Title: PSI 17/18 - Condensed Matter - Lecture 5A Reservation

Date: Jan 09, 2018 03:45 PM

URL: http://pirsa.org/18010081

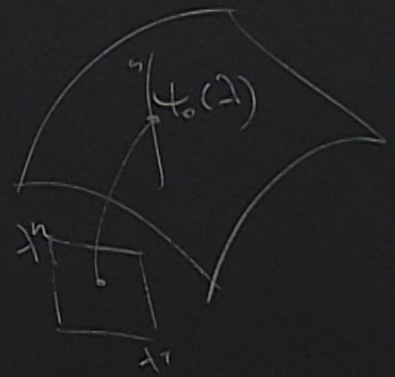
Abstract:

Differential Geometry of Quantum Phases

$H = H(\lambda)$ $\quad \lambda = (\lambda^1, \dots, \lambda^n) \in \mathbb{R}^n$

$\lambda \xrightarrow{GS} \psi_0(\lambda)$

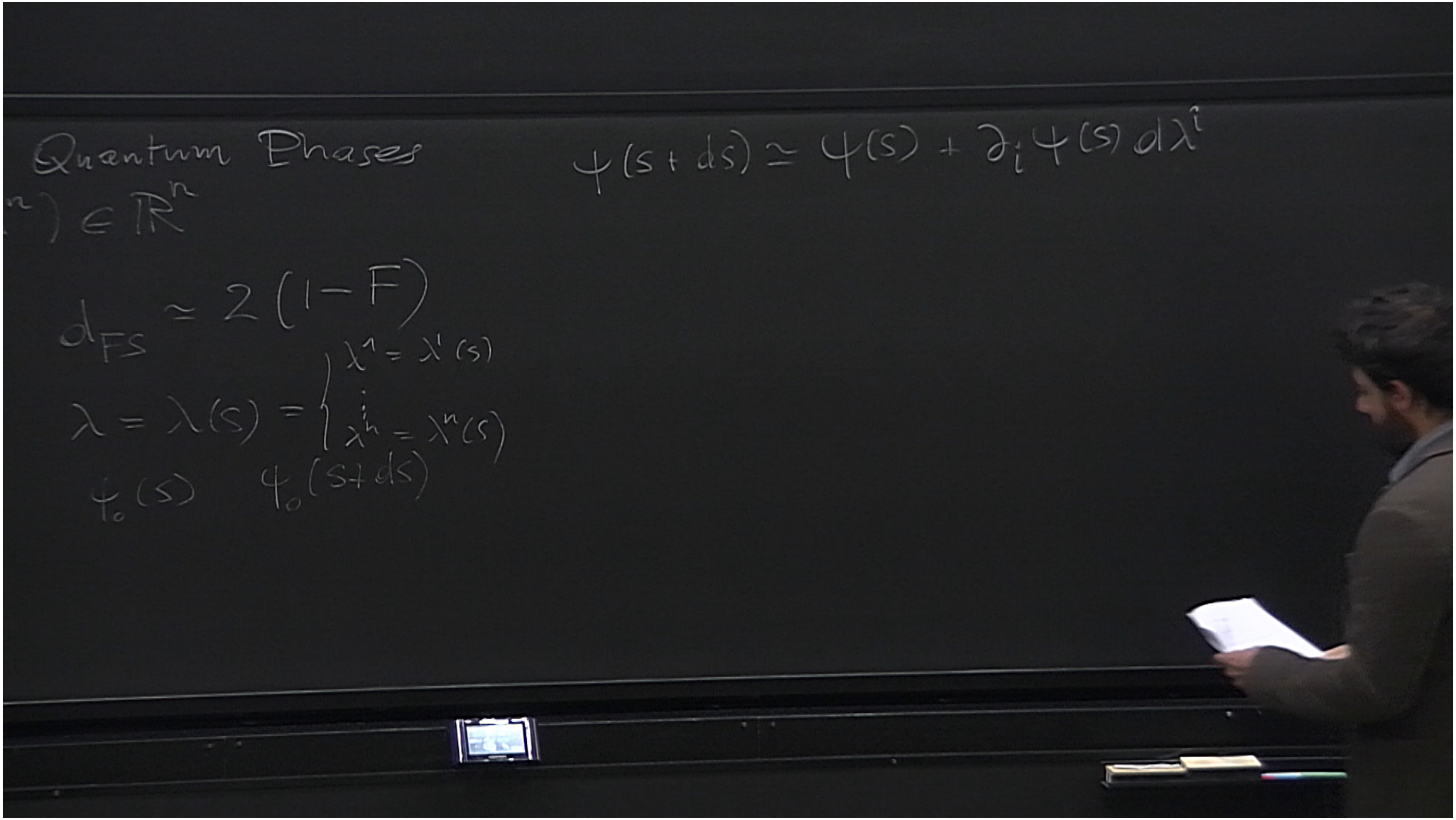
$F = |\langle \psi' | \psi^2 \rangle|$



$d_{FS} \approx 2(1-F)$

$\lambda = \lambda(s)$

$\psi_0(s)$ $\quad \psi_0(s+ds)$


Quantum Phases
$\psi(s+ds) \simeq \psi(s) + \partial_i \psi(s)\, d\lambda^i$
$\in \mathbb{R}^n$
$d_{FS} \simeq 2(1-F)$
$\lambda = \lambda(s) = \begin{cases} \lambda^1 = \lambda^1(s) \\ \vdots \\ \lambda^n = \lambda^n(s) \end{cases}$
$\psi_0(s)$ $\psi_0(s+ds)$

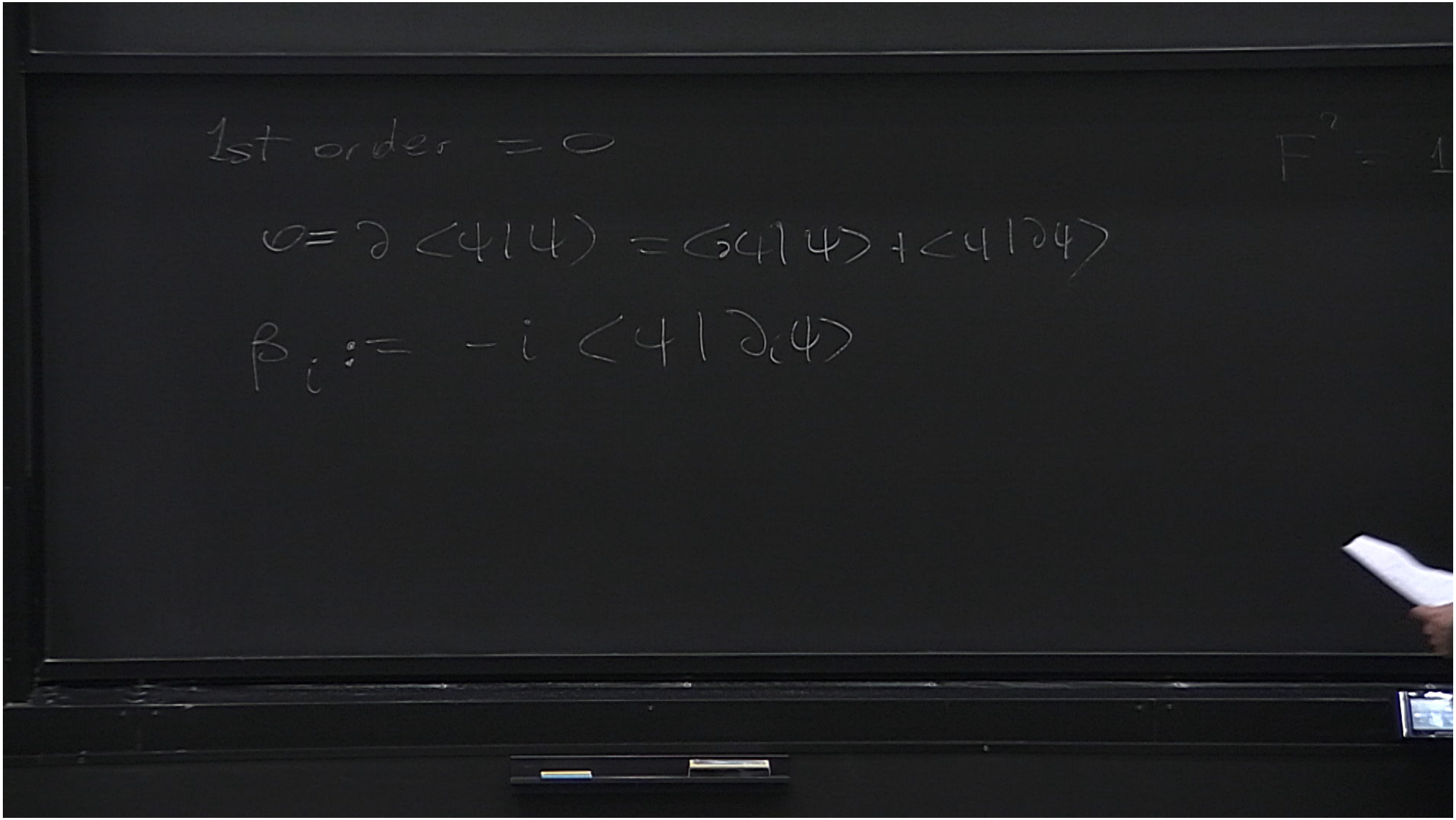
$$\psi(s+ds) \simeq \psi(s) + \partial_i \psi(s)\, dx^i + \frac{1}{2}\partial_i\partial_j \psi(s)\, dx^i dx^j + \cdots$$

