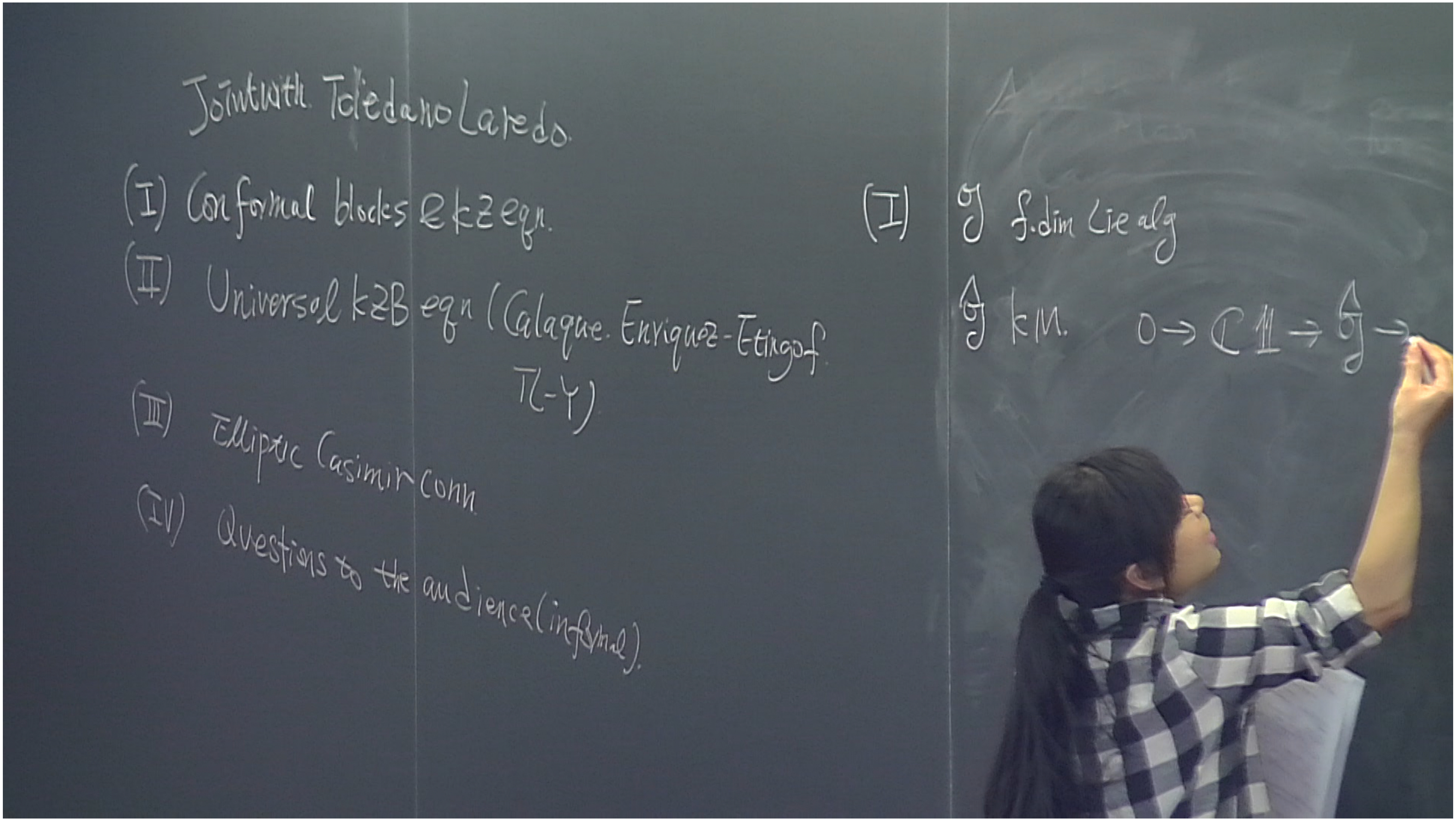
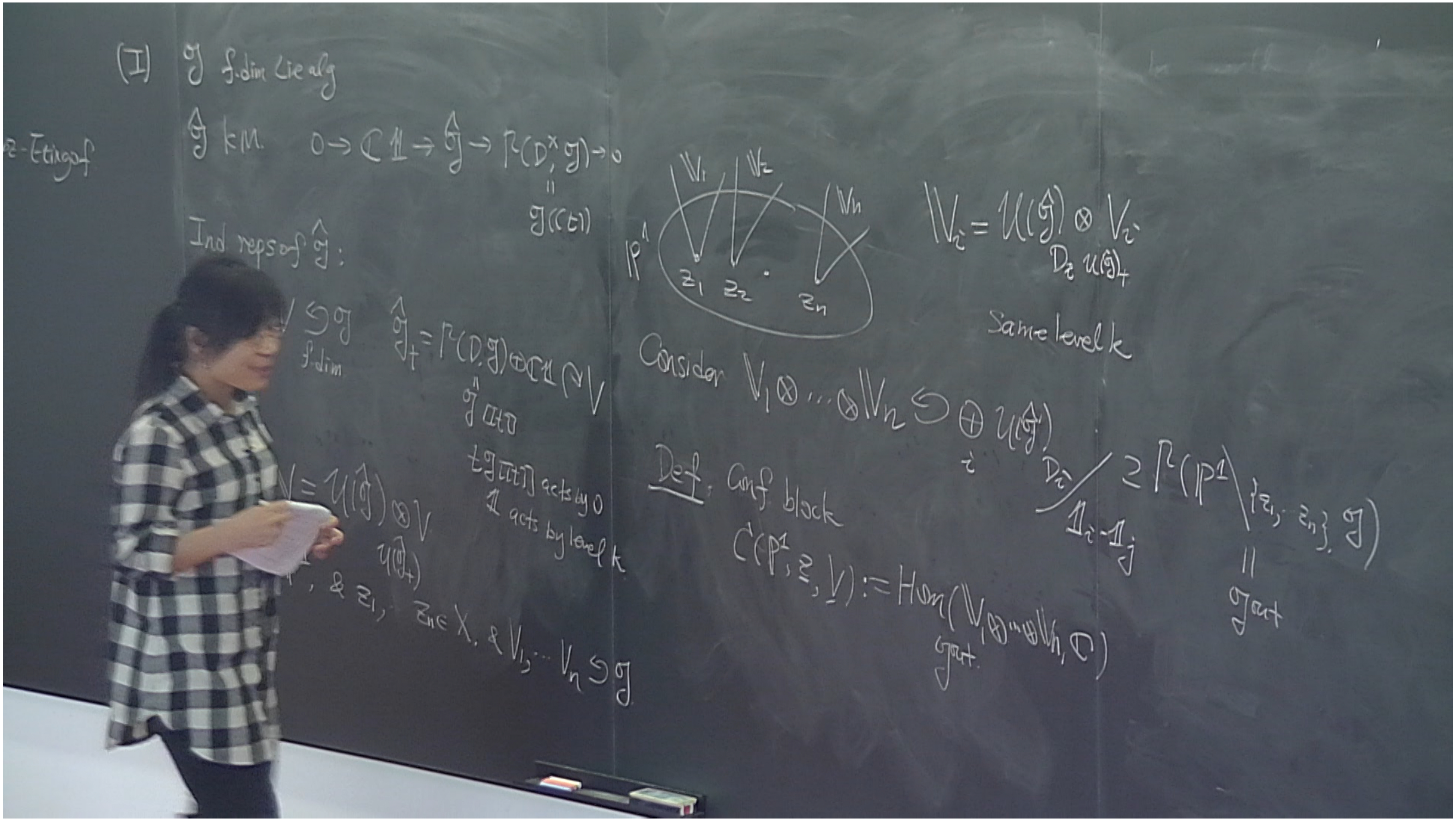
Title: Monodromy representations of elliptic braid groups

