Title: PSI 17/18 - Foundations of Quantum Mechanics - Lecture 1

Date: Jan 29, 2018 09:00 AM

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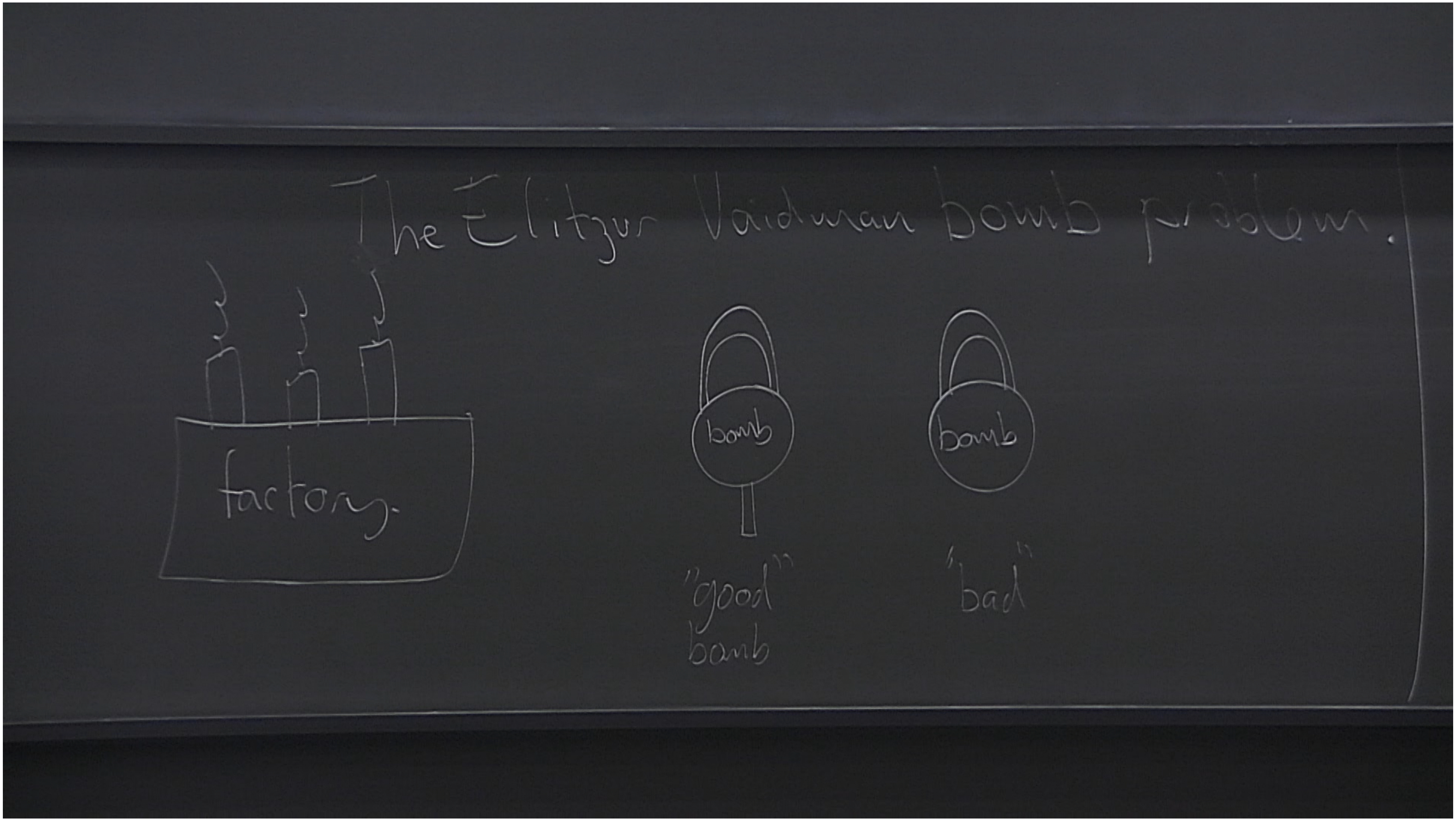
