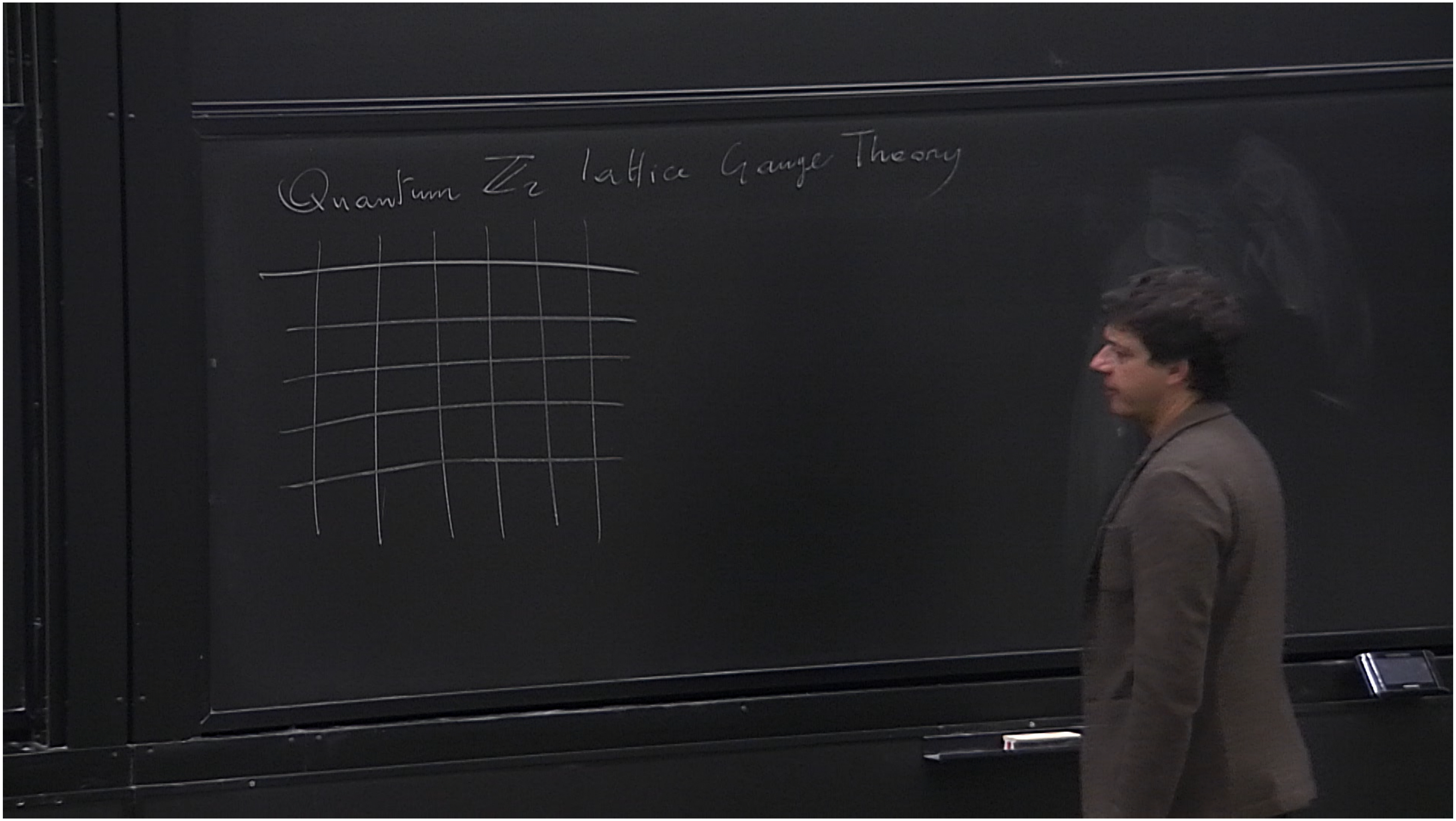
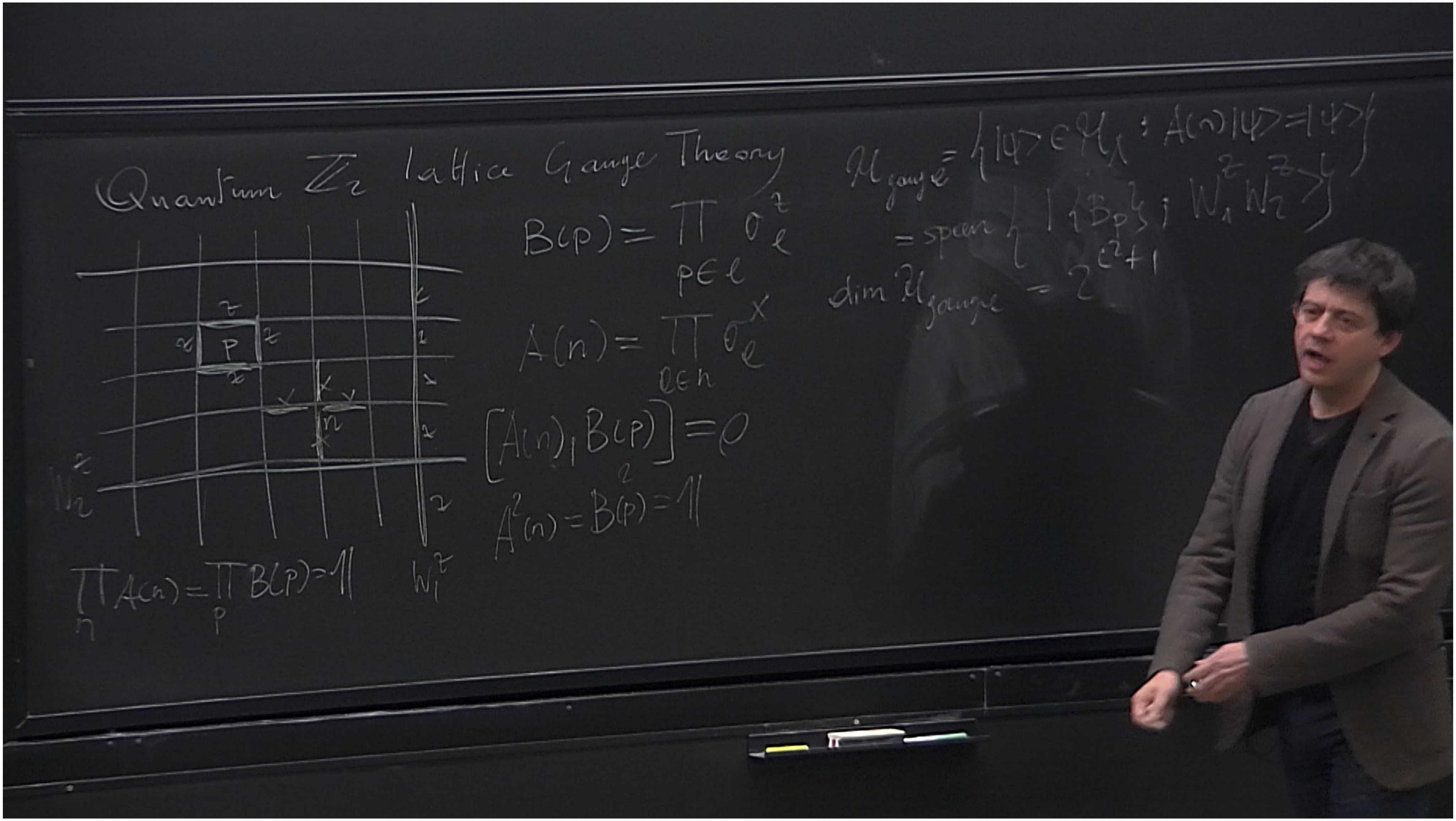
Title: PSI 17/18 - Condensed Matter - Lecture 9A Reservation