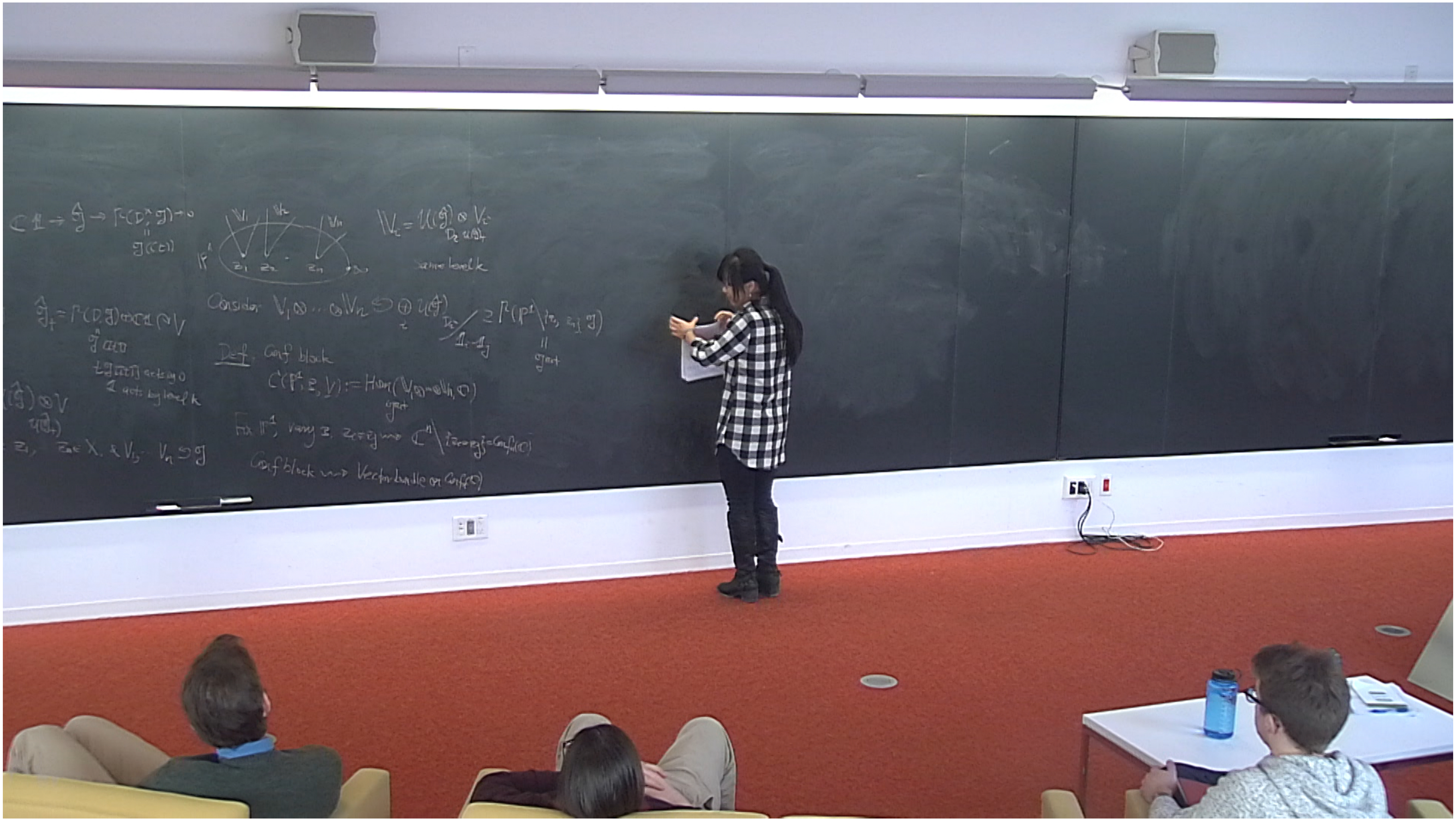
Date: Jan 15, 2018 02:00 PM

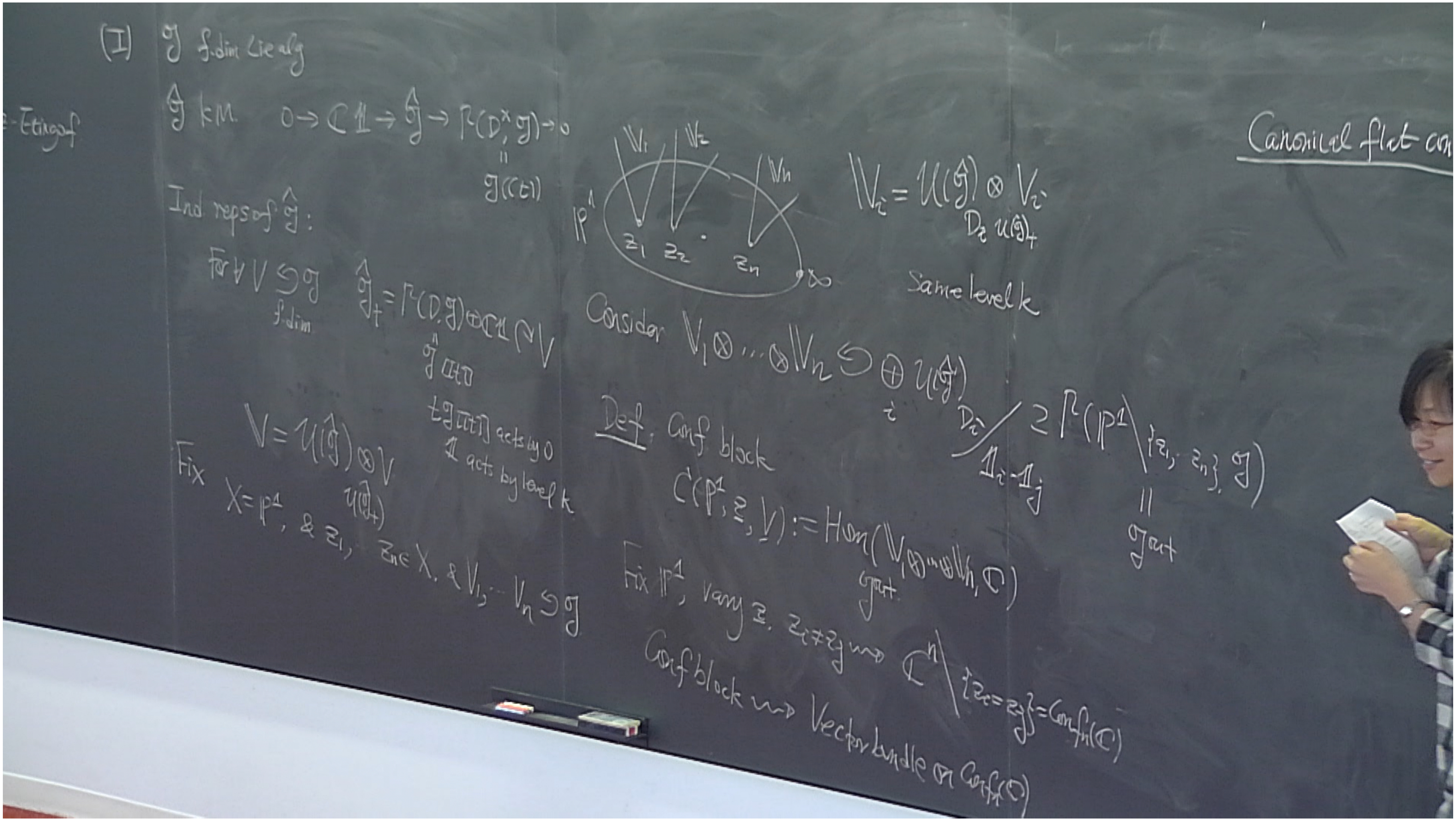
URL: http://pirsa.org/18010079

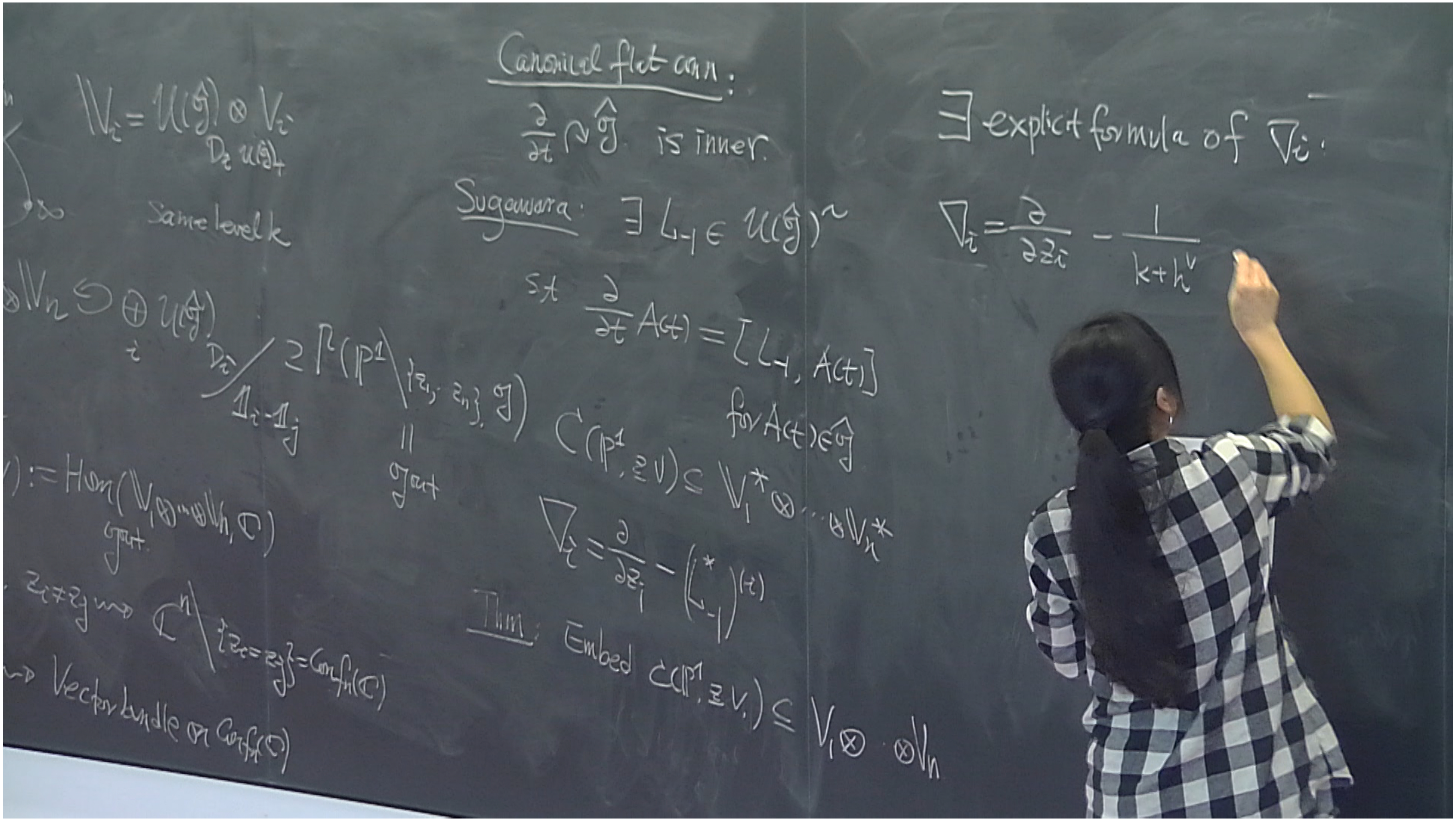
Abstract: <p>In my talk, I will briefly review the representation theoretical construction of conformal blocks attached to an affine Kac-Moody algebra and a smooth algebraic curve with marked points. I will focus on the case when the algebraic curve is an elliptic curve. The bundle of conformal blocks carries a canonical flat connection: the Knizhnik-Zamolodchikov-Bernard (KZB) equation. There are various generalizations of the KZB equation. I will talk about one generalization that constructed by myself and Toledano Laredo recently: the elliptic Casimir connection. It is a holonomic system of differential equations with regular singularities on elliptic curve with marked points, taking values in a deformation of the double current algebra g[u, v] defined by Guay. The monodromy of elliptic Casimir connection leads to interesting representations of the elliptic braid groups. </p>

