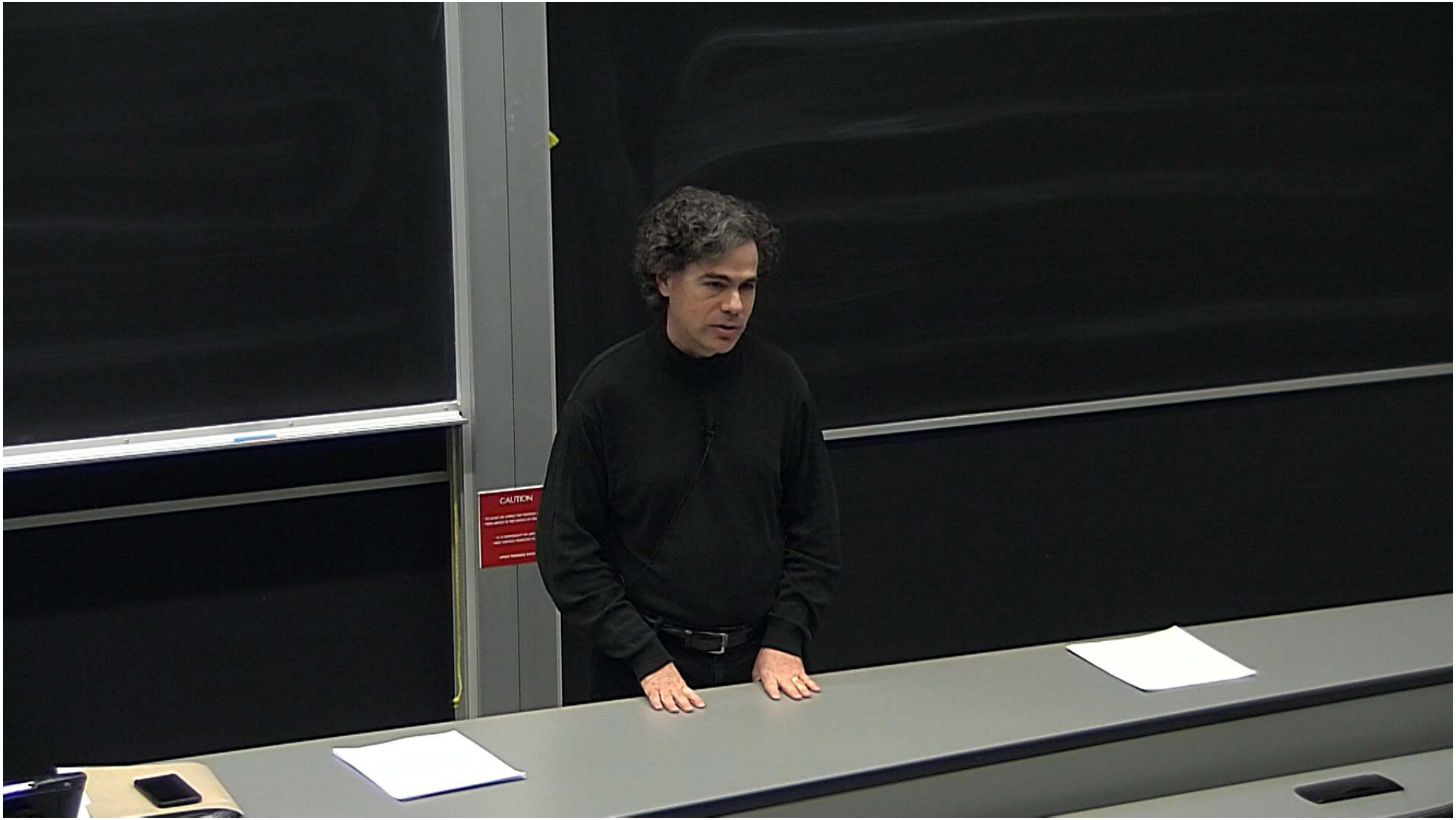
Title: IQC - Quantum Error Correction - Lecture 6

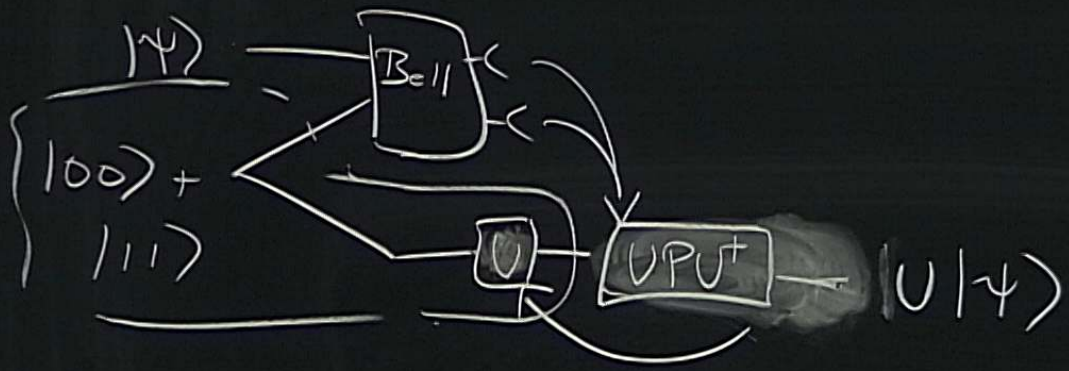
Date: Feb 08, 2018 02:00 PM

URL: http://pirsa.org/18020010

Abstract:


CAUTION

Magic state injection



$(I \otimes U)(|00\rangle + |11\rangle)$

$U \in C_1 : UPU^\dagger \in P_1$

$\forall P \in P_1 : UPU^\dagger \in C_1 \quad (U \in C_3)$

njection

Thm: C_n plus $R_{\pi/8}$ is a universal set of gates

$U^\dagger$ — $U|\psi\rangle$

$R_{\pi/8} \in C_3$

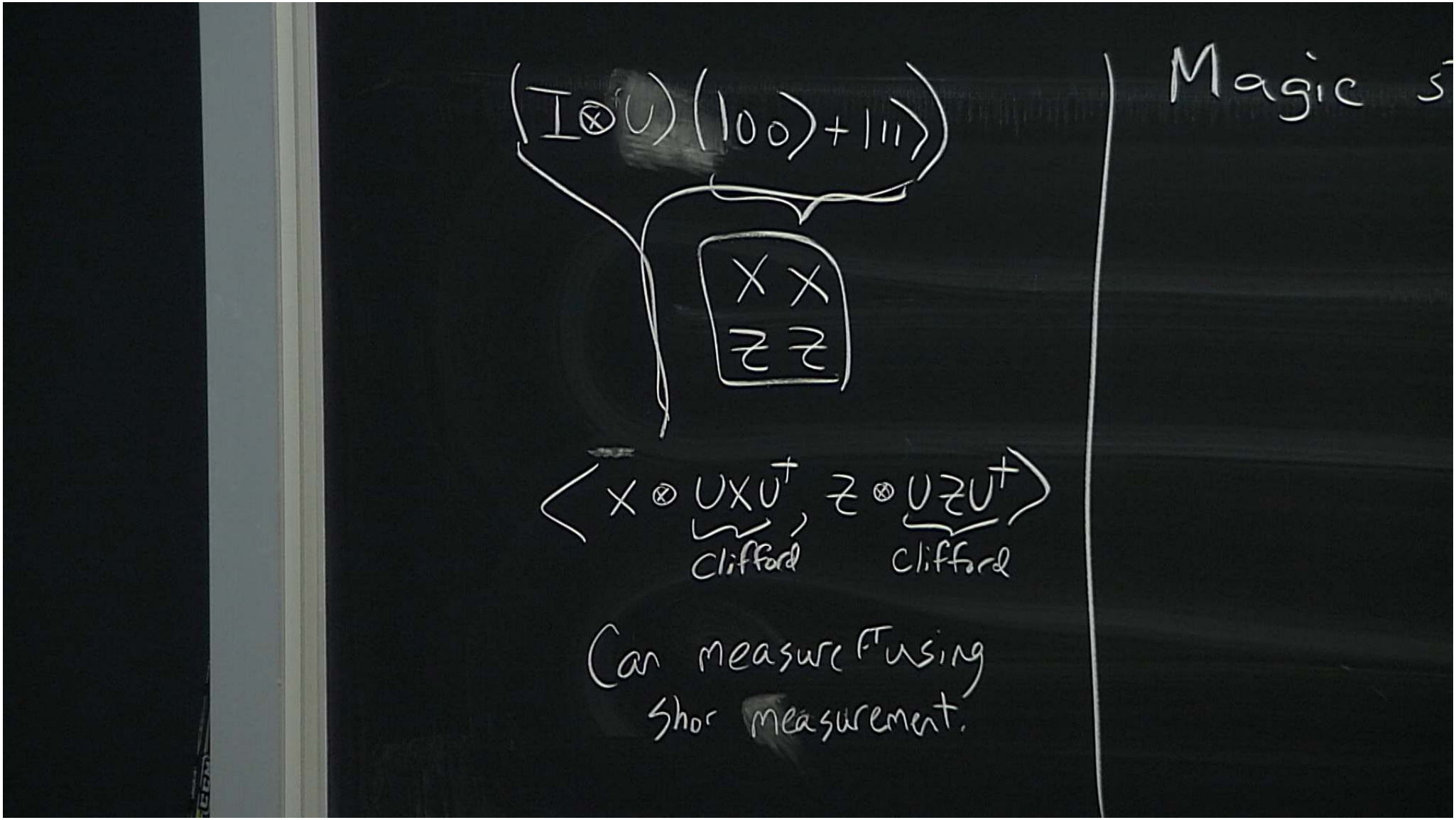
$R_{\pi/8} X R_{\pi/8}^\dagger = e^{-i\pi/4} R_{\pi/4} X$

