Title: Simulating structure formation in different environments and the application

Date: Jan 23, 2018 11:00 AM

URL: http://pirsa.org/18010083

Abstract: <p>The observables of the large-scale structure such as galaxy number density generally depends on the density environment (of a few hundred Mpc). The dependence can traditionally be studied by performing gigantic cosmological N-body simulations and measuring the observables in different density environments. Alternatively, we perform the so-called "separate universe simulations", in which the effect of the environment is absorbed into the change of the cosmological parameters. For example, an overdense region is equivalent to a universe with positive curvature, hence the structure formation changes accordingly compared to the region without overdensity. In this talk, I will introduce the "separate universe mapping", and present how the power spectrum
and halo mass function change in different density environments, which are equivalent to the squeezed bispectrum and the halo bias, respectively. I will then discuss the extension of this approach to inclusion of additional fluids such as massive neutrinos. This allows us to probe the novel scale-dependence of halo bias and squeezed bispectrum caused by different evolutions of the background overdensities of cold dark matter and the additional fluid. Finally, I will present one application of the separate universe simulations to predict the squeezed bispectrum formed by small-scale Lyman-alpha forest power spectrum and large-scale lensing convergence, and compare with the measurement from BOSS Lyman-alpha forest and Planck lensing map.</p>

Simulating structure formation in different environments and the application

Chi-Ting Chiang (蔣季庭)
C.N. Yang Institute for Theoretical Physics

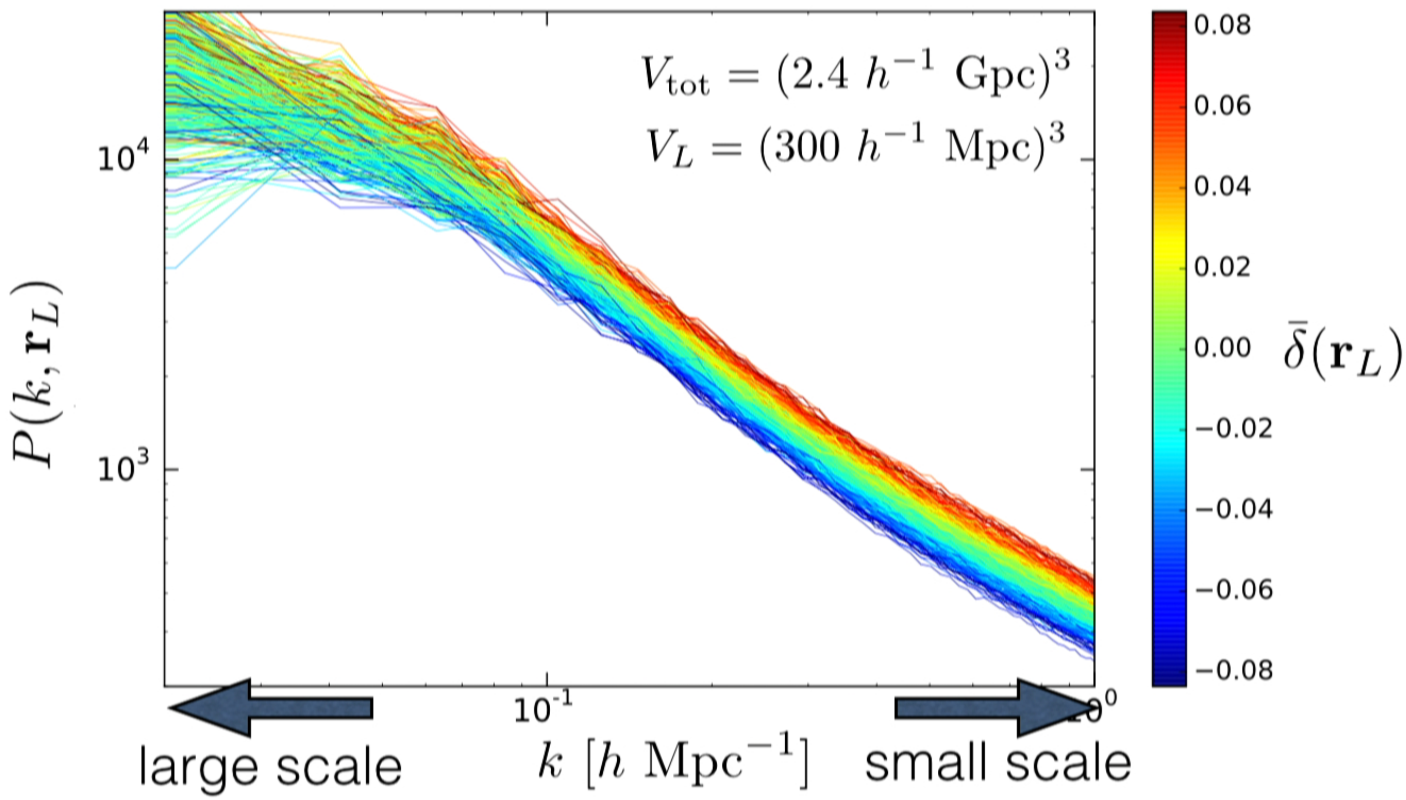
Perimeter Institute, January 23 2018

Why study large-scale structure in different environments?

- The observables of large-scale structure (such as power spectrum and halo mass function) generally depend on the environment ($\gtrsim$ a few hundred Mpc) due to gravitational evolution.

- This dependence carries additional information on top of the observables themselves that can be used to study the nonlinear gravitational evolution or to probe new physics.

Small-scale power spectra depend on large-scale densities



CTC, Wagner, Schmidt, Komatsu, 2014

Response example 1: power spectrum

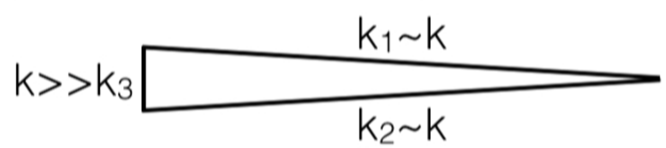
- Assume that the power spectrum depends on its density environment, and then Taylor expand it in series of the environment:

$$P(k|\delta_L) = P(k)|_{\delta_L=0} + P(k) \left.\frac{d\ln P(k)}{d\delta_L}\right|_{\delta_L=0} \delta_L + \mathcal{O}(\delta_L^2)$$

- Correlate the power spectrum with its environment:

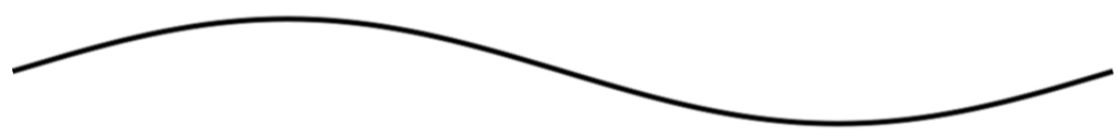
$$\langle P(k|\delta_L)\delta_L\rangle = P(k)\frac{d\ln P(k)}{d\delta_L}\langle\delta_L^2\rangle + \mathcal{O}(\delta_L^3)$$

- This is equivalent to the squeezed-limit bispectrum



How is the power spectrum affected by the environment?

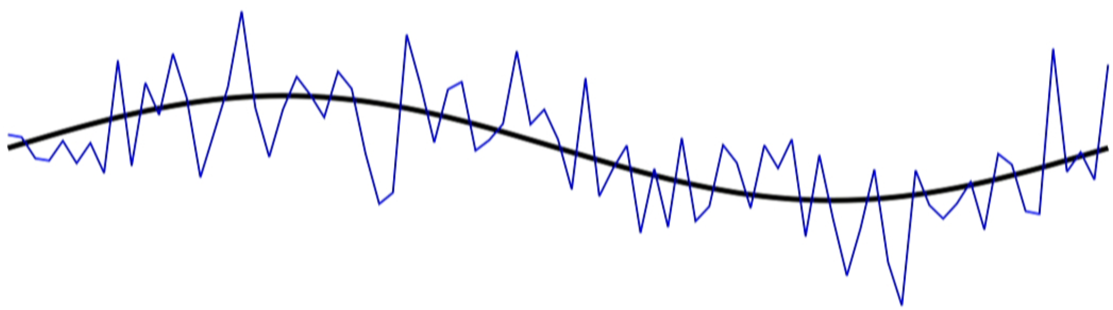
consider a long-wavelength density fluctuation



overdensity underdensity

How is the power spectrum affected by the environment?

no correlation between power spectra and environments→ squeezed-limit bispectrum is zero

