Title: Axion strings and domain walls

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Abstract: <p>I will discuss the calculation of the axion dark matter relic abundance produced by strings and domain walls in the early universe. These objects appear if the global symmetry that the axion is associated with is unbroken at the end of inflation, and in this scenario there is, in principle, a unique prediction for the axion dark matter mass. I will present results from numerical simulations that indicate that the density of strings may be significantly larger than previously thought, leading to a corresponding change in the required axion dark matter mass. I will also discuss the difficulties in computing the energy spectrum of axions produced in the physically relevant regime, and the remaining uncertainties and challenges.</p>

Axion strings and dark matter

Ed Hardy

Based on work with:
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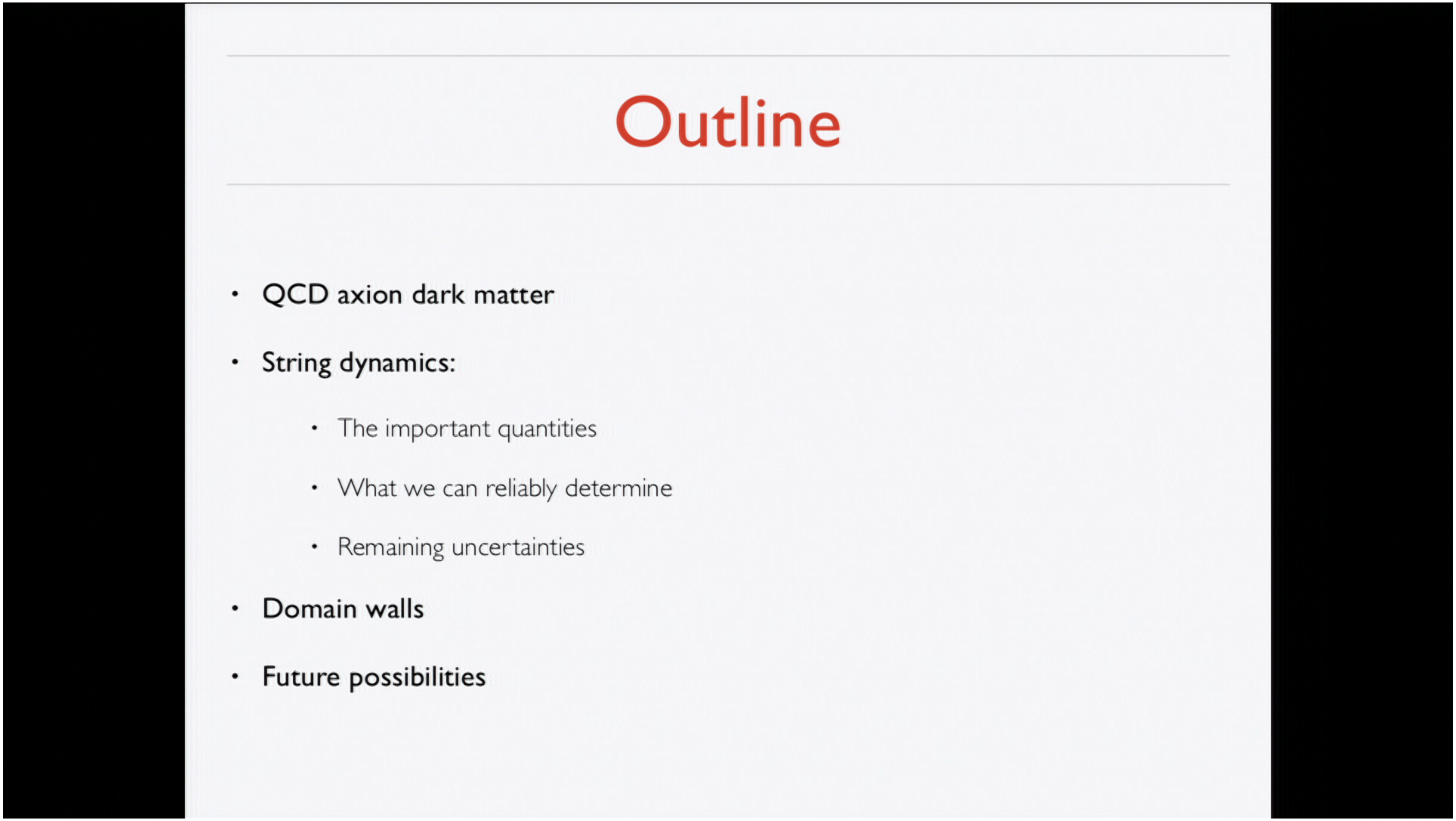


Outline

- QCD axion dark matter
- String dynamics:
  - The important quantities
  - What we can reliably determine
  - Remaining uncertainties
- Domain walls
- Future possibilities

Boundary between regimes

Depends on the details of reheating

(e.g. for quadratic inflation $m^2\phi^2 + g^2 m\phi\chi^2$; $\Gamma = g^4 m/(8\pi)$)

In the PQ breaking after inflation regime if any of:

Hubble scale during inflation	H_I
Reheating temperature when radiation domination begins	$T_{\rm RH} = \sqrt{\Gamma M_{\rm Pl}}$
Maximum temperature during perturbative inflaton decay	$T_{\rm max} \simeq \left(M_{\rm Pl}^2 H_I \Gamma\right)^{1/4}$
Effective temperature during preheating (if this occurs)	$T_{\rm pre} \simeq \sqrt{M_{\rm Pl} H_I}$

are larger than f_a

String theory axions

SUSY $\mathcal{L} \supset \mathrm{Re}\left(f\left(\Phi\right)\right) FF + i\,\mathrm{Im}\left(f\left(\Phi\right)\right) F\tilde{F}$

String theory → SUSY for consistency + no free parameters $f\left(\Phi\right) \sim \frac{\phi_1}{\Lambda} + i\frac{\phi_2}{\Lambda}$

Closed string axions:

- Never a U(1) global symmetry in the 4D EFT
- Inflation in a 4D EFT means PQ breaking before inflation

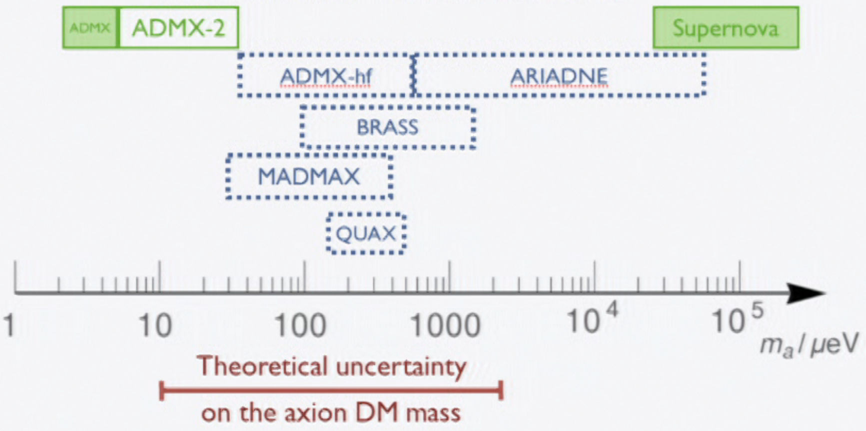
Open string axions:

- 4D EFT has a global U(1) symmetry
- High scale inflation means PQ breaking after inflation

U(1) breaking after inflation

In principle extremely predictive — unique DM axion mass

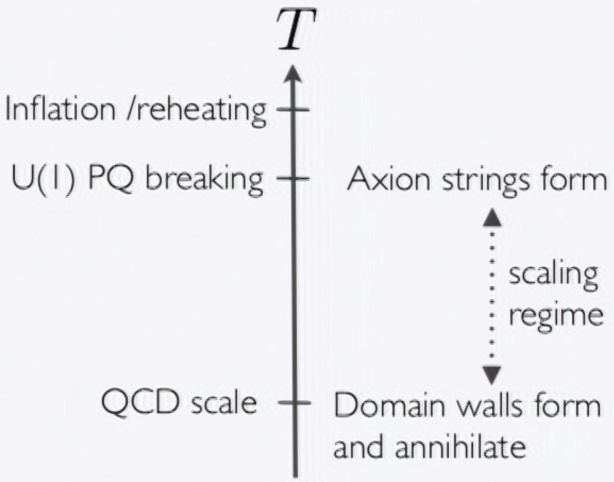
Existing data (filled) and ongoing experiments (empty),
and possible future experiments (dotted) :



