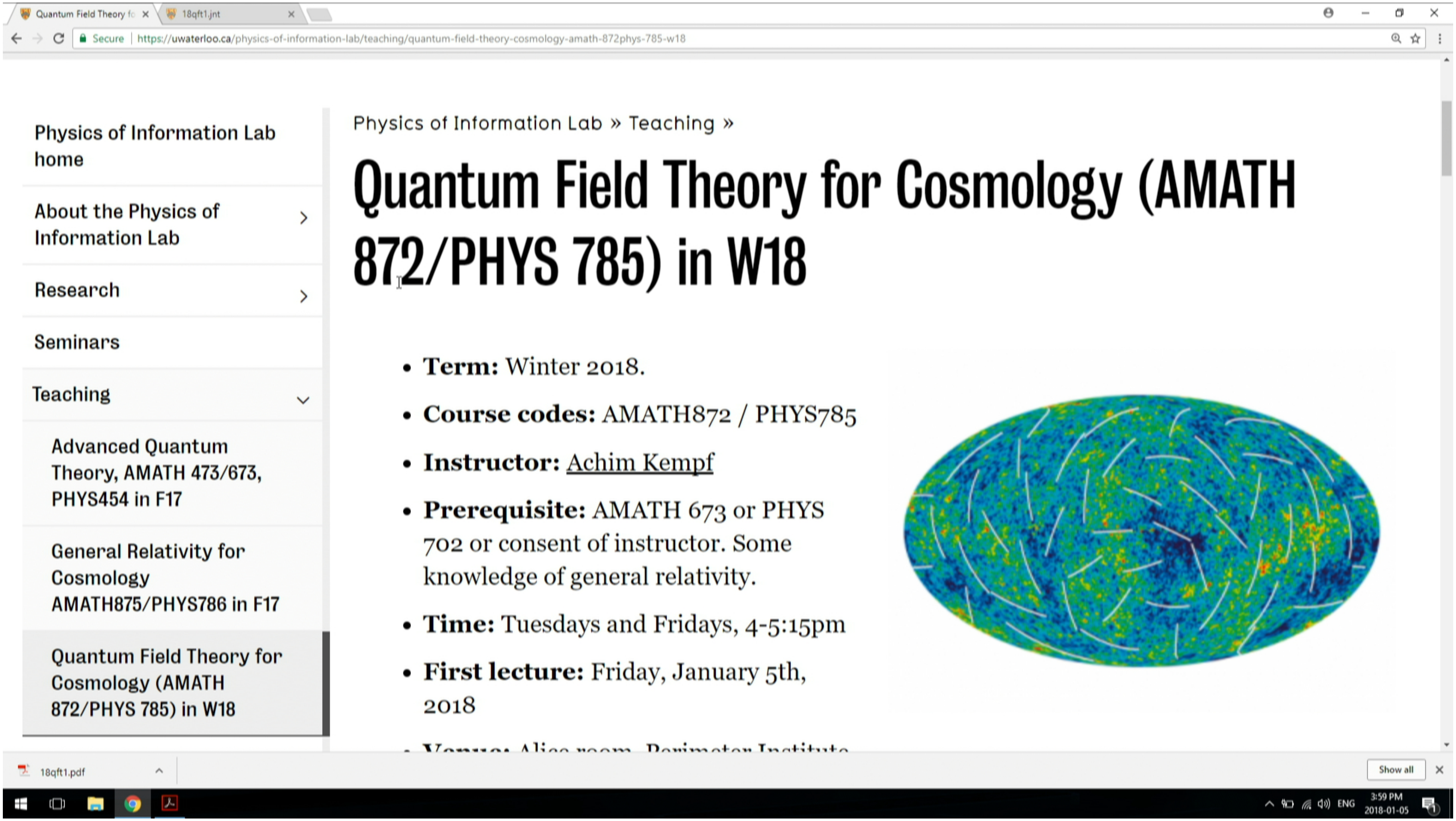
Title: Quantum Field Theory for Cosmology (AMATH872/PHYS785) - Lecture 1

Date: Jan 05, 2018 04:00 PM

URL: http://pirsa.org/18010057

Abstract:



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General Relativity for Cosmology AMATH875/PHYS786 in F17

