Title: IQC - Quantum Error Correction - Lecture 5

Date: Feb 01, 2018 02:00 PM

URL: http://pirsa.org/18020009

Abstract:

Def: Fix some QECC, corrects t errors

QECC —

r-filter is a projector onto subspace spanned by QECC with up to r Pauli errors

ideal decoder corrects errors & decodes, outputs logical (decoded) state

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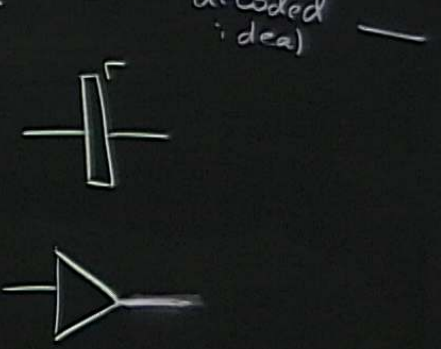

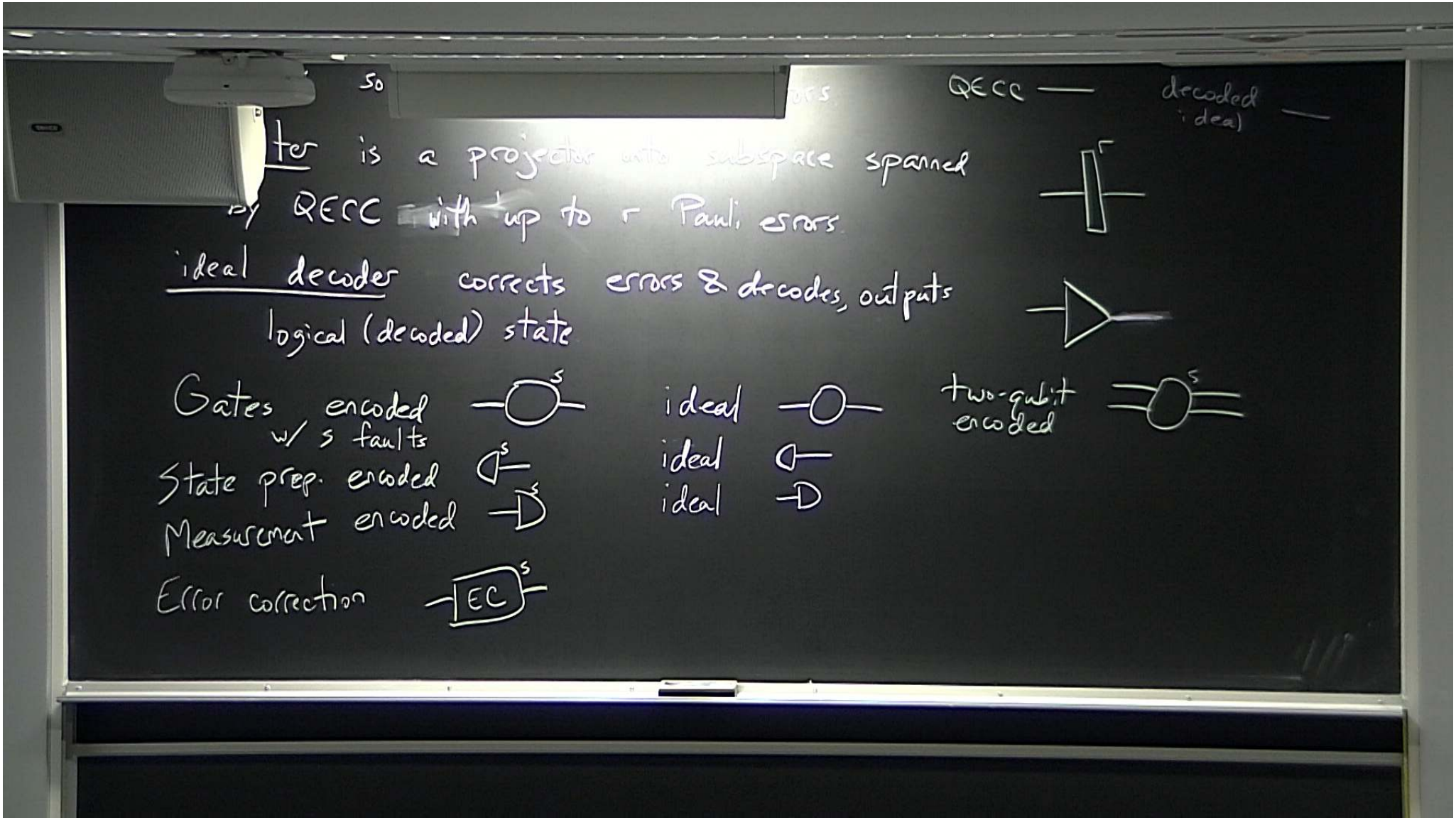
So
QECC
decoded ideal
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ideal decoder corrects errors & decodes, outputs
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Gates encoded
w/ s faults
s
ideal
two-qubit encoded
s
State prep. encoded
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Measurement encoded
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Error correction
EC
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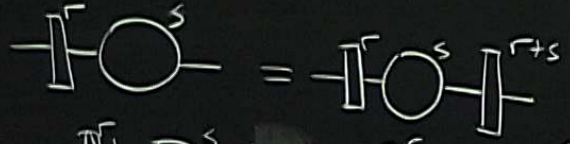
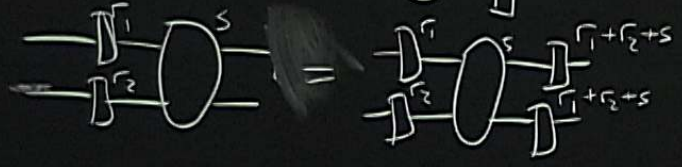
Def: The GPP (Gate error propagation property) is satisfied if