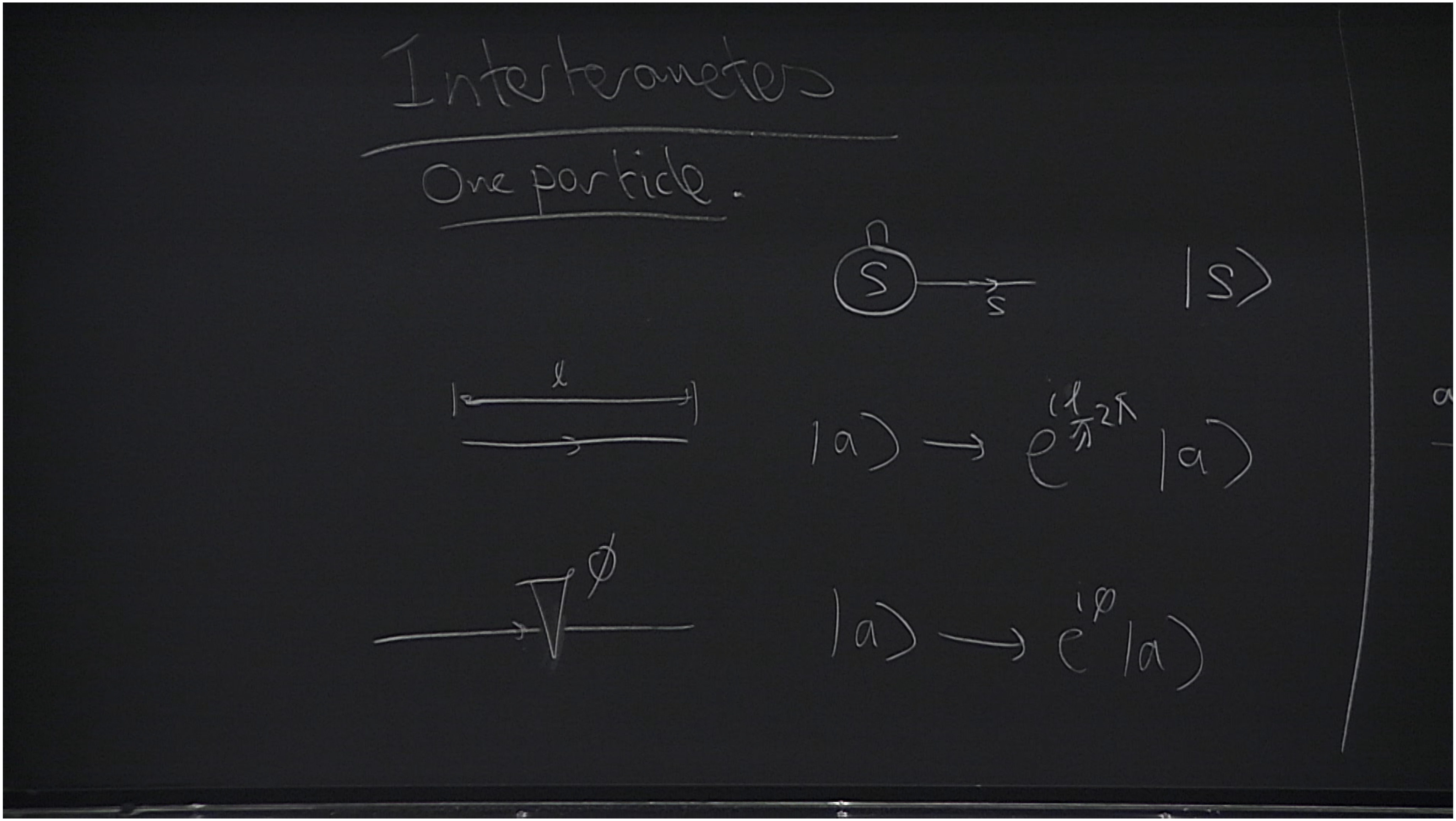
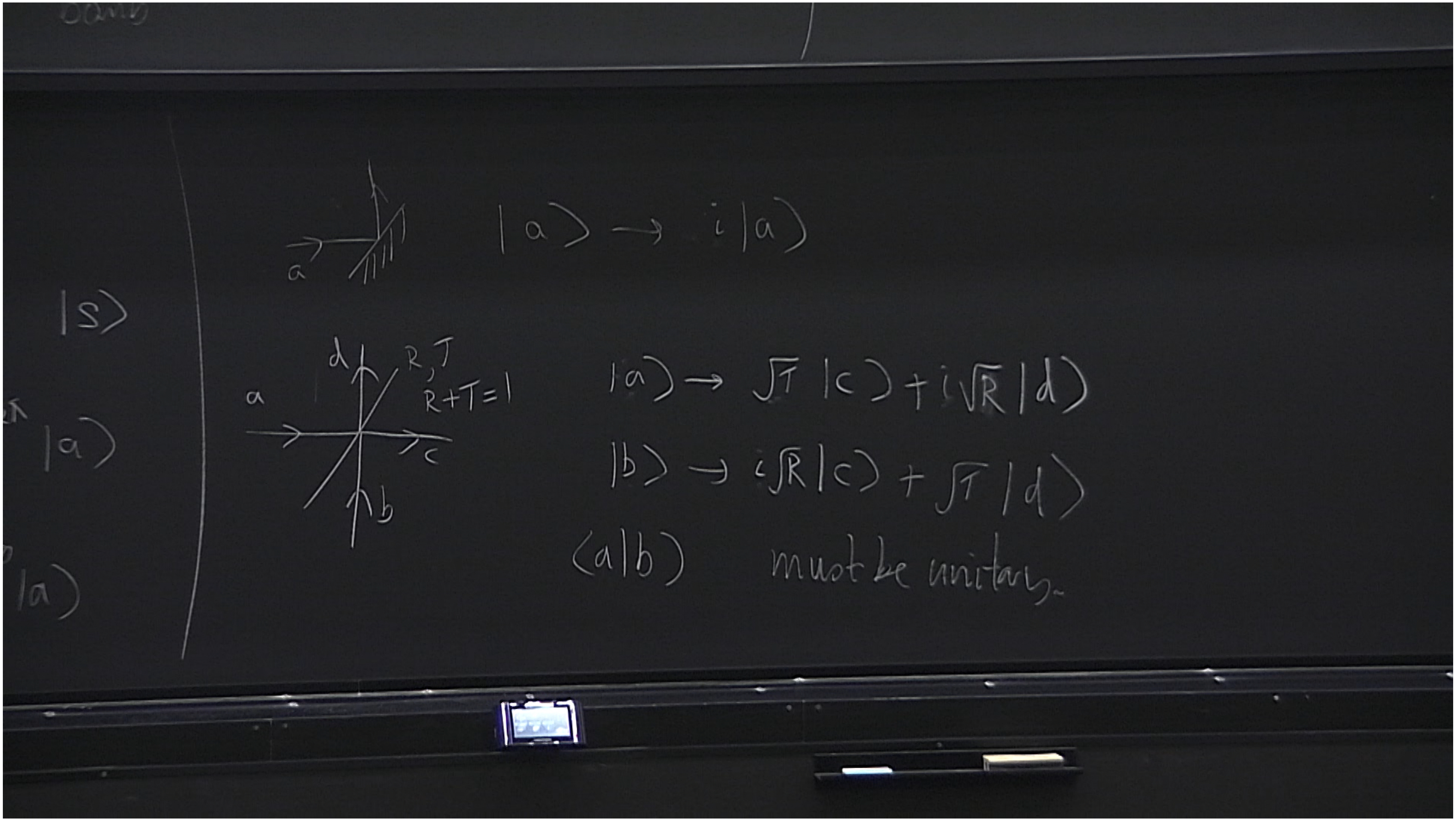
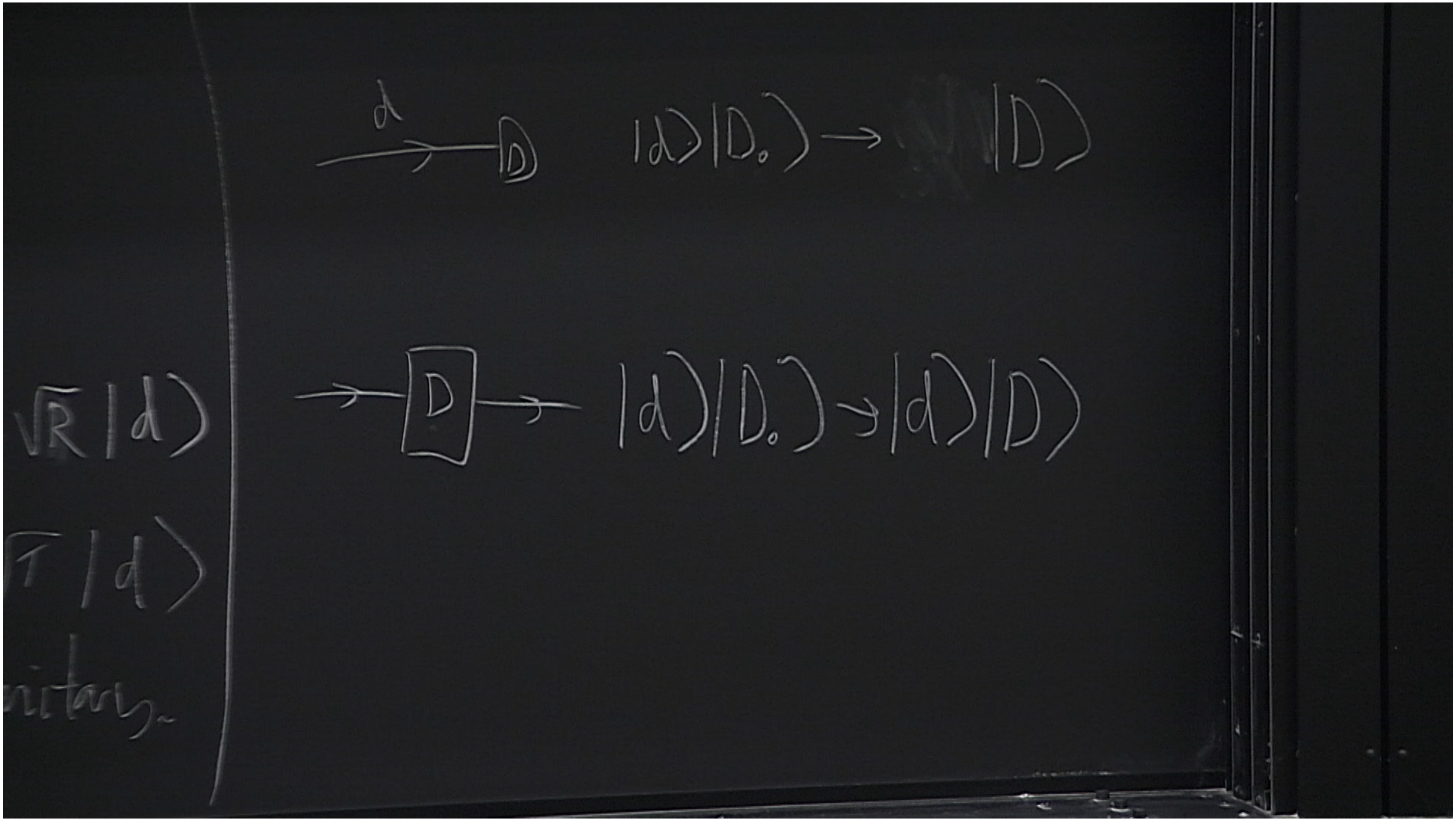
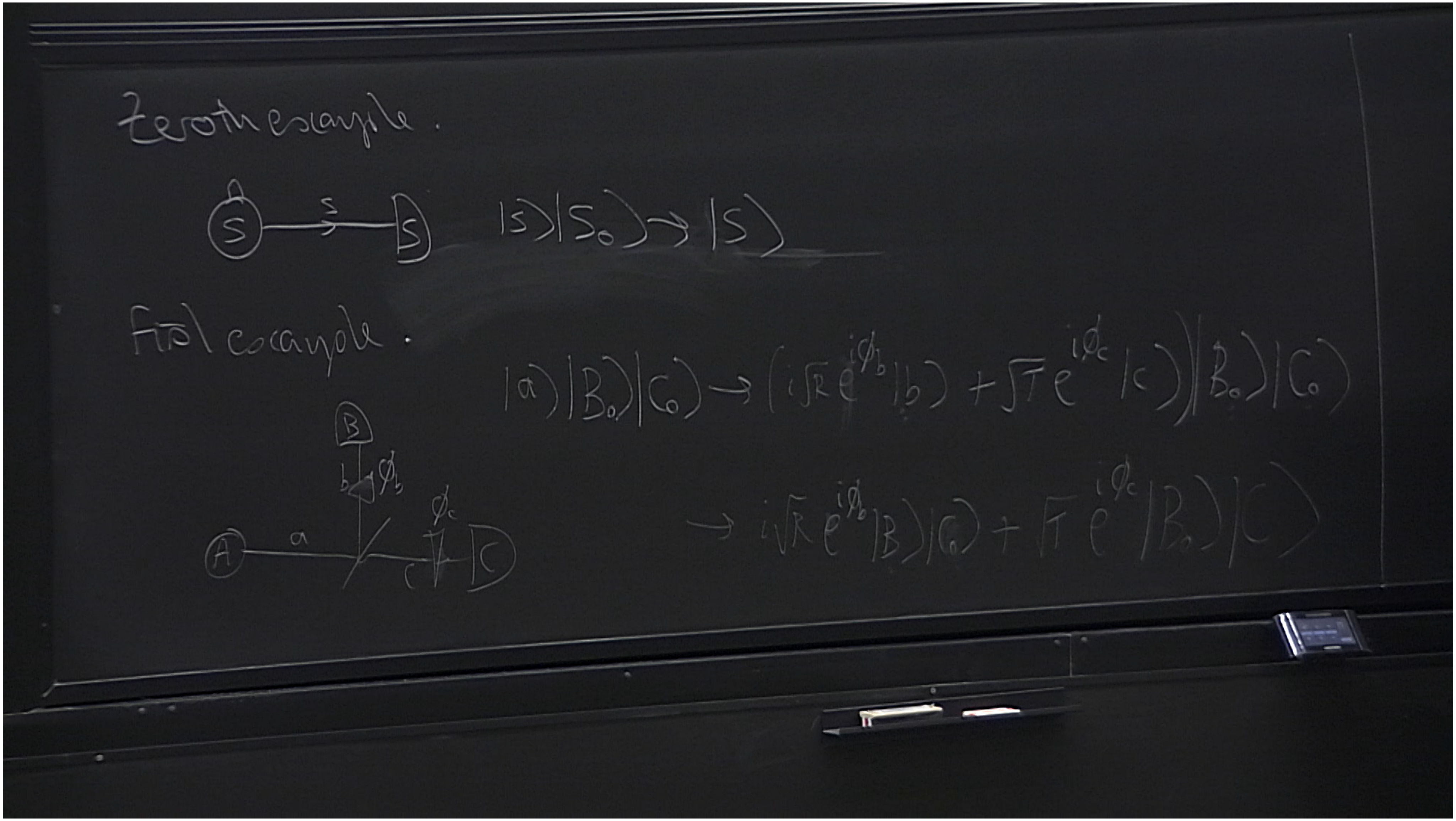
Abstract:


The Elitzur Vaidman bomb problem.
factory.
bomb
bomb
"good" bomb
"bad"


Interferometers
One particle.
S
s
$|s\rangle$
ℓ
$|a\rangle \longrightarrow e^{i\frac{\ell}{\lambda}2\pi}|a\rangle$
ϕ
$|a\rangle \longrightarrow e^{i\varphi}|a\rangle$


$|s\rangle$
$|a\rangle$
$|a\rangle$
$|a\rangle \to i|a\rangle$
R, T
$R+T=1$
$|a\rangle \to \sqrt{T}\,|c\rangle + i\sqrt{R}\,|d\rangle$
$|b\rangle \to i\sqrt{R}\,|c\rangle + \sqrt{T}\,|d\rangle$
$\langle a|b\rangle$ must be unitary


d
D
|d⟩|D₀⟩ → |D⟩
D
|d⟩|D₀⟩ → |d⟩|D⟩
√R |d⟩
|d⟩
nitary



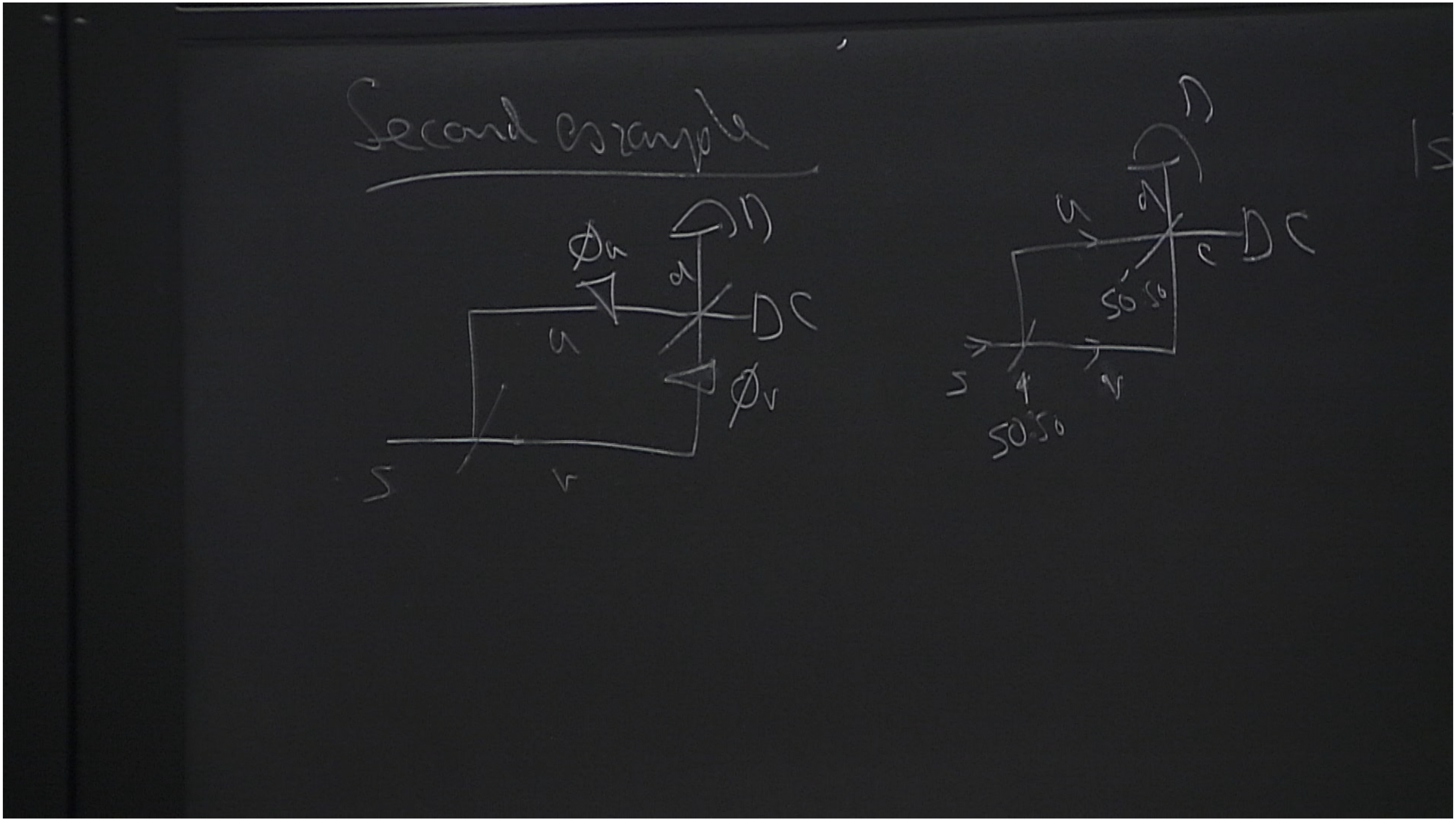
Zeroth example.
$|s\rangle|S_0\rangle \to |S\rangle$
First example.
$|a\rangle|B_0\rangle|C_0\rangle \to (i\sqrt{R}\, e^{i\phi_b}|b\rangle + \sqrt{T}\, e^{i\phi_c}|c\rangle)|B_0\rangle|C_0\rangle$
$\to i\sqrt{R}\, e^{i\phi_b}|B\rangle|C_0\rangle + \sqrt{T}\, e^{i\phi_c}|B_0\rangle|C\rangle$