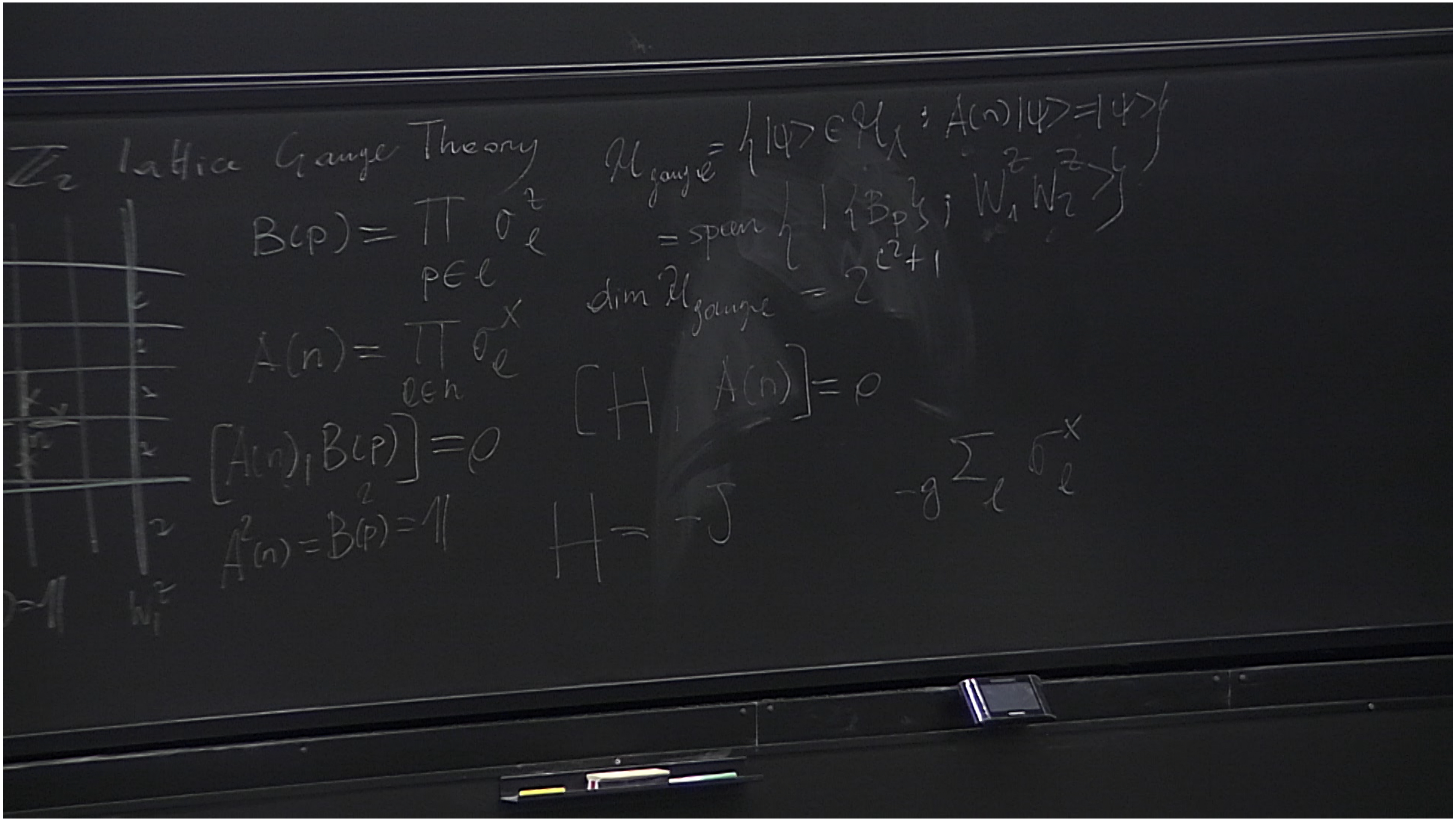
Date: Jan 15, 2018 03:45 PM

URL: http://pirsa.org/18010082

Abstract:


Quantum $\mathbb{Z}_2$ lattice Gauge Theory


Quantum Z2 lattice Gauge Theory
B(p) = ∏ σ^z_l, p∈l
A(n) = ∏ σ^x_l, l∈n
[A(n), B(p)] = 0
A²(n) = B²(p) = 1
∏_n A(n) = ∏_p B(p) = 1
W^z_1
W^z_2
H_gauge = {|ψ> ∈ H_1 : A(n)|ψ> = |ψ>}
= span{ |{B_p}; W^z_1 W^z_2> }
dim H_gauge = 2^(L²+1)


$\mathbb{Z}_2$ lattice Gauge Theory
$B(p) = \prod_{p \in \ell} \sigma^z_\ell$
$A(n) = \prod_{\ell \in n} \sigma^x_\ell$
$[A(n), B(p)] = 0$
$A^2(n) = B^2(p) = 1\!\!1$
$\mathcal{H}_{gauge} = \{ |\psi\rangle \in \mathcal{H}_\Lambda : A(n)|\psi\rangle = |\psi\rangle \}$
$= \text{span} \{ | \{B_p\} ; W^z_1 W^z_2 \rangle \}$
$\dim \mathcal{H}_{gauge} = 2^{L^2+1}$
$[H, A(n)] = 0$
$H = -J \qquad -g \sum_\ell \sigma^x_\ell$

Phases of $\mathbb{Z}_2$ Theory

$J=0$ Paramagnet $\langle \sigma^z \rangle = 0$

Unique GS + $\Delta = 2g$ $\quad \langle \sigma^z_i \sigma^z_j \rangle \sim e^{-|i-j|/\xi}$

$g=0$ $\quad H = -J \sum_p B_p$

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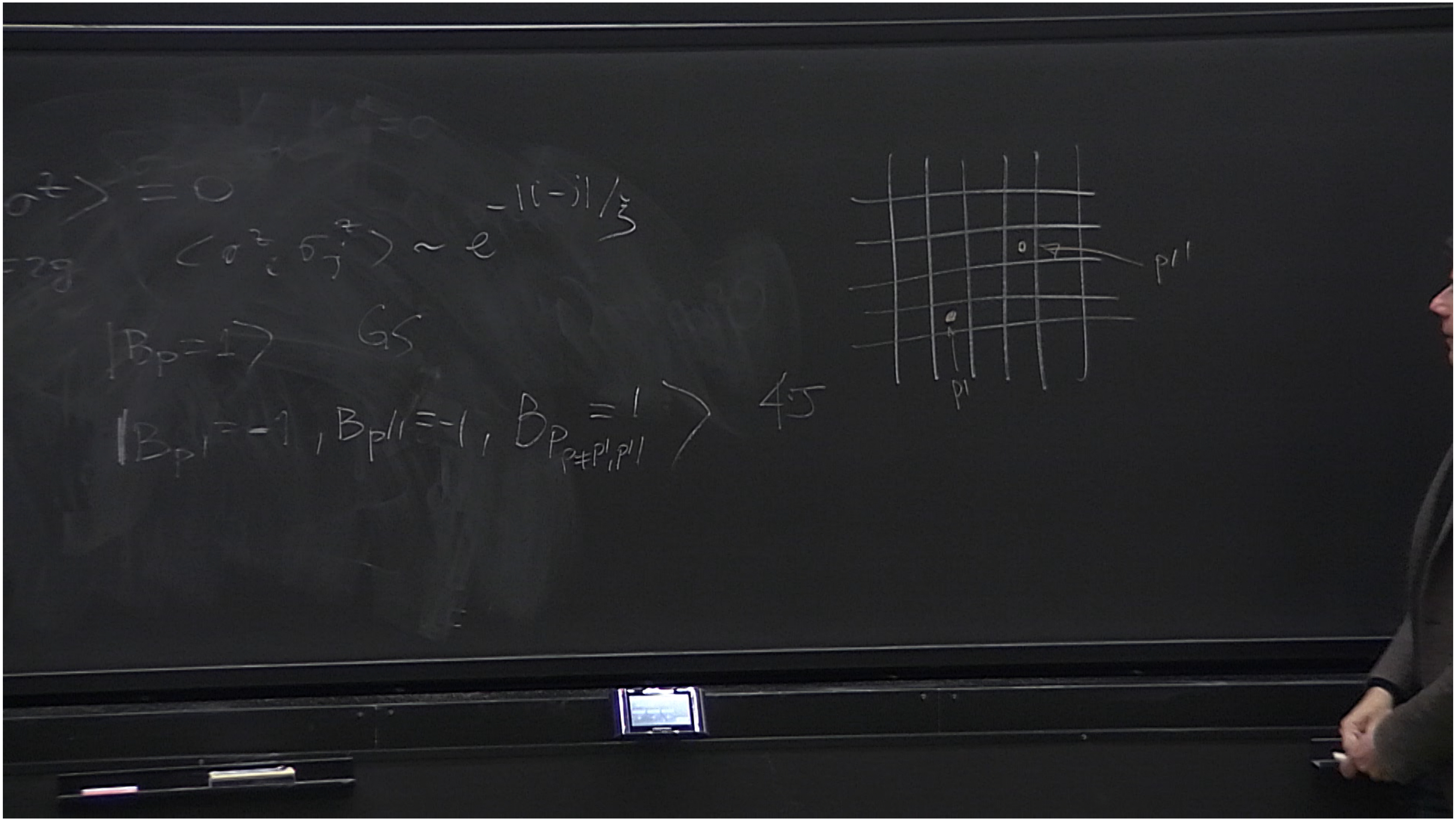
Unique GS + $\Delta = 2g$ $\quad \langle \sigma^z_i \sigma^z_j \rangle \sim e^{-|i-j|/\xi}$