Reliable prediction: interpret ongoing experiments, design future experiments

Precise agreement with an experimental discovery → minimum inflation scale

String and domain walls

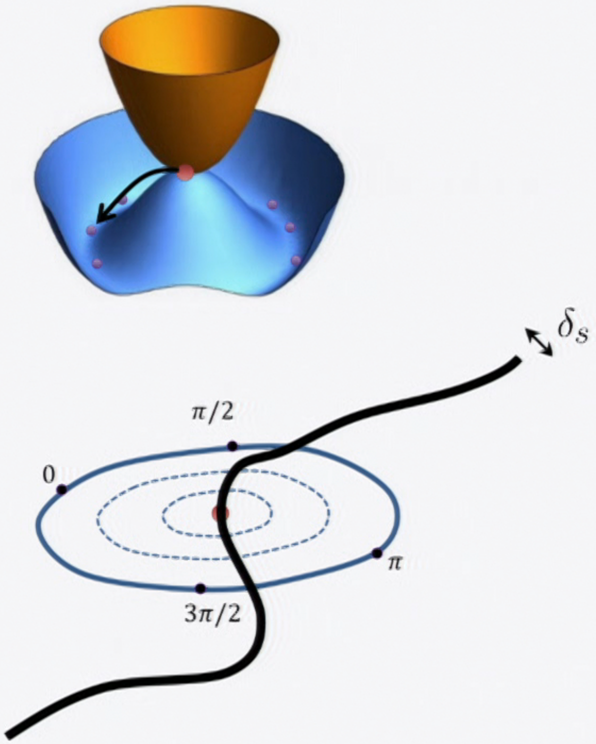


Inflation /reheating

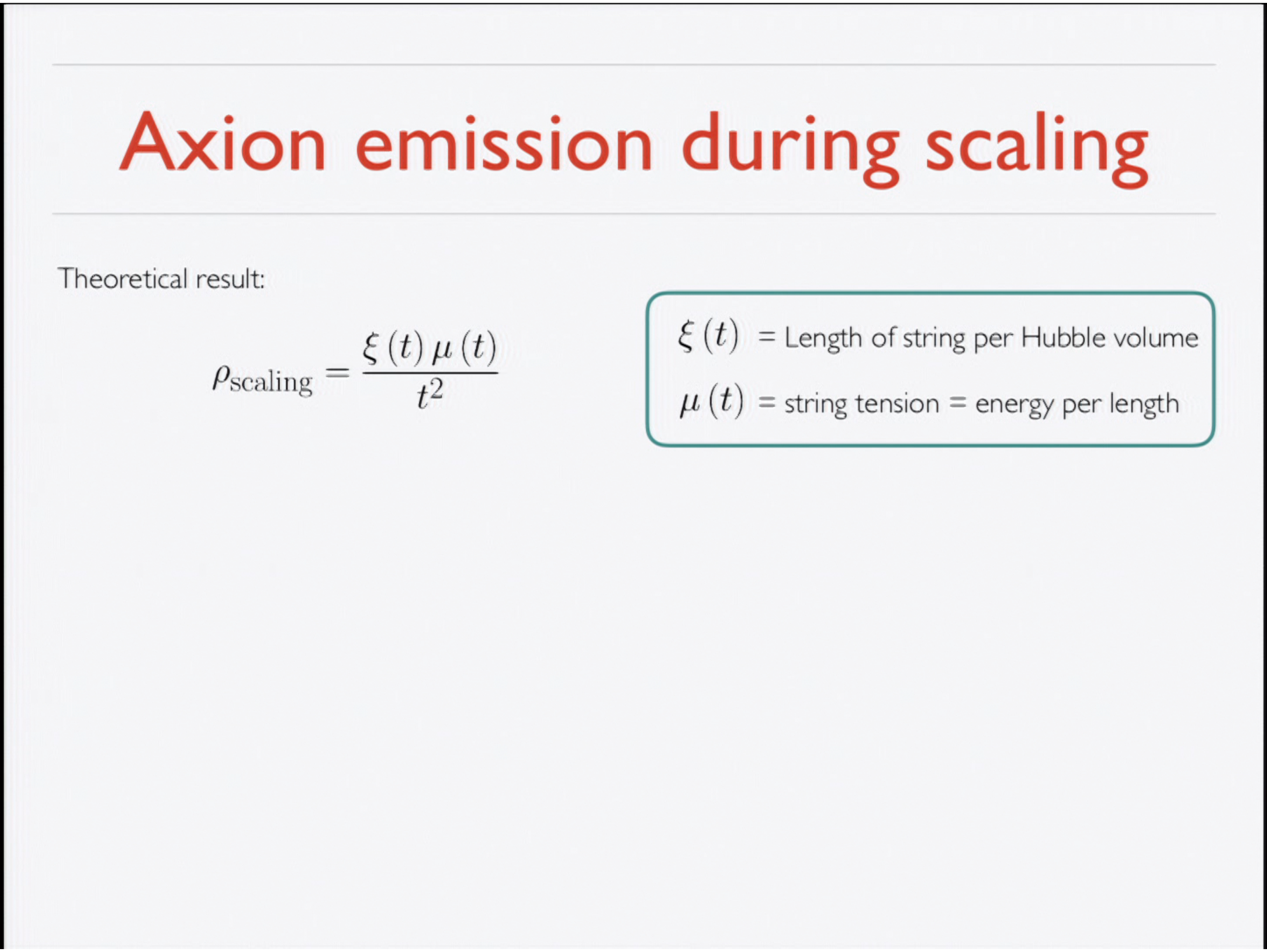
U(1) PQ breaking — Axion strings form

scaling regime

QCD scale — Domain walls form and annihilate



Significant proportion of DM axions produced by strings and domain walls



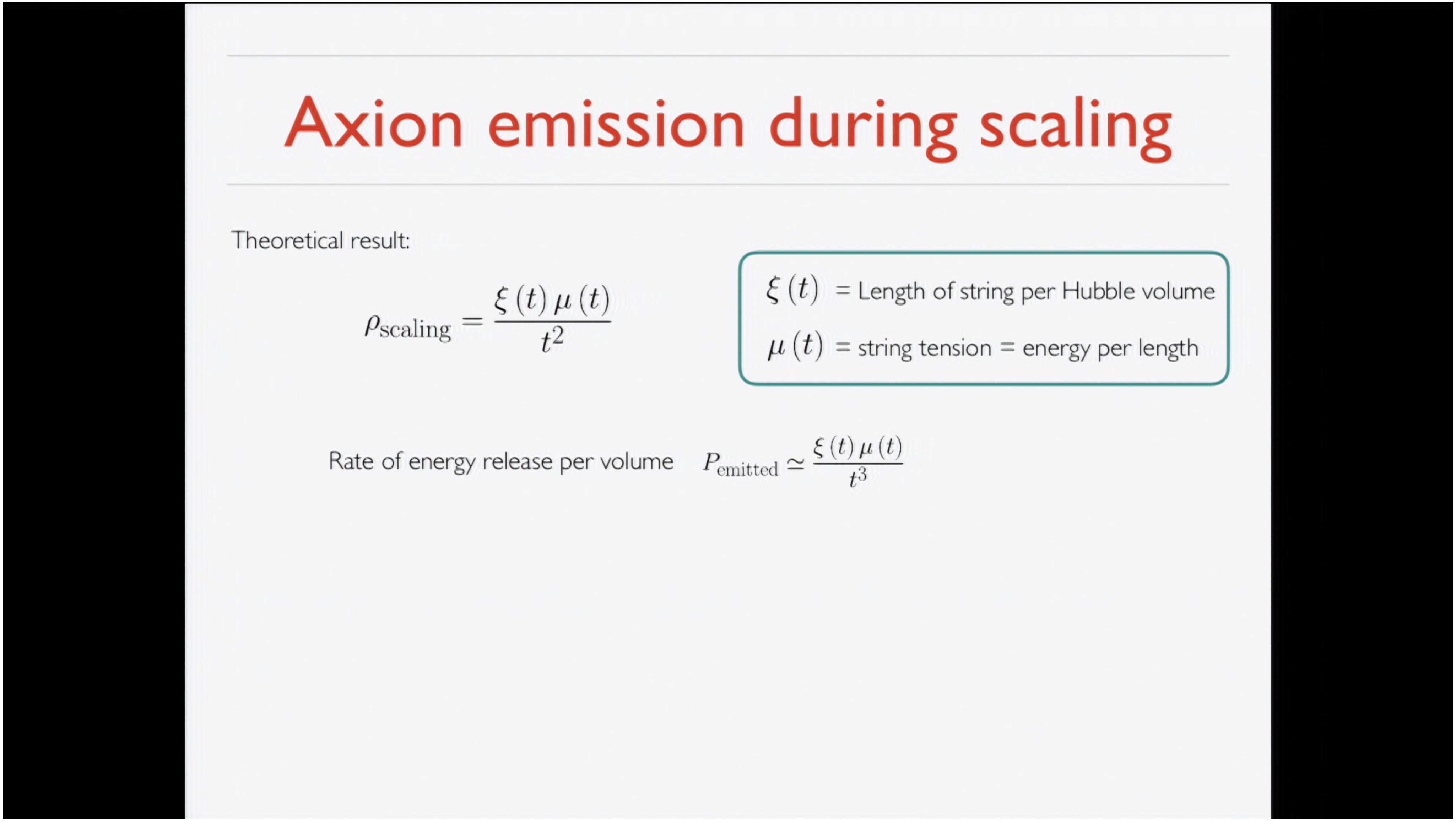
Axion emission during scaling

Theoretical result:

$$\rho_{\text{scaling}} = \frac{\xi(t)\,\mu(t)}{t^2}$$

$\xi(t)$ = Length of string per Hubble volume

$\mu(t)$ = string tension = energy per length



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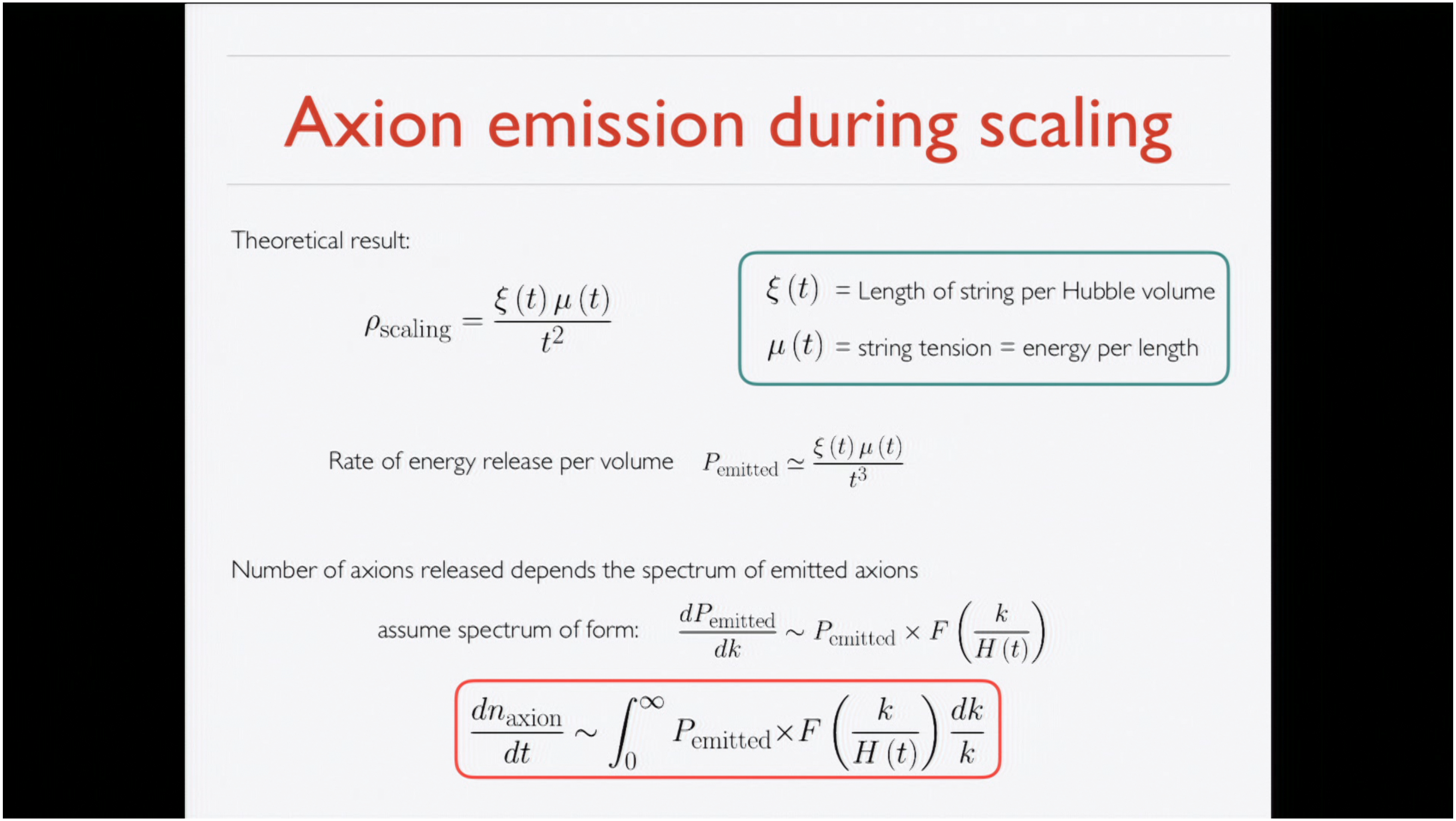
Rate of energy release per volume $P_{\text{emitted}} \simeq \frac{\xi(t)\,\mu(t)}{t^3}$