$$\mathrm{prob}(B\,C_0) = \left| i\sqrt{R}\, e^{i\phi_b} \right|^2 = R$$

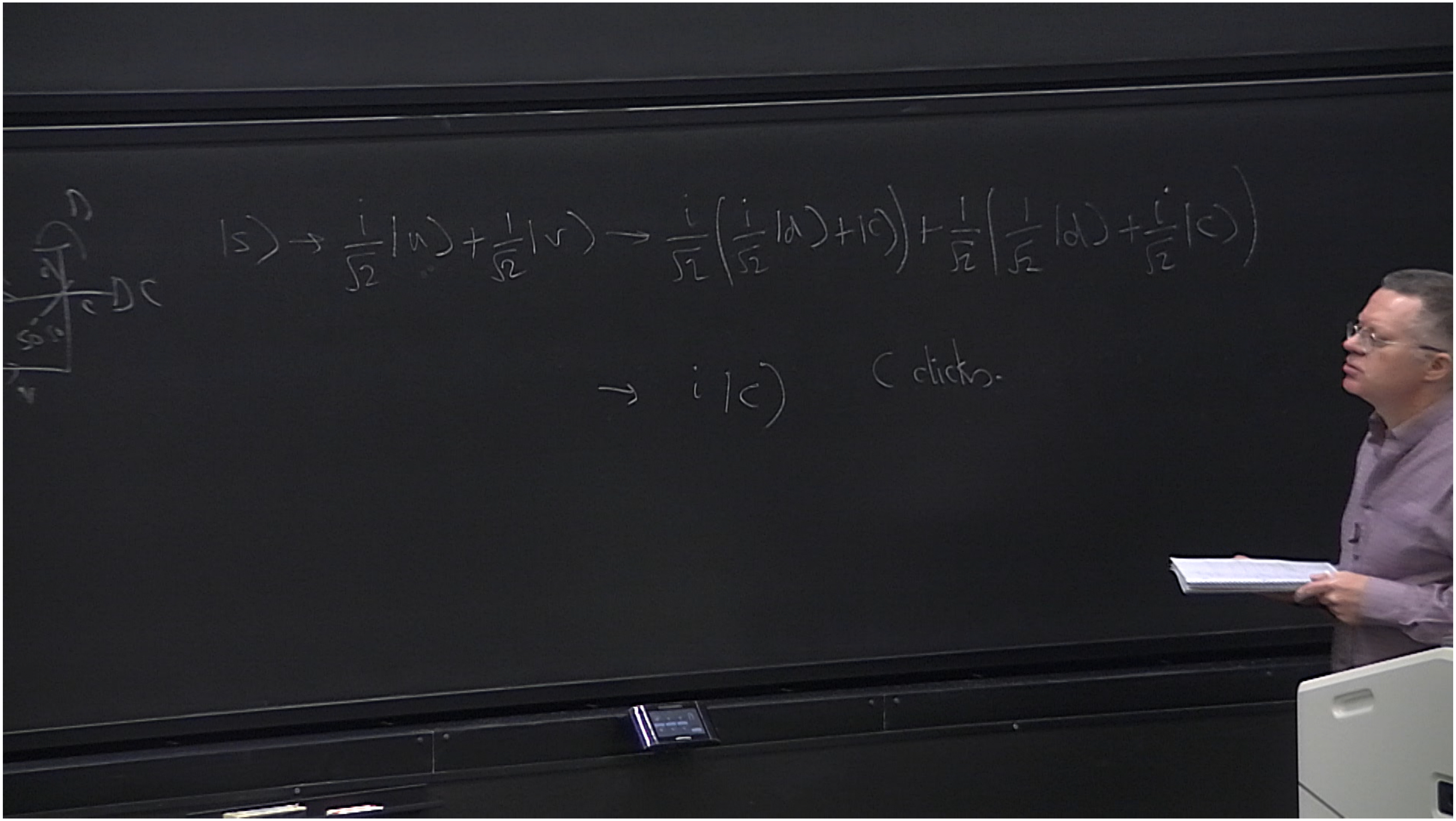
$$\mathrm{prob}(B_0\,C) = T$$

