Title: Thermodynamics and Holography for de Sitter Space

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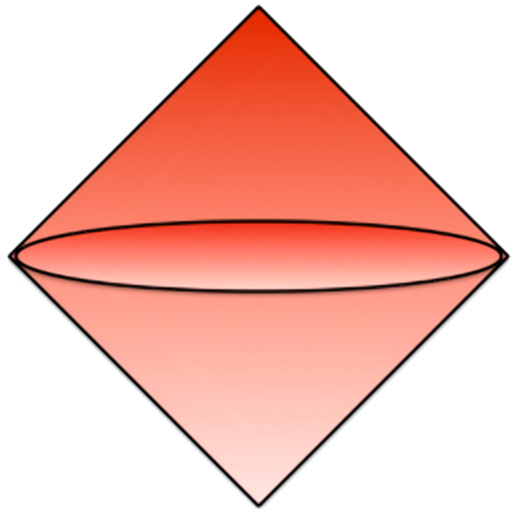
Abstract: <p>In this talk I will present new insights on a microscopic holographic theory for de Sitter space. I will focus on the static patch of dS, which describes our universe to a good approximation at late times. We use a conformal map between dS and the BTZ black hole times a sphere to relate the general microscopic properties of dS to those of symmetric product CFTs. In 2d CFT language, de Sitter space corresponds to a thermal bath of long string. The long string phenomenon exhibited by these CFTs implies that the excitation energy decreases at large distances in dS (contrary to the UV/IR relation in AdS/CFT). It also explains the smallness of the vacuum energy of dS. Similar results apply to Minkowski space and AdS below its curvature radius.</p>

Thermodynamics and Holography for de Sitter Space



Delta Institute for Theoretical Physics

University of Amsterdam

Manus Visser

Cosmology Seminar Perimeter Institute
(9 January 2018)

Motivation (1)

- At early and late times our universe is well described by dS space. Past: Inflationary patch of de Sitter.

- Future: Static patch of de Sitter.

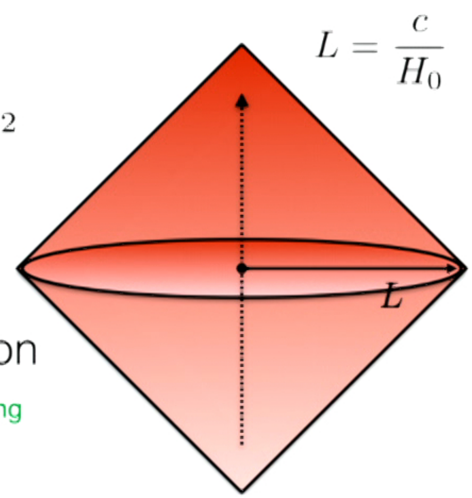
$$ds^2 = -\left(1 - R^2/L^2\right) dt^2 + \frac{dR^2}{1 - R^2/L^2} + R^2 d\Omega_{d-2}^2$$

- Cosmological horizon at: $R = L$.

- Entropy and temperature associated to horizon

Gibbons & Hawking

$$T = \frac{\hbar}{2\pi L} \qquad S = \frac{A(L)}{4G\hbar}$$



- Q: What is the microscopic interpretation of the entropy?

Motivation (2)

- Cosmological constant in de Sitter space is equivalent to introducing a vacuum energy

$$E_{\text{vac}} = \rho_{\text{vac}} V \qquad \rho_{\text{vac}} = \frac{\Lambda}{8\pi G} \sim \frac{\hbar}{(\ell_P)^{d-2}} \frac{1}{L^2}$$

- Thus the vacuum energy is determined by the IR scale! In QFT it is only set by the UV scale:

$$\rho_{\text{vac}}^{\text{QFT}} \sim \frac{\hbar}{(\ell_P)^d} \qquad \frac{\rho_{\text{vac}}^{\text{QFT}}}{\rho_{\text{vac}}} = \frac{L^2}{\ell_P^2} = 10^{120}$$

- Q: Why is the observed dark energy so small?

