Title: PSI 17/18 - Foundations of Quantum Mechanics - Lecture 2

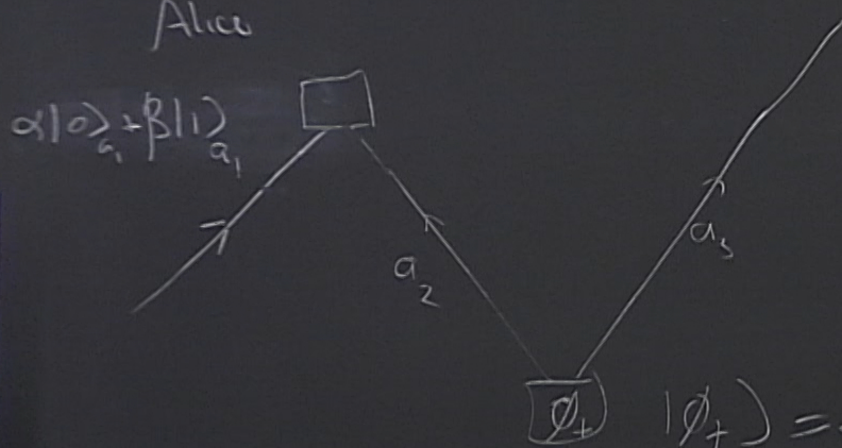
Date: Jan 30, 2018 10:15 AM

URL: http://pirsa.org/18010067

Abstract:

Quantum teleportation.

Alice



$$|\phi_+\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle_{a_2a_3} + |11\rangle_{a_2a_3}\right)$$

Bell basis.