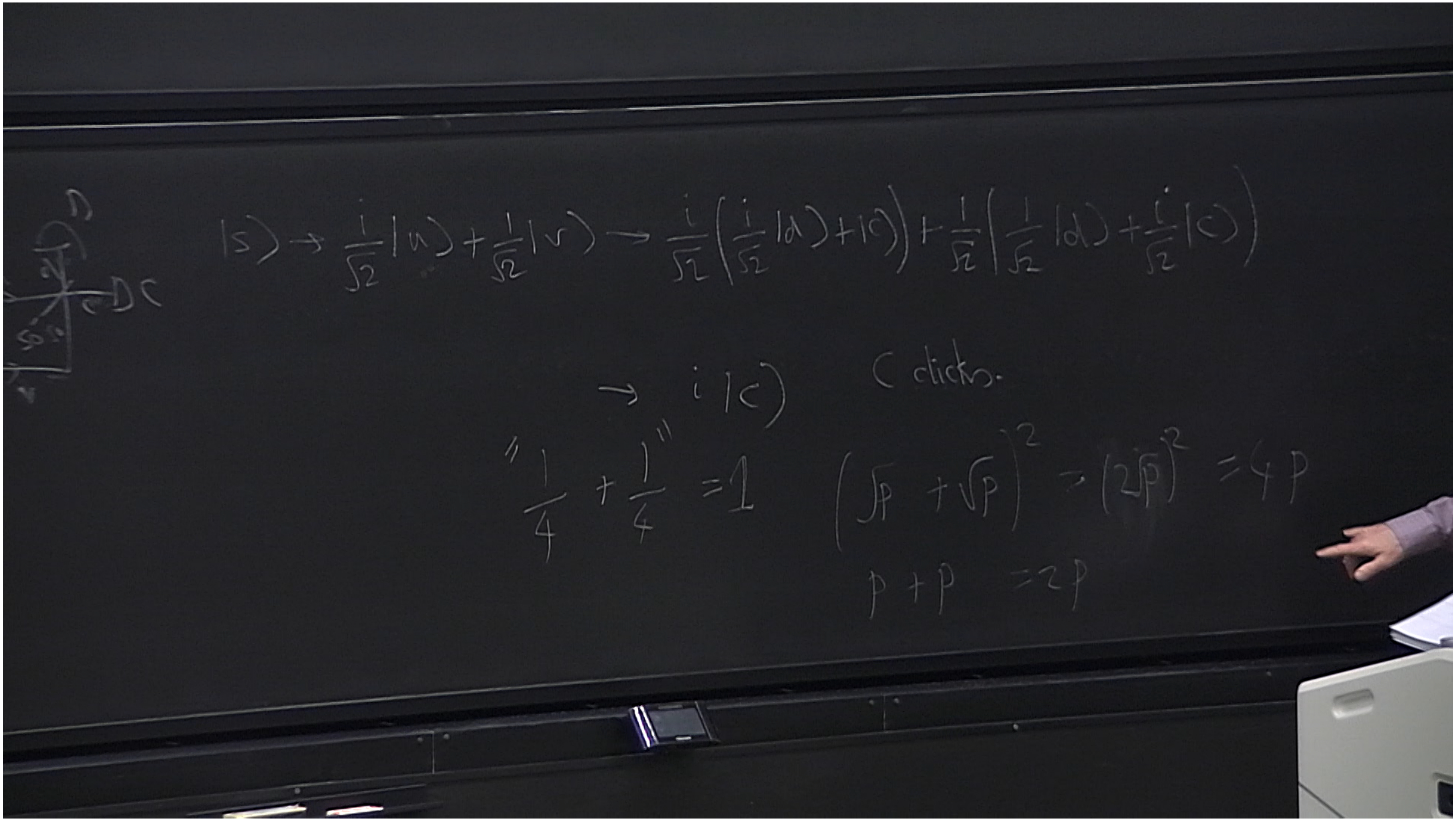
$$\mathrm{prob}(B\,C) = 0 = \mathrm{prob}(B_0\,C_0)$$

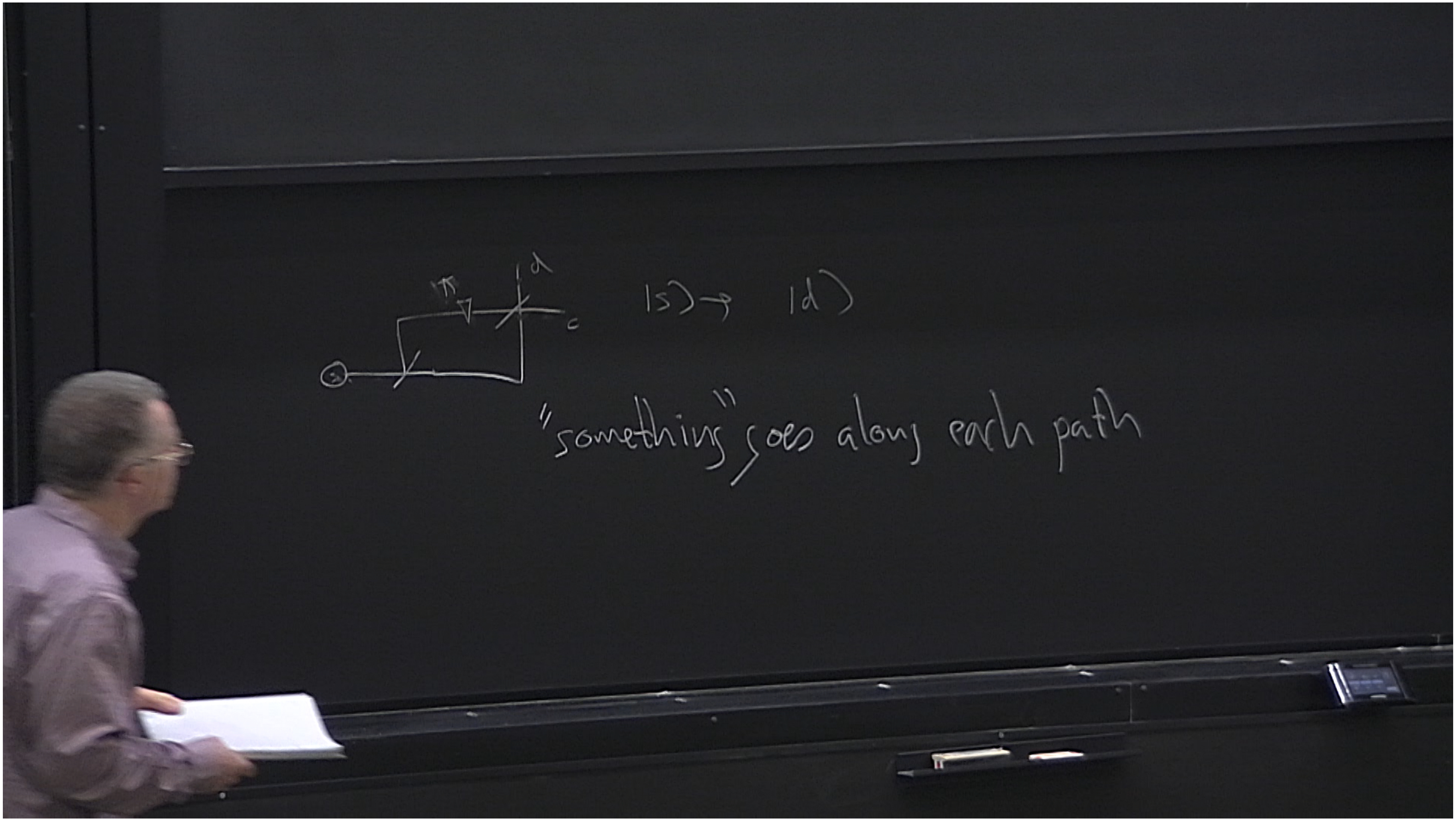
anti-correlation, no dep on $\phi_{a,b}$.