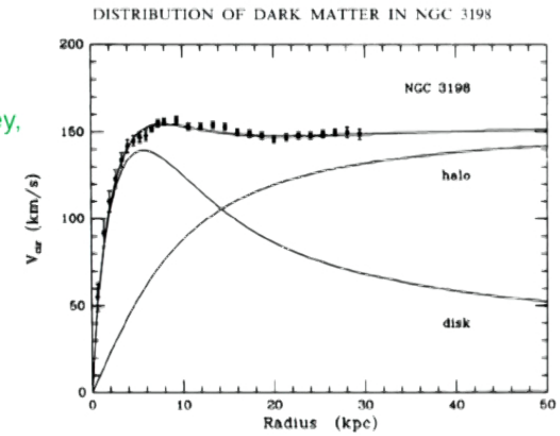
Motivation (3)

- Baryonic Tully-Fisher relation:

$$v_f^4 = a_M G M_B$$

Tully, Fisher, McGaugh, Schombert, Lelli, Famaey, Milgrom, et al.

with $a_M \approx 10^{-10}\, m/s^2 = cH_0/6$



- Cosmological acceleration scale appears in galactic physics!

- Dark matter effects start to dominate in galaxies if

$$g_B \lesssim a_M \qquad \text{or} \qquad 2\pi M_B R \lesssim \frac{A(R)}{4G}\frac{R}{L}$$

- Q: Is there a connection between dark energy and "dark matter"?
  ⟶ We need to understand the nature of dark energy better in order to understand the flattening of rotation curves.

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Plan of the talk

I. Thermodynamics of Causal Diamonds in dS

based on - arXiv:1612.04373 with Pablo Bueno, Antony Speranza, Vincent Min
- work with Ted Jacobson (to appear)

II. Towards a holographic description of dS

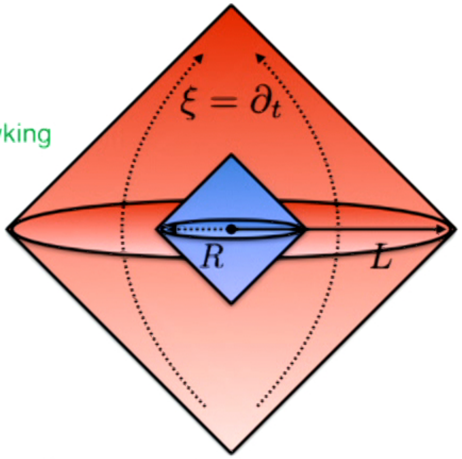
based on arXiv:1801.02589 with Erik Verlinde and Sam van Leuven

Thermodynamics of dS

- Black hole thermodynamics has taught us a lot about quantum gravity.
- Other causal horizons also have thermodynamic properties: Rindler horizon, cosmological horizon.
- First law of de Sitter spacetime: Gibbons & Hawking

$$-\delta H_\xi^m = T\delta S$$

$$H_\xi^m = \int_\Sigma \xi^a T_{ab} d\Sigma^b \qquad T = \frac{\kappa\hbar}{2\pi} \qquad S = \frac{A(L)}{4G\hbar}$$



- Q: Can this law be generalized to finite causal diamonds in de Sitter space?
A: Yes! And it also holds in flat space & AdS.

Causal Diamonds in dS

- Consider a spacelike, ball-shaped region Σ in the static patch of de Sitter space.

- A causal diamond is the intersection of the future of P with the past of P'.

- Useful coordinate system for dS:

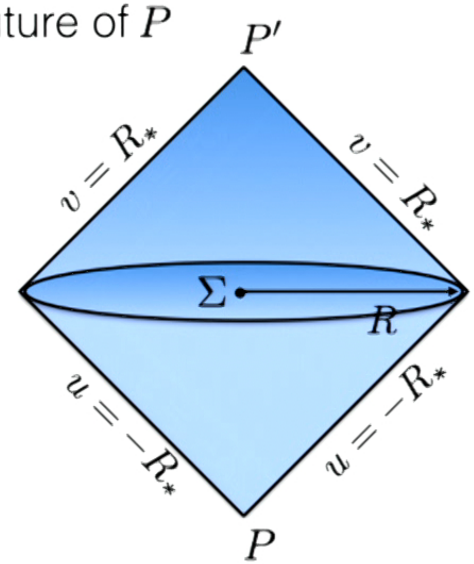
$$ds^2 = \text{sech}^2(r_*/L)\left[-dudv + L^2 \sinh^2(r_*/L) d\Omega_{d-2}^2\right]$$

where r_* is a tortoise coordinate and

$$v = t + r_*$$

$$u = t - r_*$$

are light-cone coordinates.

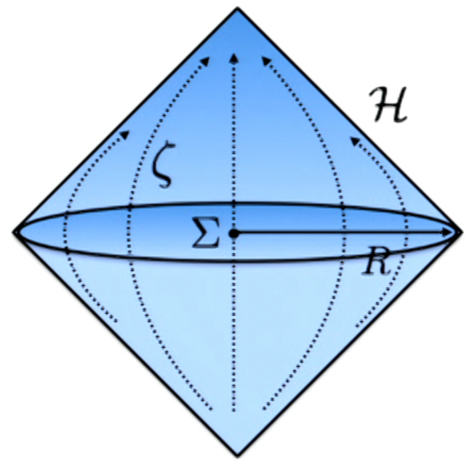


$$R = L\tanh(R_*/L)$$

Conformal isometry

- First law of de Sitter space follows from timelike Killing symmetry.
- Causal diamonds do not have a true Killing symmetry, but rather a conformal Killing symmetry

$$\mathcal{L}_\zeta g_{ab} = 2\alpha g_{ab} \qquad \alpha = \frac{1}{d}\nabla_c \zeta^c$$



- De Sitter space is conformally flat, so its conformal group is: $SO(2,d)$.
- Unique conformal Killing vector ζ which generates a flow within the diamond

$$\zeta = \frac{L}{\sinh(R_*/L)}\Big[\left(\cosh(R_*/L) - \cosh(u/L)\right)\partial_u + \left(\cosh(R_*/L) - \cosh(v/L)\right)\partial_v\Big]$$

First Law of Causal Diamonds

- Diffeomorphism Noether identity:
  (holds for any vector field!)

  Wald & Iyer

  $$\delta H_\zeta = \delta \int_{\partial\Sigma} Q_\zeta \qquad \text{or} \qquad \delta H_\zeta^m + \delta H_\zeta^g = -\frac{\kappa}{8\pi G}\delta A$$

- First law of causal diamonds for (A)dS and flat space:

  Jacobson & MV

  $$\boxed{-\delta H_\zeta^m = \frac{\kappa}{8\pi G}\left(\delta A - k\delta V\right)}$$

  'conformal Killing energy'

  $$H_\zeta^m = \int_\Sigma \zeta^a T_{ab} d\Sigma^b$$

  trace of extrinsic curvature of $\partial\Sigma$

  $$k = \frac{d-2}{R}\sqrt{1-(R/L)^2}$$

- Zeroth law: surface gravity κ is constant on $\mathcal{H}$.

  conformally invariant definition of κ: $\nabla_a\left(\zeta^b\zeta_b\right) = -2\kappa\zeta_a$

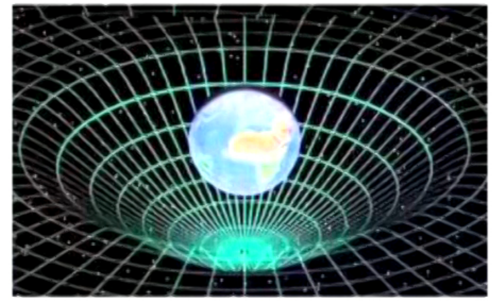
  Jacobson & Kang De Lorenzo & Perez
  Dyer, Honig, Sultana