Quantum Field Theory for Cosmology (AMATH 872/PHYS 785) in W18

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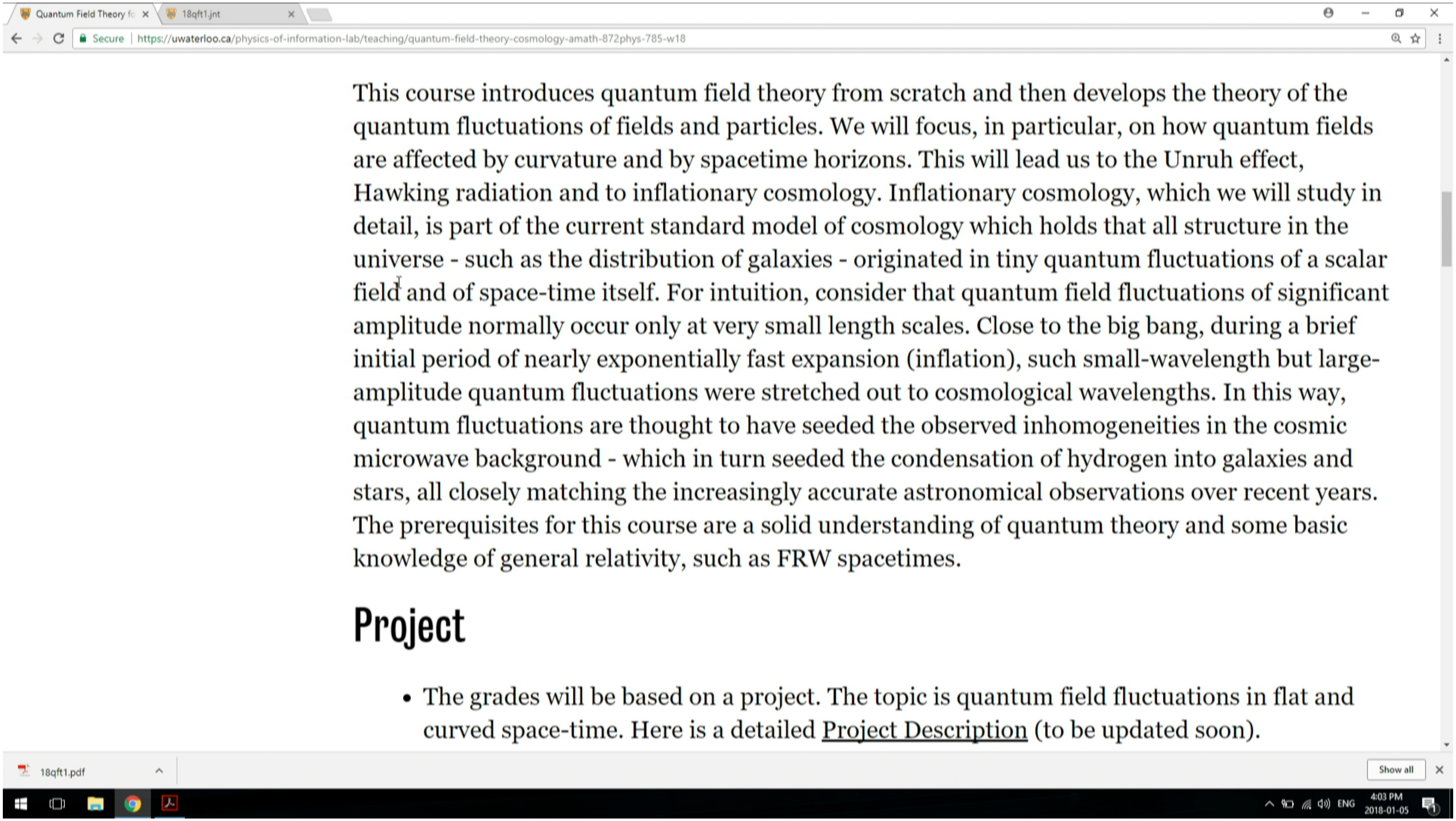
Quantum Field Theory for Cosmology (AMATH 872/PHYS 785) in W18