$\forall r, s$ s.t. $r+s \leq t$,

1-qubit gate



2-qubit gate



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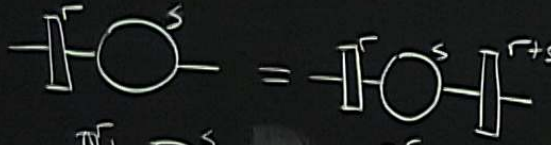

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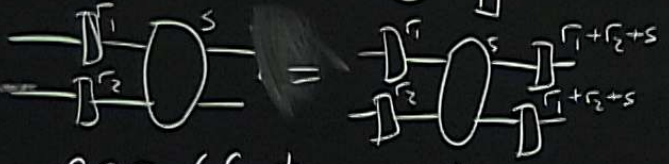

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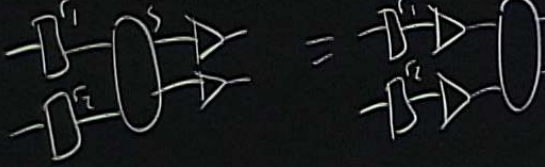

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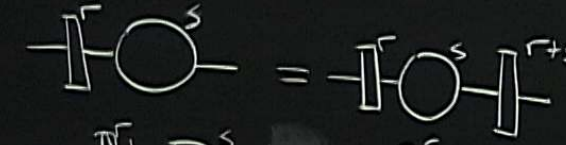

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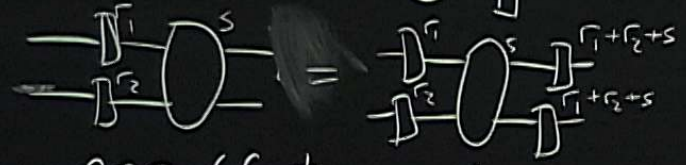

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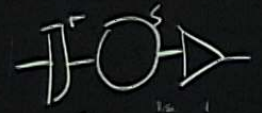

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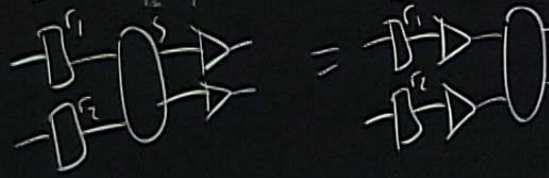

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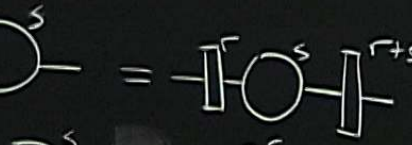

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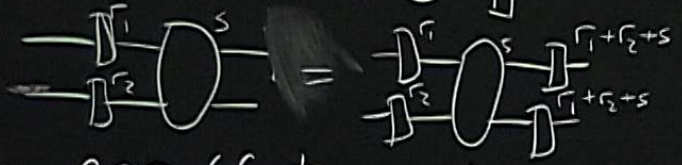

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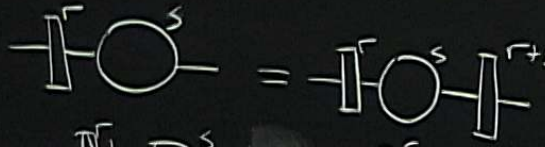
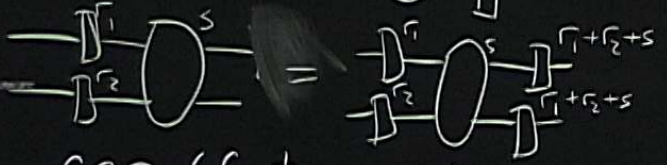


2-qubit gate



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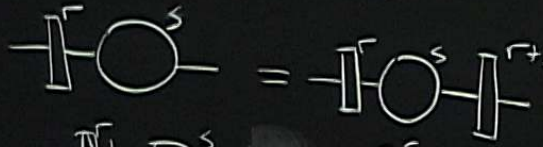

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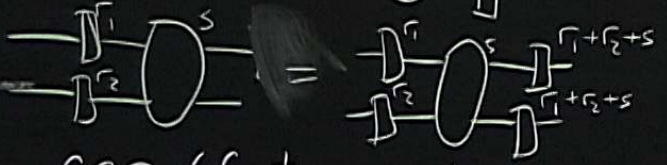

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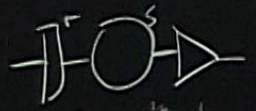

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Gates encoded w/ s faults

State prep. encoded

Measurement encoded

Error correction

ideal

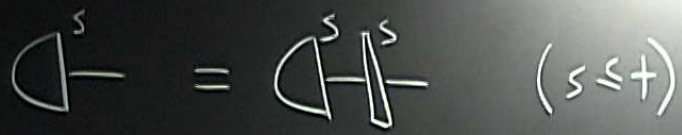
ideal

ideal

two-qubit encoded

QECC — classical decoded (ideal) —

Def: The PPP (Preparation [illegible] propagation property) is satisfied if



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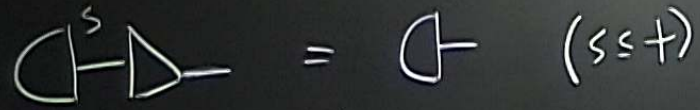
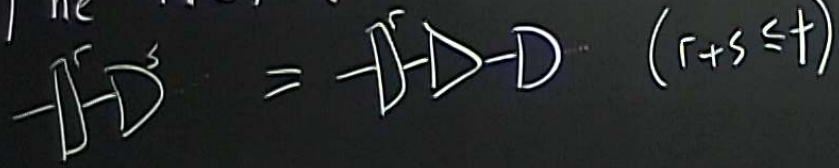
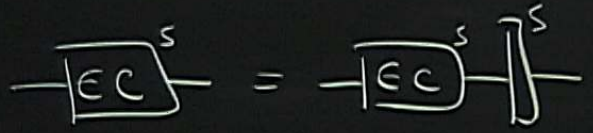