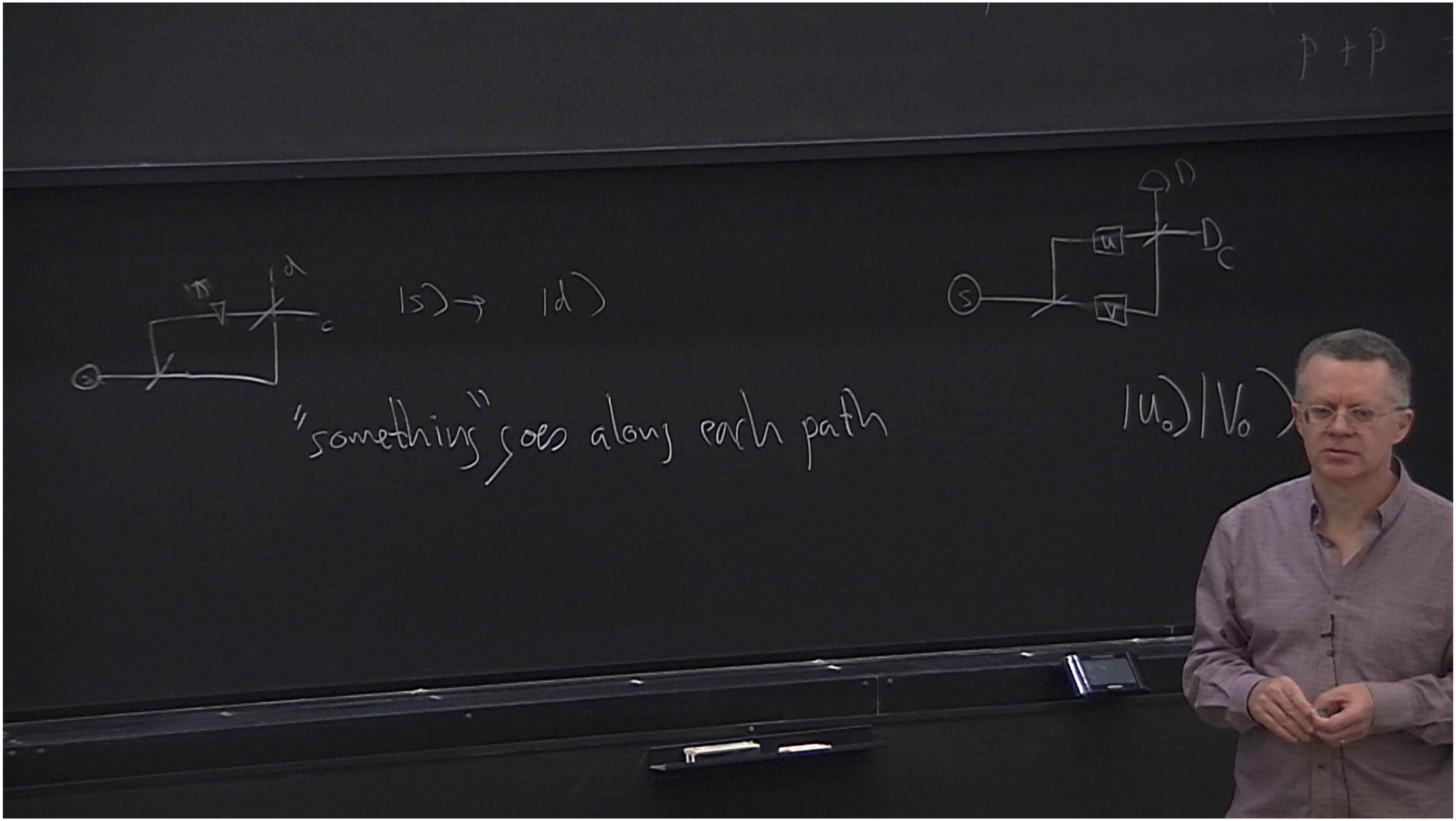


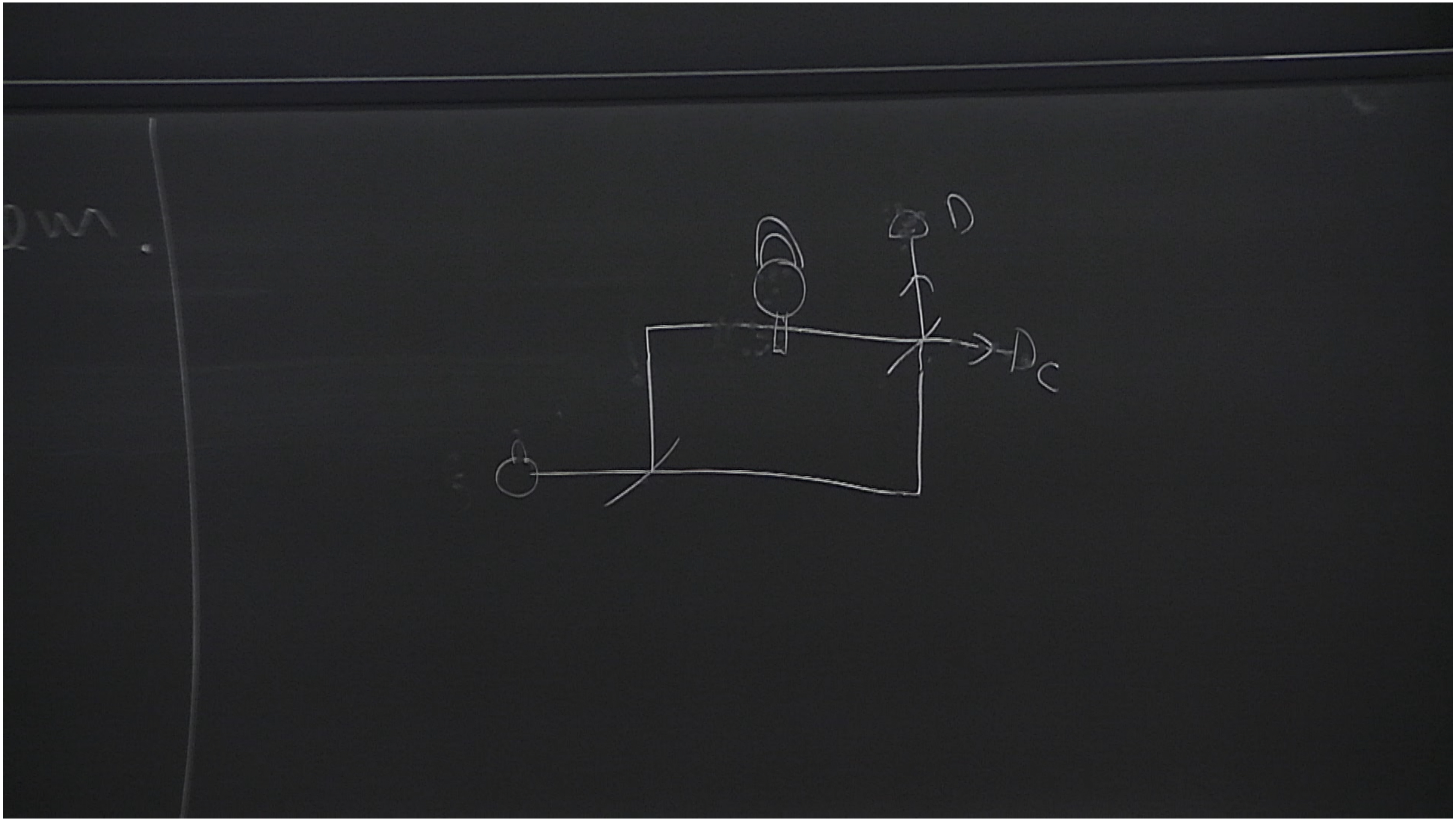
Second example
ϕ_u
D
d
u
DC
ϕ_v
S
v
D
u
d
DC
c
50:50
S
v
50:50

