Title: Universal quantum computation in thermal equilibrium

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Abstract: <p>Adiabatic quantum computation (AQC) is a method for performing universal quantum computation in the ground state of a slowly evolving local Hamiltonian, and in an ideal setting AQC is known to capture all of the computational power of the quantum circuit model. However, despite having an inherent robustness to noise as a result of the adiabatic theorem and the spectral gap of the Hamiltonian, it has been a longstanding theoretical challenge to show that fault-tolerant AQC can in principle be performed below some fixed noise threshold. There are many aspects to this challenge, including the difficulty of adapting known ideas from circuit model fault-tolerance as well as the need to develop an error model that is appropriately tailored for open system AQC. In this talk I will introduce a scheme for combining Feynman-Kitaev history state Hamiltonians with topological quantum error correction, in order to show that universal quantum computation can be encoded not only in the ground state but also in the finite temperature Gibbs state of a local Hamiltonian. Using only local interactions with bounded strength and a polynomial overhead in the number of qubits, the scheme is intended to serve as a proof of principle that universal AQC can be performed at non-zero temperature, and also to further our understanding of the complexity of highly entangled quantum systems in thermal equilibrium.</p>

Universal Quantum Computation in Thermal Equilibrium

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Physics and Computational Complexity

- Hamiltonian complexity: finding quantum ground states is as hard as guessing an accepting input of a quantum circuit.
- Key idea is to view local Hamiltonian terms as constraints that verify the correct operation of a circuit.
- Can the intricate complexity of many-body ground states survive at non-zero temperatures?
- Adiabatic preparation of ground states is a universal model of quantum computation, at least in the ideal noiseless case.
- Connecting universal QC to finite-temperature Gibbs states is a path to understanding the complexity of thermal physics and to making adiabatic computation fault-tolerant.

Outline

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  - Classical ground state computing
  - Classical self-correcting memories
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- Quantum computation in thermal equilibrium
  - Local circuit Hamiltonians $\Rightarrow$ transversal operations
  - Transversal operations $\Rightarrow$ local clocks
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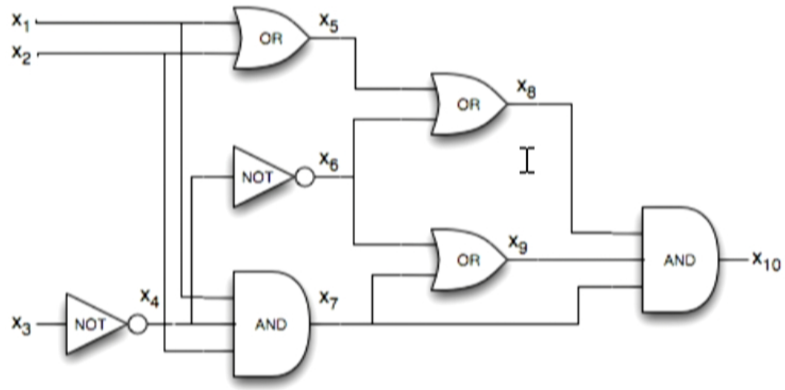
  - Self-correction in spacetime: dressing stabilizers
  - Analysis: symmetry and the global rotation
  - The 4D Fault-tolerant quantum computing laboratory
  - Summary and Outlook

Classical ground state computing

- The theory of NP-completeness grew from the connection between Boolean circuits and constraint satisfaction problems.

- The correct input / output combinations of classical gates correspond to satisfying assignments of Boolean formulas.

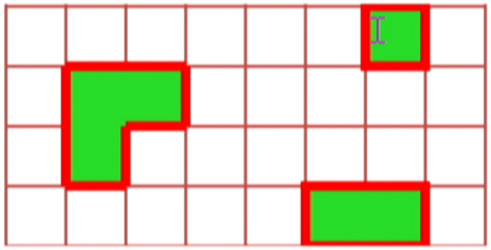


- Constrain the output of all the gates, ask if there is satisfying input? Existential quantifier: $(\exists \mathbf{x})\phi(\mathbf{x})$, nondeterministic Turing machine.

Classical self-correcting memories

- Ferromagnets and repetition codes: the Ising model
- 1D Ising model: thermal fluctuations can flip a droplet of spins, energy cost is independent of the size of the droplet

- 2D Ising model: energy cost of droplet proportional to boundary,



- At temperature T droplets of size L are supressed by $e^{-L/T}$. Ferromagnetic order at $T < T_c$, magnetization close to $\pm n$.
- Robust storage of classical information: lifetime scales exponentially in the size of the block. Hard disk drives work at room temperature!