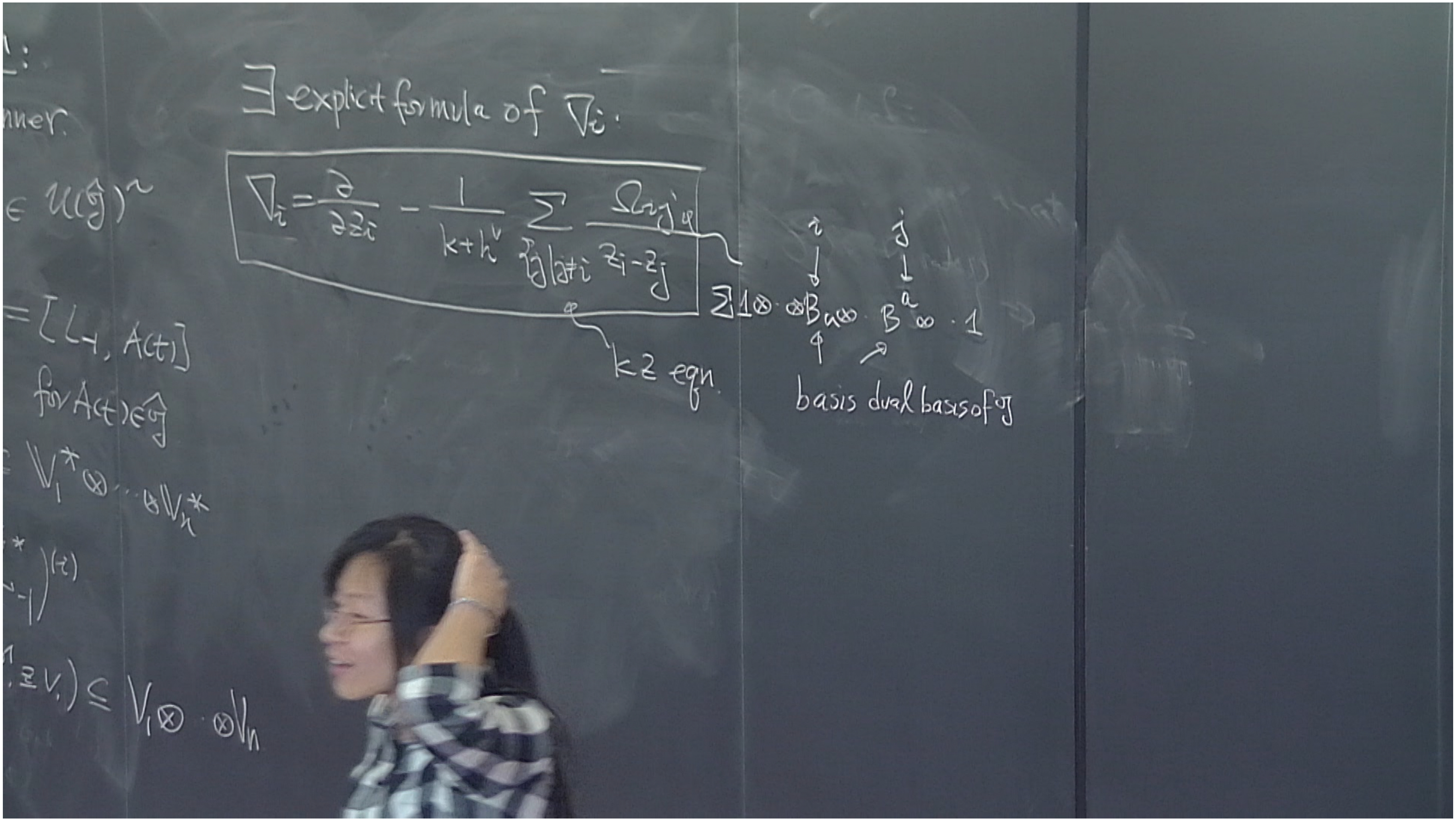

Joint with Toledano Laredo
(I) Conformal blocks & KZ eqn.
(II) Universal KZB eqn (Calaque. Enriquez-Etingof. TL-Y)
(III) Elliptic Casimir conn
(IV) Questions to the audience (informal).
(I) g f.dim Lie alg
ĝ KM. 0 → ℂ𝟙 → ĝ →

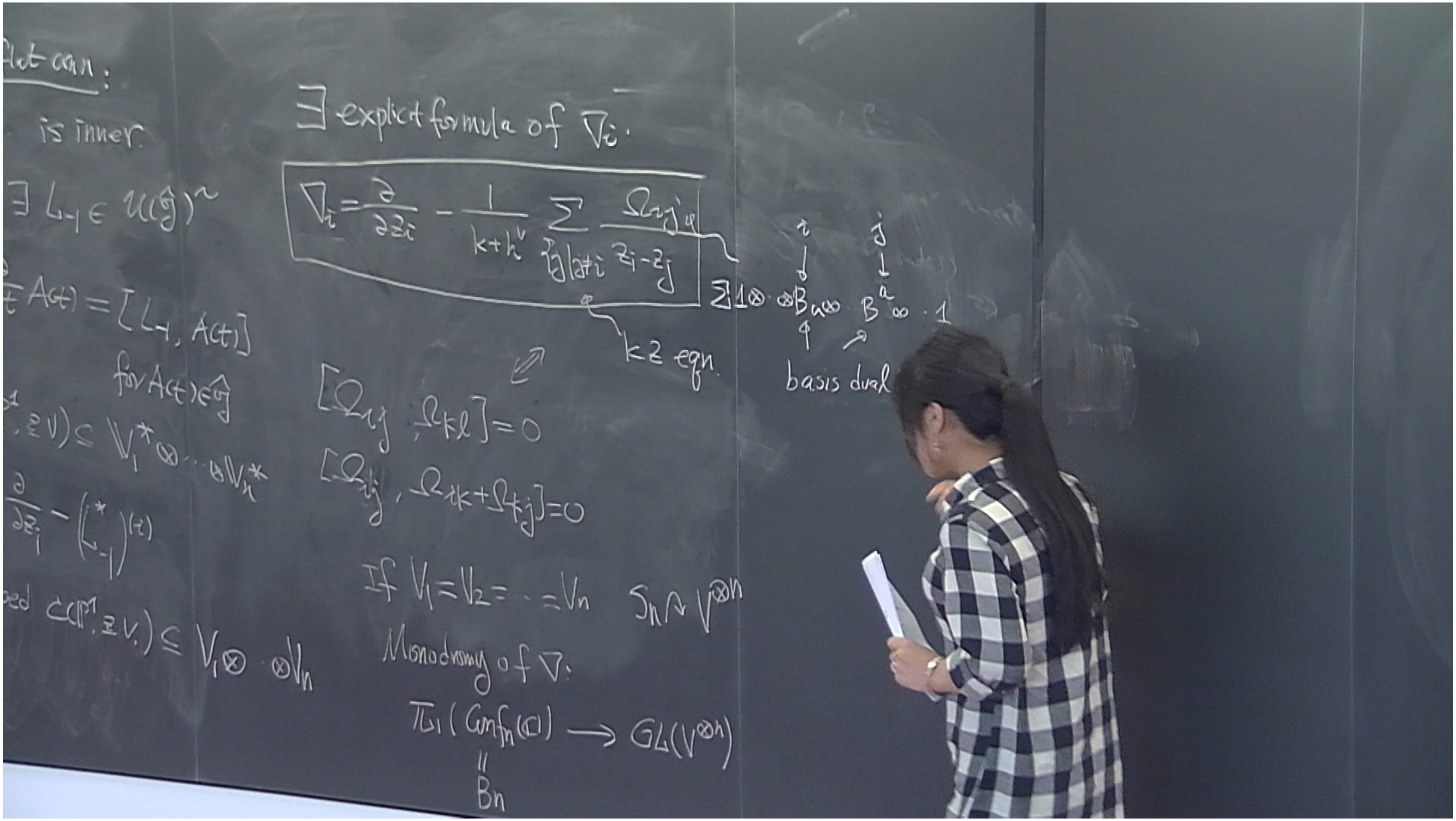

(I) g f.dim Lie alg
ĝ KM. 0 → ℂ𝟙 → ĝ → Γ(D^×, g) → 0
= g((t))
Ind reps of ĝ:
V ⟲ g f.dim
ĝ_+ = Γ(D, g) ⊕ ℂ𝟙 ⟲ V
= g[[t]]
t g[[t]] acts by 0
𝟙 acts by level k
𝕍 = U(ĝ) ⊗_{U(ĝ_+)} V
& z_1, … z_n ∈ X, & V_1, … V_n ⟲ g
-Etingof
𝕍_1 𝕍_2 𝕍_n
ℙ^1 z_1 z_2 z_n
𝕍_i = U(ĝ) ⊗_{D_i U(ĝ)_+} V_i
Same level k
Consider 𝕍_1 ⊗ … ⊗ 𝕍_n ⟲ ⊕_i U(ĝ) / D_i / 𝟙_i − 𝟙_j ⊇ Γ(ℙ^1 \ {z_1, … z_n}, g) = g_out
Def. Conf block
C(ℙ^1, z, V) := Hom_{g_out}(𝕍_1 ⊗ … ⊗ 𝕍_n, ℂ)