$$|\phi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right)$$

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right)$$

$$\langle\phi_\pm|\psi_\pm\rangle = 0$$

$$\langle\phi_+|\phi_-\rangle = 0$$

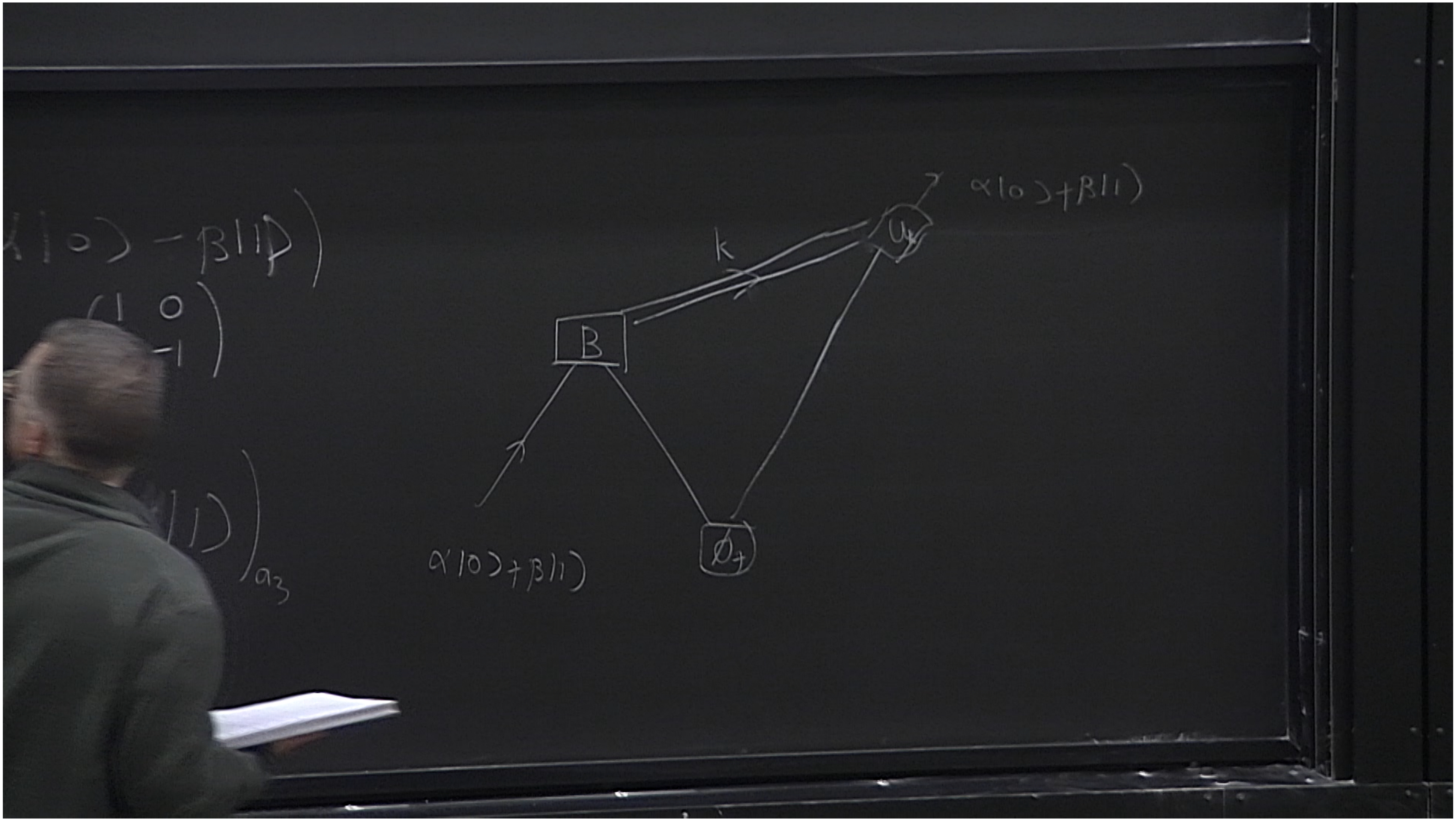
$$\langle\psi_+|\psi_-\rangle = 0$$

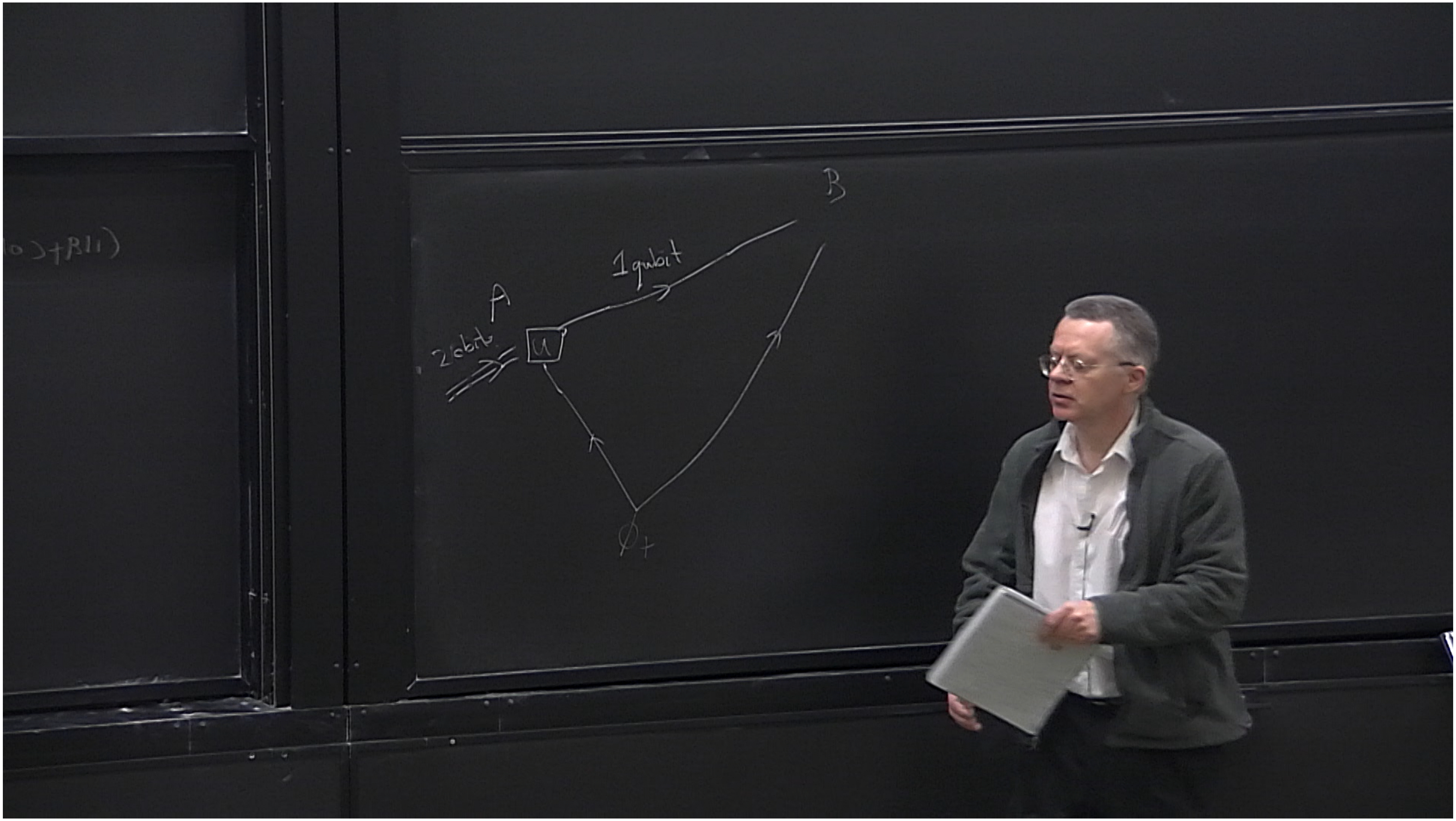
$(\alpha|0\rangle + \beta|1\rangle)|$

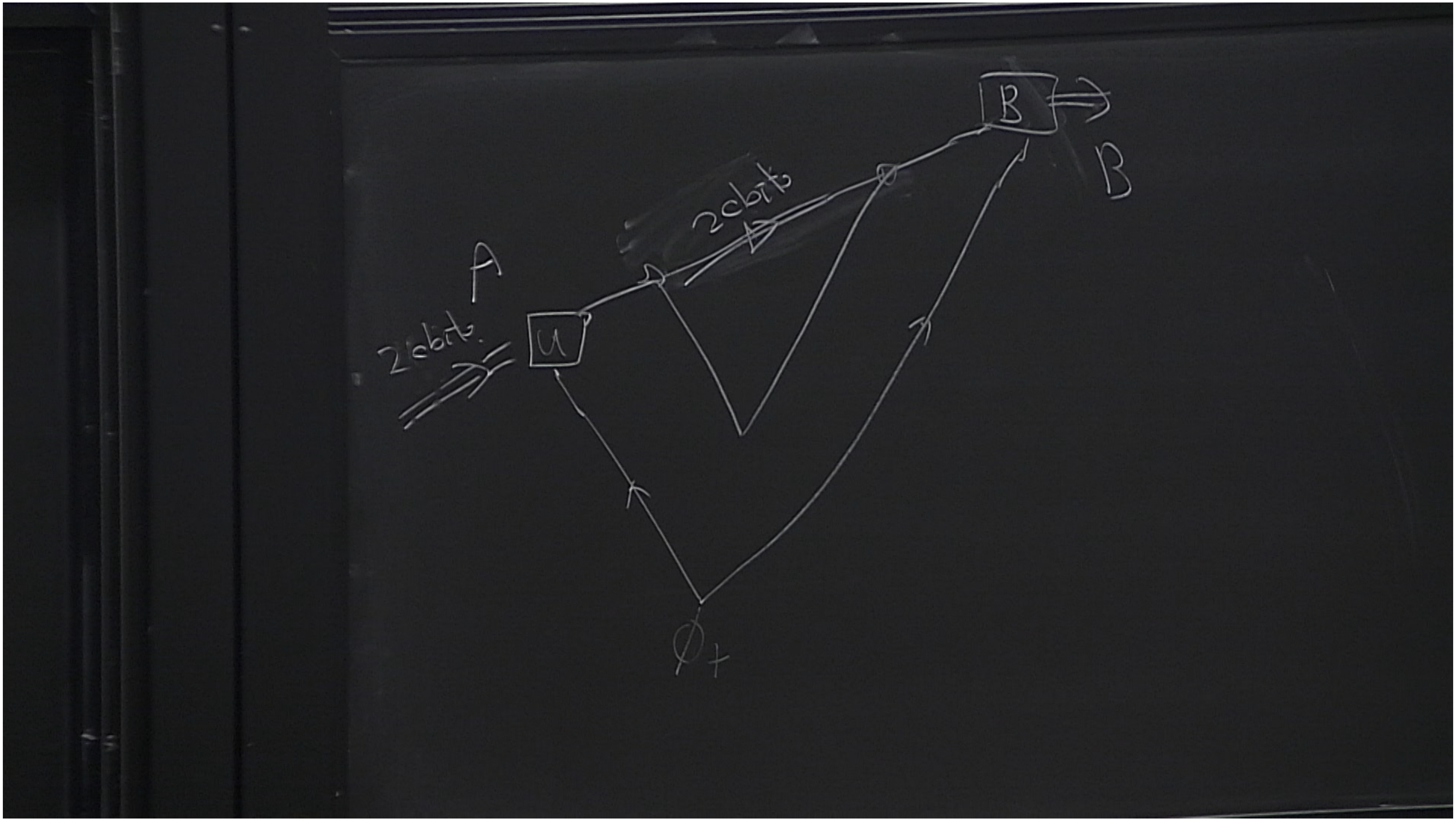
$$\langle\psi_+|\psi_-\rangle = 0$$

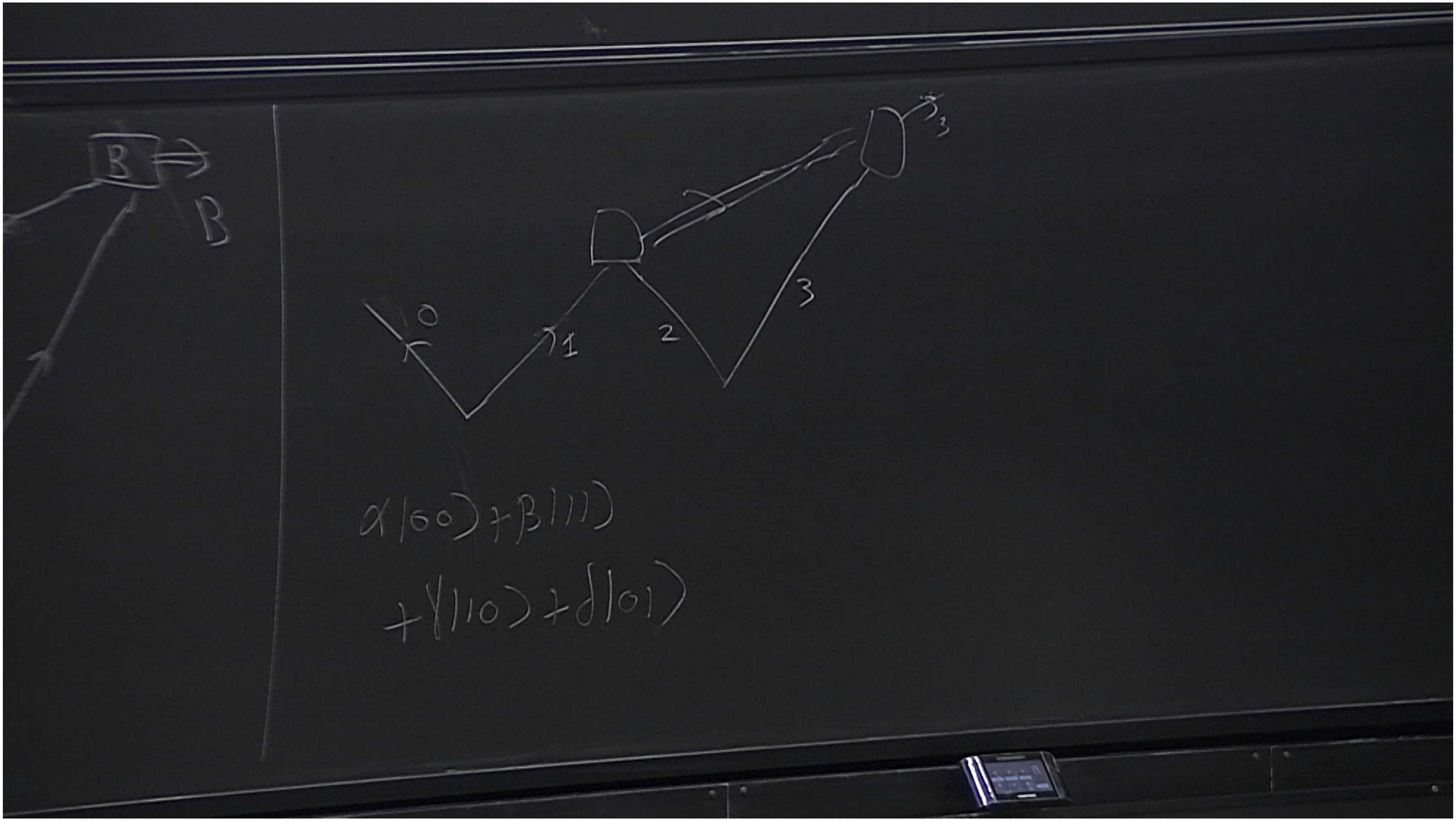
$$(\alpha|0\rangle_{a_1} + \beta|1\rangle_{a_1})|\phi_+\rangle_{a_2a_3} = \frac{1}{2}|\phi_+\rangle_{a_1a_2}\underbrace{(\alpha|0\rangle_{a_3} + \beta|1\rangle_{a_3})}_{\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}} + \frac{1}{2}|\phi_-\rangle_{a_1a_2}\underbrace{(\alpha|0\rangle - \beta|1\rangle)}_{\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}}$$

$$+\,|\psi_+\rangle_{a_1a_2}\underbrace{(\beta|0\rangle_{a_3} + \alpha|1\rangle_{a_3})}_{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}} + \frac{1}{2}|\psi_-\rangle_{a_1a_2}\underbrace{(\beta|0\rangle - \alpha|1\rangle)_{a_3}}_{\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}}$$


k
B
U_k
ϕ_+
$\alpha|0\rangle+\beta|1\rangle$
$\alpha|0\rangle+\beta|1\rangle$


B
1 qubit
A
2 cbits
U
ϕ_+


A
B
B
U
2 cbits
2 cbits
ϕ+


B
B
0
1
2
3
3
α|00⟩+β|11⟩
+γ|10⟩+δ|01⟩

The quantum Zeno effect 1977 Sudarshan Misra.