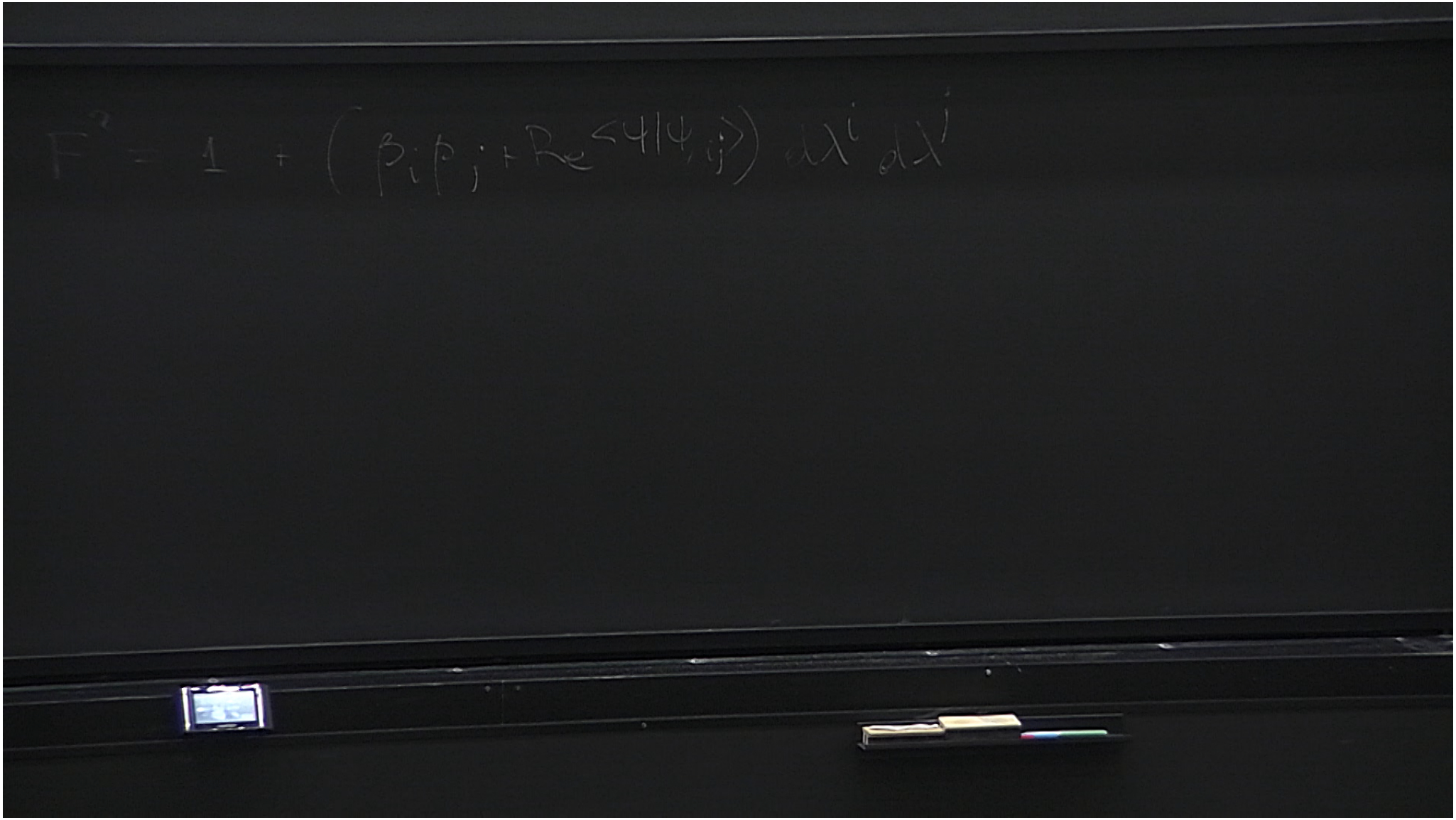
$$F^2 = \langle \psi(s) | \psi(s+ds) \rangle \langle \psi(s+ds) | \psi(s) \rangle .$$

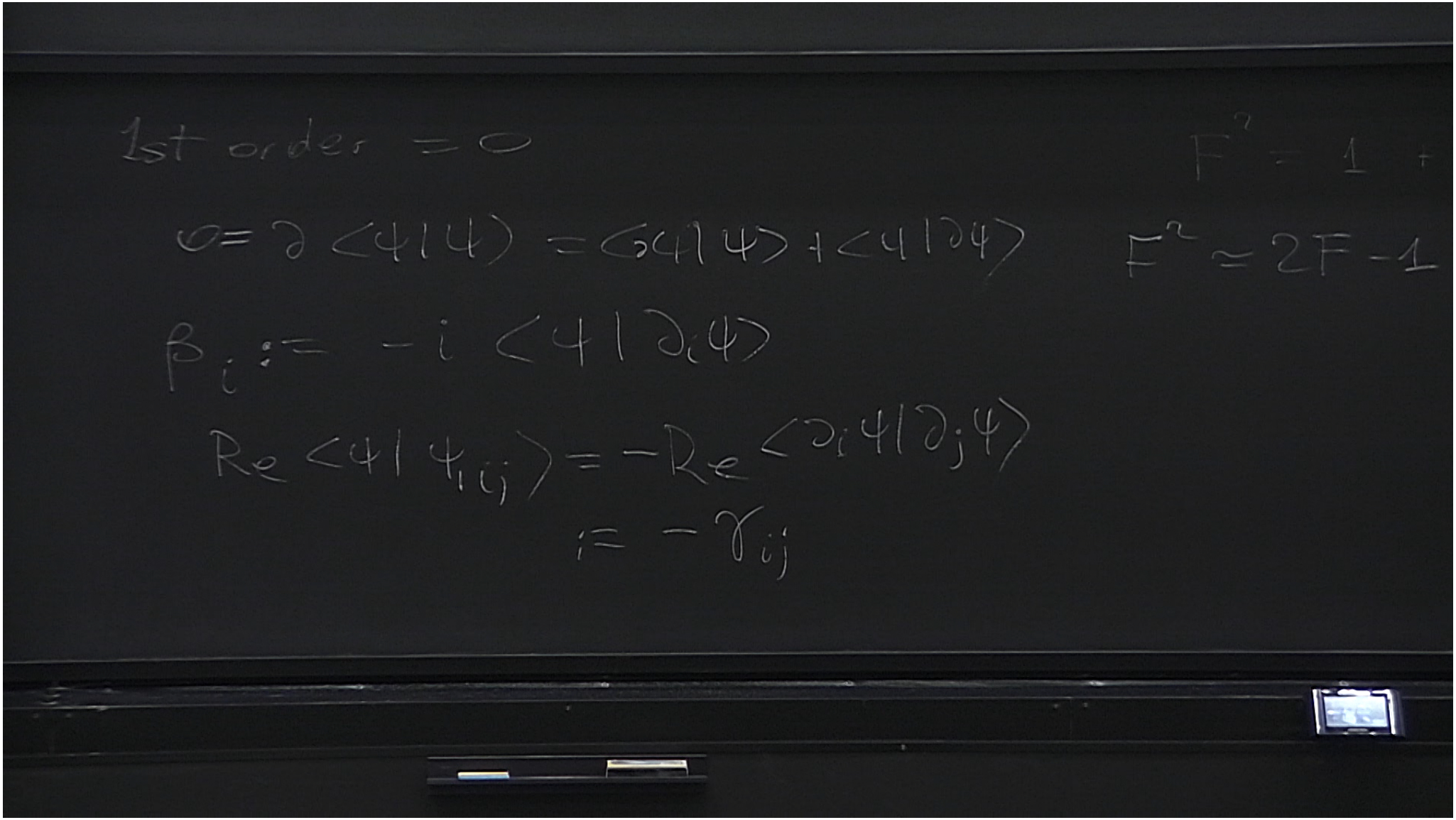
$$= 1 + \sum_i \left(\langle \partial_i \psi | \psi \rangle + \langle \psi | \partial_i \psi \rangle \right) dx^i$$

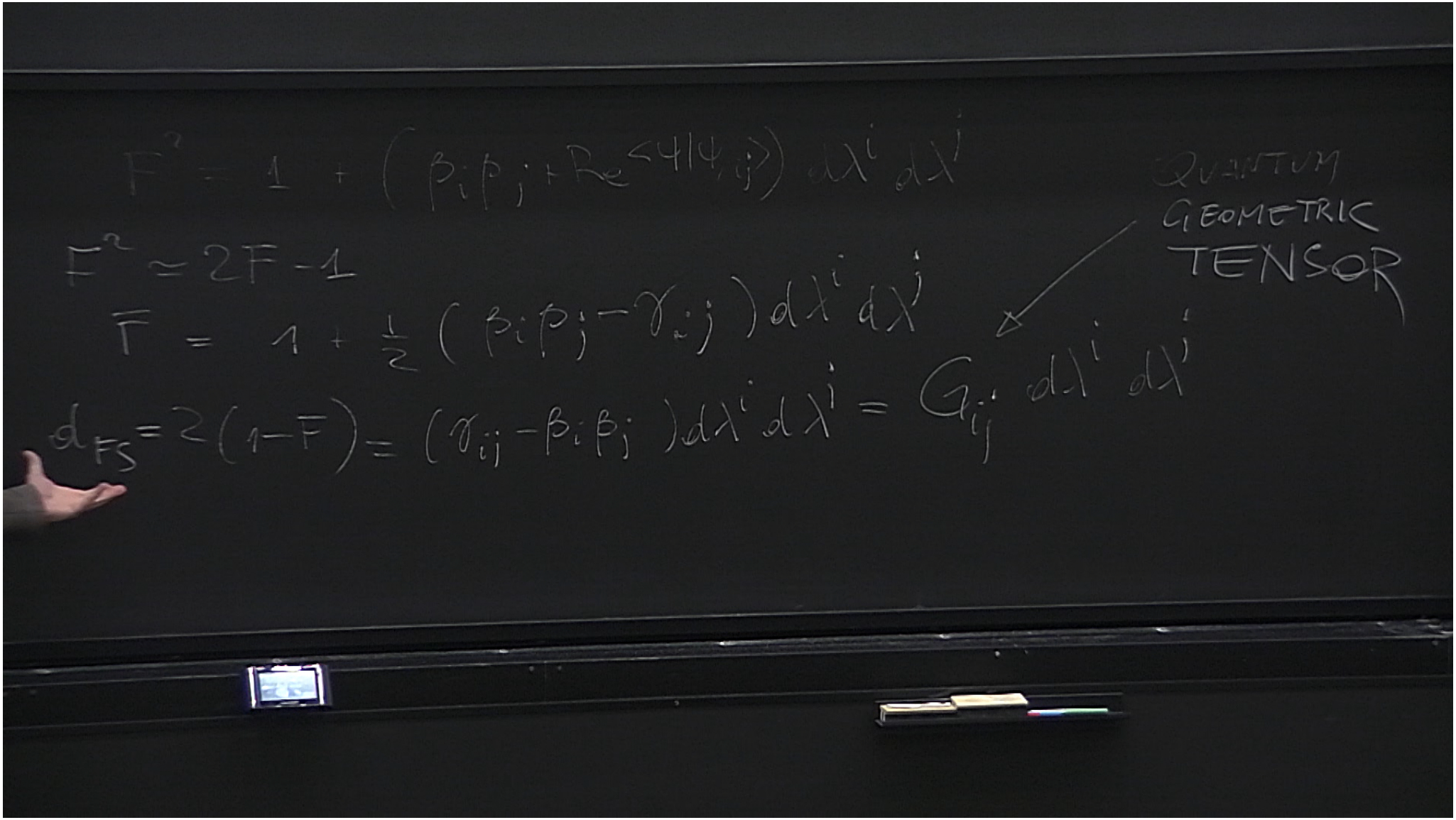
$$+ \langle \psi | \partial_i \psi \rangle \langle \partial_j \psi | \psi \rangle \, dx^i dx^j +$$