The MCP (Measurement correctness property) is satisfied if

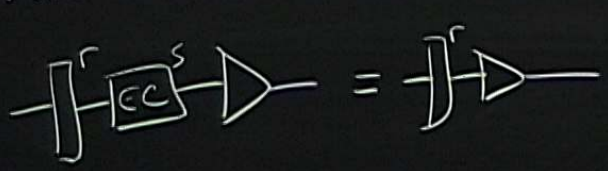


Def.: The ECRP (Error correction recovery property) is satisfied if



$(s \leq t)$

The ECCP (Error correction Correctness property) is satisfied if



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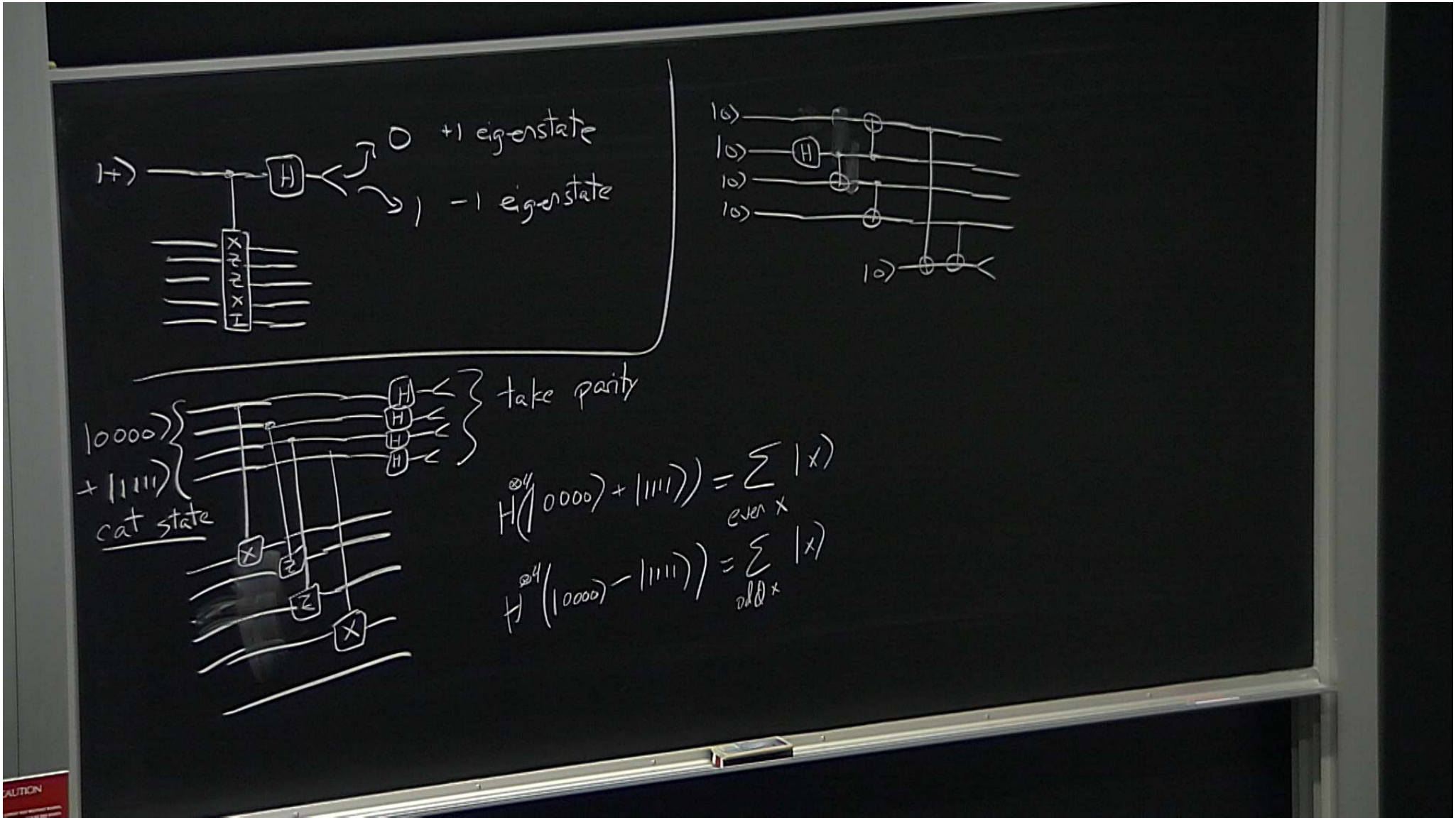
|+⟩
H
+1 eigenstate
-1 eigenstate
