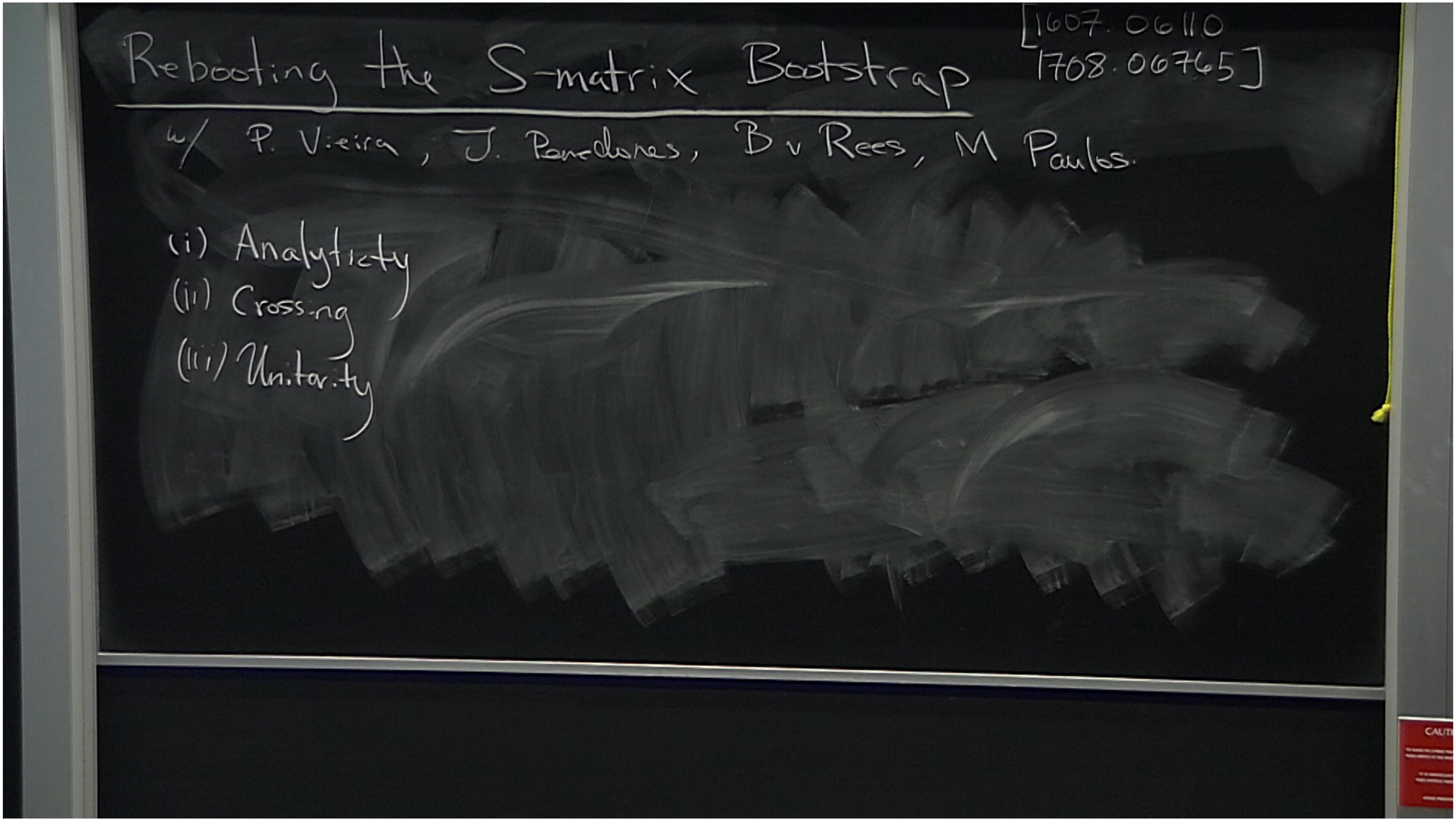
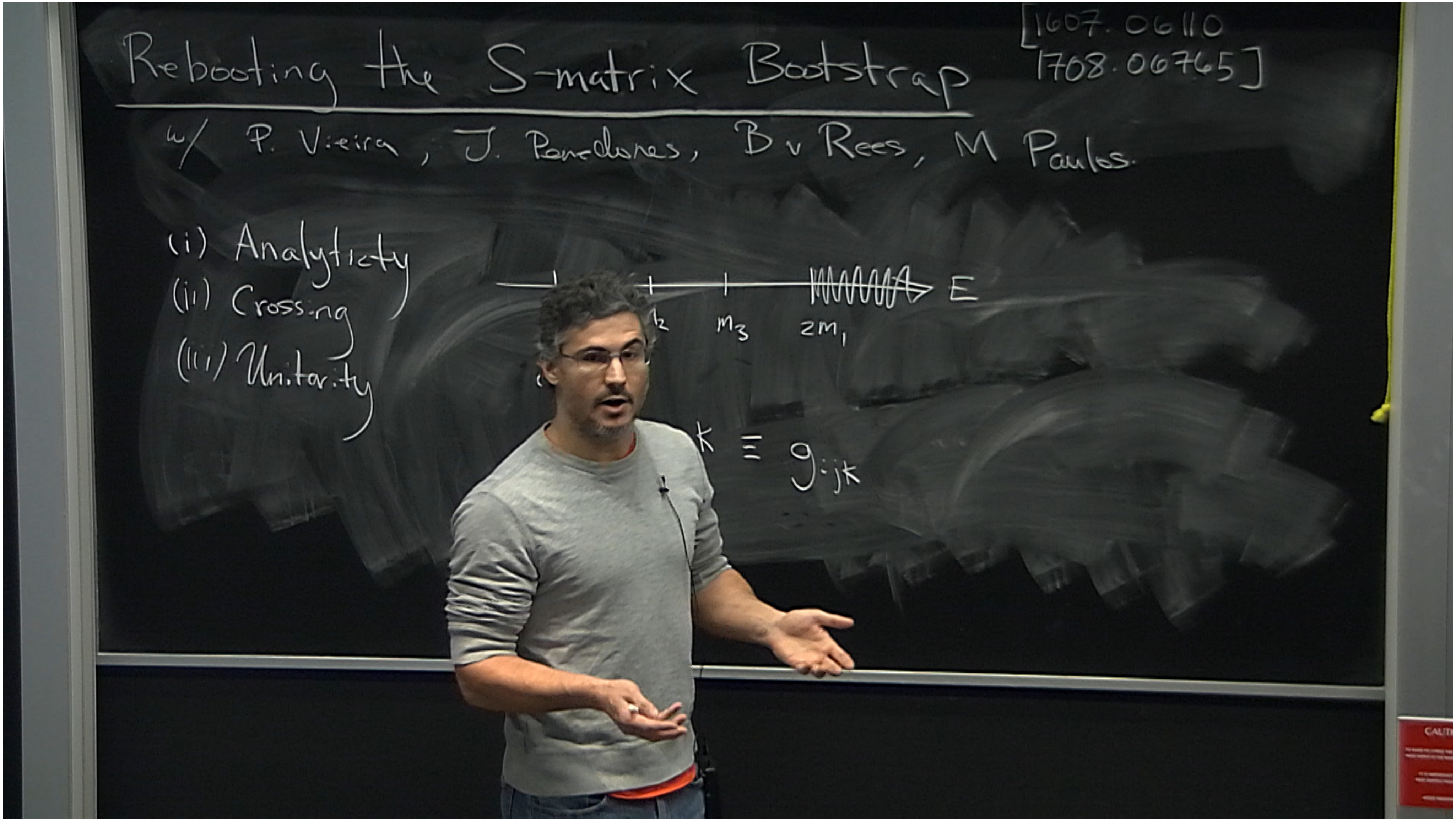
Title: Rebooting the S-matrix bootstrap