$R_{\pi/8} Z R_{\pi/8}^\dagger = Z$

$\in P_1$

$T_0 f \in C_3$ $\quad T_0 f |a,b,c\rangle = |a,b,c+ab\rangle$

$(U \in C_2)$


Magic s
(I⊗U)(|00⟩+|11⟩)
X X
Z Z
⟨X⊗UXU†, Z⊗UZU†⟩
Clifford
Clifford
Can measure F using
Shor measurement.

$+|11\rangle)$

$Z \otimes \underbrace{UZU^\dagger}_{\text{Clifford}}\rangle$

reFusing

surement.

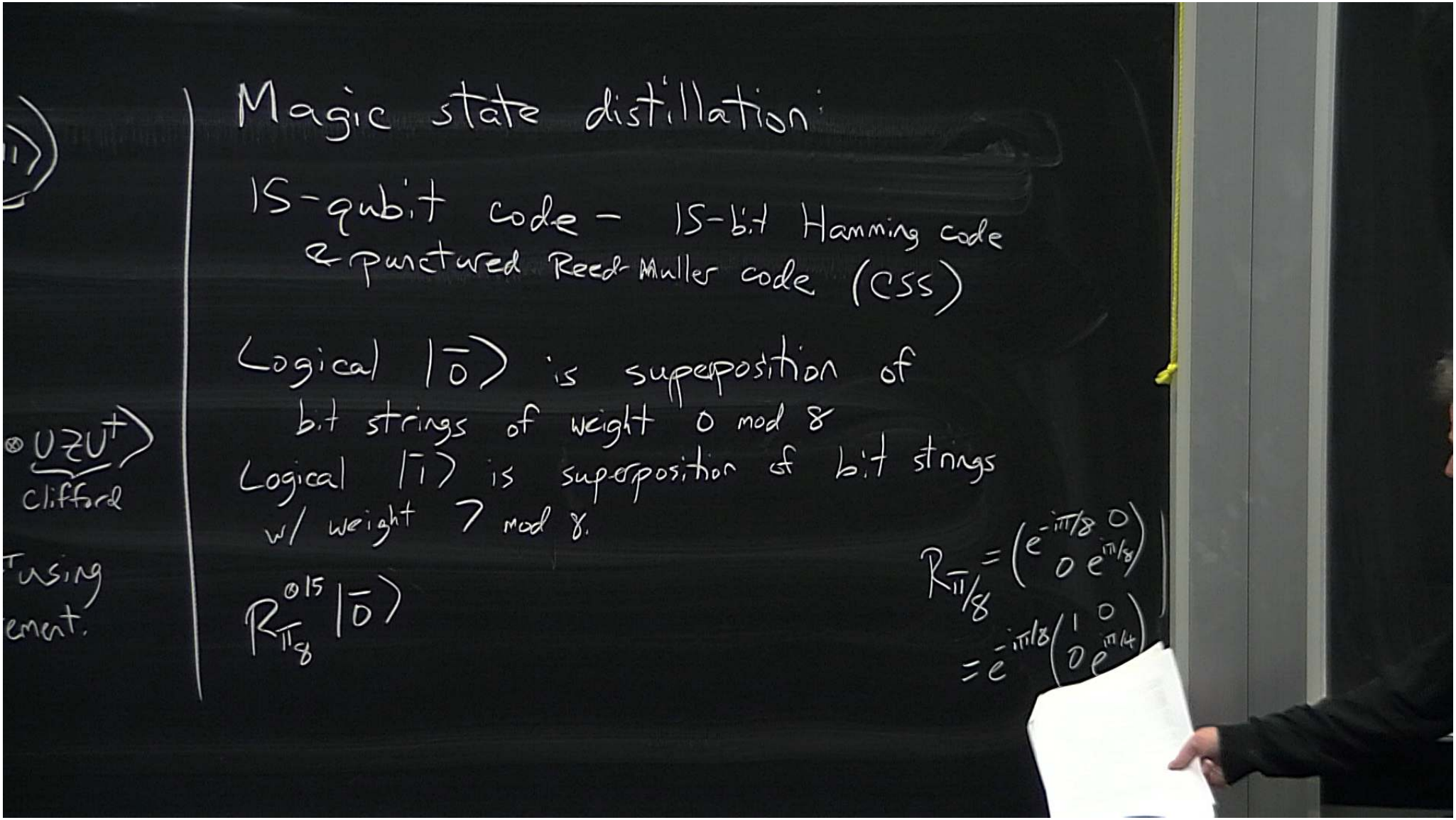
Magic state distillation:

15-qubit code – 15-bit Hamming code & punctured Reed-Muller code (CSS)

Logical $|\bar{0}\rangle$ is superposition of bit strings of weight 0 mod 8

Logical $|\bar{1}\rangle$ is superposition of bit strings w/ weight 7 mod 8.