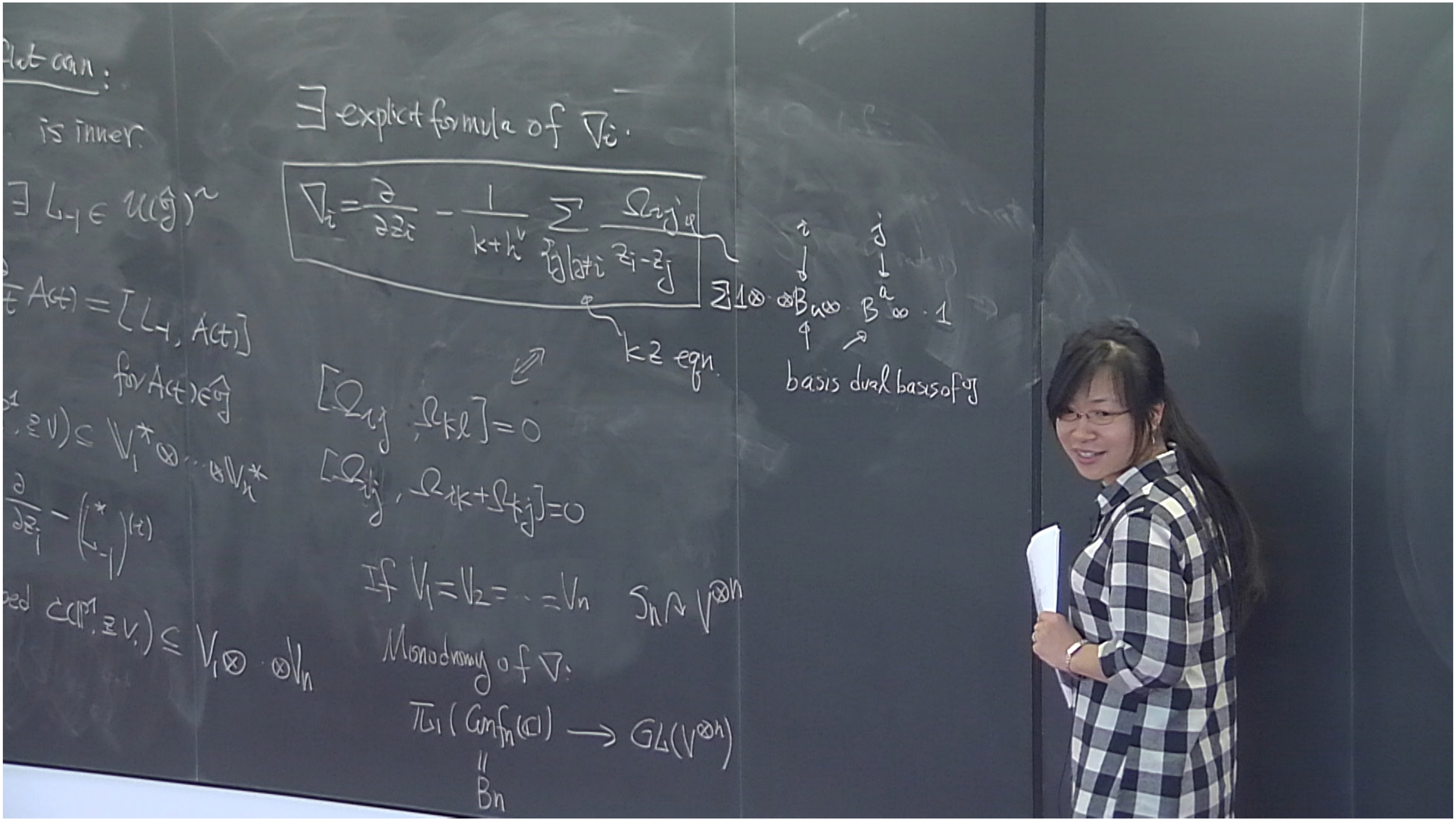

same level k
Consider
Def. Conf block
Conf block ⟶ Vector bundle on Conf


(I) g f.dim Lie alg
-Etingof
ĝ KM. 0 → ℂ1 → ĝ → Γ(D×, g) → 0
g((t))
Ind reps of ĝ:
For ∀ V ⟲ g f.dim
ĝ+ = Γ(D, g) ⊕ ℂ1 ⟲ V
ĝ ⟦t⟧
t g⟦t⟧ acts by 0
1 acts by level k
𝕍 = U(ĝ) ⊗ V
U(ĝ+)
Fix X = ℙ1, & z1, … zn ∈ X, & V1, … Vn ⟲ g
ℙ1
𝕍1 𝕍2 𝕍n
z1 z2 zn ∞
𝕍i = U(ĝ) ⊗ Vi
Di U(ĝ)+
Same level k
Consider 𝕍1 ⊗ … ⊗ 𝕍n ⟲ ⊕i U(ĝ)
Di / 1i - 1j ⊇ Γ(ℙ1 \ {z1, … zn}, g)
gout
Def. Conf block
C(ℙ1, z, V) := Hom(𝕍1 ⊗ … ⊗ 𝕍n, ℂ)
gout
Fix ℙ1, vary z. zi ≠ zj ⤳ ℂn \ {zi = zj} = Confn(ℂ)
Conf block ⤳ Vector bundle on Confn(ℂ)
Canonical flat con


Canonical flat conn:
$\frac{\partial}{\partial t} \curvearrowright \hat{\mathfrak{g}}$ is inner.
Sugawara: $\exists\, L_{-1} \in \mathcal{U}(\hat{\mathfrak{g}})^{\sim}$
s.t. $\frac{\partial}{\partial t} A(t) = [L_{-1}, A(t)]$
for $A(t) \in \hat{\mathfrak{g}}$
$C(\mathbb{P}^1, \underline{z}, V) \subseteq \mathbb{V}_1^* \otimes \cdots \otimes \mathbb{V}_n^*$
$\nabla_i = \frac{\partial}{\partial z_i} - (L_{-1}^*)^{(i)}$
Thm: Embed $C(\mathbb{P}^1, \underline{z}, V_i) \subseteq V_1 \otimes \cdots \otimes V_n$
$\exists$ explicit formula of ∇_i.
$\nabla_i = \frac{\partial}{\partial z_i} - \frac{1}{k+h^\vee}$
$\mathbb{V}_i = \mathcal{U}(\hat{\mathfrak{g}}) \otimes V_i$
$D_i\, \mathcal{U}(\hat{\mathfrak{g}})_+$
Same level k
$\mathbb{V}_n \subseteq \bigoplus_i \mathcal{U}(\hat{\mathfrak{g}})$
$\cong \Gamma(\mathbb{P}^1 \setminus \{z_1, \ldots, z_n\}, \mathfrak{g})$
$\mathfrak{g}_{out}$
$D_i / \mathbb{1}_i - \mathbb{1}_j$
$) := \mathrm{Hom}_{\mathfrak{g}_{out}}(\mathbb{V}_1 \otimes \cdots \otimes \mathbb{V}_n, \mathbb{C})$
$z_i \neq z_j \rightsquigarrow \mathbb{C}^n \setminus \{z_i = z_j\} = \mathrm{Conf}_n(\mathbb{C})$
$\rightsquigarrow$ Vector bundle on $\mathrm{Conf}_n(\mathbb{C})$


∃ explicit formula of ∇_i.
$\nabla_i = \frac{\partial}{\partial z_i} - \frac{1}{k+h^\vee} \sum_{\{j \mid j \neq i\}} \frac{\Omega_{ij}}{z_i - z_j}$
$\Sigma\, 1 \otimes \cdots \otimes B_a \otimes \cdots B^a \otimes \cdots 1$
KZ eqn.
basis dual basis of $\mathfrak{g}$
for $A(t) \in \hat{\mathfrak{g}}$


∃ explicit formula of ∇_i.
$\nabla_i = \frac{\partial}{\partial z_i} - \frac{1}{k+h^\vee} \sum_{j \mid j \neq i} \frac{\Omega_{ij}}{z_i - z_j}$
$\sum 1 \otimes \cdots \otimes B_a \otimes \cdots B^a \otimes \cdots 1$
basis dual
KZ eqn.
$[\Omega_{ij}, \Omega_{kl}] = 0$
$[\Omega_{ij}, \Omega_{ik} + \Omega_{kj}] = 0$
If $V_1 = V_2 = \cdots = V_n$ $S_n \curvearrowright V^{\otimes n}$
Monodromy of ∇:
$\pi_1(\mathrm{Conf}_n(\mathbb{C})) \longrightarrow GL(V^{\otimes n})$
B_n
is inner
$\exists L_{-1} \in \mathcal{U}(\hat{\mathfrak{g}})^\sim$
$A(t) = [L_{-1}, A(t)]$
for $A(t) \in \hat{\mathfrak{g}}$
$\subseteq V_1^* \otimes \cdots \otimes V_n^*$
$\frac{\partial}{\partial z_i} - (L_{-1}^*)^{(i)}$
$\subseteq V_1 \otimes \cdots \otimes V_n$


flat conn:
is inner
$\exists L_{-1} \in \mathcal{U}(\hat{\mathfrak{g}})^{\sim}$
$A(t) = [L_{-1}, A(t)]$
for $A(t) \in \hat{\mathfrak{g}}$
$\subseteq V_1^* \otimes \cdots \otimes V_n^*$
$\frac{\partial}{\partial z_i} - (L_{-1}^*)^{(i)}$
$\subseteq V_1 \otimes \cdots \otimes V_n$
$\exists$ explicit formula of ∇_i.
$\nabla_i = \frac{\partial}{\partial z_i} - \frac{1}{k+h^\vee} \sum_{\{j \mid j \neq i\}} \frac{\Omega_{ij}}{z_i - z_j}$
KZ eqn.
$\sum 1 \otimes \cdots \otimes B_a \otimes \cdots \otimes B^a \otimes \cdots 1$
basis dual basis of $\mathfrak{g}$
$[\Omega_{ij}, \Omega_{kl}] = 0$
$[\Omega_{ij}, \Omega_{ik} + \Omega_{kj}] = 0$
If $V_1 = V_2 = \cdots = V_n$ $S_n \curvearrowright V^{\otimes n}$
Monodromy of ∇:
$\pi_1(\mathrm{Conf}_n(\mathbb{C})) \longrightarrow GL(V^{\otimes n})$
B_n

la of ∇_i:

$$\boxed{\frac{1}{\kappa+h^\vee}\sum_{j \mid j\neq i}\frac{\Omega_{ij}}{z_i - z_j}}$$

$\sum 1\otimes\cdots\otimes B_a\otimes\cdots B^a\otimes\cdots 1$

basis dual basis of $\mathfrak{g}$

KZ eqn.

$]=0$

$+\Omega_{kj}]=0$

$=\cdots=V_n \quad S_n \curvearrowright V^{\otimes n}$

y of ∇:

$\mathrm{nf}_n(\mathbb{C}) \longrightarrow GL(V^{\otimes n})$

B_n

Thm (Kohno Drinfeld 1991)

The monodromy of KZ eqn is equiv to the R-matrix reps of $U_q(\mathfrak{g})$.

$(\mathrm{Rep}(U(\mathfrak{g})), \otimes_{KZ})$

$\cong$

$(\mathrm{Rep}(U_q(\mathfrak{g})), \otimes_R)$

Rmk

1. $\mathbb{P}^1 \rightsquigarrow E$

only change $\mathfrak{g}^E_{out} = \Gamma(E\setminus\{z_1,\ldots,z_n\}, Ad(\mathfrak{g}))$

[Felder - Wieczerkowski] $\Rightarrow$ KZB eqn.

2. $(\mathfrak{gl}_k, \mathfrak{gl}_n)$ duality

Take $\mathfrak{g}=\mathfrak{gl}_k$

$\mathfrak{gl}_n \curvearrowright \mathbb{C}^k\otimes\cdots\otimes\mathbb{C}^k \curvearrowleft \mathfrak{gl}_k$

$\alpha=\varepsilon_i-\varepsilon_j$

$C_\alpha = E_\alpha F_\alpha + F_\alpha E_\alpha + \frac{H_\alpha^2}{2}$ $\quad \Omega_{ij}$

$\in U(\mathfrak{sl}_2^\alpha) \leq U(\mathfrak{gl}_n)$

Casimir Conn [TL

$\nabla = d - \Sigma$

$d r$

☺ : Flat conn

Casimir Conn [TL, DeConcini 2002]

$$\nabla = d - \sum_{\alpha \text{ root}} \frac{C_\alpha}{\alpha} d\alpha$$

☺ : • Flat conn on $\mathfrak{h} \setminus \bigcup_\alpha$ Root hyperplane

• Trivial bundle $\mathfrak{h} \times V_{\mathfrak{g}}$

arb. reps of $\mathfrak{g}$

(II) Goal Construct an elliptic Casimir conn

Example $n=2$

base space $E \setminus \{0\}$

Vector bundle $\mathbb{C} \times V \longrightarrow \mathcal{V} = \mathbb{C} \times V / \mathbb{Z}+\tau\mathbb{Z} = \mathbb{C} \times V / (z,v) \sim (z+1, v), \ (z,v) \sim (z+\tau, e^{2\pi i X} v)$

f. dim v sp

$$\begin{array}{ccc} \mathbb{C} \times V & \longrightarrow & \mathcal{V} \\ \downarrow & & \downarrow \\ \mathbb{C} & \longrightarrow & E \end{array}$$

$X \in \mathrm{End}(V)$

Conn $\nabla = d - A(z)dz \in \Omega^1(\mathbb{C}) \otimes \mathrm{End}(V)$

s.t. ① $A(z+1) = A(z)$

② $A(z+\tau) = e^{2\pi i X} A(z)$

③ $A(z)$ has a simple pole at $z=0$

$T := \mathrm{Res}_{z=0} A(z) \in \mathrm{End}(V)$

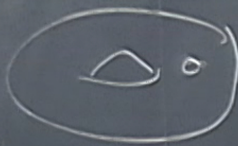
$\Rightarrow \exists !$ So...