Start with $|0\rangle_a$ at time $t=0$.

$$|\psi(\delta t)\rangle = e^{-i\hat{H}\delta t/\hbar}|0\rangle$$

$$= |0\rangle - \frac{iH\delta t}{\hbar}|0\rangle + \dots$$

Perform

Perform measurement onto $|0\rangle\langle 0| = P_0$

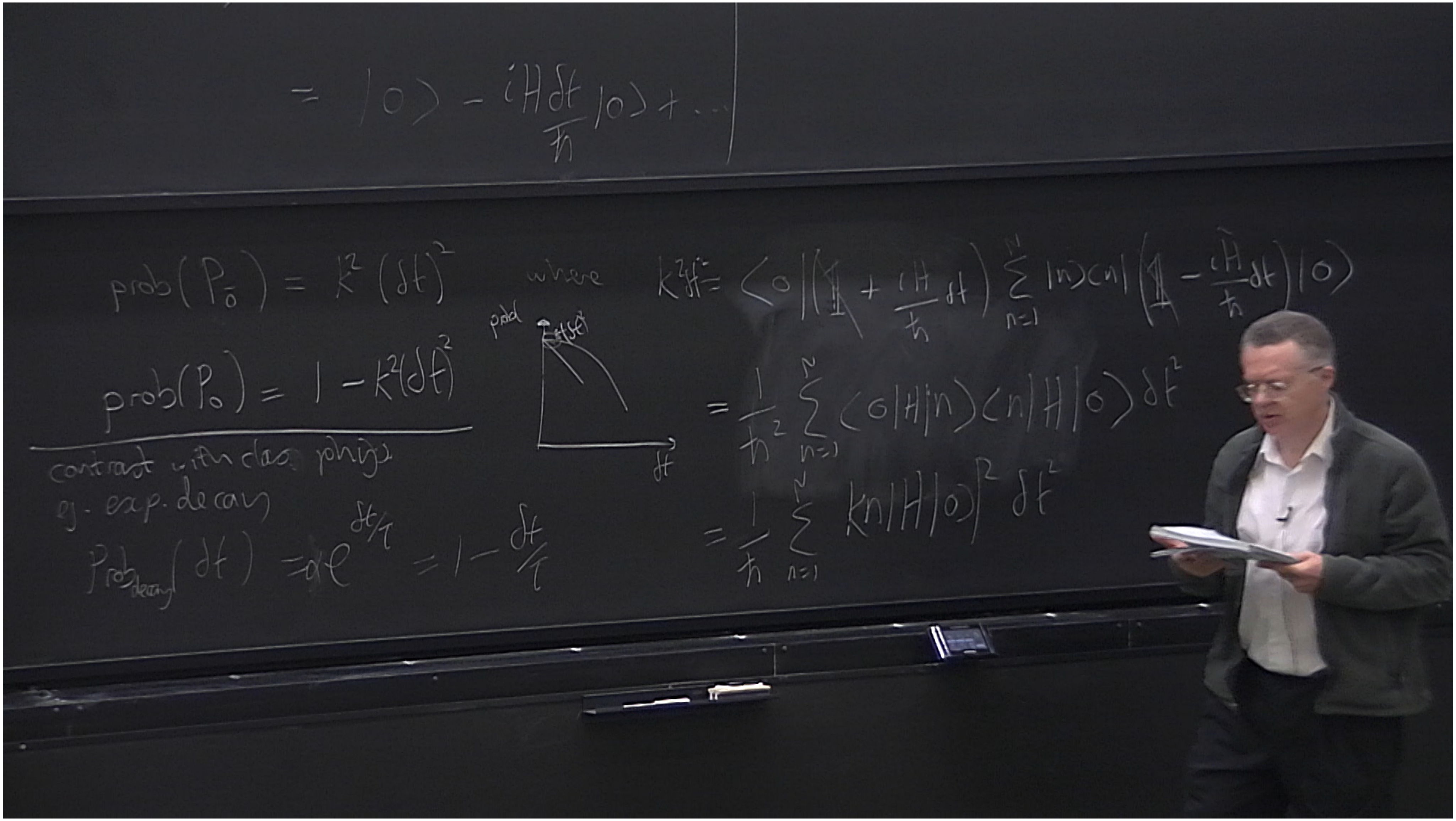
$$\mathbb{1} - |0\rangle\langle 0| = P_{\bar{0}} = \sum_{n=1}^{N} |n\rangle\langle n|$$

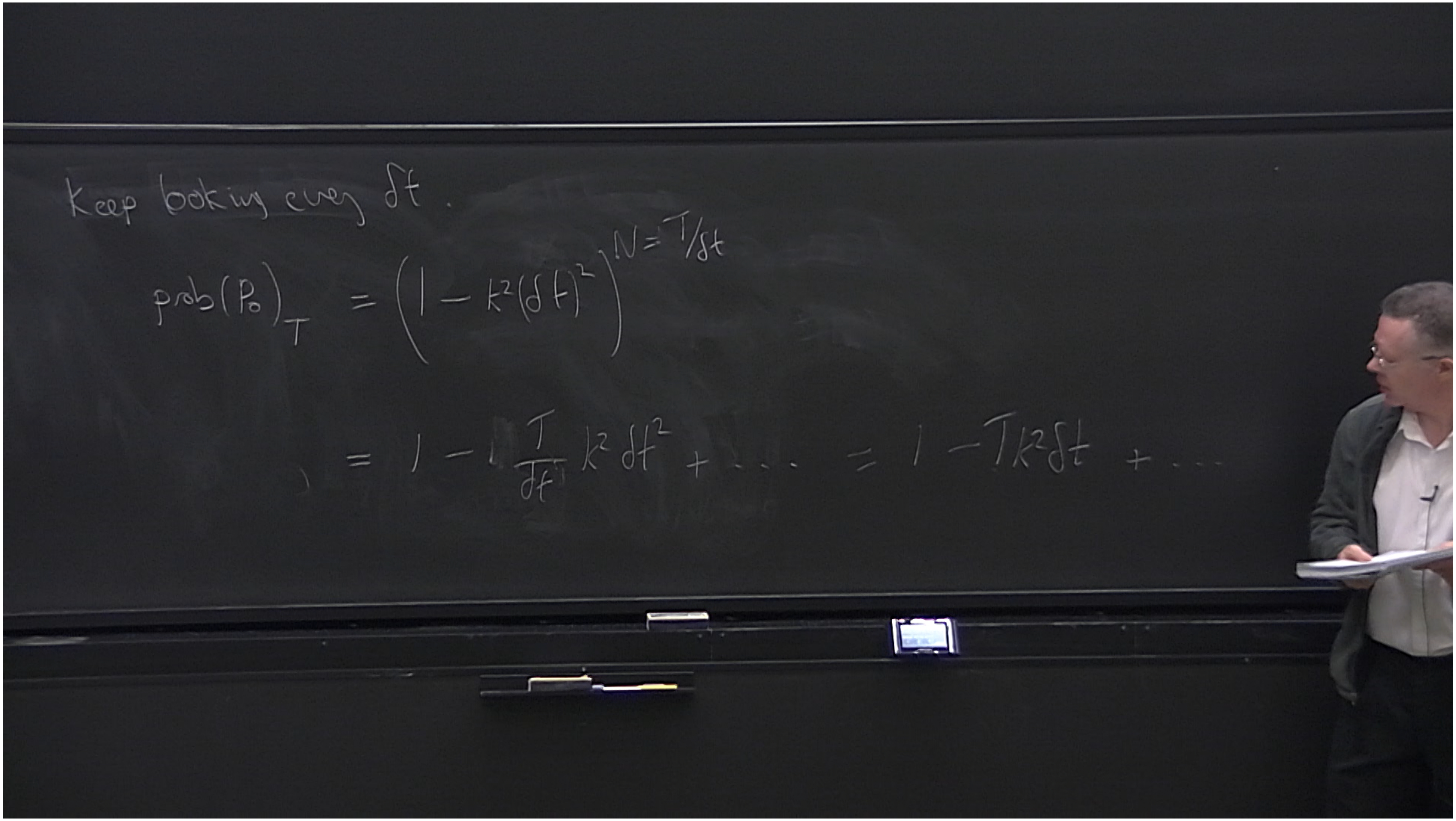
$$= |0\rangle - \frac{iH\delta t}{\hbar}|0\rangle + \dots$$