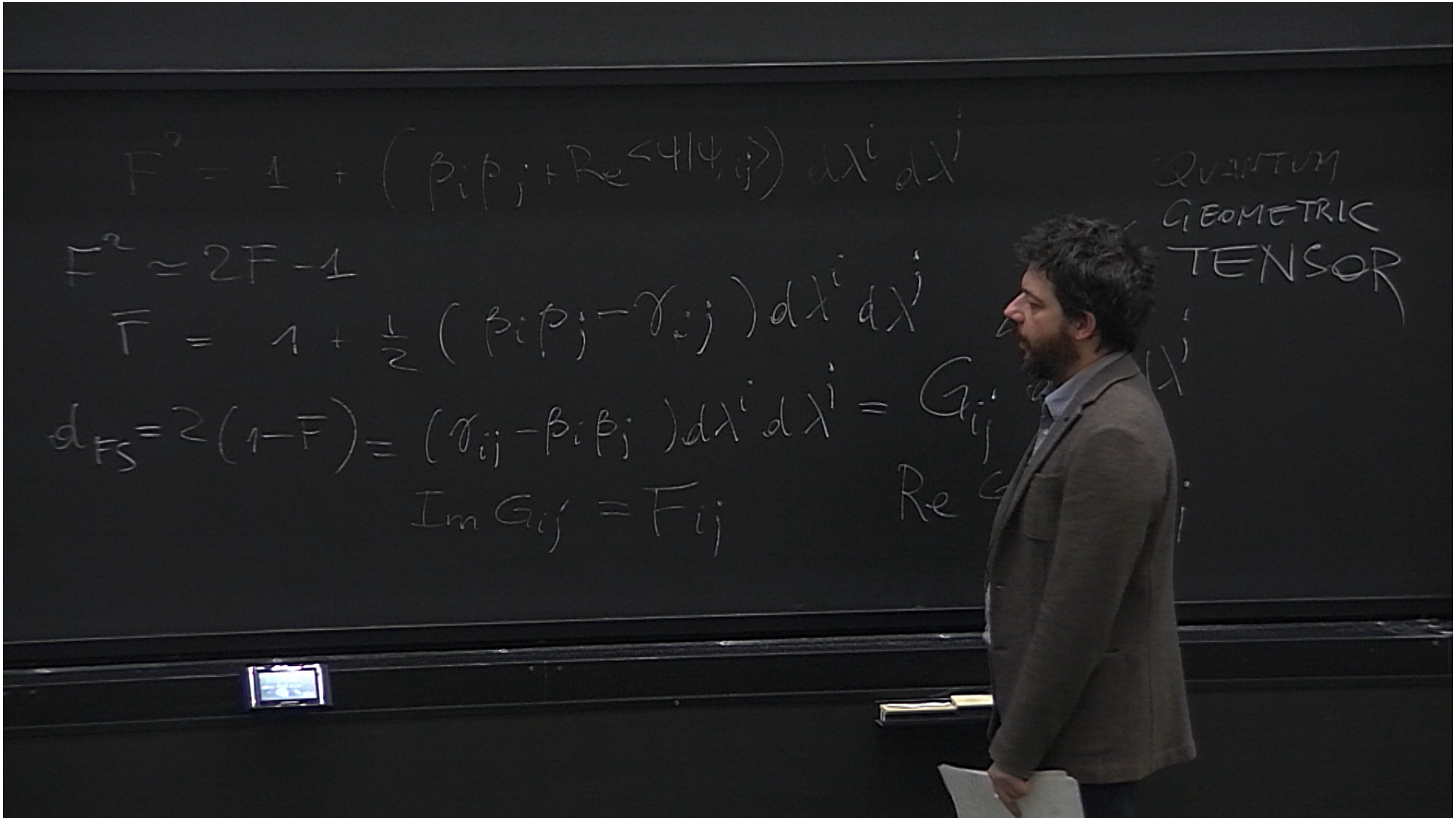
$$\frac{1}{2}\left(\langle \partial_i^2 \psi | \psi \rangle + \langle \psi | \partial_i^2 \psi \rangle \right) dx^i dx^j + \cdots$$

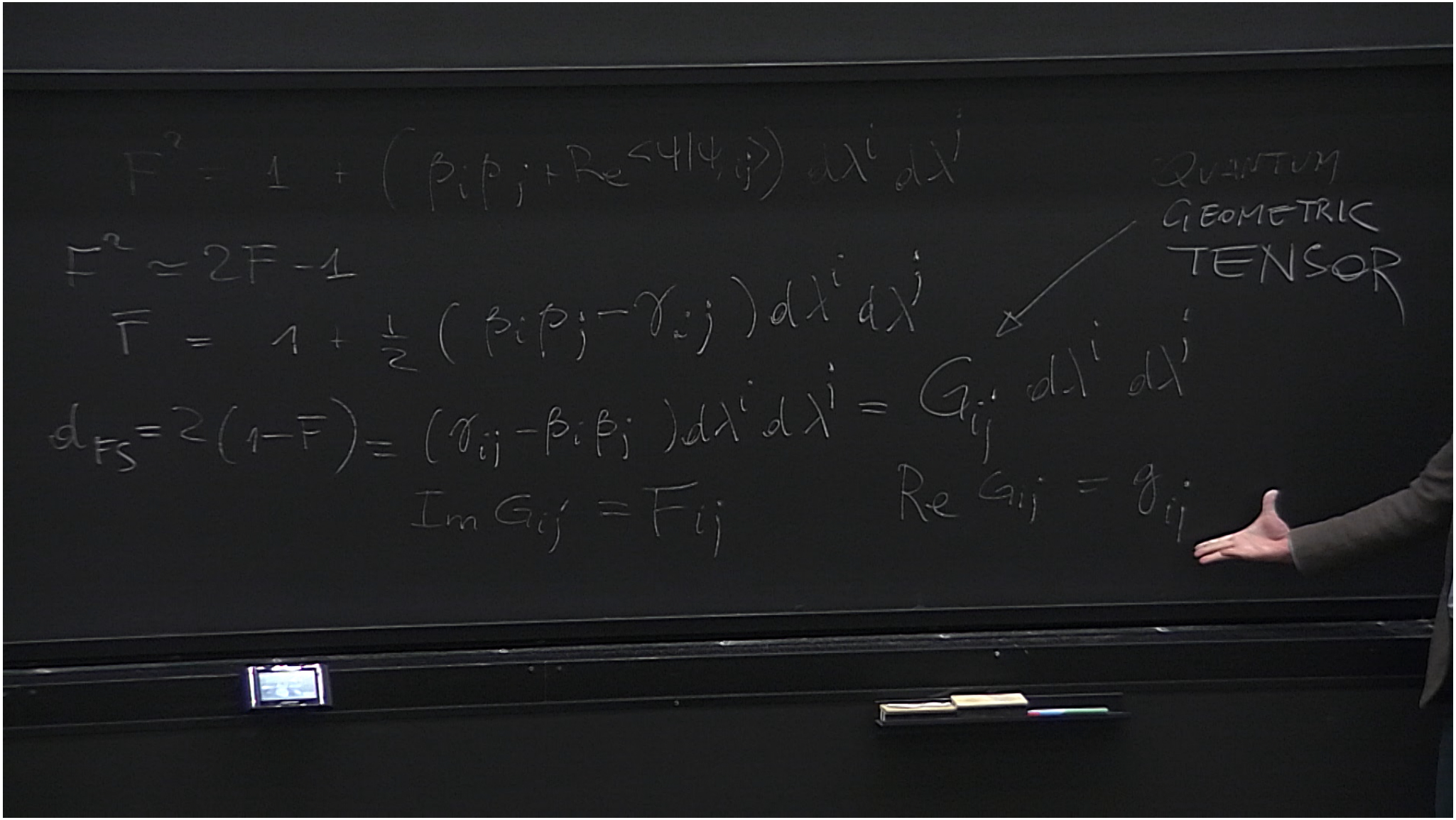

1st order = 0
$0 = \partial \langle 4 | 4 \rangle = \langle \partial 4 | 4 \rangle + \langle 4 | \partial 4 \rangle$
$p_i := -i \langle 4 | \partial_i 4 \rangle$

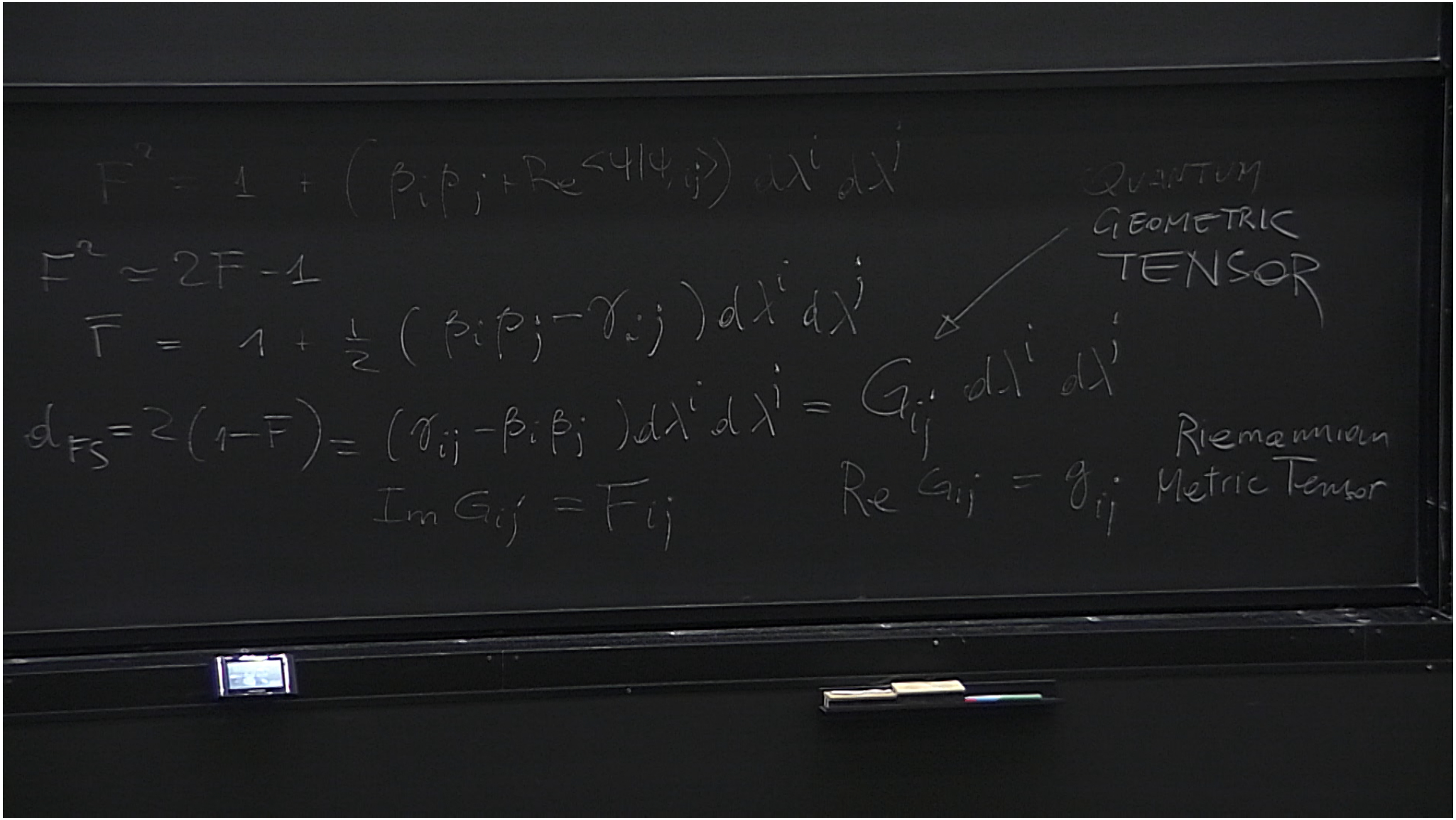

$F^2 = 1 + (\beta_i \beta_j + Re\langle \psi | \psi_{;ij} \rangle) dx^i dx^j$


1st order = 0
$0 = \partial \langle \psi | \psi \rangle = \langle \partial\psi | \psi \rangle + \langle \psi | \partial\psi \rangle$
$\beta_i := -i \langle \psi | \partial_i \psi \rangle$
$\mathrm{Re} \langle \psi | \psi_{ij} \rangle = -\mathrm{Re} \langle \partial_i \psi | \partial_j \psi \rangle$
$:= -\gamma_{ij}$
$F^2 = 1 +$
$F^2 \approx 2F - 1$