Date: Feb 08, 2018 11:00 AM

URL: http://pirsa.org/18010095

Abstract: <p>I will discuss constraints on the S-matrix of gapped, Lorentz invariant quantum field theories due to crossing symmetry, analyticity and unitarity. In particular I will bound cubic couplings, quartic couplings and scattering lengths relevant for the elastic scattering amplitude of two identical scalar particles. After a warm-up in 1+1 dimensions I will move to 3+1 dimensions. In the cases where the results can be compared with results in the older S-matrix literature they are in excellent agreement.</p>


Rebooting the S-matrix Bootstrap
[1607.06110
1708.06765]
w/ P. Vieira, J. Penedones, B v Rees, M Paulos.
(i) Analyticity
(ii) Crossing
(iii) Unitarity


Rebooting the S-matrix Bootstrap
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(i) Analyticity
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E
m3
2m1
K ≡ gijk

Rebooting the S-matrix Bootstrap [1607.06110, 1708.06765]

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(i) Analyticity
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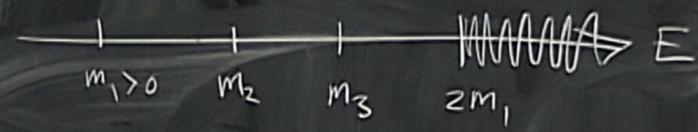
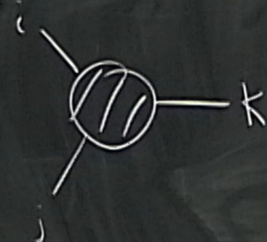
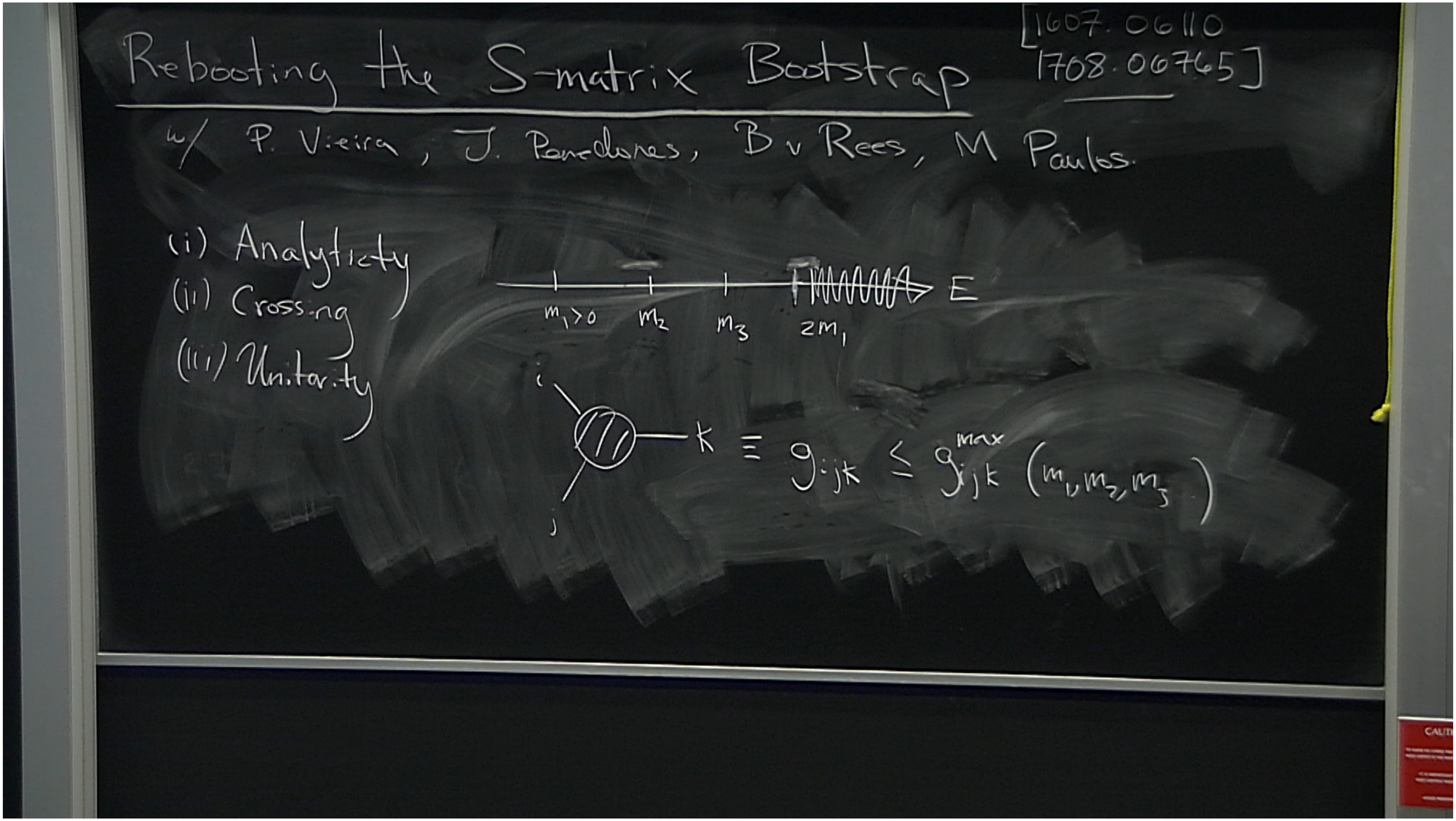