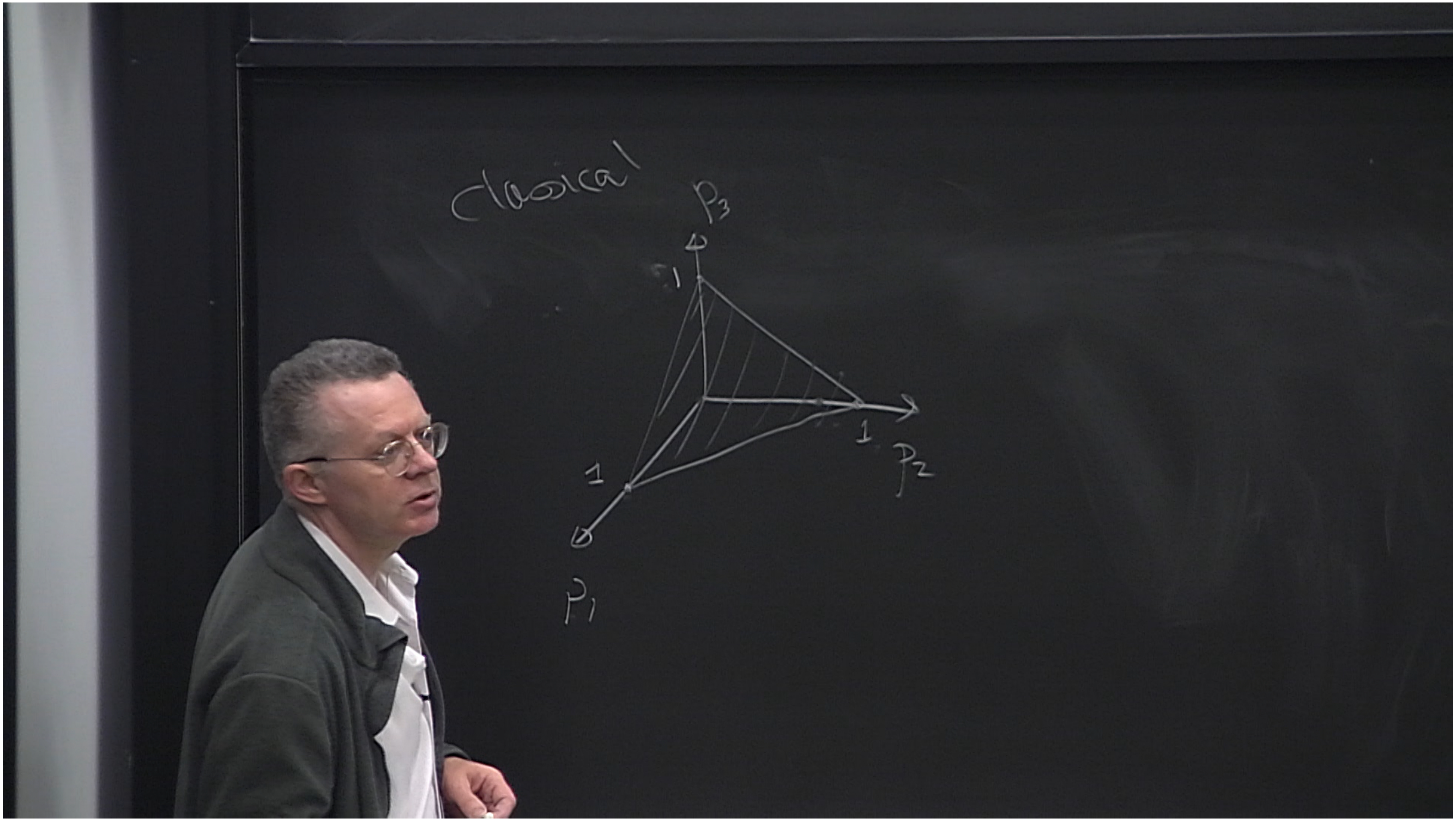
$$\text{prob}(P_{\bar{0}}) = k^2 (\delta t)^2 \quad \text{where} \quad k^2\delta t^2 = \langle 0 | \left(\mathbb{1} + \frac{iH}{\hbar}\delta t\right) \sum_{n=1}^{N} |n\rangle\langle n| \left(\mathbb{1} - \frac{iH}{\hbar}\delta t\right)|0\rangle$$

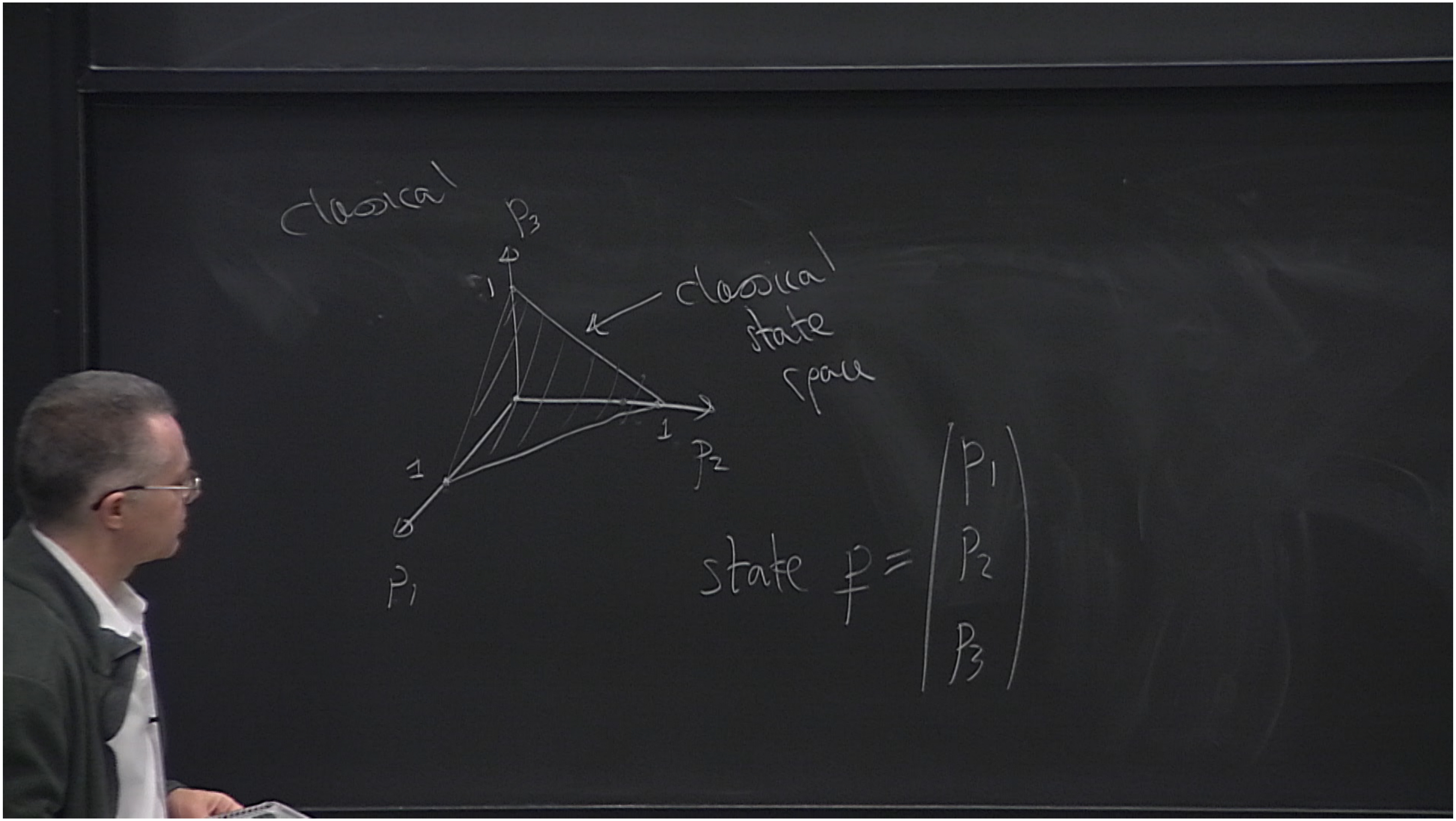
$$= \frac{1}{\hbar^2}\sum_{n=1}^{N} \langle 0|H|n\rangle\langle n|H|0\rangle \, \delta t^2$$