Classical computation in thermal equilibrium

- Encode each bit of the full input/output history of a Boolean circuit into a 2D Ising model.
- Transversal interactions: the physical spins in each logical bit couple to the corresponding spins in the neighboring logical bits.
- Full Hamiltonian: $H_{\mathrm{circuit}} + H_{\mathrm{code}}$ consists of transversal interactions and ferromagnetic Ising terms.
- Far below $T < T_c$ each logical spin will have magnetization near $\pm m$, $\implies$ low-temperature Gibbs state encodes the computation.
- Related but easier-to-analyze construction in "Making ground state classical computing fault-tolerant" (Crosson, Bacon, Brown, 2010).

Quantum ground state computing

- Encoding time steps into distinct qubits doesn't work: highly entangled states can be distinguished by local operators.

- Kitaev solved this problem by repurposing an idea from Feynman to entangle the time steps of the computation with a "clock register":

$$|\psi_T\rangle = U_T \ldots U_1 |0^n\rangle \longrightarrow |\Psi_{\text{hist}}\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} |t\rangle |\psi_t\rangle$$

- These "history states" can be checked by a local Hamiltonian:

$$H_{\text{circ}} = \underbrace{|0\rangle\langle 0| \otimes \left(\sum |1\rangle\langle 1|_i\right)}_{\text{input at t = 0}} + \sum_{t=0} H_{\text{prop}}(t) \quad , \quad |t\rangle = |\underbrace{11\ldots1}_{t \text{ times}}00\ldots0\rangle$$

$$H_{\text{prop}}(t) = \frac{1}{2}\left(|t\rangle\langle t| \otimes I + |t-1\rangle\langle t-1| \otimes I - |t\rangle\langle t-1| \otimes U_t - |t-1\rangle\langle t| \otimes U_t^\dagger\right)$$

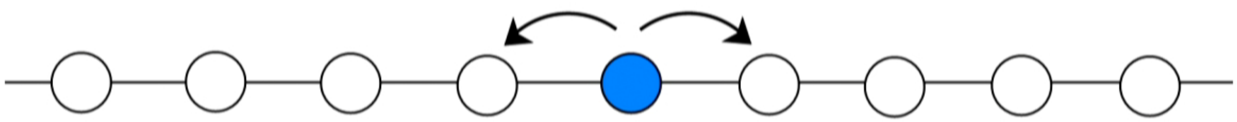
Analyzing circuit Hamiltonians

- **Analysis:** propagation Hamiltonian is unitarily equivalent to a particle hopping on a line! Define a unitary W,

$$W = \sum_{t=0}^{T} |t\rangle\langle t| \otimes U_t \ldots U_1$$

- W transforms H_{prop} into a sum of hopping terms,

$$W^\dagger H_{\mathrm{prop}} W = \sum_{t=0}^{T} \frac{1}{2} \left(|t\rangle\langle t| + |t-1\rangle\langle t-1| - |t\rangle\langle t-1| - |t-1\rangle\langle t| \right)$$



- **Diffusive random walk:** mixing time $\sim T^2$, spectral gap $\sim T^{-2}$.

Universal adiabatic computation

- Begin in an easily prepared ground state and slowly change H while remaining in the ground state by the adiabatic principle,

$$H(s) = (1-s)H_{\text{init}} + s\,H_{\text{final}} \quad , \quad 0 \leq s \leq 1$$

- Run-time estimate: $\sim \|H\|/\Delta_{\min}^{-1}$, where $\Delta = \min_s gap(H(s))$.

- Universal AQC: $H_{\text{final}} = H_{\text{init}} + H_{\text{prop}}$

- Monotonicity argument shows that the minimum spectral gap occurs at $s = 1$, so $\Delta \approx T^{-2}$ and overall run time is polynomial in n, T.

- Perturbative gadgets enable universal AQC with 2-local H,

$$H = \sum_i h_i Z_i + \sum_i \Delta_i X_i + \sum_{i,j} J_{i,j} Z_i Z_j + \sum_{i,j} K_{i,j} X_i X_j$$

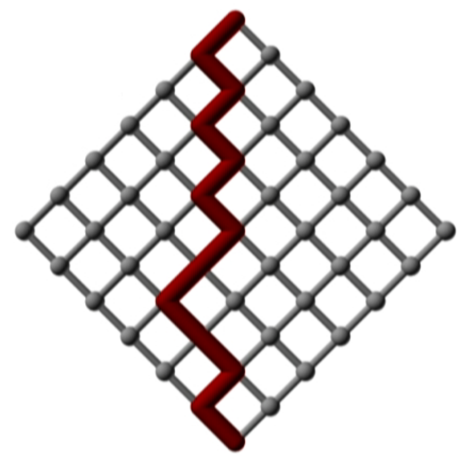
History states with local clocks

- Instead of propagating every qubit according to a global clock, assign local clock registers to the individual qubits,

$$|\boldsymbol{\tau}\rangle = |t_1 ... t_n\rangle \quad , \quad |\Psi_{\text{hist}}\rangle = \sum_{\boldsymbol{\tau}} |\boldsymbol{\tau}\rangle |\psi(\boldsymbol{\tau})\rangle$$

- Proposed to make history state Hamiltonians more realistic (Mizel, Lidar, Mitchell 2007), culminating in 2D universal AQC with 2 body interactions (Lloyd and Terhal, 2015).