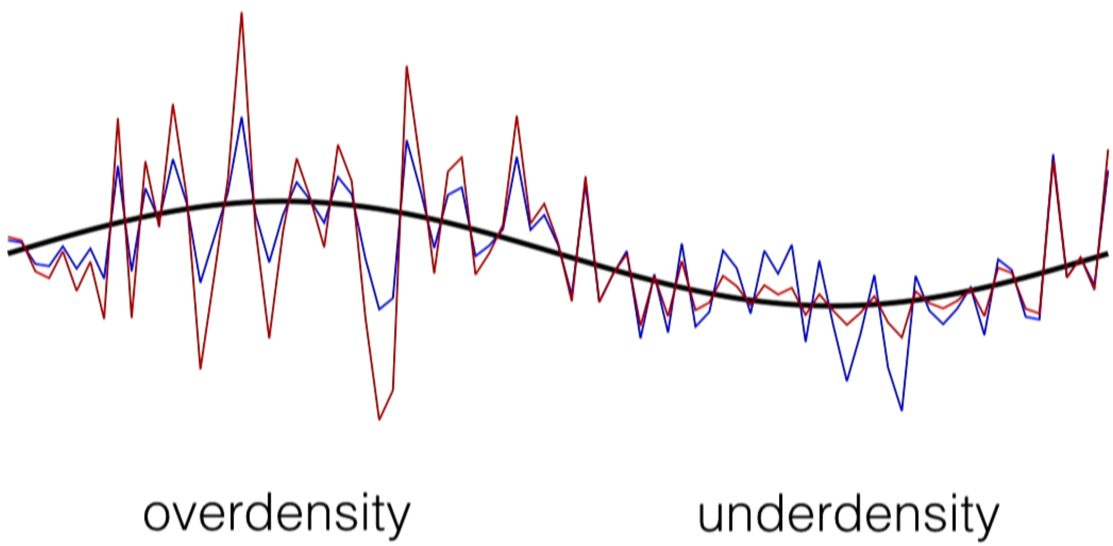


overdensity

underdensity

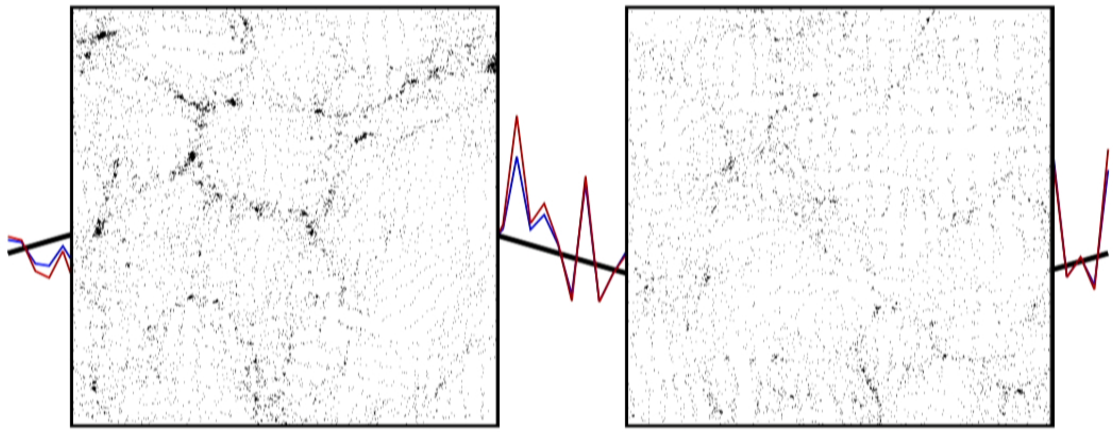
How is the power spectrum affected by the environment?

positive correlation between power spectra and environments→ squeezed-limit bispectrum is positive



How is the power spectrum affected by the environment?

positive correlation between power spectra and environments→ squeezed-limit bispectrum is positive



overdensity underdensity

Response example 2: halo mass function

- Expand the halo mass function in series of the large-scale density environment:

$$n_h(M_h|\delta_L) = n_h(M_h)|_{\delta_L=0} + \left.\frac{dn_h(M_h)}{d\delta_L}\right|_{\delta_L=0} \delta_L + \mathcal{O}(\delta_L^2)$$

- Rewrite the derivatives as the “bias” parameters for the halo number density fluctuation:

$$\delta_h(M_h|\delta_L) = \frac{n_h(M_h|\delta_L)}{n_h(M_h)} - 1 = \frac{d\ln n_h(M_h)}{d\delta_L}\delta_L + \mathcal{O}(\delta_L^2)$$
$$= b_1(M_h)\delta_L + \mathcal{O}(\delta_L^2)$$

- Large-scale halo and halo-matter power spectra:

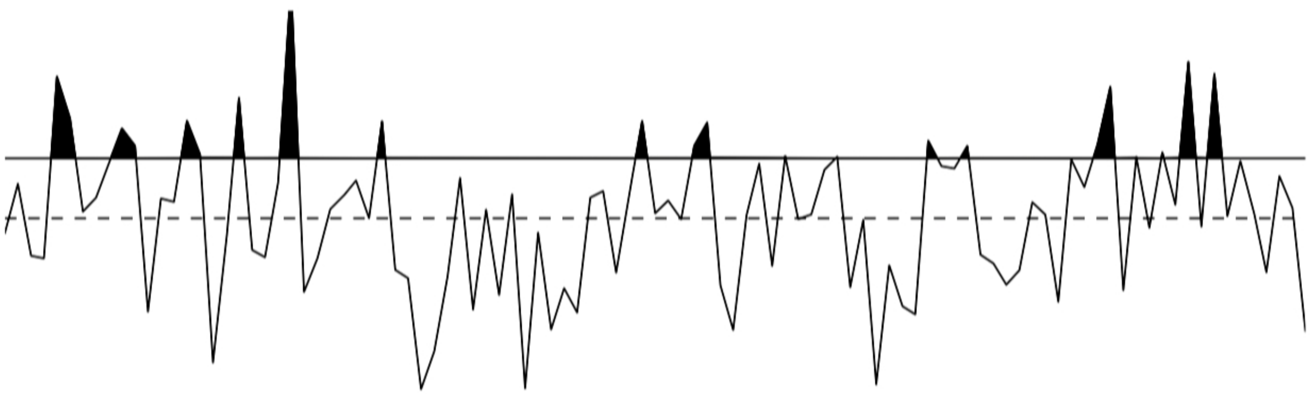
$$P_{hh} = b_1^2 P_{mm} \qquad P_{hm} = b_1 P_{mm}$$

How is the halo formation affected by the background?

Let us consider a density fluctuation in space with zero mean…

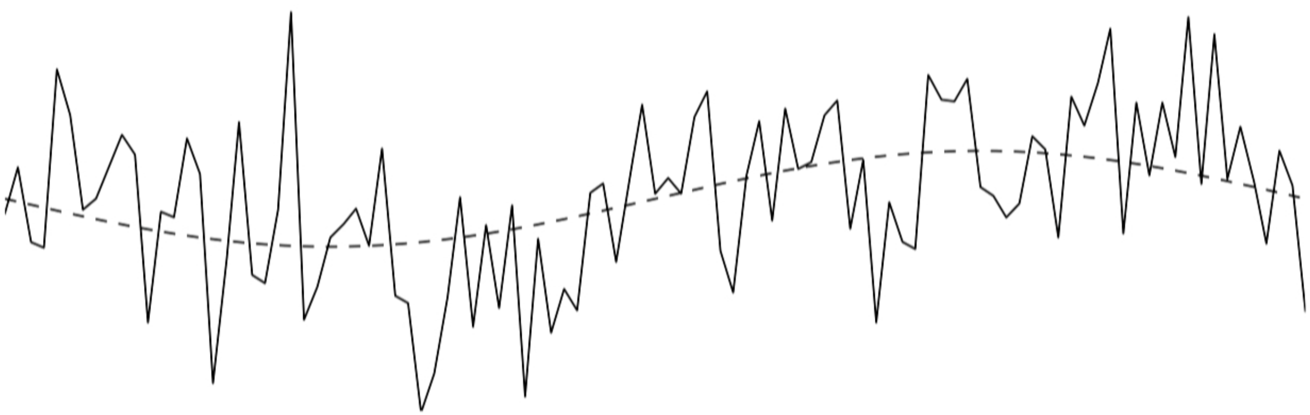
How is the halo formation affected by the background?

Once the density fluctuation exceeds the threshold, "halos" would form, as the shaded areas.



