

Title: PSI 17/18 - Foundations of Quantum Mechanics - Lecture 3

Date: Jan 31, 2018 10:15 AM

URL: <http://pirsa.org/18010068>

Abstract:

The textbook interpretation of quantum theory

Representational completeness of ψ . The rays of Hilbert space correspond one-to-one with the **physical states** of the system.

Measurement. If the Hermitian operator A with spectral projectors $\{P_k\}$ is measured, the probability of outcome k is $\langle\psi|P_k|\psi\rangle$. These **probabilities are objective -- indeterminism**.

Evolution of isolated systems. It is unitary, $|\psi\rangle \rightarrow U|\psi\rangle = e^{-\frac{i}{\hbar}Ht}|\psi\rangle$ therefore **deterministic and continuous**.

Evolution of systems undergoing measurement. If Hermitian operator A with spectral projectors $\{P_k\}$ is measured and outcome k is obtained, the physical state of the system **changes discontinuously**,

$$|\psi\rangle \rightarrow |\psi_k\rangle = \frac{P_k|\psi\rangle}{\sqrt{\langle\psi|P_k|\psi\rangle}}$$

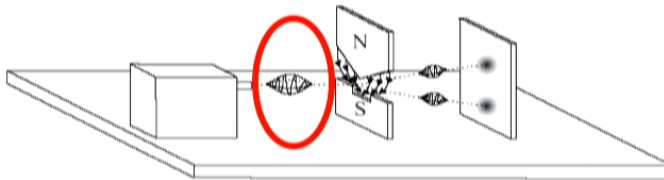
First problem: the term “measurement” is not defined in terms of the more primitive “physical states of systems”. Isn’t a measurement just another kind of physical interaction?

Can one eliminate measurement as a primitive concept and describe everything in terms of physical states?

“It would seem that the theory is exclusively concerned about "results of measurement", and has nothing to say about anything else. What exactly qualifies some physical systems to play the role of "measurer"? ”

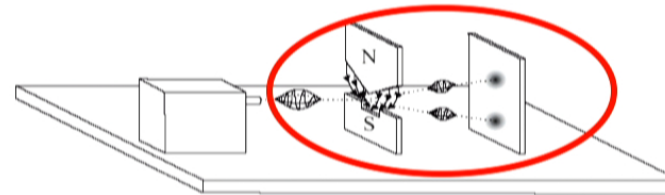
- John Bell

Inconsistencies of the textbook interpretation



By the collapse postulate
(applied to the system)

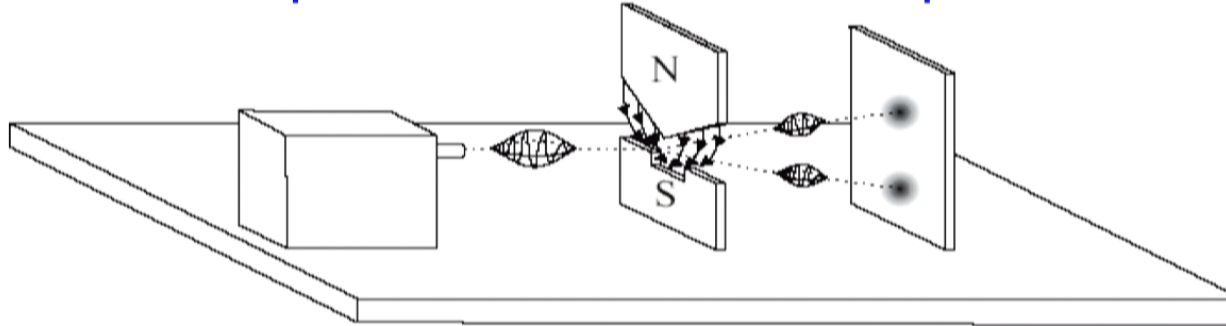
Indeterministic and
discontinuous evolution



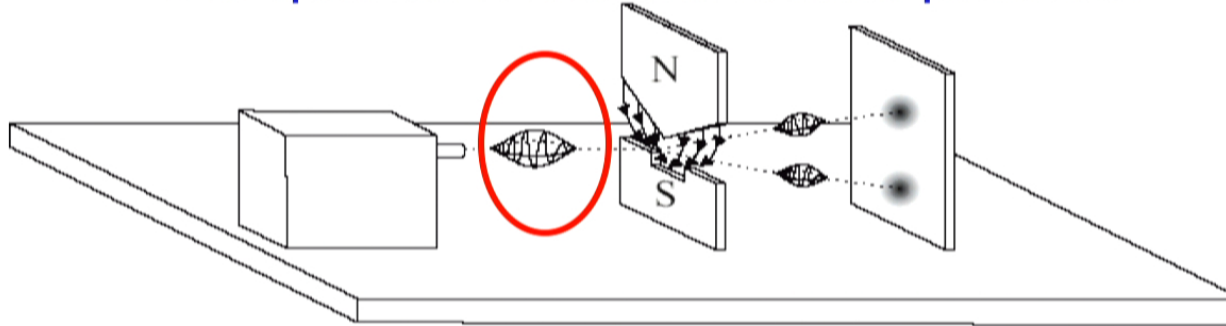
By unitary evolution postulate
(applied to isolated system that includes
the apparatus)