$A(z) = -$...

Construct an elliptic Casimir conn.

$n=2$

se space $E\setminus\{0\}$



$\{\text{dim v sp}\}$

bundle $\mathbb{C}\times V \longrightarrow \mathcal{V} = \mathbb{C}\times V - \mathbb{C}\times V \Big/ \begin{matrix}(z,v)\sim(z+1,v)\\ (z,v)\sim(z+\tau, e^{2\pi i X}v)\end{matrix}$ $\quad \mathbb{Z}+\tau\mathbb{Z}$

$$\begin{array}{ccc} \mathbb{C}\times V & & \mathcal{V} \\ \downarrow & & \downarrow \\ \mathbb{C} & \longrightarrow & E \end{array}$$

$X \in \mathrm{End}(V)$

$\nabla = d - A(z)dz \in \Omega^1(\mathbb{C})\otimes \mathrm{End}(V)$

$A(z+1) = A(z)$

$A(z+\tau) = e^{2\pi i X} A(z)$

$A(z)$ has a simple pole at $z=0$

$T := \mathrm{Res}_{z=0} A(z) \in \mathrm{End}(V)$

$\Longrightarrow \exists!$ Solution

$$A(z) = \frac{\theta(z+ad(X))}{\theta(z)\,\theta(ad(X))}(T) \in \mathcal{O}_{E\setminus\{0\}}\otimes \mathrm{End}(V)$$

$$= \underbrace{\left(\frac{\theta(z+adX)}{\theta(z)\,\theta(adX)} - \frac{1}{adX}\right)(T)}_{\text{Universal KZB of rk 1}} + \cancel{\frac{1}{adX}(T)}\ Y$$

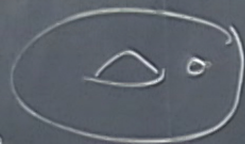
such that

$[X, Y] = T$

Construct an elliptic Casimir conn. $\Longrightarrow \exists !$ Solution

$n=2$

base space $E\setminus\{\tilde{o}\}$



bundle $\mathbb{C}\times V \xrightarrow{} \mathcal{V} = \mathbb{C}\times V - \mathbb{C}\times V \Big/ \begin{matrix}(z,v)\sim(z+1,v)\\ (z,v)\sim(z+\tau, e^{2\pi i X}v)\end{matrix}$

f.dim v sp

$\mathbb{Z}+\tau\mathbb{Z}$

$$\begin{array}{ccc} \mathbb{C}\times V & \longrightarrow & \mathcal{V} \\ \downarrow & & \downarrow \\ \mathbb{C} & \longrightarrow & E \end{array}$$

$X \in \mathrm{End}(V)$

$\nabla = d - A(z)\,dz \in \Omega^1(\mathbb{C})\otimes \mathrm{End}(V)$

1) $A(z+1) = A(z)$

2) $A(z+\tau) = e^{2\pi i X} A(z)$

3) $A(z)$ has a simple pole at $z=0$

$T := \mathrm{Res}_{z=0} A(z) \in \mathrm{End}(V)$

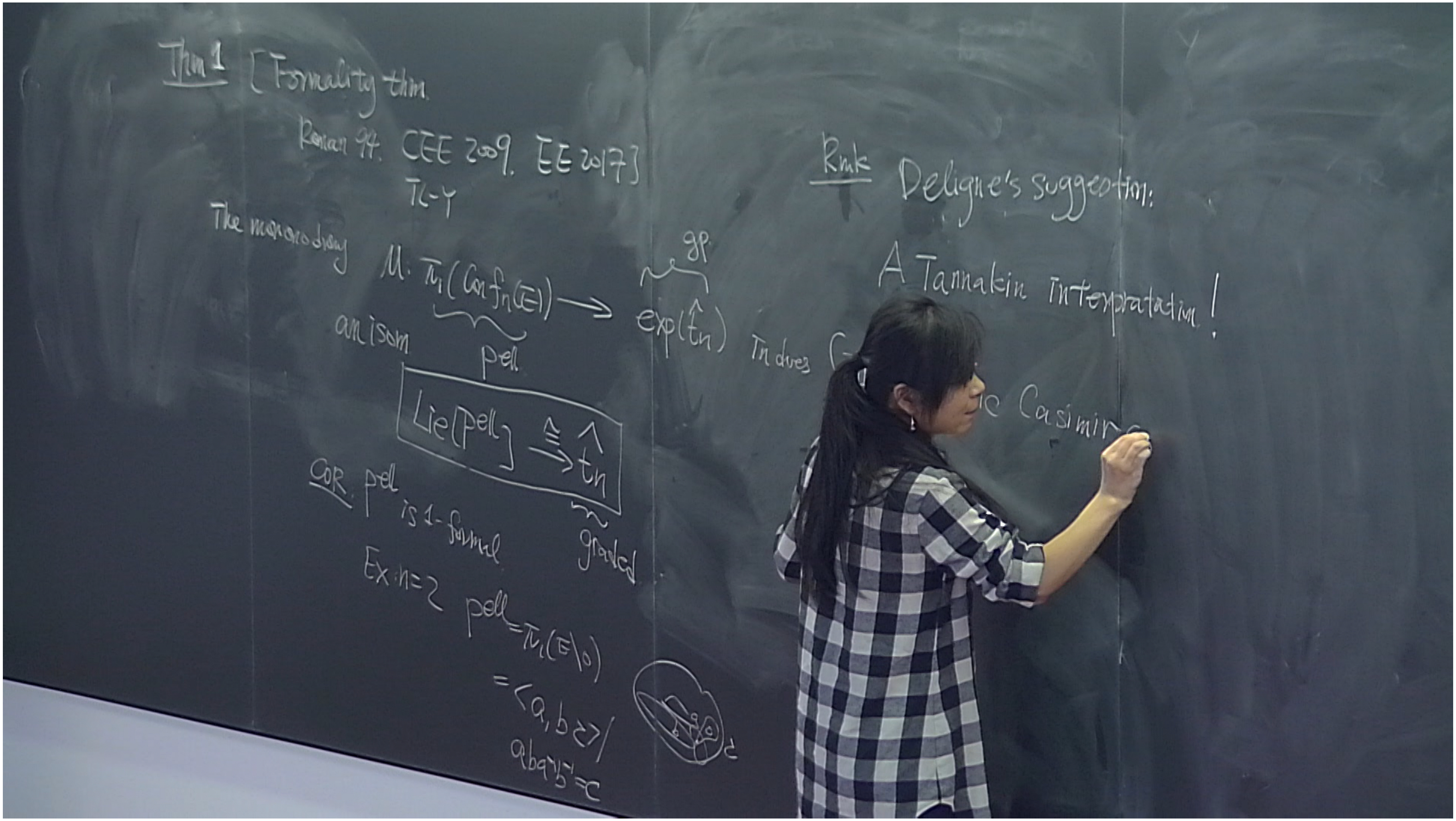
$$A(z) = \frac{\theta(z+\mathrm{ad}(X))}{\theta(z)\,\theta(\mathrm{ad}(X))}(T) \in \mathcal{O}_{E\setminus\{\tilde{o}\}}\otimes \mathrm{End}(V)$$

$$= \underbrace{\left(\frac{\theta(z+\mathrm{ad}X)}{\theta(z)\,\theta(\mathrm{ad}X)} - \frac{1}{\mathrm{ad}X}\right)(T)}_{\text{Universal KZB of rk 1}} + \cancel{\frac{1}{\mathrm{ad}X}(T)}\ Y$$

such that $[X, Y] = T$

In general

- base space $\mathrm{Conf}_n(E)$
- Holonomy Lie alg $t_n = \langle X_1, \dots X_n, Y_1, \dots Y_n, T_{ij} \rangle /$ explicit relation in deg 2, 3
- [CEE] $\exists$ the universal KZB with coeff in $\hat{t}_n$.

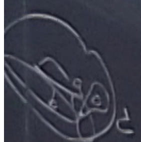

Thm 1 [Formality thm
Reman 94. CEE 2009. EE 2017]
TL-Y
The monodromy
M: π1(Conf_n(E)) → exp(t̂_n)
gp
an isom
pell
Lie(Pell) ≅ → t̂_n
graded
COR. pell is 1-formal
Ex: n=2 pell = π1(E\0)
= <a, b, c>/
aba⁻¹b⁻¹=c
Rmk Deligne's suggestion:
A Tannakin interpratation!
Tn does
Casimir

Rmk Deligne's suggestion:

A Tannakin interpretation!

$\underbrace{\exp(\hat{\mathfrak{t}}_n)}^{\text{gp}}$ Involves (III) Elliptic Casimir conn

(Specialization of KZB eqn)

