Title: Compact binary systems in massless scalar-tensor theories

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Abstract: <p>The observations of gravitational waves from coalescing compact binary systems allow us to test gravity in its strong field regime. In order to better constrain alternative theories of gravity, one has to build template waveforms for these theories. In this talk, I will present a post-Newtonian Lagrangian approach adapted to the specificities of scalar-tensor theories. I will derive the equations of motion of a compact binary system at 3PN order in harmonic coordinates. This result is primordial in order to compute the scalar and gravitational waveforms at 2PN order. I will conclude by a short discussion on tidal effects.</p>

Compact binary systems in massless scalar-tensor theories

Laura BERNARD (IST, Lisbon)

Perimeter Institute seminar

January 25$^{\text{th}}$, 2018

grit
gravitation in técnico

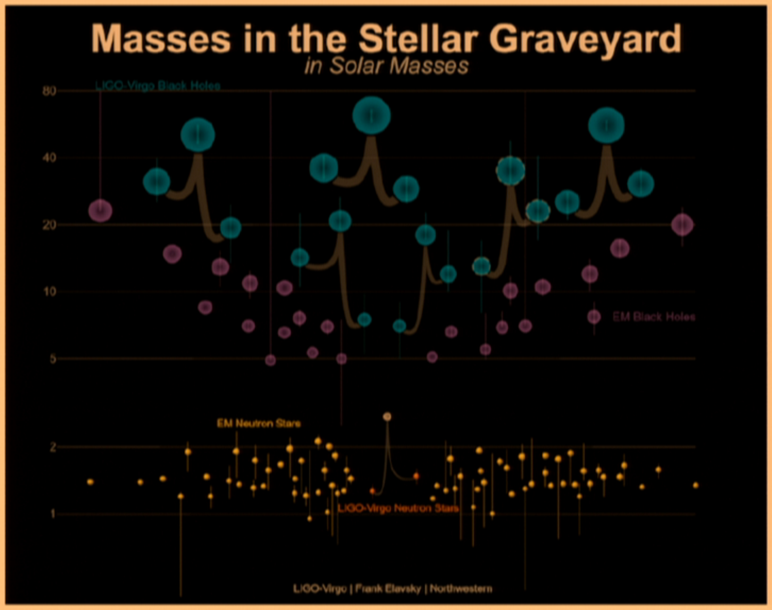
centra
multidisciplinary centre for astrophysics

1. Motivations
2. Massless scalar-tensor theories
3. The 3PN scalar-tensor Fokker Lagrangian
4. First results
5. A word on tidal effects

Weak-field regime

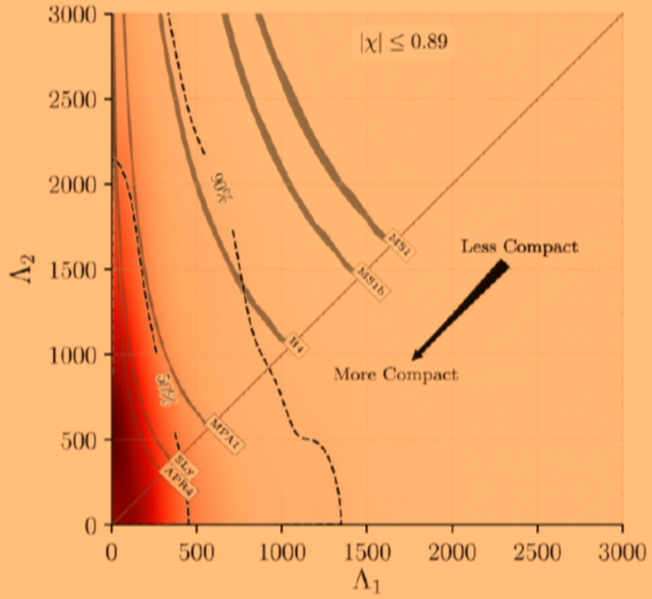
- Solar-system tests : Lunar laser ranging, etc.
- Binary pulsars tests : detection of GWs, strong constraints on GR end its extensions.

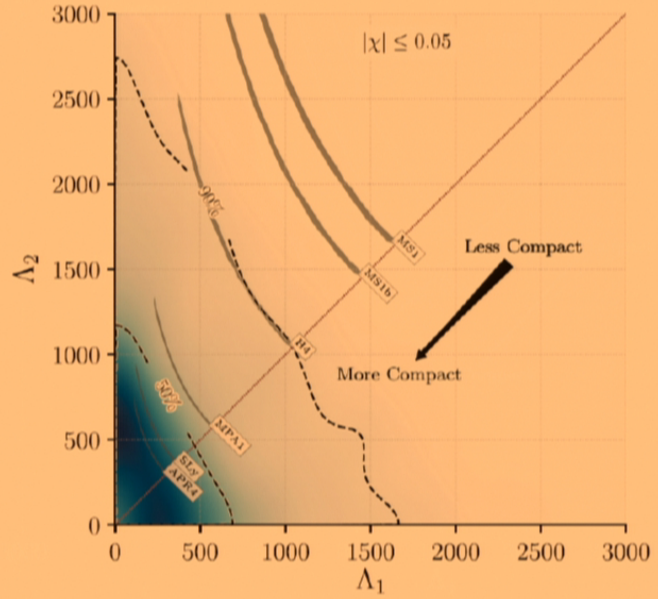
Strong field regime



Matched filtering method

- Need for accurate gravitational waveform templates for GR
- Information about the astrophysical properties of these systems
  - Formation and evolution of compact binaries,
  - Properties of BHs, equation of state of neutron stars, etc.