$g=0$ $\quad H = -J\sum_p B_p$ $\quad |B_p = 1\rangle$ GS

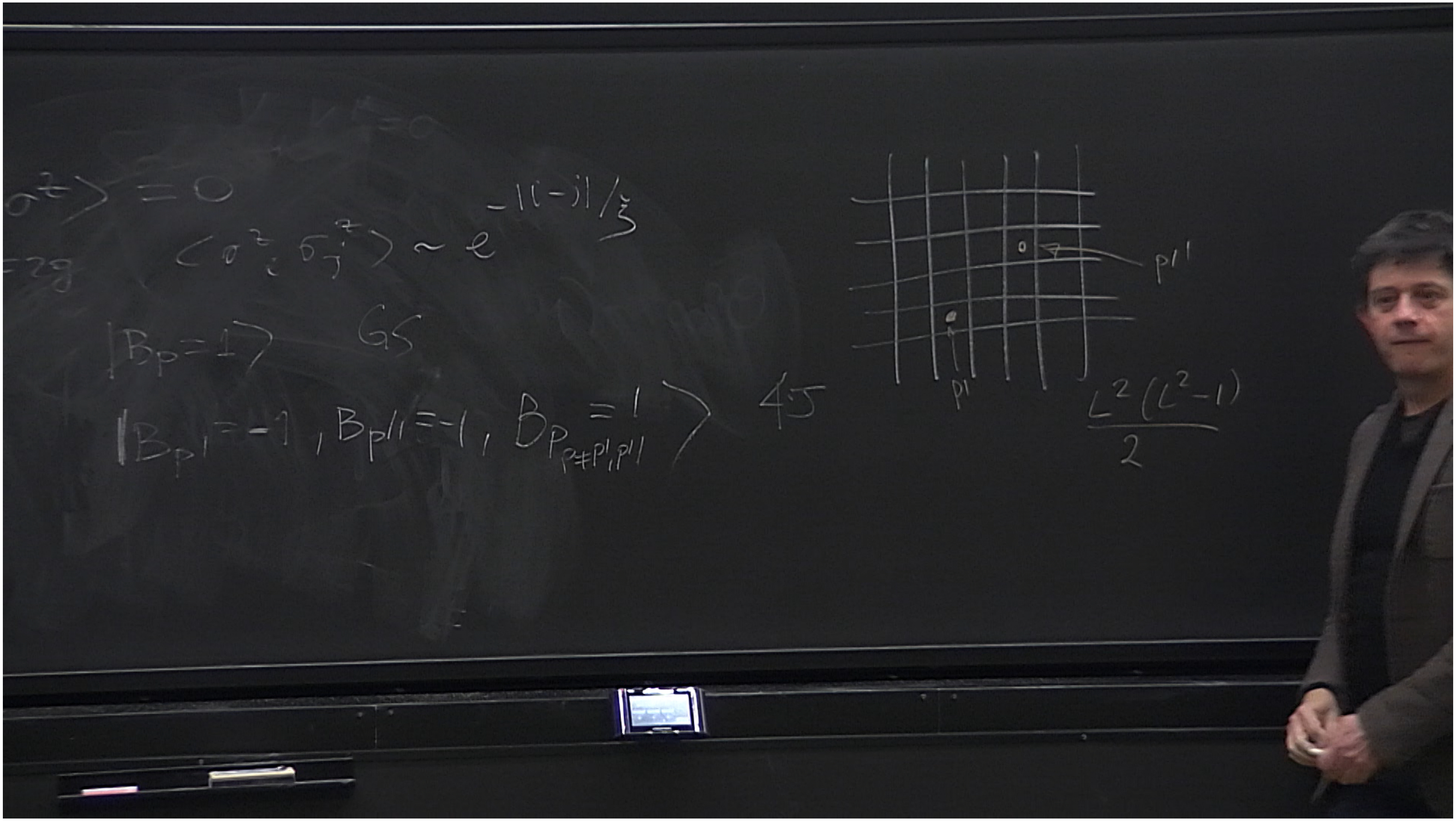
$|B_{p'} = -1, B_{p''} = -1, B_{p \neq p', p''} = 1\rangle$