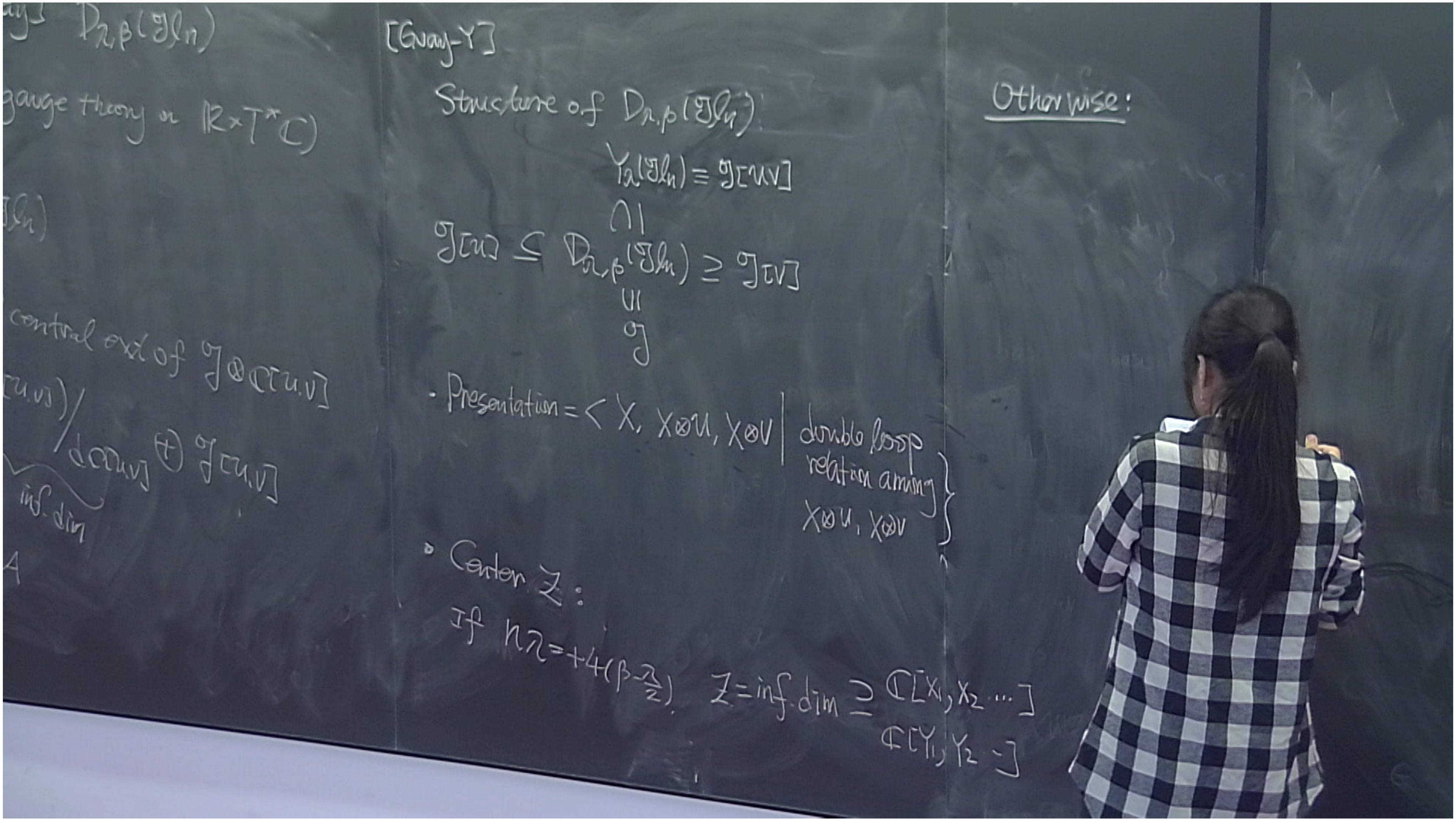


Deformed double current algebra [Guay] $D_{\lambda,\beta}(\mathfrak{gl}_n)$

- Show up in Kevin's work (4 dim gauge theory on $\mathbb{R}\times T^*\mathbb{C}$)
- rational deg. of $\ddot{\mathcal{U}}_{q_1,q_2}(\mathfrak{gl}_n)$
- $\lambda, \beta = 0$: $D_{0,0}(\mathfrak{gl}_n)$ = Universal central ext of $\mathfrak{g}\otimes\mathbb{C}[u,v]$

$$= \underbrace{\Omega^1(\mathbb{C}[u,v]) / d\mathbb{C}[u,v]}_{\text{inf. dim}} \oplus \mathfrak{g}[u,v]$$

- Schur-Weyl dual to rat DAHA



$D_{\lambda,\beta}(\mathfrak{gl}_n)$
gauge theory on $\mathbb{R}\times T^*\mathbb{C}$
central ext of $\mathfrak{g}\otimes\mathbb{C}[u,v]$
$/ d\mathbb{C}[u,v] \oplus \mathfrak{g}[u,v]$
inf. dim
[Guay-Y]
Structure of $D_{\lambda,\beta}(\mathfrak{gl}_n)$:
$Y_\lambda(\mathfrak{gl}_n) \equiv \mathfrak{g}[u]$
$\mathfrak{g}[u] \subseteq D_{\lambda,\beta}(\mathfrak{gl}_n) \supseteq \mathfrak{g}[v]$
$\mathfrak{g}$
· Presentation = < X, $X\otimes u$, $X\otimes v$ | double loop relation among $X\otimes u$, $X\otimes v$ >
· Center Z:
If $n\lambda = +4(\beta - \frac{\lambda}{2})$, Z = inf. dim $\supseteq \mathbb{C}[X_1, X_2 \dots]$ $\mathbb{C}[Y_1, Y_2 \dots]$
Otherwise:

[Guay-Y]

Structure of $D_{\hbar,\beta}(\mathfrak{gl}_n)$

$$\begin{array}{ccccc} & & Y_\hbar(\mathfrak{gl}_n) \equiv \mathfrak{g}[uv] & & \\ & & \cap\mid & & \\ \mathfrak{g}[u] & \subseteq & D_{\hbar,\beta}(\mathfrak{gl}_n) & \supseteq & \mathfrak{g}[v] \\ & & \cup\mid & & \\ & & \mathfrak{g} & & \end{array}$$

- Presentation $= \langle X,\ X\otimes u,\ X\otimes v \mid$ double loop relation among $X\otimes u,\ X\otimes v \}$

- Center Z:

If $n\hbar = \pm 4(\beta - \frac{\hbar}{2})$ $Z = \inf \dim \supseteq \mathbb{C}[X_1, X_2 \cdots]$ $\mathbb{C}[Y_1, Y_2 \cdots]$

Otherwise:

$$[H_{\beta_1}\otimes u,\ H_{\beta_2}\otimes v] \underset{q}{=} \frac{\hbar}{2} \sum_\alpha (\beta_1,\alpha)(\beta_2,\alpha) C_\alpha$$

$(\beta_1, \beta_2) = 0$

If $(\beta_1, \beta_2) \neq 0$ $\rightsquigarrow$ a Central elt in $D_{\hbar,\beta}(\mathfrak{g})$

$\to$ Symmetry on $D_{\hbar,\beta}(\mathfrak{g})$:

- $SL_2(\mathbb{C}) \curvearrowright D$: is inner! $\langle E, F, H, \delta_{2m} \rangle_{m \in \mathbb{N}}$ ($\mathfrak{sl}_2$)

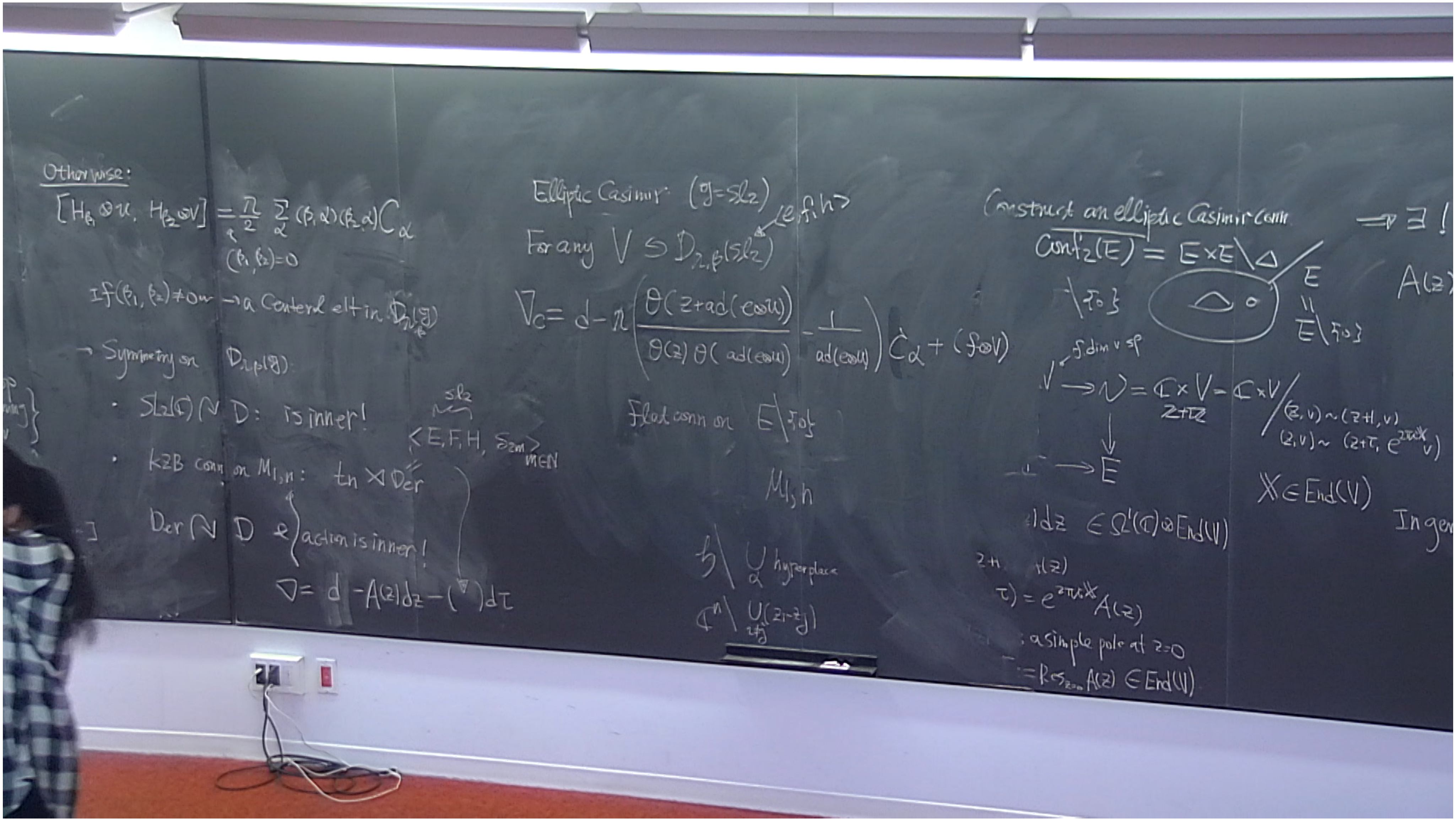
- KZB conn on $M_{1,n}$: $t_n \rtimes Der$