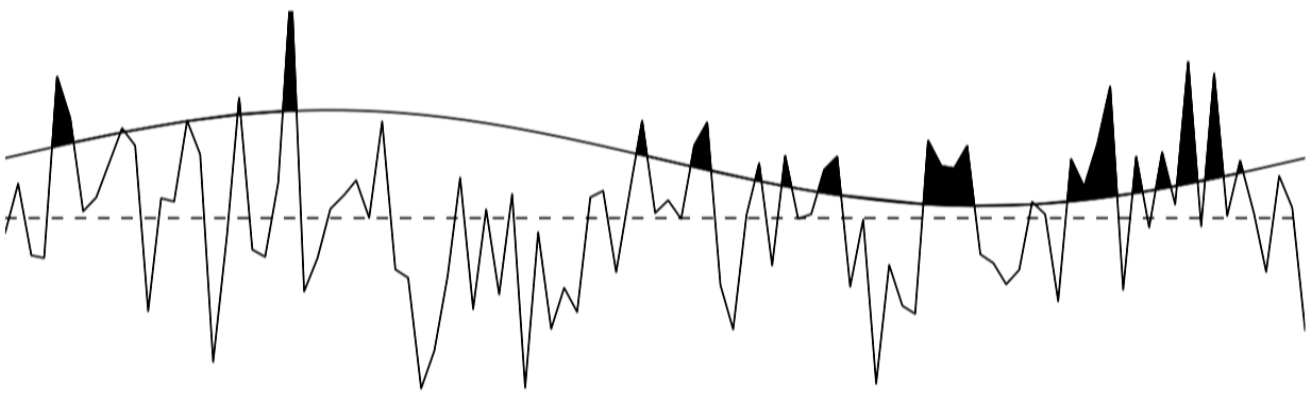
How is the halo formation affected by the background?

If there is a large-scale density variation in space…



How is the halo formation affected by the background?

Alternatively, the long-wavelength fluctuation can be viewed as the change of the threshold density locally



How can we study the environmental dependence?

- Run gigantic N-body simulations
- Identify regions with different densities
- Measure and compare observables in different density environments
- This approach however requires huge computational resources

Separate universe approach

- Absorb the long-wavelength density fluctuation into the change of **local** cosmology
  $\Rightarrow$ separate universe approach

- Perform N-body simulations in different density environments $\Rightarrow$ separate universe simulations

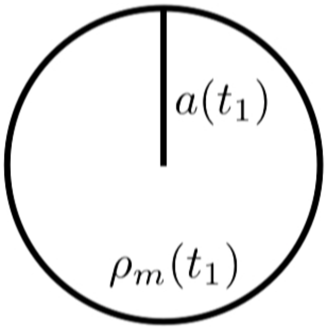
- A much cheaper way than gigantic simulations to study how the small-scale structure formation is affected by the environments

Why separate universe simulations?

- Much (much much) cheaper than gigantic simulations, and can get higher resolution
- Exact setup to test how the small-scale structure formation changes with the (infinitely) long-wavelength density fluctuations
- If we set up the initial condition cleverly, a large amount of cosmic variance is canceled

Separate universe evolution

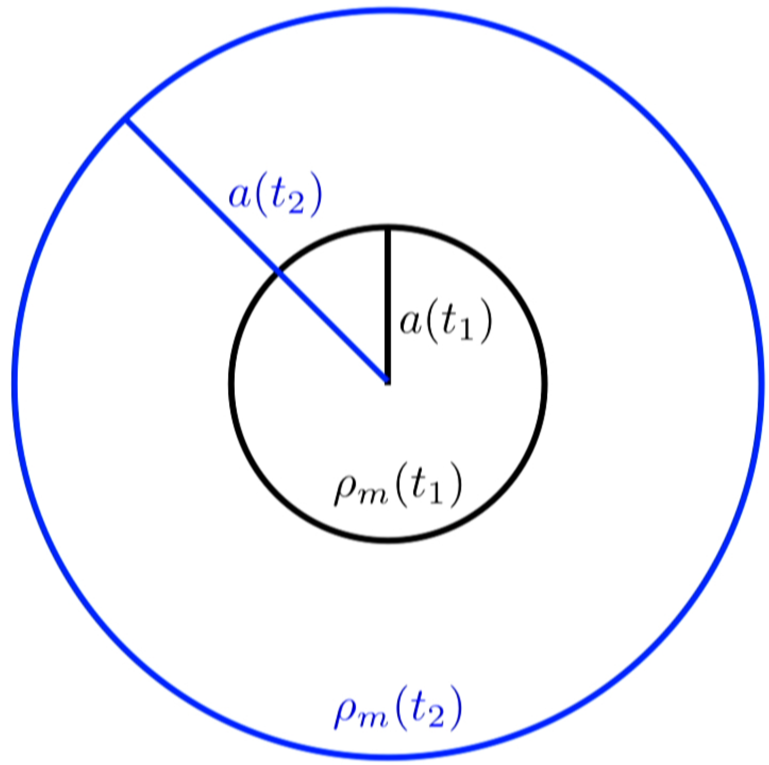
At some early time t_1 (when the overdensity is infinitesimal), the size of the universe is $a(t_1)$ with density $\rho_m(t_1)$



e.g. Sirko 2005, Baldauf et al 2011, Li et al 2014

Separate universe evolution

At some later time t_2, the fiducial universe (with cosmic mean density) expands to $a(t_2)$ with density $\rho_m(t_2)$



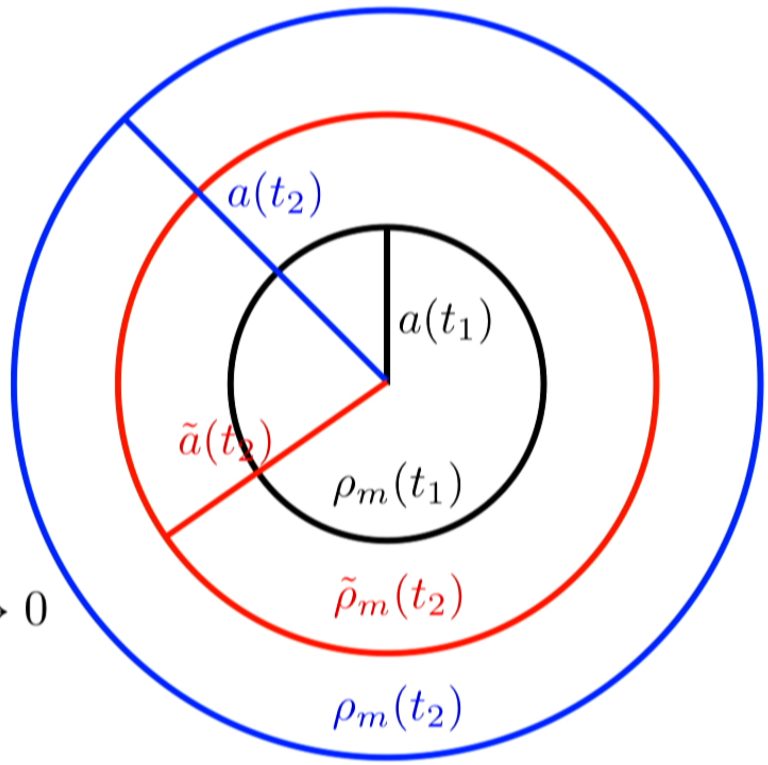
e.g. Sirko 2005, Baldauf et al 2011, Li et al 2014

Separate universe evolution

On the other hand, the overdensity $\delta_L(t_2)$ would slow down the local expansion, and so the size becomes $\tilde{a}(t_2)$ with density

$$\tilde{\rho}_m(t_2) = \rho_m(t_2)[1 + \delta_L(t_2)]$$

$\tilde{\rho}_m(t_1) = \rho_m(t_1)$ as $\delta_L(t_1) \to 0$



e.g. Sirko 2005, Baldauf et al 2011, Li et al 2014

Separate universe evolution

Since the total matter is conserved, we have

$$\tilde{\rho}_m(t_2)\tilde{a}^3(t_2) = \rho_m(t_2)a^3(t_2)$$

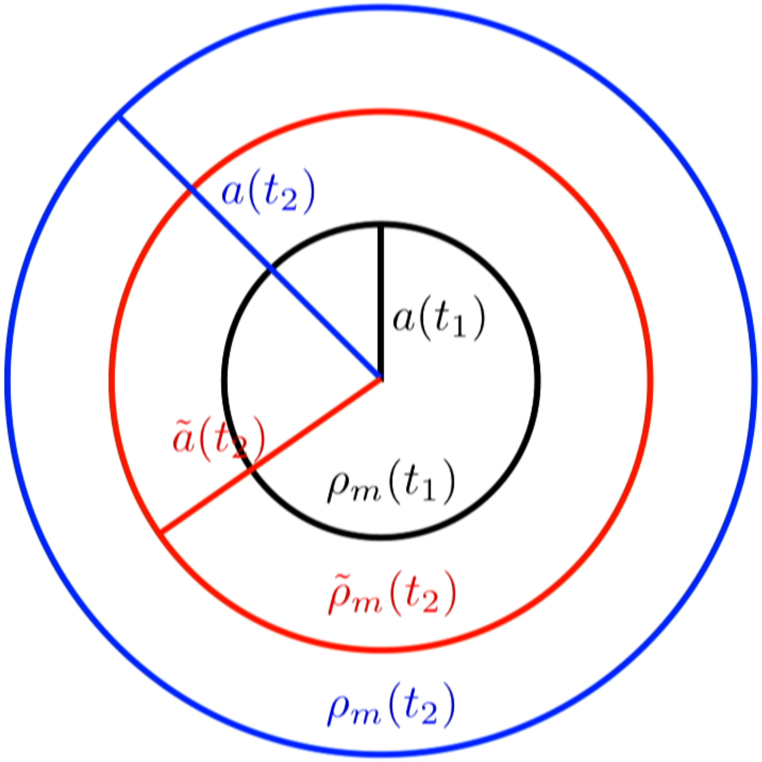
This leads to

$$\tilde{a}(t) = a(t)\left[1 - \frac{1}{3}\delta_L(t)\right]$$

and so

$$\frac{\dot{\tilde{a}}}{\tilde{a}} = \tilde{H}(t) = H(t) - \frac{1}{3}\dot{\delta}_L(t)$$