$$= \frac{1}{\hbar}\sum_{n=1}^{N} |\langle n|H|0\rangle|^2 \, \delta t^2$$

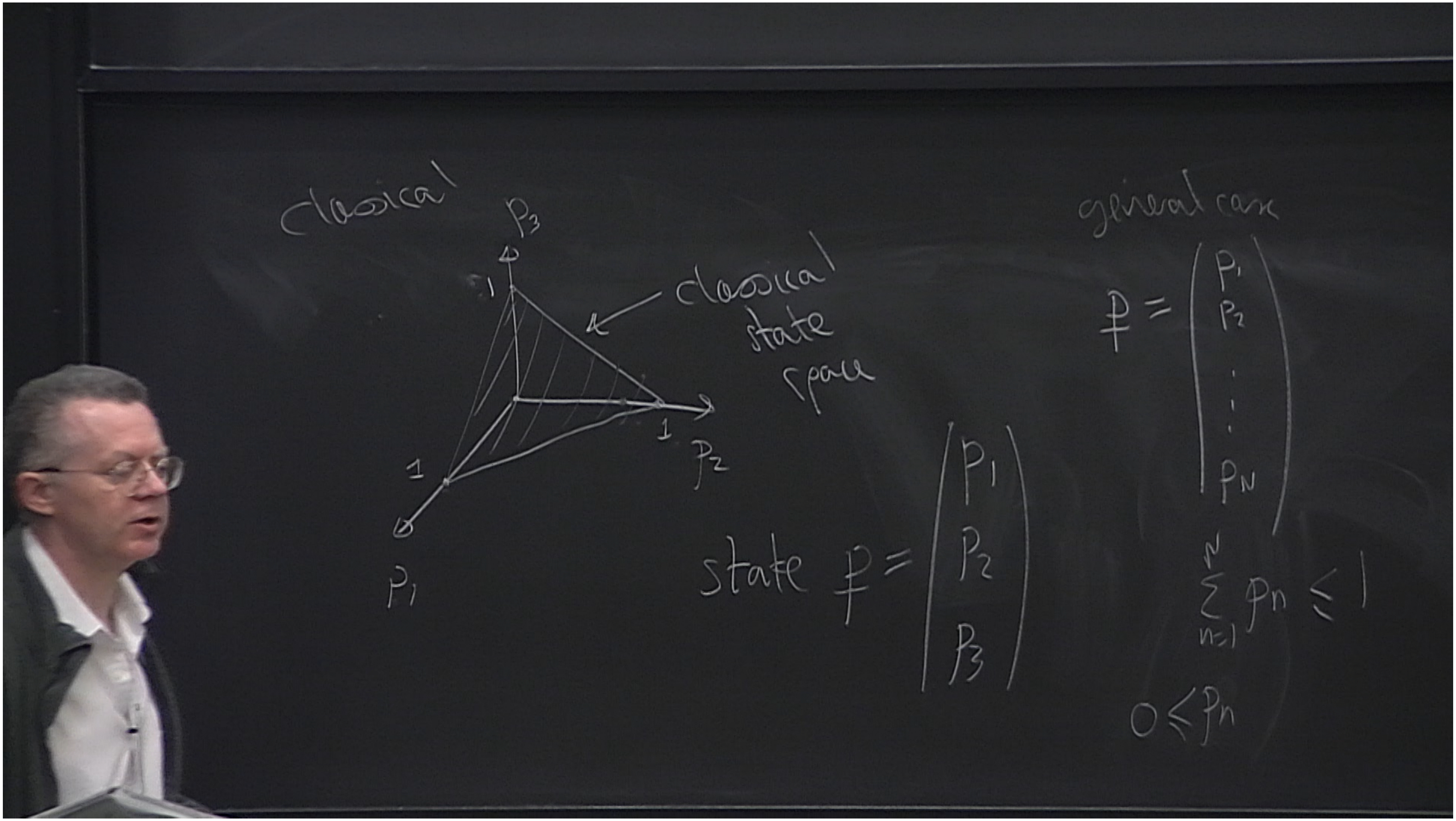

= |0⟩ − iHδt/ħ |0⟩ + ...
prob(P_0̄) = k²(δt)²
where
k²δt² = ⟨0|(𝟙 + iH/ħ δt) Σ_{n=1}^{N} |n⟩⟨n| (𝟙 − iH/ħ δt)|0⟩
prob(P_0) = 1 − k²(δt)²
= 1/ħ² Σ_{n=1}^{N} ⟨0|H|n⟩⟨n|H|0⟩ δt²
= 1/ħ Σ_{n=1}^{N} |⟨n|H|0⟩|² δt²
contrast with class. physics
eg. exp. decay
Prob_decay(δt) = e^{δt/τ} = 1 − δt/τ
prob
δt


Keep looking every δt .
$\text{prob}(P_0)_T = \left(1 - k^2(\delta t)^2\right)^{N = T/\delta t}$
$= 1 - \frac{T}{\delta t} k^2 \delta t^2 + \ldots = 1 - T k^2 \delta t + \ldots$

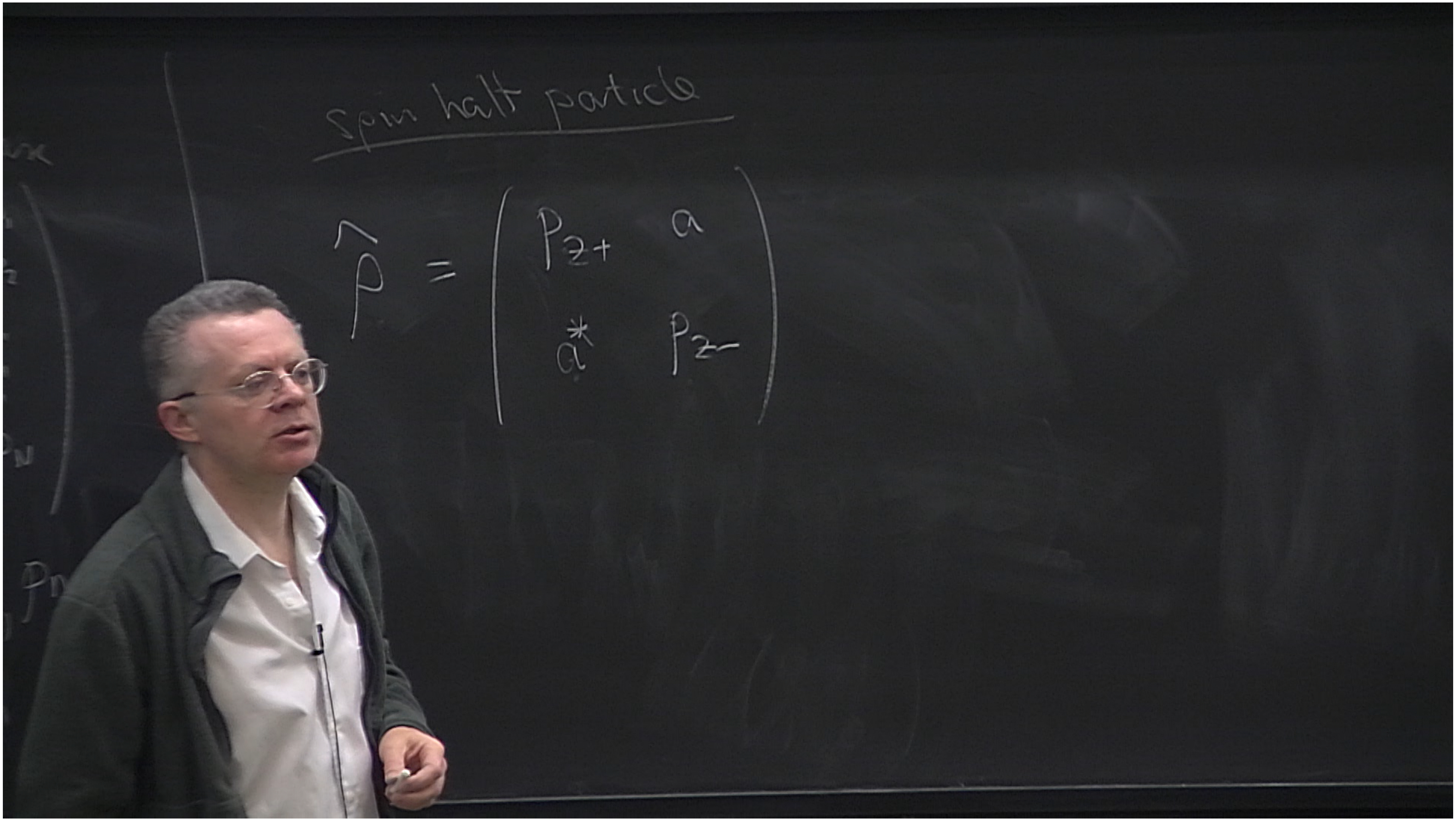

classical
p_3
1
1
1
p_2
p_1



classical
p_3
1
classical state space
1
p_2
1
p_1
state $p = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$



classical
p_3
1
classical state space
1
1
p_2
p_1
state $\underline{p} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$
general case
$\underline{p} = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{pmatrix}$
$\sum_{n=1}^{N} p_n \leq 1$
$0 \leq p_n$

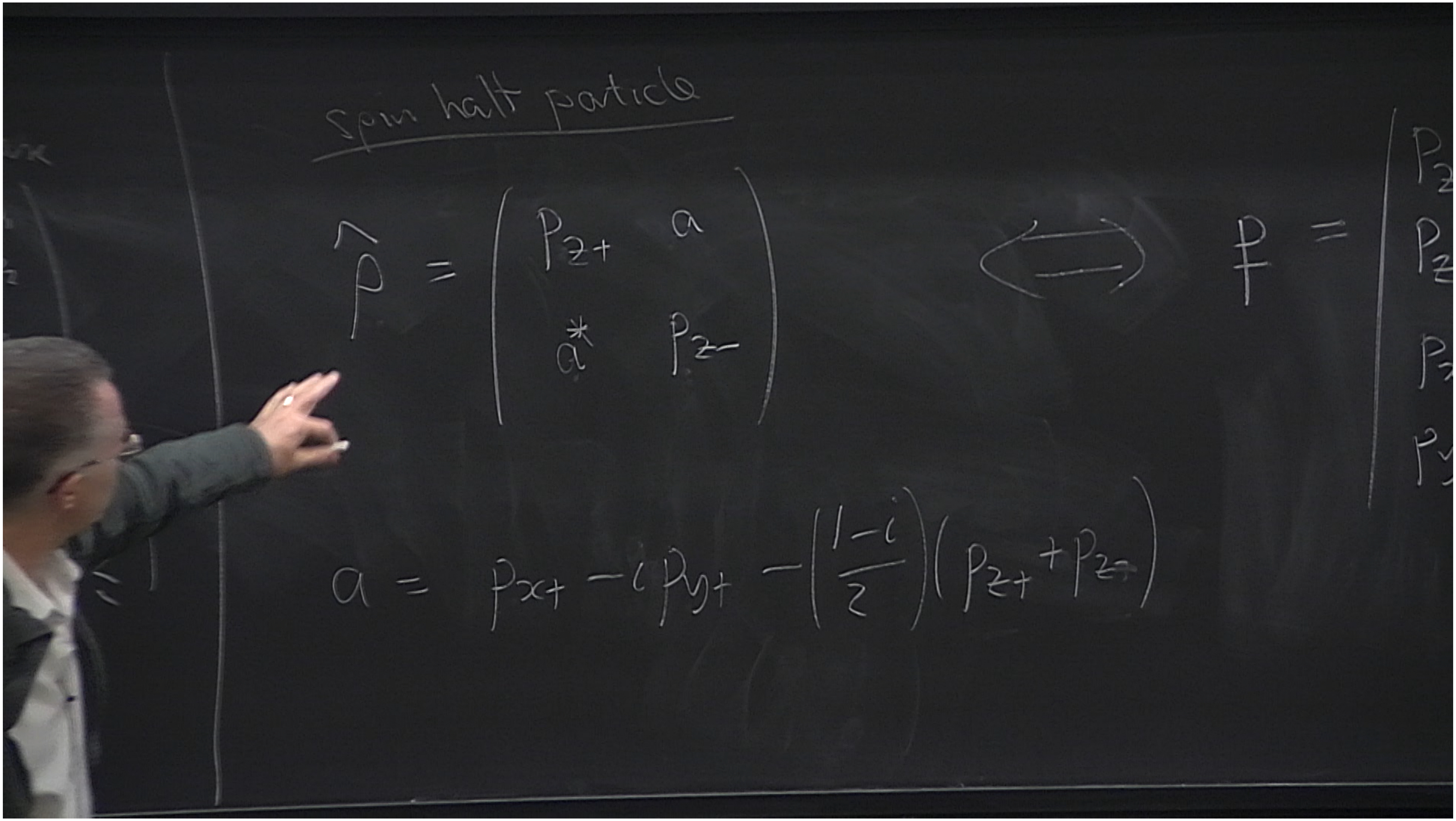

spin half particle
$\hat{\rho} = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix}$