$\equiv g_{ijk} \overset{?}{\leq} g_{ijk}^{max}(m_1, m_2, m_3)$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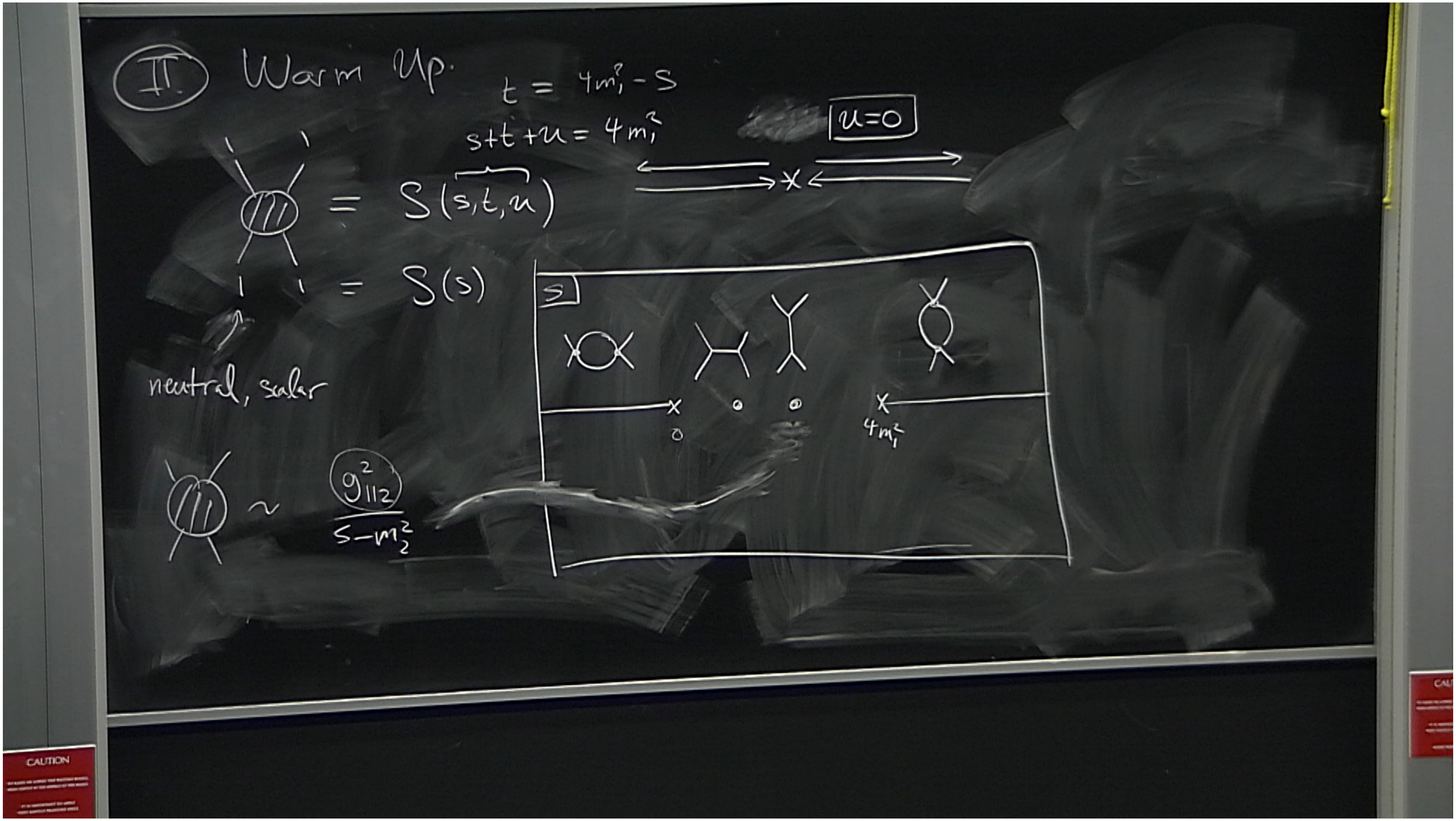
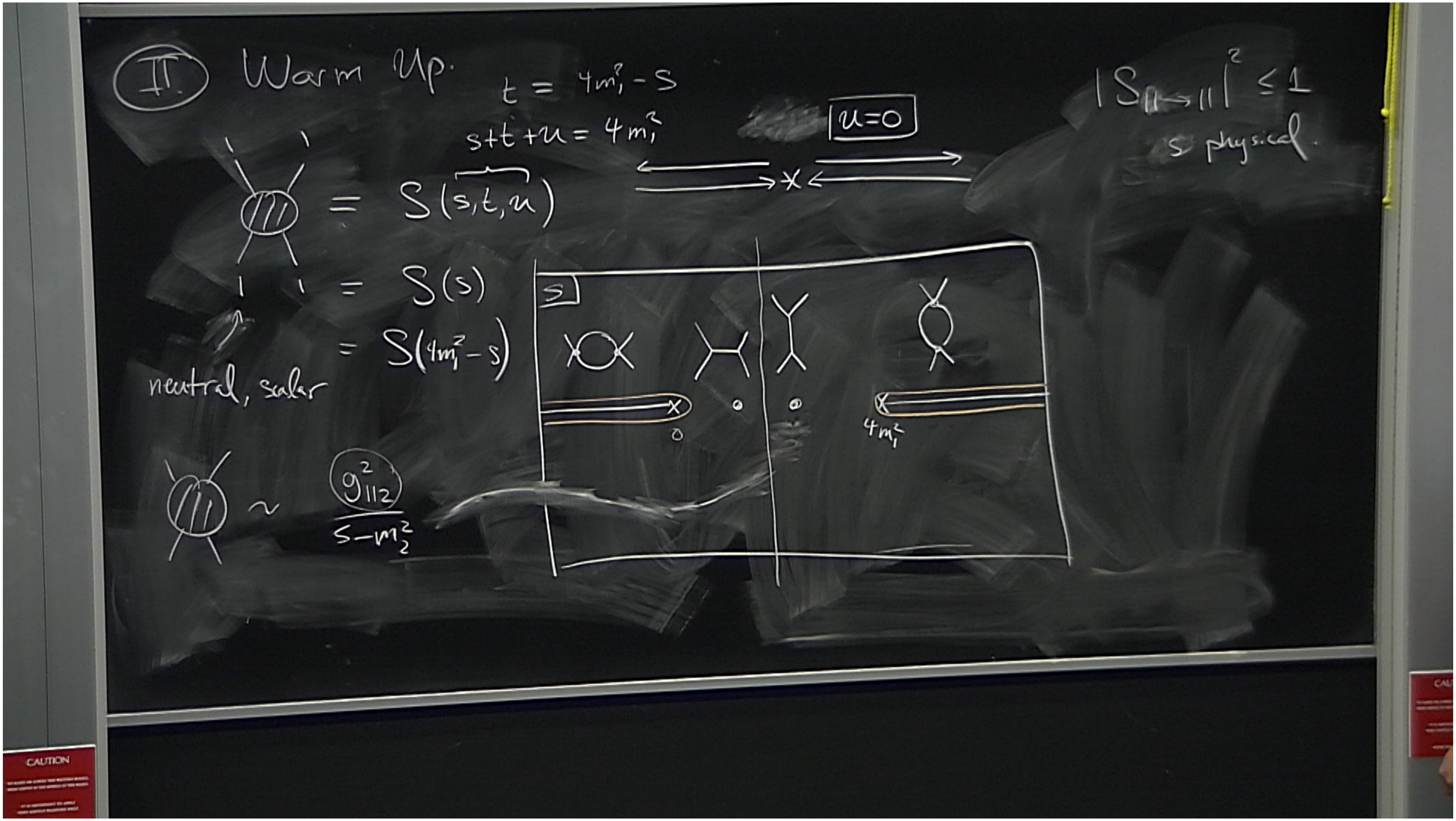
Rebooting the S-matrix Bootstrap
w/ P. Vieira, J. Penedones, B v Rees, M Paulos.
(i) Analyticity
(ii) Crossing
(iii) Unitarity
E
$m_1 > 0$ m_2 m_3 $2m_1$
i
j
k
$\equiv g_{ijk} \leq g_{ijk}^{max}(m_1, m_2, m_3)$