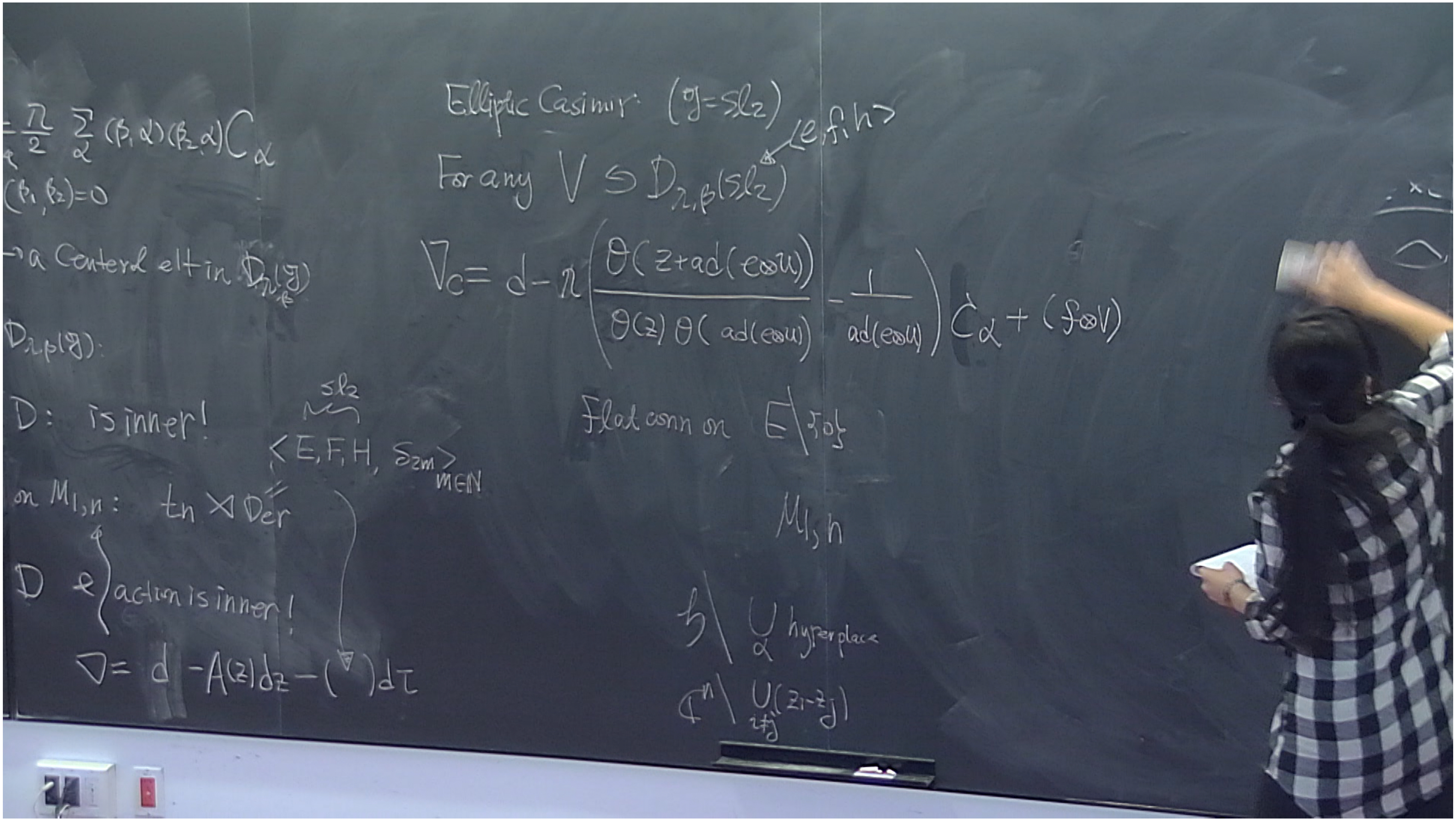
$Der \curvearrowright D$ & action is inner!

imir

- F

- Tr


Otherwise:
$[H_{\beta_1}\otimes u, H_{\beta_2}\otimes v] = \frac{\eta}{2}\sum_\alpha (\beta_1,\alpha)(\beta_2,\alpha) C_\alpha$
$(\beta_1,\beta_2)=0$
If $(\beta_1,\beta_2)\neq 0$ → a Central elt in $D_{\eta,\beta}(\mathfrak{g})$
→ Symmetry on $D_{\eta,\beta}(\mathfrak{g})$
· $Sl_2(\mathbb{Z}) \curvearrowright D$: is inner!
$\langle E, F, H, \delta_{2m}\rangle_{m\in\mathbb{N}}$
· KZB conn on $M_{1,n}$: $t_n \rtimes Der$
$Der \curvearrowright D$ & action is inner!
$\nabla = d - A(z)dz - (\ )d\tau$
Elliptic Casimir: $(\mathfrak{g}=sl_2)$
$\langle e,f,h\rangle$
For any $V \subseteq D_{\eta,\beta}(sl_2)$
$\nabla_C = d - \eta\left(\frac{\theta(z+ad(e\otimes u))}{\theta(z)\theta(ad(e\otimes u))} - \frac{1}{ad(e\otimes u)}\right)C_\alpha + (f\otimes v)$
Flat conn on $E\setminus\{0\}$
$M_{1,n}$
$\mathfrak{h}\setminus \bigcup_\alpha$ hyperplane
$\mathbb{C}^n \setminus \bigcup_{i\neq j}(z_i - z_j)$
Construct an elliptic Casimir conn
$Conf_2(E) = E\times E \setminus \Delta$
$\Rightarrow \exists!$
$A(z)$
E
$E\setminus\{0\}$
f.dim v.sp
$V \longrightarrow \mathcal{V} = \mathbb{C}\times V / \mathbb{Z}+\tau\mathbb{Z}$
$(z,v)\sim(z+1,v)$
$(z,v)\sim(z+\tau, e^{2\pi i X}v)$
$X \in End(V)$
$\longrightarrow E$
$dz \in \Omega^1(\mathbb{C})\otimes End(V)$
$\tau) = e^{2\pi i X} A(z)$
; a simple pole at $z=0$
$:= Res_{z=0} A(z) \in End(V)$
In gen



Elliptic Casimir: (g = sl2)
For any V ⊆ D_{λ,β}(sl2)
<e,f,h>
∇_C = d − λ(θ(z+ad(e⊗u)) / θ(z) θ(ad(e⊗u)) − 1/ad(e⊗u)) C_α + (f⊗V)
Flat conn on E \ {0}
M_{1,n}
h \ ∪_α hyperplane
ℂ^n \ ∪_{i≠j} (z_i − z_j)
λ/2 Σ_α (β1,α)(β2,α) C_α
(β1,β2)=0
→ a Central elt in D_{λ,β}(g)
D_{λ,β}(g):
D: is inner!
sl2
<E, F, H, δ_{2m}> m∈ℕ
on M_{1,n}: t_n ⋊ Der
D e action is inner!
∇ = d − A(z)dz − ( )dτ

$= \frac{\eta}{2}\sum_{\alpha}(\beta_1,\alpha)(\beta_2,\alpha)C_\alpha$

$(\beta_1,\beta_2)=0$

$\to$ a Central elt in $D_{\eta,\beta}(\mathfrak{g})$

$D_{\eta,\beta}(\mathfrak{g})$:

D: is inner! $\quad \langle \underbrace{E,F,H}_{sl_2},\ \delta_{2m}\rangle_{m\in\mathbb{N}}$

on $M_{1,n}$: $\quad t_n \rtimes \mathrm{Der}$

D e action is inner!

$\nabla = d - A(z)dz - (\nabla)d\tau$

Elliptic Casimir: $(\mathfrak{g}=sl_2)$ $\quad \langle e,f,h\rangle$

For any $V \in D_{\eta,\beta}(sl_2)$

$$\nabla_C = d - \eta\left(\frac{\theta(z+ad(e\otimes u))}{\theta(z)\,\theta(ad(e\otimes u))} - \frac{1}{ad(e\otimes u)}\right)\underbrace{C_\alpha}_{=\,ef+fe+\frac{h^2}{2}}d\alpha + (f\otimes V)\,d\alpha$$

$ef+fe+\frac{h^2}{2} \in U(sl_2)$

Flat conn on $E\setminus\{0\}$

$M_{1,n}$

$\mathfrak{h}\setminus\bigcup_{\alpha}$ hyperplace

$\mathbb{C}^n\setminus\bigcup_{i\neq j}(z_i-z_j)$

(IV) Question (Informal)

① KZ eqn & Fusion?

Evidence

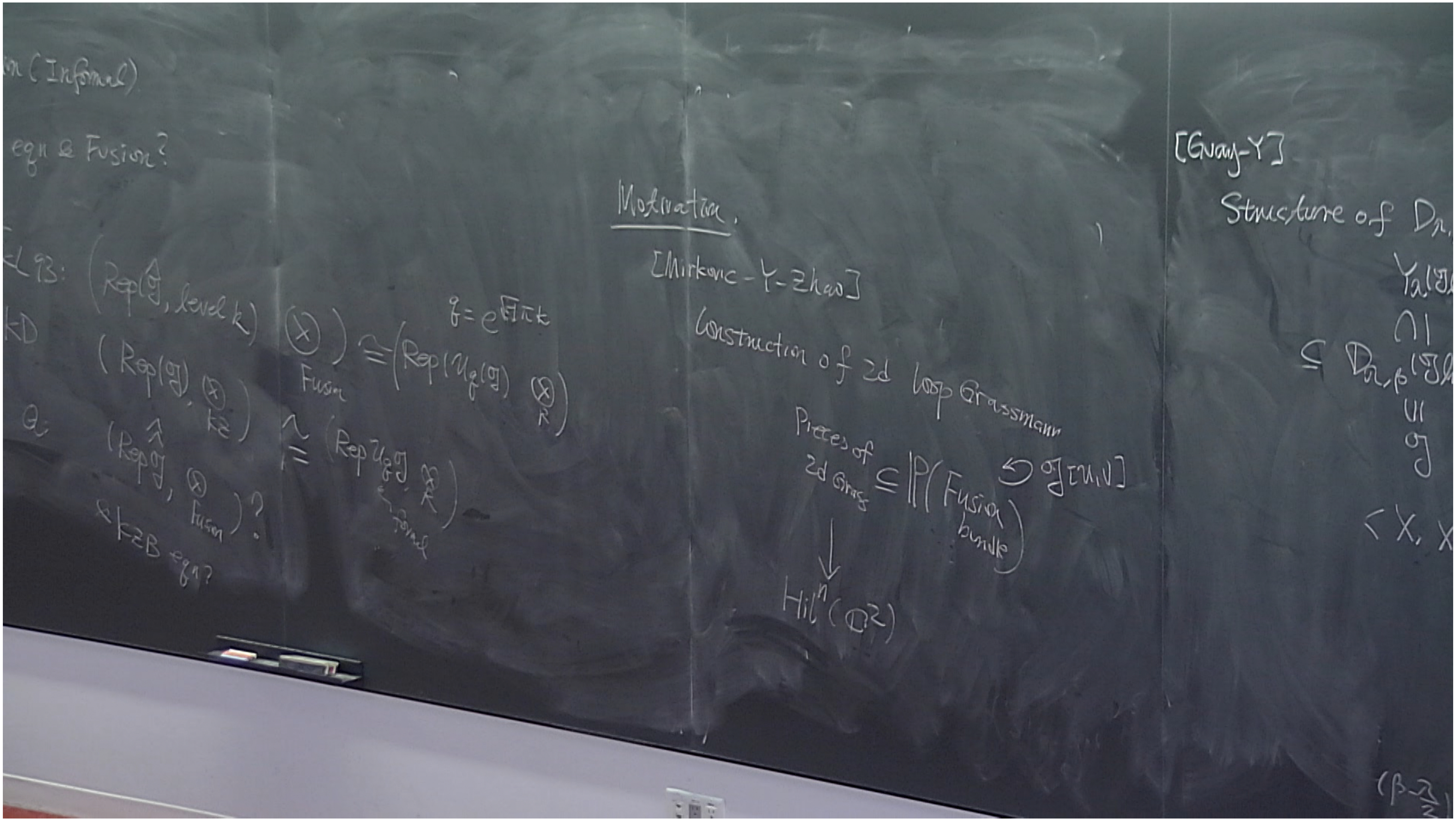
KL 93: $\left(\mathrm{Rep}(\hat{\mathfrak{g}}, \text{level } k) \underset{\text{Fusion}}{\otimes} \right) \cong \left(\mathrm{Rep}(U_q(\mathfrak{g})) \underset{R}{\otimes} \right)$ $\quad q = e^{\sqrt{-1}\pi k}$