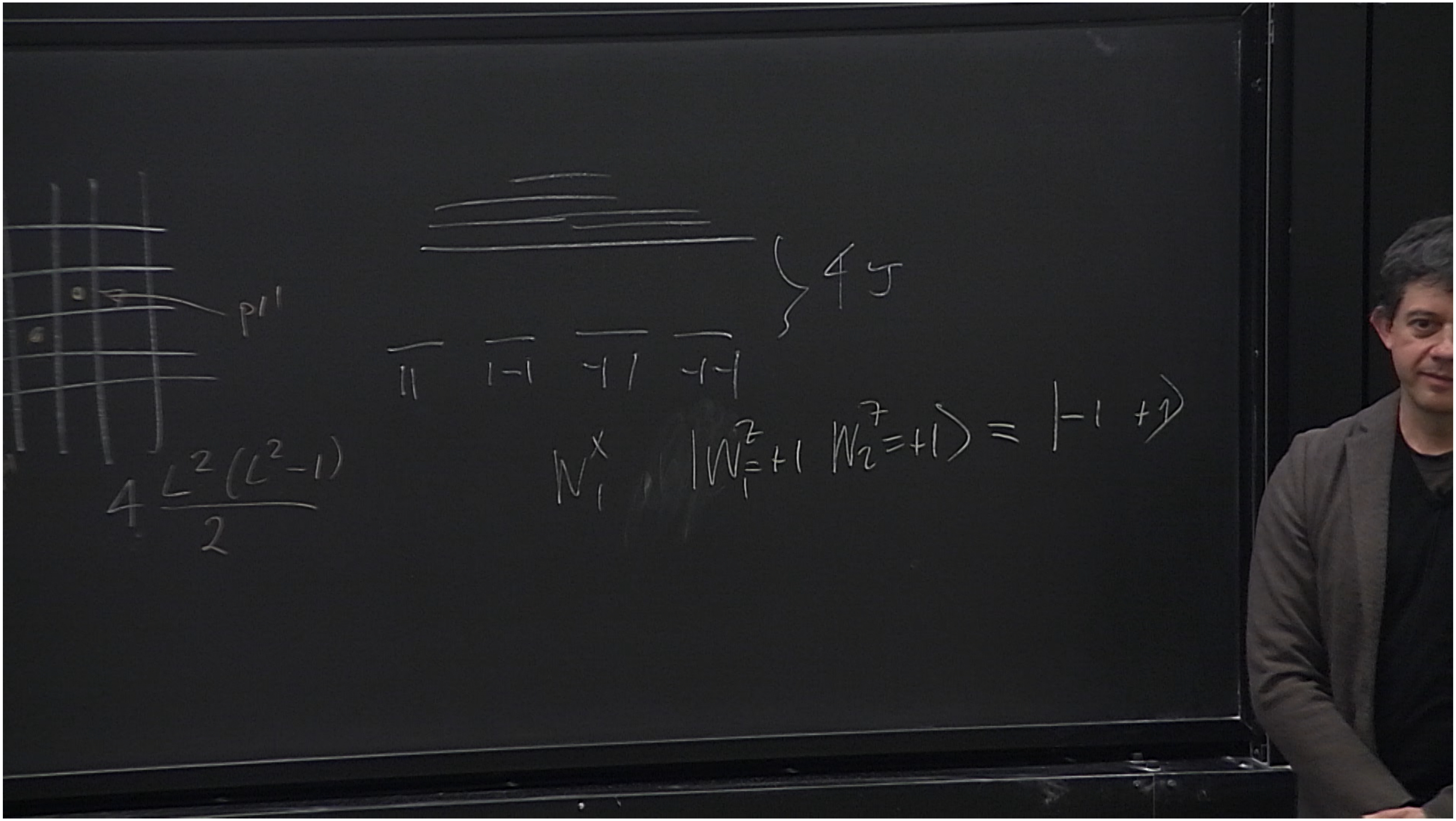
$B_p|\psi_0\rangle = |\psi_0\rangle$

$E_0 = -N^2 J$



$\sigma^z\rangle = 0$
$\langle \sigma^z_i \sigma^z_j \rangle \sim e^{-|i-j|/\xi}$
$|B_p = 1\rangle$ GS
$|B_{p'} = -1, B_{p''} = -1, B_{p_{p \neq p', p''}} = 1\rangle$ 4J
p''
p'


σ^z⟩ = 0
⟨σ^z_i σ^z_j⟩ ~ e^{-|i-j|/ξ}
|B_p = 1⟩ GS
|B_p' = -1, B_p'' = -1, B_p = 1 (p≠p',p'')⟩ 4J
p'
p''
L²(L²−1)/2



p'1
4J
11
1-1
-1 1
-1-1
4 L^2(L^2-1)/2
W_1^x |W_1^z=+1 W_2^z=+1⟩ = |-1 +1⟩

Symm. breaking ⇒ symm broken states

⇒ local operator $\langle \sigma^z \rangle \neq 0$

$$\langle \psi | O | \psi \rangle = \langle \psi | A(n)^\dagger$$

GS

Symm. breaking $\Rightarrow$ symm broken states

$\Rightarrow$ local operator $\langle \sigma^z \rangle \neq 0$

$$\langle \psi | O | \psi \rangle = \langle \psi | A^\dagger(n) \, O \, A(n) | \psi \rangle$$

GS

$$=$$

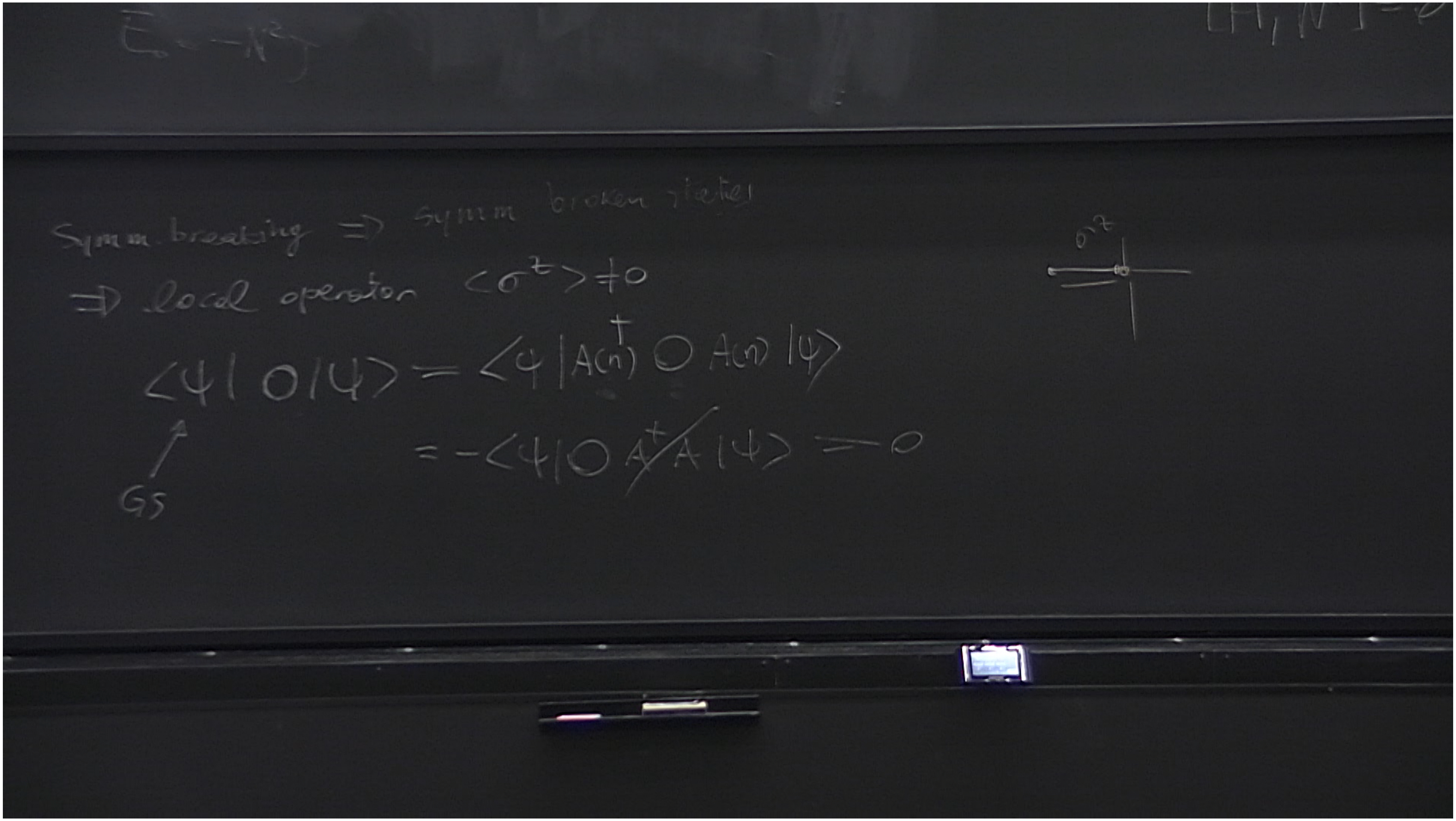
Symm. breaking $\Rightarrow$ symm broken states