First Law of Causal Diamonds

Two ways of rewriting the first law

$\Leftrightarrow$

Reformulations of the (linearized) Einstein equations



1. If volume is kept fixed, then matter decreases the area:

$$-\delta H_\zeta^m = \frac{\kappa}{8\pi G}\delta A\Big|_V$$

assuming $H_\zeta^m > 0$

2. If area is kept fixed, then matter increases the volume:

$$\delta H_\zeta^m = \frac{\kappa k}{8\pi G}\delta V\Big|_A$$

Interpretations

Interpretations of the first law of causal diamonds:

1. Assuming area is related to entanglement entropy: $\delta S_{\rm BH} = \frac{\delta A|_V}{4G\hbar}$

   Bianchi & Myers
   Ryu & Takayanagi

   Entanglement entropy of the vacuum is maximal.

   $\delta S_{\rm mat} + \delta S_{\rm BH} = 0$ with $\delta S_{\rm mat} = \frac{2\pi}{\hbar}\delta\langle H^m_\zeta\rangle$ 'entanglement equilibrium' Jacobson

2. Assuming temperature is negative: $\tilde{T} = -\frac{\hbar\kappa}{2\pi}$ Klemm & Vanzo

   Free energy of the causal diamond is minimal.

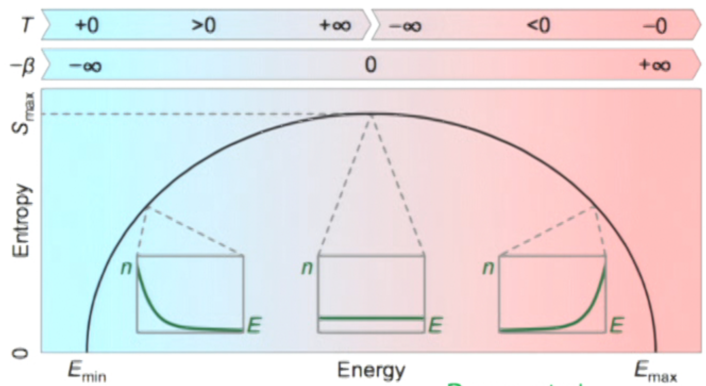
   $\delta F = 0$ where $F = H_\zeta - \tilde{T} S_{\rm BH}$

Conclusion (I)

- Causal diamonds in empty de Sitter space are equilibrium states.

$$\delta H_\zeta = \tilde{T}\delta S_{\mathrm{BH}}$$

- Adding matter decreases the gravitational entropy, and hence puts the microscopic system out of equilibrium.



Braun, et al.

Plan of the talk

I. Thermodynamics of Causal Diamonds in dS

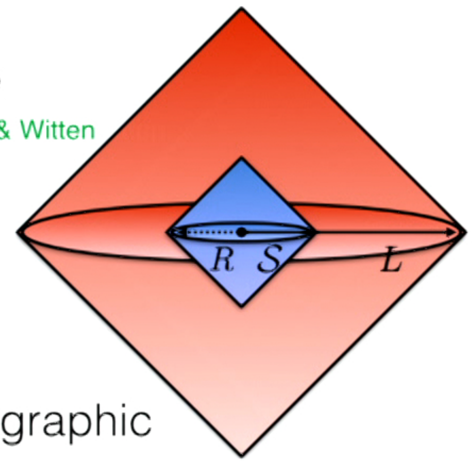
II. Towards a holographic description of dS

Holography for dS

- Holographic principle: maximal number of degrees of freedom associated to a holographic surface $\mathcal{S}$ is given by 't Hooft & Susskind

$$\mathcal{C} = \frac{A(R)}{16\pi G_d}$$

- This has been realized in anti-de Sitter space through the AdS/CFT correspondence. Susskind & Witten



- We assume there exists a microscopic holographic dual to de Sitter space.

- Q: What is the nature of the microscopic holographic degrees of freedom of dS?

