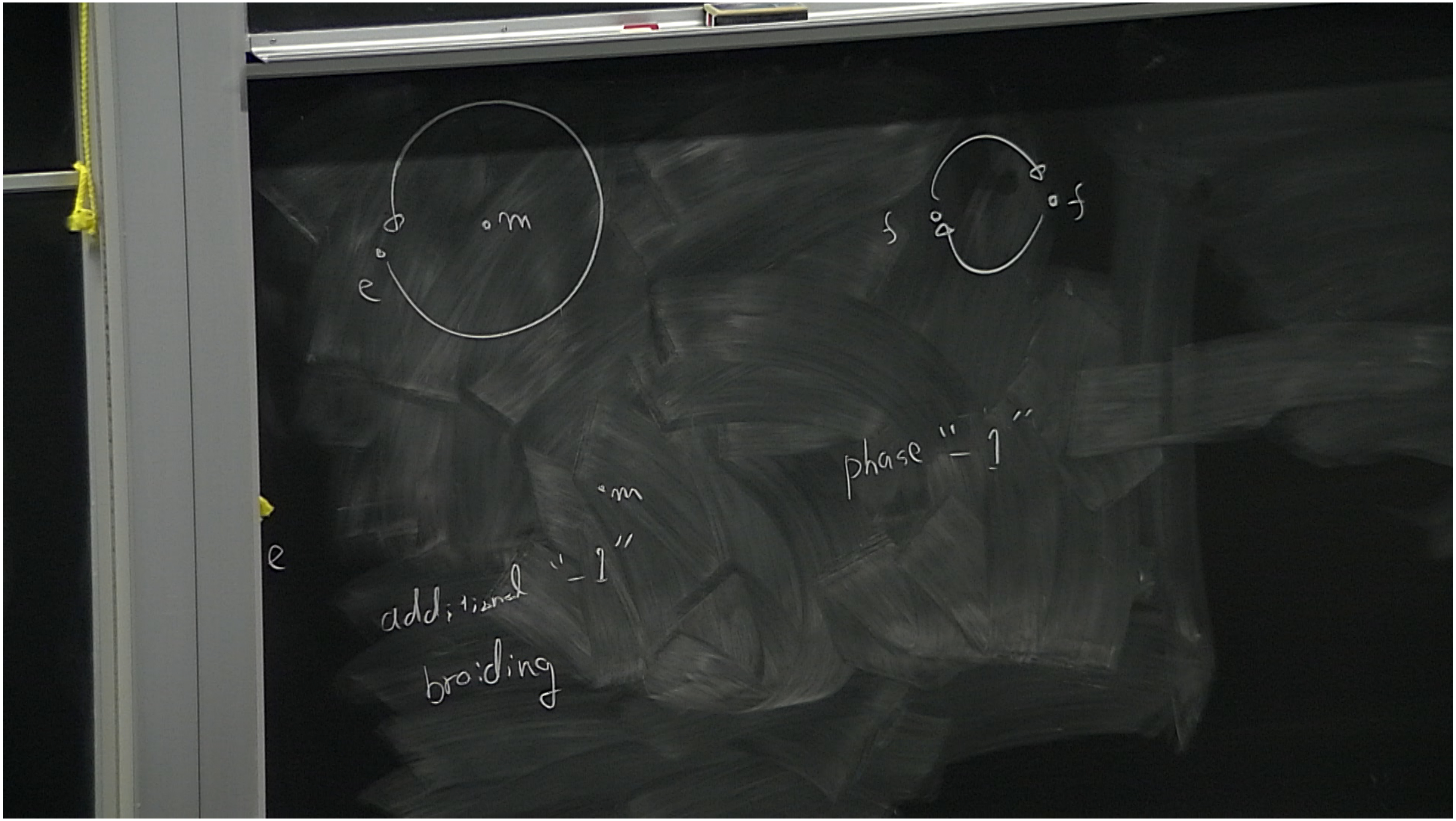
$$\prod_p b_p = 1$$

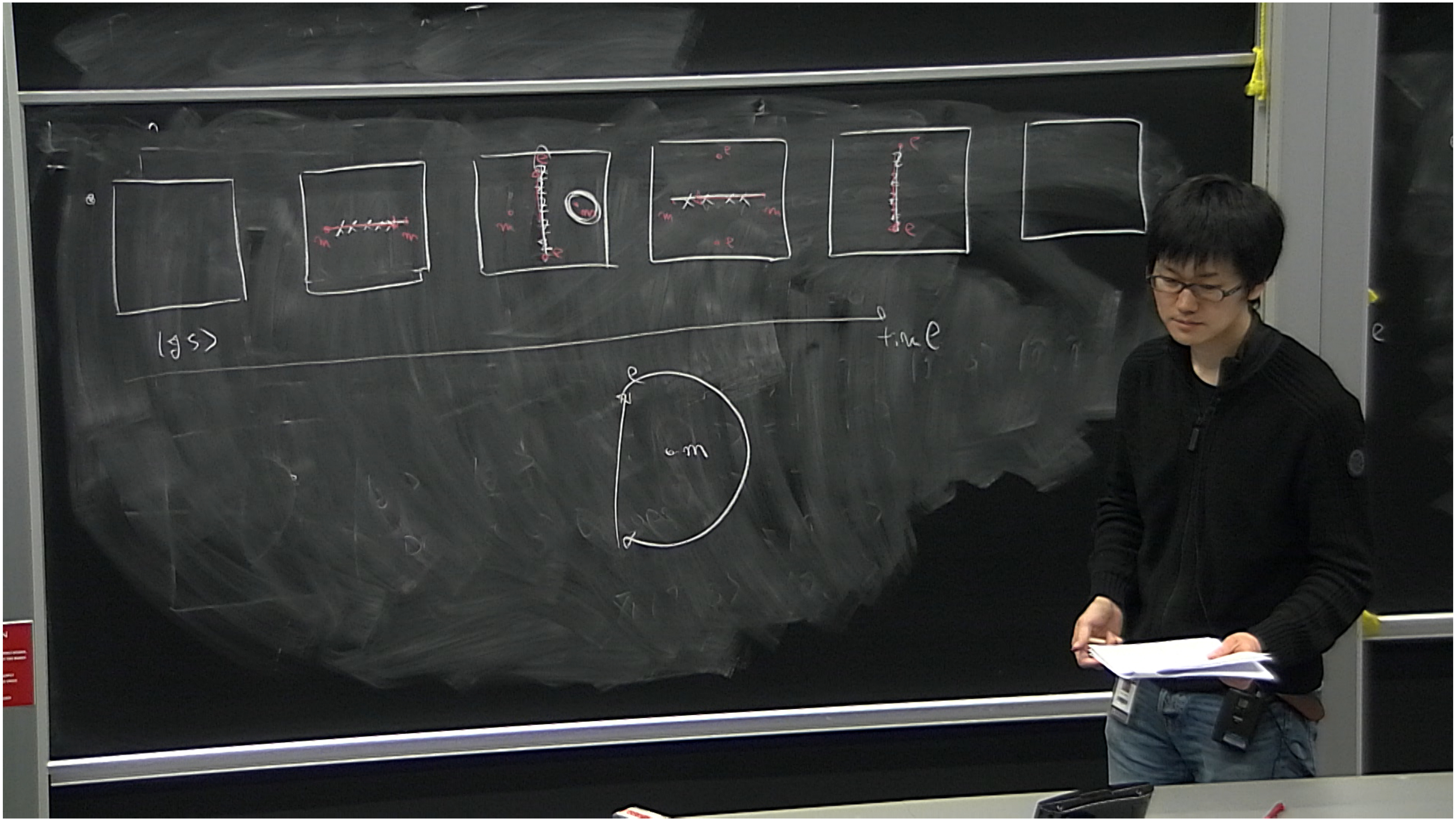
of "-1" in a_v (b_p) must be even

of e = # of m = 0 (mod 2).