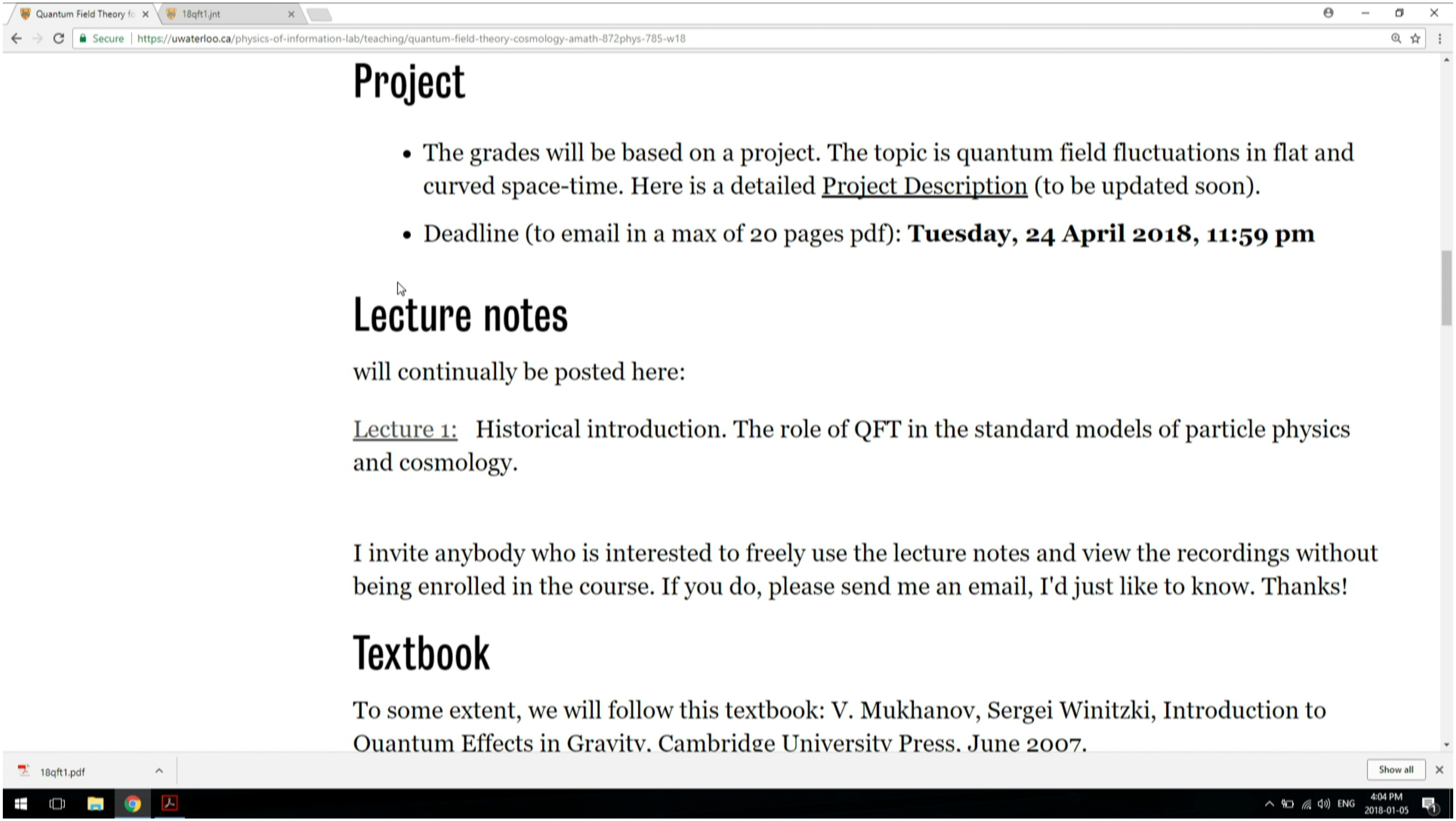
- **Term:** Winter 2018.
- **Course codes:** AMATH872 / PHYS785
- **Instructor:** Achim Kempf
- **Prerequisite:** AMATH 673 or PHYS 702 or consent of instructor. Some knowledge of general relativity.
- **Time:** Tuesdays and Fridays, 4-5:15pm
- **First lecture:** Friday, January 5th, 2018
- **Venue:** Alice room, Perimeter Institute



This course introduces quantum field theory from scratch and then develops the theory of the quantum fluctuations of fields and particles. We will focus, in particular, on how quantum fields are affected by curvature and by spacetime horizons. This will lead us to the Unruh effect, Hawking radiation and to inflationary cosmology. Inflationary cosmology, which we will study in detail, is part of the current standard model of cosmology which holds that all structure in the universe - such as the distribution of galaxies - originated in tiny quantum fluctuations of a scalar field and of space-time itself. For intuition, consider that quantum field fluctuations of significant amplitude normally occur only at very small length scales. Close to the big bang, during a brief initial period of nearly exponentially fast expansion (inflation), such small-wavelength but large-amplitude quantum fluctuations were stretched out to cosmological wavelengths. In this way, quantum fluctuations are thought to have seeded the observed inhomogeneities in the cosmic microwave background - which in turn seeded the condensation of hydrogen into galaxies and stars, all closely matching the increasingly accurate astronomical observations over recent years. The prerequisites for this course are a solid understanding of quantum theory and some basic knowledge of general relativity, such as FRW spacetimes.