II Warm Up.
t = 4m₁² - s
s+t+u = 4m₁²
u=0
= S(s,t,u)
= S(s)
neutral, scalar
s
0
4m₁²
g²₁₁₂
s-m²₂
CAUTION


II Warm Up.
t = 4m1² - s
s+t+u = 4m1²
= S(s,t,u)
= S(s)
= S(4m1² - s)
neutral, scalar
u=0
|S11→11|² ≤ 1
s physical.
s
0
4m1²
~ g112² / (s - m2²)
CAUTION

Sol.

$$f(s) = \frac{S(s)}{h(s)} \rightarrow |f(s)| \leq 1 \Rightarrow |f(m_2^2)| = \frac{g_{112}^2}{g_h^2} \leq 1 \Rightarrow g_{112}^2 \leq g_h^2$$

Res

- Same poles as S. $\text{Res}\, h(s) = g_h^2$
- No zeros
- $|h(s)| = 1$

$$\frac{\sqrt{s}\sqrt{4m_1^2 - s} + \sqrt{m_2^2}\sqrt{4m_1^2 - m_2^2}}{-}$$

$g_{112}^2 \leq \quad m_2^2 (m$

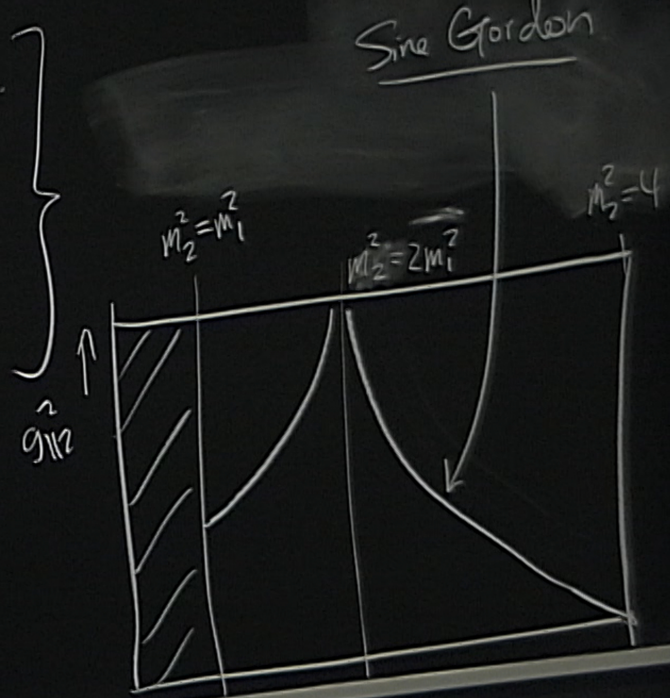
Sol.

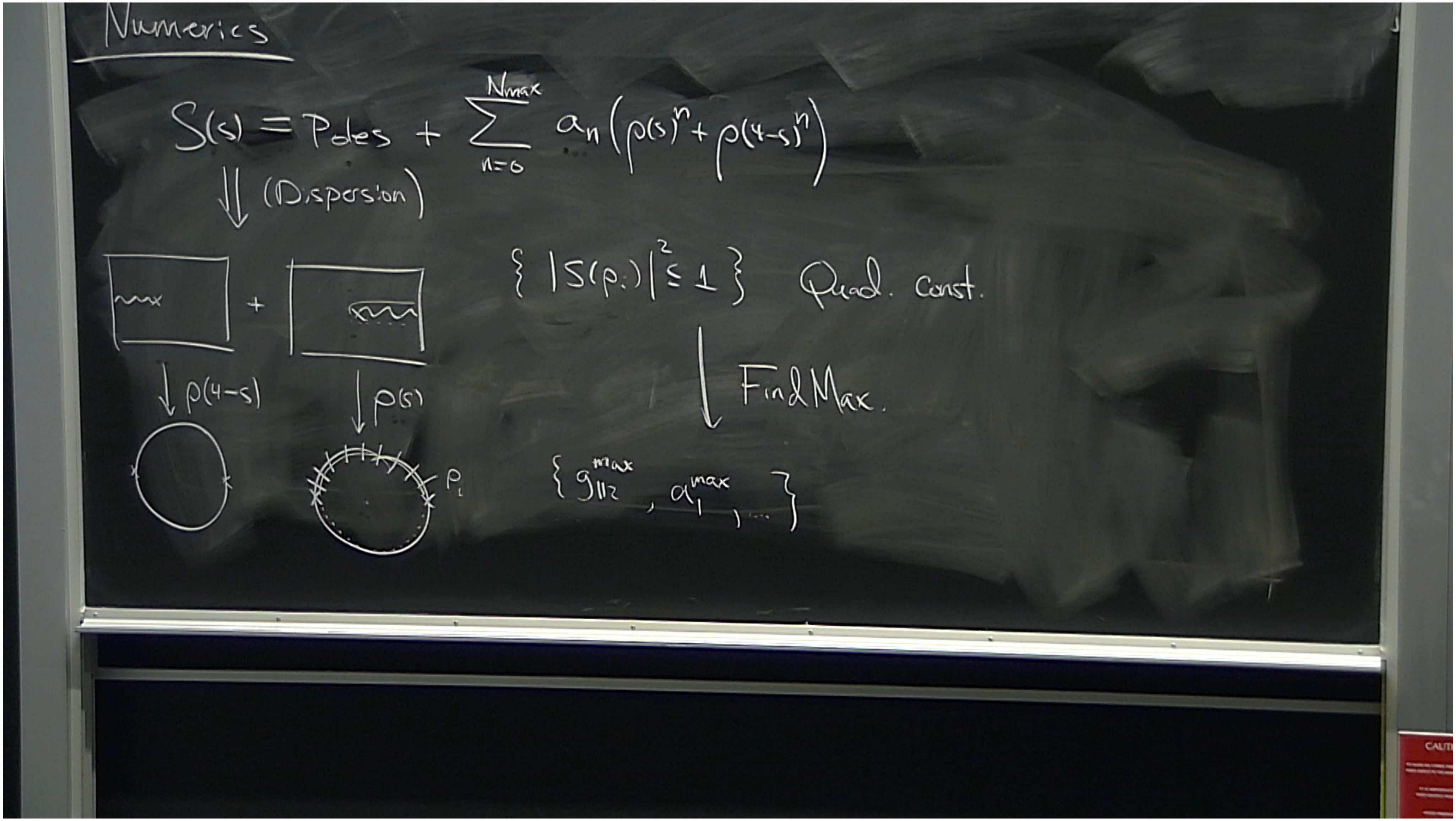
$$f(s) = \frac{S(s)}{h(s)} \rightarrow |f(s)| \leq 1 \Rightarrow |f(m_2^2)| = \frac{g_{112}^2}{g_h^2} \leq 1 \Rightarrow g_{112}^2 \leq g_h^2$$