WHAT IS CURRENTLY DONE

- Theory-independent test,
- Parametrized deviation from GR in waveforms : $h(f) = \mathcal{A}(f)\, \mathrm{e}^{i\, p(f)(1+\delta\hat{p}(f))}$

$$\delta\hat{p} = \left(\underbrace{\delta\hat{\phi}_0,\, \delta\hat{\phi}_1, \delta\hat{\phi}_2, \delta\hat{\phi}_3, \delta\hat{\phi}_4, \delta\hat{\phi}_{5l}, \delta\hat{\phi}_6, \delta\hat{\phi}_{6l}, \delta\hat{\phi}_7,}_{\text{early-inspiral stage}} \underbrace{\delta\hat{\beta}_2, \delta\hat{\beta}_3}_{\text{intermediate regime}} , \underbrace{\delta\hat{\alpha}_2, \delta\hat{\alpha}_3, \delta\hat{\alpha}_4}_{\text{merger-ringdom phase}} \right)$$

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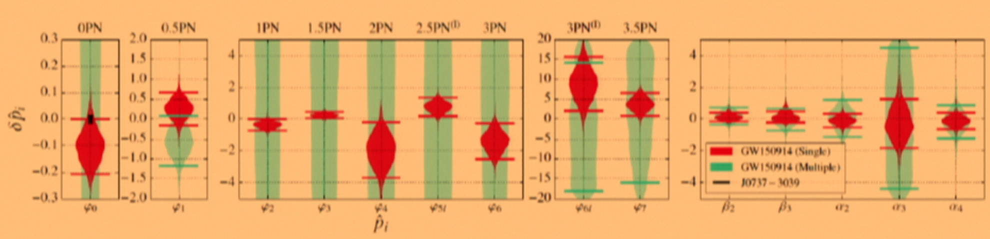
$$\delta\hat{p} = \Bigg(\underbrace{\delta\hat{\phi}_0,\ \delta\hat{\phi}_1, \delta\hat{\phi}_2, \delta\hat{\phi}_3, \delta\hat{\phi}_4, \delta\hat{\phi}_{5l}, \delta\hat{\phi}_6, \delta\hat{\phi}_{6l}, \delta\hat{\phi}_7}_{\text{early-inspiral stage}},\ \underbrace{\delta\hat{\beta}_2, \delta\hat{\beta}_3}_{\text{intermediate regime}}\ ,\ \underbrace{\delta\hat{\alpha}_2, \delta\hat{\alpha}_3, \delta\hat{\alpha}_4}_{\text{merger-ringdom phase}} \Bigg)$$



PRL116, 221101 (2016)

$\Longrightarrow$ No significant deviation from GR

Theory-dependent tests

▷ We also need accurate gravitational waveforms for **alternative theories of gravity**

WHY ?

- High energy regime : quantum completion of GR,
- Low energy regime : dark sectors
  - Accelerated expansion of the universe : cosmological constant problem,
  - Dark matter is not yet detected $\longrightarrow$ New matter and/or modified gravity ?

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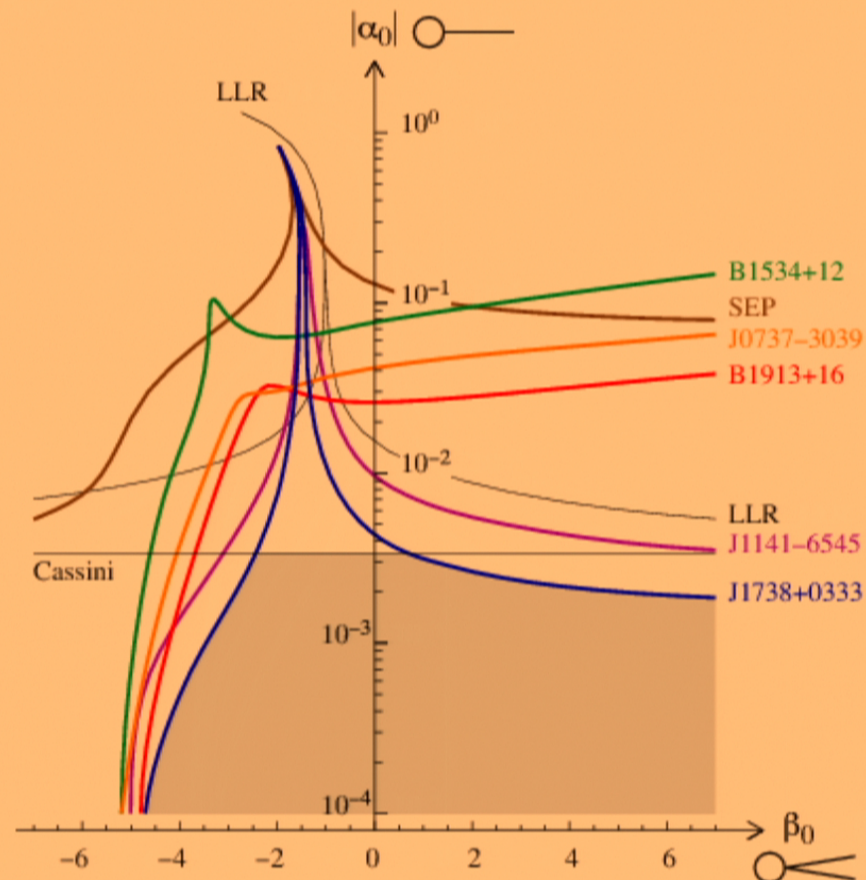
How ?