$R_{\pi_8}^{\otimes 15}|\bar{0}\rangle$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$= e^{-i\pi/8}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$
$\otimes UZU^\dagger\rangle$
Clifford
using
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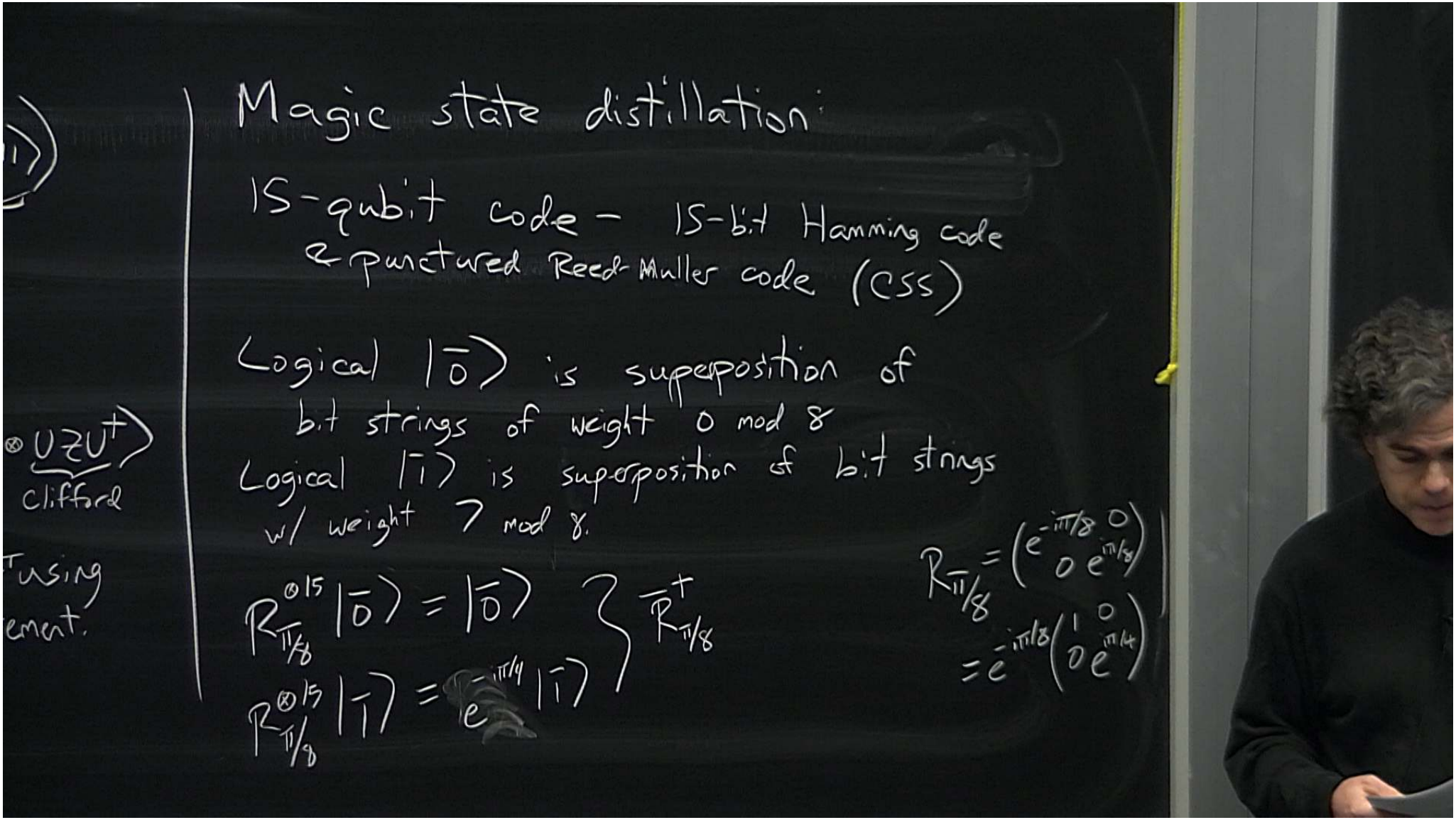
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Suppose we can do C_n reliably, make $(I \otimes R_{\pi/8})(|00\rangle + |11\rangle)$ unreliably

→ Do $R_{\pi/8}$ unreliably

→ Create $|\bar{0}\bar{0}\rangle + |\bar{1}\bar{1}\rangle$ for 15-qubit code

→ Can create $R_{\pi/8}^{\otimes 15}(|\bar{0}\bar{0}\rangle + |\bar{1}\bar{1}\rangle)$ but with $R_{\pi/8}$'s unreliable, with prob. p of error

Prob. (2 bad $R_{\pi/8}$ gates) $= O(p^2)$

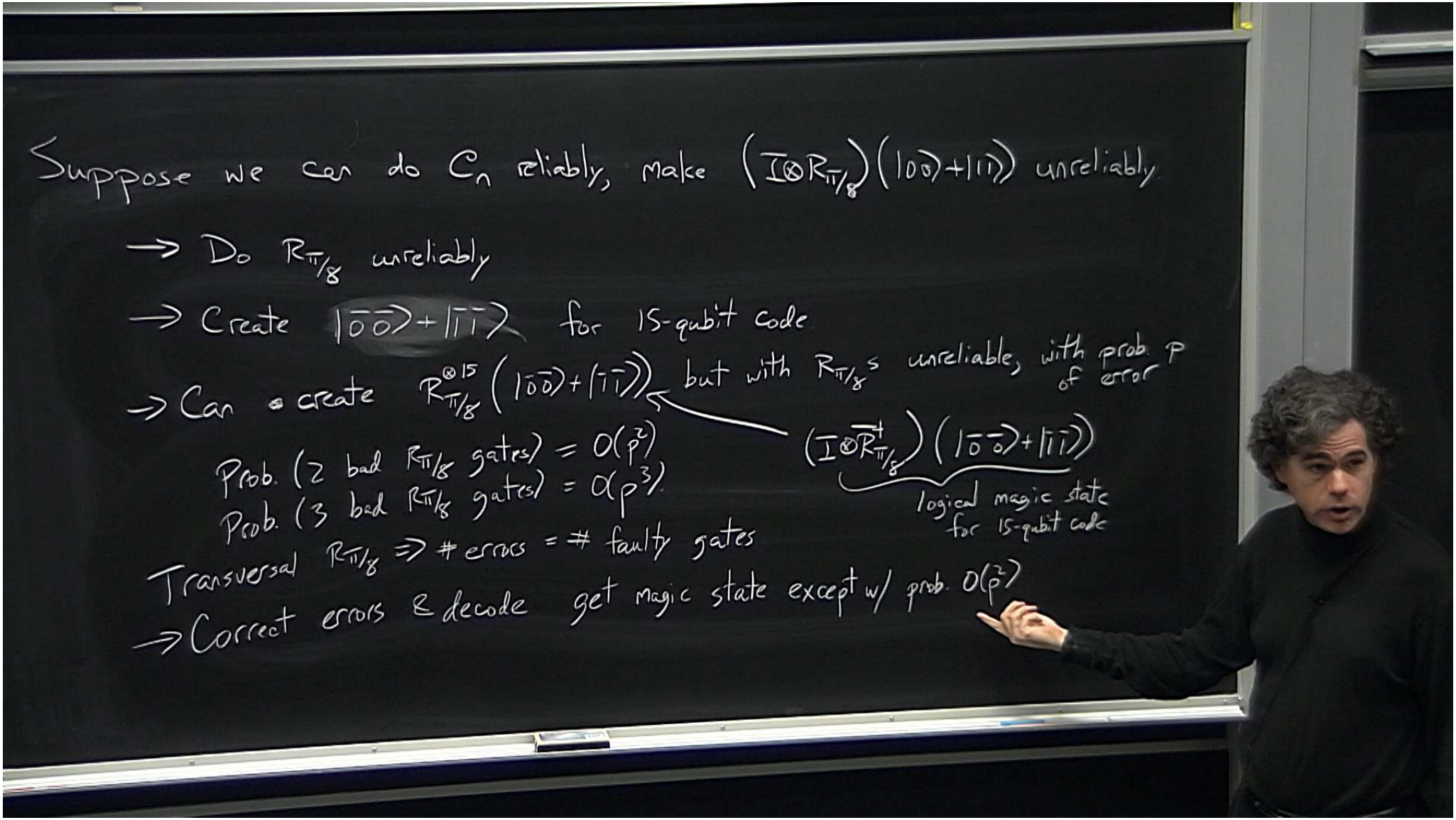
Prob. (3 bad $R_{\pi/8}$ gates) $= O(p^3)$

Transversal $R_{\pi/8}$ ⇒ # errors = # faulty gates

→ Correct errors & decode get magic state except w/

$(I \otimes \bar{R}_{\pi/8}^{\dagger})(|\bar{0}\bar{0}\rangle + |\bar{1}\bar{1}\rangle)$

logical magic state for 15-qubit code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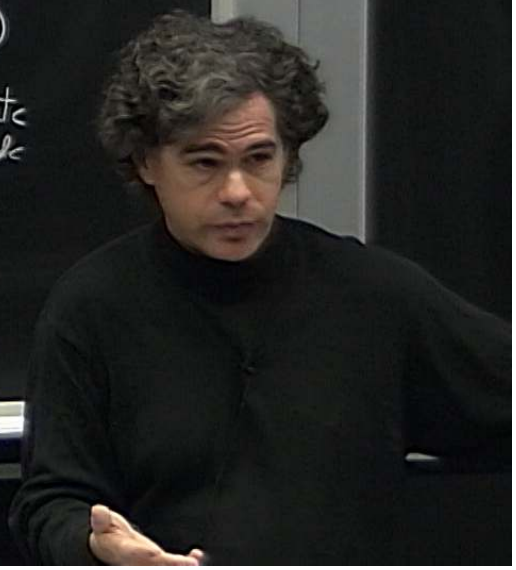
Transversal $R_{\pi/8}$ ⇒ # errors = # faulty gates