X
X
X
X
|0⟩
|0⟩
|0⟩
|0⟩
H
|0⟩
|0000⟩
+ |1111⟩
cat state
H
H
H
H
take parity
X
Z
Z
X
$H^{\otimes 4}(|0000\rangle + |1111\rangle) = \sum_{\text{even } x} |x\rangle$
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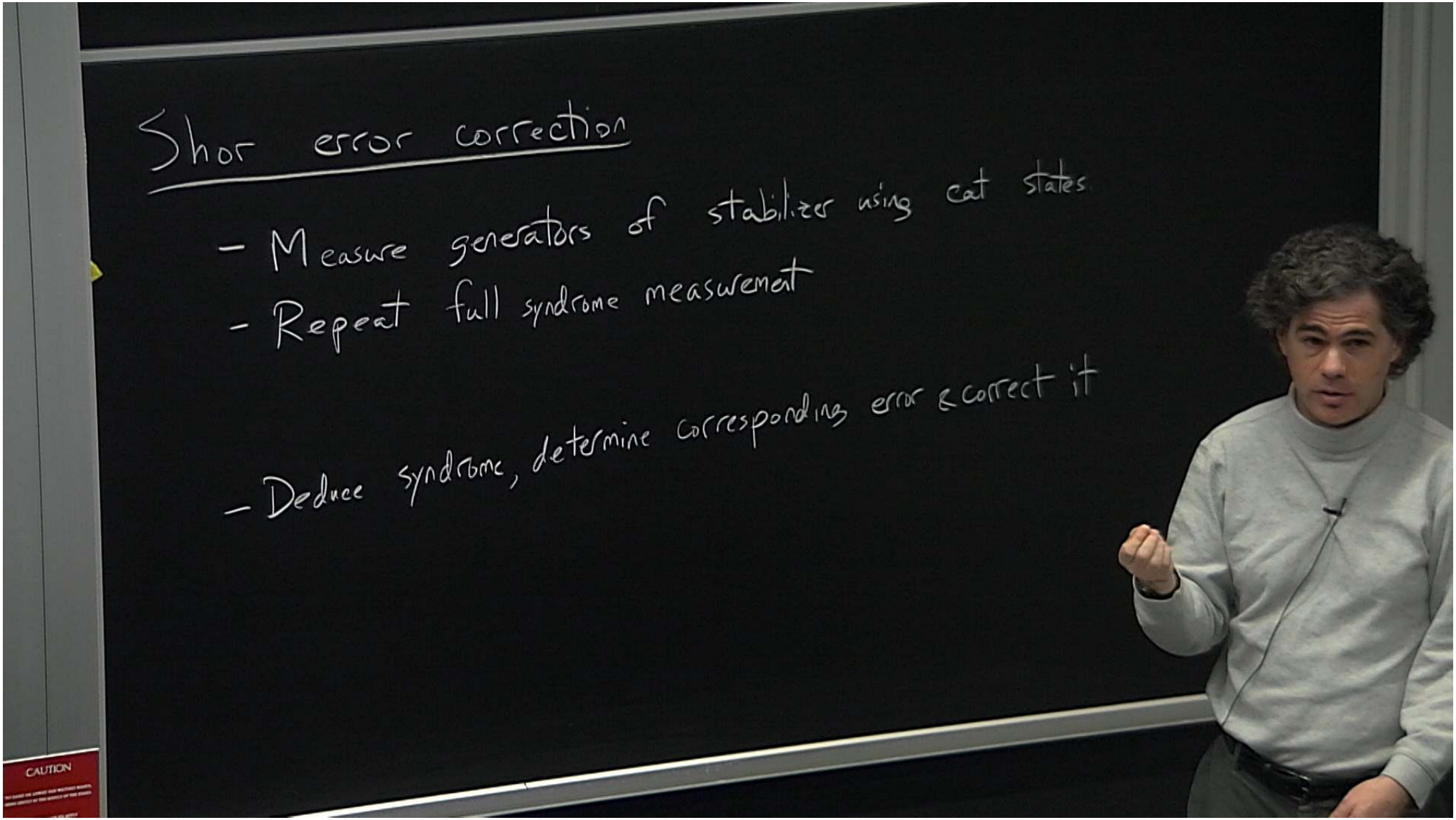
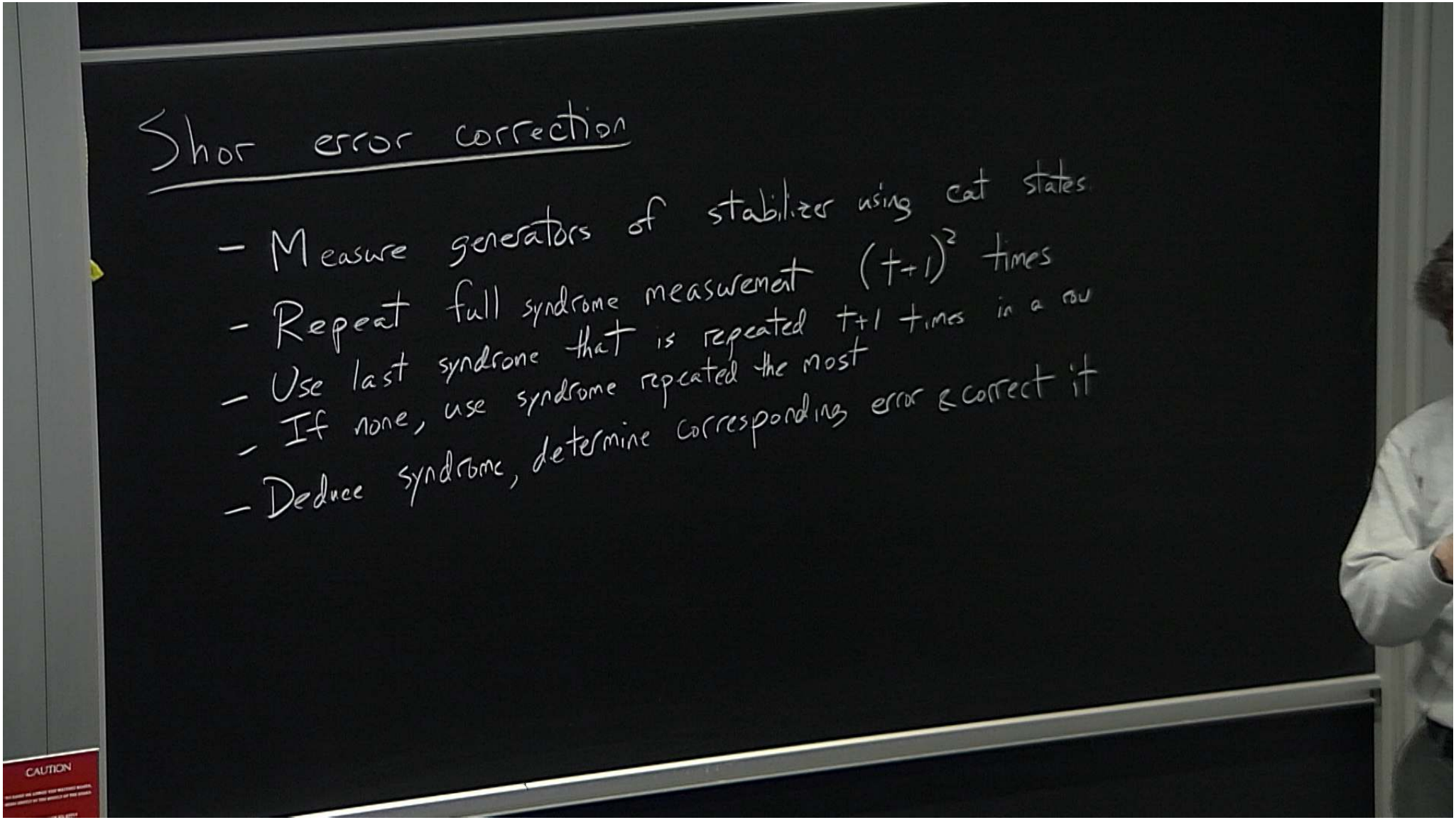
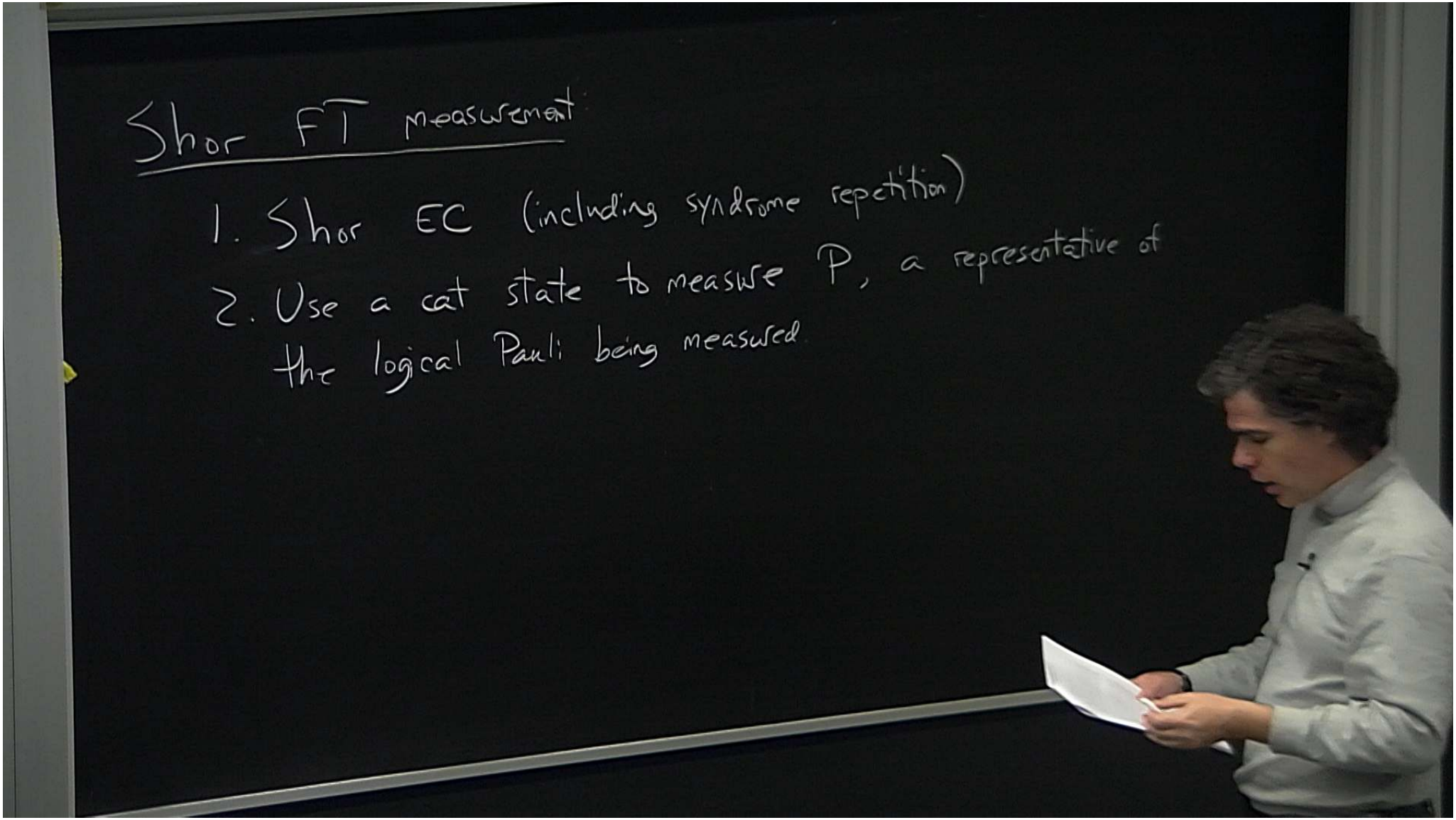
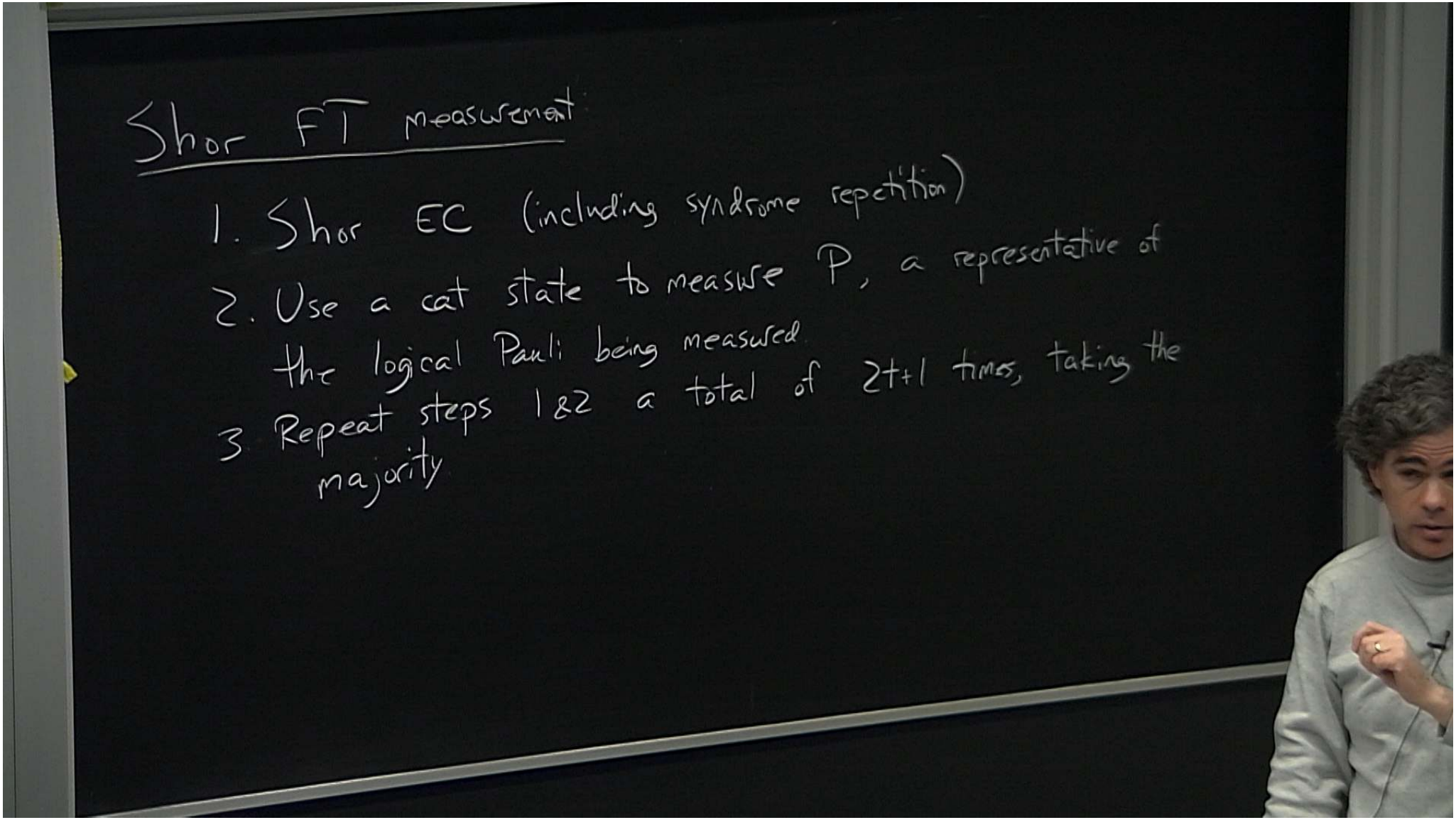