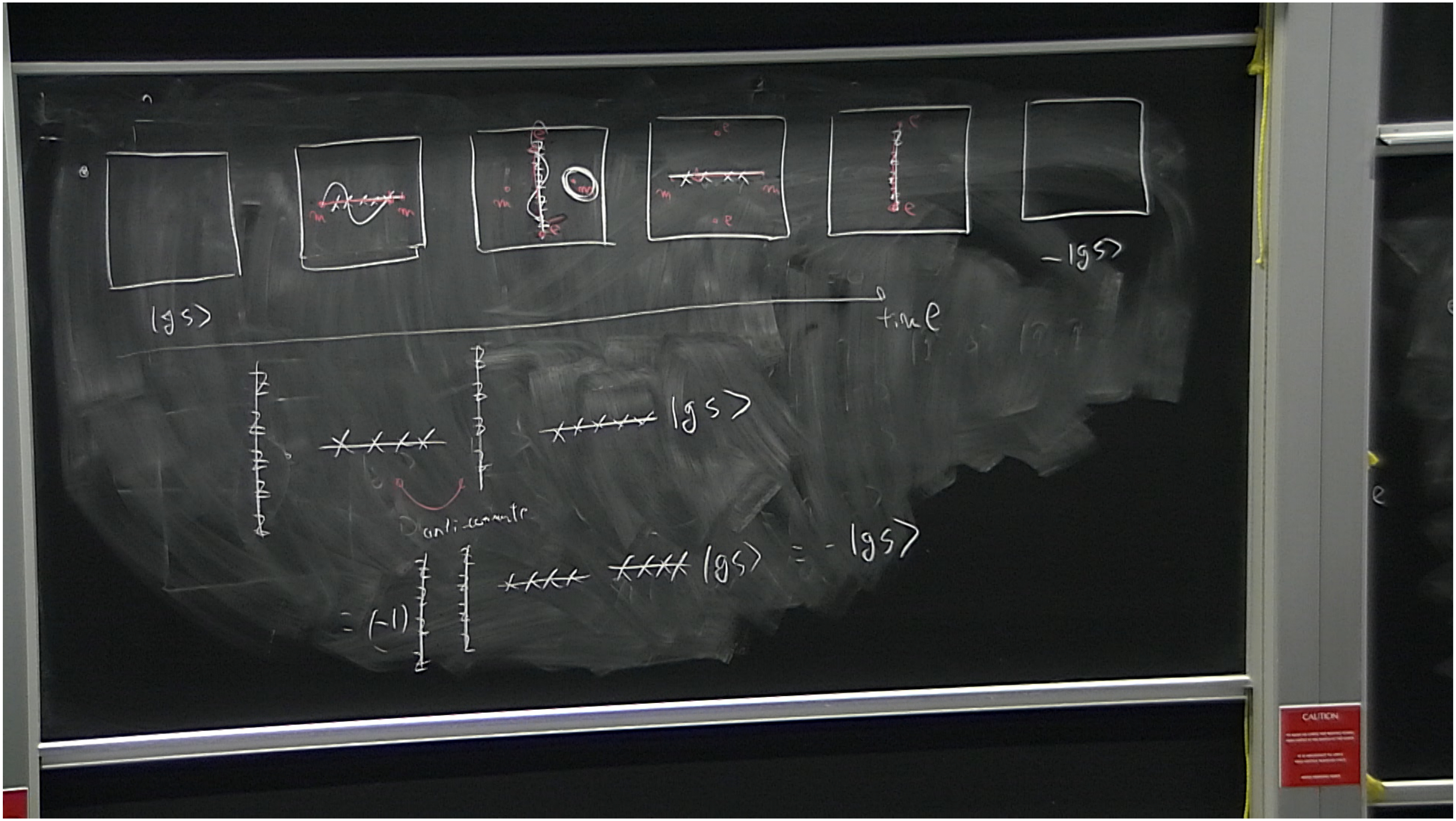

- There are many different ways to create a pair excitations
e
e
|gs> =
e
e
|gs>
|gs> = |gs>
product of Bp ops