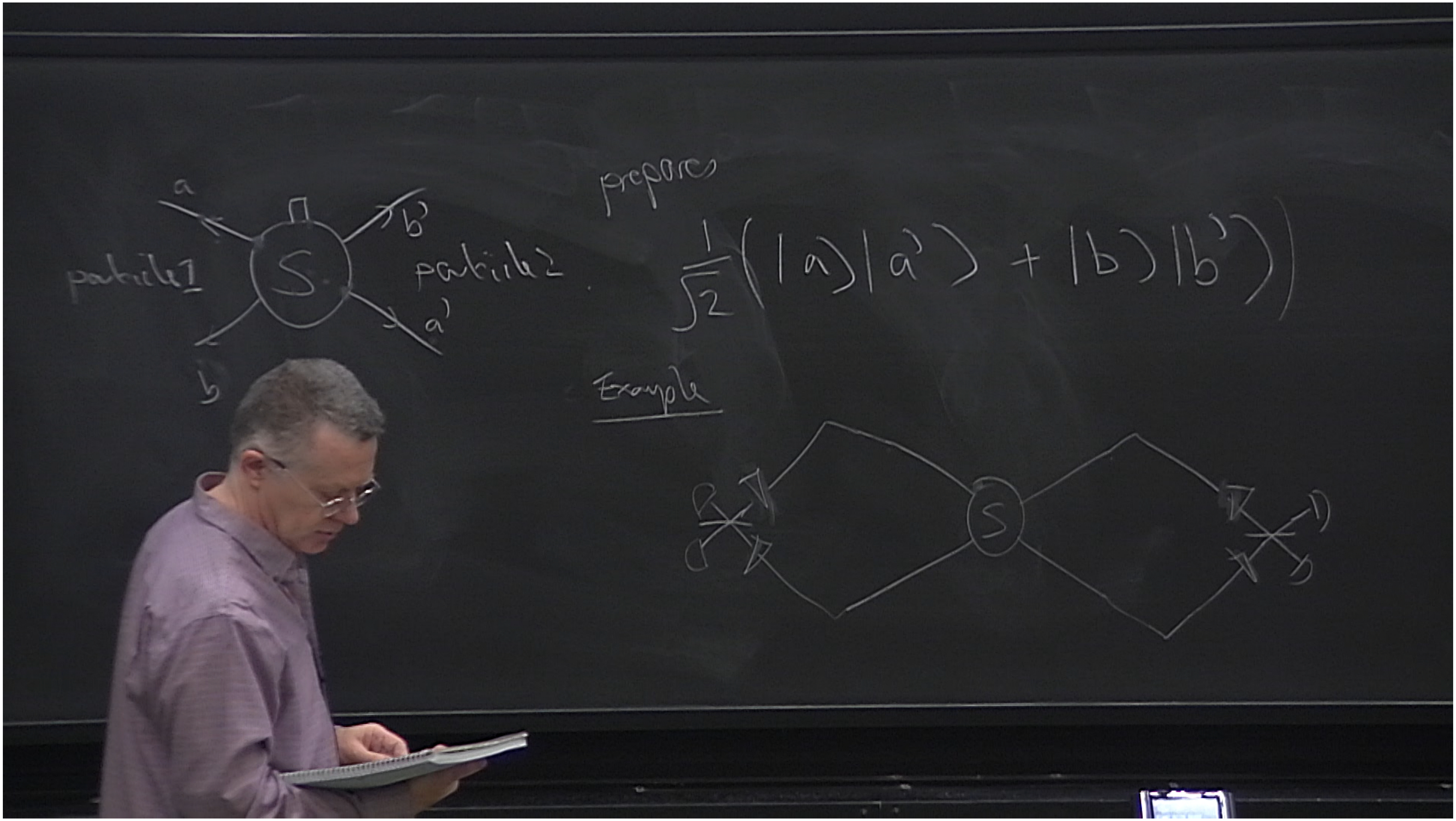

$|s\rangle \to \frac{i}{\sqrt{2}}|u\rangle + \frac{1}{\sqrt{2}}|r\rangle \to \frac{i}{\sqrt{2}}\left(\frac{i}{\sqrt{2}}|d\rangle + |c\rangle\right) + \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|d\rangle + \frac{i}{\sqrt{2}}|c\rangle\right)$
$\to i|c\rangle$
C clicks.

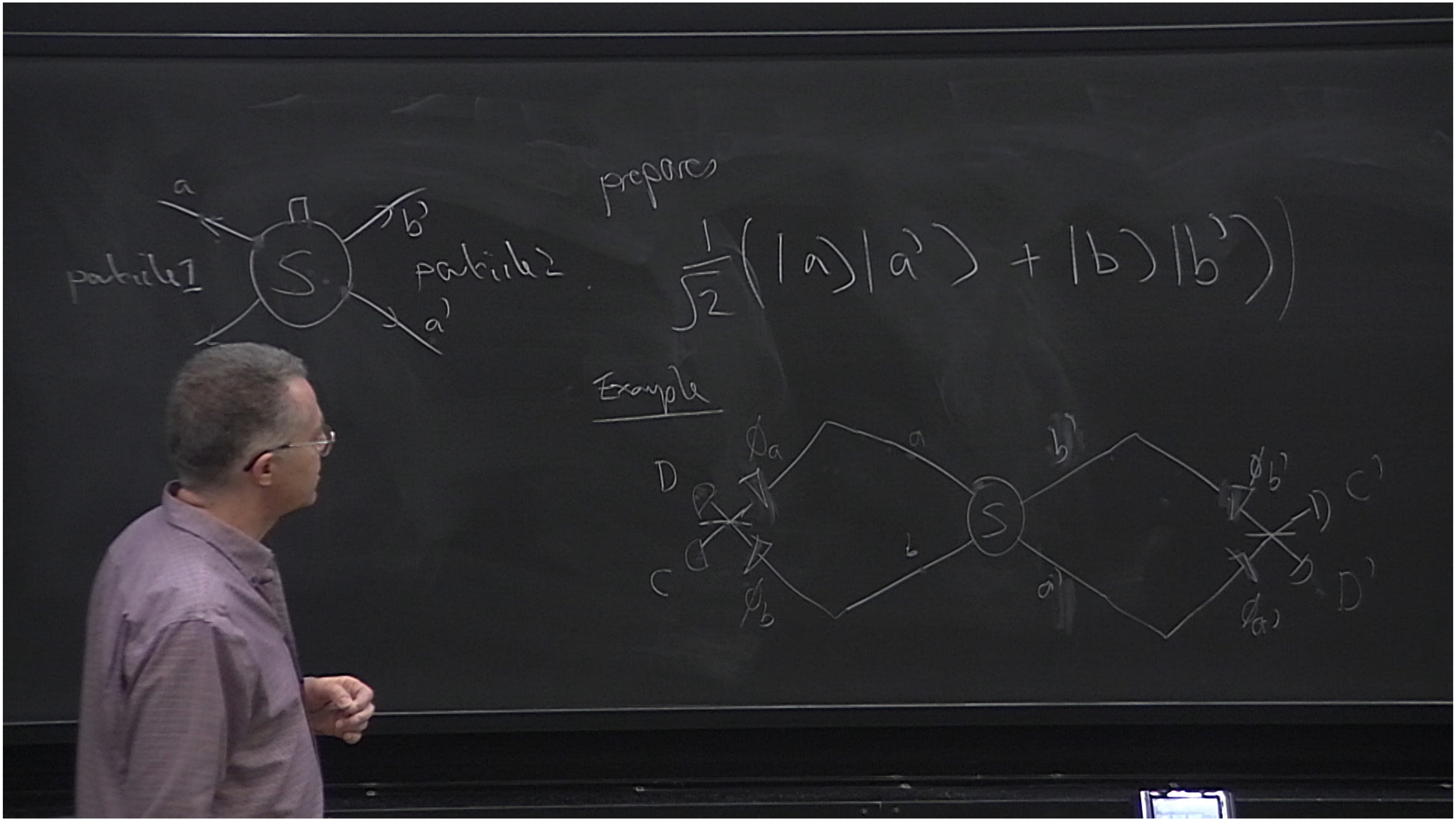

$|s\rangle \to \frac{i}{\sqrt{2}}|u\rangle + \frac{1}{\sqrt{2}}|v\rangle \to \frac{i}{\sqrt{2}}\left(\frac{i}{\sqrt{2}}|d\rangle + |c\rangle\right) + \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|d\rangle + \frac{i}{\sqrt{2}}|c\rangle\right)$
$\to i|c\rangle$
C clicks.
$"\frac{1}{4} + \frac{1}{4}" = 1$
$(\sqrt{p} + \sqrt{p})^2 = (2\sqrt{p})^2 = 4p$
$p + p = 2p$


|s⟩ → |d⟩
"something" goes along each path


|s⟩ → |d⟩
"something" goes along each path
|u₀⟩|V₀⟩
p + p


D
D_C


particle 1
S
particle 2
a
b
b'
a'
prepares
$\frac{1}{\sqrt{2}}\left(|a\rangle|a'\rangle + |b\rangle|b'\rangle\right)$
Example
S