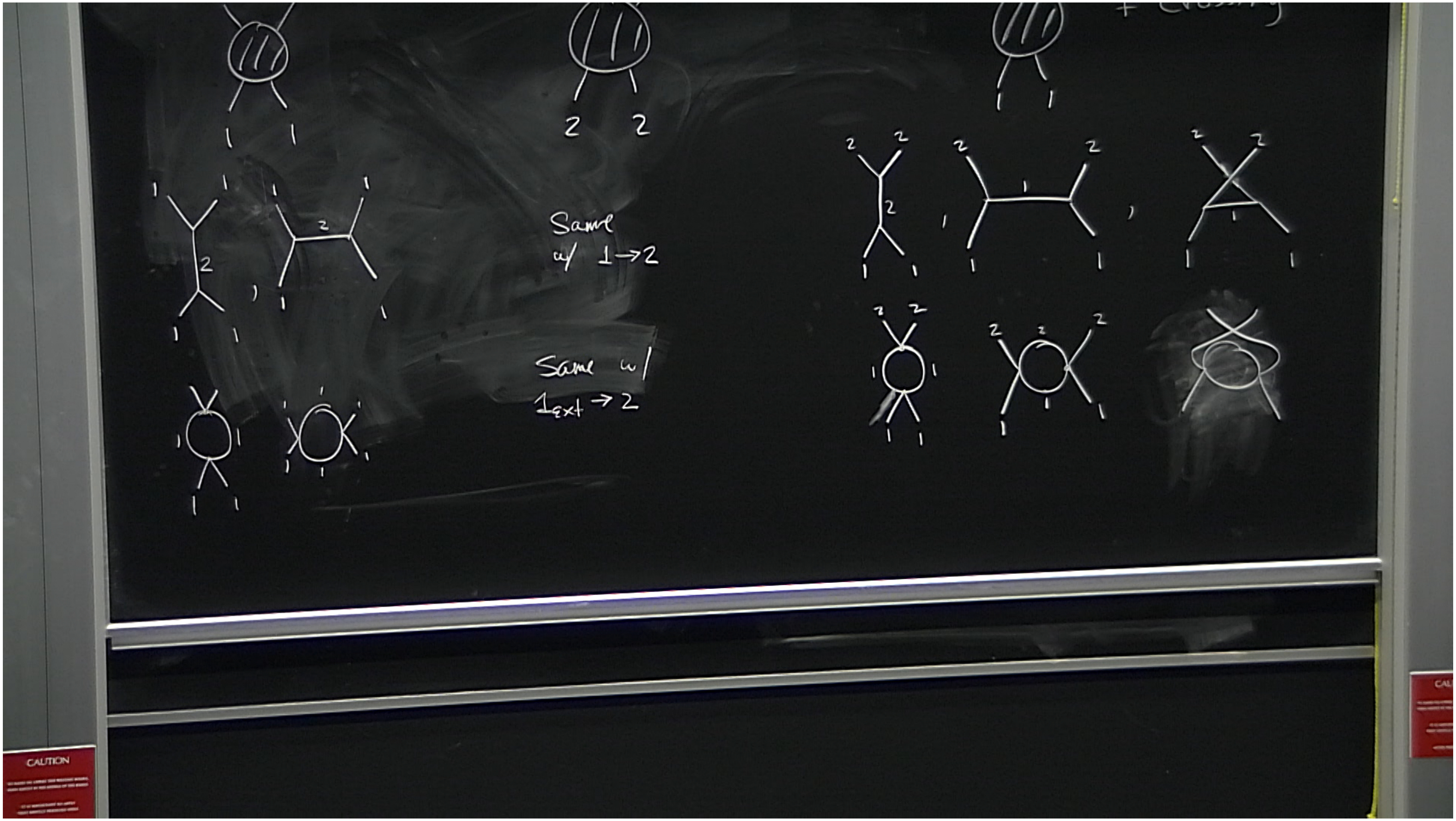
- Same poles as S. $\text{Res}\, h(s) = g_h^2$
- No zeros
- $|h(s)| = 1$