Project

- The grades will be based on a project. The topic is quantum field fluctuations in flat and curved space-time. Here is a detailed Project Description (to be updated soon).



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- The grades will be based on a project. The topic is quantum field fluctuations in flat and curved space-time. Here is a detailed Project Description (to be updated soon).
- Deadline (to email in a max of 20 pages pdf): **Tuesday, 24 April 2018, 11:59 pm**

Lecture notes

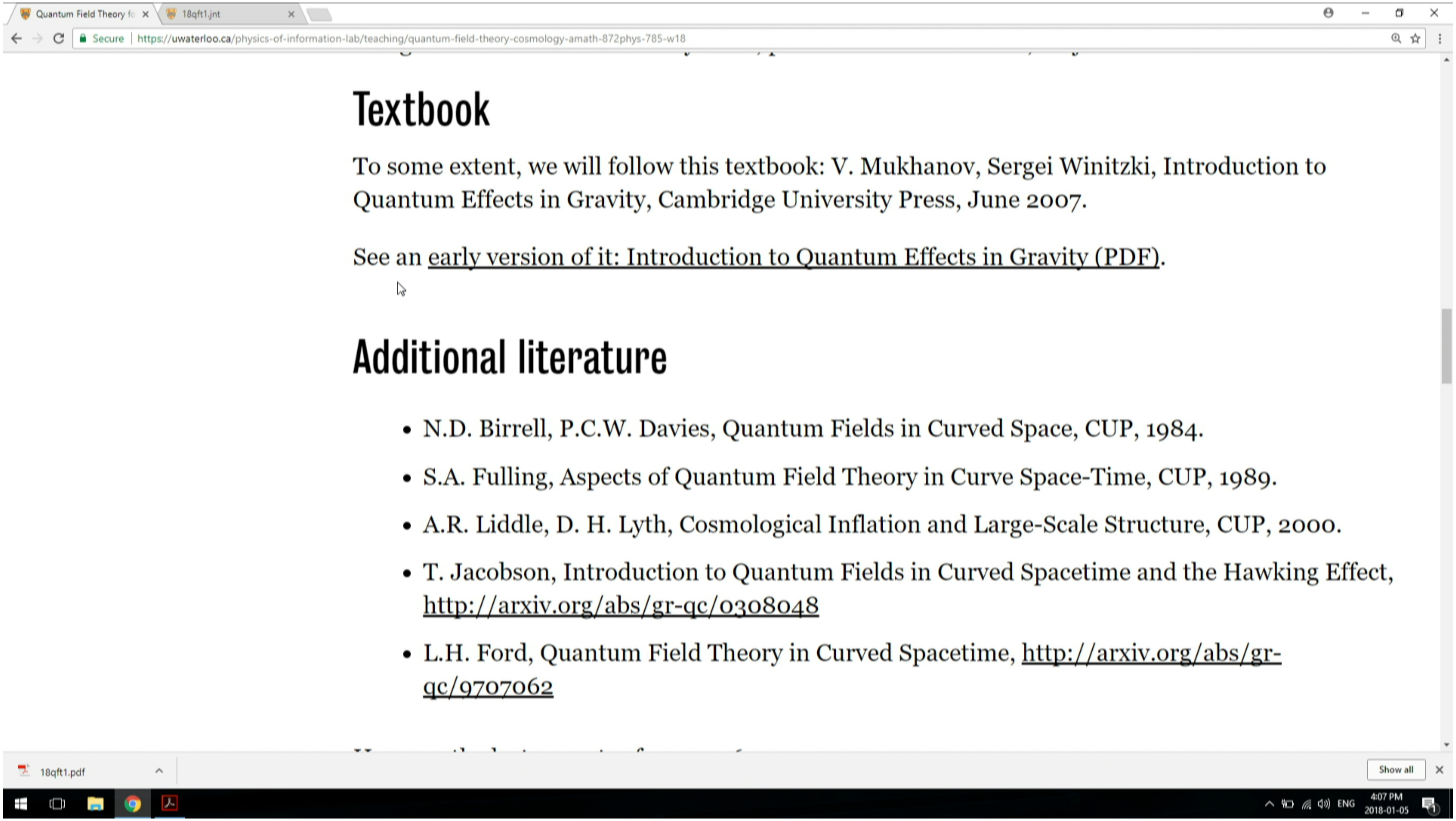
will continually be posted here:

Lecture 1: Historical introduction. The role of QFT in the standard models of particle physics and cosmology.