Shor error correction
- Measure generators of stabilizer using cat states
- Repeat full syndrome measurement
- Deduce syndrome, determine corresponding error & correct it
CAUTION


Shor error correction
- Measure generators of stabilizer using cat states
- Repeat full syndrome measurement $(t+1)^2$ times
- Use last syndrome that is repeated $t+1$ times in a row
- If none, use syndrome repeated the most
- Deduce syndrome, determine corresponding error & correct it
CAUTION


Shor FT measurement
1. Shor EC (including syndrome repetition)
2. Use a cat state to measure P, a representative of the logical Pauli being measured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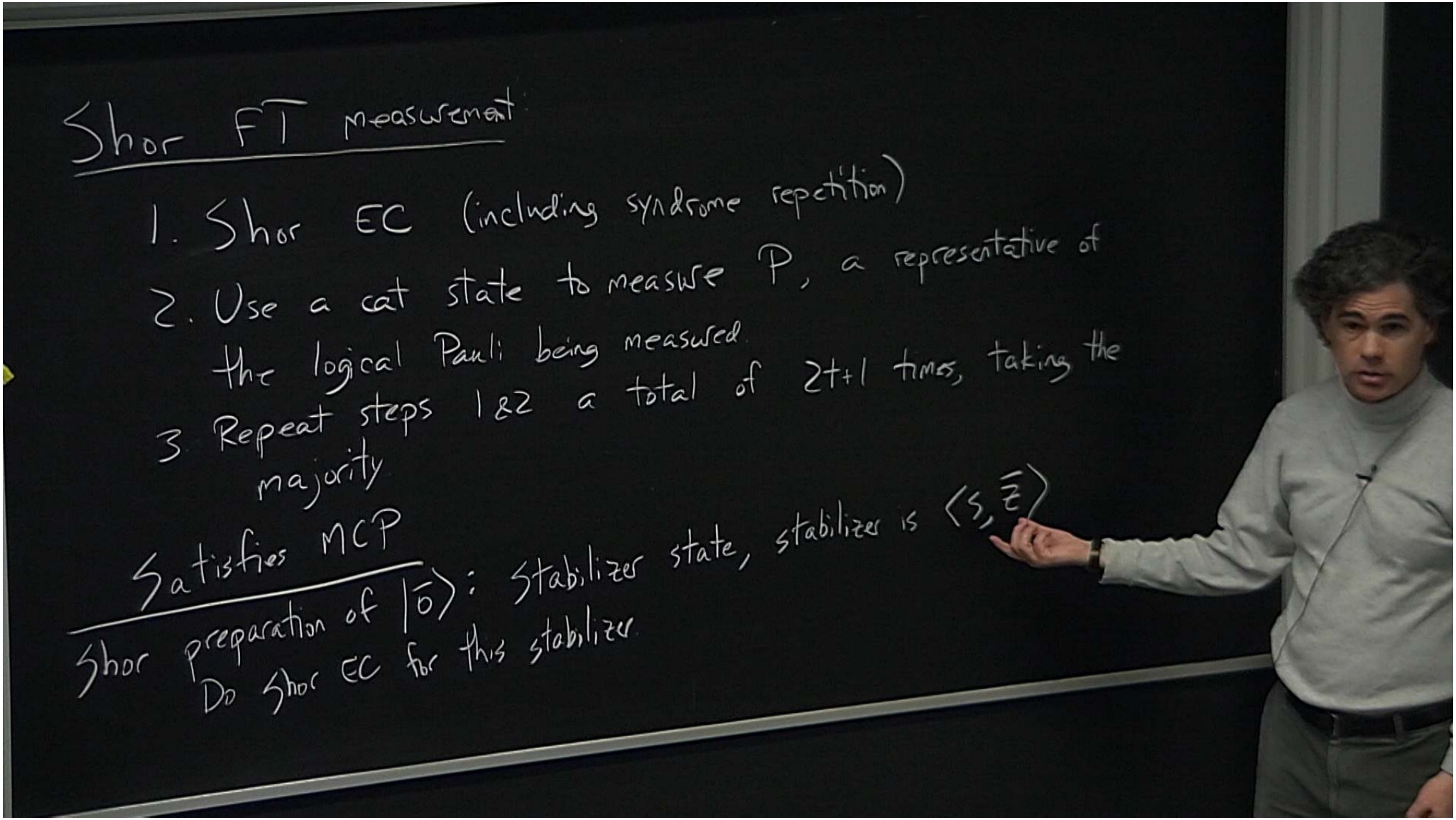
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Satisfies MCP
Shor preparation of $|\bar{0}\rangle$: Stabilizer state, stabilizer is $\langle S, \bar{Z} \rangle$
Do Shor EC for this stabilizer

For CSS codes:

Codewords $|\bar{v}\rangle = \sum_{w \in C_2^\perp} |v+w\rangle$, $v \in C_1$ $\quad C_2^\perp \subseteq C_1$

Suppose bit flip error e (bit string 1 where error)

$\longrightarrow \sum_{w \in C_2^\perp} |v+w+e\rangle$

FT Measurement: measure every qubit (logical Z)

→ get $\underbrace{v+w}_{\in C_1}+e$ for random $w \in C_2^\perp$

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→ use classical EC to decode $(v+w)+e \longrightarrow v+w$ – same coset as v

→ logical output.

For degenerate CSS code,

$d_1 < m < d$, wt $e = m$, $e \in C_2^\perp$

$X + f = x' + f' \iff f + f' \in C_2^\perp$

$(V + w) + f \not\to V + w$

$V + w + (f + f')$ same coset as V

$C_2^\perp \quad C_2^\perp$

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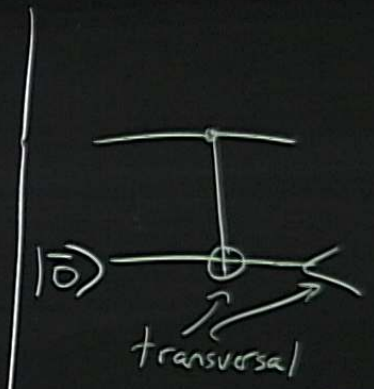