$F^2 = 1 + (\beta_i\beta_j + \mathrm{Re}\langle \partial_i\psi, \partial_j\psi\rangle)\, d\lambda^i d\lambda^j$
$F^2 \approx 2F - 1$
$F = 1 + \frac{1}{2}(\beta_i\beta_j - \gamma_{ij})\, d\lambda^i d\lambda^j$
$d_{FS} = 2(1-F) = (\gamma_{ij} - \beta_i\beta_j)\, d\lambda^i d\lambda^j = G_{ij}\, d\lambda^i d\lambda^j$
QUANTUM GEOMETRIC TENSOR


$F^2 = 1 + (\beta_i\beta_j + \mathrm{Re}\langle \partial_i \psi | \partial_j \psi \rangle)\, d\lambda^i d\lambda^j$
$F^2 \simeq 2F - 1$
$F = 1 + \frac{1}{2}(\beta_i\beta_j - \gamma_{ij})\, d\lambda^i d\lambda^j$
$d_{FS} = 2(1-F) = (\gamma_{ij} - \beta_i\beta_j)\, d\lambda^i d\lambda^j = G_{ij}$
$\mathrm{Im}\, G_{ij} = F_{ij}$
QUANTUM
GEOMETRIC
TENSOR


$F^2 = 1 + (\beta_i\beta_j + \mathrm{Re}\langle \partial_i \psi | \partial_j \psi \rangle)\, d\lambda^i d\lambda^j$
$F^2 \approx 2F - 1$
$F = 1 + \frac{1}{2}(\beta_i\beta_j - \gamma_{ij})\, d\lambda^i d\lambda^j$
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QUANTUM GEOMETRIC TENSOR
$\mathrm{Im}\, G_{ij} = F_{ij}$
$\mathrm{Re}\, G_{ij} = g_{ij}$


$F^2 = 1 + (\beta_i\beta_j + \mathrm{Re}\langle \partial_i\psi | \partial_j\psi\rangle)\, d\lambda^i d\lambda^j$
$F^2 \approx 2F - 1$
$F = 1 + \frac{1}{2}(\beta_i\beta_j - \gamma_{ij})\, d\lambda^i d\lambda^j$
$d_{FS} = 2(1-F) = (\gamma_{ij} - \beta_i\beta_j)\, d\lambda^i d\lambda^j = G_{ij}\, d\lambda^i d\lambda^j$
$\mathrm{Im}\, G_{ij} = F_{ij}$
$\mathrm{Re}\, G_{ij} = g_{ij}$
QUANTUM GEOMETRIC TENSOR
Riemannian Metric Tensor

$H(\lambda) \longrightarrow |\psi_0(\lambda)\rangle$

$|\psi_0(\lambda+\delta\lambda)\rangle =$

$\delta H = H(\lambda+\delta\lambda) - H(\lambda)$

$\mathrm{Im}\, G_{ij} = F_{ij}$ $\quad$ $\mathrm{Re}\, G_{ij} = g_{ij}$ Metric tensor

$$|\psi_0(\lambda+\delta\lambda)\rangle = |\psi_0(\lambda)\rangle + \sum_{n\neq 0} |\psi_n(\lambda)\rangle \frac{\langle\psi_n(\lambda)|\delta H|\psi_0(\lambda)\rangle}{\Delta_{0n}} + O(\delta H^2)$$

$$|\delta\lambda\rangle = |\psi_0(\lambda)\rangle + \sum_{n\neq 0} |\psi_n(\lambda)\rangle \frac{\langle\psi_n^{(0)}|\delta H|\psi_0(\lambda)\rangle}{\Delta_{0n}} + O(\delta H^2)$$

$$\underbrace{\qquad\qquad}_{|\psi\rangle}$$

$$|\psi_0(\lambda+\delta\lambda)\rangle = |\psi_0(\lambda)\rangle + \sum_{n\neq 0} |\psi_n(\lambda)\rangle\langle\psi_n(\lambda)|S+H|$$

$$\Delta_{0n}$$

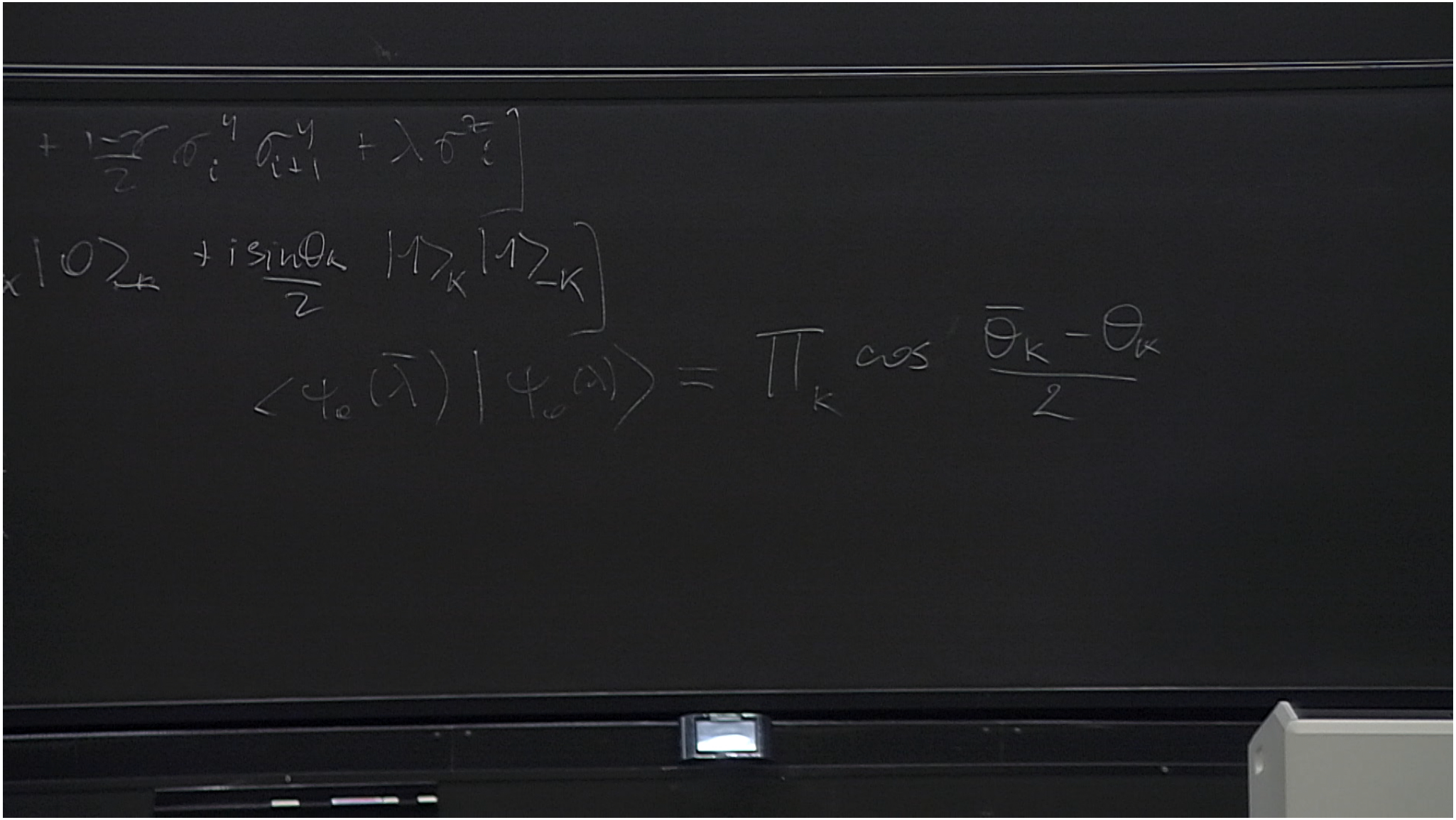
$$g_{\mu\nu} = \mathrm{Re}\left[\sum_{n\neq 0} \frac{\langle\psi_0(\lambda)|\partial_\mu H|\psi_n(\lambda)\rangle\langle\psi_n(\lambda)|\partial_\nu H|\psi_0(\lambda)\rangle}{|E_n(\lambda)-E_0(\lambda)|^2}\right]$$