I invite anybody who is interested to freely use the lecture notes and view the recordings without being enrolled in the course. If you do, please send me an email, I'd just like to know. Thanks!

Textbook

To some extent, we will follow this textbook: V. Mukhanov, Sergei Winitzki, Introduction to Quantum Effects in Gravity, Cambridge University Press, June 2007.



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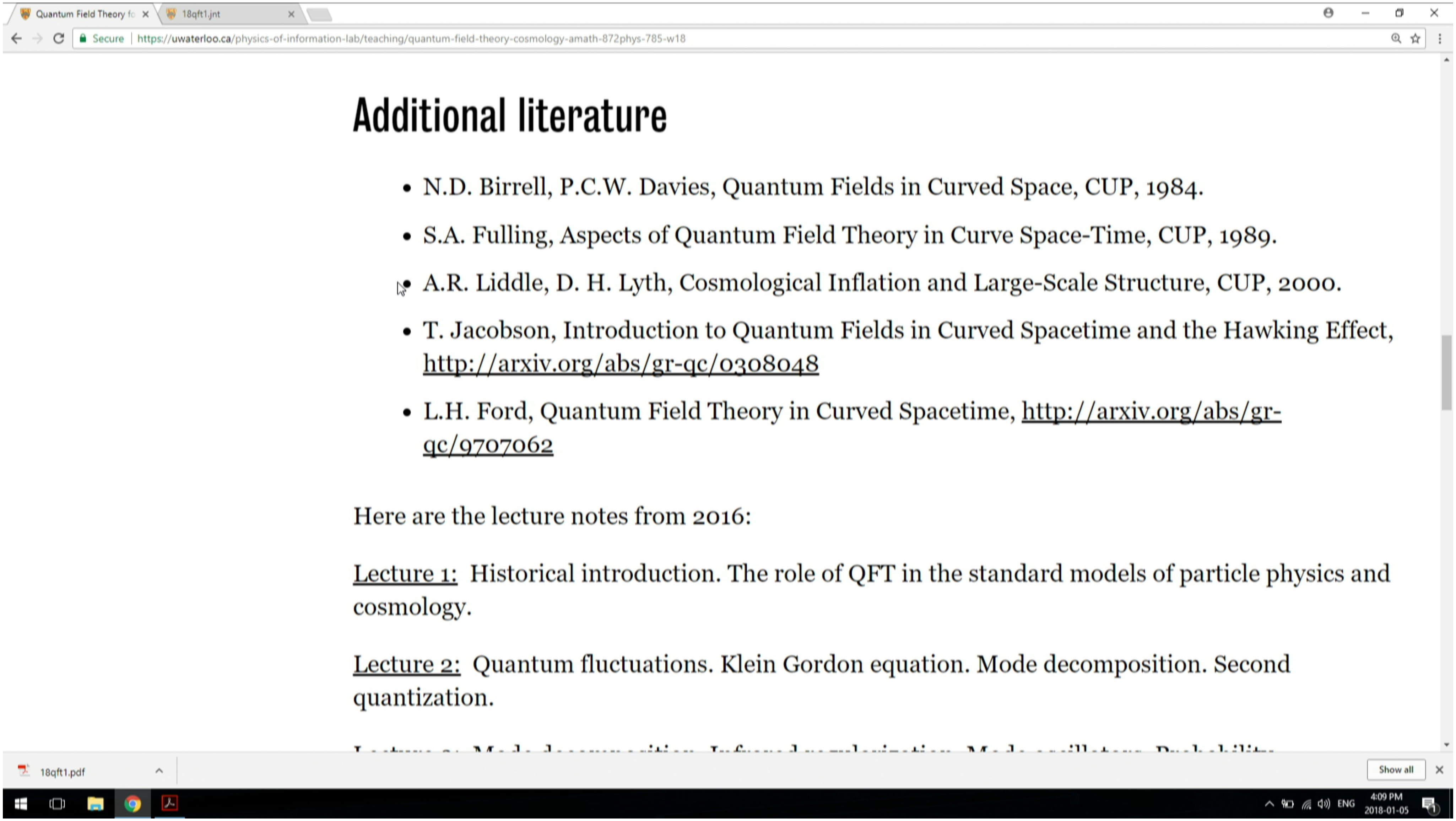
See an early version of it: Introduction to Quantum Effects in Gravity (PDF).