$$\boxed{m_1 = 1}$$

$$\boxed{g_{112}^2 \leq \frac{4 m_2^2 (m_2^2 - 4)^{3/2}}{m_2^2 - 2}}$$



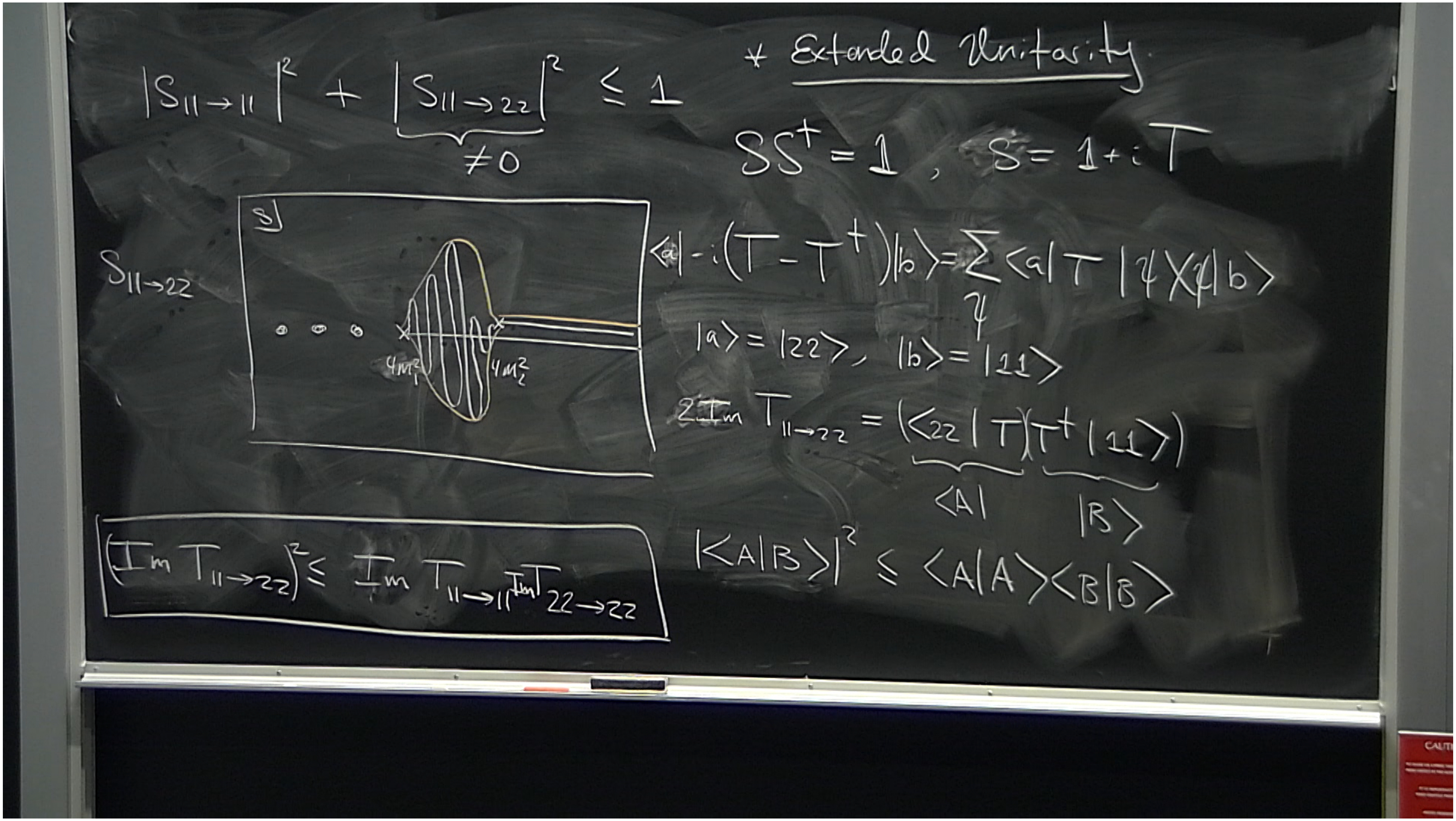

Numerics
$S(s) = \text{Poles} + \sum_{n=0}^{N_{max}} a_n \left(\rho(s)^n + \rho(4-s)^n\right)$
(Dispersion)
$\rho(4-s)$
$\rho(s)$
ρ_i
$\{ |S(\rho_i)|^2 \leq 1 \}$ Quad. Const.
Find Max.
$\{ g_{112}^{max}, a_1^{max}, \ldots \}$