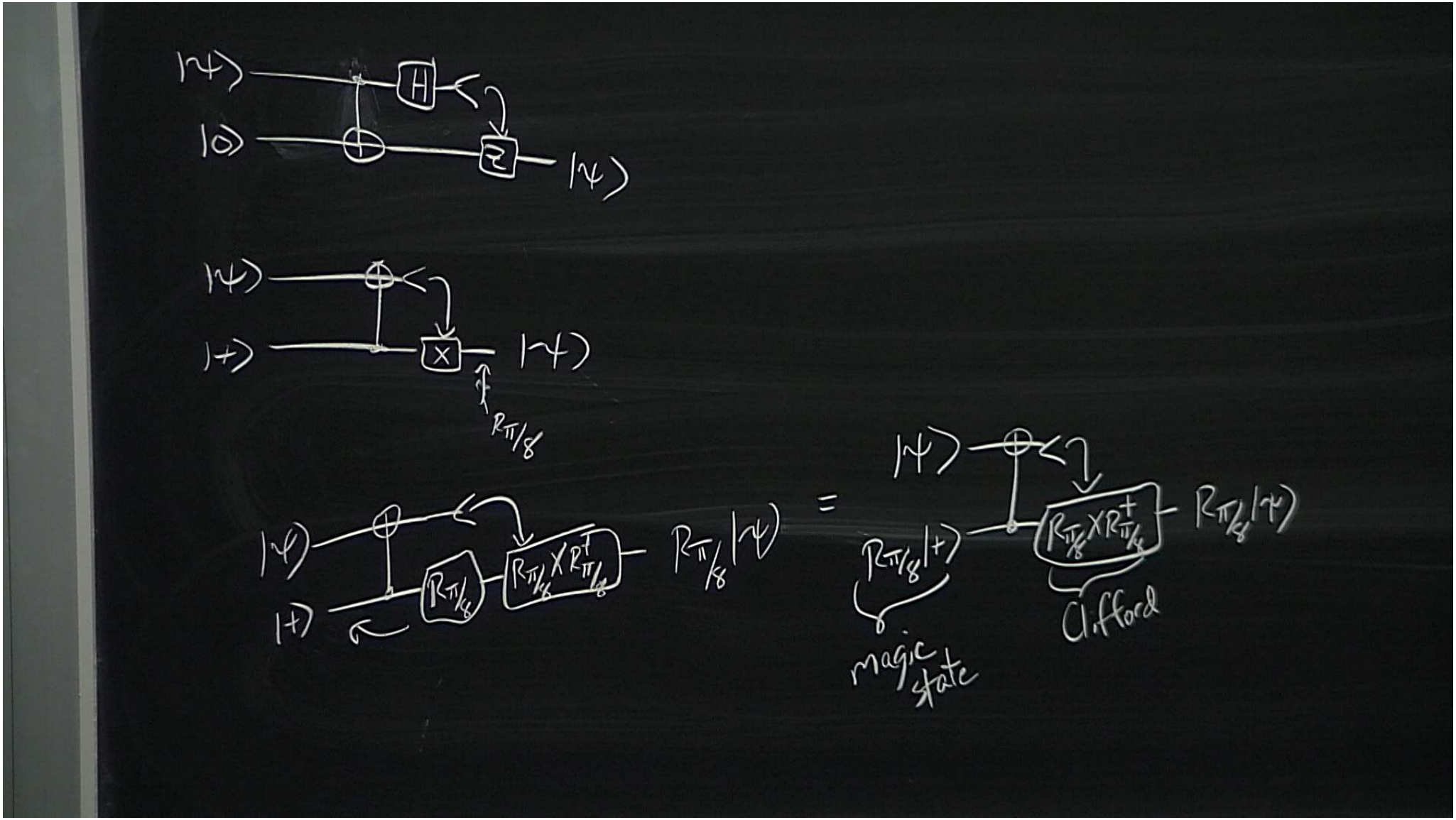
→ Correct errors & decode : get magic state except w/ prob. $O(p^2)$

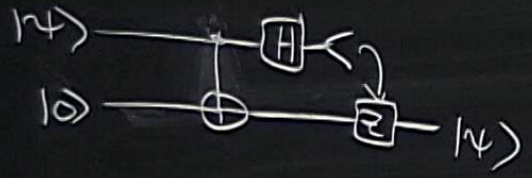
→ Detect errors & decode : " " " " " $O(p^3)$

$(I \otimes \bar{R}_{\pi/8}^{\dagger})(|\bar{0}\bar{0}\rangle + |\bar{1}\bar{1}\rangle)$

logical magic state for 15-qubit code

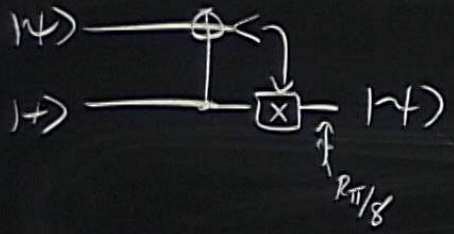


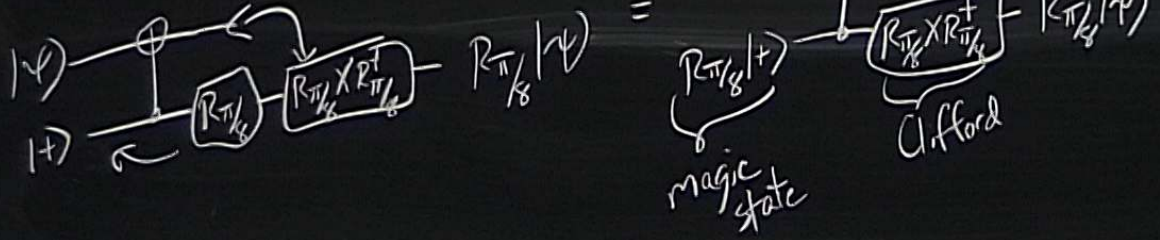

|ψ⟩
|0⟩
H
Z
|ψ⟩
|ψ⟩
|+⟩
X
|ψ⟩
$R_{\pi/8}$
|ψ⟩
|+⟩
$R_{\pi/4}$
$R_{\pi/8} X R^{\dagger}_{\pi/8}$
$R_{\pi/8}|\psi\rangle$
=
|ψ⟩
$R_{\pi/8}|+\rangle$
$R_{\pi/8} X R^{\dagger}_{\pi/8}$
$R_{\pi/8}|\psi\rangle$
magic state
Clifford



Pauli frame

- keep track of Pauli errors
- update after Clifford group gates
- reset after magic state injection