Relaxe one or several assumptions of GR

- Go to higher dimensions : Kaluza-Klein models, brane-induced gravity,
- Non diffeomorphism invariance : non-local theories,
- Lorentz violation : Einstein-aether, Horava-Lifshitz theories,
- Add extra fields : galileon theories, massive gravity.

- First introduced by Jordan, Fierz, Brans and Dicke more than 50 years ago,
- Only one additional massless scalar field, minimally coupled to gravity.
- It is the **simplest**, well motivated and most studied alternative theory of gravity,
  - It passes weak-field tests, *i.e.* in the Solar System and binary pulsar tests,
  - Strong constraints on the parameters of the theory.

$$m_A(\varphi) = \bar{m}_A \, \mathrm{e}^{\alpha_0 \varphi + \frac{1}{2}\beta_0 \varphi^2}$$



[Freire et al. (2012)]

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- It is the **simplest**, well motivated and most studied alternative theory of gravity,
  - It passes weak-field tests, *i.e.* in the Solar System and binary pulsar tests,
  - Strong constraints on the parameters of the theory.
- Binary BHs gravitational radiation indistinguishable from GR (Hawking, 1972)),
- But strong deviations from GR are expected for neutron stars (scalarization).

THE ACTION

$$S_{\mathrm{ST}} = \frac{c^3}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}g^{\alpha\beta}\partial_\alpha\partial_\beta\phi\right] + S_m\left(\mathfrak{m}, g_{\alpha\beta}\right)$$

- Metric $g_{\mu\nu}$,
- Scalar field ϕ and scalar function $\omega(\phi)$,
- Matter fields $\mathfrak{m}$, minimally coupled to the physical metric,
- No potential or mass for the scalar field.
- No direct coupling between the matter and scalar fields,

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Metric (Jordan) frame

▷ **Physical metric $g_{\alpha\beta}$** : Scalar field only coupled to the gravitational sector,

▷ Frame for physical results and observations.

CONFORMAL (EINSTEIN) FRAME

$$\tilde{g}_{\mu\nu} = \varphi\, g_{\mu\nu}\,, \qquad \varphi = \frac{\phi}{\phi_0}$$

The action becomes,

$$S_{\mathrm{ST}} = \frac{c^3\,\phi_0}{16\pi G}\int \mathrm{d}^4x\sqrt{-\tilde{g}}\left[\tilde{R} + \frac{3}{\varphi}\tilde{g}^{\alpha\beta}\nabla_\alpha\partial_\beta\varphi - \frac{9+2\omega(\phi)}{2\varphi^2}\tilde{g}^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi\right] + S_m\left(\mathfrak{m},\frac{\tilde{g}_{\alpha\beta}}{\varphi}\right)$$

- Scalar field only coupled to the matter sector.
- Simpler to do calculations.
- In vacuum : decoupling between the scalar and gravitational fields

$\Longrightarrow$ Black hole solutions are the same as in GR.

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- In ST theories : **violation of the Strong Equivalence Principle**,

▷ We need to incorporate the internal structure of compact, self-gravitationg bodies.

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- In ST theories : **violation of the Strong Equivalence Principle**,

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EARDLEY'S APPROACH

Self-gravitating bodies : the masses depend on the scalar field $M_A(\phi)$ (Eardley, 1975),

$$S_{\mathrm{m}} = -\sum_A \int \mathrm{d}t\, M_A(\phi)\, c^2 \sqrt{-g_{\alpha\beta}\frac{v_A^\alpha v_A^\beta}{c^2}}$$

Field equations

$$G_{\mu\nu} = \frac{8\pi G}{c^4\,\phi}T_{\mu\nu} + \frac{\omega(\phi)}{\phi^2}\left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial_\lambda\phi\partial^\lambda\phi\right) + \frac{1}{\phi}\left(\partial_{\mu\nu}\phi - g_{\mu\nu}\Box_g\phi\right)$$

$$\Box_g\phi = \frac{1}{3+2\omega(\phi)}\left(\frac{8\pi G}{c^4}T - \frac{16\pi G}{c^4}\phi\frac{\partial T}{\partial\phi} - \omega'(\phi)\partial_\lambda\phi\partial^\lambda\phi\right)$$

- Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}$,
- Matter stress-energy tensor : $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}$, $T \equiv g^{\mu\nu}T_{\mu\nu}$.