Same
w/ 1→2
Same w/
1ext → 2
+ crossing

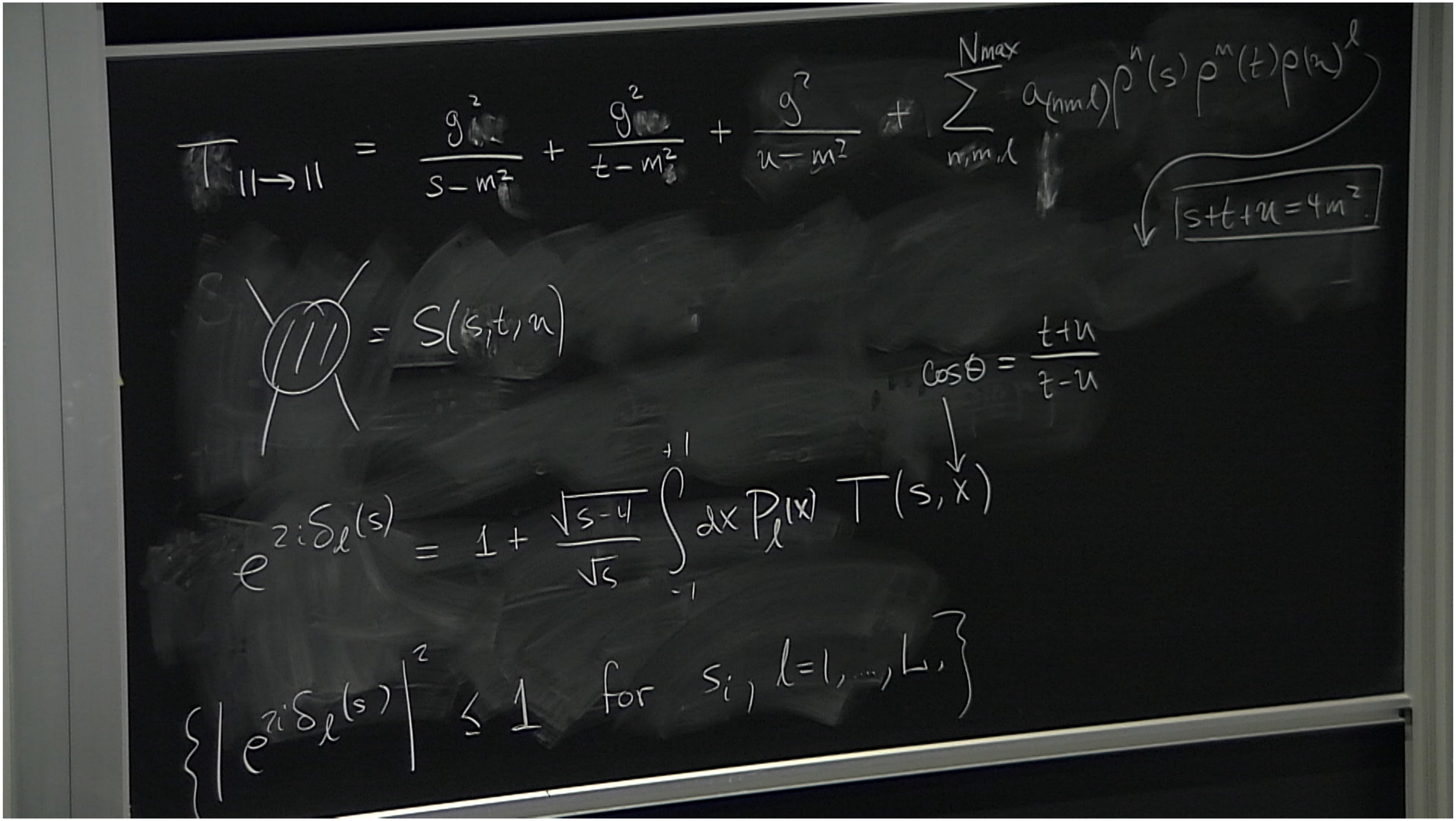
$$S_{11\to 11} = \frac{g_{112}^2}{s-m_2^2} + \frac{g_{112}^2}{t-m_2^2} + \sum_{n=0}^{\infty} a_n \left(\rho(s)^n + \rho(t)^n\right)$$

$$S_{11\to 22} = \frac{g_{112}\, g_{222}}{s-m_2^2} + \frac{g_{112}^2}{s-m_1^2} + \frac{g_{112}^2}{s-m_1^2} + \sum_{n=0}^{\infty} \left[b_n \rho(s)^n + c_n \left(\tilde{\rho}(t)^n + \tilde{\rho}(u)^n\right)\right]$$

$$S_{22\to 22} = \frac{g_{222}^2}{s-m_2^2} + \frac{g_{222}^2}{t-m_2^2} + \sum_{n=0}^{\infty} d_n \left(\rho(s)^n + \rho(t)^n\right)$$



* Extended Unitarity
|S11→11|² + |S11→22|² ≤ 1
≠ 0
SS† = 1 , S = 1 + iT
S11→22
4m1²
4m2²
⟨a| -i(T - T†)|b⟩ = Σ ⟨a| T |ψ⟩⟨ψ| b⟩
|a⟩ = |22⟩ , |b⟩ = |11⟩
2 Im T11→22 = (⟨22| T)(T† |11⟩)
⟨A|
|B⟩
|⟨A|B⟩|² ≤ ⟨A|A⟩⟨B|B⟩
(Im T11→22)² ≤ Im T11→11 Im T22→22



$T_{11\to 11} = \frac{g^2}{s-m^2} + \frac{g^2}{t-m^2} + \frac{g^2}{u-m^2} + \sum_{n,m,l}^{N_{max}} a_{(nml)} \rho^n(s) \rho^m(t) \rho(u)^l$
$s+t+u=4m^2$
$= S(s,t,u)$
$\cos\theta = \frac{t+u}{t-u}$
$e^{2i\delta_\ell(s)} = 1 + \frac{\sqrt{s-4}}{\sqrt{s}} \int_{-1}^{+1} dx\, P_\ell(x)\, T(s,x)$
$\{ |e^{2i\delta_\ell(s)}|^2 \leq 1$ for $s_i, l = 1, \ldots, L, \}$