$$\frac{\ddot{\tilde{a}}}{\tilde{a}} = \frac{\ddot{a}}{a} - \frac{1}{3}\ddot{\delta}_L - \frac{2}{3}H\dot{\delta}_L$$



e.g. Sirko 2005, Baldauf et al 2011, Li et al 2014

Separate universe mapping

- The corresponding cosmological parameters are

$$\tilde{H}_0 = H_0(1+\delta_H) \qquad \tilde{\Omega}_m = \Omega_m(1+\delta_H)^{-2}$$

$$\tilde{\Omega}_\Lambda = \Omega_\Lambda(1+\delta_H)^{-2} \quad \tilde{\Omega}_K = 1-(1+\delta_H)^{-2}$$

$$1+\delta_H = \left[1 - \frac{5}{3}\frac{\Omega_m}{D(t_0)}\delta_L(t_0)\right]^{1/2}$$

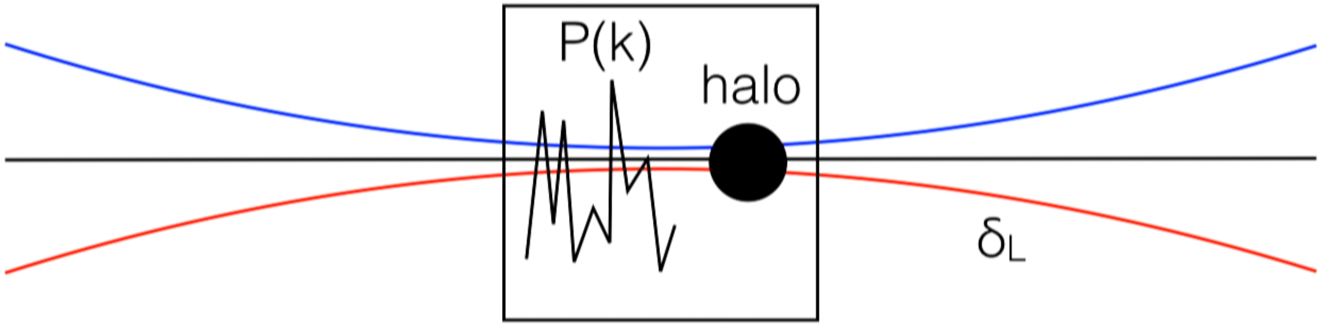
- For ΛCDM (fiducial) universe, the long-wavelength density fluctuation behaves as curvature. Namely, in the overdense (underdense) universe, the separate universe is positively (negatively) curved.

Wagner, Schmidt, ***CTC***, Komatsu, 2014

Separate universe simulations

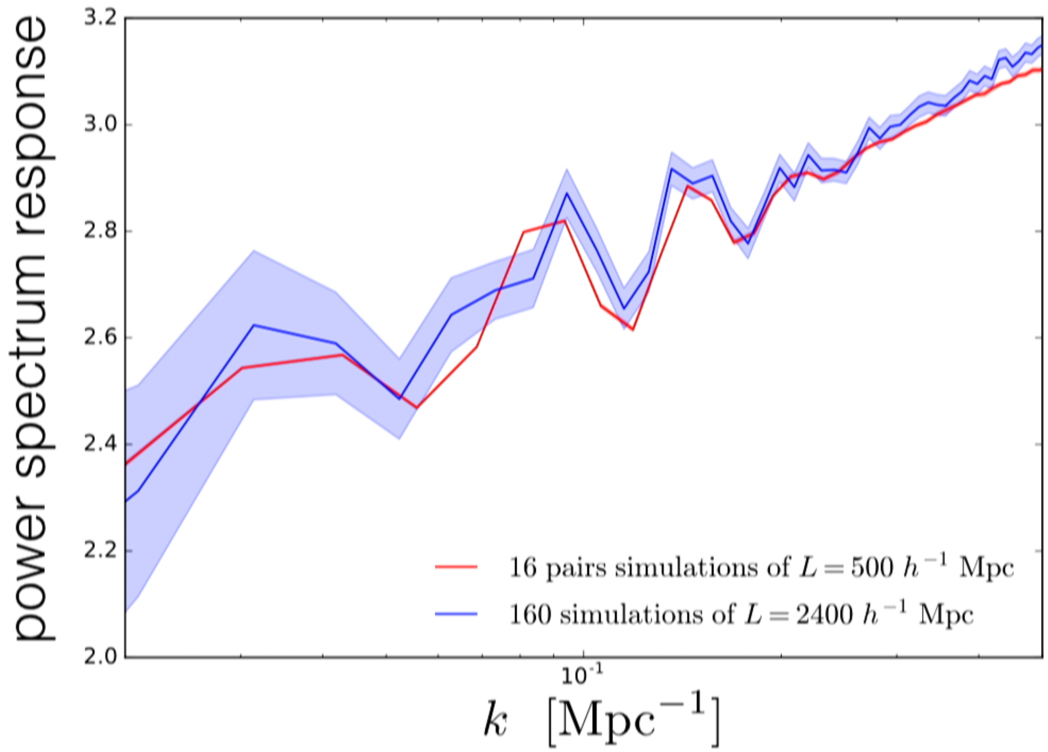
- Perform N-body simulations directly in long-wavelength overdensity and underdensity with the same random phases.
- Calibrate the response of observable A to δ_L as

$$R_A = (A_{\delta_L^+} - A_{\delta_L^-})/(2\delta_L A_{\delta_L^0})$$



Wagner, Schmidt, ***CTC***, Komatsu, 2014

Power spectrum response [squeezed-limit B(k,k,k$_L$)]



Wagner, Schmidt, ***CTC***, Komatsu, 2015

Halo finding and bias measurement

- Spherical overdensity: change the threshold density by a factor of $[1+\delta_L(t)]^{-1} \approx [1-\delta_L(t)]$

  Lazeyras, Wagner, Baldauf, Schmidt, 2015   Li, Hu, Takada, 2015

- Response bias: derivative of halo mass function from separate universe simulations

- Clustering bias: the ratio of halo-matter cross spectrum to the matter power spectrum in fiducial simulations $b_1 \sim P_{mh}/P_{mm}$

Halo finding and bias measurement

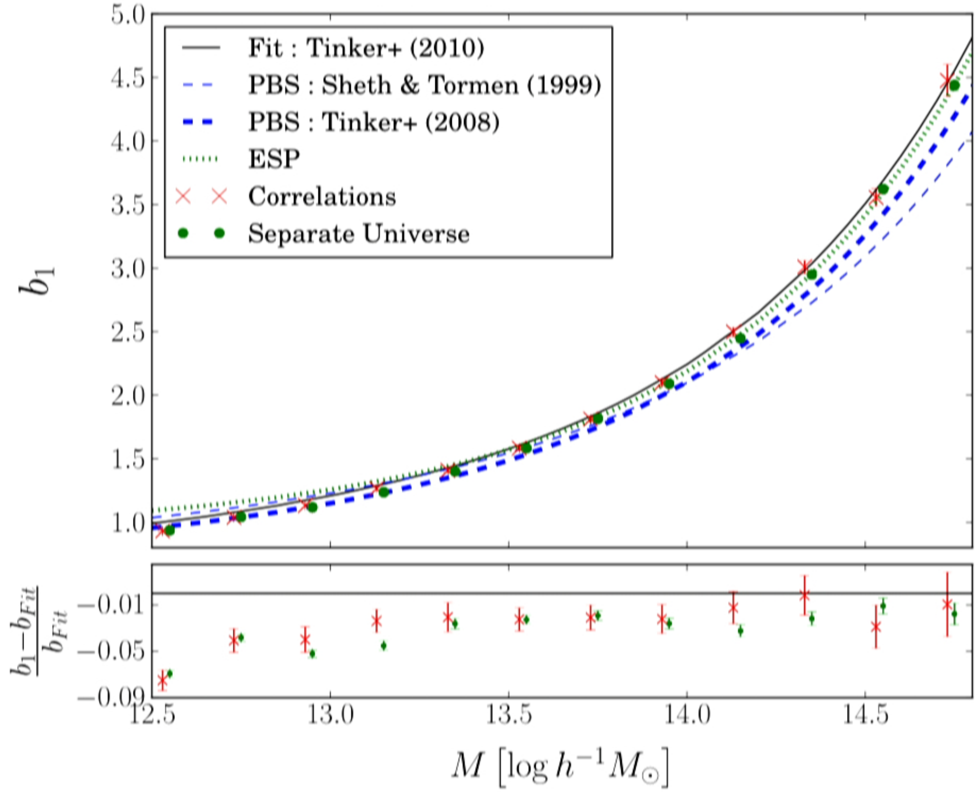
- Spherical overdensity: change the threshold density by a factor of $[1+\delta_L(t)]^{-1} \approx [1-\delta_L(t)]$

  Lazeyras, Wagner, Baldauf, Schmidt, 2015   Li, Hu, Takada, 2015

- Response bias: derivative of halo mass function from separate universe simulations

- Clustering bias: the ratio of halo-matter cross spectrum to the matter power spectrum in fiducial simulations $b_1 \sim P_{mh}/P_{mm}$

Response halo bias: b_1



Lazeyras, Wagner, Baldauf, Schmidt, 2015

What if there are neutrinos?