Additional literature

- N.D. Birrell, P.C.W. Davies, Quantum Fields in Curved Space, CUP, 1984.
- S.A. Fulling, Aspects of Quantum Field Theory in Curve Space-Time, CUP, 1989.
- A.R. Liddle, D. H. Lyth, Cosmological Inflation and Large-Scale Structure, CUP, 2000.
- T. Jacobson, Introduction to Quantum Fields in Curved Spacetime and the Hawking Effect, http://arxiv.org/abs/gr-qc/0308048
- L.H. Ford, Quantum Field Theory in Curved Spacetime, http://arxiv.org/abs/gr-qc/9707062



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- L.H. Ford, Quantum Field Theory in Curved Spacetime, http://arxiv.org/abs/gr-qc/9707062

Here are the lecture notes from 2016:

Lecture 1: Historical introduction. The role of QFT in the standard models of particle physics and cosmology.

Lecture 2: Quantum fluctuations. Klein Gordon equation. Mode decomposition. Second quantization.


Reebok
Introduction to
Quantum Effects in Gravity
V. F. Mukhanov and S. Winitzki
CAMBRIDGE
Quantum Field Theory in Curved Spacetime
Quantized Fields and Gravity
Chicago Lectures in Physics
Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics
Robert M. Wald
Aspects of Quantum Field Theory in Curved Space-Time
S. A. FULLING
London Mathematical Society Student Texts 17

Note Title

QFT for Cosmology, Achim Kempf, Lecture 1

Historical background:

☐ $\approx$ 1900:

Classical mechanics became experimentally untenable:

- Black body radiation ("Ultraviolet catastrophe")
- Photoelectric effect (Ionization depends on color, not intensity)

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☐ ≈ 1900 :

Classical mechanics became experimentally untenable:

- Black body radiation ("Ultraviolet catastrophe")
- Photoelectric effect (Ionization depends on color, not intensity)
- Stability of matter ($\Delta x \, \Delta p \geq \frac{\hbar}{2}$ implies that e^- do not spiral into the nuclei)

Finally, in 1925:

Heisenberg discovers nonrelativistic quantum mechanics (QM)

In essence:

- Equations of motion stay the same, e.g.:

$$m\ddot{\hat{x}} = -K\hat{x}$$ (harm. oscillator)

- but we have noncommutativity:

$$[\hat{x}, \underbrace{m\dot{\hat{x}}}_{=\hat{p}}] = i\hbar$$ "canonical commutation relation"

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- A few months later:

Schrödinger discovered his equation

$$i\hbar \frac{d}{dt} \Psi(x,t) = -\frac{\hbar^2}{2m} \Delta \Psi(x,t) + V(x,t) \Psi(x,t)$$

- A few more months later:

Dirac showed equivalence to Heisenberg's.

Quantization implied fundamental changes:

Math: $\left[\hat{x}(t), \hat{p}(t)\right] = i\hbar 1 \neq 0 \Rightarrow \hat{x}(t), \hat{p}(t)$ not number-valued.

Q: Could $\hat{x}(t), \hat{p}(t)$ take values in finite dimensional matrices?

A: No: If $\hat{x}(t), \hat{p}(t)$ were $N \times N$ matrices, then:

$$\boxed{Tr\left([\hat{x}, \hat{p}]\right) = Tr(i\hbar 1) \Rightarrow 0 = i\hbar N} \quad ↯$$

$\Rightarrow \hat{x}(t), \hat{p}(t)$ must not have well-defined trace, i.e., must act on ∞ dim. Hilbert space, i.e., must be operator-valued.

Physics: $\Delta x_i \Delta p_j \geq \frac{\hbar}{2} \delta_{ij}$

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Physics: $\Delta x_i \Delta p_j \geq \frac{\hbar}{2}\delta_{ij}$

⤳ Uncertainty, i.e. "quantum fluctuations", are seen as being part of nature.

- **But:** Nonrelativistic quantum mechanics, i.e.,

$$[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij} \quad \text{and} \quad i\hbar\frac{d}{dt}\hat{f}(\hat{x},\hat{p}) = [\hat{f}(\hat{x},\hat{p}), \hat{H}]$$

soon became unsatisfactory.

- **Why?** QM is not consistent with special relativity:

E.g. typical momentum of e^- in ground state of H-atom corresponds to $\approx 1\%$ of speed of light.

$\Rightarrow$ The effects of special relativity were soon spectroscopically measurable.

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E.g. typical momentum of e^- in ground state of H-atom corresponds to $\approx 1\%$ of speed of light.

$\Rightarrow$ The effects of special relativity were soon spectroscopically measurable.