$p_n \leq 1$

spin half particle

$$\hat{\rho} = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix}$$

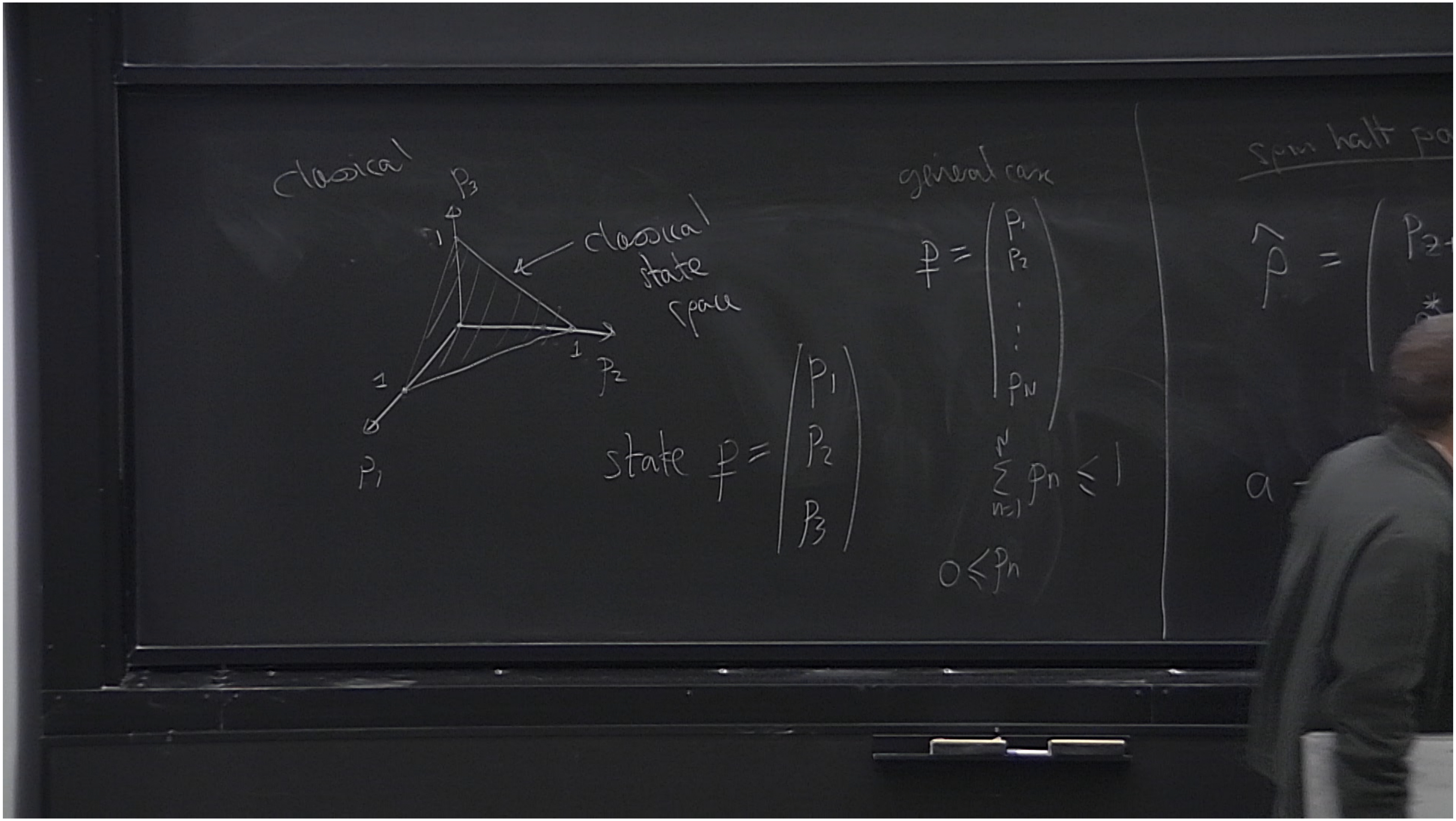
$$a = p_{x+} - i\,p_{y+} - \left(\frac{1-i}{2}\right)\left(p_{z+} + p_{z-}\right)$$

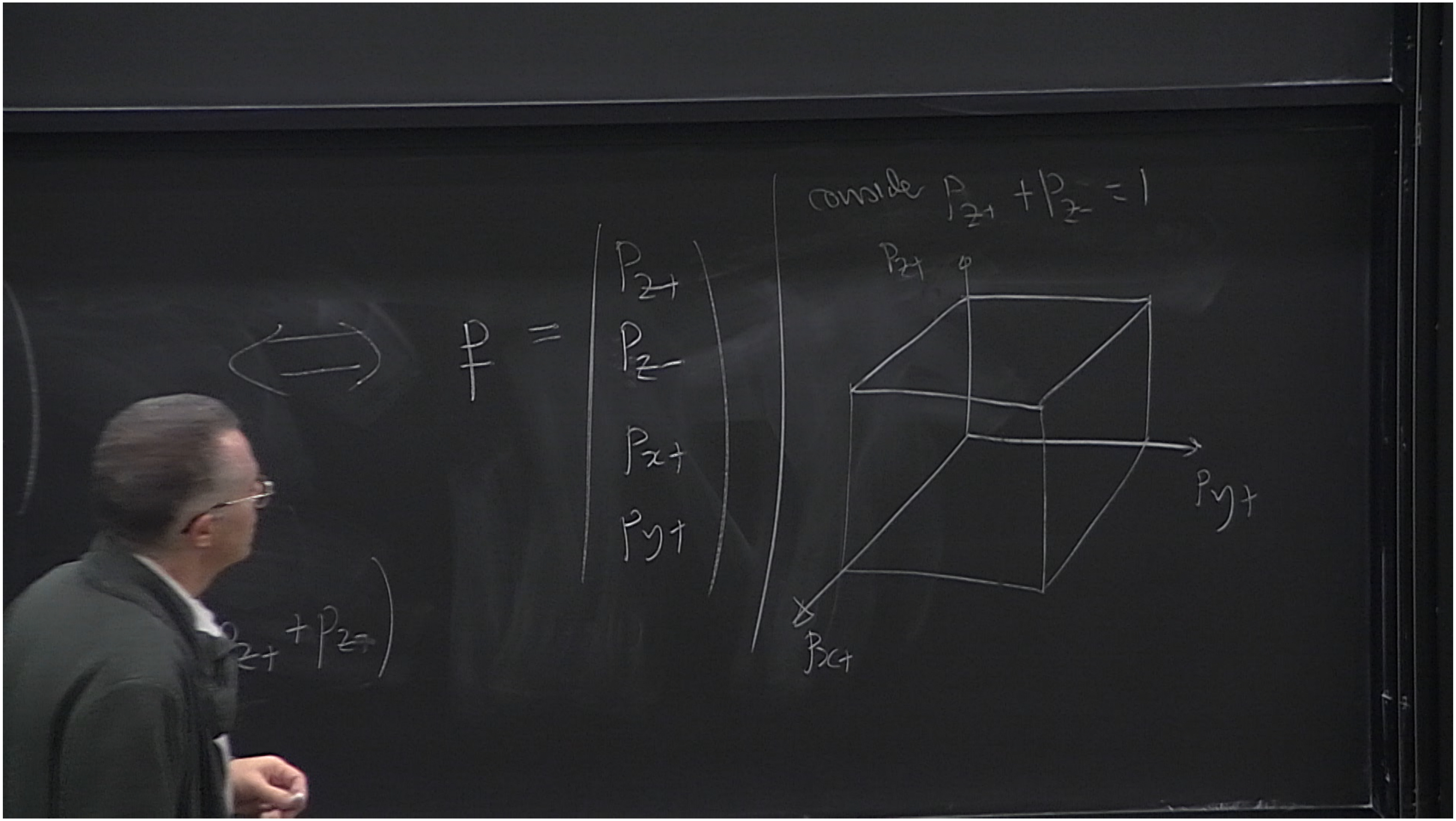

spin half particle
$\hat{\rho} = \begin{pmatrix} p_{z+} & a \\ a^* & p_{z-} \end{pmatrix} \Longleftrightarrow p =$
$a = p_{x+} - i p_{y+} - \left(\frac{1-i}{2}\right)(p_{z+} + p_{z-})$