Deterministic and
continuous evolution

The quantum measurement problem



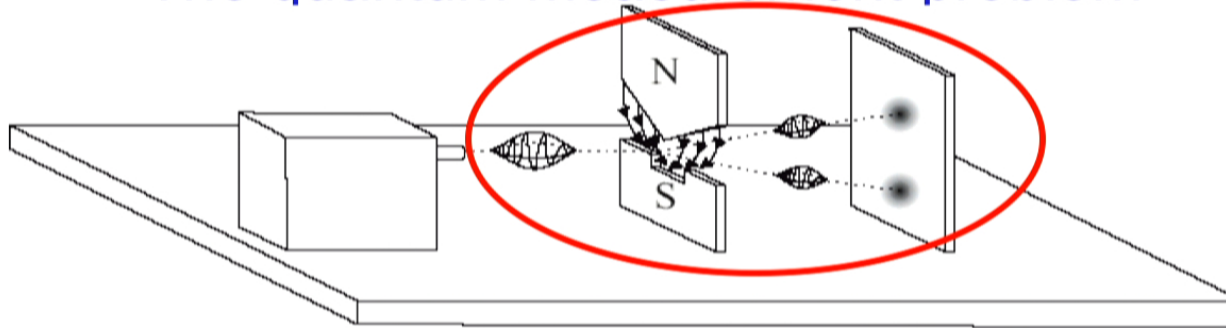
The quantum measurement problem



If the measurement apparatus is treated **externally**

$$\begin{aligned} a|\uparrow\rangle + b|\downarrow\rangle &\rightarrow |\uparrow\rangle \text{ with probability } |a|^2 \\ &\rightarrow |\downarrow\rangle \text{ with probability } |b|^2 \end{aligned}$$

The quantum measurement problem



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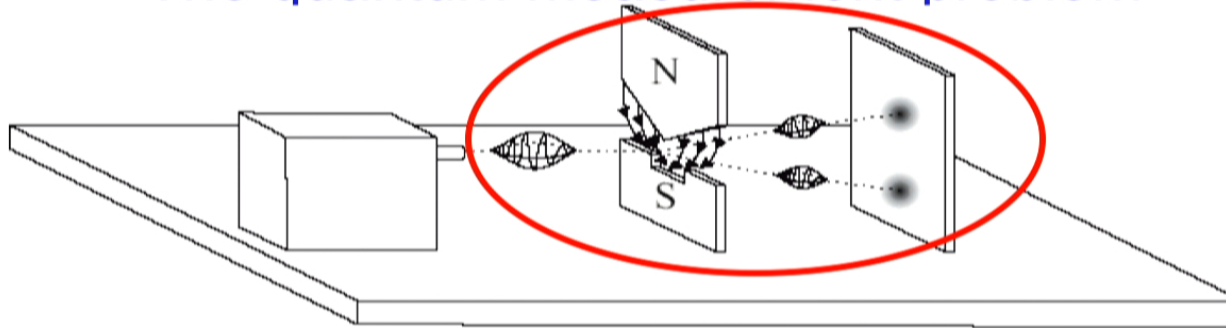
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If the measurement apparatus is treated **internally**

$$|\uparrow\rangle \otimes |\text{"ready"}\rangle \rightarrow U(|\uparrow\rangle \otimes |\text{"ready"}\rangle) = |\uparrow\rangle \otimes |\text{"up"}\rangle$$

$$|\downarrow\rangle \otimes |\text{"ready"}\rangle \rightarrow U(|\downarrow\rangle \otimes |\text{"ready"}\rangle) = |\downarrow\rangle \otimes |\text{"down"}\rangle$$

The quantum measurement problem



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$$U \text{ is a linear operator } U(a|\psi\rangle + b|\phi\rangle) = aU|\psi\rangle + bU|\phi\rangle$$

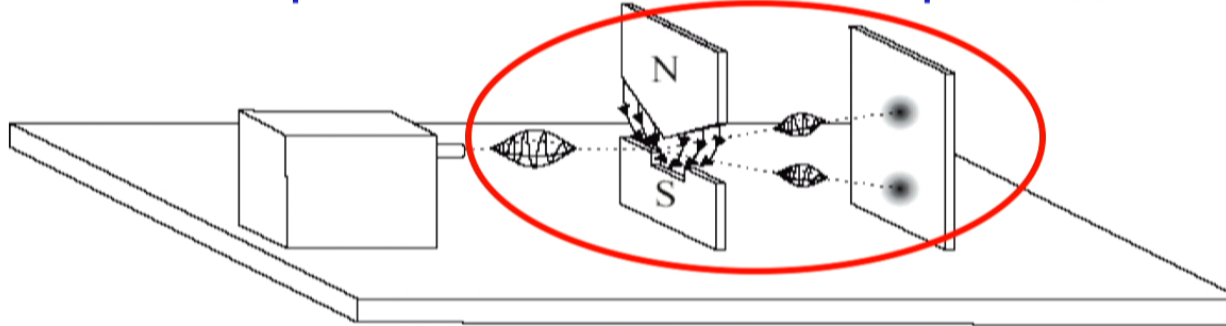
$$\begin{aligned} (a|\uparrow\rangle + b|\downarrow\rangle) \otimes |\text{"ready"}\rangle &\rightarrow U[a|\uparrow\rangle \otimes |\text{"ready"}\rangle + b|\downarrow\rangle \otimes |\text{"ready"}\rangle] \\ &= a|\uparrow\rangle \otimes |\text{"up"}\rangle + b|\downarrow\rangle \otimes |\text{"down"}\rangle \end{aligned}$$