$\Rightarrow$ measurable contradiction to QM!

- Attempts to find a covariant generalization of the Schrödinger equation led to:
  - "Dirac Equation"
  - "Klein Gordon Equation" (see later)

- They had some success, but suffer serious problems too:
  - Energy not bounded from below ⇒ "instability"
  - Unitarity of time evolution unclear
  - Also: It remained unclear how particle creations

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- They had some success, but suffer serious problems too:
  - Energy not bounded from below => "instability"
  - Unitarity of time evolution unclear
  - Also: It remained unclear how particle creation and annihilation processes could be calculated.

- Thus, a new idea was needed!

The idea of 2^{nd} quantization: (Heisenberg and others, 1930s)

- Observation:

In QM, all is subject to quantum fluctuations and therefore to uncertainty - except for the wave function $\psi(x,t)$:

Namely:

As in classical theories, if the wave function's initial conditions are known, then the equation of motion (say the Schrödinger, Klein Gordon or

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Namely:

As in classical theories, if the wave function's initial conditions are known, then the equation of motion (say the Schrödinger, Klein Gordon or Dirac equation) determins the evolution of $\Psi(x,t)$ without any uncertainty.

- **Idea:**

In 2nd quantization, quantize ψ !

- **Program:**

Similar to $\hat{p}_i = \dot{\hat{x}}_i$ (in suitable units)

introduce a "momentum wave function":

$$\hat{\pi}(x,t) = \dot{\hat{\psi}}(x,t)$$

Then, similar to $[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$, require:

$$\left[\hat{\psi}(x,t), \hat{\pi}(x',t)\right] = i\hbar\,\delta(x-x')$$

- **Success!**

- **Program:**

Similar to $\hat{p}_i = \dot{\hat{x}}_i$ (in suitable units)

introduce a "momentum wave function":

$$\hat{\pi}(x,t) = \dot{\hat{\varphi}}(x,t)$$

Then, similar to $[\hat{x}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$, require:

$$\left[\hat{\varphi}(x,t), \hat{\pi}(x',t)\right] = i\hbar\,\delta(x-x')$$

- **Success!**

Problems with energy positivity, unitarity etc can be solved.

☐ Consequences:

Math:

→ $\hat{\phi}(x,t)$ and $\hat{\pi}(x,t)$ can no longer be number-valued.

→ For each x and t the "value"

$$\hat{\phi}(x,t)$$

is an operator on a Hilbert space!

Notice: (Recall: The eqns of motion stay the same also in 1st quantization)

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Notice:

(Recall: The eqns of motion stay the same also in 1st quantization)

The equations of motion (Schrödinger, Klein Gordon or Dirac equation) stay the same only now with $\hat{\varphi}, \hat{\pi}$ noncommutative.

Physics:

we'll need to discuss that

$$\Delta\Psi(x,t)\,\Delta\Pi(x',t) \geq \frac{\hbar}{2}\,\delta^3(x-x')$$

$x = (x_1, x_2, x_3)$

⇒ The "wave function" is now subject to quantum fluctuations and uncertainty!

⇒ New phenoma now predicted and described:

1.) Regarding particles:

Particle creation/annihilation ↙ (E.g. norm of wave fctn i.e. particle number no longer fixed)

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Particle creation/annihilation ↙ (E.g. norm of wave fctn i.e. particle number no longer fixed)

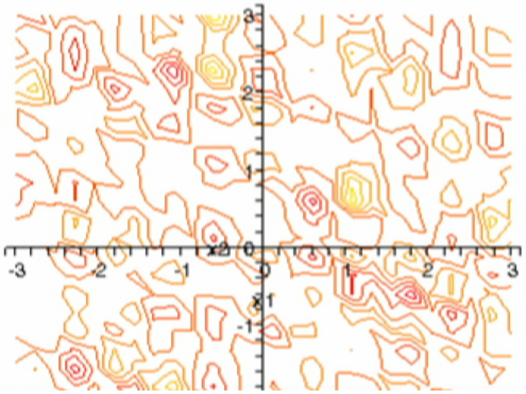
Existence of antiparticles ← (the negative energy (or mass) states can be interpreted as particles propagating backwards in time, thus to us appearing to have positive energy (or mass).)