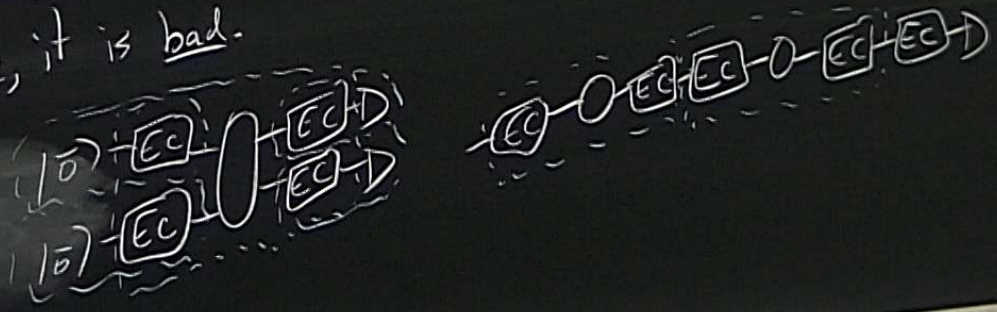


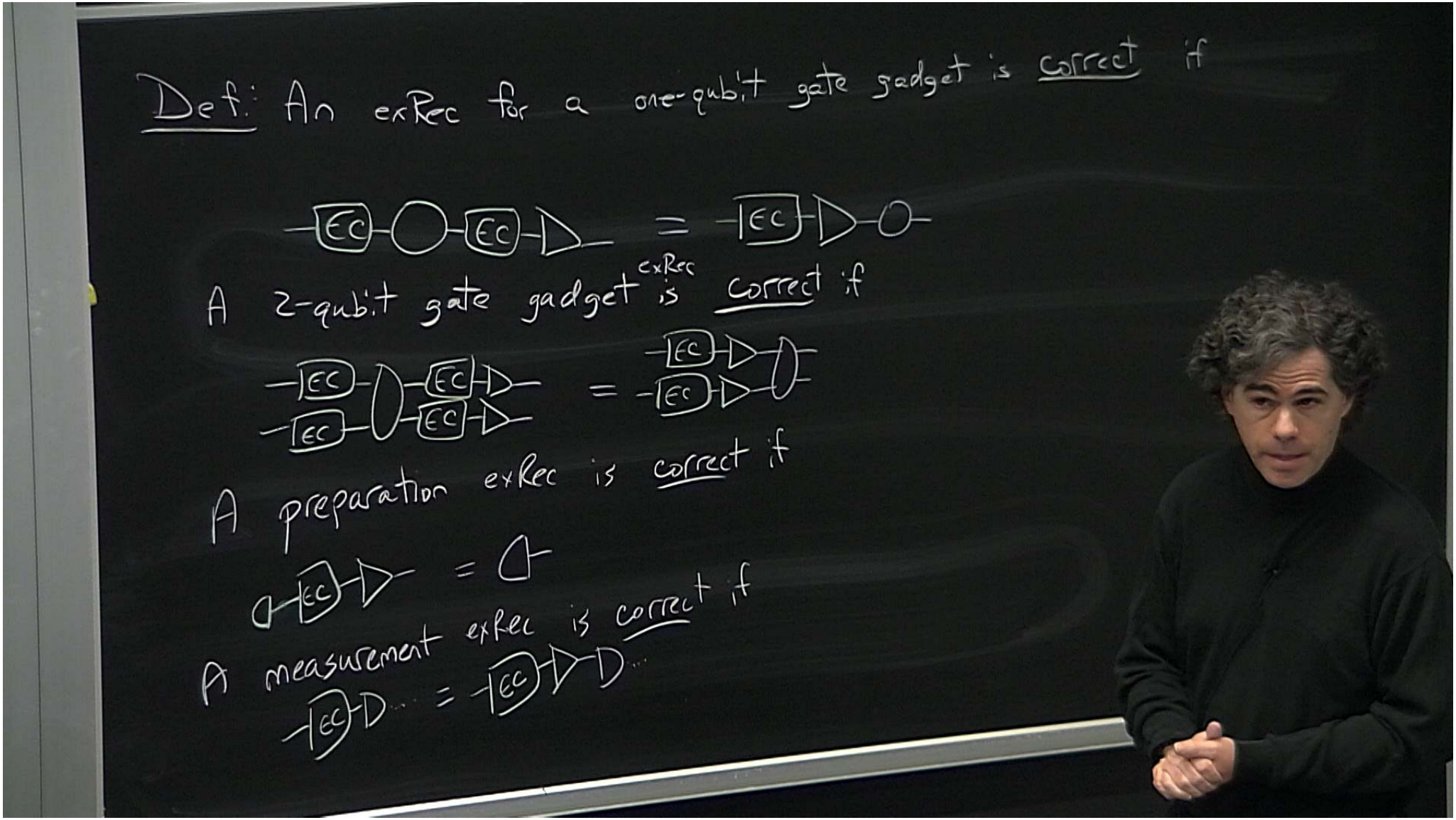


Def: An exRec (extended rectangle) consists of one of the following:

- A gate or wait gadget plus EC gadgets before & after
- A state preparation gadget plus EC gadget(s) after
- A measurement gadget plus EC gadget(s) before

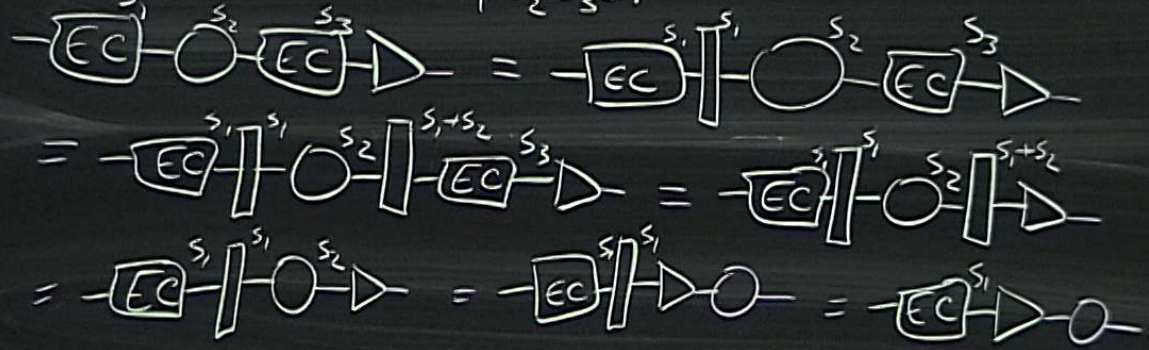
If the QECC corrects t errors, an exRec is **good** if it has $\leq t$ faults.
Otherwise, it is **bad**.




Def: An exRec for a one-qubit gate gadget is correct if
EC
EC
=
EC
A 2-qubit gate gadget exRec is correct if
EC
EC
EC
EC
=
EC
EC
A preparation exRec is correct if
EC
=
A measurement exRec is correct if
EC
D...
=
EC
D...

Thm: A good exRec is correct.

Proof:



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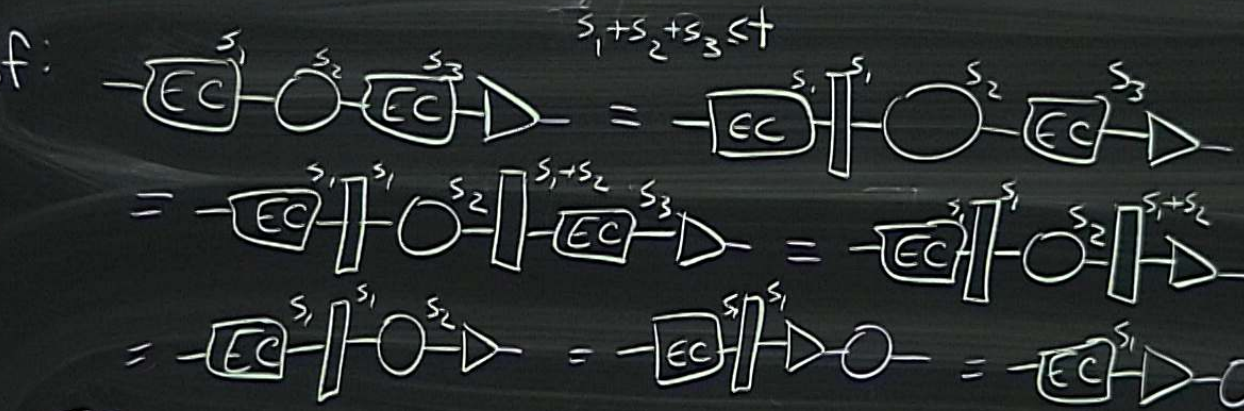
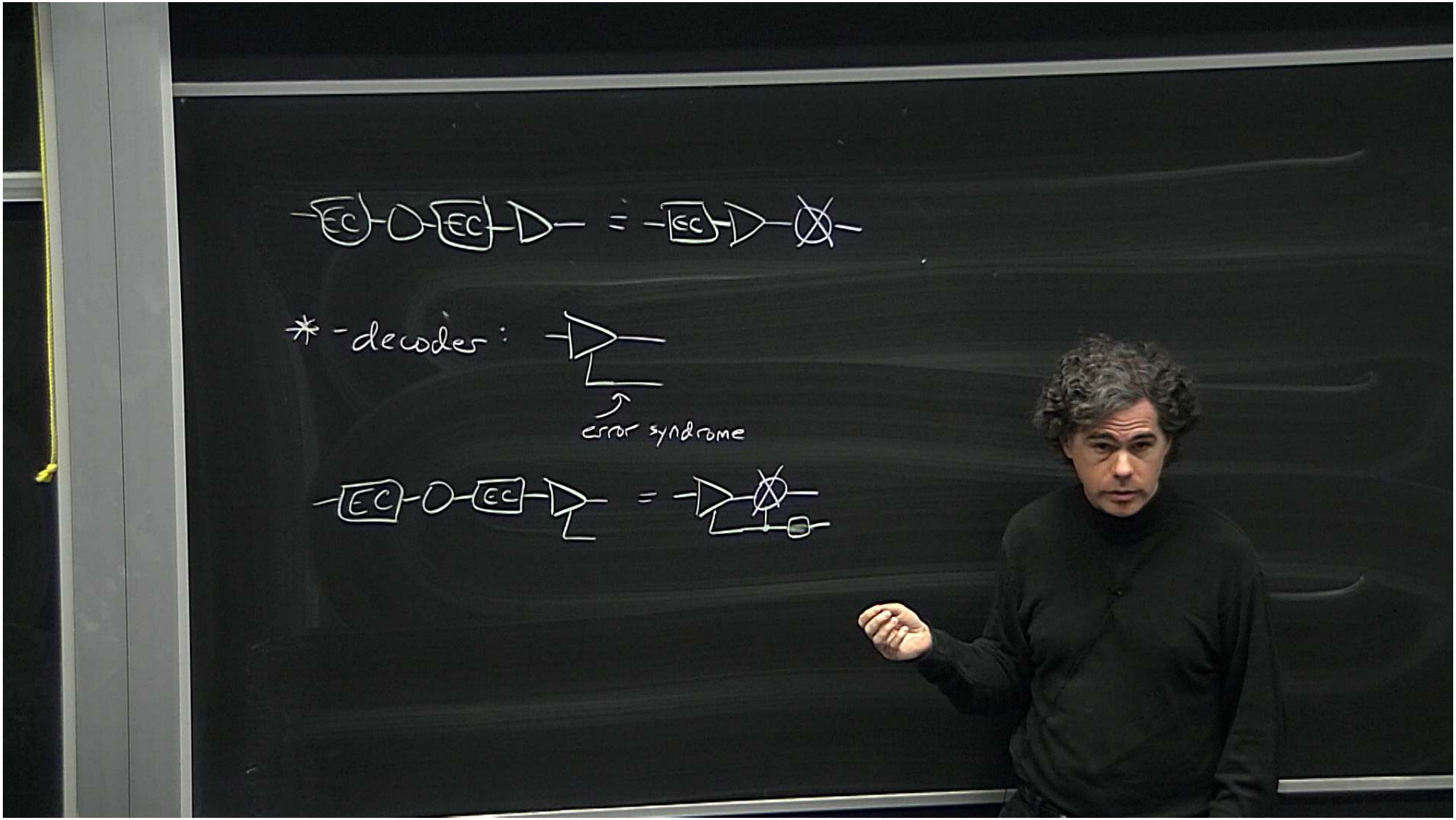
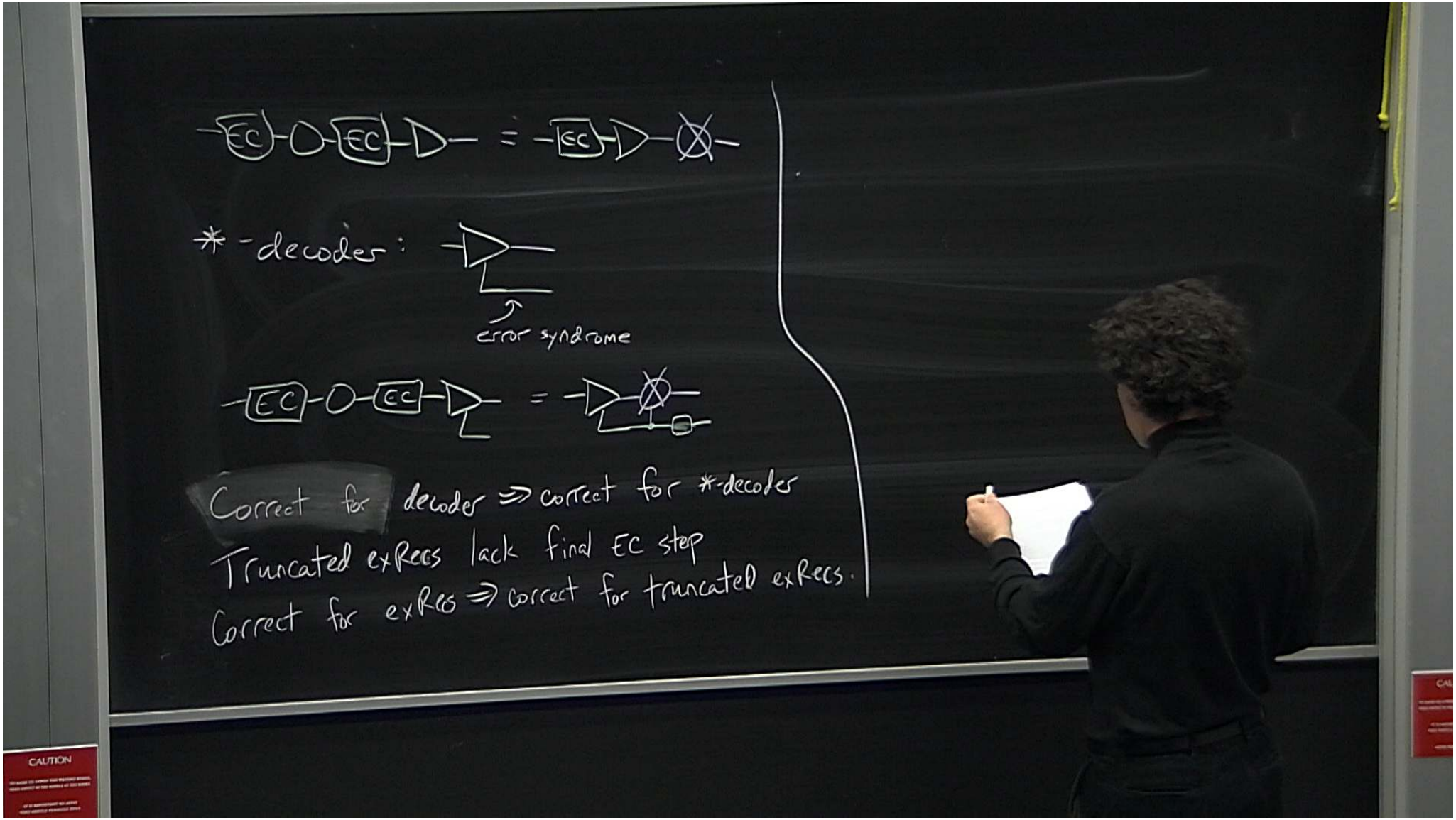
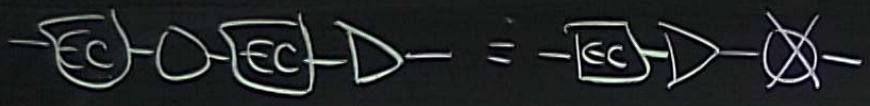


Cor: If all exRecs in a FT circuit are good, the circuit is equivalent to the ideal circuit.


EC
EC
D
=
EC
D
*-decoder:
error syndrome
EC
EC
=


EC
EC
D
=
EC
D
*-decoder:
error syndrome
EC
EC
=
Correct for decoder ⇒ correct for *-decoder
Truncated exRecs lack final EC step
Correct for exRec ⇒ correct for truncated exRecs.
CAUTION