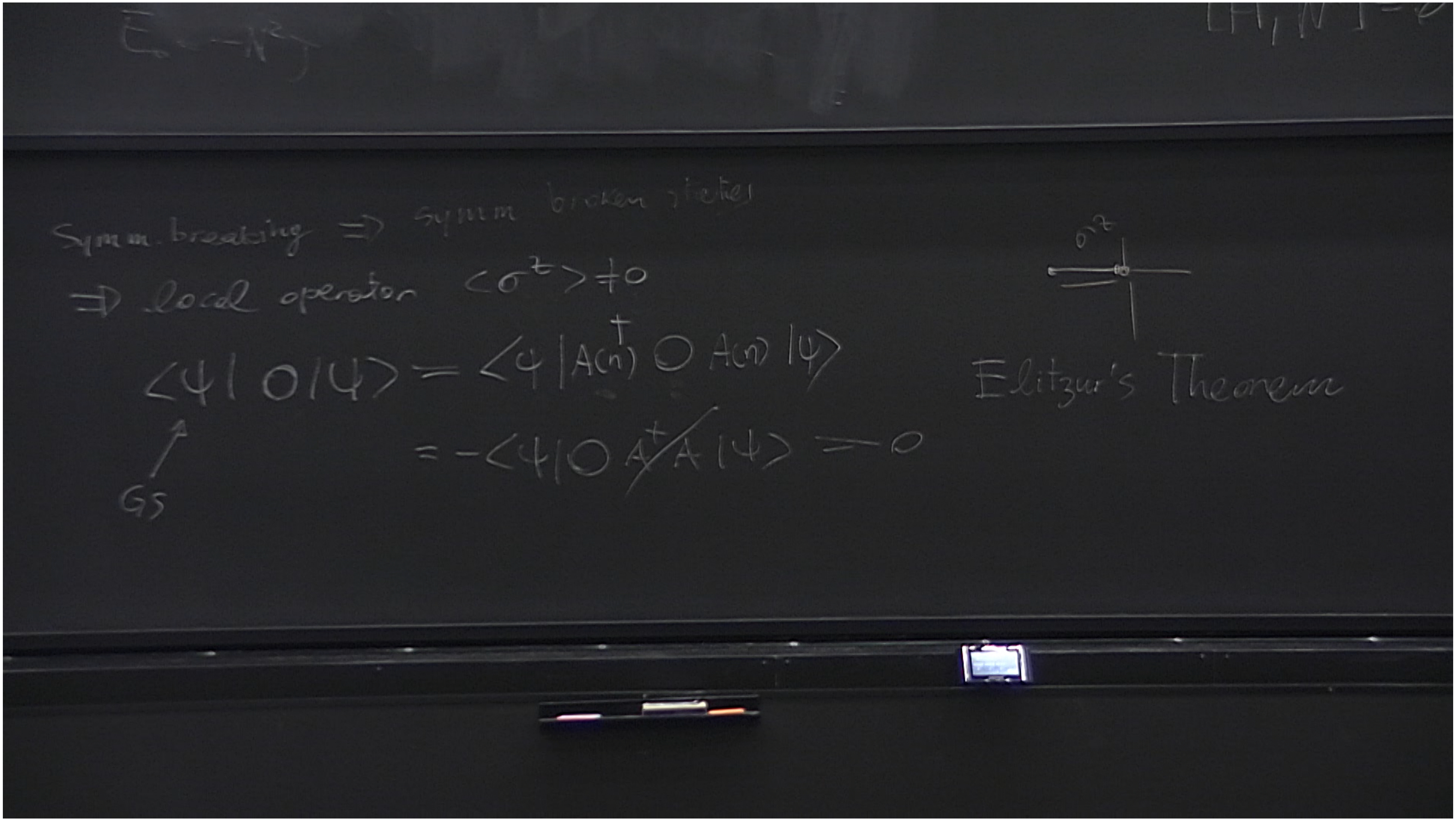
$\Rightarrow$ local operator $\langle \sigma^z \rangle \neq 0$

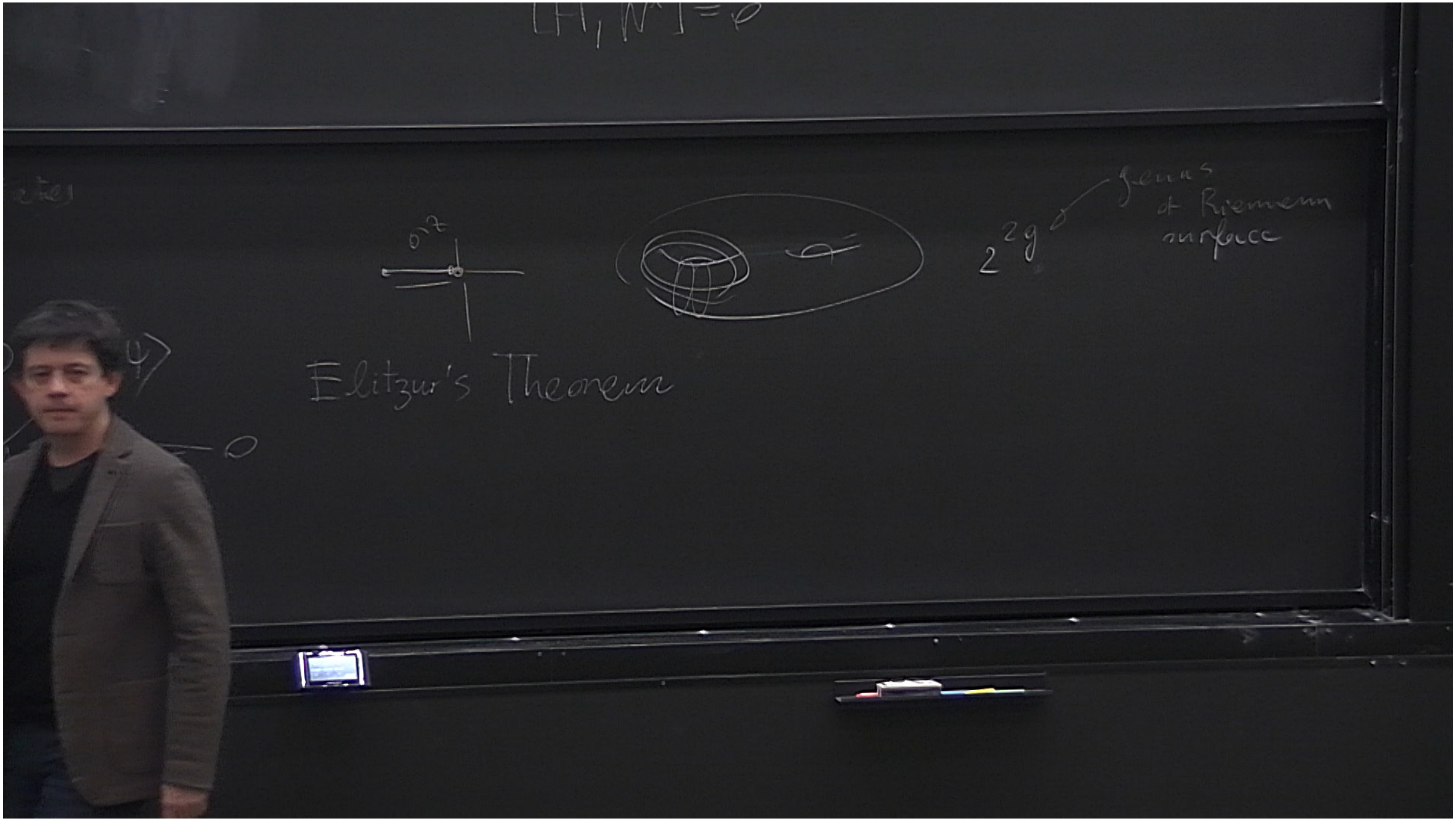
$$\langle \psi | O | \psi \rangle = \langle \psi | A(n)^\dagger \, O \, A(n) | \psi \rangle$$