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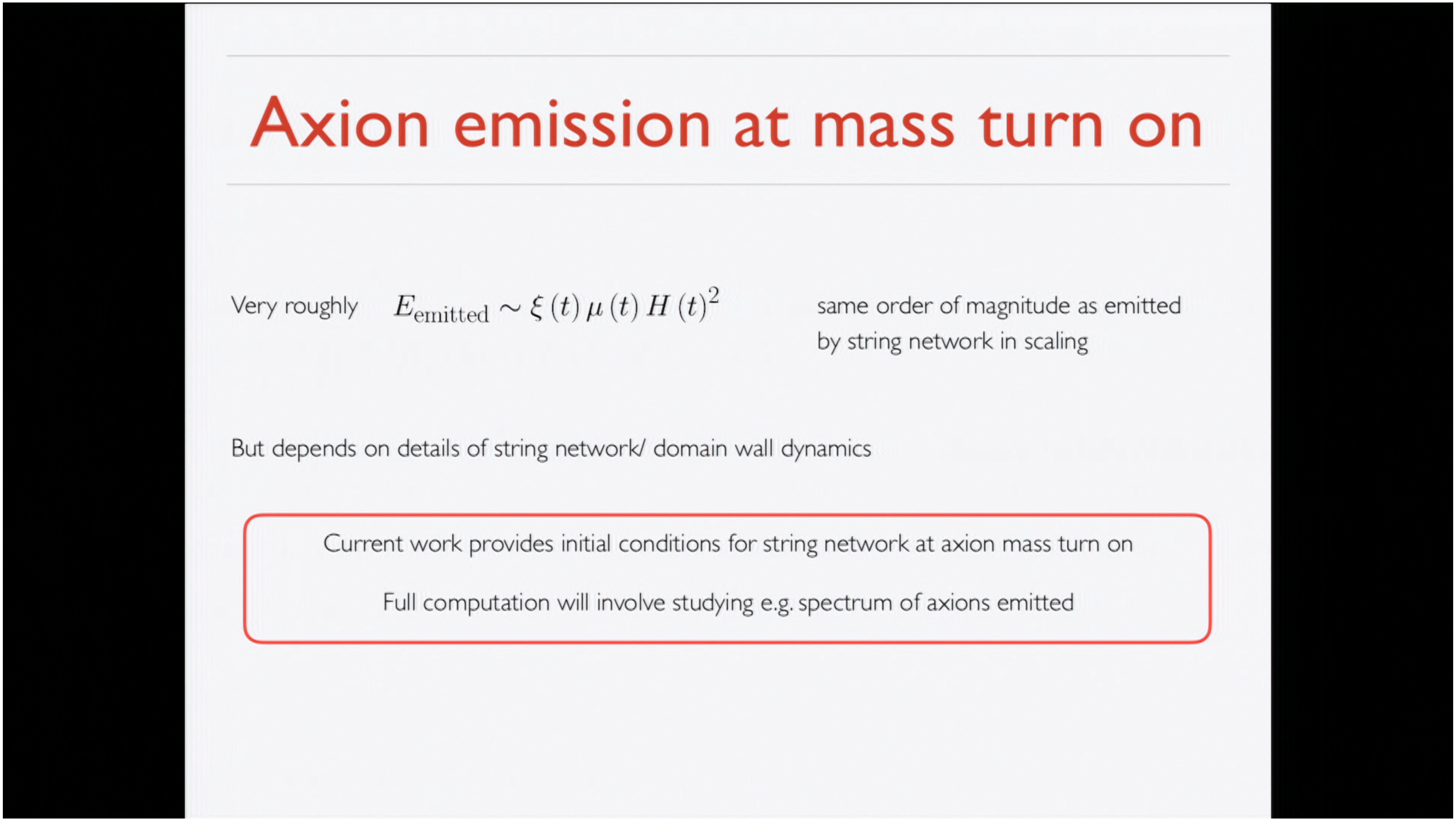
Rate of energy release per volume $P_{\text{emitted}} \simeq \dfrac{\xi(t)\,\mu(t)}{t^3}$

Number of axions released depends the spectrum of emitted axions

assume spectrum of form: $\dfrac{dP_{\text{emitted}}}{dk} \sim P_{\text{emitted}} \times F\left(\dfrac{k}{H(t)}\right)$

$$\boxed{\frac{dn_{\text{axion}}}{dt} \sim \int_0^\infty P_{\text{emitted}} \times F\left(\frac{k}{H(t)}\right)\frac{dk}{k}}$$



Axion emission at mass turn on

Very roughly $E_{\text{emitted}} \sim \xi(t)\,\mu(t)\,H(t)^2$

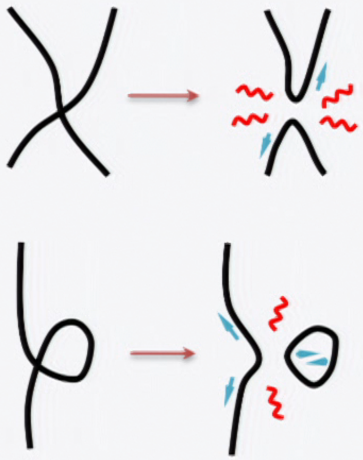
same order of magnitude as emitted by string network in scaling

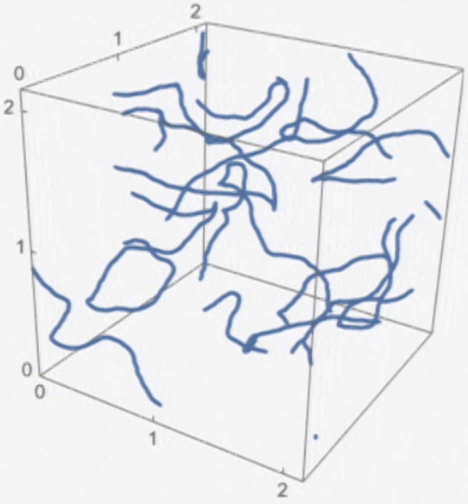
But depends on details of string network/ domain wall dynamics

Current work provides initial conditions for string network at axion mass turn on

Full computation will involve studying e.g. spectrum of axions emitted

String dynamics



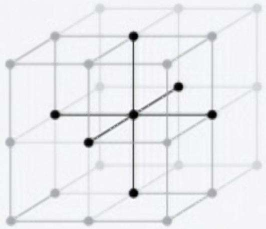


Hard to study analytically, can help with qualitative understanding, but full network has complicated interactions and dynamics

Instead resort to numerical simulations

Numerical simulation

Simulate full complex scalar field on a lattice (no benefit to simulating just the axion field)



Evolve using algorithm that is 4th order in space, 2nd order in time

Parallelised (relatively simple way, order 1 improvements with significantly more work?)

Identify strings by looking at field change around loops in different 2D planes

➡ group identified lattice points and form strings

Why it's hard

Large separation of scale

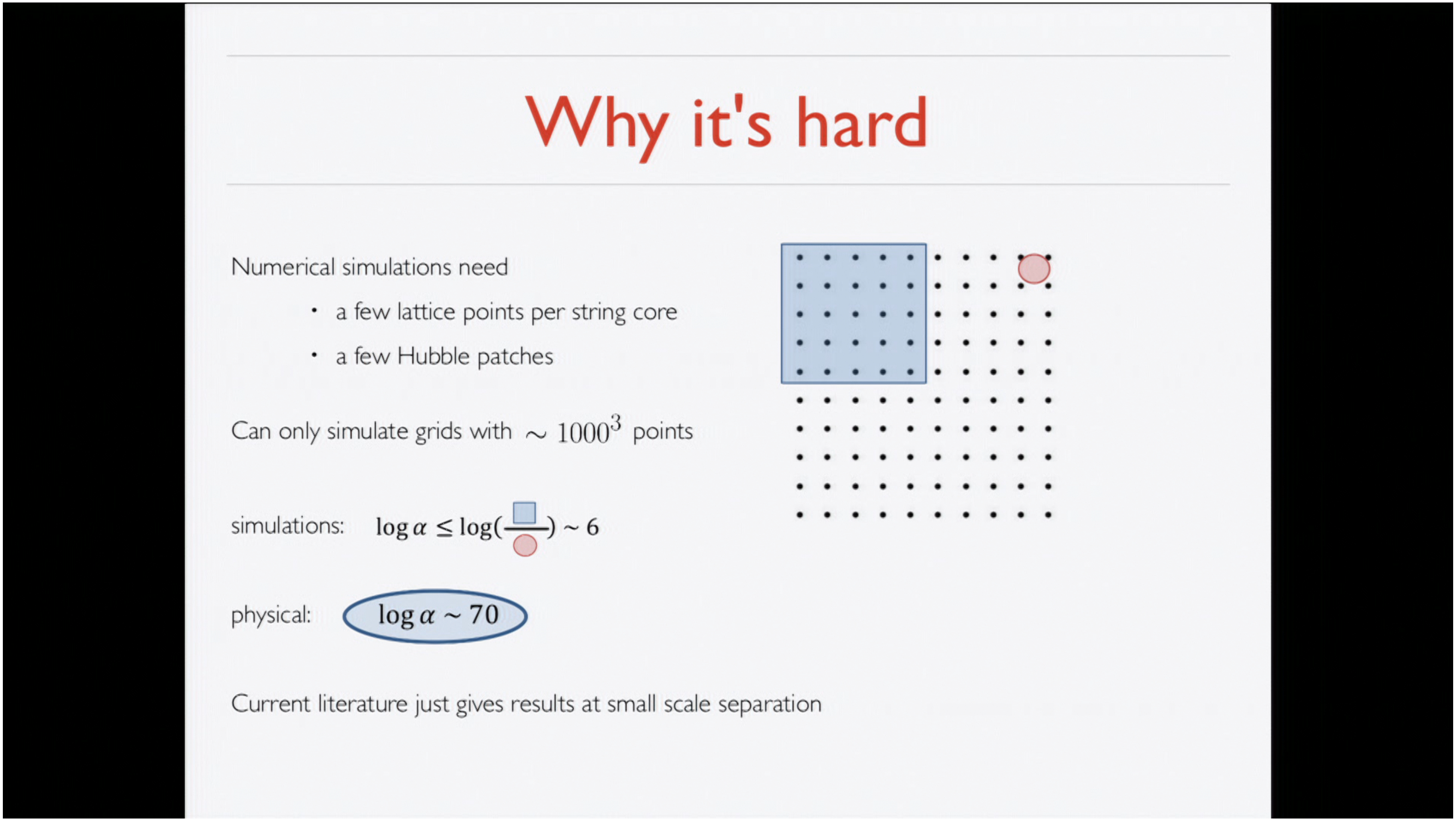
- String core is very thin $\delta_s \simeq \frac{1}{f_a}$
- Hubble distance is much larger $H^{-1} \simeq \frac{M_{\rm pl}}{T^2} \simeq \frac{M_{\rm pl}}{\Lambda_{\rm QCD}^2}$

String tension depends on the ratio of string core size and Hubble scale

$$\mu\left(t\right) \simeq \pi f_a^2 \log\left(\frac{H\left(t\right)^{-1}}{\delta_s}\right) =: \pi f_a^2 \log\left(\alpha\left(t\right)\right)$$