- In ΛCDM universe, the responses are independent of the wavelength of linear δ_L we put in, since the linear growth is scale independent.
- Massive neutrinos possess a free-streaming scale k_{fs}, and the growth of CDM with wavenumber k_L is stronger for $k_L < k_{fs}$ than $k_L > k_{fs}$.
- Depending on what wavelength of δ_L we put in, the expansion histories and small-scale structure formation are different for different k_L.

Evolution of long-wavelength CDM +baryon fluctuation with neutrinos

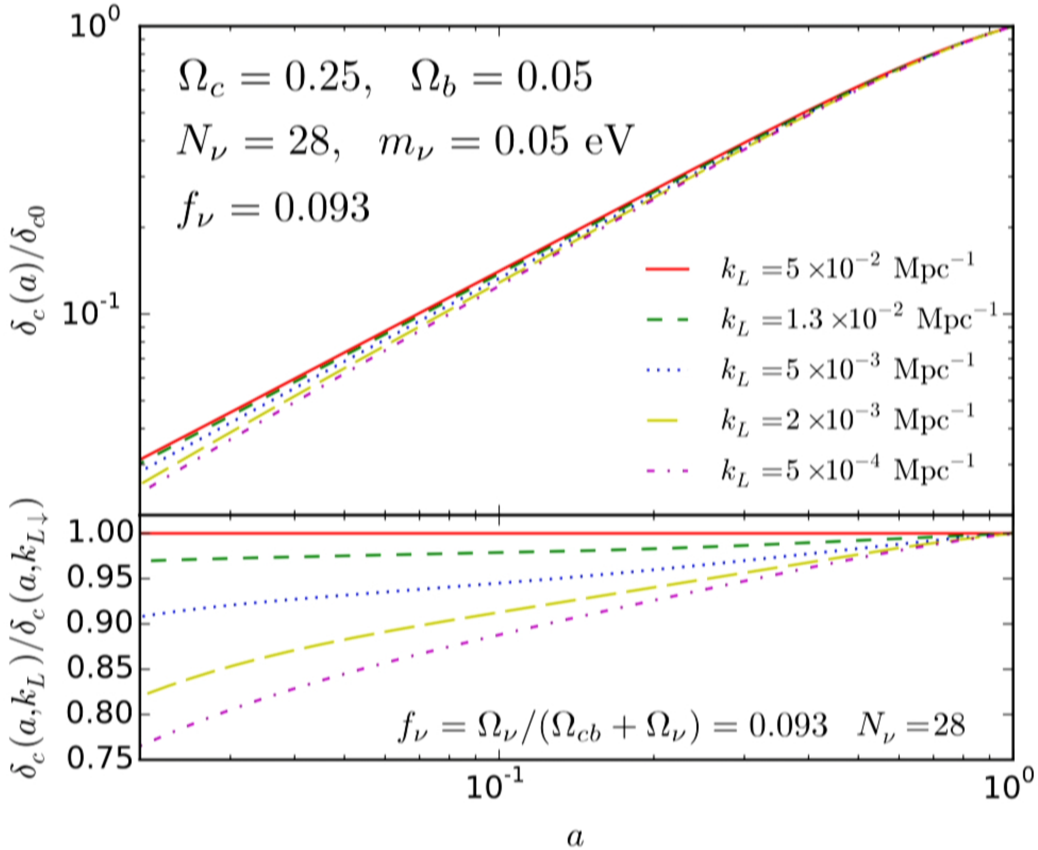
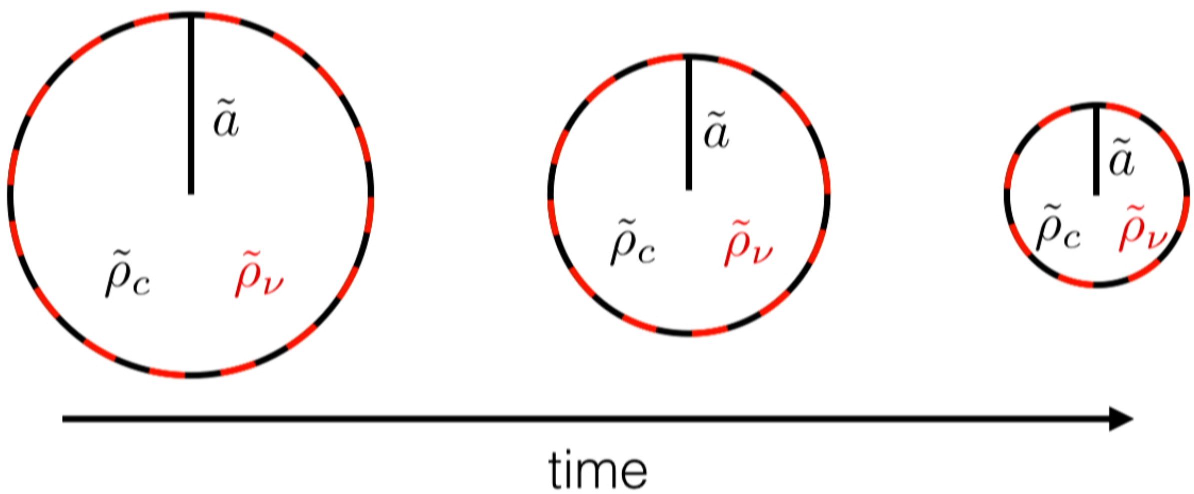


Illustration of separate universe with massive neutrinos

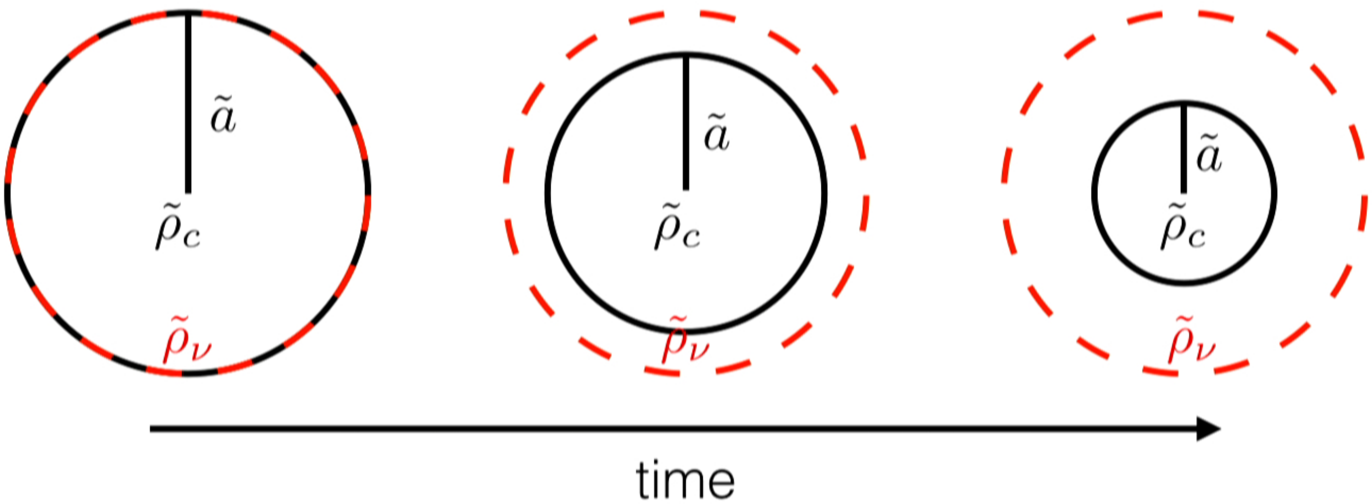
above the neutrino free-streaming scale ($k_L < k_{fs}$)
CDM and neutrino grow coherently