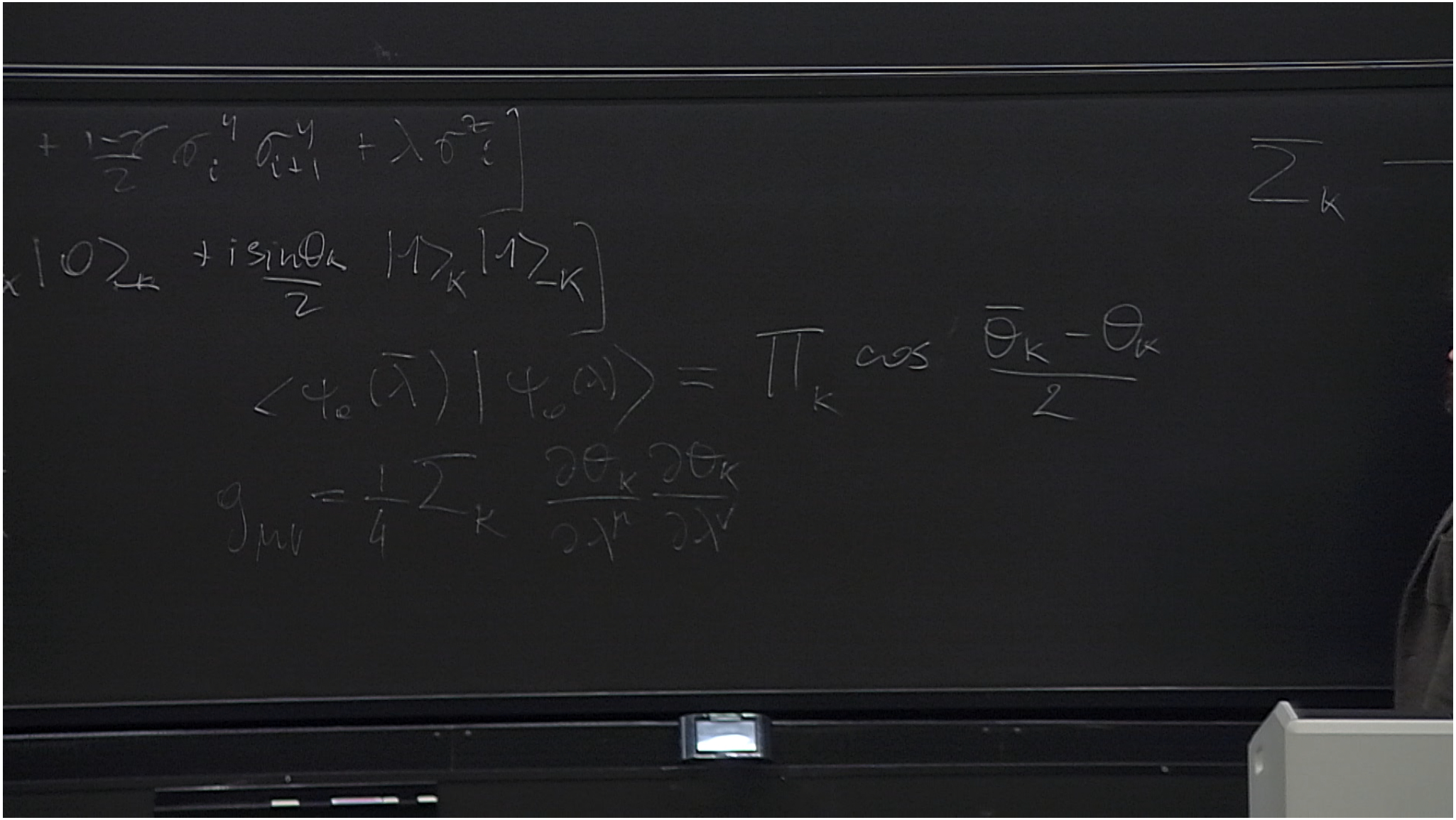
$$H = -\sum_i \left[\frac{1+\gamma}{2} \sigma_i^x \sigma_{i+1}^x + \frac{1-\gamma}{2} \sigma_i^y \sigma_{i+1}^y + \lambda \sigma_i^z \right]$$

$$|\Omega\rangle = \bigotimes_k \left[\cos\frac{\theta_k}{2} |0\rangle_k |0\rangle_{-k} + i \sin\frac{\theta_k}{2} |1\rangle_k |1\rangle_{-k} \right]$$

$$\Lambda_k = \sqrt{\varepsilon_k^2 + \delta_k^2} \qquad \theta_k = \frac{\varepsilon_k}{\Lambda_k}$$

$$\delta_k = \gamma \sin k$$


$+\frac{1-\gamma}{2}\sigma_i^y\sigma_{i+1}^y+\lambda\sigma_i^z\Big]$
$|0\rangle_{-k}+i\sin\theta_k\,|1\rangle_k|1\rangle_{-k}\Big]$
$\langle\psi_0(\bar{\lambda})|\psi_0(\lambda)\rangle=\prod_k\cos\frac{\bar{\theta}_k-\theta_k}{2}$


$+\frac{1-\gamma}{2}\sigma_i^4\sigma_{i+1}^4+\lambda\sigma_i^z]$
$\sum_k$
$|0_k\rangle+i\sin\theta_k\,|1_k\rangle|1_{-k}\rangle]$
$\langle\psi_0(\bar{\lambda})|\psi_0(\lambda)\rangle=\prod_k\cos\frac{\bar{\theta}_k-\theta_k}{2}$
$g_{\mu\nu}=\frac{1}{4}\sum_k\frac{\partial\theta_k}{\partial\lambda^\mu}\frac{\partial\theta_k}{\partial\lambda^\nu}$

$$\sum_k \longrightarrow \frac{L}{2\pi}\int_0^{\pi} dx$$

$$= \prod_k \cos\frac{\bar{\theta}_k - \theta_k}{2}$$

magn field

$|h|<1$

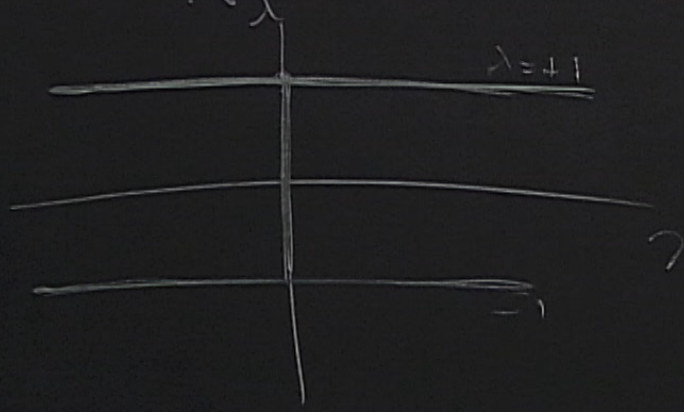
$\frac{\partial\theta_k}{\partial x}$ → Integral

$$g_{\mu\nu} = \frac{N}{16|\gamma|}\begin{pmatrix} \frac{1}{1-x^2} & 0 \\ 0 & \frac{1}{(1+\gamma)^2} \end{pmatrix}$$