2.) Regarding fields:

Even in the lowest energy state (i.e. no particles, i.e. in the Vacuum, the statement

$$\bar{\psi}(x,t) = \langle \text{vacuum} | \hat{\psi}(x,t) | \text{vacuum} \rangle = 0$$

allows for the values of $\hat{\psi}(x,t)$ when measured,



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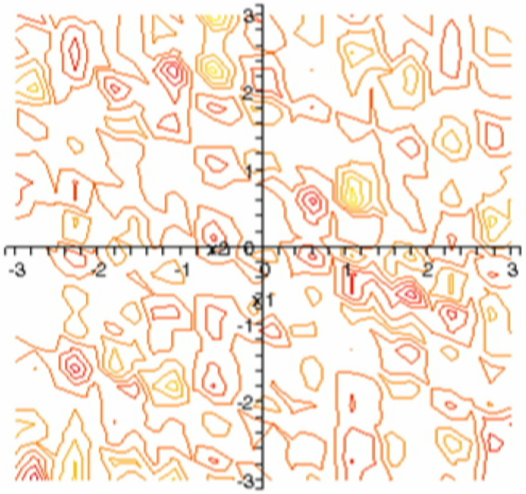
allows for the values of $\hat{\psi}(x,t)$ when measured, to fluctuate:



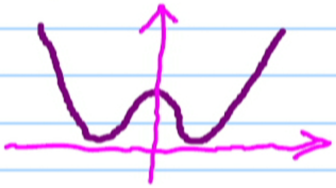
⇝ 2 main uses of quantum field theory:

1) The Standard Model of Particle Physics

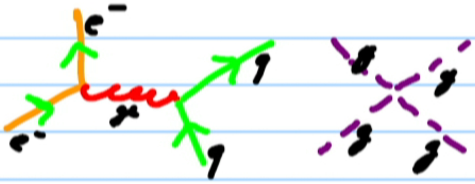
- * EM, weak and strong forces

- * Screening, anti-screening
  and renormalization

- * How fundamentally massless particles can effectively acquire a mass:
  "Spontaneous symmetry breaking"
  Namely: Groundstate has less symmetry than the action:
  "Higgs" particle.



1) The Standard Model of Particle Physics



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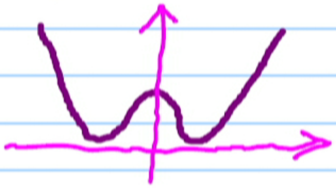

- How fundamentally massless particles can effectively acquire a mass:
  "Spontaneous symmetry breaking"
  Namely: Groundstate has less symmetry than the action:
  "Higgs" particle.

- Anomalies: Quantum fluctuations reduce symmetry of the action itself.
  The contrain the Standard Model of Particle Physics's structure.

2) The Standard Model of Cosmology

(the aim of this course)

Classical General Relativity + QFT

↳ Mostly

i.e.: Accelerations, curvature, horizons + QFT

* Unruh Effect: What is a "particle"?

* Hawking Effect: Can nature destroy information?

* Cosmic Inflation: Where did it all come from?

(the aim of this course)

Classical General Relativity + QFT

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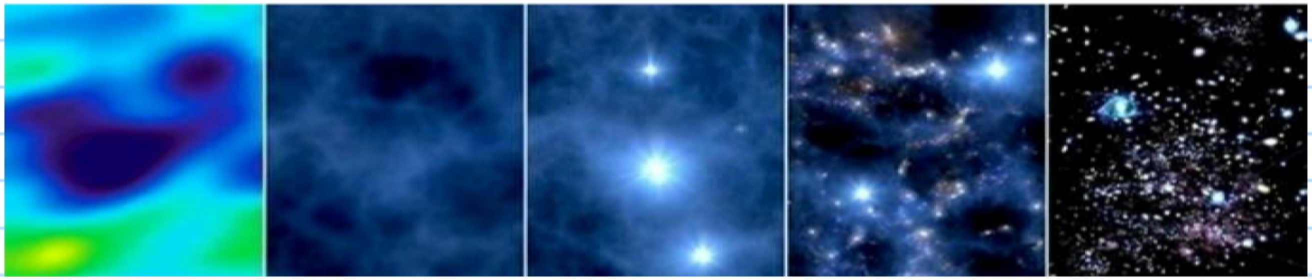
- * Unruh Effect: What is a "particle"?
- * Hawking Effect: Can nature destroy information?
- * Cosmic Inflation: Where did it all come from?

Cosmic Inflation:

- A local quantum fluctuation of high potential $V(\phi)$ may occur.
- Acting as temporary cosm. constant, may spawn a rapidly-expanding daughter universe
- Finally, $V(\phi) \to 0$, energy goes into particle production: plasma
- Rapid expansion amplified quantum field fluctuations.
- These fluctuations imprinted on primordial plasma, seeding galaxy formation:



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Our target