Hu, ***CTC***, Li, LoVerde, 2016

Illustration of separate universe with massive neutrinos

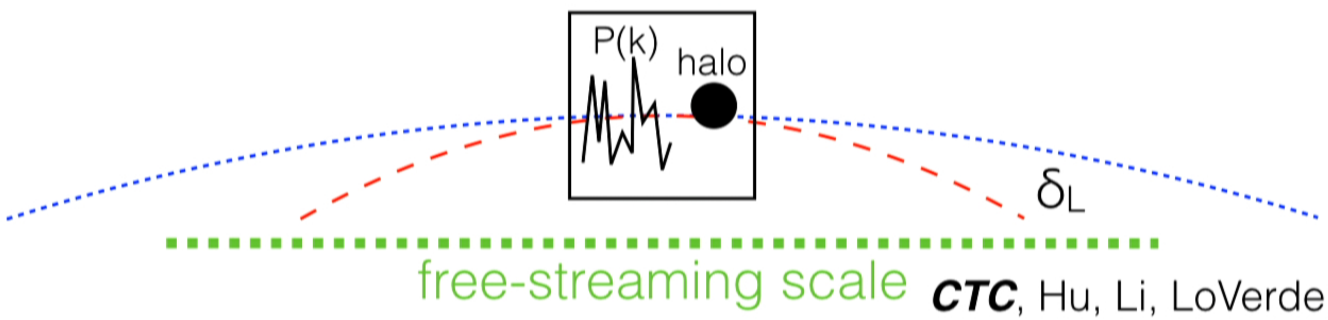
below the neutrino free-streaming scale ($k_L > k_{fs}$)
neutrino is smooth and only CDM clusters



Hu, ***CTC***, Li, LoVerde, 2016

Separate universe simulations with massive neutrinos

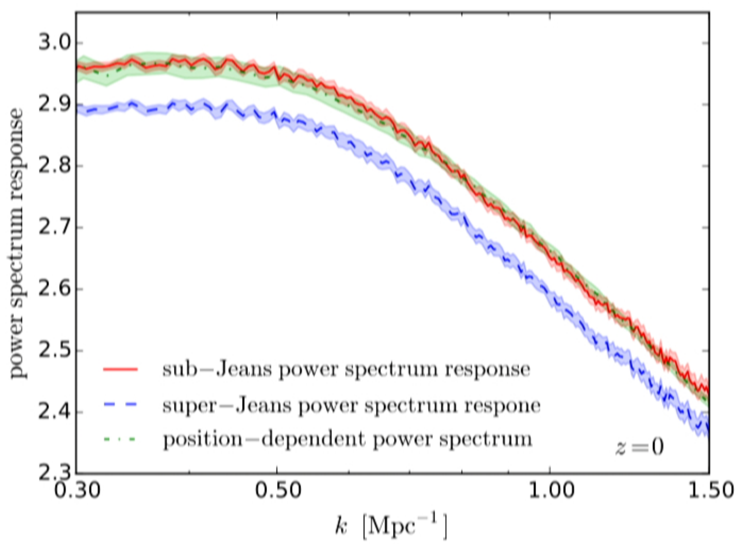
- Finding cosmological parameters in the separate universe is non-trivial due to different evolution of CDM and neutrino, but we can still run simulations with a different expansion by $\tilde{H}(t) = H(t)\left[1 - \frac{1}{3}\frac{d\delta_L(t)}{d\ln a}\right]$
- Within the simulation box we simulate only the dynamics of CDM, so the neutrino effect is only modeled at the background.



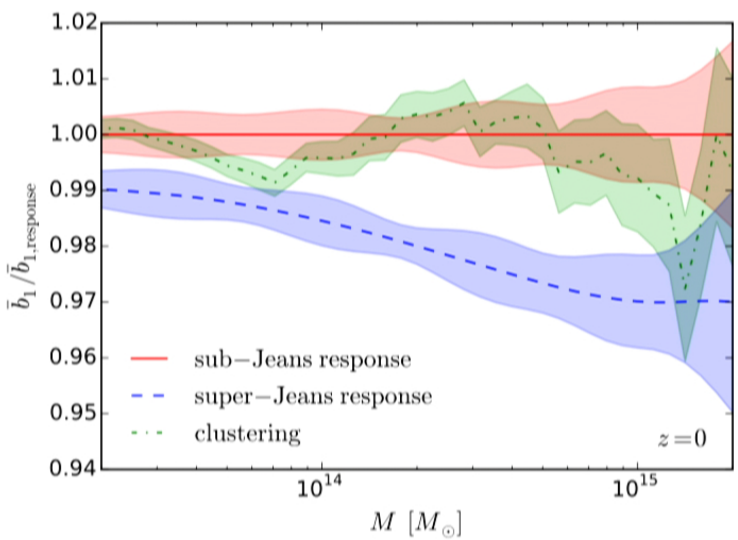
CTC, Hu, Li, LoVerde, 2017

Validation for separate universe simulation with quintessence

power spectrum response
(squeezed-limit bispectrum)



halo mass function response
(response halo bias)



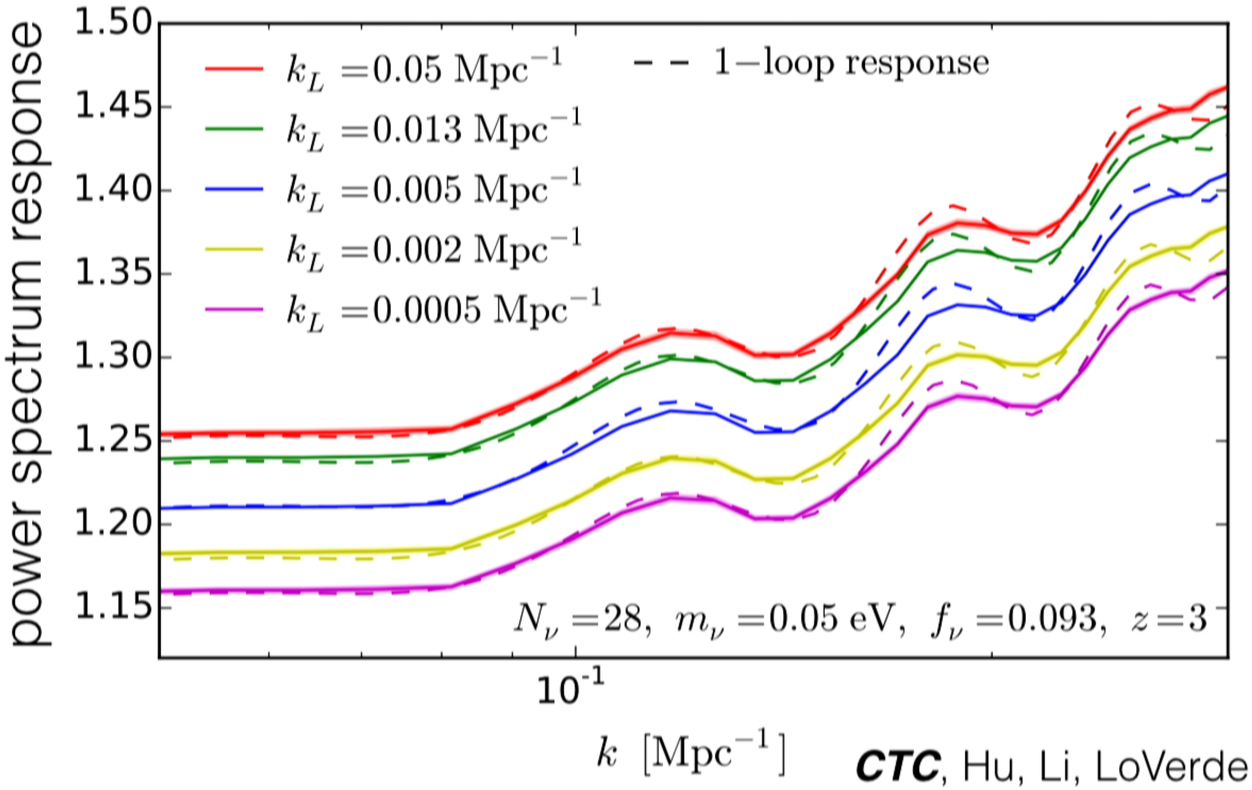
CTC, Li, Hu, LoVerde, 2016

Separate universe simulations with massive neutrinos

- To avoid the neutrino nonlinear clustering in the simulation box and to assure that the neutrino free-streaming length is on linear scale, we choose m_ν=0.05 eV (free-streaming length is ~200 Mpc).
- To make $f_\nu=\Omega_\nu/(\Omega_\nu+\Omega_{cb})$ large enough so that the effect can be detected with a handful amount of simulations, we set N_ν=14 and 28 (instead of 3), which leads to $f_\nu \approx 0.05$ and 0.1.
- We set $\delta_{L0}=\pm 0.01$ for k_L=0.0005, 0.002, 0.005, 0.013, and 0.05 Mpc^{-1}, and the evolution of δ_L is solved by CLASS/CAMB.