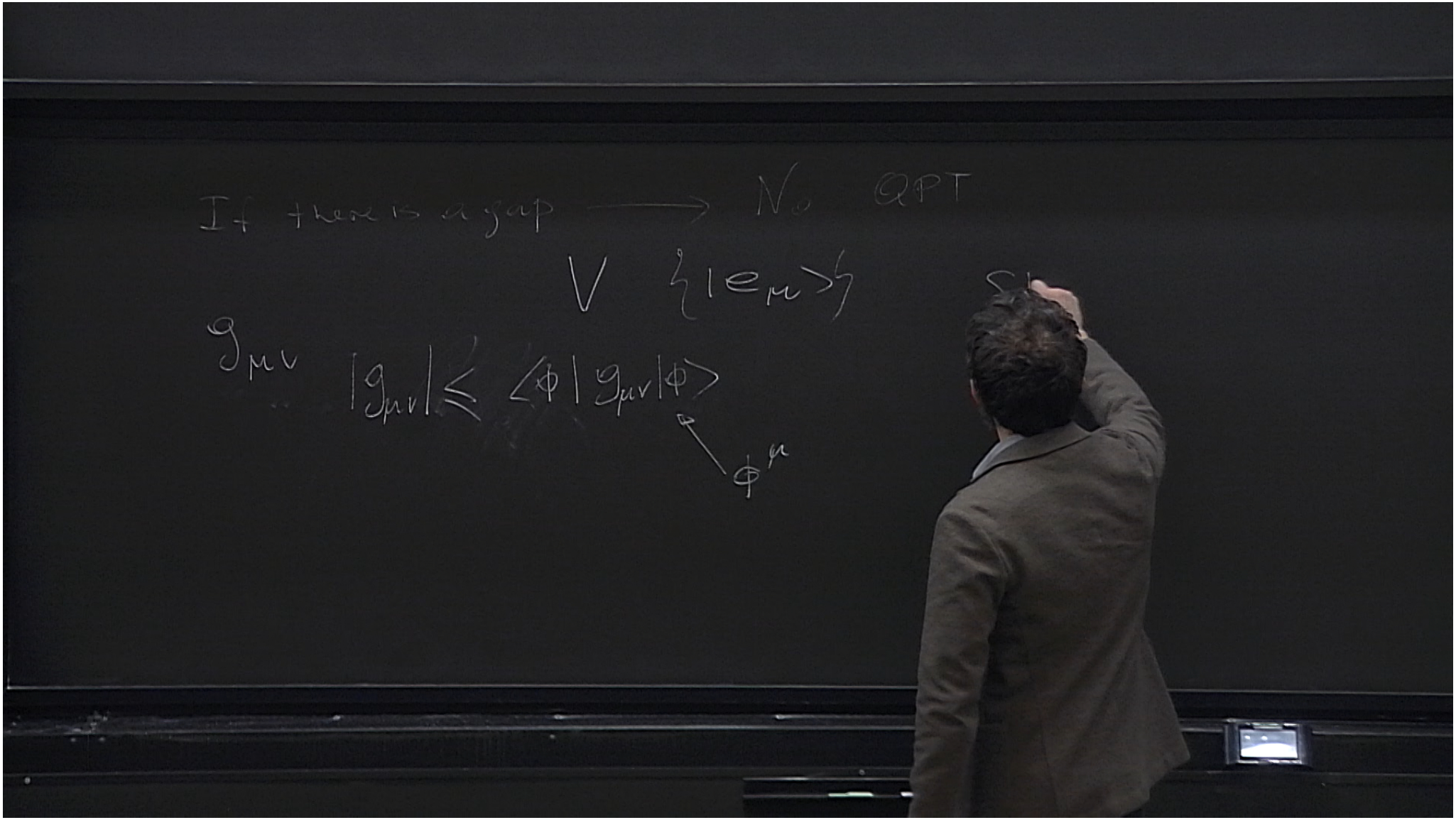
$\frac{g_{\mu\nu}}{N_\lambda}$

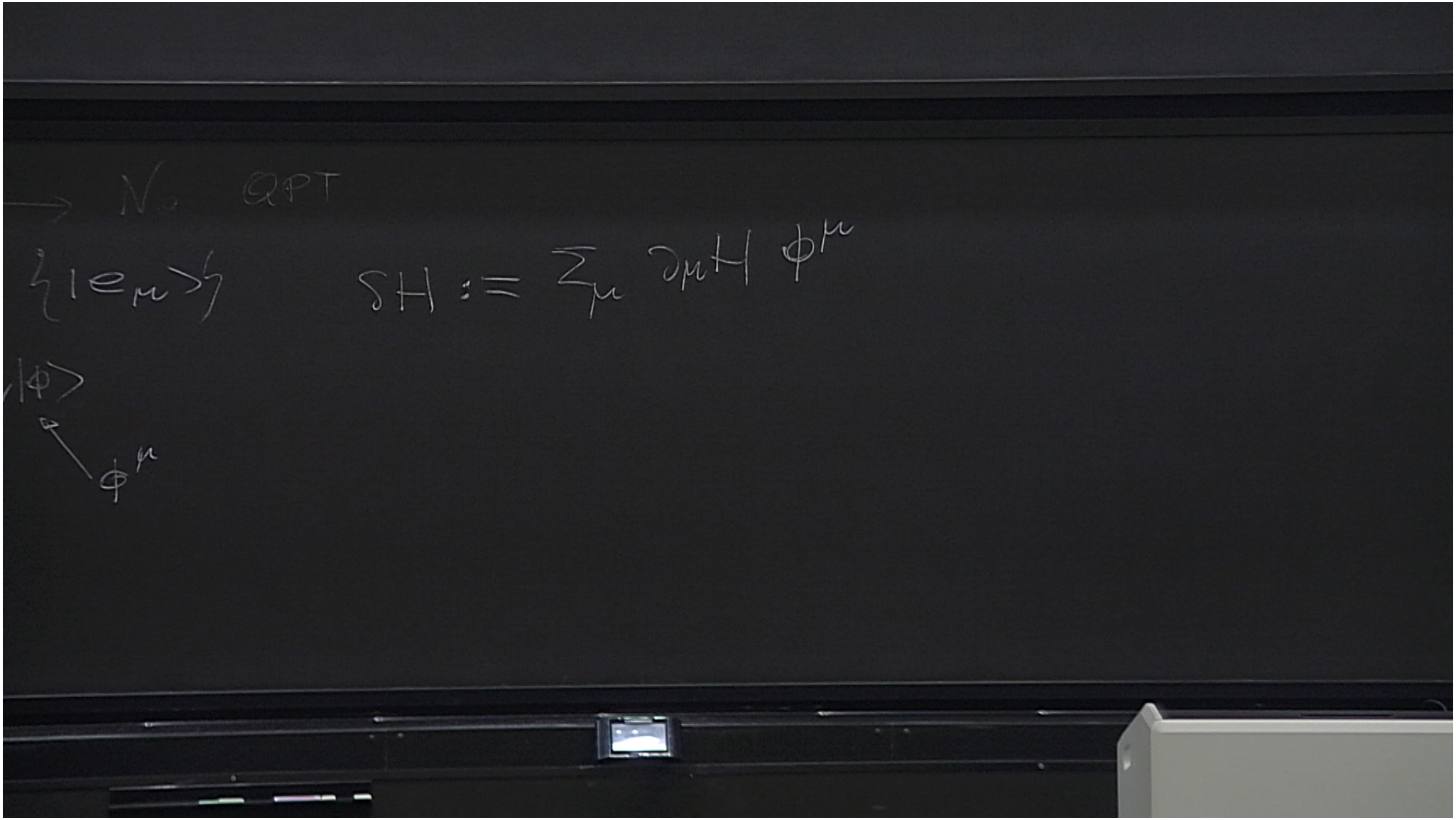
$R = R^a{}_a$

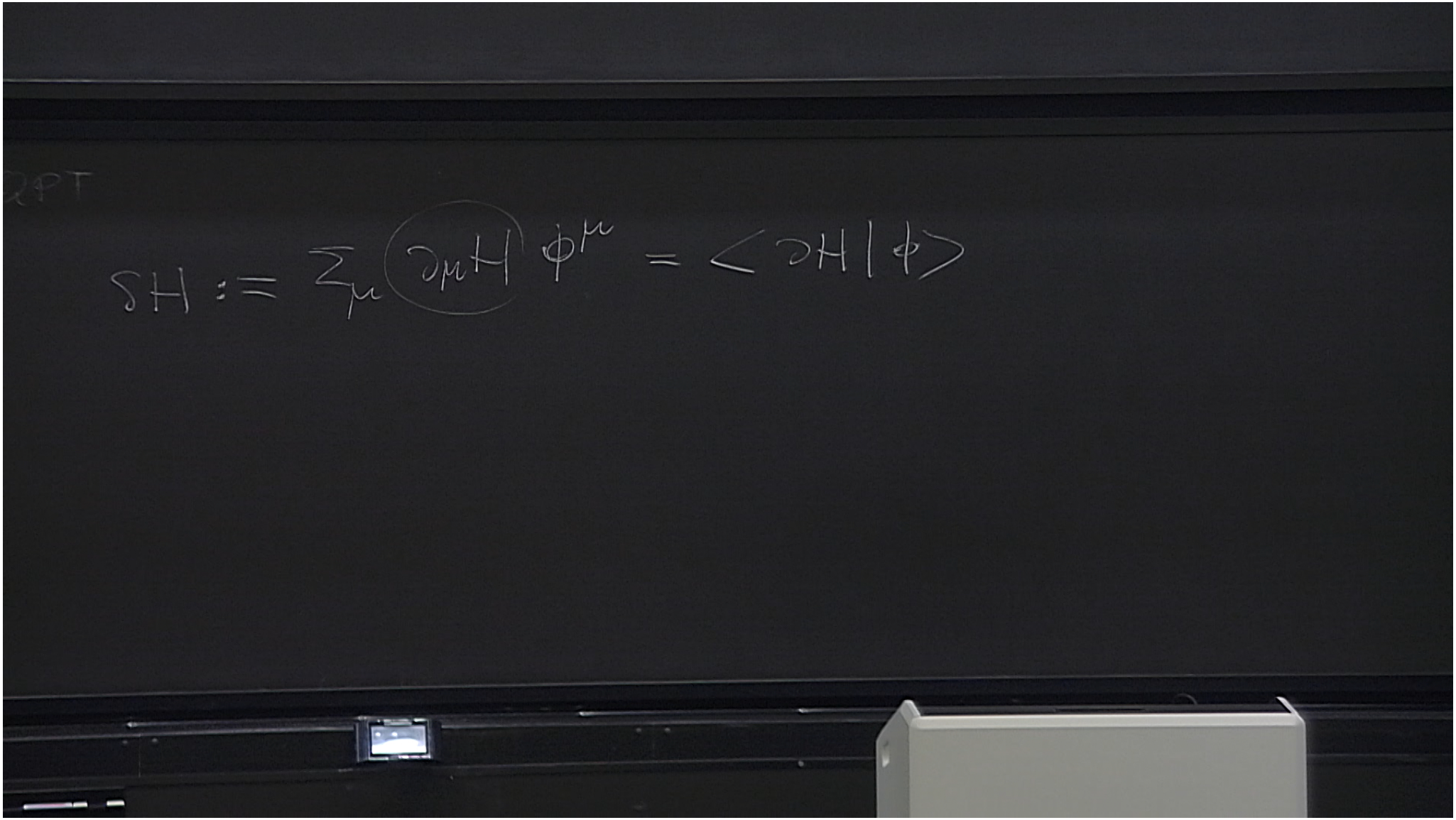
$\begin{cases} |\lambda| < 1 & -\frac{16}{N} \; \frac{1+|\gamma|}{|\gamma|} \\ |\lambda| > 1 & \frac{16}{N} \; \frac{|\lambda| + \sqrt{\lambda^2+\gamma^2-1}}{\sqrt{\lambda^2+\gamma^2-1}} \end{cases}$