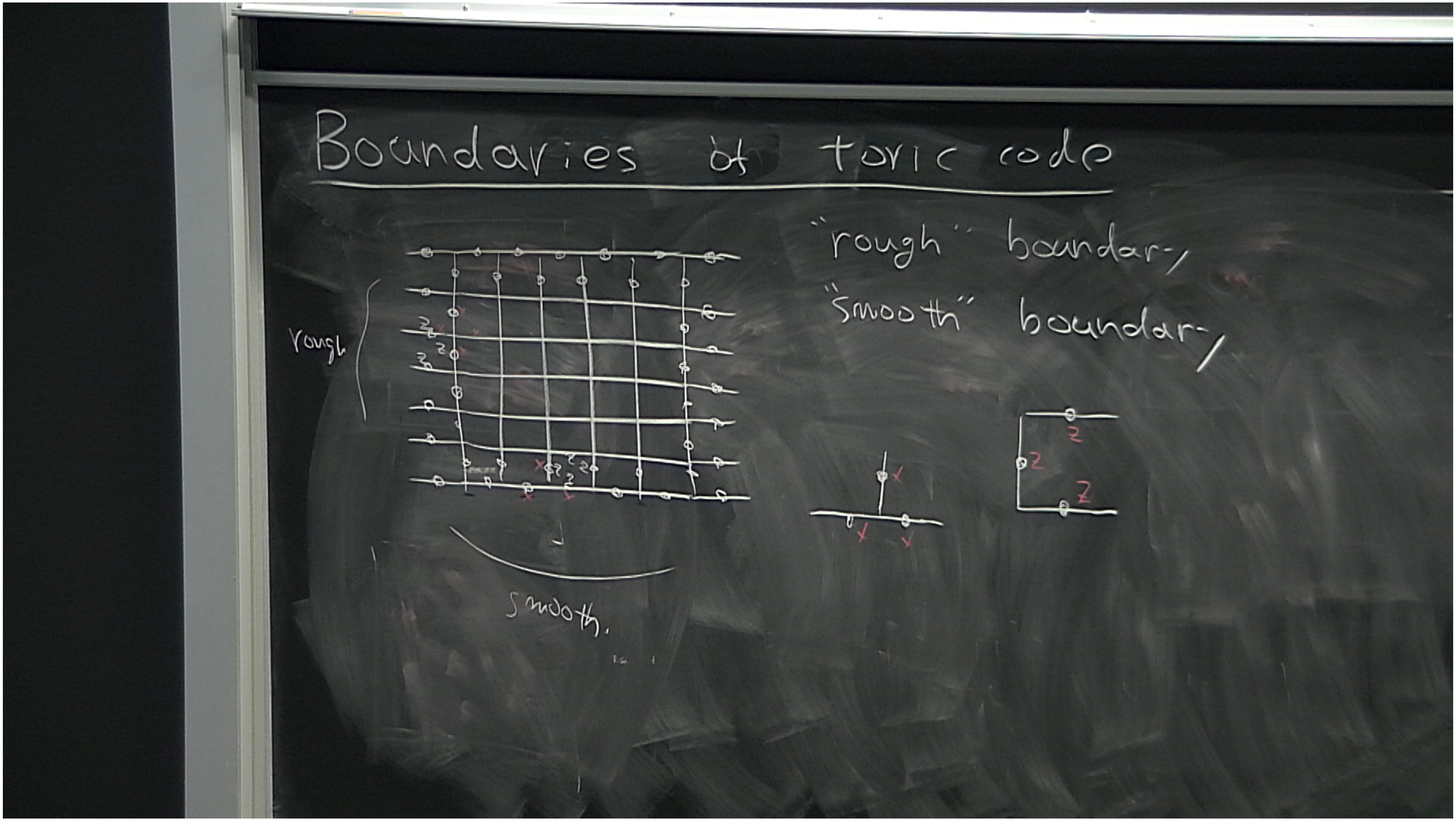

Local indistinguishability
e
e
pair creation
e
e
propagation
pair annihilation
CAUTION