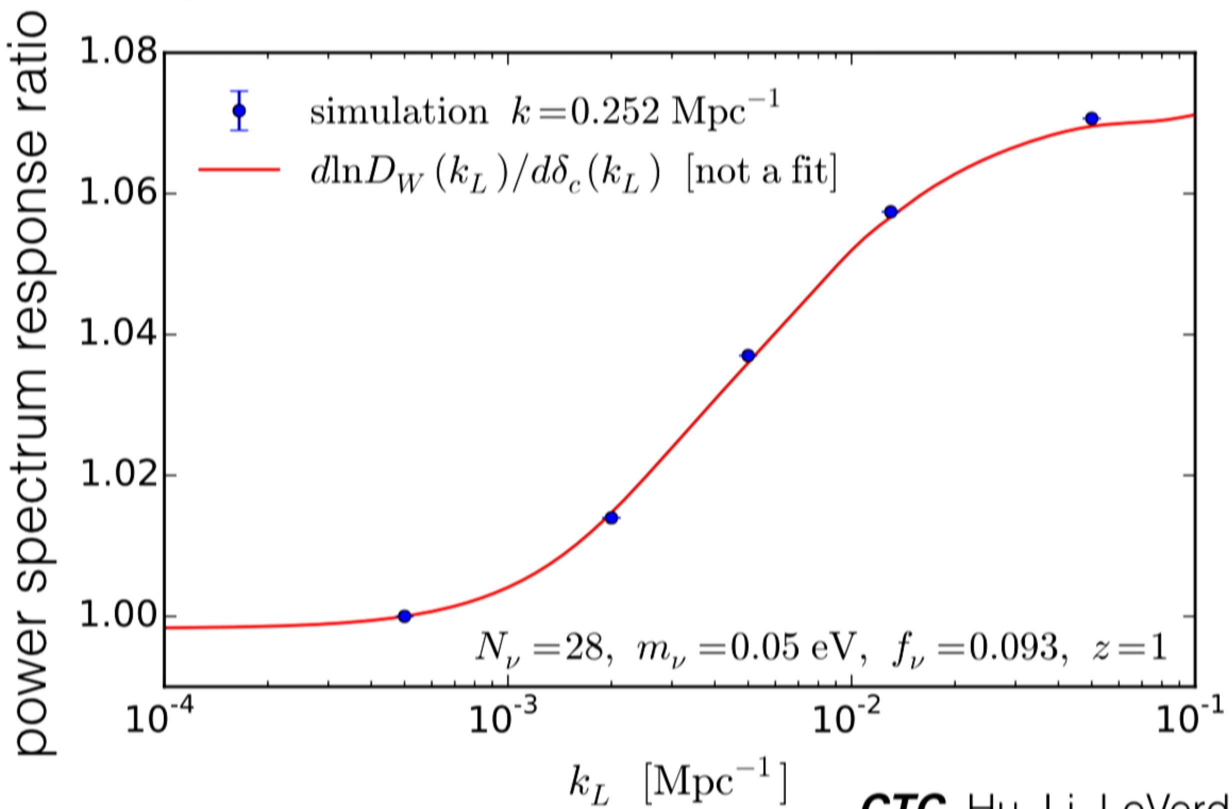
CTC, Hu, Li, LoVerde, 2017

Power spectrum response [squeezed-limit B(k,k,k$_L$)]



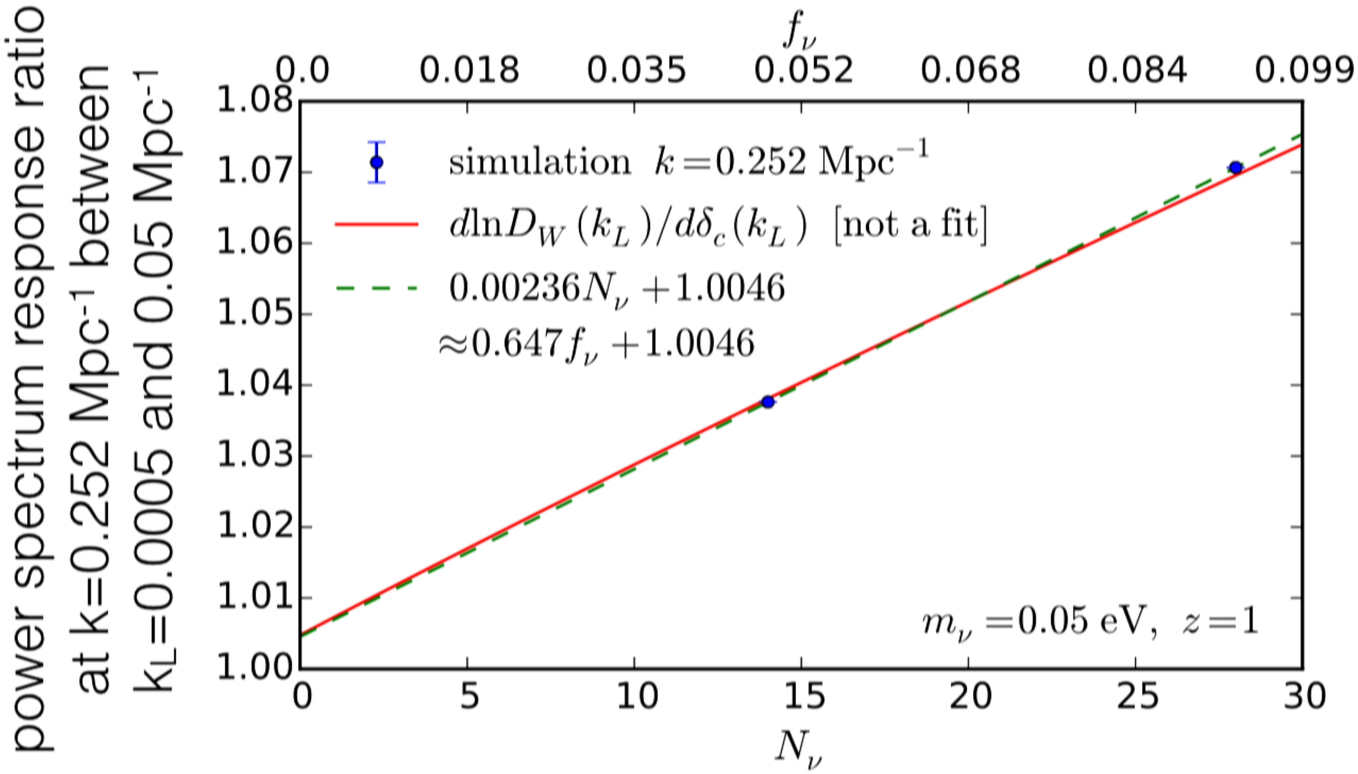
CTC, Hu, Li, LoVerde, 2017

Power spectrum response [squeezed-limit B(k,k,k$_L$)]