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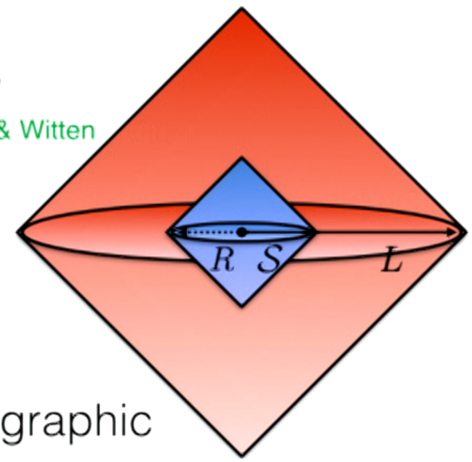


Conformal map

Anninos, Hartnoll, Hofman

- Conformal map between: $dS_d \times S^1 \cong BTZ \times S^{d-2}$

$$ds^2 = -\left(1 - R^2/L^2\right) dt^2 + \frac{dR^2}{1 - R^2/L^2} + R^2 d\Omega_{d-2}^2 + L^2 d\phi^2$$

$$ds^2 = \Omega^2 d\tilde{s}^2 \qquad \Omega = \frac{R}{L} = \frac{L}{r}$$

Banados, Teitelboim, Zanelli

$$d\tilde{s}^2 = -\left(r^2/L^2 - 1\right) dt^2 + \frac{dr^2}{r^2/L^2 - 1} + r^2 d\phi^2 + L^2 d\Omega_{d-2}^2$$

- Idea: use conformal mapping to relate general features of the microscopic theories on holographic screens in both spacetimes, e.g. the number of d.o.f. $\mathcal{C}$ and the excitation energy per d.of. ϵ . The holographic dual of BTZ is well understood through AdS/CFT.

Conjecture

- Conjecture: The holographic theories for two spacetimes that are conformally related have identical microscopic properties when the Weyl factor equals one on the corresponding holographic screens.

- Example: note that $\Omega = 1$ if $r = R = L$.
  BTZ central charge coincides with total number of d.o.f. in dS

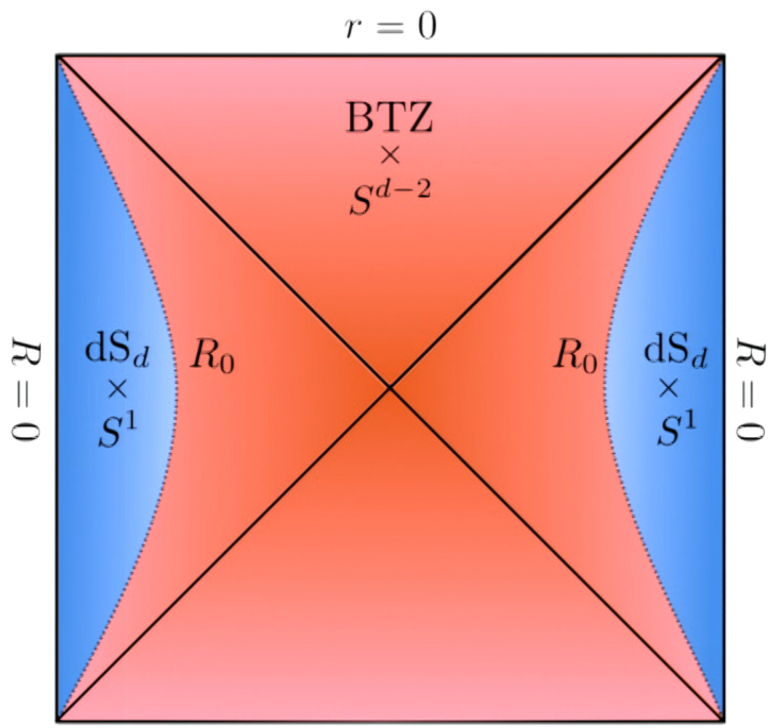
Brown & Henneaux

$$\frac{c}{12} = \frac{2\pi L}{16\pi G_3} = \frac{A(L)}{16\pi G_d} = \mathcal{C}(L)$$

where we used:

$$\frac{1}{G_3} = \frac{A(L)}{G_{d+1}} \qquad \frac{1}{G_d} = \frac{2\pi L}{G_{d+1}}$$