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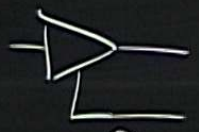


error syndrome

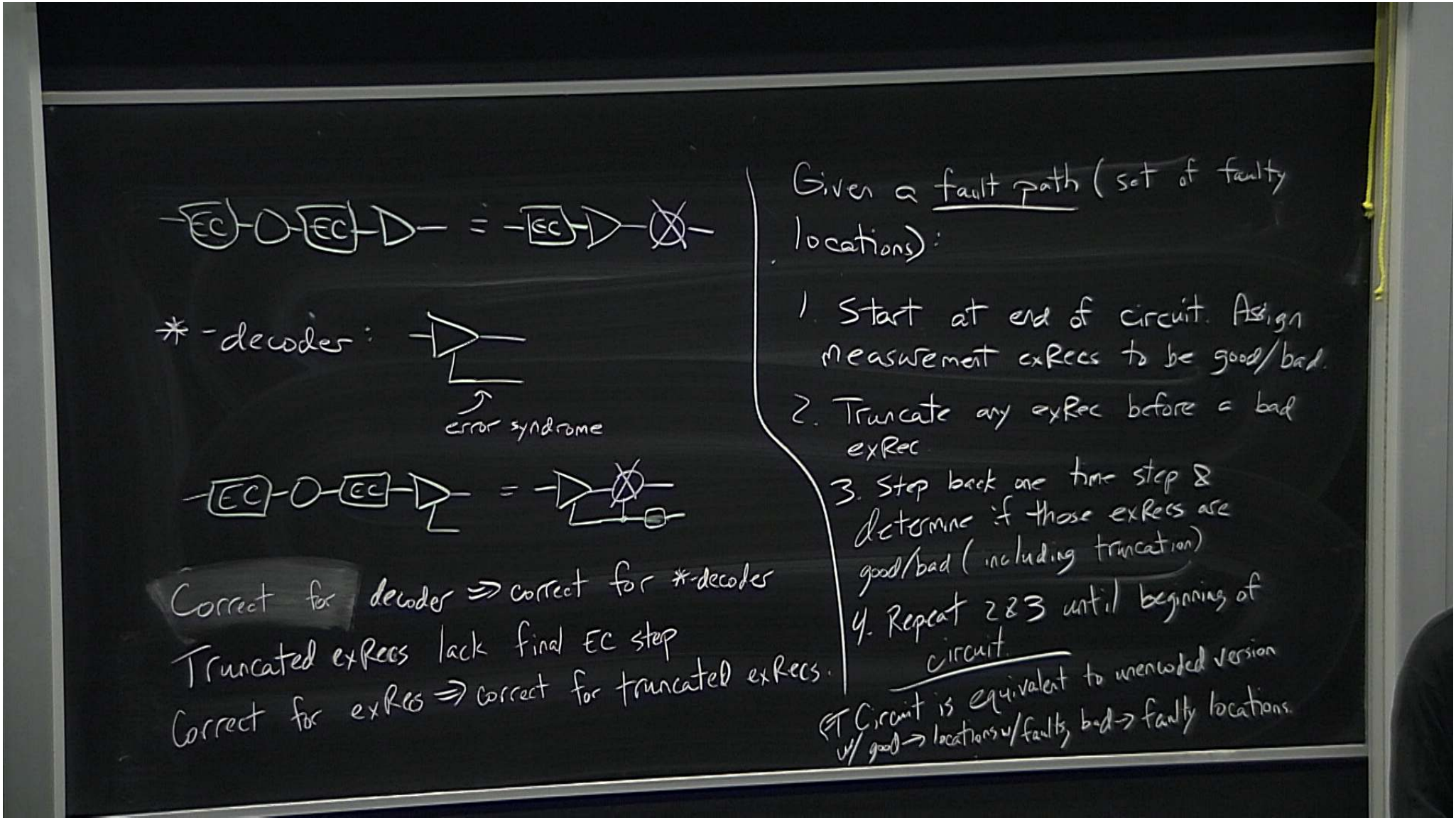
Correct for decoder ⇒ correct for *-decoder

Truncated exRecs lack final EC step

Correct for exRecs ⇒ correct for truncated exRecs.

Given a fault path (set of faulty locations):

1. Start at end of circuit. Assign measurement exRecs to be good/bad.
2. Truncate any exRec before a bad exRec
3. Step back one time step & determine if those exRecs are good/bad (including truncation)
4. Repeat 2&3 until beginning of circuit.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4. Repeat 2&3 until beginning of circuit
⇒ Circuit is equivalent to unencoded version
w/ good → locations w/ faults, bad → faulty locations.

Def: An error model is a *local stochastic noise model* if w/ probability $p_{\bar{\Phi}}$ we have the fault path $\bar{\Phi}$, st

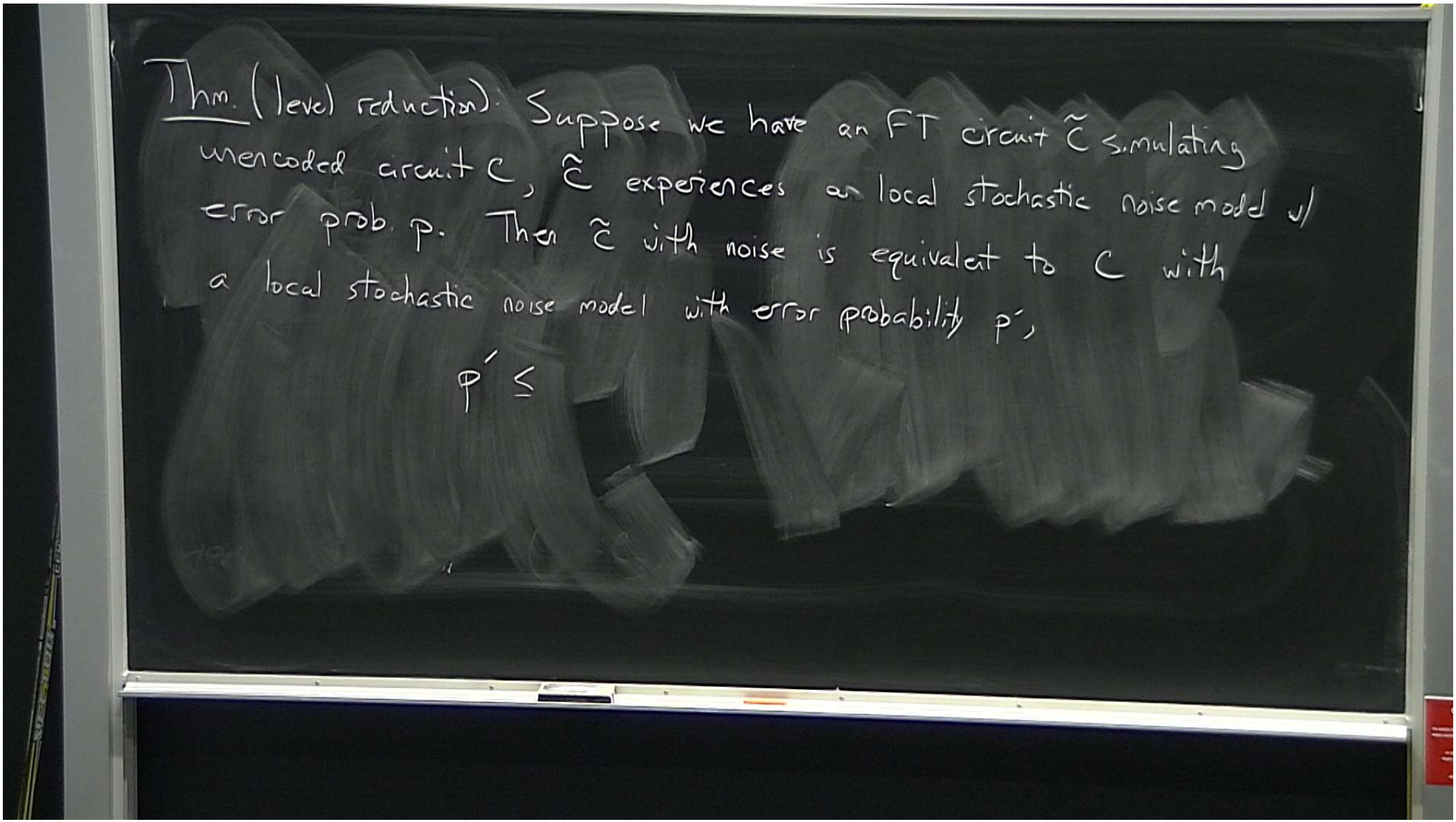
$$\text{Prob.}\left(S \subseteq \bar{\Phi}\right) \leq p^{|S|} \quad \forall S$$

S is a set of locations

$$\text{Prob.}\left(S \subseteq \bar{\Phi}\right) = \sum_{\bar{\Phi} \mid S \subseteq \bar{\Phi}} p_{\bar{\Phi}}$$

We call p *the error probability*

The errors can be controlled by a shared persistent environment.