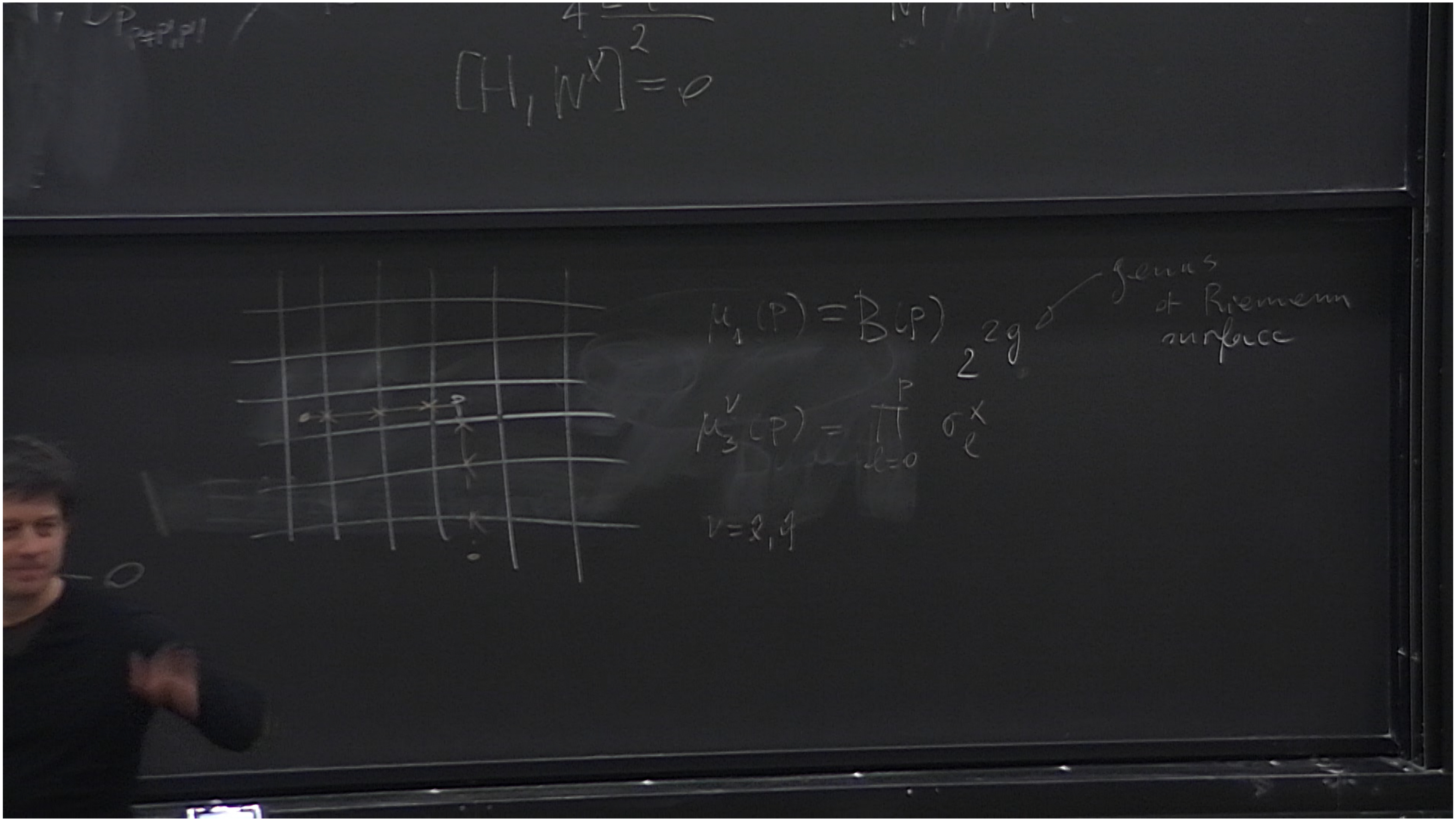
$$= - \langle \psi | O \, A^\dagger A | \psi \rangle$$