$$S_{\hat{z}} = (+1)|\uparrow\rangle\langle\uparrow| + (-1)|\downarrow\rangle\langle\downarrow|$$

$$(S_{\hat{z}} \otimes I)(a|\uparrow\rangle \otimes |\text{"up"}\rangle + b|\downarrow\rangle \otimes |\text{"down"}\rangle)$$

$$a(S_{\hat{z}}|\uparrow\rangle) \otimes |\text{"up"}\rangle + b(S_{\hat{z}}|\downarrow\rangle) \otimes |\text{"down"}\rangle$$

The quantum measurement problem



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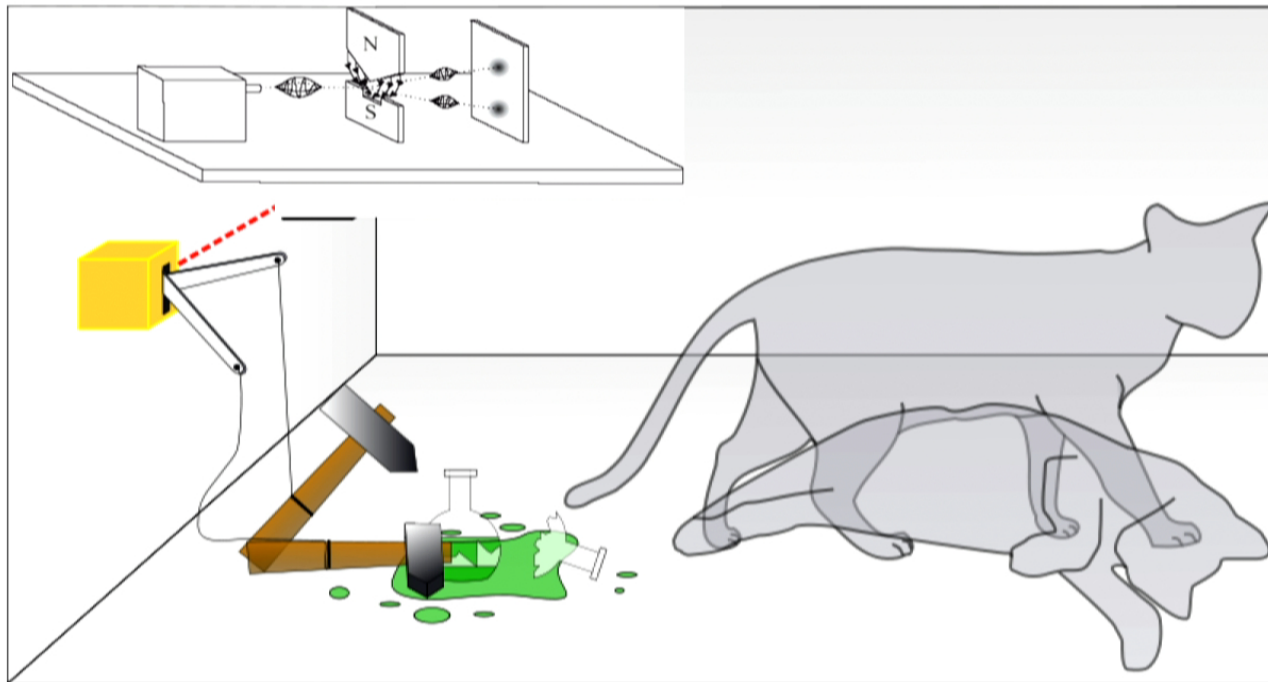
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$$\rho^{SA} = |a|^2 |\uparrow\rangle\langle\uparrow| \otimes | \text{"up"} \rangle \langle \text{"up"} | \\ + |b|^2 |\downarrow\rangle\langle\downarrow| \otimes | \text{"down"} \rangle \langle \text{"down"} |$$

$$\rho^{SAE} = |\uparrow\rangle\langle\uparrow| \otimes | \text{"up"} \rangle \langle \text{"up"} | \otimes \rho_E$$

Responses to the measurement problem

1. Deny realism
 - Purely operational account of quantum theory
2. Deny the universality of unitary dynamics
 - Dynamical collapse theories
3. Deny that ψ is a complete representation of reality
 - Hidden variable models
 - Models of reality beyond hidden variables?
4. Deny indeterminism and discontinuity, except as subjective illusions
 - Everett's relative state interpretation, or "many worlds"

Note: Logical consistency is a low bar. We will see a number of other desiderata that one should want an interpretation of QT to satisfy.