Thm. (level reduction): Suppose we have an FT circuit $\tilde{C}$ simulating unencoded circuit C, $\tilde{C}$ experiences a local stochastic noise model w/ error prob. p. Then $\tilde{C}$ with noise is equivalent to C with a local stochastic noise model with error probability p',
$p' \leq$

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$$p' \leq \binom{A}{t+1} p^{t+1}$$

A is # locations in the biggest exRec

$R^{\otimes 15}_{T/8}|\bar{T}\rangle = e$

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$$p' \leq \binom{A}{t+1} p^{t+1}$$

A is # locations in the biggest exRec

Proof: w/ s faulty locations in C, s faulty exRecs

How many sets of locations in $\tilde{C}$ · at most $\binom{A}{t+1}^s$ Prob of those locations being faulty is $p^{s(t+1)}$

Thm: (Threshold thm): $\exists$ threshold error rate p_T s.t., in a system subjected to local stochastic noise w/ error prob. $p < p_T$, then $\forall \varepsilon > 0$, $\forall$ circuits C w/ T locations, $\exists$ FT encoding $\tilde{C}$ of C s.t. the output probability distribution of $\tilde{C}$ is w/in statistical distance ε of that of C, $\tilde{C}$ uses time, gates, # qubits that are greater than C by a $\text{poly}\log(T/\varepsilon)$ factor.

Proof: Use concatenated FT protocol w/ L levels.

$$p \xrightarrow{\text{FT}} p' \le \binom{A}{t+1} p^{t+1} = p_T \left(\frac{p}{p_T}\right)^{t+1} \qquad \frac{1}{p_T} = \binom{A}{t+1} \Rightarrow p_T = \frac{1}{\binom{A}{t+1}^{1/t}}$$

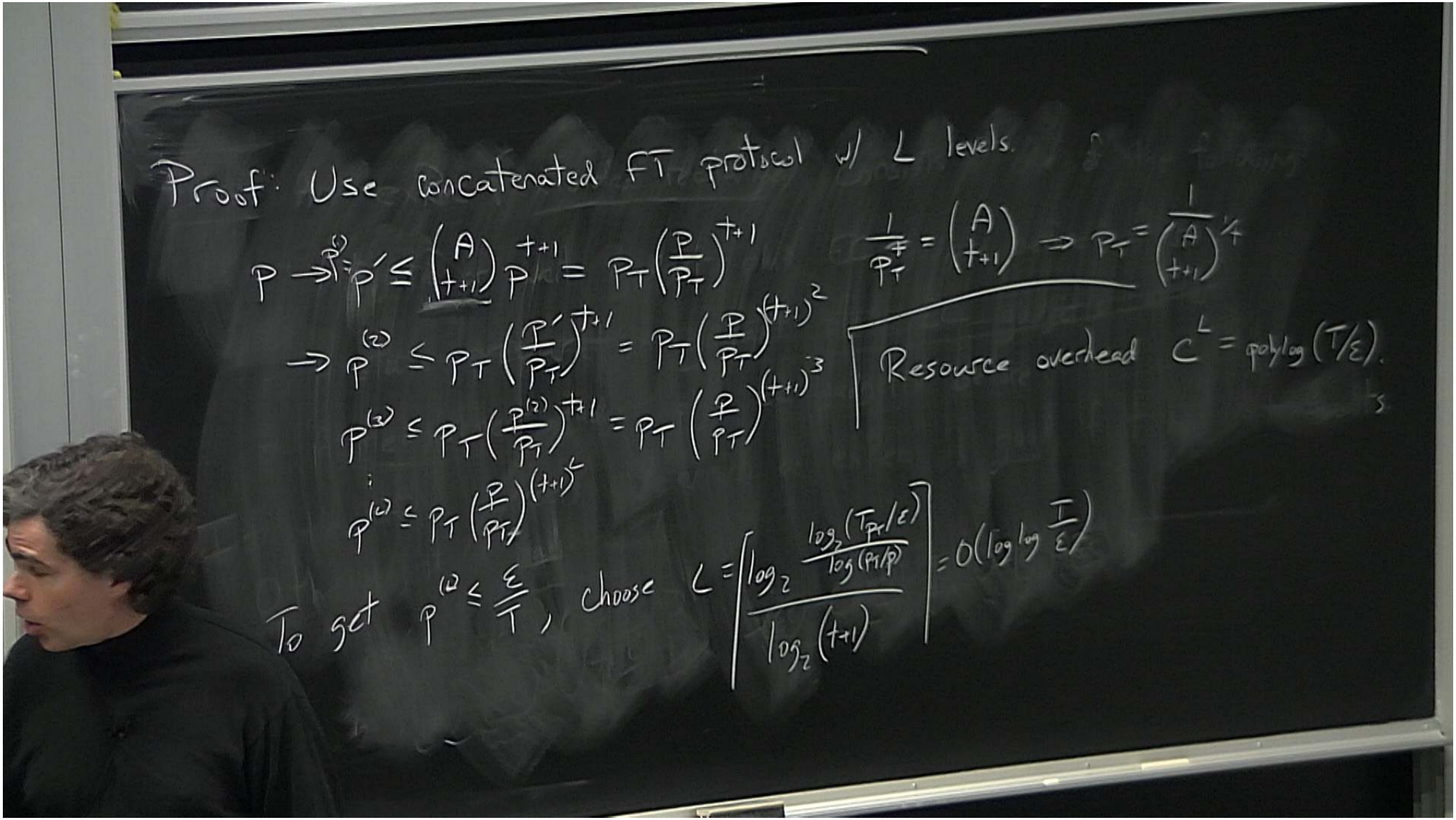
$$\Rightarrow p^{(2)} \le p_T \left(\frac{p'}{p_T}\right)^{t+1} = p_T \left(\frac{p}{p_T}\right)^{(t+1)^2}$$

$$p^{(3)} \le p_T \left(\frac{p^{(2)}}{p_T}\right)^{t+1} = p_T \left(\frac{p}{p_T}\right)^{(t+1)^3}$$

$$\vdots$$

$$p^{(L)} \le p_T \left(\frac{p}{p_T}\right)^{(t+1)^L}$$

To get $p^{(L)} \le \frac{\varepsilon}{T}$, choose $L = \left\lceil \frac{\log_2 \frac{\log_2(T p_T/\varepsilon)}{\log(p_T/e)}}{\log_2(t+1)} \right\rceil = O\left(\log\log \frac{T}{\varepsilon}\right)$


Proof: Use concatenated FT protocol w/ L levels.
$p \to p^{(1)} = p' \le \binom{A}{t+1} p^{t+1} = p_T \left(\frac{p}{p_T}\right)^{t+1}$
$\frac{1}{p_T} = \binom{A}{t+1} \Rightarrow p_T = \left(\frac{1}{\binom{A}{t+1}}\right)^{1/t}$
$\to p^{(2)} \le p_T \left(\frac{p'}{p_T}\right)^{t+1} = p_T \left(\frac{p}{p_T}\right)^{(t+1)^2}$
$p^{(3)} \le p_T \left(\frac{p^{(2)}}{p_T}\right)^{t+1} = p_T \left(\frac{p}{p_T}\right)^{(t+1)^3}$
$p^{(L)} \le p_T \left(\frac{p}{p_T}\right)^{(t+1)^L}$
Resource overhead $C^L = \text{poly}\log(T/\varepsilon)$.
To get $p^{(L)} \le \frac{\varepsilon}{T}$, choose $L = \left\lceil \frac{\log_2 \frac{\log_2(T p_T/\varepsilon)}{\log(p_T/p)}}{\log_2(t+1)} \right\rceil = O(\log\log \frac{T}{\varepsilon})$

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w/ concatenated 7-qubit code, $p_T \geq 2.7 \times 10^{-5}$

best known simulations suggest $p_T \approx 5\%$

Assumptions in threshold theorem:

Gates:

1. Particular set of gates (H, $R_{\pi/8}$, CNOT): not necessary
2. Two-qubits gates between all pairs of qubits: not necessary
3. Produce $|0\rangle$ states on demand: necessary, sort of
4. Measurements during the computation & reliable classical computation: not necessary
5. Maximal parallelism: necessary

Assumptions about errors:

1. Each location has same error rate: not necessary
2. Bounded correlation: necessary
3. No leakage errors: not necessary
4. Adversarial errors: cannot gain advantage
5. Stochastic errors: not necessary, but there are some models of non-Markovian errors for which no proof is known