Physical scale separation $\alpha \sim 10^{30}$

→ $\log\alpha \simeq 70$



Why it's hard

Numerical simulations need

- a few lattice points per string core
- a few Hubble patches

Can only simulate grids with $\sim 1000^3$ points

simulations: $\log\alpha \leq \log(\frac{\square}{\bigcirc}) \sim 6$

physical: $\boxed{\log\alpha \sim 70}$

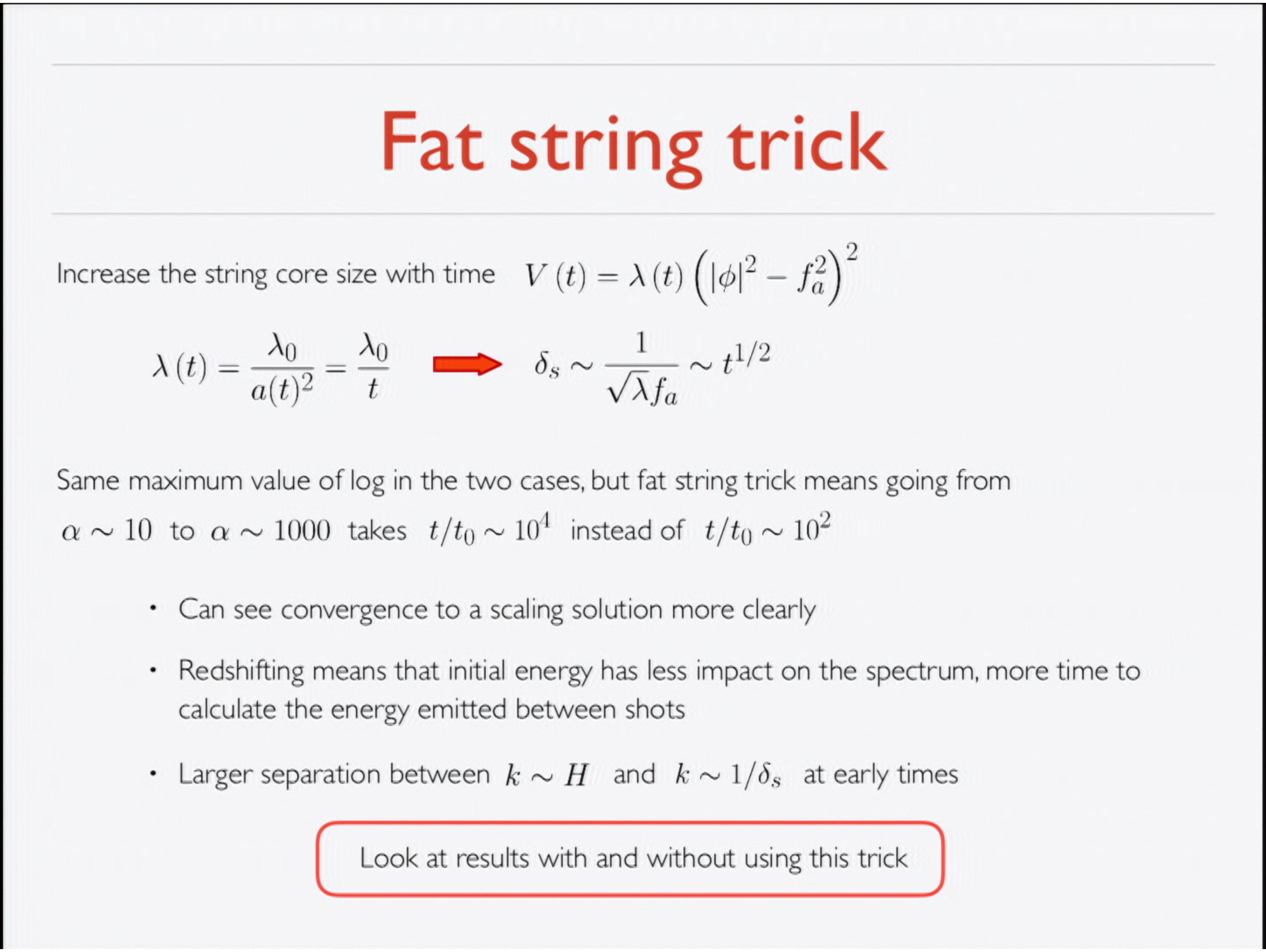
Current literature just gives results at small scale separation

Fat string trick

Increase the string core size with time $V(t) = \lambda(t)\left(|\phi|^2 - f_a^2\right)^2$

$$\lambda(t) = \frac{\lambda_0}{a(t)^2} = \frac{\lambda_0}{t} \quad \Rightarrow \quad \delta_s \sim \frac{1}{\sqrt{\lambda} f_a} \sim t^{1/2}$$

Same maximum value of log in the two cases, but fat string trick means going from $\alpha \sim 10$ to $\alpha \sim 1000$ takes $t/t_0 \sim 10^4$ instead of $t/t_0 \sim 10^2$



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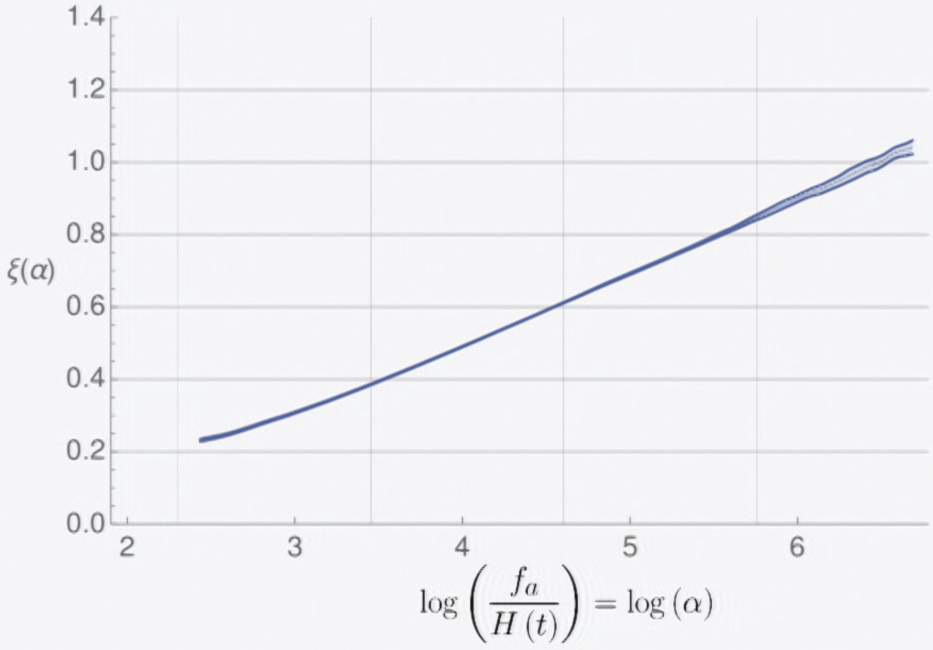
- Can see convergence to a scaling solution more clearly
- Redshifting means that initial energy has less impact on the spectrum, more time to calculate the energy emitted between shots
- Larger separation between $k \sim H$ and $k \sim 1/\delta_s$ at early times

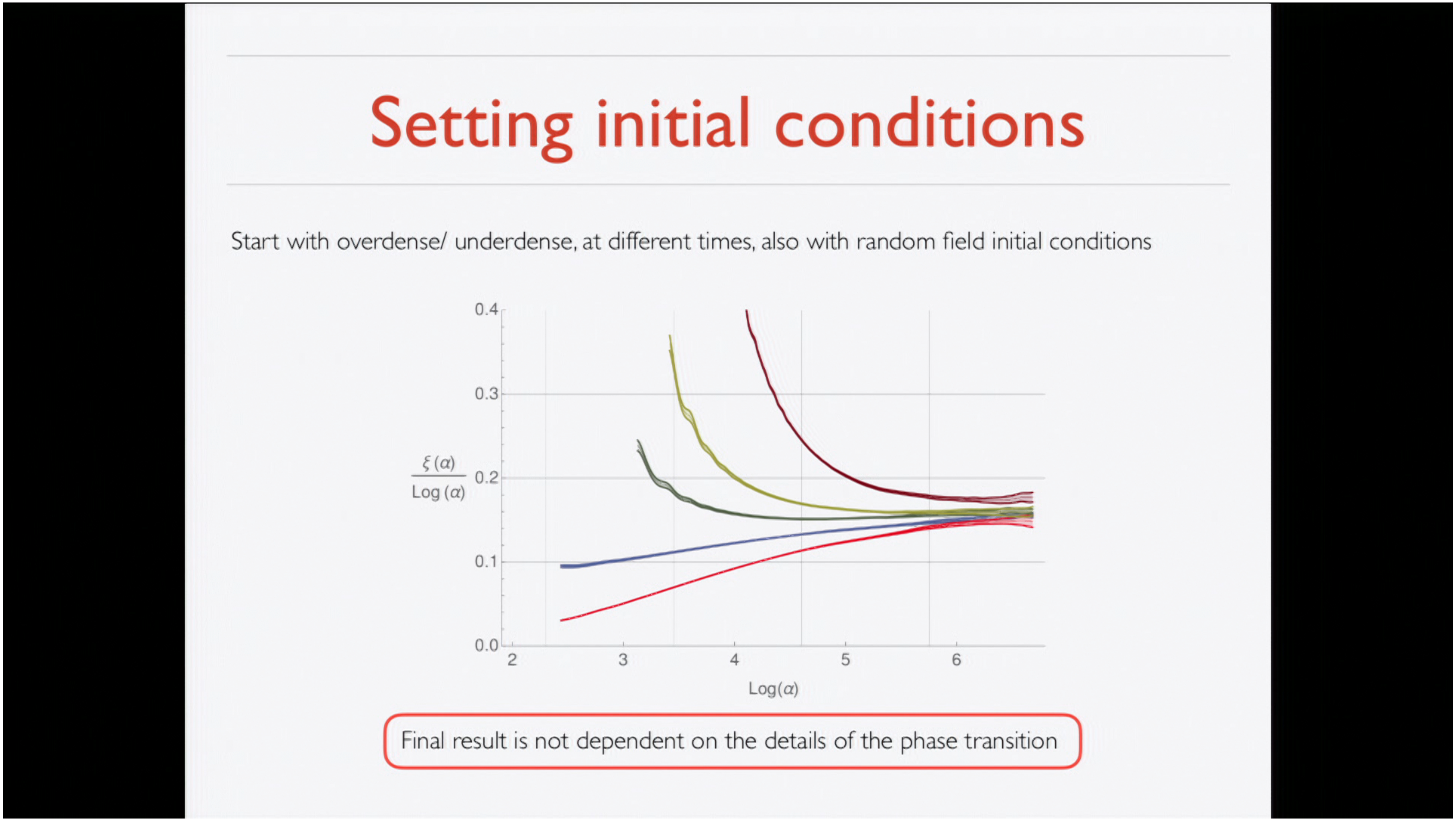
Look at results with and without using this trick

String length per Hubble volume

Find a log increase, theoretically plausible since the tension is increasing



If extrapolation is valid, grows to ~10 at QCD scale

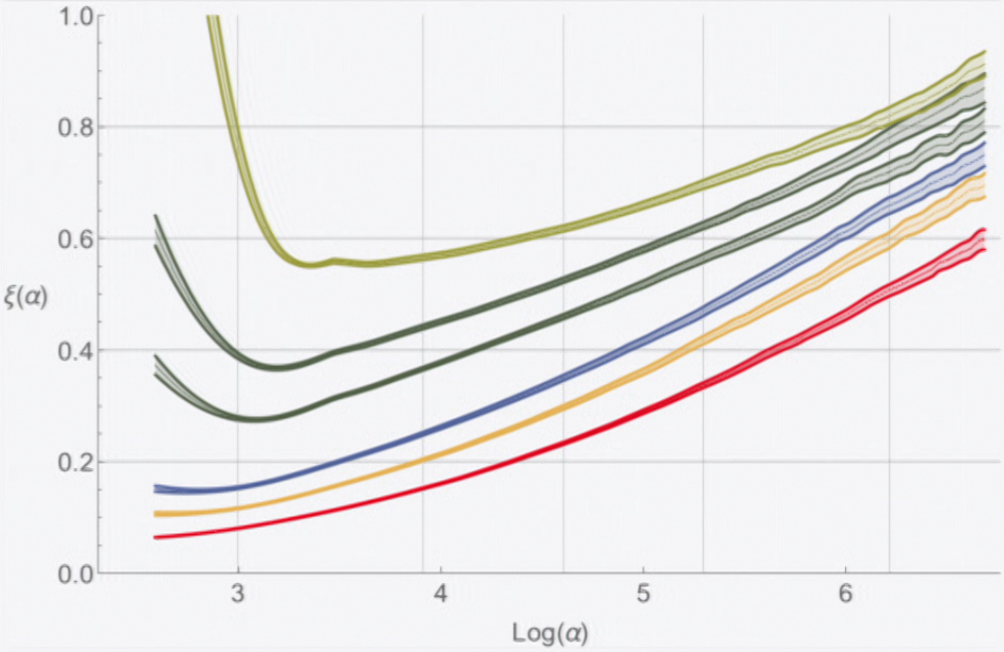


Setting initial conditions

Start with overdense/ underdense, at different times, also with random field initial conditions