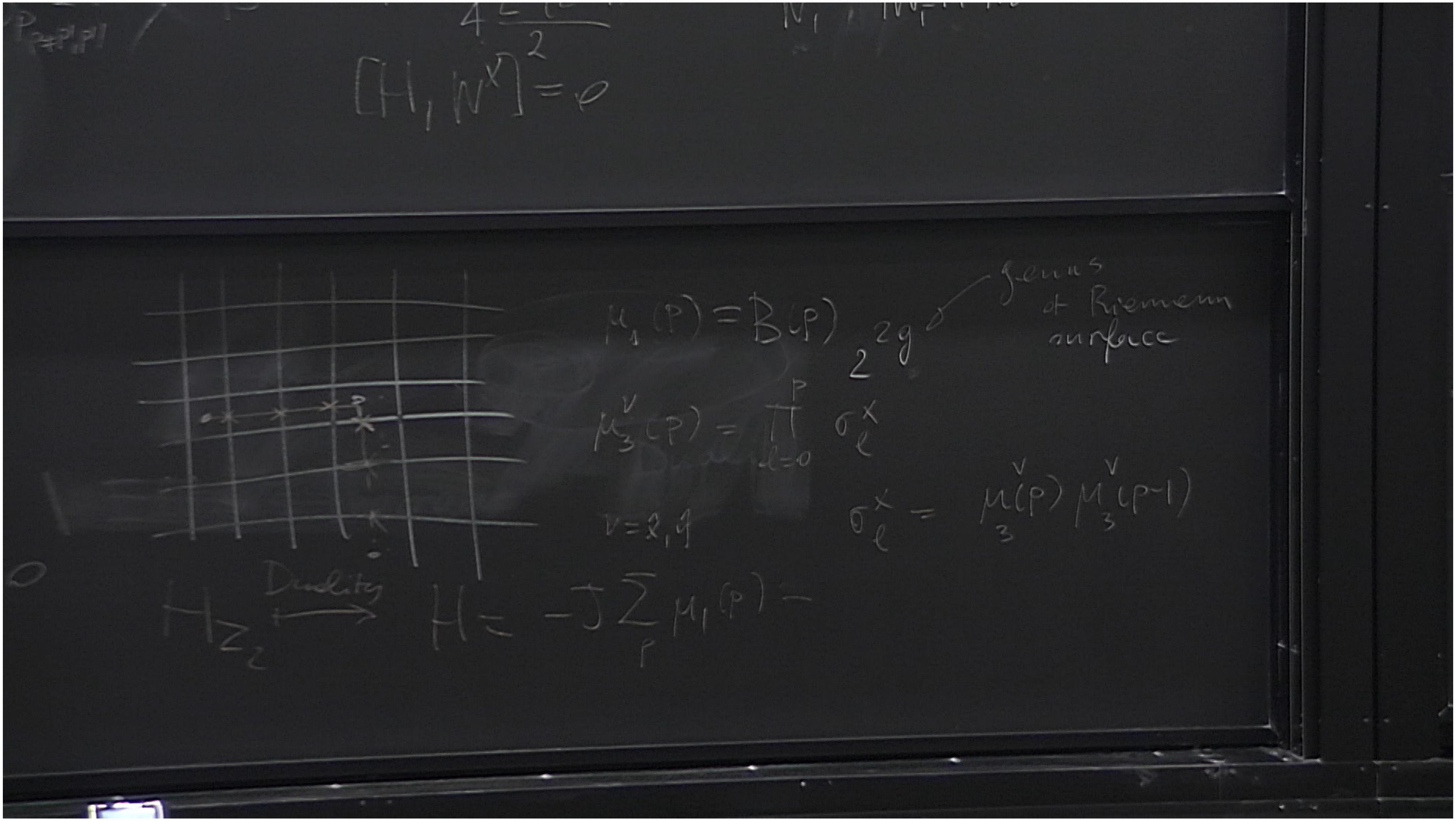
GS

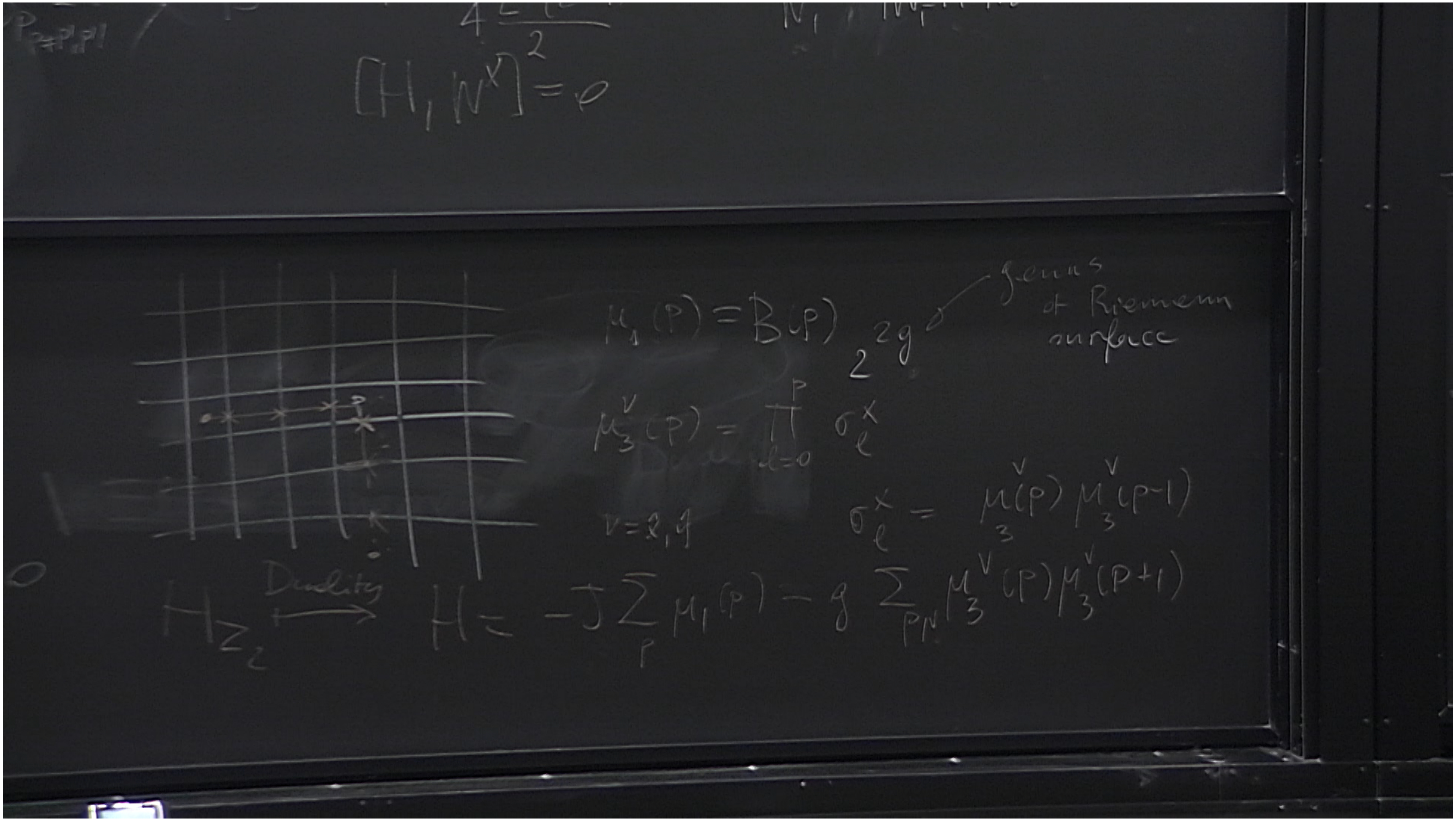

E0 = -N^2 J
[H, N] = 0
Symm. breaking ⇒ symm broken states
⇒ local operator ⟨σ^z⟩ ≠ 0
⟨ψ|O|ψ⟩ = ⟨ψ|A(n)† O A(n)|ψ⟩
= -⟨ψ|O A† A|ψ⟩ = 0
GS
σ^z

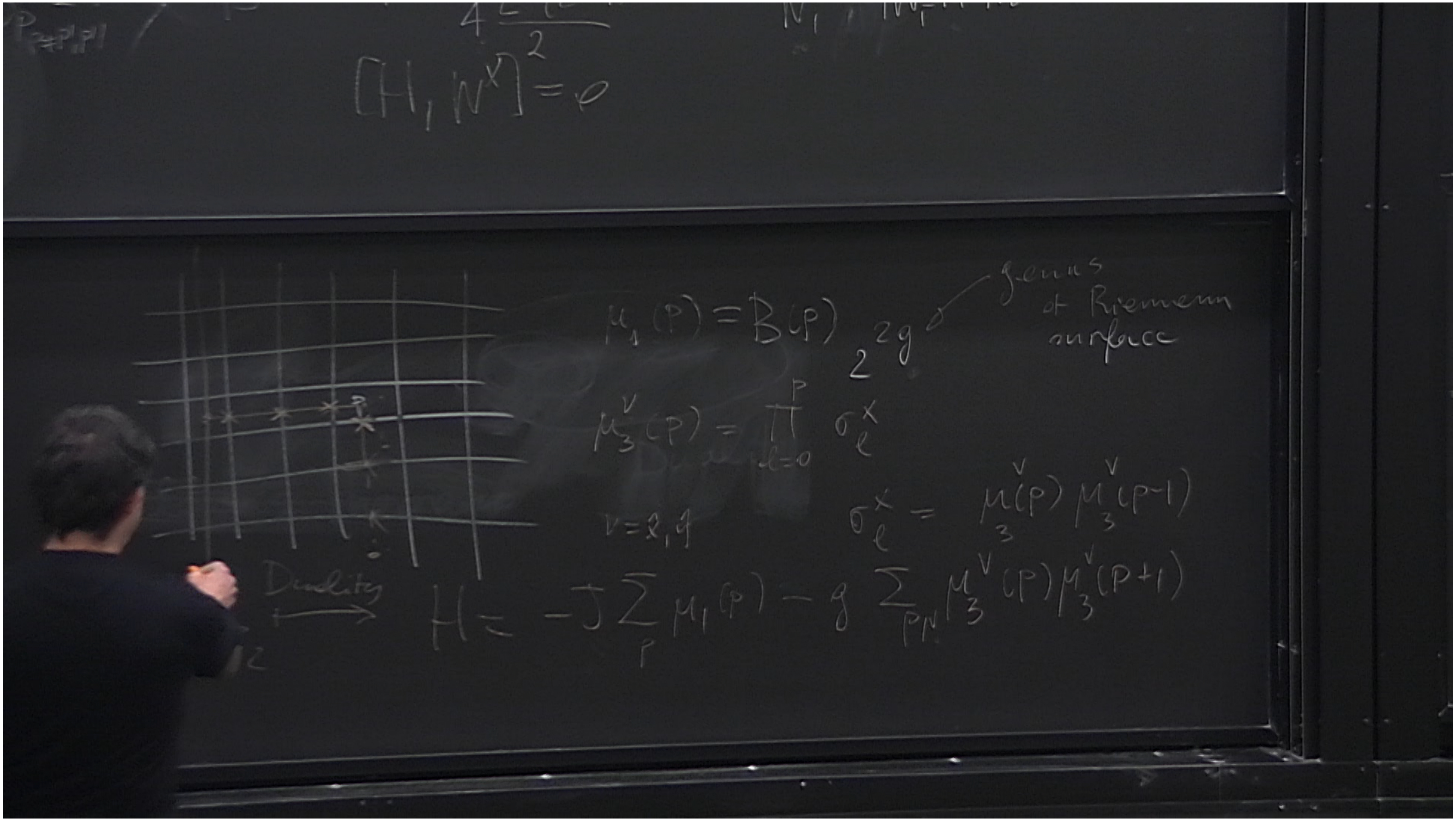

Symm. breaking ⇒ symm broken states
⇒ local operator $\langle \sigma^z \rangle \neq 0$
$\langle \psi | O | \psi \rangle = \langle \psi | A(n)^\dagger O A(n) | \psi \rangle$
$= -\langle \psi | O \cancel{A^\dagger A} | \psi \rangle = 0$
GS
σ^z
Elitzur's Theorem


Elitzur's Theorem
2^{2g}
genus
of Riemann
surface


[H, N^x] = 0
μ_1(P) = B(P)
2^2g
genus of Riemann surface
μ_3^ν(P) = ∏_{l=0}^{P} σ_l^x
ν = 2, 4


[H, N^x]^2 = 0
μ_1(p) = B(p)
2^{2g}
genus of Riemann surface
μ_3^ν(p) = ∏_{l=0}^{p} σ_l^x
ν = 2, 4
σ_l^x = μ_3^ν(p) μ_3^ν(p-1)
H_{Z_2} —Duality→ H = -J Σ_p μ_1(p) -


[H, N^x] = 0
μ_1(p) = B(p)
2^{2g}
genus of Riemann surface
μ_3^v(p) = ∏_{l=0}^{p} σ_l^x
v = x, y
σ_l^x = μ_3^v(p) μ_3^v(p-1)
H_{Z_2}
Duality
H = -J Σ_p μ_1(p) - g Σ_{p,v} μ_3^v(p) μ_3^v(p+1)


[H, N^x] = 0
μ1(P) = B(P)
2^2g
genus of Riemann surface
μ3^v(P) = ∏ σl^x, l=0 … P
v = x, y
σl^x = μ3^v(P) μ3^v(P-1)
Duality
H = -J Σp μ1(p) - g Σp,v μ3^v(p) μ3^v(p+1)