Instead of a hopping particle, the Hamiltonian is unitarily equivalent to the diffusion of a string or membrane.

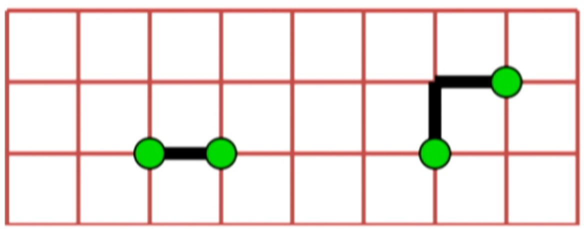


Topological quantum error correction

- Quantum codes require local indistinguishability $\implies$ topological order (toric code) instead of symmetry-breaking order (Ising model).

$$H_{\text{code}} = -\sum_{s\in\mathcal{S}} H_s \quad , \quad \mathcal{S} = \{ \text{ stabilizer generators } \}$$

- 2D toric code analogous to 1D Ising model: thermal fluctuations create pairs of anyons connected by a string. No additional cost to growing the string $\implies$ constant energy cost for a logical error.

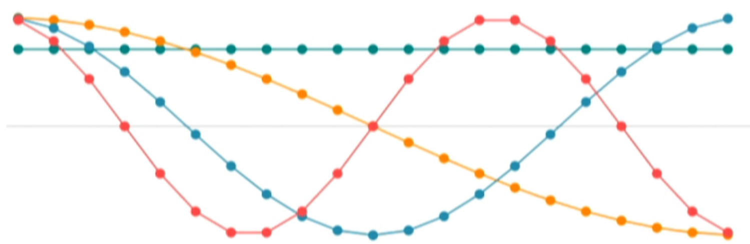


- 4D toric code: logical operators are 2D membranes, energy cost scales like the 1D boundary so errors supressed by $e^{-L/T}$.

- Open question: can finite temperature topological order exist in 3D?

Error supression in adiabatic computation (previous work)

- Simply replace X, Z with logical $\bar{X}, \bar{Z}$ for some quantum error correcting code? 4-local operators to supress 1-local noise (JFS' 05)
- JFS scheme is not scalable. Logical operators for codes with macroscopic distance are incompatible with local Hamiltonian terms.
- How about turning a fault-tolerant quantum circuit $U_1, \ldots, U_T$ into a circuit Hamiltonian? Could help with control errors (Lloyd '08)



- Lloyd also pointed out that the unitary equivalence to a hopping particle means that excited states still encode the valid circuit history. But this breaks down at $E = 1$ when input term is violated.

History states with topological quantum codes

- Each logical qubit $Q_1, \ldots, Q_n$ in the history state is made of physical qubits $q_{i,1}, \ldots, q_{i,m}$. Each physical qubit $q_{i,j}$ has its own clock $t_{i,j}$.
- Just as in the classical case, both the computation and the code stabilizers are enforced by local Hamiltonian terms.

$$H = \sum_{\tau} H_{\text{prop}}(\tau) + \sum_{\tau} H_{\text{code}}(\tau)$$

- H_{prop} needs to consist of local gates, and H_{code} needs to accomodate the propagation of the circuit without frustration.
- We consider codes with universal sets of local operations e.g. transversal gates + gate teleportation.
- Gate teleportation uses logical measurement and classical post-processing, which will all be part of the history state.

Transversal unitaries in a local Hamiltonian

- Transversal operations are used in fault-tolerance to locally perform logical operations on codes with a large distance:

$$U[Q_{\mathrm{logical}}] = \bigotimes_{q} U[q_{\mathrm{physical}}]$$

- Logical operations $U[Q_i, Q_j]$ can be transversally implemented by local Hamiltonian terms by using local clocks,

$$H_{\mathrm{prop}}[\mathbf{t}_{Q_i}, \mathbf{t}_{Q_j}, Q_i, Q_j] \longrightarrow \sum_{q_i \in Q_i, q_j \in Q_j} H_{\mathrm{prop}}[t_{q_i}, t_{q_j}, q_i, q_j]$$

- Challenge: advancing all physical clocks in a logical qubit at once would not be local. Advancing them one at a time would violate terms in H_{code}. Dressed stabilizers are the solution!