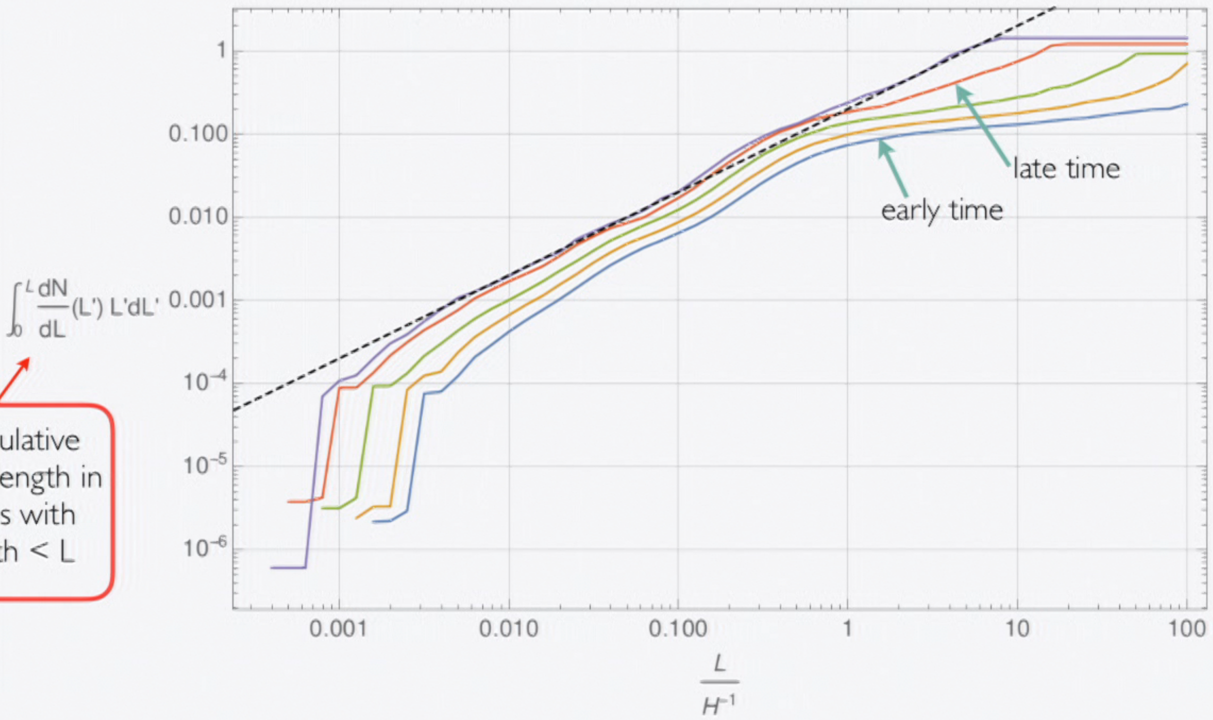
Final result is not dependent on the details of the phase transition

Scaling solution: non-fat



Parameters of the scaling solution slightly different, otherwise physics seems similar

Distribution of loop length

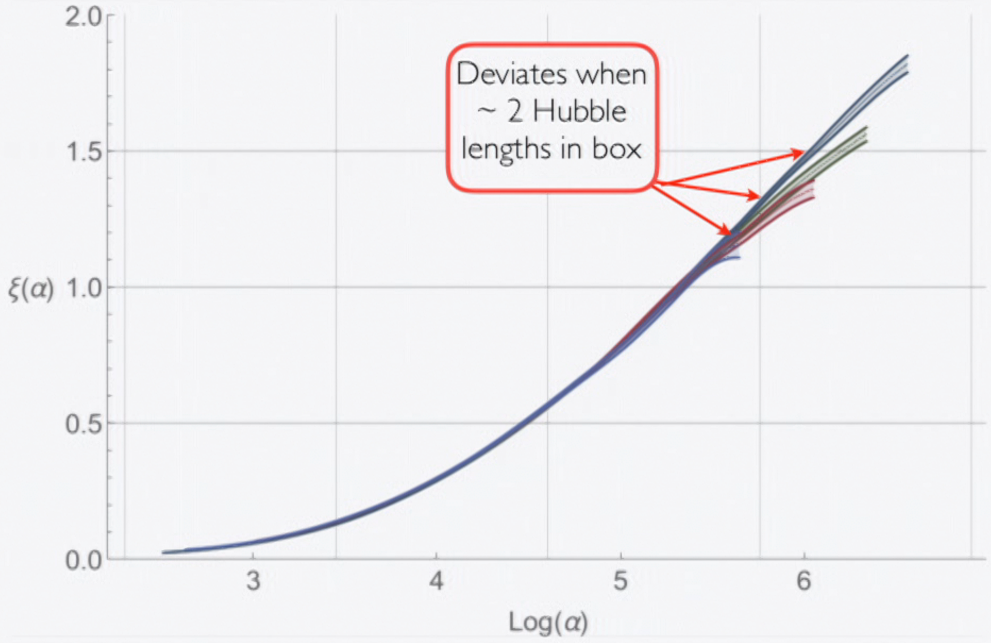


Cumulative string length in loops with length < L

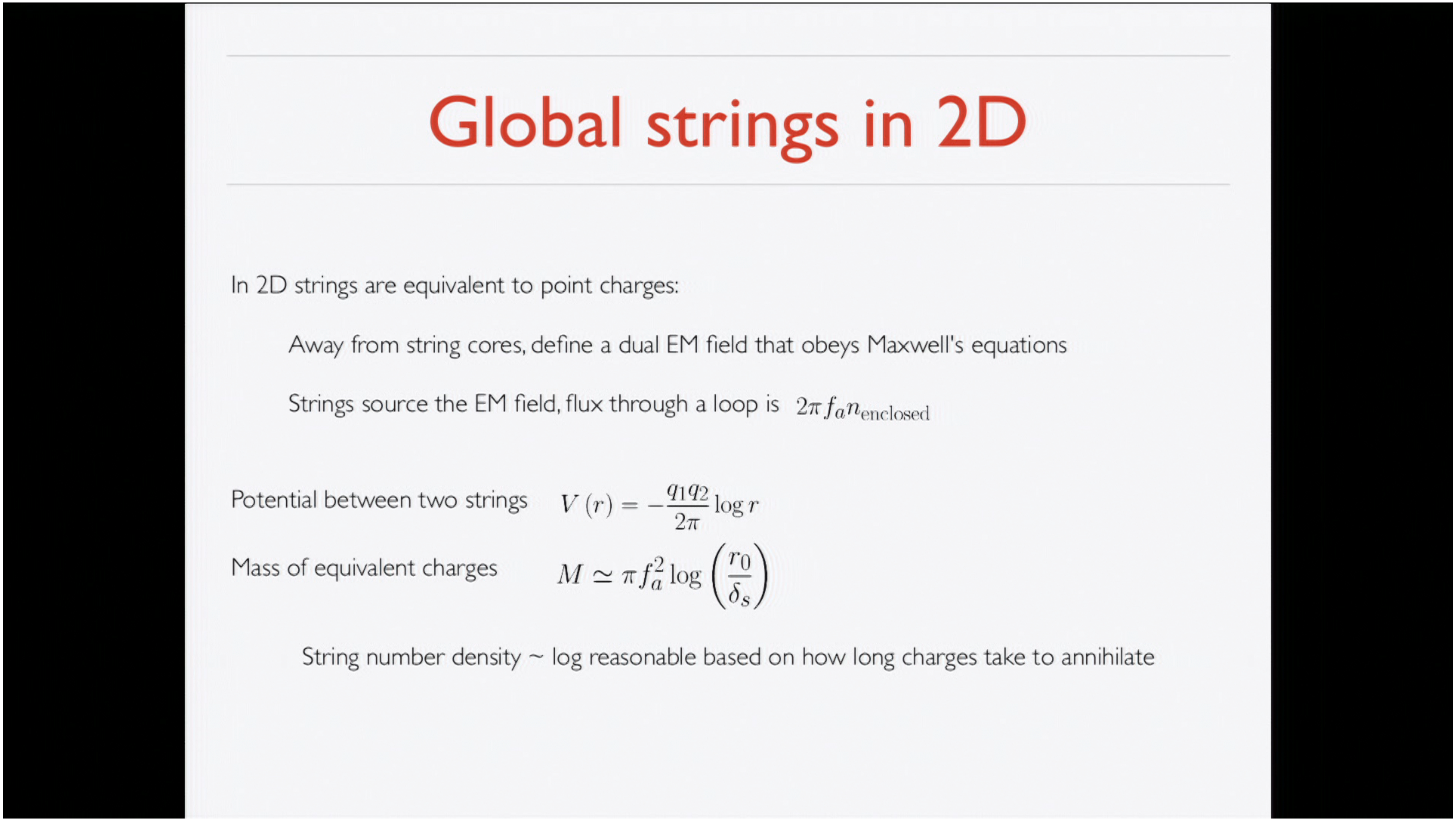
A scaling solution is established

Numerical checks

E.g. number of Hubble patches at end of simulation



Deviates when ~ 2 Hubble lengths in box



Global strings in 2D

In 2D strings are equivalent to point charges:

Away from string cores, define a dual EM field that obeys Maxwell's equations

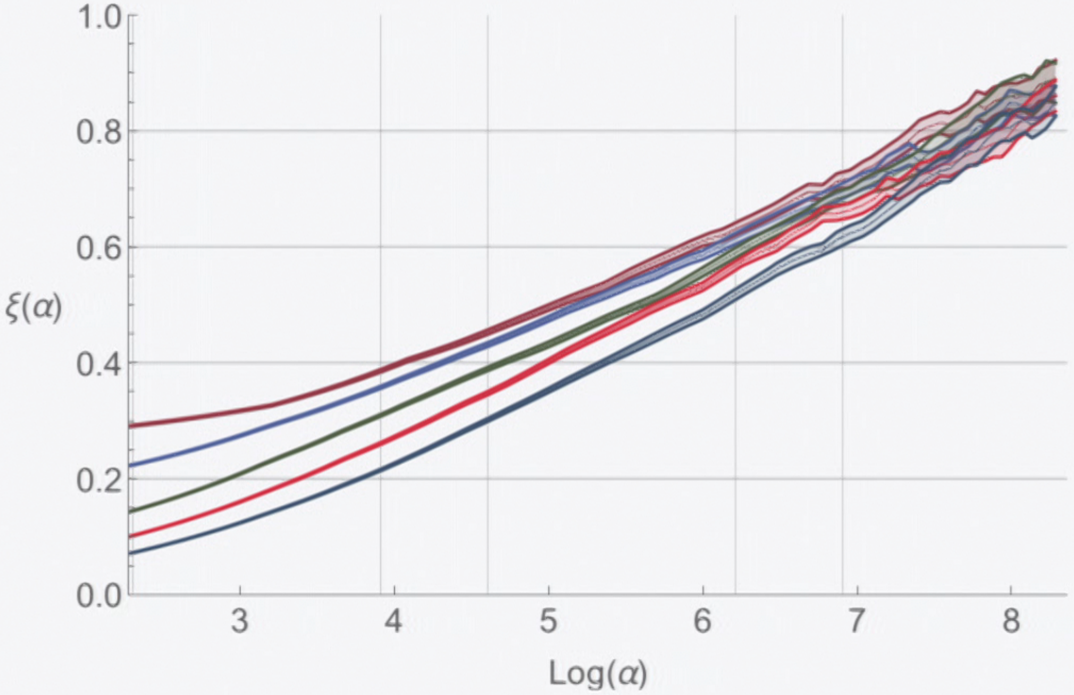
Strings source the EM field, flux through a loop is $2\pi f_a n_{\mathrm{enclosed}}$

Potential between two strings $V(r) = -\frac{q_1 q_2}{2\pi} \log r$

Mass of equivalent charges $M \simeq \pi f_a^2 \log\left(\frac{r_0}{\delta_s}\right)$