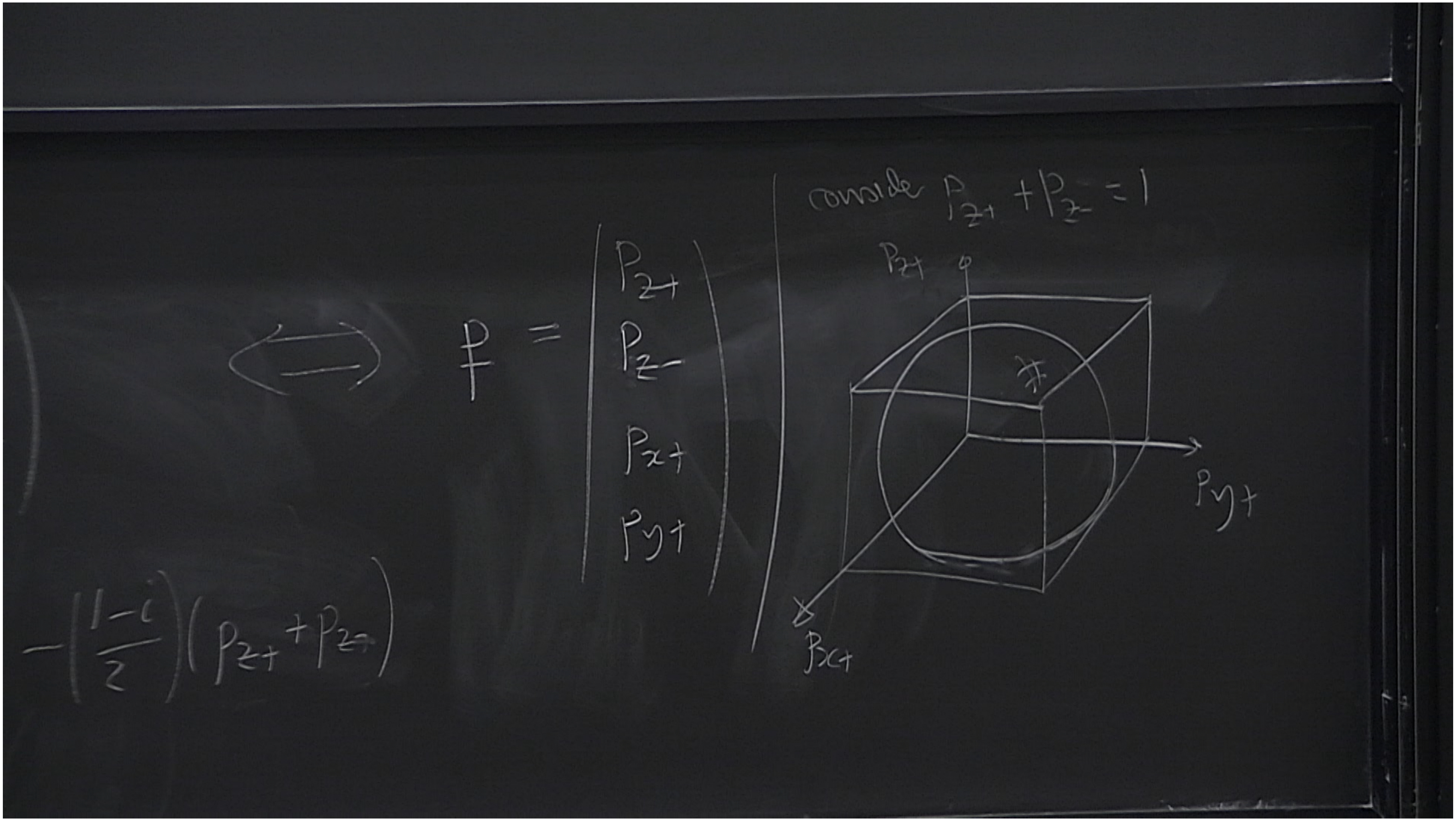
spin half particle

$$\hat{\rho} = \begin{pmatrix} p_{z+} & a \\ a^{*} & p_{z-} \end{pmatrix} \iff \underline{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$$

$$a = p_{x+} - i\, p_{y+} - \left(\frac{1-i}{2}\right)\left(p_{z+} + p_{z-}\right)$$

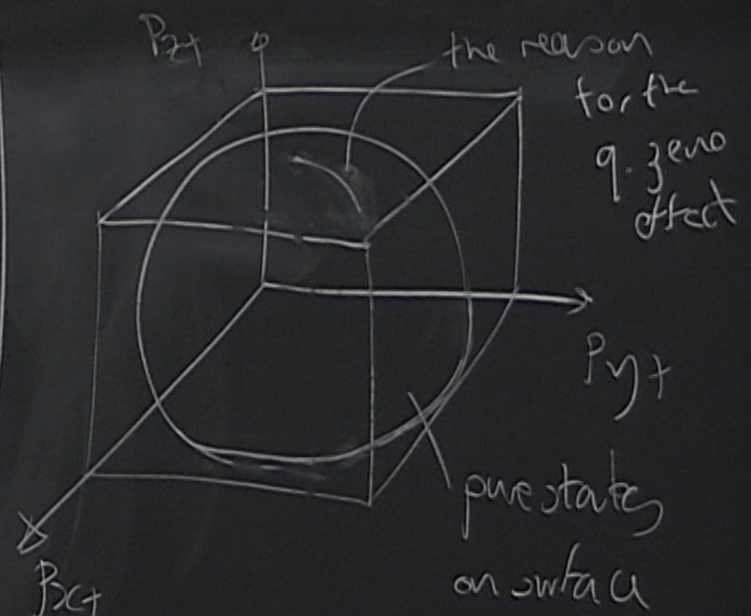

classical
classical state space
state
general case


consider $P_{z+} + P_{z-} = 1$
$P = \begin{pmatrix} P_{z+} \\ P_{z-} \\ P_{x+} \\ P_{y+} \end{pmatrix}$
P_{z+}
P_{y+}
P_{x+}
$P_{z+} + P_{z-}$

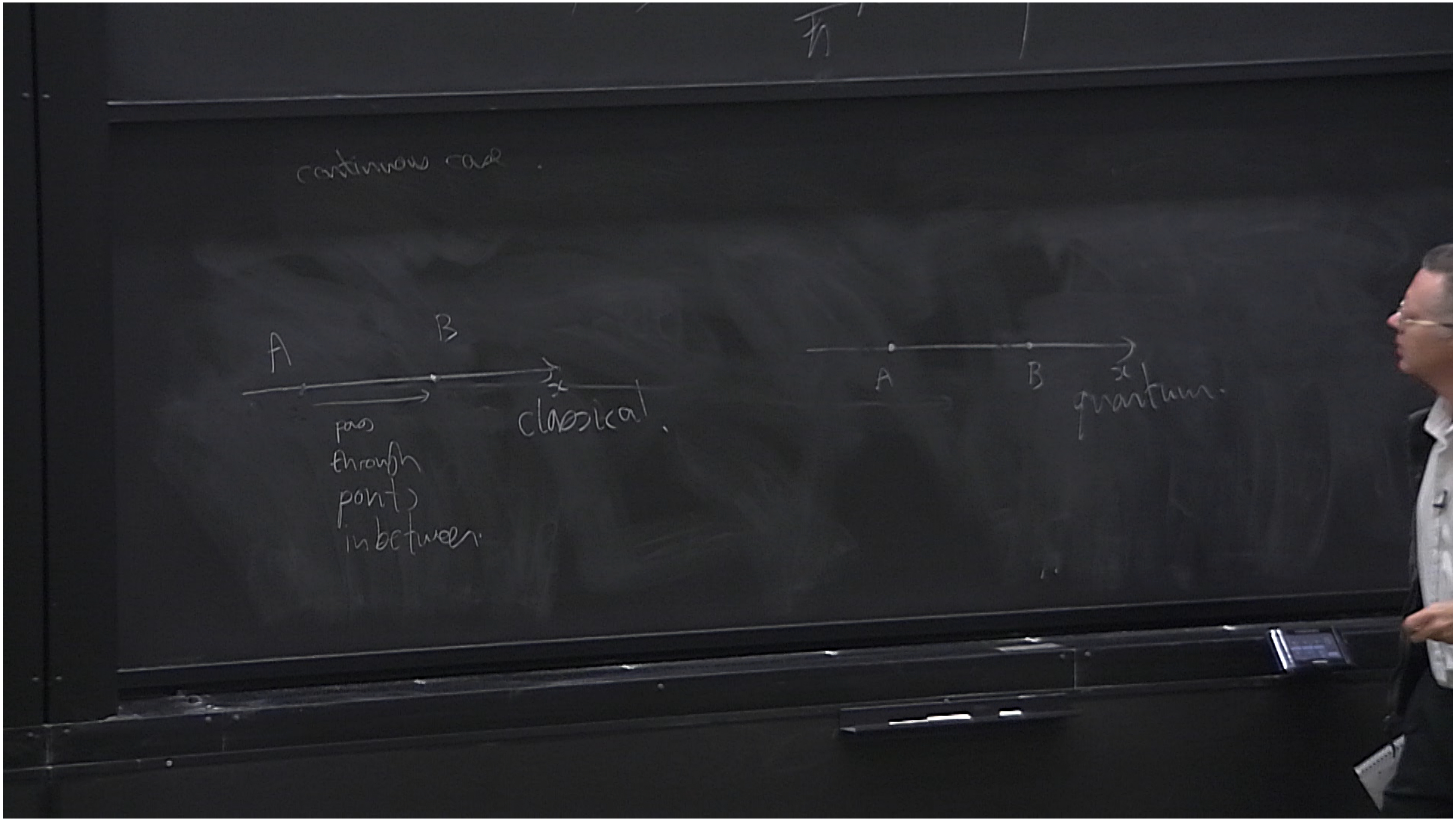

consider $P_{z+} + P_{z-} = 1$
$P = \begin{pmatrix} P_{z+} \\ P_{z-} \\ P_{x+} \\ P_{y+} \end{pmatrix}$
$-\left(\frac{1-i}{2}\right)\left(P_{z+} + P_{z+}\right)$
P_{z+}
P_{y+}
P_{x+}