Dressing stabilizers to avoid frustration

- We need to tell the stabilizers "what time it is" so that they can accomodate diffusive propagation without frustration,

$$|t_{s_1}, \ldots, t_{s_m}\rangle\langle t_{s_1}, \ldots, t_{s_m}| \otimes H_s(t_{s_1}, \ldots, t_{s_m})$$

- Stabilizers that act on "staggered" time configurations in which not all clocks are equal are rotated by local unitaries to advance the qubits that are lagging behind (or getting ahead),

$$|\mathbf{t}_s\rangle\langle\mathbf{t}_s| \otimes H_s(\mathbf{t}) := \left(\bigotimes_{k\in s} |t_k\rangle\langle t_k|_{t_k}\right) \bigotimes \left(\prod_{t_k} U^{\dagger}_{t_k,t}[q_k]\right) H_s \left(\prod_{t_k} U_{t_k,t}[q_k]\right)$$

- Dressing for two qubit gates intertwines stabilizers from distinct logical qubits, but terms remain k-local.
- It suffices to limit staggering to a constant window, adding rigidity to the membrane on the scale of the stabilizers.

Universal sets of local operations

- 6D color code is self-correcting and admits transversal $\{T, CNOT, X, Z\}$. Teleported Hadamard completes the set.
- To teleport H it suffices to (1) prepare logical X,Z , (2) measure X, Z, (3) apply X, Z conditioned on measurement outcomes.
- Replace projective measurement $\Pi_0 + \Pi_1 = I$ of the physical qubits with coherent unitaries onto the classical ancillas:

$$|\psi\rangle|0\rangle \longrightarrow \Pi_0|\psi\rangle|0\rangle + \Pi_1|\psi\rangle|1\rangle$$

- Each physical qubit is measured by a "classical wire". The classical wire is a logical ancilla encoded in the repetition code.
- Classical post-processing is global takes poly time. The rest of the computation "waits around" for this to be done.

Analysis of the Gibbs state

- The entire Hamiltonian is unitarily equivalent to a diffusing membrane and a static code Hamiltonian, the dressing disappears:

$$W = \sum_{\boldsymbol{\tau}\,\mathrm{valid}} U(\mathbf{0} \to \boldsymbol{\tau})|\boldsymbol{\tau}\rangle\langle\boldsymbol{\tau}| \quad , \quad W^\dagger H W = H_{\mathrm{membrane}} \otimes I + I \otimes H_{\mathrm{code}}$$

- Thermal stability from the static code Hamiltonian at $T < T_c$.
- Analyzing H_{membrane} is challenging, but circular symmetry makes all valid time configurations equally likely in every eigenstate.
- Transversal input term acts on classical logical ancillas, and the topological code is initialized by coherent measurements.
- Diagonal elements of the thermal density matrix of H in the time register basis have the form

$$|\boldsymbol{\tau}\rangle\langle\boldsymbol{\tau}| \otimes W(\boldsymbol{\tau})\rho_{\mathrm{code}} W^\dagger(\boldsymbol{\tau})$$

Misspecification errors in local H terms: $\|W_{\mathrm{actual}} - W_{\mathrm{ideal}}\|$ is small because W is a fault-tolerant circuit.

The 4D spacetime view of active error correction

- Consider the history state of a fault-tolerant quantum computer e.g. surface code qubits connected to a classical PC.
- Instead of a code Hamiltonian, such a scheme depends on actively measuring and correcting stabilizers.
- There is no energetic protection of the qubits, *but there is energetic protection from the materials in the classical PC.*
- *Active error correction is possible because we dump entropy from quantum computers into classical self-correcting memories.*
- 4D self-correcting memory from the history state of 3D FT-QC. Quest for planar FT-QC architectures relates to self-correction in 3D!

Summary and Outlook

- Universal quantum computation in a finite temperature Gibbs state of a k-local Hamiltonian ($k < 1000$) with polynomial overhead.
- Can we efficiently prepare these Gibbs states? Lower bound the gap of H_{membrane}? Open system dynamics?
- Is it QMA-complete to decide the thermal energy?
- $H_{\mathrm{circuit}} + H_{\mathrm{code}}$ is like "kinetic + potential", the missing ingredient ("potential") was the spacetime view of interactions.
- Is time an illusion? Everything we know is consistent with living in a history state universe. Wheeler-DeWitt equation, $H|\psi\rangle = 0$?
- Thank you for your attention!