Far from the system : $\phi = \phi_0 = \text{cst}$ and $\omega(\phi_0) = \omega_0$,

- Sensitivities : $s_A = \left.\frac{\mathrm{d}\ln M_A(\phi)}{\mathrm{d}\ln\phi}\right|_0$, and all higher order derivatives,
  - Black holes : $s_A = 1/2$
  - Neutron stars : $s_A \sim 0.2$ (depends on the eos),
  - related to the scalar charge $\alpha_A \propto 1 - 2s_A$.
- Derivatives of the scalar function $\omega(\phi)$, *i.e.* $\left.\frac{\mathrm{d}\omega}{\mathrm{d}\phi}\right|_0$.

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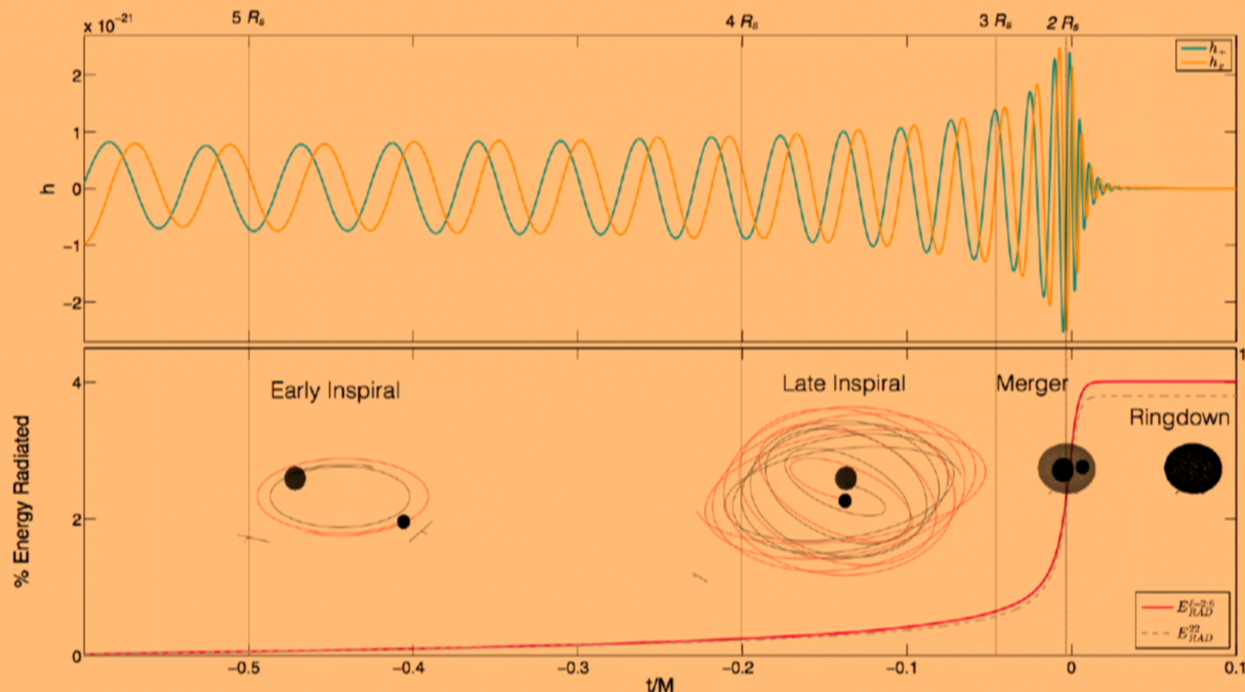
NEWTONIAN RESULT

$$\mathbf{a_{1,\,N}} = -\frac{\tilde{G}\alpha\, m_2}{r_{12}^2}\mathbf{n_{12}}$$

- ST parameters : $\tilde{G} = \frac{G(4+2\omega_0)}{\phi_0(3+2\omega_0)}$, $\alpha = \frac{2+\omega_0-s_1-s_2+2s_1s_2}{2+\omega_0}$,

▷ **Indistinguishable from GR**, effective gravitational constant $G_{\text{eff}} = \tilde{G}\alpha$.

The complete waveform



post-Newtonian formalism   numerical relativity   perturbation theory

Post-Newtonian source

- Isolated, compact, **slowly moving** and **weakly stressed** source,

$$\epsilon \equiv \frac{v^2}{c^2} \sim \frac{Gm}{rc^2} \ll 1$$

- We can develop perturbatively the dynamics in $\epsilon \sim \frac{v^2}{c^2} \ll 1$.

post-Newtonian order : $1\mathrm{PN} = \mathcal{O}\left(\frac{1}{c^2}\right) \equiv \mathcal{O}\left(2\right)$.