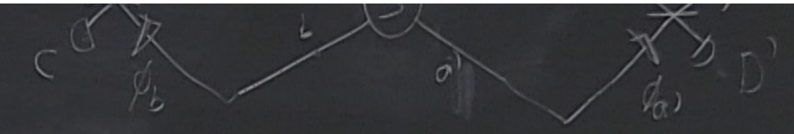


particle 1
S
particle 2
a
b'
a'
prepares
$\frac{1}{\sqrt{2}}\left(|a\rangle|a'\rangle + |b\rangle|b'\rangle\right)$
Example
ϕ_a
ϕ_b
$\phi_{b'}$
$\phi_{a'}$
D
C
C'
D'

The no cloning theorem

Imagine a machine M that clones states in $\mathcal{H}_{a_1}$

$$|\psi\rangle_{a_1}|M\rangle_{b_3} \xrightarrow{U} |\psi\rangle_{a_1}|\psi\rangle_{a_2}|M_\psi\rangle_{c_4}$$



The no cloning theorem

Imagine a machine M that clones states in $\mathcal{H}_a$

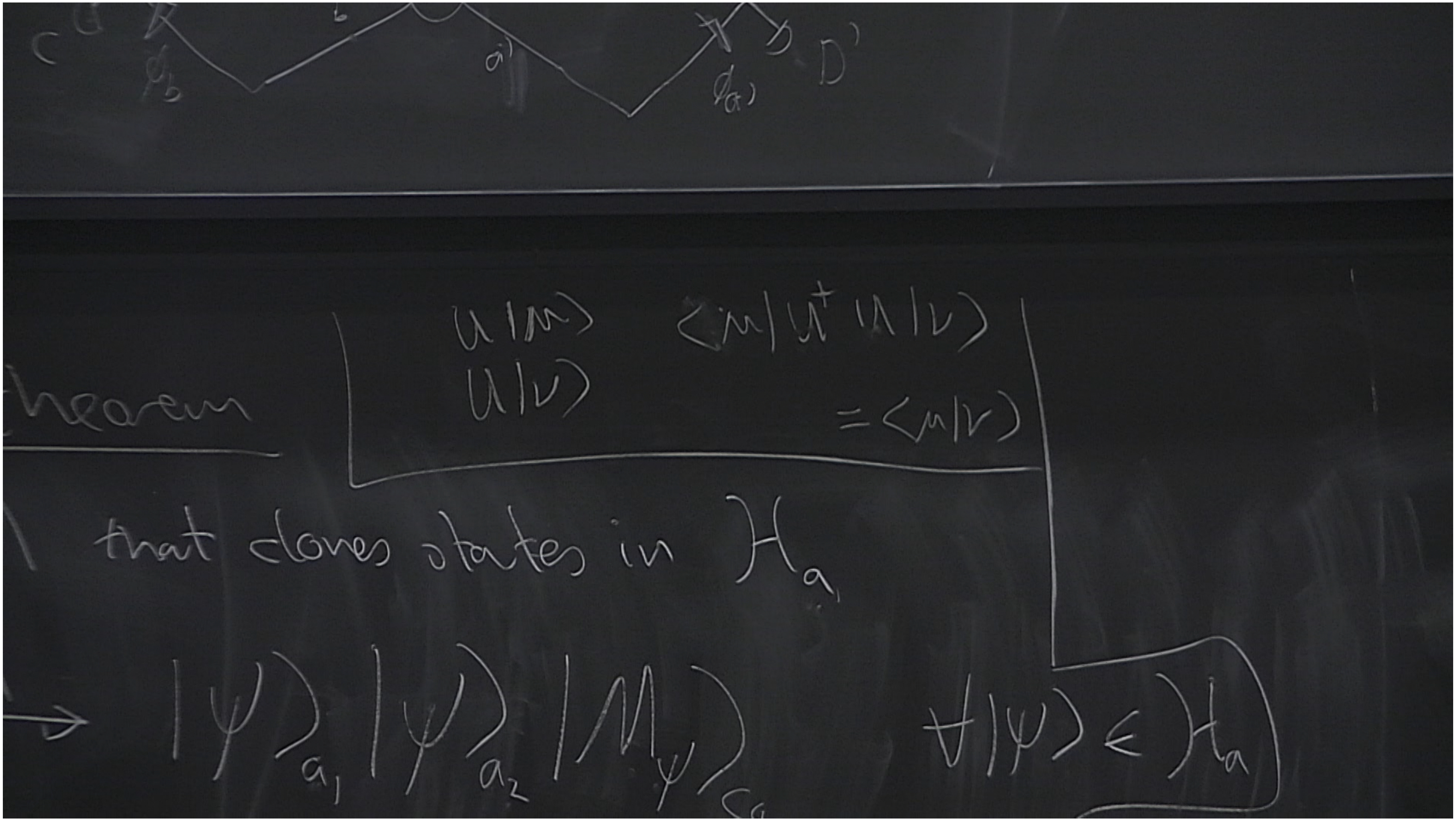
$$|\psi\rangle_{a_1}|M\rangle_{b_3} \xrightarrow{U} |\psi\rangle_{a_1}|\psi\rangle_{a_2}|M_\psi\rangle_{c_4} \qquad \forall |\psi\rangle \in \mathcal{H}_a$$

The no cloning theorem

Imagine a machine M that clones states in $\mathcal{H}_a$

$$|\psi\rangle_{a_1} |M\rangle_{b_3} \xrightarrow{U} |\psi\rangle_{a_1} |\psi\rangle_{a_2} |M_\psi\rangle_{c_4} \qquad \forall |\psi\rangle \in \mathcal{H}_a$$

$$|\varphi\rangle_{a_1} |M\rangle_{b_3} \xrightarrow{U} |\varphi\rangle_{a_1} |\varphi\rangle_{a_2} |M_\varphi\rangle_{c_4}$$


theorem
$U|\mu\rangle$ $\langle\mu|U^\dagger U|\nu\rangle$
$U|\nu\rangle$
$=\langle\mu|\nu\rangle$
that clones states in $\mathcal{H}_{a_1}$
$\rightarrow |\psi\rangle_{a_1}|\psi\rangle_{a_2}|M_\psi\rangle$
$\forall|\psi\rangle\in\mathcal{H}_a$

$\ldots \mu|\nu\rangle$

$= \langle\mu|\nu\rangle$

take inner product

$$\langle\varphi|\psi\rangle \overbrace{\langle m|m\rangle}^{=1} = \langle\varphi|\psi\rangle^2 \langle M_\varphi|M_\psi\rangle$$

either $\langle\varphi|\psi\rangle = 0$

or $1 = \langle\varphi|\psi\rangle\langle M_\varphi|M_\psi\rangle$

$$|\langle\varphi|\psi\rangle| = 1$$

$\forall |\psi\rangle \in \mathcal{H}_a$