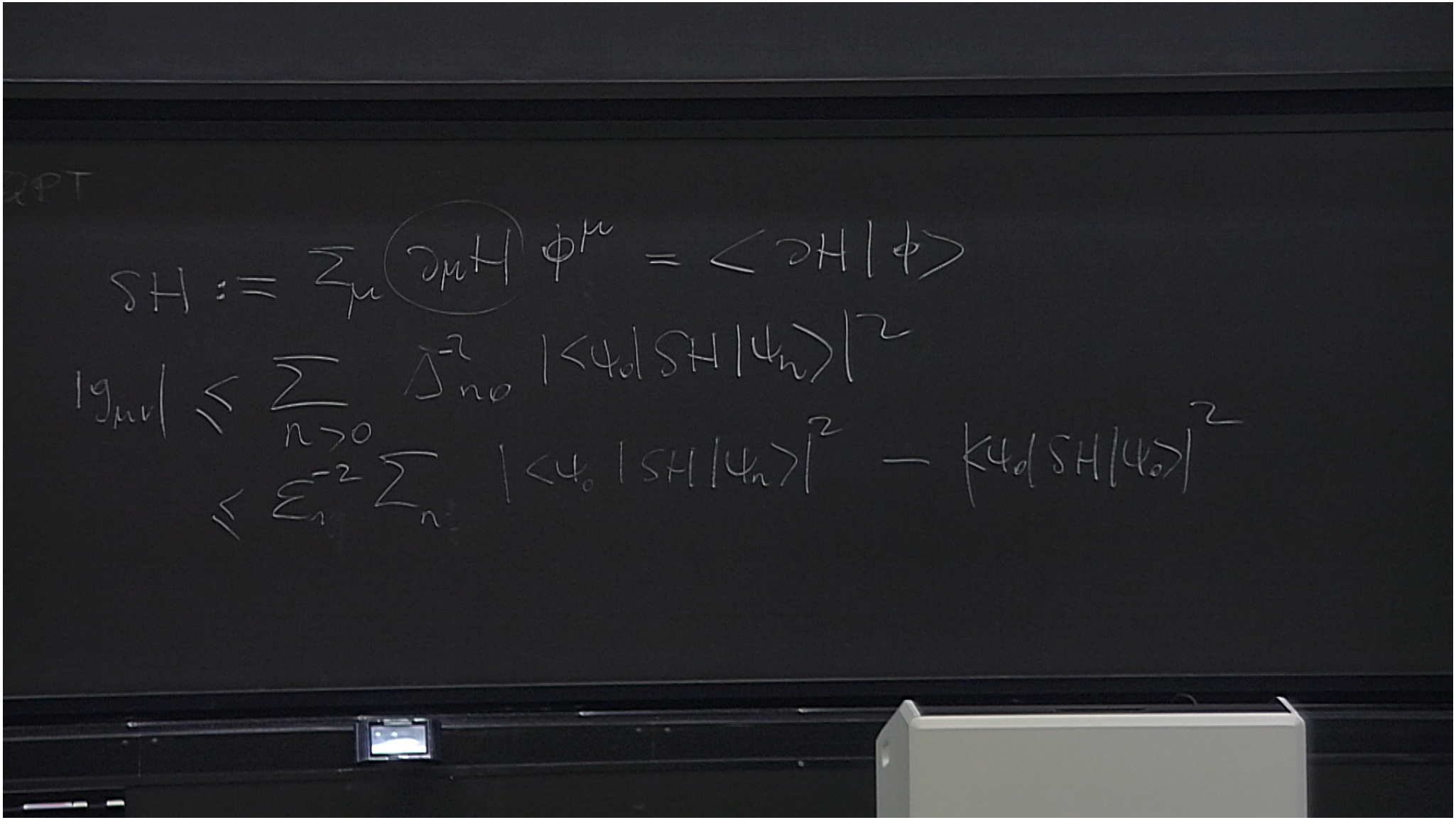


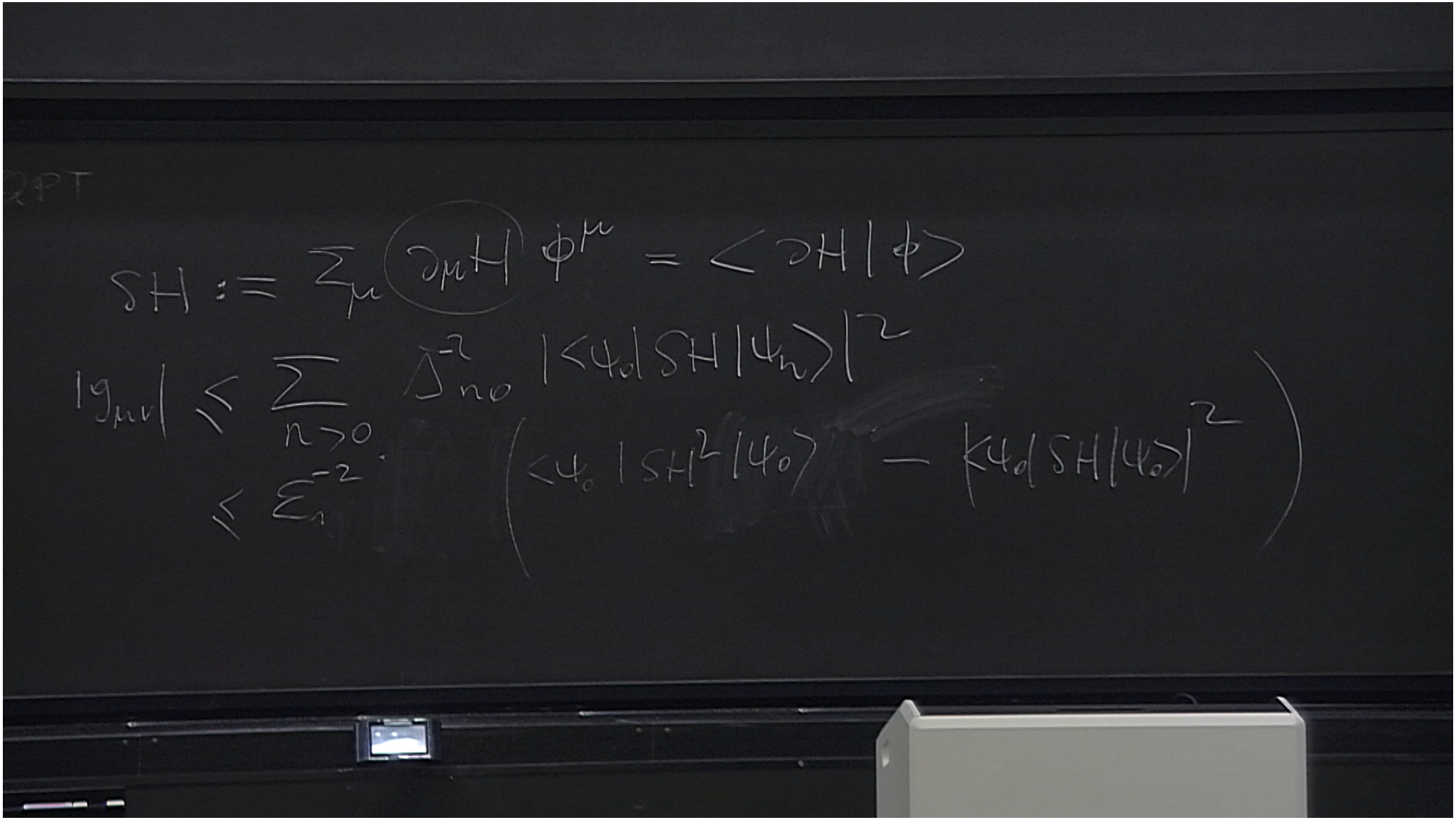
R is discontinuous $\lambda = \pm 1$

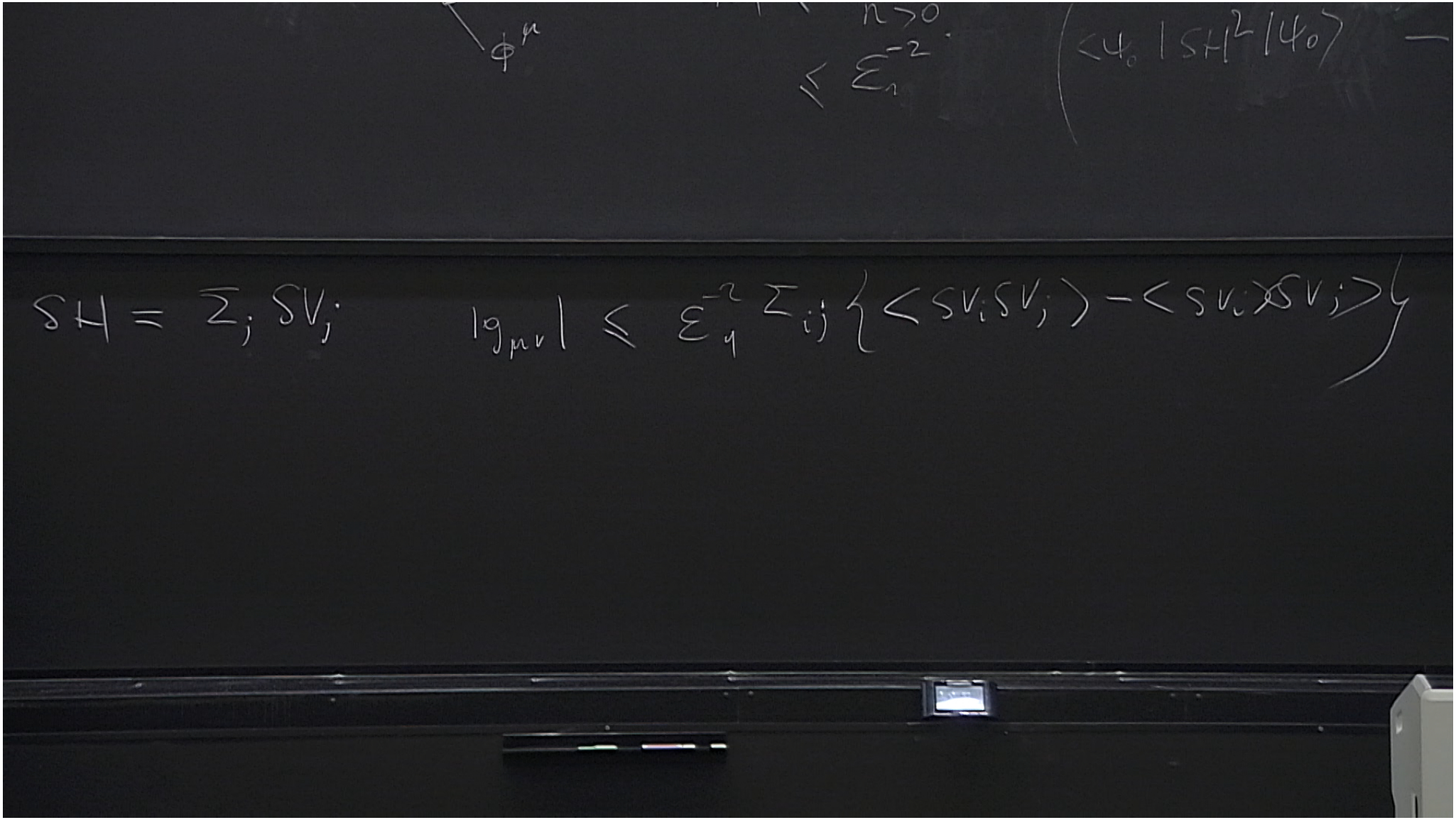

If there is a gap ⟶ No QPT
V {|e_μ⟩}
g_μν |g_μν| ≤ ⟨Φ| g_μν |Φ⟩
Φ^μ

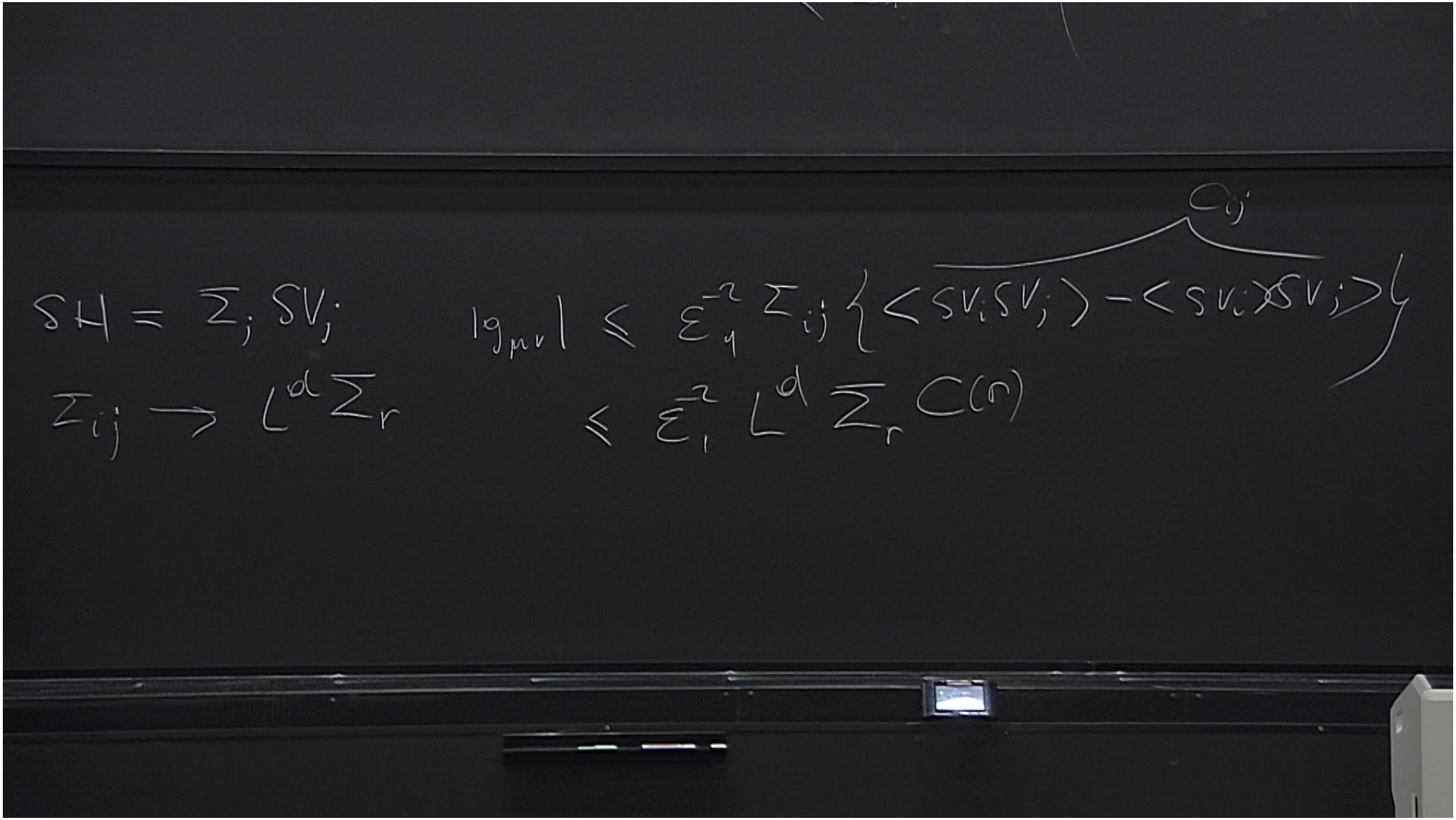

N. QPT
$\{|e_\mu\rangle\}$
$\delta H := \sum_\mu \partial_\mu H \, \phi^\mu$
$|\phi\rangle$
ϕ^μ


QPT
$\delta H := \sum_\mu \partial_\mu H \, \phi^\mu = \langle \partial H | \phi \rangle$