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STATE OF THE ART IN GENERAL RELATIVITY

- Gravitational waveform up to 3.5PN, including spin effects,
- Energy flux at 4.5PN,
- **Equations of motion at 4PN :**
  - **Full result without any ambiguity parameter in harmonic coordinates**,
  - Full result with one ambiguity parameter in ADM coordinates,
  - Partial result in effective field theory,

▷ Dynamics at 4.5PN, and gravitational waveform at 4PN : on-going.

Scalar tensor waveforms

- Equations of motion at 2.5PN [Mirshekari & Will, 2013],
- Tensor gravitational waveform to 2PN [Lang, 2013],
- Scalar waveform to 1.5PN : **it starts at -0.5PN** [Lang, 2014],
- Energy flux to 1PN beyond the leading order : **it starts at -1PN** [Lang, 2014],

$$\frac{\mathrm{d}E_{\text{dipole}}}{\mathrm{d}t} = \frac{4m\nu^2}{3rc^3}\left(\frac{\tilde{G}\alpha m}{r}\right)^3 \frac{(s_2 - s_1)^2}{\alpha(4+2\omega_0)}$$

SCALAR TENSOR WAVEFORMS

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SOME REMARKS

- So far done using the DIRE method [Pati & Will, 2000] ,
- Flux and gravitational waveform at 2PN : on-going (A. Heffernan, C. Will),

▷ We need the EoM at 3PN .

- **In the near zone :** post-Newtonian expansion

$$\bar{h}^{\mu\nu} = \sum_{m=2}^{\infty} \frac{1}{c^m} \bar{h}^{\mu\nu}_m \,, \quad \text{with} \quad \Box \bar{h}^{\mu\nu}_m = 16\pi G \, \bar{\tau}^{\mu\nu}_m \,,$$

$$\bar{\psi} = \sum_{m=2}^{\infty} \frac{1}{c^m} \bar{\psi}_m \,, \quad \text{with} \quad \Box \bar{\psi}_m = -8\pi G \, \bar{\tau}^{(s)}_m$$

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- **In the wave zone :** multipolar expansion

$$\mathcal{M}(h)^{\alpha\beta} = \sum_{n=1}^{\infty} G^n h^{\alpha\beta}_{(n)} \,, \quad \text{with} \quad \Box h^{\alpha\beta}_{(n)} = \Lambda^{\alpha\beta}_n \left[h_{(1)}, \ldots, h_{(n-1)}; \psi \right] ,$$

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- **Buffer zone** $\Longrightarrow$ matching between the near zone and far zone solutions :

$$\overline{\mathcal{M}(h)} = \mathcal{M}\left(\bar{h}\right) \quad \text{everywhere,}$$

$$\overline{\mathcal{M}(\psi)} = \mathcal{M}\left(\bar{\psi}\right) \quad \text{everywhere.}$$

WHAT IS THE FOKKER LAGRANGIAN ?

- Replace the gravitational degrees of freedom by their solution

$$S_{\text{Fokker}}\left[y_A, v_A, \dots\right] = S\left[g_{\text{sol}}\left(y_B, v_B, \dots\right), \phi_{\text{sol}}\left(y_B, v_B, \dots\right); v_A\right]$$

- **Generalized Lagrangian** : dependent on the accelerations and their derivatives,
- It describes exactly the dynamics of the original action.

$$\frac{\delta S_{\text{Fokker}}}{\delta y_A^i} = \left.\frac{\delta S}{\delta y_A^i}\right|_{g=g_{\text{sol}},\phi=\phi_{\text{sol}}} + \underbrace{\left.\frac{\delta S}{\delta F}\right|_{g=g_{\text{sol}},\phi=\phi_{\text{sol}}} \frac{\delta F}{\delta y_A^i}}_{=0}, \qquad F = g\,,\phi$$

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WHY A FOKKER LAGRANGIAN ?

- Only for the conservative part,
- **The "$\mathbf{n+2}$" method** : we need to know the metric at only half the order we would have expected,

$$\mathcal{O}(n+2) \quad \text{instead of} \quad \mathcal{O}(2n)\,.$$

The gravitational part

- Rescaled scalar field : $\varphi = \frac{\phi}{\phi_0}$,
- From the conformal metric $\tilde{g}_{\mu\nu} = \varphi g_{\mu\nu}$ to the gothic metric $\tilde{\mathfrak{g}}^{\mu\nu} = \sqrt{\tilde{g}}\tilde{g}^{\mu\nu}$,

$$S_{\mathrm{ST}} = \frac{c^3\phi_0}{32\pi G}\int \mathrm{d}^4x \Bigg[-\frac{1}{2}\left(\tilde{\mathfrak{g}}_{\mu\sigma}\tilde{\mathfrak{g}}_{\mu\rho} - \frac{1}{2}\tilde{\mathfrak{g}}_{\mu\nu}\tilde{\mathfrak{g}}_{\rho\sigma}\right)\tilde{\mathfrak{g}}^{\lambda\gamma}\partial_\lambda\tilde{\mathfrak{g}}^{\mu\nu}\partial_\gamma\tilde{\mathfrak{g}}^{\rho\sigma} + \tilde{\mathfrak{g}}_{\mu\nu}\left(\partial_\sigma\tilde{\mathfrak{g}}^{\rho\mu}\partial_\rho\tilde{\mathfrak{g}}^{\sigma\nu} - \partial_\rho\tilde{\mathfrak{g}}^{\rho\mu}\partial_\sigma\tilde{\mathfrak{g}}^{\sigma\nu}\right) - \frac{3+2\omega}{\varphi^2}\tilde{\mathfrak{g}}^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi\Bigg]$$