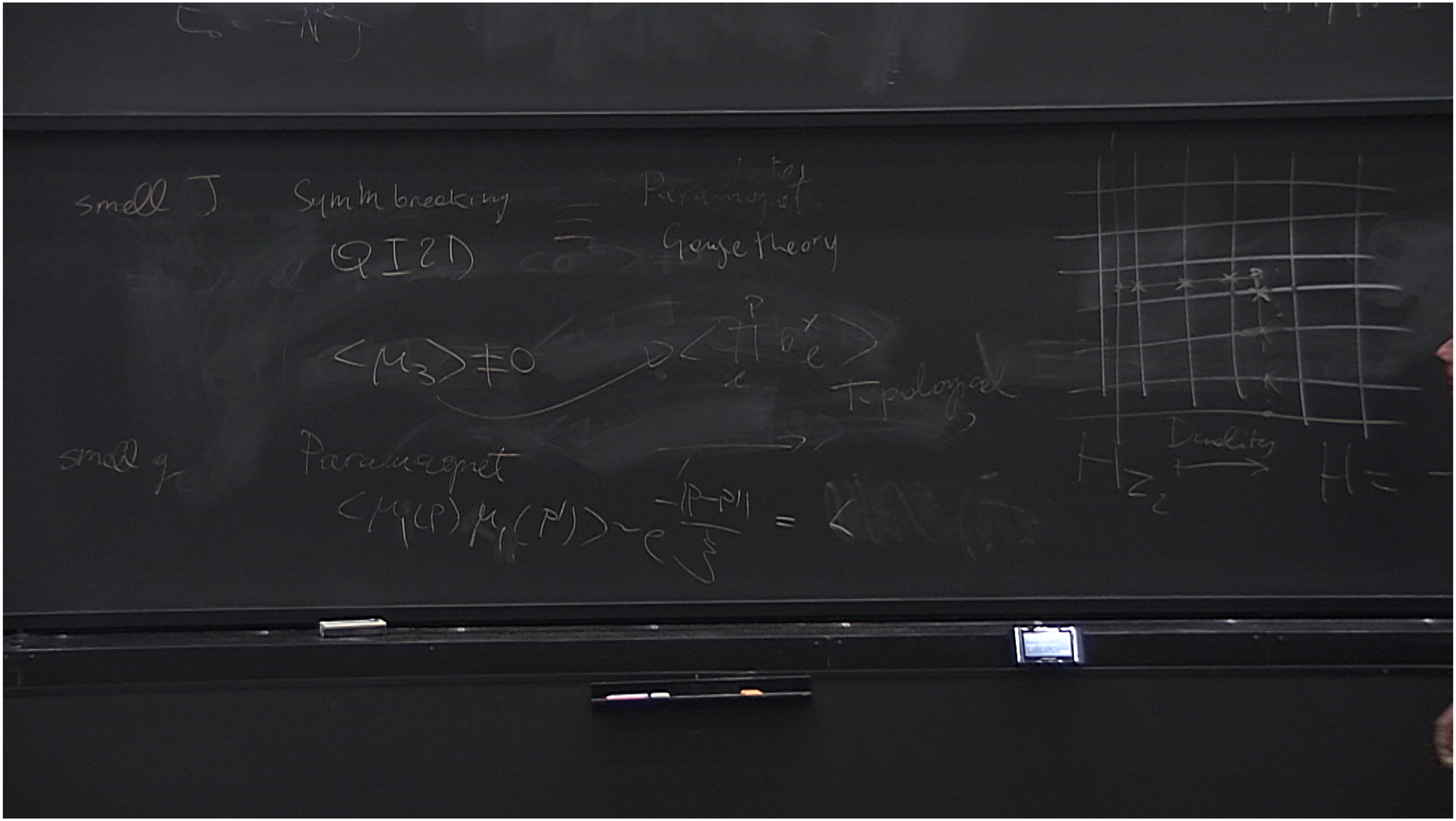
small J Symm breaking = Paramagnet

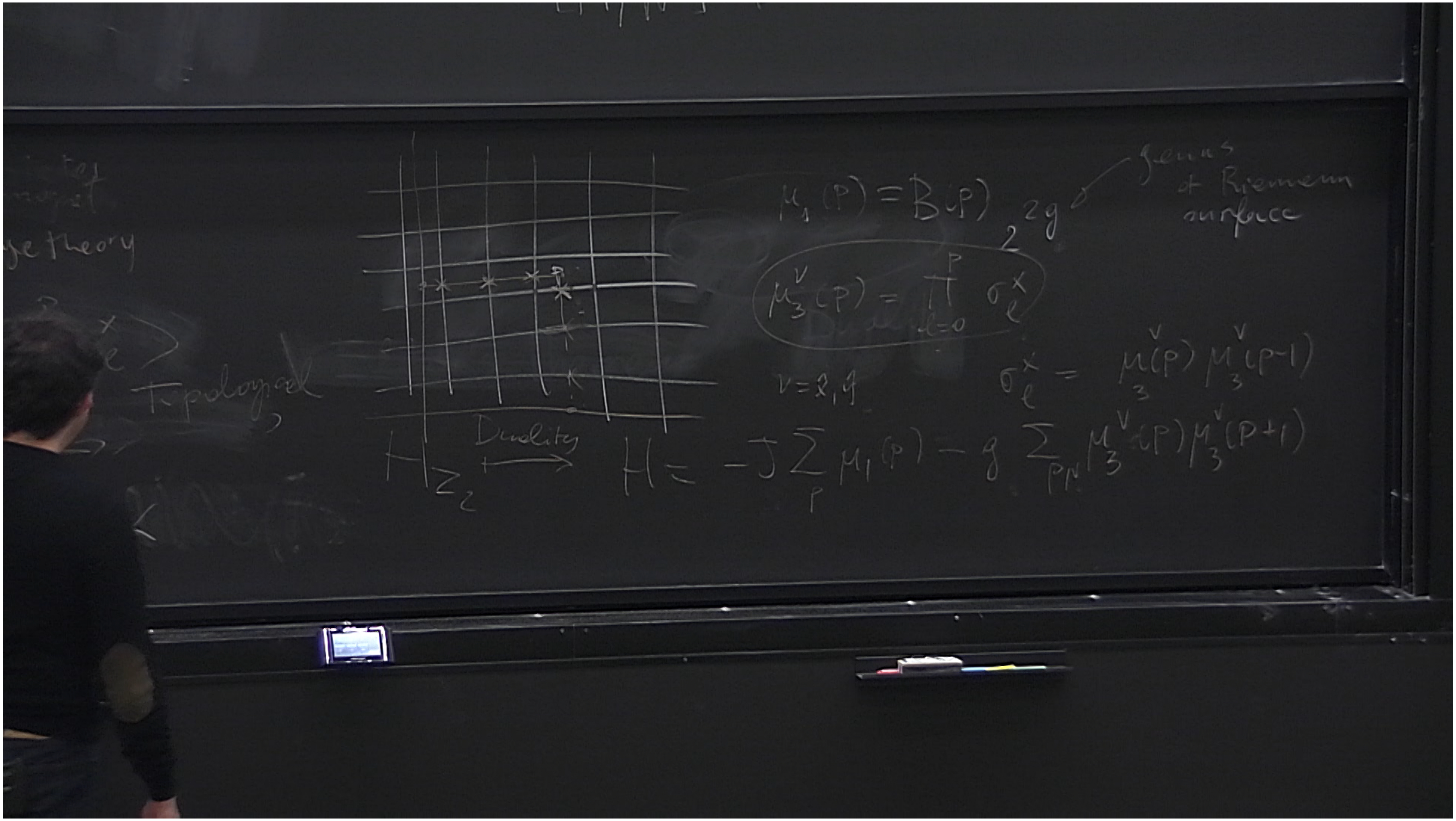
(QI2D) = Gauge theory

$$\langle \mu_3 \rangle \neq 0 \qquad \left\langle \prod_{l}^{P} \sigma^x_l \right\rangle$$

small g Paramagnet

$$\langle \mu_3(P)\, \mu_3(P') \rangle \sim e^{-\frac{|P-P'|}{\xi}}$$


small J
Symm breaking
(QI2D)
Paramagnet
Gauge theory
<μ3> ≠ 0
Topological
small g
Paramagnet
Duality


genus of Riemann surface
$\mu_1(p) = B(p)$
2^{2g}
$\mu_3^v(p) = \prod_{l=0}^{p} \sigma_l^x$
$\sigma_l^x = \mu_3^v(p)\,\mu_3^v(p-1)$
Duality
$H = -J\sum_p \mu_1(p) - g \sum_{p} \mu_3^v(p)\,\mu_3^v(p+1)$
Topological
ge theory