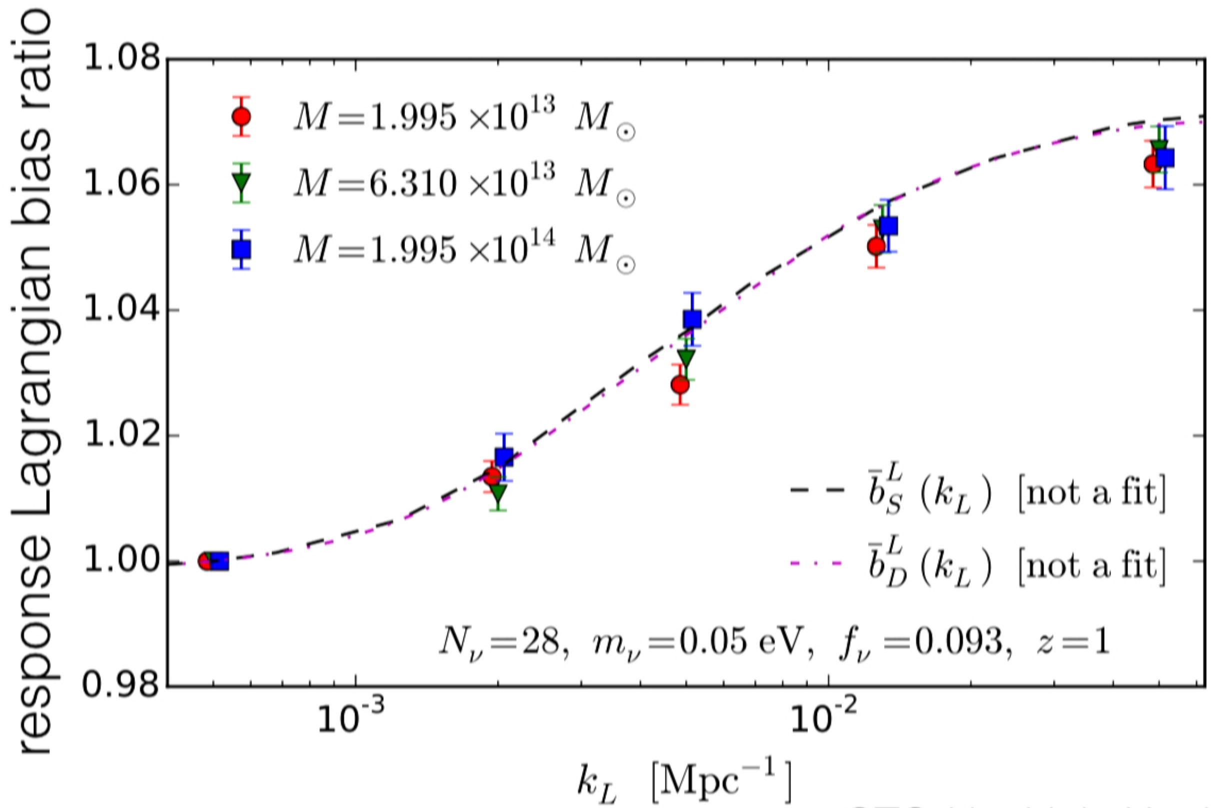
CTC, Hu, Li, LoVerde, 2017

Dependence on f_ν



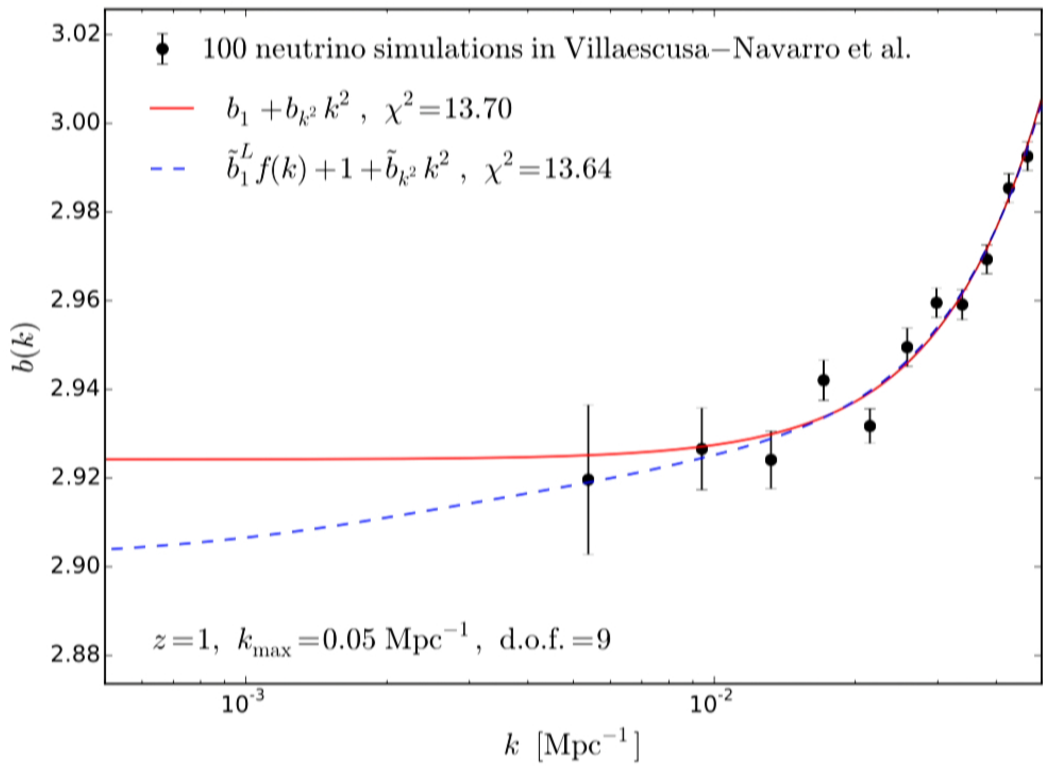
CTC, Hu, Li, LoVerde, 2017

Scale-dependent halo bias



CTC, Hu, Li, LoVerde, 2017

Comparison with neutrino particle simulations (N_ν=3, m_ν=0.05eV)



Other application: squeezed bispectrum model

- Squeezed bispectrum contains one long and two short modes.
- If the long mode of interest is greater than ~100 Mpc, then it can be treated as linear mode.
- The evolution and response of the short modes can be characterized by separate universe simulations.
- This is a good ansatz for building the squeezed bispectrum model.

CTC, Slosar, 2017

Squeezed bispectrum formed by different observables

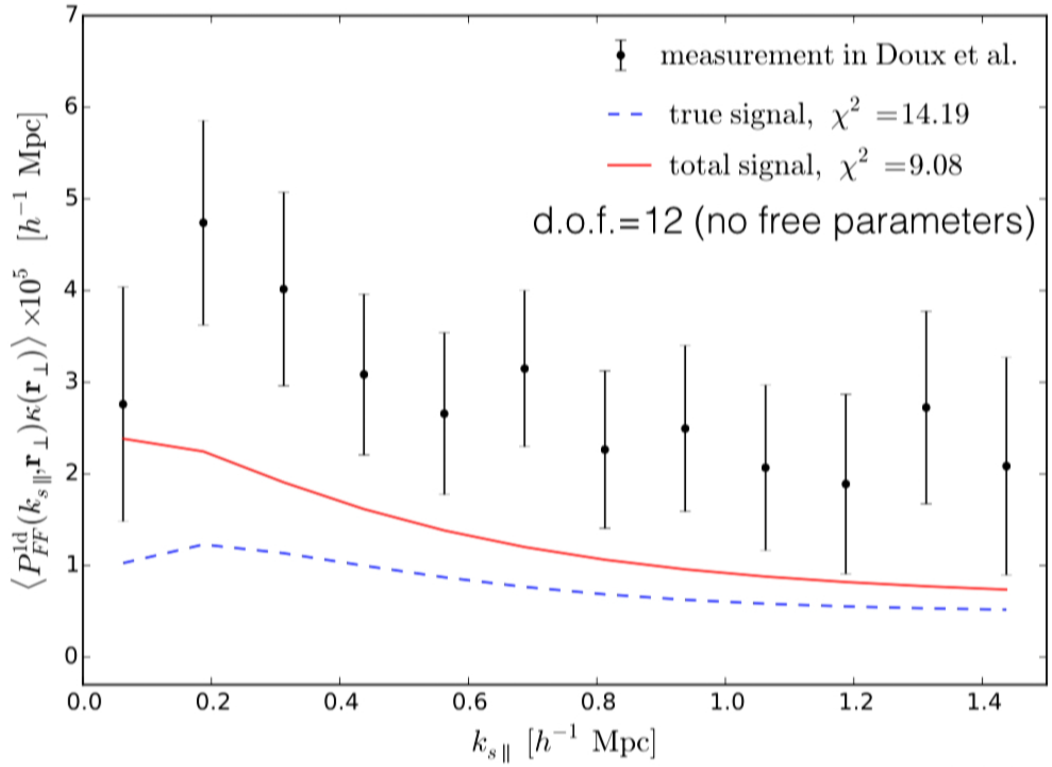
- Consider the correlation of small-scale Lyα forest power spectrum and the lensing convergence.

- The signal can be schematically written down as
$$B^{\mathrm{sq}}_{FF\kappa}(k_S, k_L) = \langle P_{FF}(k_S)\kappa\rangle = \frac{dP_{FF}(k_S)}{d\delta} P_{\delta\kappa}(k_L)$$

- $dP_{FF}(k_S)/d\delta$ can be measured from hydrodynamic separate universe simulations, and $P_{\delta\kappa}(k_L)$ can be computed analytically.

CTC, Slosar, 2017

Comparison with measurement of BOSS cross Planck



CTC, Slosar, 2017

Other combinations

- Consider the correlation between Lyα forest power spectrum and quasar overdensity.

- Quasar perturbation can be decomposed into density, gradient of peculiar velocity along the line-of-sight, and the potential due to primordial non-Gaussianity as

$$\delta_q(\mathbf{k}_L) = b_\delta \delta(\mathbf{k}_L) + b_\eta \eta(\mathbf{k}_L) + b_\phi \phi(\mathbf{k}_L)$$

- We thus need to consider different responses of Lyα power spectrum to δ, η, and ϕ.

CTC, Cieplak, Schmidt, Slosar, 2017

Conclusions

- Separate universe simulations are powerful to study the responses of small-scale observables to the large-scale environment.
- Using this technique, we find scale-dependent bias and squeezed bispectrum in νΛCDM cosmology.
  - Small-scale neutrino clustering is ignored in the box
  - Long mode may have similar length as the box
  - Work in progress to probe the scale-dependence with other simulation techniques (particle/hybrid) with gigantic box (~2 h^{-1}Gpc).
- The separate universe approach is very useful to construct the squeezed bispectrum model.