▷ gauge-fixing term $-\frac{1}{2}\tilde{g}_{\mu\nu}\tilde{\Gamma}^\mu\tilde{\Gamma}^\nu \longrightarrow$ harmonic coordinates $\partial_\nu h^{\mu\nu} = 0$

The matter part

$$S_{\mathrm{m}} = -\sum_A \int \mathrm{d}t\, M_A(\phi)\, c^2 \sqrt{-g_{\alpha\beta}\frac{v_A^\alpha v_A^\beta}{c^2}}$$

▷ depends on the scalar field through the masses and the physical metric $g_{\alpha\beta}$

Post-Newtonian formalism : from GR to ST theories

- Perturbed metric $h^{\mu\nu} = \tilde{\mathfrak{g}}^{\mu\nu} - \eta^{\mu\nu}$ and scalar field $\psi = \varphi - 1$,
- At leading order $(h, \psi) = (h^{00ii} = h^{00} + h^{ii}, h^{0i}, h^{ij};\, \psi) = \mathcal{O}\,(2, 3, 4;\, 2)$

CANCELLATIONS BETWEEN GRAVITATIONAL AND MATTER TERMS

- consider the solution (n odd)

$$\overline{h}^{\mu\nu}_n = \mathcal{O}(n+1, n+2, n+1; n+1),$$

such that

$$\frac{\delta S_{\mathsf{F}}}{\delta \overline{h}^{00ii}}\left[\overline{h}_n\right] = \mathcal{O}(n-1),\ \frac{\delta S_{\mathsf{F}}}{\delta \overline{h}^{0i}}\left[\overline{h}_n\right] = \mathcal{O}(n),\ \frac{\delta S_{\mathsf{F}}}{\delta \overline{h}^{ij}}\left[\overline{h}_n\right] = \mathcal{O}(n-1),\ \frac{\delta S_{\mathsf{F}}}{\delta \overline{\psi}}\left[\overline{h}_n\right] = \mathcal{O}(n-1)$$

- define the rests

$$\overline{r}_{n+2} = (\overline{r}^{00ii}_{n+3}, \overline{r}^{0i}_{n+4}, \overline{r}^{ij}_{n+3}, \overline{r}^{\psi}_{n+3}) = \mathcal{O}(n+3, n+4, n+3; n+3),$$

- expand the action,

$$S_{\mathsf{F}}\left[\overline{h}\right] = S_{\mathsf{F}}\left[\overline{h}_n\right] + \int \left[\frac{\delta S_{\mathsf{F}}}{\delta \overline{h}^{00ii}}\left[\overline{h}_n\right]\overline{r}^{00ii}_{n+3} + \frac{\delta S_{\mathsf{F}}}{\delta \overline{h}^{0i}}\left[\overline{h}_n\right]\overline{r}^{0i}_{n+4} + \frac{\delta S_{\mathsf{F}}}{\delta \overline{h}^{ij}}\left[\overline{h}_n\right]\overline{r}^{ij}_{n+3} + \frac{\delta S_{\mathsf{F}}}{\delta \overline{\psi}}\left[\overline{h}_n\right]\overline{r}^{\psi}_{n+3} + \cdots\right]$$

$$= \mathcal{O}\left(\frac{1}{c^{2n}}\right).$$

▷ **For ST at 3PN :** $\mathcal{O}\left(4, 5, 4;\ 4\right)$

- Perturbed metric $h^{\mu\nu} = \tilde{\mathfrak{g}}^{\mu\nu} - \eta^{\mu\nu}$ and scalar field $\psi = \varphi - 1$,
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The "$n+2$" method in ST : $\mathcal{O}(4, 5, 4;\ 4)$

$$h^{00ii} = -\frac{4V}{c^2} - \frac{8V^2}{c^4} + \mathcal{O}\left(\frac{1}{c^6}\right),$$
$$h^{0i} = -\frac{4V^i}{c^3} - \frac{8}{c^5}\left(R_i + VV^i\right) + \mathcal{O}\left(\frac{1}{c^7}\right),$$
$$h^{ij} = -\frac{4}{c^4}\left(W_{ij} - \frac{1}{2}\delta_{ij}W\right) + \mathcal{O}\left(\frac{1}{c^6}\right)$$
$$\psi = -\frac{2\psi_{(0)}}{c^2} + \frac{2}{c^4}\left(1 - \frac{\phi_0\omega_0'}{3 + 2\omega_0}\right)\psi_{(0)}^2 + \mathcal{O}\left(\frac{1}{c^6}\right),$$

▷ We need V, V^i and $\psi_{(0)}$ at 1PN and R^i, W_{ij} at N,

$$\Delta W_{ij} = -\frac{4\pi G}{\phi_0}\left(\sigma_{ij} - \delta_{ij}\sigma_{kk}\right) - \partial_i V\,\partial_j V - (3 + 2\omega_0)\partial_i\psi_{(0)}\,\partial_j\psi_{(0)}$$

1. Compute the (local) Fokker Lagrangian in 3 dimensions :
   - using Hadamard-type regularisation for both infrared and ultraviolet divergences.