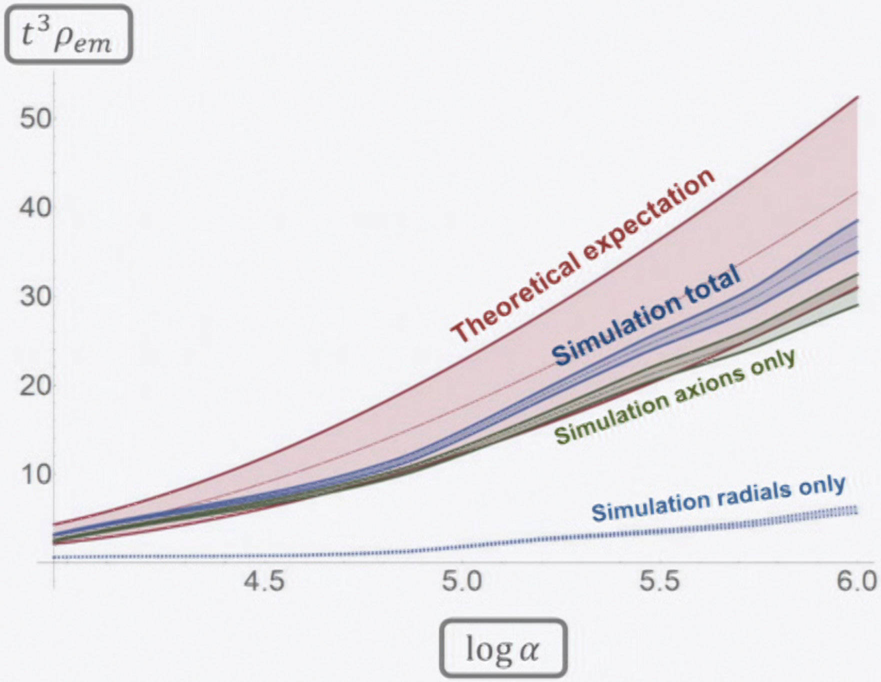
String number density ~ log reasonable based on how long charges take to annihilate

Global strings in 2D



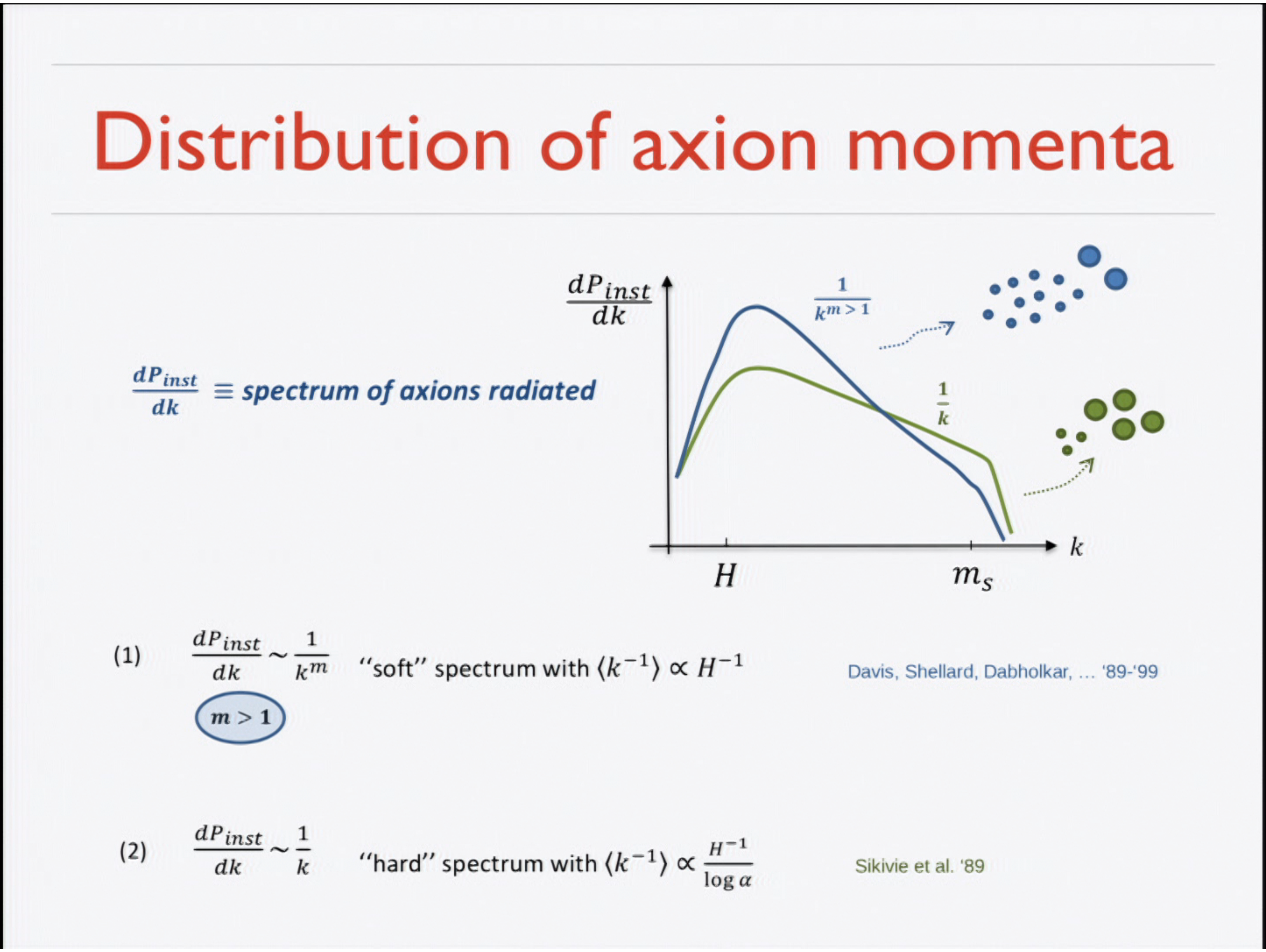
Energy emitted by strings

Theoretical "prediction" from measured $\xi(t)$ and expectation that $\mu(t) \simeq \pi f_a^2 \log\left(c\frac{H^{-1}}{\delta_s}\right)$



$$\rho_{em} = \frac{\delta(\rho_{free} - \rho_{st})}{\delta t}$$

Significant proportion of energy going into the scalar radial modes



Distribution of axion momenta

$\frac{dP_{inst}}{dk}$ ≡ ***spectrum of axions radiated***

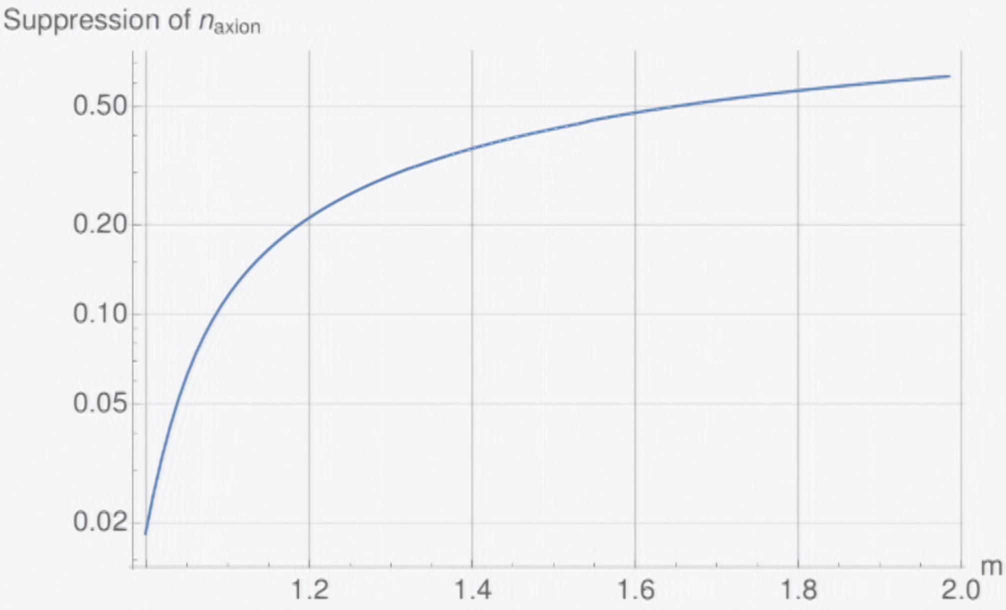
(1) $\frac{dP_{inst}}{dk} \sim \frac{1}{k^m}$ "soft" spectrum with $\langle k^{-1} \rangle \propto H^{-1}$ Davis, Shellard, Dabholkar, … '89-'99

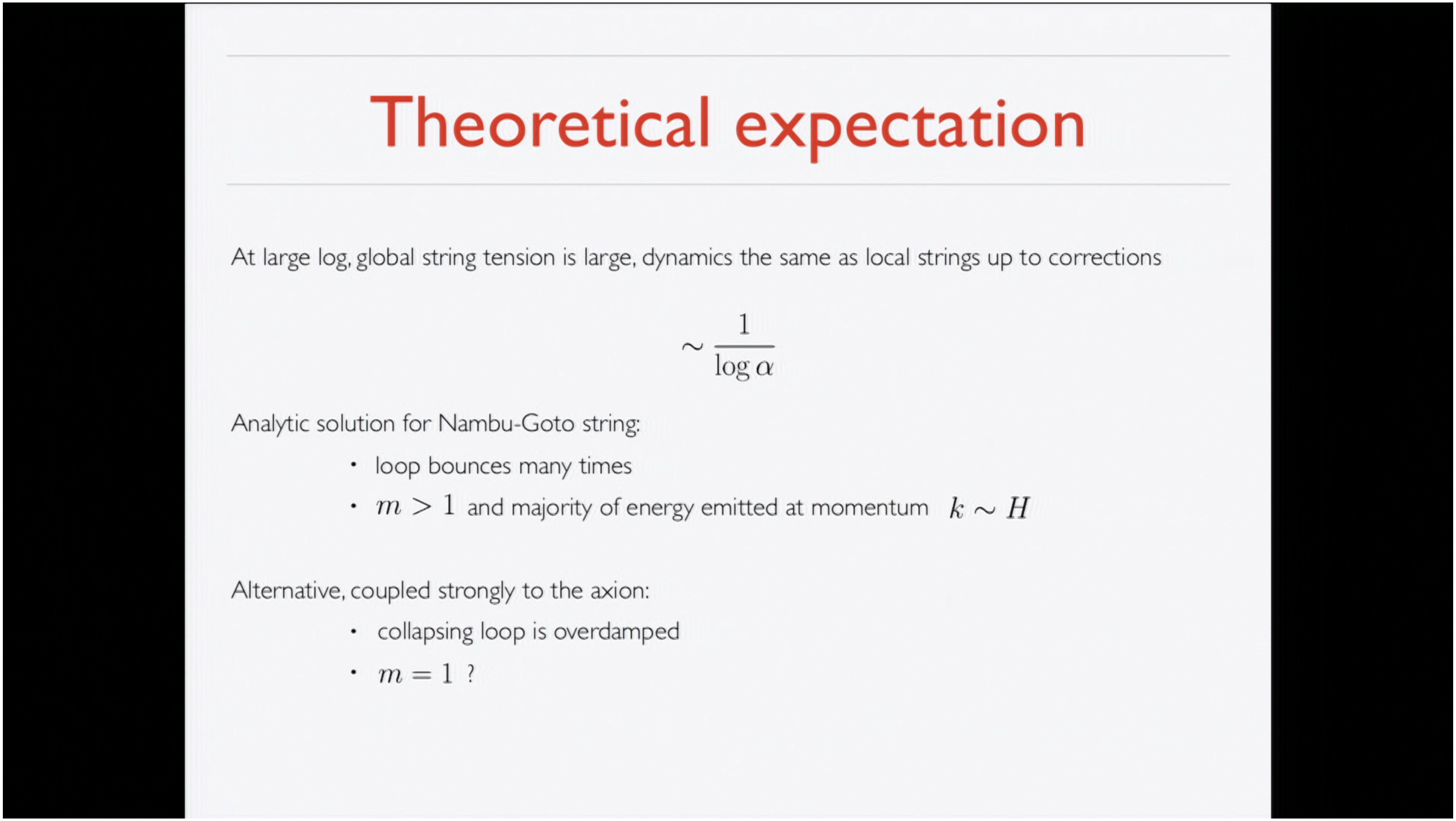
$m > 1$

(2) $\frac{dP_{inst}}{dk} \sim \frac{1}{k}$ "hard" spectrum with $\langle k^{-1} \rangle \propto \frac{H^{-1}}{\log \alpha}$ Sikivie et al. '89