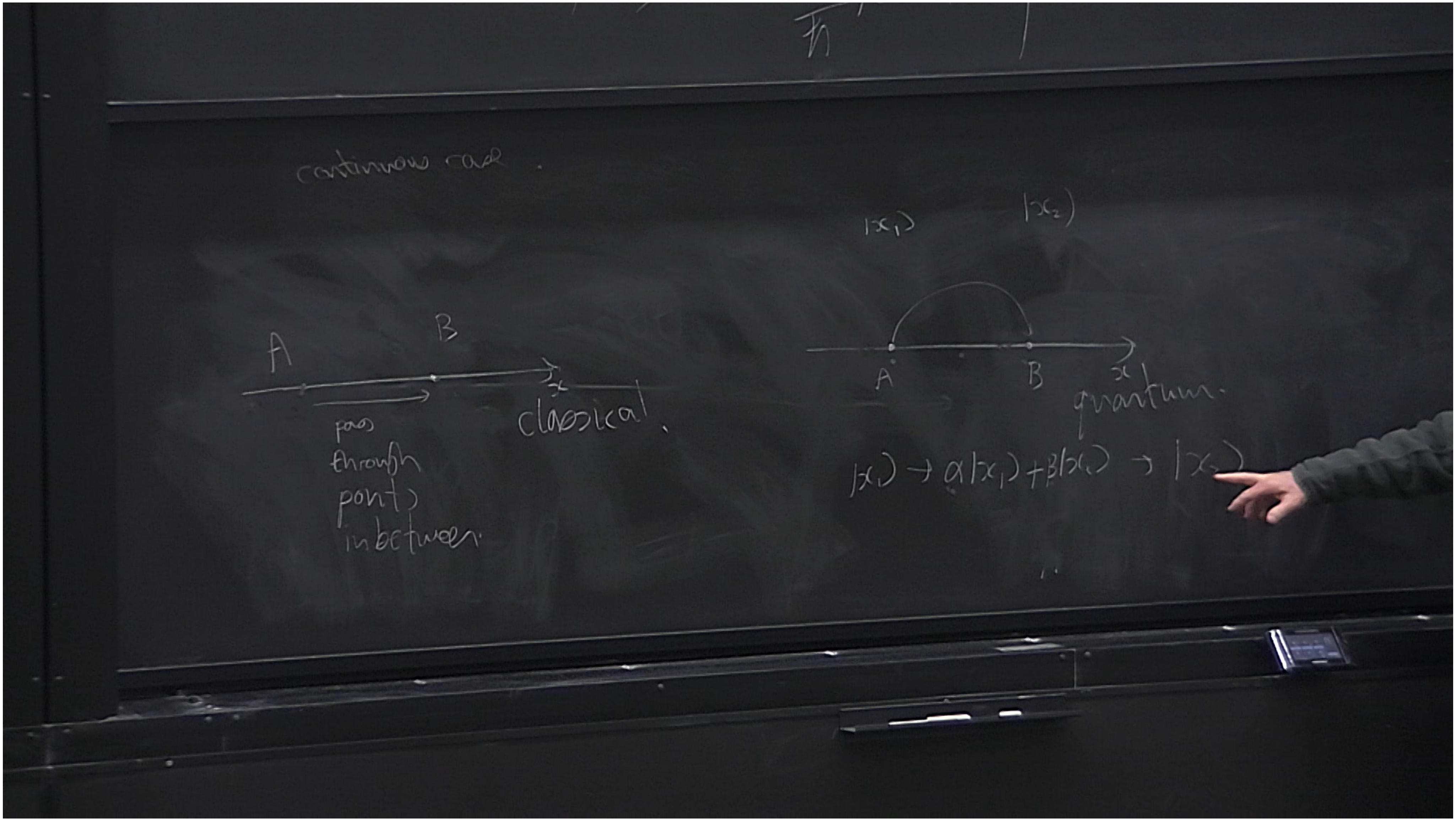
consider $P_{z+} + P_{z-} = 1$

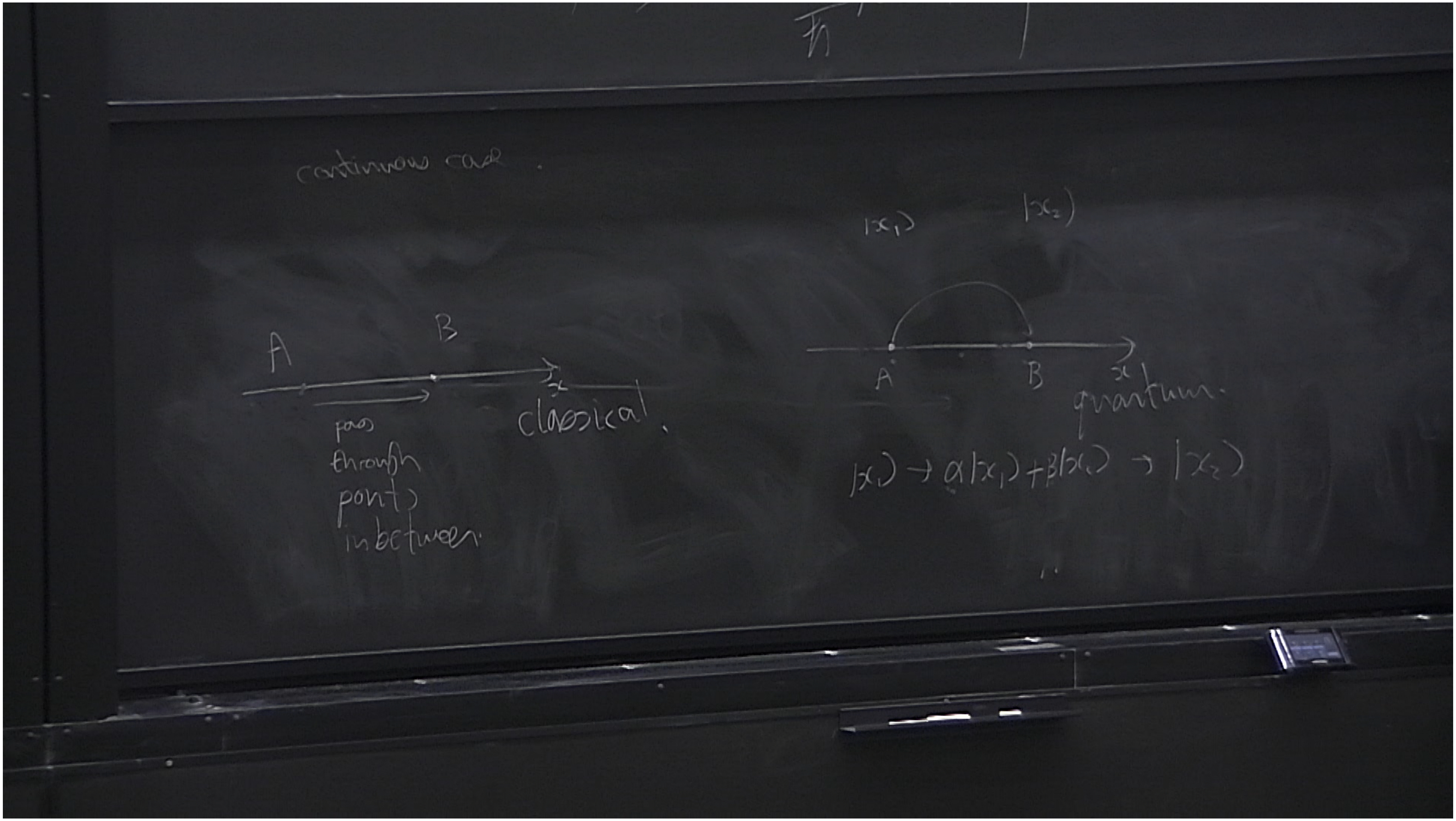
$$\Longleftrightarrow \quad \underline{P} = \begin{pmatrix} P_{z+} \\ P_{z-} \\ P_{x+} \\ P_{y+} \end{pmatrix}$$



$$-\left(\frac{1-i}{2}\right)\left(P_{z+} + P_{z-}\right)$$


continuous case.
A
B
x
pass
through
points
inbetween.
classical.
A
B
x
quantum.


continuous case
A
B
x
pass
through
points
inbetween
classical
|x1)
|x2)
A
B
x
quantum
|x1) → α|x1) + β|x2) → |x2)


continuous case.
A
B
x
pass through points inbetween.
classical.
|x1⟩
|x2⟩
A
B
x
quantum.
|x1⟩ → α|x1⟩ + β|x2⟩ → |x2⟩