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   - ▷ using Hadamard-type regularisation for both infrared and ultraviolet divergences.

2. Treat all the divergences by **dimensional regularisation,**

1- Compute the 3PN ST Lagrangian using Hadamard regularisation

UV divergences

- Compact bodies modelised by point particles , *i.e.* $\delta^{(3)}\left(\mathbf{x}-\mathbf{y}_A(t)\right)$

  $\Longrightarrow$ divergences at the position of the particles.

$$I = Pf_{l_1,l_2} \int \mathrm{d}^3x \, \overline{\mathcal{L}}$$

- **Two constants of regularisation l_1 and l_2.**

1- Compute the 3PN ST Lagrangian using Hadamard regularisation

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IR divergence

- Post-Newtonian solution valid only in the near zone $\Longrightarrow$ divergences at infinity,

$$I = \mathrm{FP}_{B=0} \int \mathrm{d}^3x \left(\frac{r}{\mathbf{r_0}}\right)^B \overline{\mathcal{L}}$$

- **at 3PN in ST contributions !**
  - **constant of regularisation $\mathbf{r_0}$** : does not vanish through a shift,
  - vanishes in the GR limit ($\omega_0 \to \infty$) and when $s_1 = s_2$ or $s_{1\,\mathrm{or}\,2} = \frac{1}{2}$ (BHs).

2- Use dimensional regularisation for the UV divergences

1. In $d = 3 + \varepsilon$ spatial dimensions :
   - $G \to G\, \ell_0^{d-3}$,
   - we expand all functions near $r_1 \to 0$:

$$F^{(d)}(\mathbf{x}) = \sum_{p \geqslant -p_0} \sum_{q=-q_0}^{q_1} r_1^{p+q\varepsilon} \, f_{p,q}^{(\varepsilon)}(\mathbf{n})$$

2. Compute the difference between HR and DR through the formula

$$\mathcal{D}I = \frac{1}{\varepsilon} \sum_{q=q_0}^{q_1} \left[\frac{1}{q+1} + \varepsilon \ln l_1 \right] \langle f_{-3,q}^{(\varepsilon)} \rangle + (1 \leftrightarrow 2)$$

3. Expand the Lagrangian in the limit $\varepsilon \to 0$.

2- Use dimensional regularisation for the IR divergences

- We expand all functions near $r \to \infty$:

$$F^{(d)}(\mathbf{x}) = \sum_{p \geqslant -p_0} \sum_{q=-q_0}^{q_1} \frac{1}{r^p} \left(\frac{\ell_0}{r}\right)^{q\varepsilon} f^{(\varepsilon)}_{p,q}(\mathbf{n})$$

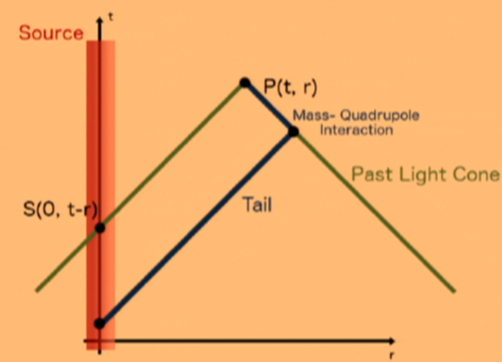
- Compute the difference between HR and DR through the formula

$$\mathcal{D}I = \sum_q \left[\frac{1}{(q-1)\varepsilon} - \ln\left(\frac{r_0}{\ell_0}\right)\right] \int \mathrm{d}\Omega_{2+\varepsilon}\, f^{(\varepsilon)}_{3,q}(\mathbf{n}) + \mathcal{O}(\varepsilon)$$

- Expand the Lagrangian in the limit $\varepsilon \to 0$.

Result

- No more the constant r_0 : ok
- **Presence of a pole $\frac{1}{\varepsilon}$** : does not vanish through a redefinition of the trajectory of the particles !



A scalar tail effect

- Presence of te IR pole $\Longrightarrow$ Non-local tail terms in the conservative dynamics at 3PN :

$$L_{\text{tail}} = \frac{2G^2M}{3c^6}(3+2\omega_0)I_i^{(2)}(t)\int_0^{+\infty}\mathrm{d}t\left[\ln\left(\frac{c\sqrt{\bar{q}}\tau}{2\ell_0}\right)-\frac{1}{2\varepsilon}-\frac{5}{4(3+2\omega_0)}+\frac{23}{12}\right]I_i^{(3)}(t-\tau)$$

▷ Exactly compensate the pole $1/\varepsilon$ from the dimensional regularisation of the IR divergences.