Gluing of dS and BTZ



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Rescaled BTZ metric

- Master formula for family of Weyl rescaled BTZ metrics:

$$ds^2 = \frac{R^2}{L^2}\left[-\left(\frac{r^2}{L^2} - \frac{\Delta - c/12}{c/12}\right)dt^2 + \left(\frac{r^2}{L^2} - \frac{\Delta - c/12}{c/12}\right)^{-1} dr^2 + r^2 d\phi^2 + L^2 d\Omega_{d-2}^2\right]$$

$$\begin{aligned} \Delta = 0 \quad &: \quad \text{anti-de Sitter space}\,, \\ \Delta = \frac{c}{12} &: \quad \text{Minkowski space}\,, \\ \Delta = \frac{c}{6} \quad &: \quad \text{de Sitter space}\,. \end{aligned} \qquad \frac{R}{L} = \frac{L}{r}$$

- Rescaling of the metric changes the curvature radius. Take $R = R_0$. The central charge now depends on the radius R_0

$$\frac{c(R_0)}{12} = \frac{2\pi R_0}{16\pi G_3} = \frac{A(R_0)}{16\pi G_d}\frac{R_0}{L}$$

Reversal of UV/IR relation

- What do we learn from the conformal map?

- Typical energy required to excite UV d.o.f. $\epsilon = 1/\delta$

  In AdS/CFT: UV-IR correspondence $\delta = \frac{L^2}{r} \longrightarrow \epsilon = \frac{r}{L^2}$

- After the conformal map UV cut-off energy becomes

  $\epsilon \sim \frac{1}{R}$ since $\frac{r}{L} = \frac{R}{L}$

- Thus, *UV-IR relation is reversed for dS!* (and for sub-AdS)
  Large distances (=IR) in the bulk correspond to low energies (=IR) in the microscopic theory. The holographic principle then implies that the number of d.o.f. *increases* from UV to the IR.

Long string interpretation

- Reduce the size of the transverse S^1: Kaluza-Klein circle. This introduces a conical defect in the BTZ geometry.

$$\phi \equiv \phi + 2\pi/N \qquad\qquad N = L/\ell \gg 1$$

- $BTZ \times S^{d-2}$ metric

$$ds^2 = -\left(r^2/L^2 - 1\right) dt^2 + \frac{dr^2}{r^2/L^2 - 1} + r^2 d\phi^2 + L^2 d\Omega_{d-2}^2$$

$$= N^2 \left[-\left(\hat{r}^2/\ell^2 - 1/N^2\right) dt^2 + \frac{d\hat{r}^2}{\hat{r}^2/\ell^2 - 1/N^2} + \hat{r}^2 d\hat{\phi}^2 + \ell^2 d\Omega_{d-2}^2 \right]$$

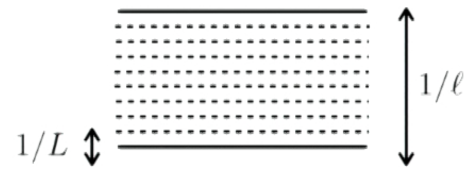
- This BTZ geometry is dual to the twisted sector of a symmetric product 2d CFT

$$\hat{r} = r/N^2$$
$$\hat{\phi} = N\phi$$

$$CFT^N/S_N \qquad\qquad \Delta_\ell = \frac{c_\ell}{12}\left(1 - 1/N^2\right)$$

Long string interpretation

- Long string length: L
  Short string length: ℓ



- The long string phenomenon has two effects

  1. fractionated spectrum: $\Delta_L - \frac{c_L}{12} = N\left(\Delta_\ell - \frac{c_\ell}{12}\right)$
  2. reduction of central charge: $c_L = \frac{1}{N} c_\ell$