QPT
$\delta H := \sum_\mu \partial_\mu H \, \phi^\mu = \langle \partial H | \phi \rangle$
$|g_{\mu\nu}| \leq \sum_{n>0} \Delta_{n0}^{-2} |\langle \psi_0 | \delta H | \psi_n \rangle|^2$
$\leq \varepsilon_1^{-2} \sum_n |\langle \psi_0 | \delta H | \psi_n \rangle|^2 - |\langle \psi_0 | \delta H | \psi_0 \rangle|^2$


QPT
$\delta H := \sum_\mu \partial_\mu H \, \phi^\mu = \langle \partial H | \phi \rangle$
$|g_{\mu\nu}| \leq \sum_{n>0} \Delta_{n0}^{-2} \, |\langle \psi_0 | \delta H | \psi_n \rangle|^2$
$\leq \varepsilon_1^{-2} \cdot \left(\langle \psi_0 | \delta H^2 | \psi_0 \rangle - |\langle \psi_0 | \delta H | \psi_0 \rangle|^2 \right)$


ϕ^n
$n>0$.
$\leq \varepsilon_1^{-2}$.
$\langle 4_0 | SH^2 | 4_0 \rangle$ —
$SH = \Sigma_j SV_j$
$|g_{\mu\nu}| \leq \varepsilon_1^{-2} \Sigma_{ij} \{ \langle SV_i SV_j \rangle - \langle SV_i \rangle \langle SV_j \rangle \}$


$\delta H = \sum_j \delta V_j$
$\Sigma_{ij} \to L^{\alpha} \Sigma_r$
$|g_{\mu\nu}| \leq \varepsilon_1^{-2} \Sigma_{ij} \{ \langle \delta V_i \delta V_j \rangle - \langle \delta V_i \rangle \langle \delta V_j \rangle \}$
C_{ij}
$\leq \varepsilon_1^{-2} L^{d} \Sigma_r C(n)$

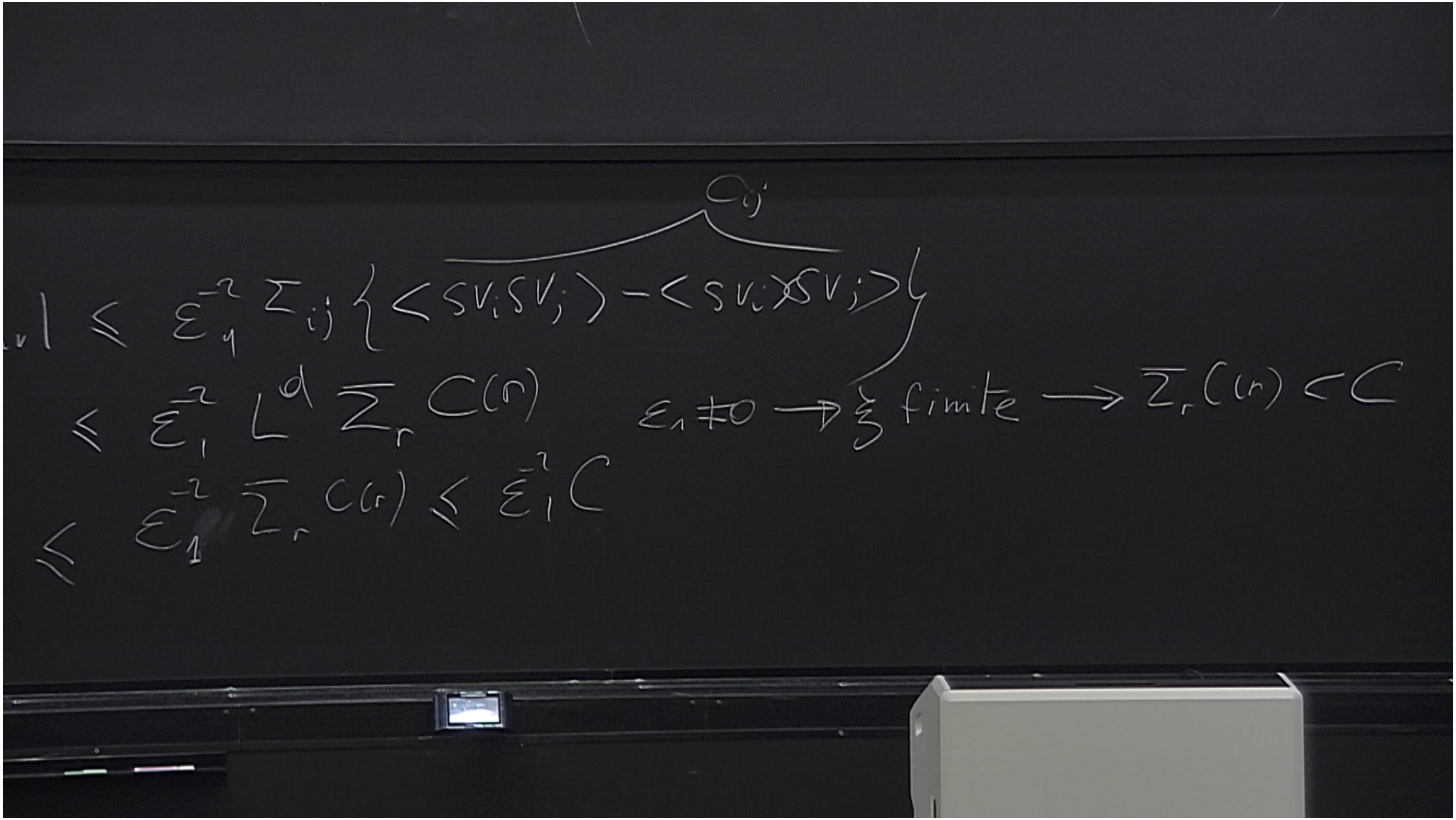
$$\delta H = \sum_j \delta V_j$$

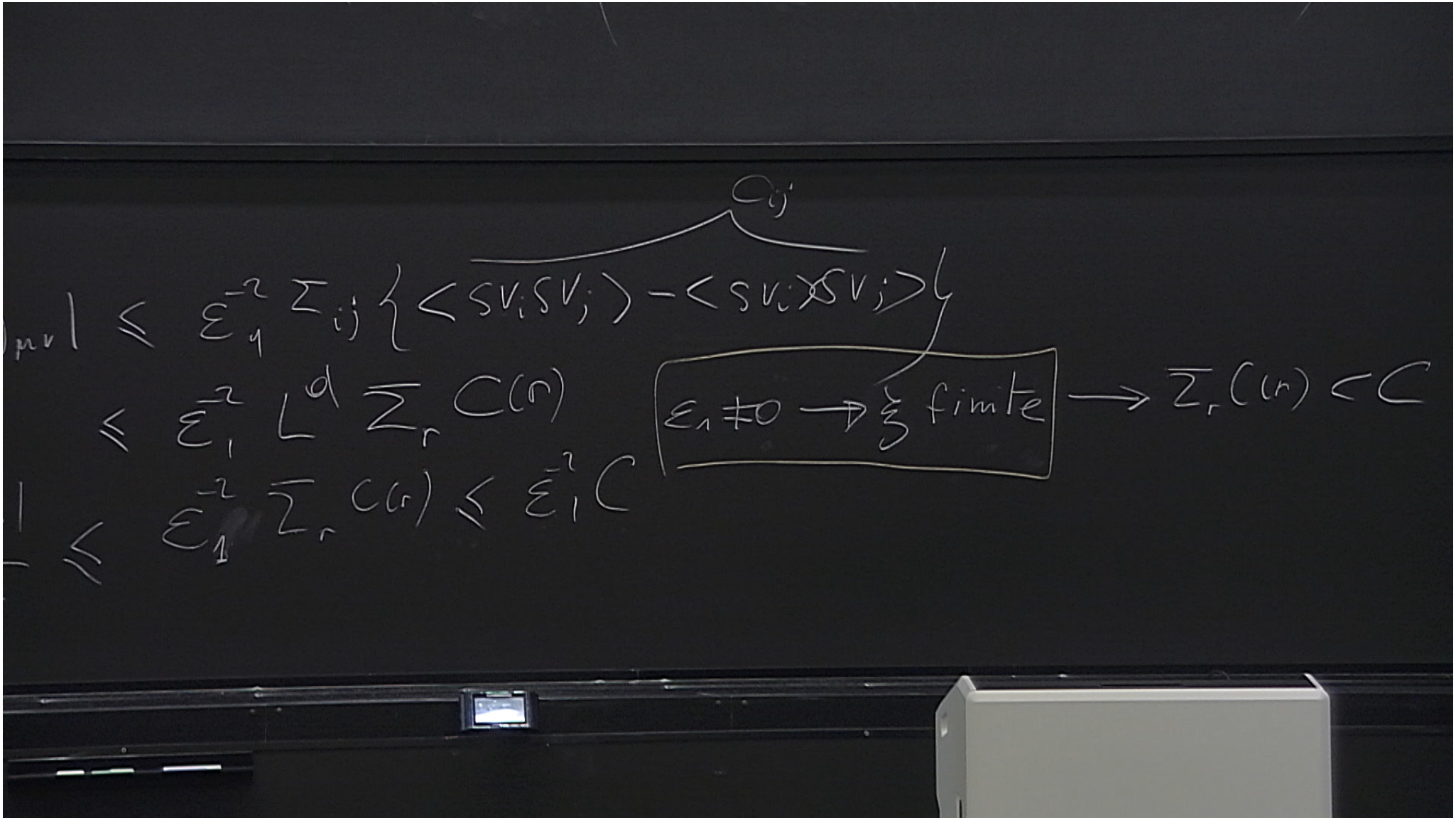
$$\Sigma_{ij} \to L^d \Sigma_r$$

$$|g_{\mu\nu}| \leq \varepsilon_1^{-2} \Sigma_{ij} \{ \overbrace{\langle \delta V_i \delta V_j \rangle - \langle \delta V_i \rangle \langle \delta V_j \rangle}^{C_{ij}} \}$$

$$\leq \varepsilon_1^{-2} L^d \Sigma_r C(r)$$

$$\frac{|g_{\mu\nu}|}{L^d} \leq \varepsilon_i^{-2} \Sigma_r C(r)$$


C_{ij}
$\leq \varepsilon_1^{-2} \Sigma_{ij} \{ \langle sv_i sv_j \rangle - \langle sv_i \otimes sv_j \rangle \}$
$\leq \varepsilon_1^{-2} L^d \Sigma_r C(r)$
$\varepsilon_1 \neq 0 \rightarrow \xi$ finite $\rightarrow \Sigma_r C(r) < C$
$\leq \varepsilon_1^{-2} \Sigma_r C(r) \leq \varepsilon_1^{-2} C$


C_{ij}
$\leq \varepsilon_1^{-2} \Sigma_{ij} \{ \langle sv_i sv_j \rangle - \langle sv_i \rangle \langle sv_j \rangle \}$
$\leq \varepsilon_1^{-2} L^d \Sigma_r C(r)$
$\leq \varepsilon_1^{-2} \Sigma_r C(r) \leq \varepsilon_1^{-2} C$
$\varepsilon_1 \neq 0 \rightarrow \xi$ finite
$\rightarrow \Sigma_r C(r) < C$