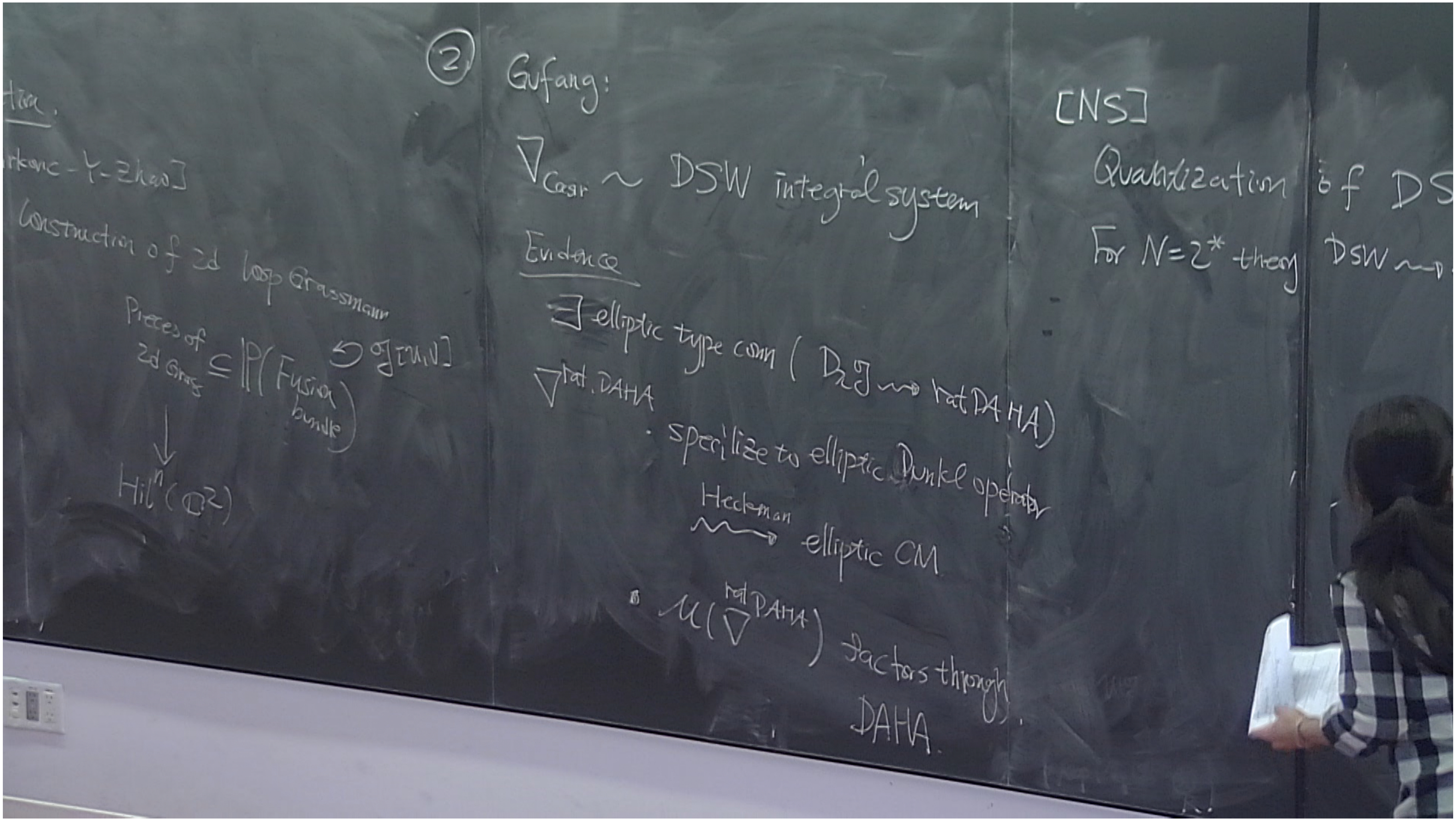
KD: $\left(\mathrm{Rep}(\mathfrak{g}), \underset{KZ}{\otimes} \right) \sim$

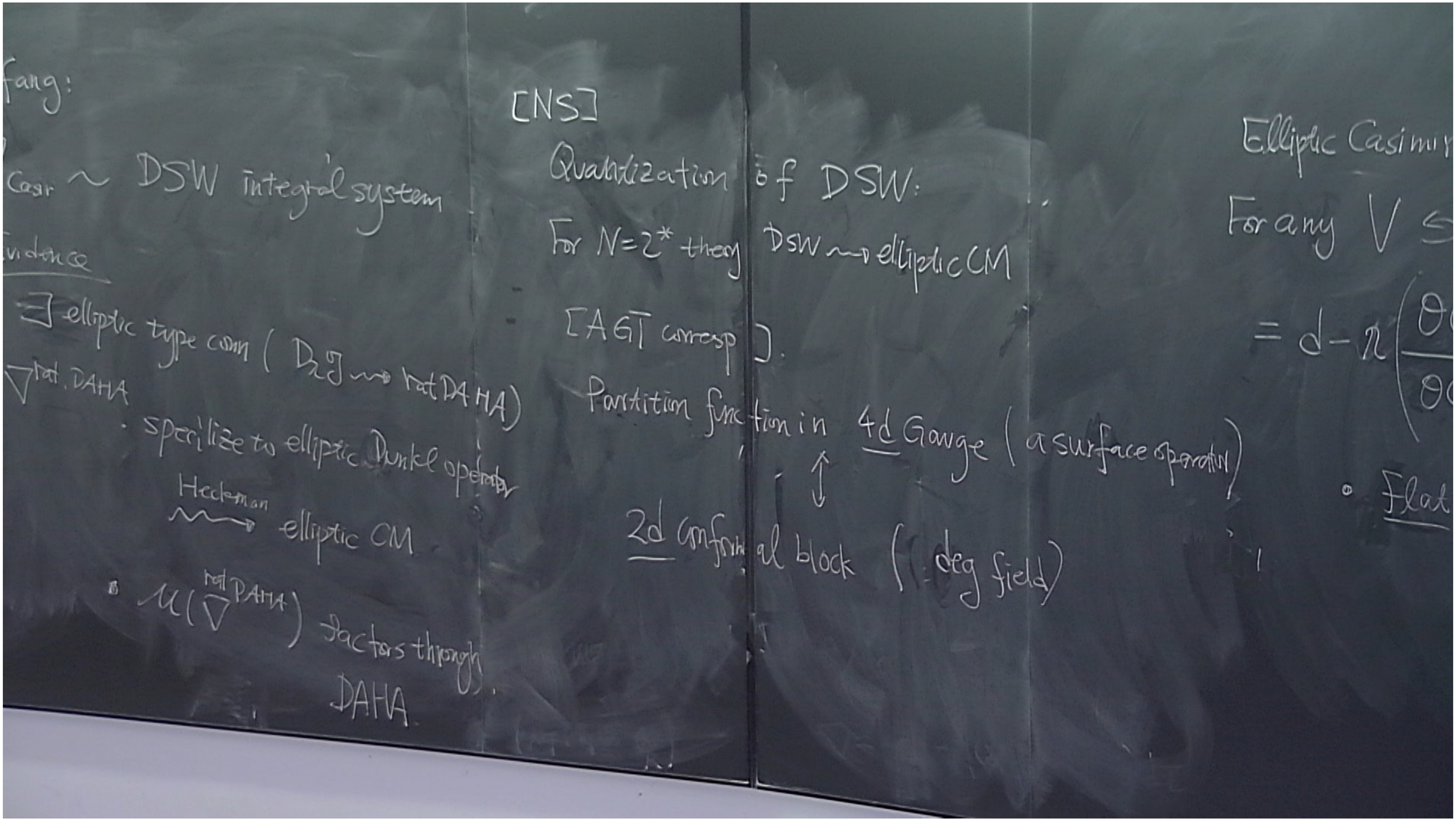
Q: $\left(\mathrm{Rep}\,\hat{\mathfrak{g}}, \underset{\text{Fusion}}{\otimes} \right)$? $\cong \left(\mathrm{Rep}\, U_q\mathfrak{g}, \underset{R}{\otimes} \right)$ ← formal

& KZB eqn?

Motiva


(Informal)
eqn & Fusion?
(Rep($\hat{\mathfrak{g}}$, level k) ⊗) ≅ (Rep($U_q(\mathfrak{g})$) ⊗ R)
$q = e^{\sqrt{-1}\pi k}$
(Rep($\mathfrak{g}$), ⊗ KZ)
Fusion
(Rep $\hat{\mathfrak{g}}$, ⊗ Fusion)?
& KZB eqn?
Motivation.
[Mirkovic - Y - Zhao]
Construction of 2d loop Grassmann
Pieces of 2d Grass ⊆ ℙ(Fusion bundle) ↺ $\mathfrak{g}[u,v]$
$Hil^n(\mathbb{C}^2)$
[Guay-Y]
Structure of


[…arkov-Y-Zhao]
Construction of 2d loop Grassmann
Pieces of 2d Grass ⊆ P(Fusion bundle) ↺ g[u,v]
Hil^n(C^2)
(2) Gufang:
∇_Casr ~ DSW integral system
Evidence
∃ elliptic type conn (D_2 g ⇝ rat DAHA)
∇^rat. DAHA
· specilize to elliptic Dunkl operator
Heckman ⇝ elliptic CM
· M(∇^rat DAHA) factors through DAHA.
[NS]
Quantization of DS…
For N=2* theory DSW ⇝ …


fang:
Casr ~ DSW integral system
vidence
∃ elliptic type conn ($D_2 g$ ⇝ rat DAHA)
$\nabla^{rat.\ DAHA}$
· specilize to elliptic Dunkl operator
Heckman ⇝ elliptic CM
· $\mathcal{M}(\nabla^{rat\ DAHA})$ factors through DAHA.
[NS]
Quantization of DSW:
For $N=2^*$ theory DSW ⇝ elliptic CM
[AGT corresp].
Partition function in 4d Gauge (a surface operator)
↕
2d conformal block (deg field)
Elliptic Casimir
For any V
$= d - \hbar \left(\frac{\theta}{\theta} \right.$
· Flat