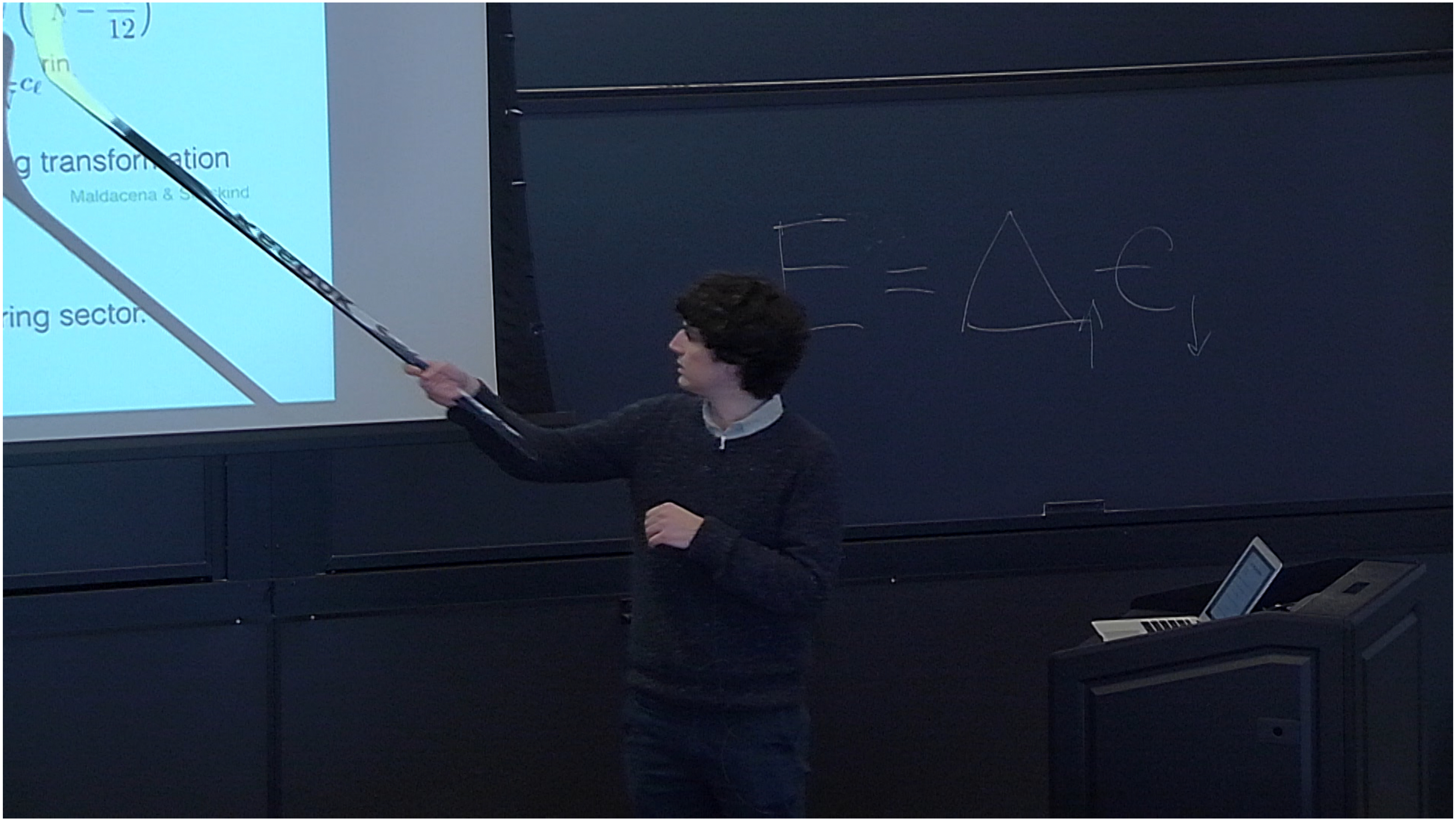
- Cardy formula is invariant under the long string transformation

Maldacena & Susskind

$$S = 4\pi\sqrt{\frac{c}{12}\left(\Delta - \frac{c}{12}\right)}$$

- Thermal entropy is dominated by the long string sector.


string transformation
Maldacena & Susskind
g string sector.


g transformation
Maldacena & S kind
ring sector.