Impact on relic density

Suppression of number density compared to $m = \infty$





Theoretical expectation

At large log, global string tension is large, dynamics the same as local strings up to corrections

$$\sim \frac{1}{\log \alpha}$$

Analytic solution for Nambu-Goto string:

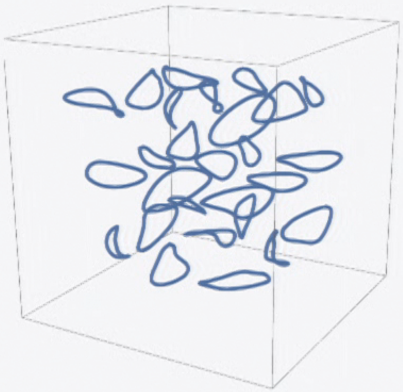
- loop bounces many times
- $m > 1$ and majority of energy emitted at momentum $k \sim H$

Alternative, coupled strongly to the axion:

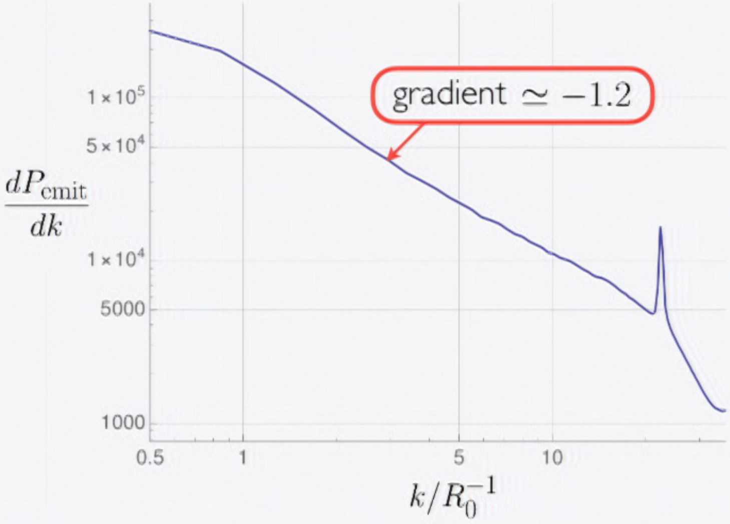
- collapsing loop is overdamped
- $m = 1$?

Collapsing loops

Ensemble of non-circular loops

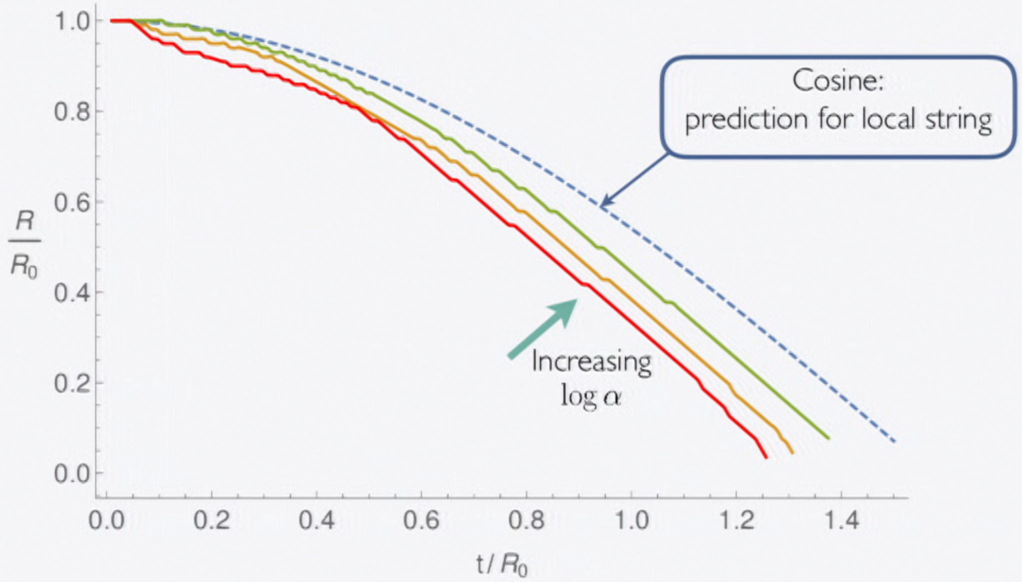


Spectrum emitted



Power law with $m > 1$ so predicts emission dominantly at $k \sim H$

Collapse time for different log



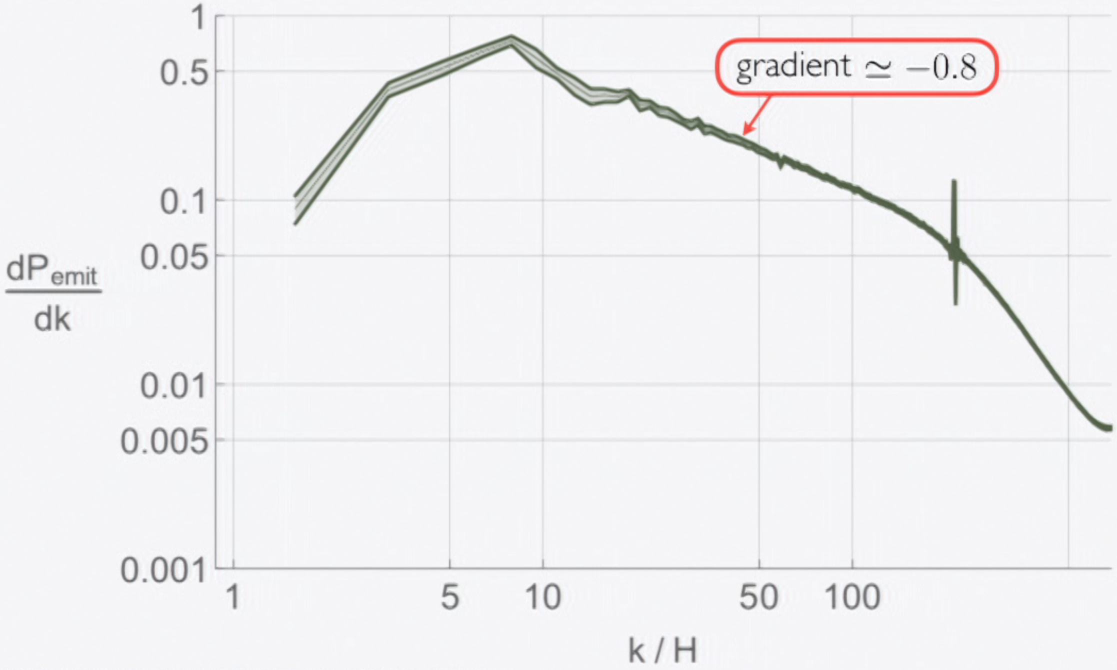
Cosine:
prediction for local string

Increasing
$\log\alpha$

Convergence towards the local string cosine prediction

Spectrum from the network

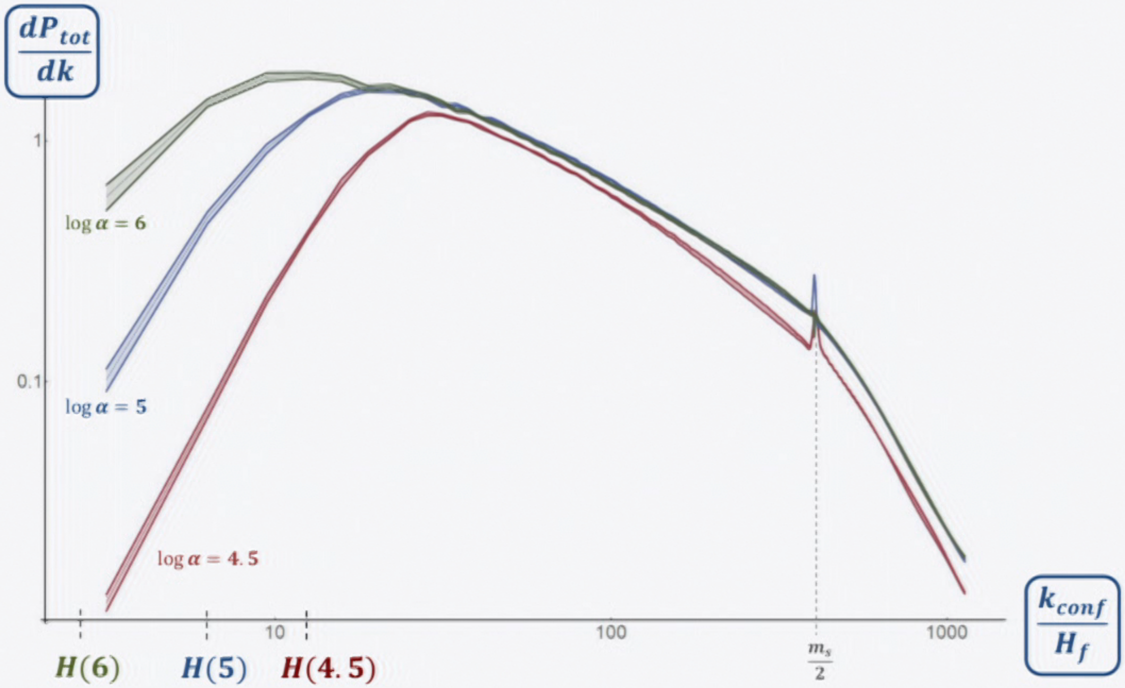
Look at the energy emitted in a small time interval