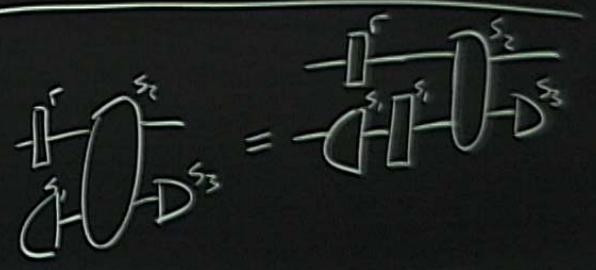

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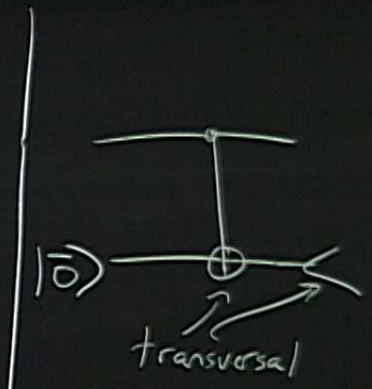

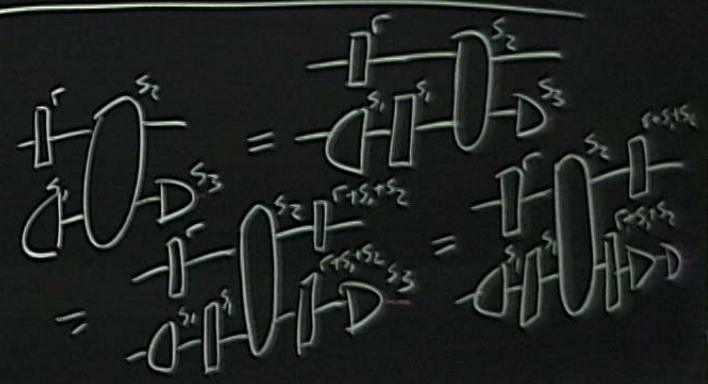

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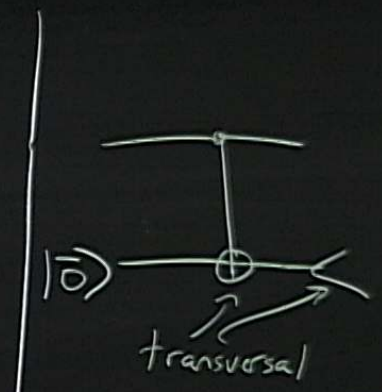
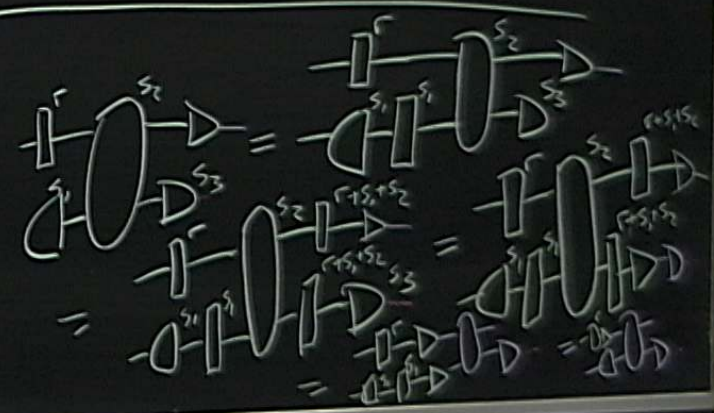
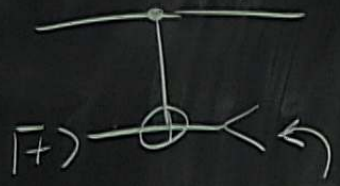




Steane EC:

Bit flip



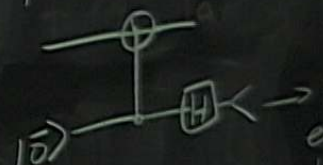
random codeword of C_2 w/ bit flip errors $e = f + g$

from faults in circuit & pre-existing errors in code

from faults in circuit

$$\sum_{w \in C_2^\perp} |v + w + f\rangle \rightarrow \sum_{w \in C_2^\perp} |v + w + f + e\rangle = \sum_{w \in C_2^\perp} |v + w + g\rangle$$

Phase



→ deduce phase error from classical EC

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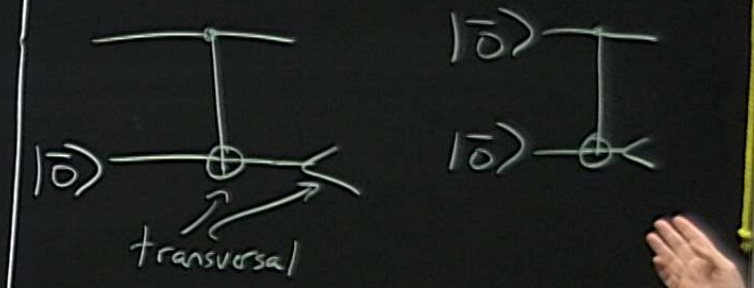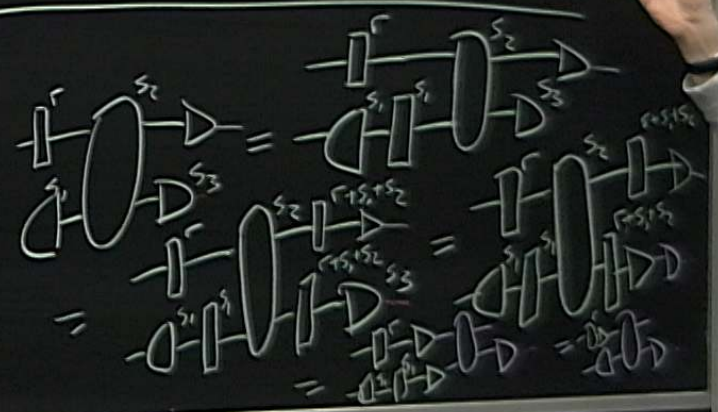
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