- New effect in ST theories, due to the fact that the scalar field flux starts at -1PN.

Equations of motion at 2PN

- Easy and "quick" calculation : $\mathcal{O}(4, 3, 4; 4)$, only Hadamard regularisation,
- Confirmation of the previous result by Mirshekari & Will (2013).

At 3PN with Hadamard regularisation (3 dimensions)

- Fokker Lagrangian using Hadamard regularisation,
- Tail term using a Hadamard-type regularisation,
- Some consistency checks :
  - GR limit : $\omega_0 \to \infty \Longrightarrow$ GR result,
  - Two black hole limit : $s_1 = s_2 = \frac{1}{2} \Longrightarrow$ indistinguishable from GR.

IMPLEMENTATION OF THE DIMENSIONAL REGULARISATION

- Dimensional regularisation for the UV and IR divergences,
- 3PN scalar tail term in d dimensions,
- Tests of the results and regularisation procedure
  - GR and 2-black-hole limits : ok
  - Renormalisation of the trajectories $\Longleftrightarrow$ the poles disappear : ok
  - Lorentz invariance : ok

ON-GOING CALCULATIONS

- Poincaré group symmetry $\longrightarrow$ 10 conserved quantities
  - Energy, angular momentum, linear momentum, center-of-mass
  - Carefull treatment of the non-local contribution

On-going calculations

- Poincaré group symmetry $\longrightarrow$ 10 conserved quantities
  - Energy, angular momentum, linear momentum, center-of-mass
  - Carefull treatment of the non-local contribution

What's next ?

▷ Ready to use eom to be incorporated in the scalar waveform and the scalar flux at 2PN,

▷ Incorporate the tidal effects (important for neutron stars).

Finite-size effects in GR

$$S_{\text{extended bodies}} = S_{\text{p.p.}} + \int \left(k_2 C^2_{\mu\nu\rho\sigma} + k_4 C^2 u^2 + \text{etc.}\right) c\,\mathrm{d}s$$

$C^{\mu}_{\nu\rho\sigma}$: Weil tensor, $C \sim R^{-2}$.

- $k \sim mR^4 \sim m\left(\frac{Gm}{c^2}\right)^4 \Longrightarrow$ corrections $kc^2C^2 \sim \mathcal{O}\left(\frac{1}{c^{10}}\right)$: starts at 5PN.
- Tidal Love number $Q_{ij} = -\lambda_2\,\mathcal{E}_{ij}$, with $\lambda_2 \sim R^5$

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MORE PRECISELY

- Electric-type tidal Love number k_l^{E} : $Q_L = -\frac{2(l-2)!}{(2l-1)!!}\, k_l^{\mathrm{E}} R_{2l+1} \mathcal{E}_L$ with $\mathcal{E}_{a_1\cdots a_l} \propto C_{a_1 0 a_2 0; a_3\cdots a_l}$,
- Similar definition for the magnetic-type tidal Love number k_l^{B},

▷ **Tidal love numbers for black holes are zero.**

Finite-size effects in scalar-tensor theories

$$S_{\mathrm{m}} = -\sum_A \int \mathrm{d}t\, m[\varphi]\, c^2 \sqrt{-g_{\alpha\beta}\frac{v_A^\alpha v_A^\beta}{c^2}}$$

with $m(\varphi) \longrightarrow m[\varphi, \tilde{g}_{\mu\nu}] = m(\varphi) + N(\varphi)\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \text{etc.}$

- with $N \sim mR^2 \sim m\left(\frac{Gm}{c^2}\right)^2$

▷ corrections to the motion $N[\varphi]c^2(\partial\varphi)^2 \sim \mathcal{O}\left(\frac{1}{c^6}\right)$: **starts at 3PN !**

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Some properties

- 3 types of TLNs : $k_L^{\rm E}$, $k_L^{\rm B}$ and $k_L^{\rm S}$,
- In Brans-Dicke theory ($\omega = cste$) tidal Love number for BHs are zeros,
- Compute tidal Love numbers for ST theories in general.

Equations of motion at 3PN in scalar-tensor theories

- Equations of motion at 3PN in harmonic coordinates ,
- Conserved quantities : in progress.

EQUATIONS OF MOTION AT 3PN IN SCALAR-TENSOR THEORIES

- Equations of motion at 3PN in harmonic coordinates ,
- Conserved quantities : in progress.

PROSPECTS

- Waveform for ST theories at 2.5PN,
- Tidal effects (start at 3PN or even at lower order for ST theories),
- Construct a full IMR waveform,
  - include dynamical scalarisation
  - EOB waveform
- Comparison with gravitational self-force results and numerical relativity in ST theories.

WE NEED TO BUILD PRECISE GRAVITATIONAL WAVEFORM TEMPLATES FOR **ALTERNATIVE THEORIES OF GRAVITY !**