Much more UV dominated than expected

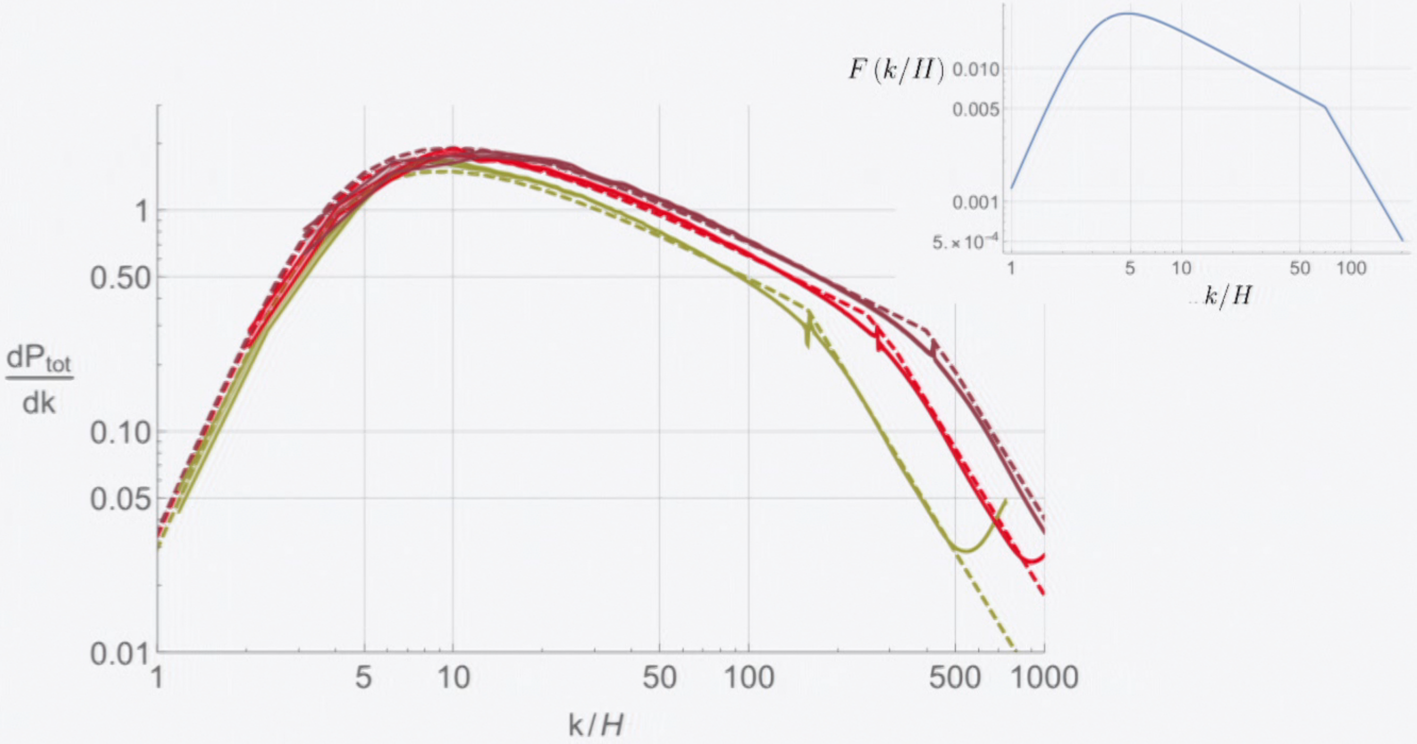
Spectrum from the network

Can also measure the total spectrum in the simulation



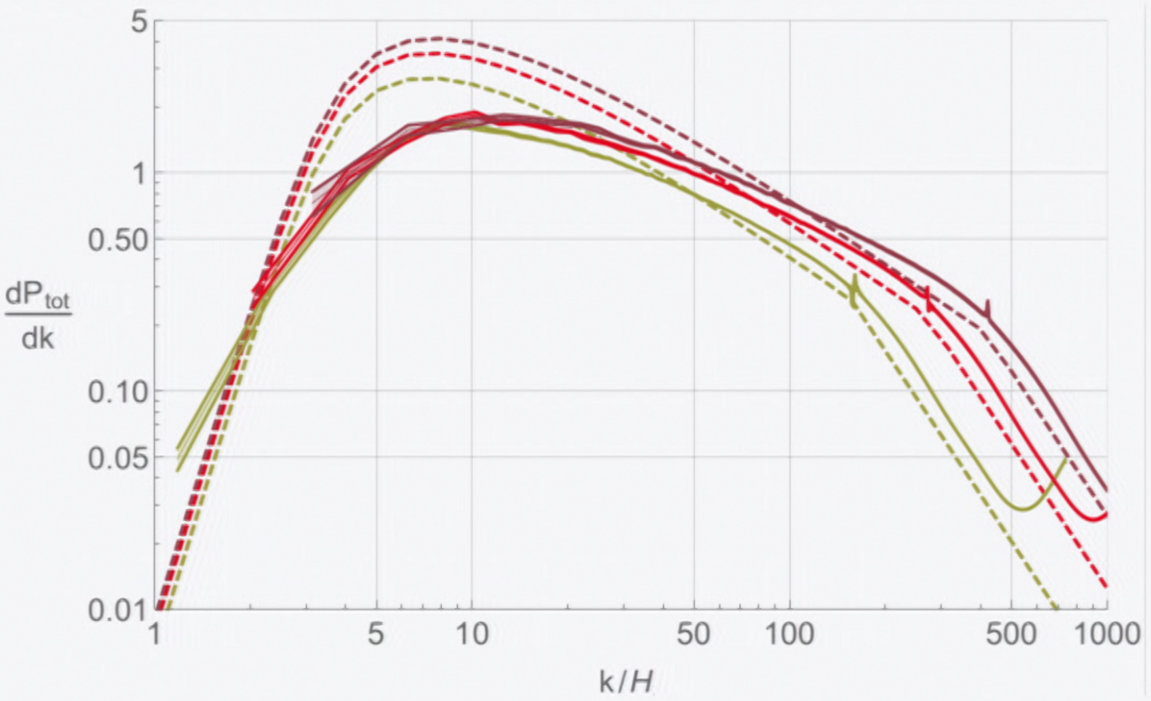
Spectrum from the network

Good agreement with prediction from $\xi(t)$ and instantaneous emission spectrum



Spectrum from the network

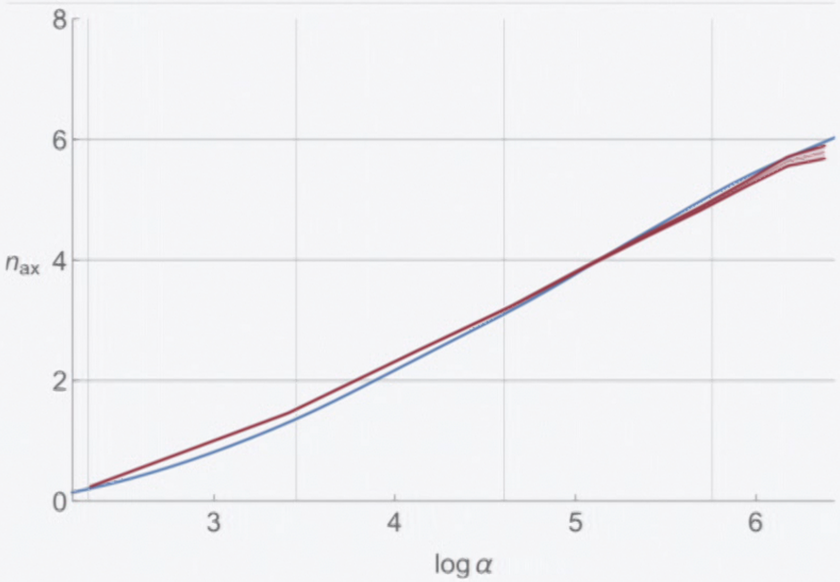
Strong disagreement with predicted spectrum assuming $m = 1$



Consistency check

Number density of axions can also be predicted

More IR dominated than the energy in the system



All consistent with a more UV dominated power spectrum than expected in the physical regime

Domain walls

To get a final result, also need to study the dynamics of domain walls

Depends on the anomaly coefficient:

- $N = 1$, unstable, automatically decay
- $N > 1$, stable in the absence of extra PQ breaking, current simulations seems marginally ruled out unless fine-tuned

Domain walls

Axion mass becomes cosmologically relevant when

$$m_a\left(T_0\right) \simeq H\left(T_0\right)$$

Subsequently it increases fast, and quickly $m_a\left(T\right) \gg H\left(T_0\right)$

But typical size of domain walls still $\sim 1/H\left(T_0\right)$, momentum of lowest harmonics $\sim H\left(T_0\right)$
emission at higher harmonics strongly suppressed

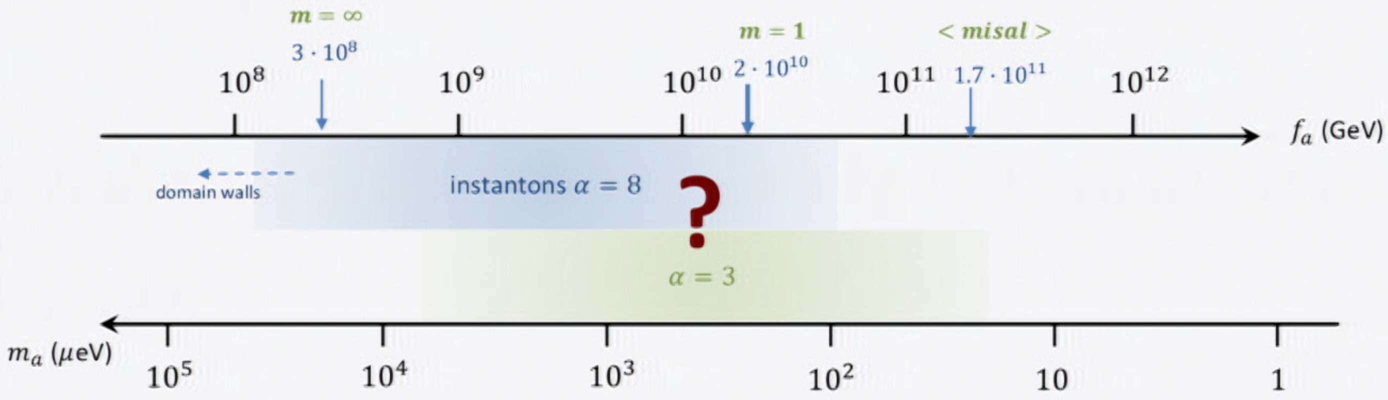
Could this delay the destruction of the domain wall network? Potentially a big effect on the relic abundance?

Impact on the relic abundance

- Extrapolation of $\xi(t) \sim \log\left(\frac{f_a}{H(t)}\right)$ is plausible (but a dynamical change is not ruled out)
- Unfortunately cannot reliably determine m from the full simulation

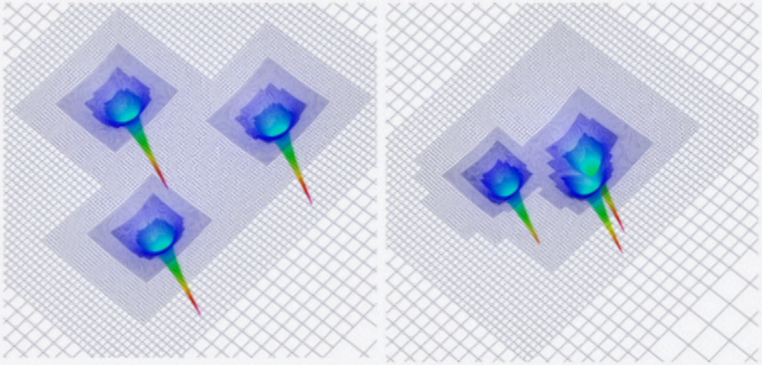
$$\frac{n_{\text{strings}}}{n_{\text{misal}}} \simeq 0.8\,\xi\left(T = \Lambda_{\text{QCD}}\right)\ f(m)$$

$$\boxed{\begin{gathered} f(m=1) = 1 \\ f(m \to \infty) = \log\alpha \sim 70 \end{gathered}}$$



Possible improvements

- Bigger computers, running for longer, lead to relatively little gain in the range of $\log(\alpha)$
- Effective field theory approach is tempting: carry out a simulation where the degrees of freedom are evolving strings
- Might be possible to parameterise the probability of passing through, rate that curves straighten out etc. but not straightforward
- Adaptive mesh, win a factor of 10?



Previous literature

Hiramatsu, Kawasaki, Saikawa, Toyokazu Sekiguchi , e.g. arXiv:1202.5851

- Extract the spectrum at small scale separation
- But are looking at the region to the right of the string core peak
- Find m >> 1 (which might be physically correct but not justified from their analysis)
- Find $\xi(t) \sim 1$ (since at small scale separation)
- Use this to compute relic abundance

Moore, arxiv:1509.00026

- Simulation at small tension and extracts axion number density directly
- No extrapolation
- Results are compatible with our measurements of $\xi(t)$ and spectrum, but not physically reliable

Conclusions

- If PQ symmetry is broken after inflation there is in principle a unique axion DM prediction, this is experimentally important and related to early universe cosmology
- Exact calculation is challenging, and cannot directly study physically relevant regime
- String density increases ~ $\log(\alpha)$ in the regime we can study
- Emitted spectrum does not match theoretical expectation for the physically relevant regime
- More than order of magnitude changes in axion DM mass compared to previous results

Thanks