Volume law for entropy

- The entropy of the BTZ black hole matches the entropy of dS. Both are computed by the Cardy formula. Take $R = L$.

$$S = 4\pi\sqrt{\frac{c_L(L)}{12}\left(\Delta_L - \frac{c_L(L)}{12}\right)} = \frac{A(L)}{4G_d} \quad \text{since} \quad \Delta_L = c_L(L)/6$$

- Thus, in 2d CFT language, dS space corresponds to a thermal gas of long strings at temperature $1/L$.

- At a finite radius R the entropy satisfies a volume law Verlinde

$$S = 4\pi\sqrt{\frac{c_L(R)}{12}\left(\Delta_L - \frac{c_L(R)}{12}\right)} = \frac{A(R)}{4G_d}\frac{R}{L} = \frac{V(R)}{V_0}$$

⟶ dS entropy is divided over the volume!

Vacuum energy of dS

- In dS each long string typically only carries its lowest energy excitation mode $1/L$.

- Vacuum (dark) energy of dS can be interpreted as the energy associated to these excitations

$$E = \left(\Delta_L - \frac{c_L(R)}{12}\right)\epsilon_L \quad \text{with} \quad \Delta_L = c_L(R)/6$$

$$E_{\rm vac} = +\frac{c_L(R)}{12}\,\epsilon_L = +\frac{(d-1)(d-2)}{16\pi G_d L^2}V(R)$$

- Vacuum energy associated to short strings $\quad \frac{L^2}{\ell_P^2} = 10^{120}$

$$E_{\rm vac}^{\rm UV} = +\frac{c_\ell(R)}{12}\,\epsilon_\ell = +\frac{(d-1)(d-2)}{16\pi G_d \ell^2}V(R)$$

- The long string phenomenon explains why observed $E_{\rm vac}$ is L^2/ℓ^2 times lower than $E_{\rm vac}^{\rm UV}$.

Conclusion (II)

- De Sitter space is dual to a thermal bath of long strings, and its entropy is counted by the Cardy formula.
- The number of degrees of freedom in the dual system increases towards the IR, opposite the Wilsonian intuition. This is due to the long string phenomenon.
- The long string phenomenon also explains the smallness of the value of dark energy (although not the value itself).
- The non-equilibrium dynamics of the microscopic d.o.f. of dS might lead to an additional gravitational force at long distances and large time scales.

Apparent dark matter formula

- By computing the elastic energy associated to this stress, Verlinde finds the following formula that relates apparent dark matter to baryonic matter

$$\int_0^R \frac{GM_D^2(R')}{R'^2} dR' = a_M M_B(R) R$$

- For point masses, i.e. $M_B \neq M_B(R)$, this is equivalent to the Baryonic Tully-Fisher relation

$$v_f^4 = a_M G M_B$$

Effect of matter on dark energy

Verlinde

- Positive dark energy leads to a thermal volume law for entropy

$$S_{DE} = \frac{A(R)}{4G_d}\frac{R}{L} = \frac{V(R)}{V_0} \quad \text{with} \quad V_0 = \frac{4GL}{d-1}$$

- By adding matter to dS, the volume increases and the energy density decreases. Total energy and entropy stay the same. Volume of space which does not contain dark energy: $V_M(L)$

$$2\pi ML = \frac{V_M(L)}{V_0}$$

- We conjecture that matter removes the following amount of entropy from the microscopic system in a finite region of radius R

$$S_M = 2\pi MR = \frac{V_M(R)}{V_0}$$

Dark matter and dark energy

- Dark matter effect start to dominate if

$$2\pi M_B R \lesssim \frac{A(R)}{4G}\frac{R}{L} \qquad \text{or} \qquad S_M \lesssim S_{DE}$$

- Verlinde's proposal: the entropy density turns the dark energy into a dynamical medium. If the elastic medium is at *equilibrium*, then it is stress free, and everything is well described by GR. This is the case in empty dS.

- If part of the dark energy is turned into actual matter, a residual stress is created in the system. The medium is *out of equilibrium* and it takes a long (Hubble) time for the system to equilibrate. This stress leads to modifications of GR, precisely when the dark energy starts to dominate.

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