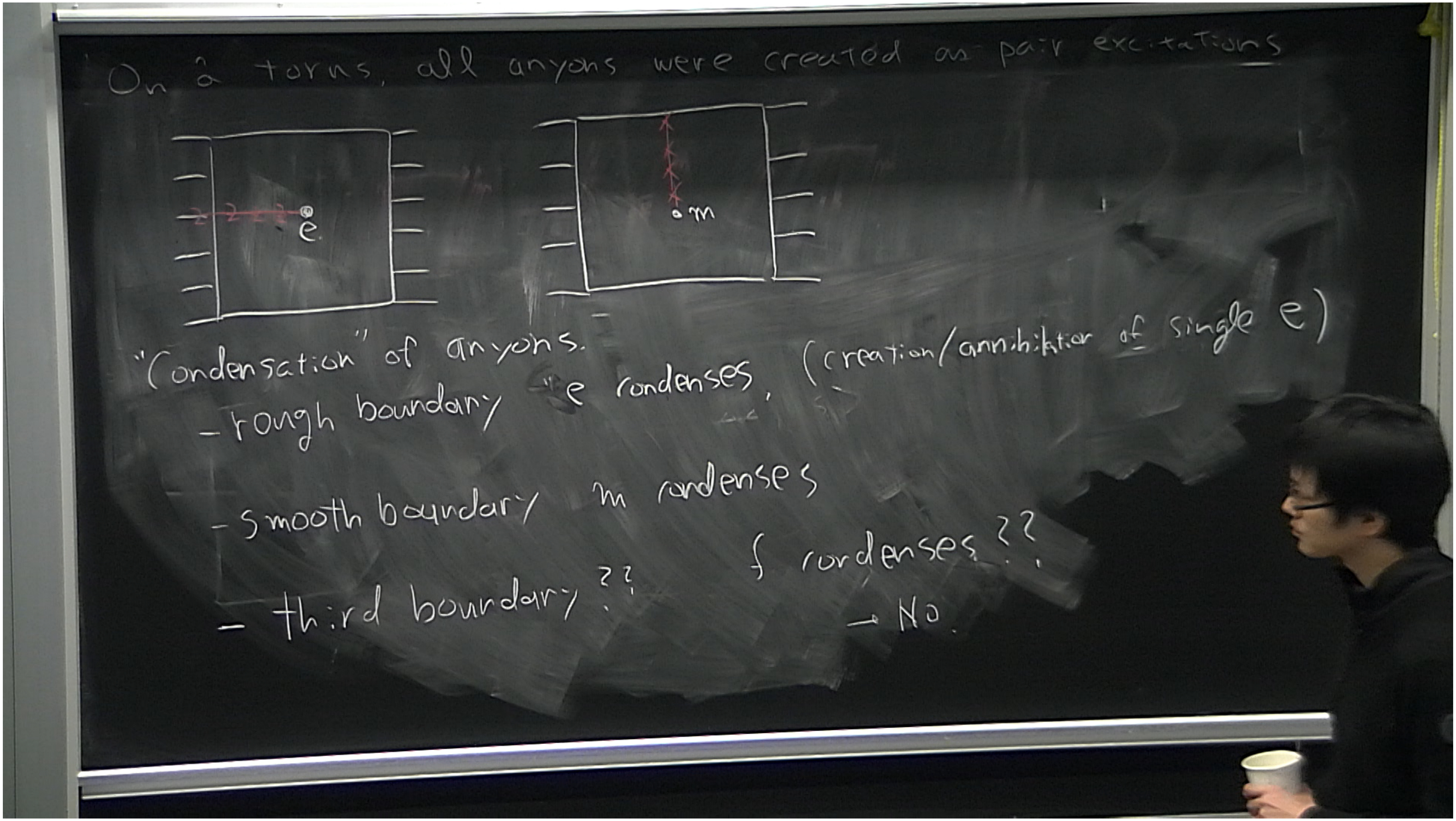

m
e
f
f
phase "−1"
m
e
additional "−1"
braiding

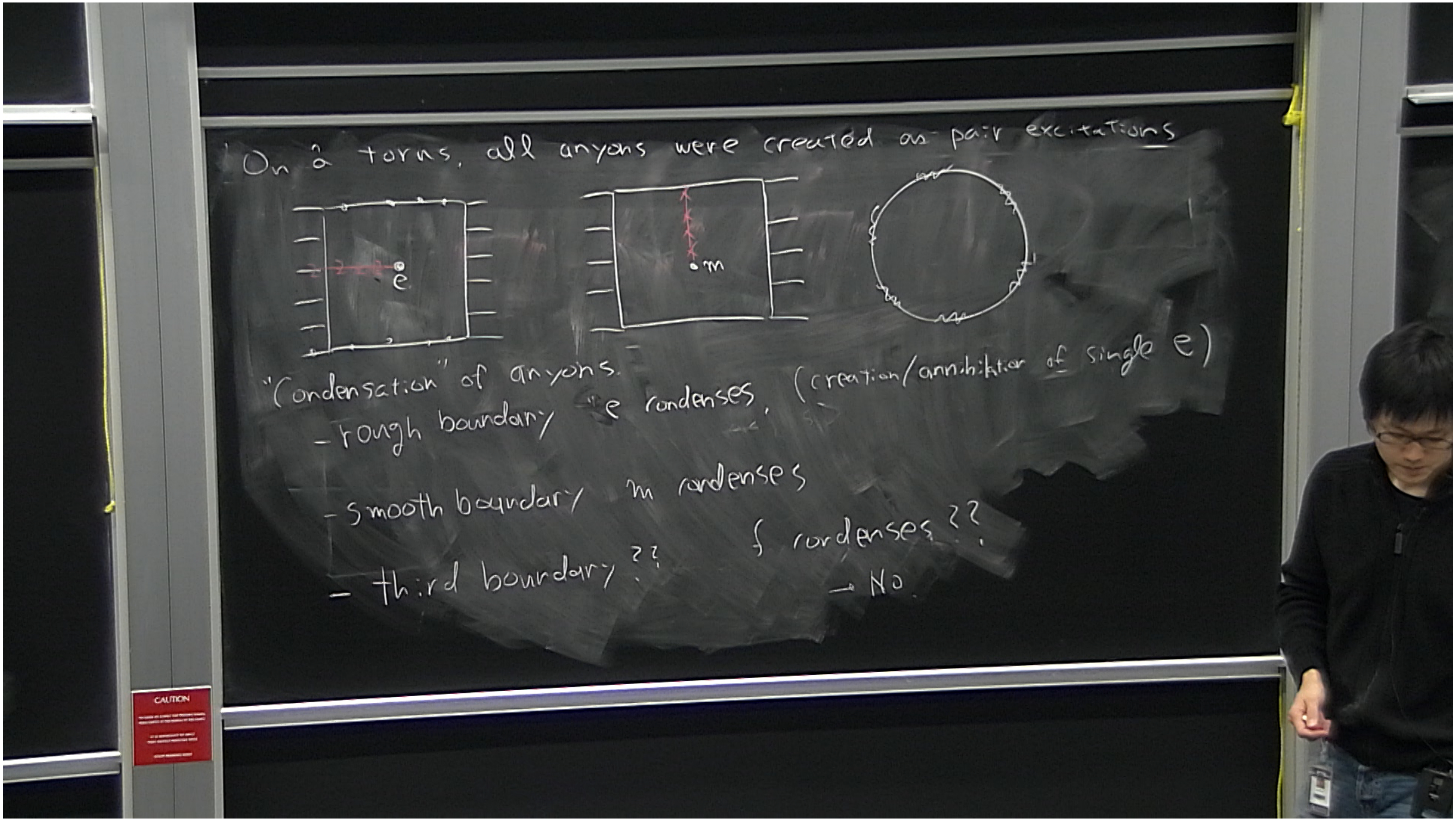


|gs>
time
e
m


|gs>
time
-|gs>
|gs>
anti-commute
|gs> = -|gs>
= (-1)


Boundaries of toric code
"rough" boundary
"smooth" boundary
rough
smooth.


On a torus, all anyons were created as pair excitations
e
m
"Condensation" of anyons.
- rough boundary e condenses, (creation/annihilation of single e)
- smooth boundary m condenses
- third boundary ?? f condenses ??
→ No.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